Human Capital, Business Cycles and Asset Pricing

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Discussed by Urban Jermann
• Contribution: RBC model with human capital, study of asset pricing and business cycle implications

• Key model features:
  – Human capital accumulation, human capital and productivity shocks, labor-leisure-education trade-off, human capital in utility function
  – Time-to-plan friction for human capital and physical capital
  – Production economy, distinguishes between output, consumption and dividends
• Most prominent findings:
  
  – higher equity premium, upward sloping yield curve, cyclical variations in equity returns
  
  – matches business cycle statistics
Why is the equity premium larger than in standard model?

Time-to-plan

- Time-to-plan is a version of time-to-build
  \[ K_{t+1} = (1 - \delta_K) K_t + S_{1,t} \]
  \[ I_t = \phi^K_j S_{J,t} + \phi^K_{J-1} S_{J-1,t} \ldots + \phi^K_2 S_{2,t} + \phi^K_1 S_{1,t} \]
- Time-to-build
  \[ \phi^K_j = \frac{1}{J} \]
- Time-to-plan (here)
  \[ \phi^K_4 = 0.01, \text{ and other } \phi^K_{j<4} = 0.33 \]
Percentage Deviations From Unshocked Steady-State Paths
After a 1 Percent Unexpected Increase in the Level of Technology in Period 1

Chart 1  Output

Chart 2  Consumption

Chart 3  Investment

Chart 4  Capital Stock
Why is the equity premium larger than in standard model?

**Human capital (shocks) in utility function**

Assume “endowment” version of this model: no endogenous capital accumulation ($U = I = 0$) and no labor/leisure choice.

Then,

$$D_t = \theta A_t^K K^\theta (H_t N)^{1-\theta},$$
$$H_t = A_t^H H_{t-1}$$

and

$$D_{t+1}/D_t = \left(\frac{A_{t+1}^K}{A_t^K}\right) \cdot (A_{t+1}^H)^{1-\theta}$$
With Cobb-Douglas utility

\[ V(C_t, H_t, L_t) = \left( \frac{1}{1 - \gamma} \right) \left[ C_t^\alpha (H_t L^v)^{1-\alpha} \right]^{1-\gamma}, \]

and because with the given assumptions

\[ D_t = \theta C_t \]

we have SDF

\[
\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left( \left[ \frac{A^K_{t+1}}{A^K_t} \right] \left( A^H_{t+1} \right)^{1-\theta} \right)^{\alpha(1-\gamma)-1} \left( A^H_{t+1} \right)^{(1-\alpha)(1-\gamma)}
\]
that is

$$\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left( \frac{D_{t+1}}{D_t} \right)^{\alpha(1-\gamma)-1} (A_{t+1}^H)^{(1-\alpha)(1-\gamma)}$$

but if $H_t$ is taken out of utility then we would have

$$\beta \frac{V_1(C_{t+1}, H_{t+1}, L_{t+1})}{V_1(C_t, H_t, L_t)} = \beta \left( \frac{D_{t+1}}{D_t} \right)^{\alpha(1-\gamma)-1}$$
• With IID lognormal $A^H$ shocks, then

$$\log(R^K/R^f) = a \cdot \sigma^2,$$

with shocks in utility : 

$$a = \alpha \gamma + (1 - \alpha) + \frac{(\gamma - 1)(1 - \alpha)}{1 - \theta},$$

without shocks in utility : 

$$a' = \alpha \gamma + (1 - \alpha)$$

• With $1 - \theta = .64$ and $\alpha \gamma + (1 - \alpha) = 5$,

Assume $\alpha \in (.2 : .5)$, which implies $\gamma \in (9 : 21)$

$$\gamma = 9 \quad \rightarrow \quad \frac{a}{a'} = 2.25$$

$$\gamma = 21 \quad \rightarrow \quad \frac{a}{a'} = 6$$
Calibration

• A lot of free parameters

\[ \sigma_H, \psi_H, \varphi_{HK}, \rho, \nu, \kappa, \gamma \]

• Business cycles properties

  – Investment standard deviation relative to output std:
    here 1.87, US economy 2.93

  – Standard RBC better than “Standard” here (?)
    here 1.82, King+Rebelo (1999) 2.95
Countercyclical risk aversion and numerical methods (?)

- Paper uses log-linear approximations of first-order conditions

\[ \lambda_t = a_c \cdot c_t + a_h \cdot h_t + a_l \cdot l_t \]

where \( x_t \equiv \ln \left( \frac{X_t}{X} \right) \)

\[ RRA_t \equiv \frac{\partial \log \left( \frac{\partial V_t}{\partial C_t} \right)}{\partial \log C_t} \]

\[ \begin{align*}
\quad & = \frac{\partial}{\partial c_t} \left( a_c \cdot c_t + a_h \cdot h_t + a_l \cdot l_t \right) \\
\quad & = a_c
\end{align*} \]