DIFFERENCES IN OPINION IN AN INTERNATIONAL FINANCIAL MARKET EQUILIBRIUM
by Bernard Dumas, Karen Lewis and Emilio Osambela

Discussed by Urban Jermann
Contribution

- To build a 2 country model with *Differences of Opinion* to study a set of well documented empirical regularities
  - Solve numerically a rich model
Model: Output processes

\[
\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t} \, dt + \sigma_\delta \, dZ_{i,t}^\delta
\]

\[
df_{i,t} = -\zeta \left( f_{i,t} - \overline{f} \right) \, dt + \sigma_f \, dZ_{i,t}^f, \quad i = A, B
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Model: Output processes

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Model: Differences of Opinion about signals

Output

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\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t} dt + \sigma_\delta dZ_{i,t}^\delta
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df_{i,t} = -\zeta (f_{i,t} - \bar{f}) dt + \sigma_f dZ_{i,t}^f, \ i = A, B
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Model: Differences of Opinion about signals

- Output

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\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t} dt + \sigma_\delta dZ_{i,t}^\delta
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df_{i,t} = -\zeta (f_{i,t} - \bar{f}) dt + \sigma_f dZ_{i,t}^f, \; i = A, B
\]

- Signals and Differences of Opinion

Domestic output: \[
ds_{i,t} = \phi dZ_{i,t}^f + \sqrt{1 - \phi^2} dZ_{i,t}^s
\]

Foreign output: \[
ds_{i,t} = dZ_{i,t}^s, \; i = A, B
\]
Model

- Estimate/filter expected growth rates from observables

Domestic:  \[ d\hat{f}_{i,t} = -\zeta \left( \hat{f}_{i,t} - \bar{f} \right) dt + \frac{\gamma_i}{\sigma_\delta^2} \left( \frac{d\delta_{i,t}}{\delta_{i,t}} - \hat{f}_{i,t} dt \right) + \phi \sigma_f ds_{i,t} \]

Foreign:  \[ d\hat{f}_{i,t} = -\zeta \left( \hat{f}_{i,t} - \bar{f} \right) dt + \frac{\gamma_i}{\sigma_\delta^2} \left( \frac{d\delta_{i,t}}{\delta_{i,t}} - \hat{f}_{i,t} dt \right) \]

with variance of conditional expectation

\[ \gamma^\neq > \gamma^= \]
Model

- Change from probability B to A: "Sentiment Risk"

\[
\frac{d\eta_t}{\eta_t} = -\frac{1}{\sigma^2} \left\{ \begin{aligned}
&\left( \hat{f}_{A,t}^B - \hat{f}_{A,t}^A \right) \left[ \frac{d\delta_{A,t}}{\delta_{A,t}} - \hat{f}_{A,t}^B \, dt \right] \\
+&\left( \hat{f}_{B,t}^B - \hat{f}_{B,t}^A \right) \left[ \frac{d\delta_{B,t}}{\delta_{B,t}} - \hat{f}_{B,t}^B \, dt \right]
\end{aligned} \right\}
\]
Change from probability B to A: "Sentiment Risk"

\[
\frac{d\eta_t}{\eta_t} = -\frac{1}{\sigma^2_\delta} \left\{ \begin{align*}
\left( \hat{f}_{A,t}^B - \hat{f}_{A,t}^A \right) \left[ \frac{d\delta_{A,t}}{\delta_{A,t}} - \hat{f}_{A,t}^B dt \right] \\
+ \left( \hat{f}_{B,t}^B - \hat{f}_{B,t}^A \right) \left[ \frac{d\delta_{B,t}}{\delta_{B,t}} - \hat{f}_{B,t}^B dt \right]
\end{align*} \right\}
\]

\[\eta_t \approx \frac{\text{probability}^A}{\text{probability}^B}\]
Model: Identical preferences and complete markets

- Country B solves

\[
\max_{c_B} E_0^B \int_0^\infty e^{-\rho t} \frac{1}{\alpha} c_B^\alpha_t \, dt \\
\text{s.t. } E_0^B \int_0^\infty \xi_t^B (c_B,t - \delta_B,t) \, dt \leq 0
\]
Model: Identical preferences and complete markets

- Country B solves
  \[
  \max_{c_B} E^B_0 \int_0^\infty e^{-\rho t} \frac{1}{\alpha} c_B^\alpha, t \, dt
  \]
  s.t. \[ E^B_0 \int_0^\infty \xi^B_t \left( c_B, t - \delta_B, t \right) \, dt \leq 0 \]

- Country A solves
  \[
  \max_{c_A} E^B_0 \int_0^\infty \eta_t \cdot e^{-\rho t} \frac{1}{\alpha} c_A^\alpha, t \, dt
  \]
  s.t. \[ E^B_0 \int_0^\infty \xi^B_t \left( c_A, t - \delta_A, t \right) \, dt \leq 0 \]
Model: Identical preferences and complete markets

- Country B solves

$$\max_{c_B} E_0^B \int_0^\infty e^{-\rho t} \frac{1}{\alpha} c_{B,t}^\alpha \, dt$$

s.t. $$E_0^B \int_0^\infty \xi_t^B (c_{B,t} - \delta_{B,t}) \, dt \leq 0$$

- Country A solves

$$\max_{c_A} E_0^B \int_0^\infty \eta_t \cdot e^{-\rho t} \frac{1}{\alpha} c_{A,t}^\alpha \, dt$$

s.t. $$E_0^B \int_0^\infty \xi_t^B (c_{A,t} - \delta_{A,t}) \, dt \leq 0$$

- Solution

$$c_{B,t} = \left( \lambda_B e^{\rho t} \xi_t^B \right)^{-\frac{1}{1-\alpha}}$$

and

$$c_{A,t} = \eta_t^{\frac{1}{1-\alpha}} \left( \lambda_A e^{\rho t} \xi_t^B \right)^{-\frac{1}{1-\alpha}}$$

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Model: Solution cont’d

- Pricing measure

\[ \xi_t^B = e^{-\rho t} \left[ \left( \frac{\eta_t}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left( \frac{1}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1} \]
Model: Solution cont’d

- Pricing measure

\[ \zeta^B_t = e^{-\rho t} \left[ \left( \frac{\eta_t}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left( \frac{1}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1} \]

- Next steps:
Model: Solution cont’d

- Pricing measure

\[ \zeta^B_t = e^{-\rho t} \left[ \left( \frac{\eta_t}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left( \frac{1}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1} \]

- Next steps:
  - Implement allocation with 2 stocks, 2 futures contracts on signals and 1 risk free deposit
Model: Solution cont’d

- Pricing measure

\[ \zeta^B_t = e^{-\rho t} \left[ \left( \frac{\eta_t}{\lambda^A} \right)^{1}\frac{1}{1-\alpha} + \left( \frac{1}{\lambda^B} \right)^{1}\frac{1}{1-\alpha} \right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1} \]

- Next steps:
  - Implement allocation with 2 stocks, 2 futures contracts on signals and 1 risk free deposit
  - Solve for prices and portfolio strategies that finance optimal implied wealth process

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Model: Solution cont’d

- Pricing measure

\[ \zeta^B_t = e^{-\rho t} \left[ \left( \frac{\eta_t}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left( \frac{1}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1} \]

- Next steps:
  - Implement allocation with 2 stocks, 2 futures contracts on signals and 1 risk free deposit
  - Solve for prices and portfolio strategies that finance optimal implied wealth process

- Sequential solution method makes this tractable

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Sources of Heterogenous Beliefs

- Asymmetric Information versus (Stubborn) Differences in Opinion
Sources of Heterogenous Beliefs

- Asymmetric Information versus (Stubborn) Differences in Opinion
- What is the difference?
Sources of Heterogenous Beliefs

- Asymmetric Information versus (Stubborn) Differences in Opinion
- What is the difference?
- Advantages and disadvantages
Consider more basic models

\[ \frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t} dt + \sigma_{\delta} dZ_{i,t}^\delta \]

\[ df_{i,t} = -\zeta (f_{i,t} - \bar{f}) dt + \sigma_f dZ_{i,t}^f, \ i = A, B \]

Domestic output : \[ ds_{i,t} = \phi dZ_{i,t}^f + \sqrt{1 - \phi^2} dZ_{i,t}^s \]

Foreign output : \[ ds_{i,t} = dZ_{i,t}^s, \ i = A, B \]
Home Bias: Findings

Share of foreign stocks held by domestic investors ($Z$)
Relatively weak effect. The example assumes $\phi = .95$, implying that 90% of signal variance is useful information, and 10% is noise.
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What drives this result?
Home Bias: Comments and Questions

- Relatively weak effect. The example assumes $\phi = .95$, implying that 90% of signal variance is useful information, and 10% is noise.
- What drives this result?
- Can home bias ever become a foreign bias?
Econometrician is assumed to know true probabilities
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Under Econometrician’s measure, the two CAPMs hold equally well

\[ \hat{\mu}^E_{S_i} - r = (1 - \alpha) \text{cov} \left( \frac{dS_i}{S_i}, \frac{dc_A}{c_A} \right) - \text{cov} \left( \frac{dS_i}{S_i}, \frac{d\eta_A}{\eta_A} \right); \quad i = A, B \]

\[ \hat{\mu}^E_{S_i} - r = (1 - \alpha) \text{cov} \left( \frac{dS_i}{S_i}, \frac{dc_B}{c_B} \right) - \text{cov} \left( \frac{dS_i}{S_i}, \frac{d\eta_B}{\eta_B} \right); \quad i = A, B \]
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Interpretation: "One being national and the other a world factor"
This two-factor model holds too:

\[
\hat{\mu}_S - r = \left(1 - \alpha\right) \text{cov} \left(\frac{dS_i}{S_i}, \frac{d(c_A+c_B)}{c_A+c_B}\right) - \text{cov} \left(\frac{dS_i}{S_i}, \frac{c_A}{c_A+c_B} \frac{d\tilde{\eta}_A}{\eta_A} + \frac{c_B}{c_A+c_B} \frac{d\tilde{\eta}_B}{\eta_B}\right)
\]

\(i = A, B,\)

but both factors seem global

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Model without frictions also has 3 CAPMs

\[ \mu_{S_i} - r = (1 - \alpha) \text{cov} \left( \frac{dS_i}{S_i}, \frac{dc_A}{c_A} \right) ; \ i = A, B \]

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\[ \mu_{S_i} - r = (1 - \alpha) \text{cov} \left( \frac{dS_i}{S_i}, \frac{d(c_A + c_B)}{c_A + c_B} \right) ; \ i = A, B \]

the model implies that consumption is perfectly correlated across countries
Overall

- Very interesting paper!
- I would like a more systematic quantitative evaluation, maybe with a more basic model version