

## Handout 10: Applying the CAPM to Capital Budgeting

Corporate Finance, Sections 001 and 002

Previously we calculated net present value given a discount rate  $r$ , without discussing where  $r$  came from. One major reason why we are interested in the CAPM is it tells us what  $r$  should be.

- Result 1:

Consider a firm that has no debt. Consider a project in the same line of business as the firm's existing projects. Assume that the CAPM is true.

Then the cost of capital of the firm is

$$\bar{R} = R_f + \beta(\bar{R}_M - R_f)$$

where  $\beta$  is  $\text{Cov}(R, R_M)/\sigma_M^2$ , calculated for returns  $R$  on the firm's equity.

In other words, when deciding to take a project with cash flow  $C_0$  today and expected future cash flows  $\bar{C}_1, \bar{C}_2, \dots$ , the firm should calculate the NPV of the project as:

$$\text{NPV} = C_0 + \sum_{t=1}^{\infty} \frac{\bar{C}_t}{(1 + \bar{R})^t}.$$

so  $\bar{R}$  is the discount rate for the future payoffs. To maximize firm value, the firm should accept the project if  $\text{NPV} > 0$ , otherwise the firm should reject the project.

Result 1 says that the discount rate should come from the CAPM. Why is this true? To understand why result 1 is true, we review the basics of the NPV decision in the single payoff case:

- Suppose the firm has access to a project that costs  $-C_0 = \$1000$  this year and will pay  $C_1$  *for sure* next year.
- What is the alternative to investing in the project? Investing in the riskless asset  $R_f$  (or equivalently, putting money in bank).

- If the firm invests at  $R_f$ , the payoff is  $\$1000(1 + R_f)$  next year
- The firm should take the project only if it offers a greater payoff than investing in the the riskfree asset, i.e. only if

$$C_1 > \$1000(1 + R_f)$$

This is the same as asking whether

$$\text{NPV} = C_0 + C_1/(1 + R_f) = -1000 + C_1/(1 + R_f) > 0.$$

This justifies the NPV rule for riskfree payoffs. Now suppose that the payoff next year is risky. Its expected value is  $\bar{C}_1$ . How does the argument change?

- The firm should only invest in the project if it offers a higher return than the alternative *given the level of risk*. It is not enough that the project offers a higher return than the riskfree rate because a riskless payoff is more valuable than a risky payoff, all else equal.
- The firm could put the  $\$1000$  in a stock portfolio that has the same  $\beta$  as the firm. The stock portfolio would then have the same risk as the project. According to the CAPM, the rate of return on the portfolio must be

$$\bar{R} = R_f + \beta(\bar{R}_M - R_f)$$

- Therefore if the firm puts  $\$1000$  into this portfolio, the expected payoff will be

$$\$1000(1 + \bar{R})$$

- The firm should take the project only if it offers greater value than investing in the portfolio. That is, only if  $\bar{C}_1 > C_0(1 + \bar{R})$ . Clearly this is the same as asking

$$\text{NPV} = C_0 + \bar{C}_1/(1 + \bar{R}) = -1000 + \bar{C}_1/(1 + \bar{R}) > 0.$$

If the NPV is greater than zero, accepting the project will increase the value of the firm (if the firm invests in the asset with return  $R$ , the value of the firm is unchanged).

Note: We use  $\beta$  to measure the amount of risk in the project. Why? Because if shareholders hold well-diversified portfolios only systematic risk matters to them. An increase in idiosyncratic risk will be diversified away.

The multi-payoff case works similarly to the single-payoff case. We have just shown Result 1.

In Result 1 we have made two important simplifying assumptions. The first is that the project is in the same line of business as the firm. The second is that the firm has no debt. Making both of these assumptions allowed us to use the  $\beta$  estimated from the firm's stock. If either of these assumptions do not hold, then it is not correct to discount using the required rate of return on the firm's stock.

Similar reasoning to Result 1 tells us that if a firm is embarking on a project in a new line of business, the firm should use the  $\beta$  for this new line of business, rather than the  $\beta$  estimated from the firm's stock. The  $\beta$  estimated from the firm's stock represents the risk of the existing businesses, not the new business.

For example, the computer software industry has a  $\beta$  of about 1.25. If a software company decides to invest in building hardware, the company should not use a  $\beta$  of 1.25. Rather the company should use the  $\beta$  for the hardware industry of (about) 1.80. The higher  $\beta$  reflects the fact that, in entering the hardware industry, the firm is taking on a riskier project and thus the cash flows need to be discounted at a higher rate.

What happens when the firm contains debt as well as equity? This leads us to Result 2:

- Result 2

Assume the market value of the firm's debt is  $D$  and the market value of the firm's equity is  $E$ . Let

$\beta_E$  = beta on firm's equity

$\beta_D$  = beta on firm's debt

Assume there are no taxes and that the CAPM is true. Consider a project in the same line of business as the firm's existing projects. Then the cost of capital for that project equals

$$\bar{R}_A = \frac{E}{D+E} \bar{R}_E + \frac{D}{D+E} \bar{R}_D \quad (1)$$

where  $\bar{R}_E$  is the equity cost of capital:

$$\bar{R}_E = R_f + \beta_E(\bar{R}_M - R_f)$$

and  $\bar{R}_D$  is the debt cost of capital:

$$\bar{R}_D = R_f + \beta_D(\bar{R}_M - R_f).$$

Note: The rate of return  $\bar{R}_A$  is the *Weighted Average Cost of Capital* (or WACC).

Why is  $\bar{R}_A$  the proper discount rate to use? Because it is the required rate of return on the firm's assets. For concreteness, consider Citigroup. We can think of Citi's assets as a portfolio of Citi's debt and Citi's equity. The portfolio weights are

$$X_E = \frac{E}{D+E}$$

for Citi's equity and

$$X_D = \frac{D}{D+E}$$

for Citi's debt. As we learned from portfolio theory, the expected return on this portfolio is

$$\bar{R}_A = X_E \bar{R}_E + X_D \bar{R}_D.$$

So (1) follows from substituting in  $X_E$  and  $X_D$ .

Because the new project has the same risk profile as the existing projects,  $\bar{R}_A$  must also be the required rate of return on the new project. This shows Result 2.

Using formulas from portfolio theory, we can derive an equation for the  $\beta$  of Citi as a whole. This is sometimes useful in calculating WACC. We call this the asset  $\beta$  and label it  $\beta_A$ . Because the portfolio  $\beta$  is the weighted average of the underlying  $\beta$ s,

$$\beta_A = \frac{E}{D+E} \beta_E + \frac{D}{D+E} \beta_D \quad (2)$$

This gives us another way of showing (1). By the CAPM,

$$\bar{R}_A = R_f + \beta_A (\bar{R}_M - R_f)$$

But we know that  $\beta_A$  is just a weighted average of  $\beta_E$  and  $\beta_D$ . Substituting in:

$$\bar{R}_A = R_f + \left( \frac{E}{D+E} \beta_E + \frac{D}{D+E} \beta_D \right) (\bar{R}_M - R_f)$$

Expanding out this equation: (Note that  $R_f = \frac{E}{D+E} R_f + \frac{D}{D+E} R_f$ )

$$\begin{aligned} \bar{R}_A &= \frac{E}{D+E} (R_f + \beta_E (\bar{R}_M - R_f)) + \frac{D}{D+E} (R_f + \beta_D (\bar{R}_M - R_f)) \\ &= \frac{E}{D+E} \bar{R}_E + \frac{D}{D+E} \bar{R}_D \end{aligned}$$

Which is just equation (1).

Thus there are two equivalent ways of calculating the WACC.

1. From  $\beta_E$  and  $\beta_D$ , calculate  $R_E$  and  $R_D$  using the CAPM. Then calculate  $\bar{R}_A$  by (1).
2. From  $\beta_E$  and  $\beta_D$ , calculate  $\beta_A$  using (2). Then calculate  $\bar{R}_A$  using the CAPM.

We have just shown that these two methods are guaranteed to give you the same answer.