# ASSET PRICING IN THE FREQUENCY DOMAIN: THEORY AND EMPIRICS

by Ian Dew-Becker and Stefano Giglio

Discussed by Urban Jermann

Study asset prices in the frequency domain

- Study asset prices in the frequency domain
- Theoretical results: Frequency domain representation for SDF with applications

- Study asset prices in the frequency domain
- Theoretical results: Frequency domain representation for SDF with applications
- Empirical results: Estimation of risk prices by frequency from cross-section, find significantly priced low frequency risk

- Study asset prices in the frequency domain
- Theoretical results: Frequency domain representation for SDF with applications
- Empirical results: Estimation of risk prices by frequency from cross-section, find significantly priced low frequency risk
- Analysis suggests EZ closer to data than Habits

# Theory

Focus on innovations in SDF with time-non-separabilities

$$\Delta E_{t+1} m_{t+1} = \Delta E_{t+1} \sum_{k=0}^{\infty} z_k \cdot x_{t+1+k}$$

with priced variable  $x_t$ 

# Theory

Focus on innovations in SDF with time-non-separabilities

$$\Delta E_{t+1} m_{t+1} = \Delta E_{t+1} \sum_{k=0}^{\infty} z_k \cdot x_{t+1+k}$$

with priced variable  $x_t$ 

Assume a vector MA process for state variables

$$x_{t}=B_{1}X_{t}=B_{1}\Gamma\left( L\right) \varepsilon_{t}$$

# Theory

Focus on innovations in SDF with time-non-separabilities

$$\Delta E_{t+1} m_{t+1} = \Delta E_{t+1} \sum_{k=0}^{\infty} z_k \cdot x_{t+1+k}$$

with priced variable  $x_t$ 

Assume a vector MA process for state variables

$$x_t = B_1 X_t = B_1 \Gamma(L) \varepsilon_t$$

0

$$\Delta E_{t+1} m_{t+1} = -\sum_{j} \left( \sum_{k=0}^{\infty} \underbrace{z_{k}}_{\mathsf{Price of risk}} \cdot \underbrace{g_{j,k}}_{\mathsf{Imp rsp.}} \right) \varepsilon_{j,t+1}$$

3 / 19

#### Result 1

$$\Delta E_{t+1} m_{t+1} = -\sum_{j} \left( \sum_{k=0}^{\infty} z_{k} g_{j,k} \right) \varepsilon_{j,t+1}$$
$$= -\sum_{j} \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} Z(\omega) G_{j}(\omega) d\omega \right) \varepsilon_{j,t+1}$$

with

$$Z(\omega) \equiv z_0 + 2 \sum_{k=1}^{\infty} z_k \cos(\omega k)$$
  
 $G_j(\omega) \equiv \sum_{k=1}^{\infty} g_{j,k} \cos(\omega k)$ 

◆ロト ◆昼ト ◆豆ト ■ りへ○

4 / 19

#### Derivation

• #1: Discrete-time Fourier Transform, and Inverse

$$ilde{G}_{j}\left(\omega
ight)\equiv\sum_{k=0}^{\infty}\mathrm{e}^{-i\omega k}g_{j,k}$$
, and  $g_{j,k}=rac{1}{2\pi}\int_{-\pi}^{\pi} ilde{G}_{j}\left(\omega
ight)\mathrm{e}^{-i\omega k}d\omega$ 

#### Derivation

• #1: Discrete-time Fourier Transform, and Inverse

$$\tilde{G}_{j}\left(\omega\right)\equiv\sum_{k=0}^{\infty}\mathrm{e}^{-i\omega k}g_{j,k}$$
, and  $g_{j,k}=rac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{G}_{j}\left(\omega\right)\mathrm{e}^{-i\omega k}d\omega$ 

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \sum_{k=0}^{\infty} z_k \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{G}_j(\omega) e^{i\omega k} d\omega \right)$$

#### Derivation

• #1: Discrete-time Fourier Transform, and Inverse

$$\tilde{G}_{j}\left(\omega
ight)\equiv\sum_{k=0}^{\infty}\mathrm{e}^{-i\omega k}g_{j,k}$$
, and  $g_{j,k}=rac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{G}_{j}\left(\omega
ight)\mathrm{e}^{-i\omega k}d\omega$ 

 $\sum_{k=0}^{\infty} z_k g_{j,k} = \sum_{k=0}^{\infty} z_k \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{G}_j(\omega) e^{i\omega k} d\omega \right)$ 

• #2: Using fact that  $g_{j,k} = 0$ , for k < 0, and algebra

$$\sum_{k=0}^{\infty} z_k \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{G}_j(\omega) 2 \cos(\omega k) d\omega \right)$$

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{G}_j(\omega) \underbrace{\left[z_0 + \sum_{k=1}^{\infty} z_k 2 \cos(\omega k)\right]}_{\equiv Z(\omega)} d\omega,$$

$$Z(\omega) \equiv$$
 'weighting function'

6 / 19

Discussed by Urban Jermann

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_j(\omega) \underbrace{\left[z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)\right]}_{\equiv Z(\omega)} d\omega$$

with

$$G_{j}\left(\omega\right)=re\left( ilde{G}_{j}\left(\omega
ight)
ight)=\sum_{k=1}^{\infty}g_{j,k}\cos\left(\omega k
ight)$$

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_j(\omega) \underbrace{\left[z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)\right]}_{\equiv Z(\omega)} d\omega$$

with

$$G_{j}\left(\omega\right)=re\left( ilde{G}_{j}\left(\omega\right)
ight)=\sum_{k=1}^{\infty}g_{j,k}\cos\left(\omega k
ight)$$

•  $G_{j}(\omega)$  and  $Z(\omega)$  are real

Discussed by Urban Jermann

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_j(\omega) \underbrace{\left[z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)\right]}_{\equiv Z(\omega)} d\omega$$

with

$$G_{j}\left(\omega\right)=re\left( ilde{G}_{j}\left(\omega
ight)
ight)=\sum_{k=1}^{\infty}g_{j,k}\cos\left(\omega k
ight)$$

- $G_{j}\left(\omega\right)$  and  $Z\left(\omega\right)$  are real
- Frequency response function:  $H(\omega) = |H(w)| \cdot e^{-i\psi(\omega)}$

Discussed by Urban Jermann

$$\sum_{k=0}^{\infty} z_k g_{j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} G_j(\omega) \underbrace{\left[z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)\right]}_{\equiv Z(\omega)} d\omega$$

with

$$G_{j}\left(\omega
ight)=re\left( ilde{G}_{j}\left(\omega
ight)
ight)=\sum_{k=1}^{\infty}g_{j,k}\cos\left(\omega k
ight)$$

- $G_i(\omega)$  and  $Z(\omega)$  are real
- Frequency response function:  $H(\omega) = |H(w)| \cdot e^{-i\psi(\omega)}$
- Need Inverse "Dew-Becker-Giglio" Transform

$$z_{k} = \frac{\frac{1}{2\pi} \int_{-\pi}^{\pi} Z(\omega) d\omega \text{ for } k = 0}{\frac{1}{\pi} \int_{-\pi}^{\pi} Z(\omega) \cos(\omega k) d\omega \text{ for } k > 0}$$

←□ → ←□ → ← = → ← = → へ ○

## Alternative derivation

#1: Parseval's theorem

$$\sum_{k=-\infty}^{\infty} z_k g_k = \int_{-\pi}^{\pi} Z(\omega) \, \overline{G(\omega)} d\omega$$

$$Z(\omega) = \sum_{k=-\infty}^{\infty} z_k e^{-i\omega k}$$
,  $G(\omega) = \sum_{k=-\infty}^{\infty} g_k e^{-i\omega k}$ 



## Alternative derivation

#1: Parseval's theorem

$$\sum_{k=-\infty}^{\infty} z_{k} g_{k} = \int_{-\pi}^{\pi} Z(\omega) \, \overline{G(\omega)} d\omega$$

$$Z(\omega) = \sum_{k=-\infty}^{\infty} z_k e^{-i\omega k}$$
,  $G(\omega) = \sum_{k=-\infty}^{\infty} g_k e^{-i\omega k}$ 

•  $Z(\omega)$  is complex, because  $\sum_{k=0}^{\infty} z_k L^{-k}$ 

→ロト ◆回 ト ◆ 差 ト ◆ 差 ト ・ 差 ・ 夕 Q (\*)

$$\sum_{k=-\infty}^{\infty} z_k g_k = \sum_{k=-\infty}^{\infty} z_k^{[2Sym]} g_k$$
, because  $g_k = 0$ , for  $k < 0$ 

$$\sum_{k=-\infty}^{\infty}z_kg_k=\sum_{k=-\infty}^{\infty}z_k^{[2Sym]}g_k$$
 , because  $g_k=0$  , for  $k<0$ 

Frequency response function

$$Z(\omega) = \sum_{k=-\infty}^{\infty} z_k^{[2Sym]} e^{-i\omega k} = z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)$$

$$\sum_{k=-\infty}^{\infty}z_kg_k=\sum_{k=-\infty}^{\infty}z_k^{[2Sym]}g_k$$
 , because  $g_k=0$  , for  $k<0$ 

Frequency response function

$$Z(\omega) = \sum_{k=-\infty}^{\infty} z_k^{[2Sym]} e^{-i\omega k} = z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)$$

• This is the "Dew-Becker-Giglio" Weighting Function!

<ロ > < 回 > < 回 > < 巨 > < 巨 > 三 9 < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ い < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 つ < 0 い < 0 つ < 0 つ < 0 つ < 0 つ < 0 い < 0 つ < 0 い < 0 つ < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0 い < 0

$$\sum_{k=-\infty}^{\infty}z_kg_k=\sum_{k=-\infty}^{\infty}z_k^{[2Sym]}g_k$$
 , because  $g_k=0$  , for  $k<0$ 

Frequency response function

$$Z(\omega) = \sum_{k=-\infty}^{\infty} z_k^{[2Sym]} e^{-i\omega k} = z_0 + \sum_{k=1}^{\infty} z_k 2\cos(\omega k)$$

- This is the "Dew-Becker-Giglio" Weighting Function!
- Need Inverse D-B-G Transform, split weights

CRRA

$$E_{t+1}m_{t+1} = -\alpha E_{t+1}\Delta c_{t+1}$$
$$Z(\omega) = \alpha$$

CRRA

$$E_{t+1}m_{t+1} = -\alpha E_{t+1}\Delta c_{t+1}$$
$$Z(\omega) = \alpha$$

Habits

$$u\left(.\right) = \frac{1}{1-\alpha} \left(C_t - bC_{t-1}\right)^{1-\alpha}$$

CRRA

$$E_{t+1}m_{t+1} = -\alpha E_{t+1}\Delta c_{t+1}$$
 $Z(\omega) = \alpha$ 

Habits

$$u(.) = \frac{1}{1-\alpha} \left( C_t - bC_{t-1} \right)^{1-\alpha}$$

$$\exp(m_{t+1}) = \beta \frac{(C_{t+1} - bC_t)^{-\alpha} - E_{t+1}b(C_{t+2} - bC_{t+1})^{-\alpha}}{(C_t - bC_{t-1})^{-\alpha} - E_tb(C_{t+1} - bC_t)^{-\alpha}}$$

CRRA

$$E_{t+1}m_{t+1} = -\alpha E_{t+1}\Delta c_{t+1}$$
 $Z(\omega) = \alpha$ 

Habits

$$u(.) = \frac{1}{1-\alpha} \left( C_t - bC_{t-1} \right)^{1-\alpha}$$

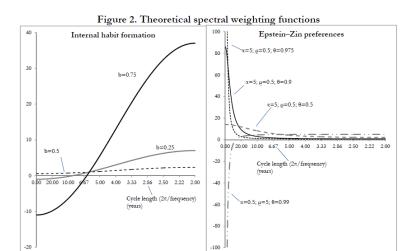
0

$$\exp(m_{t+1}) = \beta \frac{(C_{t+1} - bC_t)^{-\alpha} - E_{t+1}b(C_{t+2} - bC_{t+1})^{-\alpha}}{(C_t - bC_{t-1})^{-\alpha} - E_tb(C_{t+1} - bC_t)^{-\alpha}}$$

0

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left( b \left( 1 - b \right)^{-2} + 1 \right) E_{t+1} \Delta c_{t+1} \\ -b \left( 1 - b \right)^{-2} E_{t+1} \Delta c_{t+2} \end{bmatrix}$$

4 D > 4 D > 4 E > 4 E > E



• If  $E_{t+1}\Delta c_{t+2} = E_{t+1}\Delta c_{t+1}$ : very persistent shock

• If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$E_{t+1}m_{t+1} \approx -\alpha \left[ \begin{pmatrix} b(1-b)^{-2} + 1 \end{pmatrix} E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2} E_{t+1}\Delta c_{t+2} \end{pmatrix} \right]$$
$$= -\alpha E_{t+1}\Delta c_{t+1}$$

• If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

0

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix}$$
  
=  $-\alpha E_{t+1}\Delta c_{t+1}$ 

ullet if  $E_{t+1}\Delta c_{t+2}=-E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

• If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

•

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix} \\ = -\alpha E_{t+1}\Delta c_{t+1}$$

ullet if  $E_{t+1}\Delta c_{t+2}=-E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

0

$$E_{t+1}m_{t+1} \approx -\alpha \left[\frac{2b}{(1-b)^2}\right] E_{t+1}\Delta c_{t+1}$$

$$= -\alpha \left[40\right] E_{t+1}\Delta c_{t+1} \quad \text{for } b = .8$$

←□ → ←□ → ← □ → □ → ○ へ ○ ○

• Consumption durability b < 0

- Consumption durability b < 0
- ullet If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

- Consumption durability b < 0
- ullet If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix} \\ = -\alpha E_{t+1}\Delta c_{t+1}$$

- Consumption durability b < 0
- ullet If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix}$$
  
=  $-\alpha E_{t+1}\Delta c_{t+1}$ 

• if  $E_{t+1}\Delta c_{t+2} = -E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

4 U > 4 @ > 4 E > 4 E > E 990

0

- Consumption durability b < 0
- If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix}$$
  
=  $-\alpha E_{t+1}\Delta c_{t+1}$ 

• if  $E_{t+1}\Delta c_{t+2} = -E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

$$E_{t+1}m_{t+1} \approx \alpha \left[ \frac{2b}{(1-b)^2} \right] = -\alpha \left[ \frac{1.6}{(1.8)^2} \right] E_{t+1} \Delta c_{t+1}$$
  
=  $-\alpha \left[ 0.5 \right] E_{t+1} \Delta c_{t+1}$ 

4 D > 4 B > 4 E > E 999

0

0

• Consumption durability b < 0

0

0

ullet If  $E_{t+1}\Delta c_{t+2}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$E_{t+1}m_{t+1} \approx -\alpha \begin{bmatrix} \left(b(1-b)^{-2}+1\right)E_{t+1}\Delta c_{t+1} \\ -b(1-b)^{-2}E_{t+1}\Delta c_{t+2} \end{bmatrix}$$
  
=  $-\alpha E_{t+1}\Delta c_{t+1}$ 

ullet if  $E_{t+1}\Delta c_{t+2}=-E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

$$E_{t+1}m_{t+1} \approx \alpha \left[\frac{2b}{(1-b)^2}\right] = -\alpha \left[\frac{1.6}{(1.8)^2}\right] E_{t+1}\Delta c_{t+1}$$
  
=  $-\alpha \left[0.5\right] E_{t+1}\Delta c_{t+1}$ 

• External habits, only innovations in  $\Delta c_{t+1}$ , spectrum is flat

$$\exp\left(m_{t+1}
ight) = eta rac{\left(C_{t+1} - bar{C}_{t}
ight)^{-lpha}}{\left(C_{t} - bar{C}_{t-1}
ight)^{-lpha}}$$

13 / 19

Discussed by Urban Jermann

• If  $E_{t+1}\Delta c_{t+1+j}=E_{t+1}\Delta c_{t+1}$ : very persistent shock

• If  $E_{t+1}\Delta c_{t+1+j}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$\Delta E_{t+1} m_{t+1} \approx -\left(\rho \Delta E_{t+1} c_{t+1} + (a-\rho) \Delta E_{t+1} \Delta c_{t+1} \sum_{j=0}^{\infty} \theta^{j}\right)$$

$$= -\left(\rho + (a-\rho) \frac{1}{1-\theta}\right) \Delta E_{t+1} c_{t+1}$$

$$= -\left(.5 + (5-.5) \frac{1}{.025}\right) \Delta E_{t+1} c_{t+1}$$

$$= -[180] \Delta E_{t+1} c_{t+1}$$

• If  $E_{t+1}\Delta c_{t+1+j}=E_{t+1}\Delta c_{t+1}$ : very persistent shock

$$\Delta E_{t+1} m_{t+1} \approx -\left(\rho \Delta E_{t+1} c_{t+1} + (a-\rho) \Delta E_{t+1} \Delta c_{t+1} \sum_{j=0}^{\infty} \theta^{j}\right)$$

$$= -\left(\rho + (a-\rho) \frac{1}{1-\theta}\right) \Delta E_{t+1} c_{t+1}$$

$$= -\left(.5 + (5-.5) \frac{1}{.025}\right) \Delta E_{t+1} c_{t+1}$$

$$= -[180] \Delta E_{t+1} c_{t+1}$$

ullet if  $E_{t+1}\Delta c_{t+2}=-E_{t+1}\Delta c_{t+1}$  : shock reversed immediately

4 D > 4 B > 4 E > 4 E > 4 C + 4 Q C

• If  $E_{t+1}\Delta c_{t+1+j}=E_{t+1}\Delta c_{t+1}$  : very persistent shock

$$\Delta E_{t+1} m_{t+1} \approx -\left(\rho \Delta E_{t+1} c_{t+1} + (a-\rho) \Delta E_{t+1} \Delta c_{t+1} \sum_{j=0}^{\infty} \theta^{j}\right)$$

$$= -\left(\rho + (a-\rho) \frac{1}{1-\theta}\right) \Delta E_{t+1} c_{t+1}$$

$$= -\left(.5 + (5-.5) \frac{1}{.025}\right) \Delta E_{t+1} c_{t+1}$$

$$= -[180] \Delta E_{t+1} c_{t+1}$$

• if  $E_{t+1}\Delta c_{t+2} = -E_{t+1}\Delta c_{t+1}$ : shock reversed immediately

$$\Delta E_{t+1} m_{t+1} \approx -\rho \Delta E_{t+1} c_{t+1}$$

$$= -0.5 \Delta E_{t+1} c_{t+1}$$

4 D > 4 B > 4 E > 4 E > 9 Q C

# Estimates of spectral weighting functions

Parameterize spectral functions

## Estimates of spectral weighting functions

- Parameterize spectral functions
- Utility basis

$$Z^{U}(\omega) = q_1 \sum_{j=1}^{\infty} \theta^{j} \cos(\omega j) + q_2 + q_3 \cos(\omega)$$

EZ :  $q_1, q_2 > 0, q_3 = 0$ 

Internal Habit :  $q_3 < 0$ ,  $q_2 > 0$ ,  $q_3 = 0$ 

# Estimates of spectral weighting functions

- Parameterize spectral functions
- Utility basis

$$Z^{U}\left(\omega
ight) = q_{1}\sum_{j=1}^{\infty} heta^{j}\cos\left(\omega j
ight) + q_{2} + q_{3}\cos\left(\omega
ight)$$
EZ:  $q_{1},q_{2}>0,\ q_{3}=0$ 

Internal Habit :  $q_3 < 0$ ,  $q_2 > 0$ ,  $q_3 = 0$ 

Bandpass basis

$$Z^{BP}(\omega) = q_1 Z^{(\infty,32 \text{quarters})} + q_2 Z^{(32,6)} + q_3 Z^{(6,2)}$$

4 D > 4 A > 4 B > 4 B > B 9 9 0

#### Apply Inverse Transform

$$\Delta E_{t+1} m_{t+1} = -\bar{q}' \left( \sum_{j=0}^{\infty} \underbrace{H_j^{U,BP}}_{\mathsf{Freq.func.}} B_1 \Phi^j \right) \underbrace{(X_{t+1} - \Phi X_t)}_{\mathsf{VAR \ state \ var.}}$$

$$= -\bar{q}' u_{t+1}$$

Apply Inverse Transform

$$\Delta E_{t+1} m_{t+1} = -\bar{q}' \left( \sum_{j=0}^{\infty} \underbrace{H_j^{U,BP}}_{\mathsf{Freq.func.}} B_1 \Phi^j \right) \underbrace{(X_{t+1} - \Phi X_t)}_{\mathsf{VAR \ state \ var.}}$$

$$= -\bar{q}' u_{t+1}$$

Moment conditions

$$E\left(R_{i,t} - R_{t-1}^f\right) = -E\left(-\bar{q}'u_{t+1}, r_{i,t} - r_{t-1}^f\right)$$

## Results

Portfolios:	FF25				
Basis:	Bandpass	t-stat	Utility (0.975)	t-stat	
Consumption q1 q2 q3	269 -431 138	2.47 ** -1.17 0.33	555.47 -442.65 616.12	1.66 * -0.44 0.32	



### Results

Portfolios:	FF25					
Basis:	Bandpass	t-stat	Utility (0.975)	t-stat		
Consumption q1 growth q2 q3	269 -431 138	2.47 ** -1.17 0.33	555.47 -442.65 616.12	1.66 * -0.44 0.32		

Epstein-Zin Risk Aversion coefficient implied by Utility basis

$$q_1 = (\alpha - \rho) 2$$

Risk Aversion: 
$$\alpha = \frac{555.47}{2} + \rho > 277.7$$



Discussed by Urban Jermann

### Other comments

Comparison with Yu (2012)

### Other comments

- Comparison with Yu (2012)
- Cross-section: Drivers of results, and literature (ex: Parker and Julliard (2005), Hansen, Heaton, Lee & Roussanov (2007))

## Conclusion

Nice paper!