Handout 14: MM with Corporate Taxes

Corporate Finance, Sections 001 and 002

Previously we showed that when there are no taxes or other capital market imperfections, the value of the firm was independent of the percent of debt and equity in the capital structure. How do taxes change our conclusions? Under the U.S. tax code, and the tax code in many other countries, interest payments on debt are tax deductable. As we will see, this implies that capital structure will no longer be irrelevant to the value of the firm.

Proposition: Assume corporate income is taxed, but there are no other market imperfections. Then *the value of the firm is the value if the firm were all equity financed, plus the present value of the tax shield.*

This statement says that taking on debt can increase the value of the corporation, through the present value of the tax shield. It is illustrated in the following example.

Example: Suppose there are two firms, identical in every way except for capital structure. One firm is unlevered (all-equity). We will call it U. The other is levered. We will call it L. The levered firm has borrowed \$4000 to be paid back in equal installments in perpetuity. We start by assuming cash flows on both firms are certain. Because cash flows are certain, the discount rate for both firms is the riskfree rate, say, 10%. Note that this implies an annual interest payment of \$400 for the levered firm. Income statements for these firms are as follows:

	U	L
Operating Income ($)	1000	1000
Interest ($)	–	-$400
Pre-tax Income ($)	1000	600
Tax at 35% ($)	-350	-210
Equity income ($)	650	390
Total cash flows to investors ($)	650	790

Suppose the operating income is expected to continue in perpetuity. Then

$$\begin{aligned} E_U &\equiv \text{Value of Equity for Unlevered Firm} \\ &= \frac{\$650}{1.10} + \frac{\$650}{1.10^2} + \cdots \\ &= \frac{\$650}{.10} = \$6500 \end{aligned}$$

The total value of the unlevered firm is

$$V_U = E_U + D_U = \$6500 + 0 = \$6500$$

How does this compare with the value of the levered firm? We first compute the value of equity for the levered firm:

$$\begin{aligned} E_L &\equiv \text{Value of Equity for the Levered Firm} \\ &= \frac{\$390}{1.10} + \frac{\$390}{1.10^2} + \cdots \\ &= \frac{\$390}{.10} = \$3900. \end{aligned}$$

The levered firm also includes debt, so we need to compute its value too:

$$\begin{aligned} D_L &\equiv \text{Value of Debt for the Levered Firm} \\ &= \frac{\$400}{1.10} + \frac{\$400}{1.10^2} + \cdots \\ &= \frac{\$400}{.10} = \$4000 \end{aligned}$$

The total value of the levered firm is

$$V_L = E_L + D_L = \$3900 + \$4000 = \$7900$$

This is our main result: the total value of the levered firm is higher because leverage has allowed this firm to shield some of its income from taxes.

The difference between the value of the levered and the unlevered firm is the total income that has been shielded from taxes, multiplied by the tax rate, and discounted back to the present:

$$\text{PV of tax shield} = \frac{.35 \times 400}{1 + .10} + \frac{.35 \times 400}{1.10^2} + \cdots = \frac{.35 \times 400}{.10} = \$1400$$

We have seen that the value of the levered firm and the value of the unlevered firm are related by

$$\begin{aligned} V_L &= V_U + \text{PV of the tax shield} \qquad (1) \\ &= \$6500 + \$1400 = \$7900 \end{aligned}$$

Notes:

1. This example illustrated proposition I under certainty. When there is uncertainty, the argument remains the same. However, the tax shield now needs to be valued at a different discount rate. The most common assumption is that the risk of the tax shield is the same as the risk of the debt. Therefore, the tax shield should be discounted at r_D. Suppose the firm as debt D. Then

   $$\text{PV of tax shield} = \frac{T_C r_D D}{1 + r_D} + \frac{T_C r_D D}{(1 + r_D)^2} + \cdots = \frac{T_C r_D D}{r_D} = T_C D.$$

   Therefore we can write Equation (1) as

   $$V_L = V_U + T_C D \qquad (2)$$

2. The calculation assumed that debt was perpetual. This implies that interest equals $r_D D$. If debt is not perpetual, then Equation (2) cannot be applied (and in general, the value of the tax shield will depend on r_D). In many circumstances this formula is a good approximation.