Risk, Unemployment, and the Stock Market: A Rare-Events-Based Explanation of Labor Market Volatility *

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Abstract

What is the driving force behind the cyclical behavior of unemployment and vacancies? What is the relation between job creation incentives of firms and stock market valuations? This paper proposes an explanation of labor market volatility based on time-varying risk, modeled using a small and variable probability of an economic disaster. We characterize recessions as periods of high discount rates applied to future cash flows from a new hire and low expected growth. As a result, investment in hiring falls, leading to higher unemployment and lower stock market valuations. The presence of disaster risk generates a realistic magnitude of the equity premium, while time variation in disaster probability can quantitatively generate a high volatility in vacancies and unemployment. The model can thus explain the comovement of unemployment and stock market valuations present in the data.

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1 Introduction

The Diamond-Mortensen-Pissarides (DMP) model of search and matching offers an intriguing theory of labor market fluctuations based on the job creation incentives of employers (Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994), Pissarides (2000)). When the contribution of a new hire to firm value decreases, employers reduce investment in hiring, decreasing the number of vacancies and, in turn, increasing unemployment. Due to the glut of jobseekers in the labor market, vacancies become easier for employers to fill. Therefore, unemployment stabilizes at a higher level and the number of vacancies at a lower level. That is, labor market tightness (defined as the ratio of vacancies to unemployment) decreases until the payoff to hiring changes again.

While the mechanism of the DMP model is intuitively promising, there is a fundamental question concerning the model: what is the driving force behind the cyclical behavior of job creation incentives? In the canonical DMP model and numerous successor models, the driving force is labor productivity. However, explaining labor market volatility based on productivity fluctuations is difficult, because unemployment and vacancies are much more volatile than labor productivity (Shimer (2005)). Furthermore, unemployment does not track the movements of labor productivity, as is particularly apparent in the last three recessions. Rather, these recent data suggest a link between unemployment and stock market valuations (Hall (2014)).

This paper proposes time-varying risk as the common source of equilibrium fluctuations in the labor market and the stock market. We characterize risk by a small and time-varying probability of a large drop in productivity, namely, an economic disaster. In recessions, expected growth rates are low and risk premia are high. High risk premia imply high discount rates that firms apply to future cash flows from a new hire. Therefore, even if labor productivity remains constant, the expectation of low future productivity, along with high discount rates, affects job creation incentives of firms. An increase in risk leads the labor market equilibrium to shift to a lower point on the vacancy-unemployment locus (the Beveridge curve), with higher unemployment and lower vacancy openings. At the same time, stock market valuations decline.
Our model generates a high volatility in unemployment and vacancies, along with a strong negative correlation between the two. This is consistent with U.S. data. We calibrate wage dynamics to match the behavior of the labor share in the data and find that matching the observed low response of wages to labor market conditions is crucial for both labor market volatility and realistic behavior of financial markets. Furthermore, the search and matching friction in the labor market and time-varying disaster risk result in a realistic equity premium and stock return volatility. Because the labor market and the stock market are driven by the same force, the price of the aggregate stock market and labor market tightness are highly correlated, while the correlation between labor productivity and tightness is realistically low.

Our paper is related to three strands of literature. First, since Shimer (2005) showed that the DMP model with standard parameter values implies small movements in unemployment and vacancies, a strand of literature has further developed the model to generate large responses of unemployment to aggregate shocks. In these papers, the aggregate shock driving the labor market is labor productivity. Hagedorn and Manovskii (2008) argue that a calibration of the model with low bargaining power of workers and a flow value of unemployment close to labor productivity can reconcile unemployment volatility in the DMP model with the data. Other papers suggest alternatives to the Nash bargaining assumption in the canonical DMP model to render wages less responsive to productivity shocks so that they do not rapidly adjust downward following a negative shock, leading to little destruction of job creation incentives (Hall (2005), Hall and Milgrom (2008), Gertler and Trigari (2009)). Our paper departs from these in that we do not rely on time-varying labor market productivity as a driver of labor market tightness, which leads to a counterfactually high correlation between these variables. Furthermore, we also derive implications for the stock market, and explain the equity premium and volatility puzzles.

Second, the present work relates to ones that embeds the DMP model into the real business cycle framework, with a representative risk averse household that makes investment and consumption decision. In the standard real business cycle (RBC) model (Kydland and Prescott (1982)), employment is driven by the marginal rate of substitution between consumption and leisure, and, because the labor market is frictionless, no vacancies go unfilled. Merz (1995) and Andolfatto
(1996) observe that this model has counterfactual predictions for the correlations of productivity and employment, and build a model that incorporates RBC features and search and matching. These models captures the lead-lag relationship between employment and productivity while having more realistic implications for wages and unemployment compared to the baseline RBC model. In this paper, we also document the lead-lag relationship between productivity and labor market tightness in the period that this literature analyzes (1959 - 1988). However, our empirical analysis shows that this lead-lag relation is absent in more recent data. These papers do not study asset pricing implications.

Third, our paper is related to the literature on asset prices in dynamic production economies. In these models, as in the RBC framework described above, consumption and dividend dynamics are endogenously determined by the optimal equilibrium policy of a representative firm. This contrasts with the more standard asset-pricing approach of assuming an endowment economy, in which consumption and dividends are taken as given. The main difficulty in production economies is endogenous consumption smoothing. While higher risk aversion raises the equity premium in an endowment economy, this leads to even smoother consumption in production economies resulting in very little fluctuation in marginal utility. One way of overcoming this problem is to assume alternative preferences, for example, habit formation as in Jermann (1998), though these can lead to highly volatile riskfree rates. Another approach is to allow for rare disasters. Gourio (2012) studies the implications of time-varying disaster risk modeled as large drops in productivity and destruction of physical capital in a business cycle model with recursive preferences and capital adjustment costs. Gourio’s model can explain the observed co-movement between investment and risk premia. However, unlevered equity returns have little volatility, and thus the premium on unlevered equity is low. This model can be reconciled with the observed equity premium by adding financial leverage, but the leverage ratio must be high in comparison with the data. Also, as in RBC models with frictionless labor markets, Gourio’s model does not explain unemployment. Petrosky-Nadeau, Zhang, and Kuehn (2013) build a model where rare disasters arise endogenously through a series of negative productivity realizations. Like our paper, they make use of the DMP model, but with a very different aim and implementation. Their paper incorporates a calibration
of Nash-bargained wages similar to Hagedorn and Manovskii (2008), leading to wages that are high and rigid. Moreover, their specification of marginal vacancy opening costs includes a fixed component, implying that it costs more to post a vacancy when labor conditions are slack and thus when output is low. Finally, they assume that workers separate from their jobs at a rate that is high compared with the data. The combination of a high separation rate, fixed marginal costs of vacancy openings and high and inelastic wages amplify negative shocks to productivity and produce a negatively skewed output and consumption distribution. Like other DMP-based models described above, their model implies that labor market tightness is (counterfactually) driven by productivity. Furthermore, while their model can match the equity premium, the fact that their simulations contain consumption disasters make it unclear whether the model can match the high stock market volatility and low consumption volatility that characterize the U.S. postwar data.

The paper is organized as follows. Section 2 provides empirical evidence about the relation between the labor market, labor productivity and the stock market. Section 3 presents the model and illustrates the mechanism in a simplified version. Section 4 discusses the quantitative results from the benchmark calibration and alternative calibrations. Section 5 concludes.

2 Labor Market, Labor Productivity and Stock Market Valuations

The literature succeeding the canonical DMP model uses labor productivity as the driving force of labor market fluctuations. However, this approach has faced two main challenges. First, explaining highly volatile unemployment and vacancy rates is difficult the volatility of productivity is very low compared to the high volatility of unemployment and vacancy rates, wages in the DMP model are highly pro-cyclical, which offsets the effect of productivity shocks on job creation incentives. Second, unemployment does not always track movements in productivity in the data. Therefore, Hall (2014) argues that an alternative source of time variation in job creation incentives is the variation in discount rates that firms apply to future benefits from a new hire, which is the same
discount rate that firms apply to all future cash flows determining firm value. This section provides some empirical evidence on the interplay between the labor market, productivity and the stock market.

Figure 1 plots the time series of the cyclically adjusted price-earnings ratio \((P/E)\) constructed by Robert Shiller and the price-productivity ratio \((P/Z)\).\(^1\) We construct \(P/Z\) by dividing the real price of S&P 500 by real output per person in the non-farm business sector. The correlation between the quarterly observations of these series is 0.97 for the period from 1951 to 2013. We introduce \(P/Z\) as a reliable valuation ratio because it has an obvious counterpart in the model presented in Section 3.

In Figure 2, we plot the time series of deviations of labor productivity \(Z\) and the vacancy-unemployment ratio \(V/U\), the key variable describing the labor market in the DMP model, from an HP trend.\(^2\) \(Z\) never deviates by more than 5 percent from trend, while \(V/U\) is highly volatile and deviates up to one log point from trend. There is some evidence that \(Z\) leads \(V/U\) by one year, with a correlation of 0.31 between \(V/U\) and lagged \(Z\).\(^3\) This correlation is 0.62 in the subsample before 1985 and -0.09 after 1985 while the contemporaneous correlation is 0.47 until 1985 and -0.36 after 1985. Specifically, the behavior of these two series seems similar in the recessions of the early 1960s and early 1980s. The positive correlation turns into a negative correlation from 1986 onwards.\(^4\) In particular, the productivity boom of late 1990s and early 2000s, with small interruptions before and after the recession of the early 2000s, is associated with a downturn in the labor market. We observe a striking pattern in the Great Recession, which was followed by a boom in productivity in 2009, but the labor market has not recovered tracking productivity. This confirms the observation of Hall (2014) regarding the challenge in explaining labor market fluctuations based on labor productivity shocks, particularly during the recessions of the last 28

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\(^1\)Appendix F describes the data sources used in this paper.

\(^2\)Following Shimer (2005) we use a low frequency HP filter with smoothing parameter 10\(^5\) throughout to capture business cycle fluctuations. All results are robust to using an HP filter with smoothing parameter 1,600.

\(^3\)One year is the lag length for productivity with which \(V/U\) has the maximum correlation. However, this depends on the sample period. Consistent with Hagedorn and Manovskii (2011), we find that the lag length with maximum correlation is two quarters in the data until 2004.

\(^4\)Shimer (2005) uses data up to 2003 and notes the negative correlation in the period from 1986 to 2003. (footnote 10).
years.\textsuperscript{5}

Figure 3 shows the time series of $V/U$ and $P/Z$. The behavior of these indicators is similar from the mid-1980s to 2013. Furthermore, deviations of $P/Z$ from trend are fairly volatile, with deviations up to 0.5 log points below trend. There is no obvious lead-lag relation between $V/U$ and $P/Z$. The highest correlation is between $V/U$ and lagged $P/Z$ by 2 quarters with 0.57, and the contemporaneous correlation is 0.47. In the period from 1986 to 2013, the contemporaneous correlation of 0.71 is the maximum among correlations with leads and lags. Figure 4 plots the time series of vacancies $V$ and $P/Z$, and the pattern is similar to $V/U$.

We confirm these observations by contemporaneous regressions of labor market variables on $Z$ and $P/Z$.\textsuperscript{6} The correlation between $Z$ and $P/Z$ is 0.00 in the entire sample, 0.30 from 1951 to 1985 and -0.28 from 1986 to 2013. The time series behavior of vacancies, unemployment and the vacancy-unemployment ratio in the post-war period seems to be associated with the behavior of the price-productivity ratio. In particular, productivity and vacancies are significantly negatively correlated between 1986 and 2013 while $P/Z$ explains 51\% of the variance in $V/U$. This fraction is 5\% in the subsample until 1985 and 22\% in the entire period from 1951 to 2013. The coefficient on $Z$ is not significantly different from zero when $P/Z$ is included in regressions except in the subsample until 1985. These results suggest that the driving force behind the labor market is not simply productivity; stock market valuations also play a key role in explaining labor market volatility.

Mortensen and Nagypal (2007) and Hall and Milgrom (2008) argue that models based on productivity shocks should not be evaluated by their ability to explain all of labor market volatility but by their ability to explain the portion of labor market volatility that is empirically explained by productivity shocks because these models do not consider alternative forces, such as discount rate.

\textsuperscript{5}The inconsistent behavior of labor productivity with the baseline business cycle model has been documented and addressed by the literature. Galí and Van Rens (2014) document the vanishing procyclicality of labor productivity and high volatility of employment. Hagedorn and Manovskii (2011) argue that the correlation between labor market tightness can be partially reconciled with the data using output per person from the Current Population Survey instead of the Current Employment Statistics by the Bureau of Labor Statistics. McGrattan and Prescott (2012) argue that a model with intangible capital and nonneutral technology can potentially recover the role of the productivity channel.

\textsuperscript{6}We find that the lagged values of $Z$ and $P/Z$ increase the goodness of fit but do not change qualitatively the results. We run regressions using log deviations of all variables from trend.
and separation rate fluctuations. However, our empirical analysis suggests that the productivity channel cannot qualitatively explain the directional changes in $V/U$ in the last 28 years, although it has high explanatory power from 1951 to 1985. $Z$ and $V/U$ are significantly negatively correlated in the subsample after 1985. Periods of low $V/U$ do not coincide with periods of low $Z$.

This evidence motivates a united mechanism of job creation incentives and stock market valuations. In recessions, even if productivity does not change, future productivity expectations are low and uncertainty is high. This leads to lower stock market valuations and a lower present value of a new hire, increasing unemployment. This is the key mechanism of the model we present in Section 3.

3 Model

We present a parsimonious model that establishes the relation between equity market valuations and labor market quantities in Section 3.1. Using minimal assumptions, we show that the value of a representative firm facing a labor market characterized by a search friction is closely linked to unemployment and vacancies in the aggregate economy, and illustrate that this result provides an accurate description of the interplay between the stock market and the labor market observed in the data. In the remaining subsections, we specify preferences, technology and the search friction in the labor market leading to the characterization of general equilibrium. The specification of preferences and the productivity process allow us to characterize and quantitatively assess the ability of the model to jointly explain volatility in the stock market and the labor market as well as the relation between the two. We highlight the role of preference parameters in Section 3.6 in establishing that stock market valuations and labor market tightness are low in times of high uncertainty.
3.1 Equity Valuation and the Labor Market

Let $M_{t+1}$ denote the representative household’s stochastic discount factor. The representative firm produces output $Y_t$ with technology $F(Z_t, N_t)$ where $Z_t$ is the non-negative level of aggregate productivity and $N_t$ is employment in the economy and lies between 0 and 1. $W_t = W(Z_t, N_t, V_t)$ denotes the aggregate wage rate where $V_t$ is the vacancy rate. Furthermore, the representative firm incurs costs $\kappa_t$ per vacancy opening. As a result, the firm pays dividends $D_t$, which is what remains from output after paying wages and investing in hiring:

$$D_t = Z_t N_t - W_t N_t - \kappa_t V_t.$$  \hfill (1)

The hiring flow is determined according to the matching function $m(N_t, V_t)$. The following result establishes a general relation between the stock market and the labor market.

**Result 1.** Assume that the production function is given by a linear technology

$$F(Z_t, N_t) = Z_t N_t$$  \hfill (2)

Furthermore, the matching function takes the Cobb-Douglas form so that the vacancy-filling rate $q(\theta_t)$ is a decreasing function of the vacancy-unemployment ratio $\theta_t$. The representative firm solves the following problem:

$$\max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau}$$  \hfill (3)

subject to

$$N_{t+1} = (1 - s) N_t + q(\theta_t) V_t,$$  \hfill (4)

where $D_{t+\tau}$ is as defined in (1) and the representative firm takes $M_{t+1}$, $q(\theta_t)$ and $W_t$ as given.

The first order conditions of the firm’s problem imply the following ex-dividend value of the firm.

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7We use the representative agent framework throughout the model. Appendix A shows the properties of a market, populated by households with identical preferences and firms facing the same aggregate labor market, that aggregate to our representative agent model.

8We show in Section 3.3.1 that the vacancy filling rate $q(\theta_t) = m(N_t, V_t)/V_t$ becomes a decreasing function of the vacancy-unemployment ratio $\theta = V_t/U_t$ if the matching function takes a Cobb-Douglas form. We impose the Cobb-Douglas form on the matching function for a better illustration of the relation between the equity value and the vacancy-unemployment ratio. The results hold also if we leave the vacancy filling rate as a function of $N_t$ and $V_t$ rather than $\theta_t$ only.
where $\frac{\kappa_t}{q(\theta_t)}$ is equal to the Lagrange multiplier on the aggregate law of motion for employment.

Further assume that vacancy costs are proportional to productivity, namely, $\kappa_t = \kappa Z_t$, which implies:

$$\frac{P_t}{Z_t} = \frac{\kappa}{q(\theta_t)} N_{t+1}. \quad (6)$$

Appendix C illustrates the proof of Result 1. We document the joint time series behavior of the price-productivity ratio and the labor market tightness in Section 2. The value of the representative firm $P_t$ corresponds to the value of the aggregate stock market. In this parsimonious framework, the price-productivity ratio is purely driven by labor market quantities. The model predicts that equity valuations are higher when the number of vacancy openings is high, leading to a lower vacancy filling rate and higher employment in the next period. Furthermore, equity valuations are low when unemployment is high leading to a higher vacancy filling rate and higher employment in the next period.

The accuracy of this relation can be assessed by plugging in the time series of the price-productivity ratio and the unemployment rate into (6) which gives a model-implied time series of the vacancy rate. Accordingly, Figure 5 illustrates that the model-implied values of the vacancy-unemployment ratio are closely aligned with the corresponding data values. The result in (6) successfully describes the relation between the stock market and the labor market. This motivates that the economic force behind the high volatility of in the labor market is likely to be the same as the factors of high stock market volatility. Therefore, risk factors driving price formation in the aggregate stock market should also drive the volatility in the labor market through the cyclical behavior of firm hiring decisions.

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9The proportionality of vacancy costs in productivity also guarantees the existence of a balanced growth path in our fully specified equilibrium model.

10Figure 5 uses the parameter values for the matching function that are introduced in the calibration in Section 4.
3.2 The Representative Household

Following Merz (1995) and Gertler and Trigari (2009), we assume that the representative household is a continuum of members who provide one another with perfect consumption insurance.\footnote{Appendix A illustrates the implications of the perfect consumption insurance assumption in an economy populated by agents with identical preferences.} We normalize the size of the labor force to one.\footnote{This assumption implies that our model focuses on the transition between employment and unemployment rather than between in and out of labor force.} The household maximizes utility over consumption, characterized by the recursive utility function introduced by Kreps and Porteus (1978) and Epstein and Zin (1989):

\[ J_t = \left[C_t^{1-\frac{1}{\psi}} + \beta \left( \mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{1-\frac{1}{\psi}} \right]^{1-\frac{1}{\psi}}, \tag{7} \]

where \( \beta \) is the time discount factor, \( \gamma \) is relative risk aversion and \( \psi \) is often interpreted as elasticity of intertemporal substitution (EIS). In case of \( \gamma = 1/\psi \), recursive preferences collapse to power utility. The recursive utility function implies that the stochastic discount factor takes the following form:

\[ M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left( \frac{J_{t+1}}{\mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right]^{1-\gamma}} \right)^{1-\gamma}. \tag{8} \]

3.3 The Labor Market

3.3.1 Search Friction

The labor market is characterized by the DMP model of search and matching. The representative firm posts a number of job vacancies \( V_t \geq 0 \). The hiring flow is determined according to the matching function \( m(N_t, V_t) \), where \( N_t \) is employment in the economy and lies between 0 and 1. We assume that the matching function takes the following Cobb-Douglas form:

\[ m(N_t, V_t) = \xi (1 - N_t)^\eta V_t^{1-\eta}, \tag{9} \]
where $\xi$ is matching efficiency and $\eta$ is the unemployment elasticity of the hiring flow. As a result, the aggregate law of motion for employment is given by

$$N_{t+1} = (1-s)N_t + m(N_t, V_t), \tag{10}$$

where $s$ is the separation rate.\textsuperscript{13} The unemployment rate in the economy is defined by $U_t = 1-N_t$. The probability of finding a job for an unemployed worker is $m(N_t, V_t)/U_t = \xi\theta_t^{1-\eta}$. Accordingly, we define the job-finding rate $f(\theta_t)$ to be

$$f(\theta_t) = \xi\theta_t^{1-\eta}. \tag{11}$$

Analogously, the probability of filling a vacancy posted by the representative firm is $m(N_t, V_t)/V_t = \xi\theta_t^{-\eta}$ which corresponds to the vacancy-filling rate $q(\theta_t)$ in the economy:

$$q(\theta_t) = \xi\theta_t^{-\eta}. \tag{12}$$

The functional form of $f$ and $q$ provide useful insights about the mechanism of the DMP model. The job-finding rate is increasing, and the vacancy-filling rate is decreasing in the vacancy-unemployment ratio. In times of high labor market tightness, namely, when vacancy rate is high and/or unemployment rate is low, the probability of finding a job per unit time increases, whereas filling a vacancy takes more time.

Finally, the representative firm incurs costs $\kappa_t$ per vacancy opening. As a result, aggregate investment in hiring is $\kappa_t V_t$.

### 3.3.2 Wages

The canonical DMP model assumes that wages are determined by Nash bargaining between the employer and the jobseeker. Both parties observe the surplus of job creation and the bargaining power of the jobseeker is equal to the fraction of the surplus the jobseeker receives. Pissarides\textsuperscript{13} The assumption of $V_t > 0$ implies that the maximum drop in employment level is $s$.\textsuperscript{11}
shows that the Nash-bargained wage, $W_t^N$, is given by

$$W_t^N = (1 - B)b_t + B(Z_t + \kappa_t \bar{\theta}),$$

(13)

where $0 \leq B \leq 1$ is worker’s bargaining power and $b_t$ is the flow value of unemployment. The worker threatens the employer to leave the wage bargain and continue to search while receiving the flow value of unemployment. The Nash-bargained wage can be interpreted as the weighted average of two components: the opportunity cost of employment and a term that represents the contribution of the worker to the firm’s profits. If the bargaining power of the worker is high, the firm has to pay a higher fraction of the output the worker produces as wage. Furthermore, the worker receives a higher fraction of the foregone costs by the firm by employing the worker which prevents incurring vacancy costs. The compensation of the worker due to foregone vacancy costs is higher if labor market tightness is high, making it easier for the worker to find a job and more difficult for the firm to fill the vacancy in case the worker leaves the firm.

Although the Nash-bargained wage is a convenient formulation from a modeling perspective, Shimer (2005) shows that it is too flexible to account for the high volatility in the labor market. Furthermore, the labor share of output is not as responsive to labor market tightness as the Nash-bargained wage implies. Therefore, we use a wage rule introduced by Hall (2005) that features stickiness:

$$W_t = \nu W_t^N + (1 - \nu)W_t^I,$$

(14)

where

$$W_t^I = (1 - B)b_t + B(Z_t + \kappa_t \bar{\theta}).$$

(15)

The parameter $\nu$ controls the degree of tightness insulation. With $\nu = 1$, we are back in the Nash bargaining case. With $\nu = 0$, wages do not respond to labor market tightness. The resulting wage remains sensitive to productivity but loses some of its sensitivity to tightness. Furthermore, this formulation allows a direct comparison between versions of the model with and without tightness.

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14Appendix B shows that the canonical DMP wage equation holds in our model.

15See Section 4.4 for details.
insulated wages. The parameters $B$, $\kappa$ and $\nu$ jointly determine the dynamics of wages given the dynamics of productivity and labor market tightness. We will calibrate these parameters to match the behavior of the labor share in postwar U.S. data.

3.4 The Representative Firm

3.4.1 Technology

The representative firm produces output $Y_t$ with technology $F(Z_t, N_t)$ given in (2). In normal times, $\log Z_t$ follows a random walk with drift. In every period, there is a small and time-varying probability of a disaster. Thus,

$$
\log Z_{t+1} = \log Z_t + \mu + \epsilon_{t+1} + d_{t+1} \zeta_{t+1},
$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. The parameter $d_{t+1}$ is the disaster indicator:

$$
d_{t+1} = \begin{cases} 
1 & \text{with probability } \lambda_t \\
0 & \text{with probability } 1 - \lambda_t,
\end{cases}
$$

where $\lambda_t$ is the conditional disaster probability. Disaster probability dynamics are given by

$$
\log \lambda_t = \rho_\lambda \log \lambda_{t-1} + \left(1 - \rho_\lambda\right) \log \bar{\lambda} + \epsilon_{\lambda, t},
$$

where $\epsilon_{\lambda, t} \sim N(0, \sigma_\lambda^2)$. Furthermore, $\zeta$ is the size of the downward jump in productivity, has a time-invariant distribution and takes only negative values. Disaster probability, disaster size and productivity shocks are independent.

3.4.2 Equity Returns

The representative firm maximized the present value of dividends as described in Section 3.1. Appendix C shows that the first-order conditions of the firm imply the fundamental asset-pricing

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16 We use a finite-state Markov process to approximate this process in our numerical calibration with all nodes smaller than one. See
equation $\mathbb{E}_t [M_{t+1} R_{t+1}] = 1$, where the return is given by

$$R_{t+1} = \frac{Z_{t+1} - W_{t+1} + (1 - s) \frac{\kappa_{t+1}}{q(\theta_{t+1})}}{\frac{\kappa_t}{q(\theta_t)}}. \quad (18)$$

This is the return on investment in hiring and, equivalently, the equity return of the unlevered representative firm, as shown in Appendix C.

An investigation of the return expression in (18) conveys the intuition that $R_{t+1}$ is the return on hiring. The proportional cost of vacancy opening is $\kappa_t$ while the marginal cost depends on how many vacancies are filled conditional on the number of vacancy openings. For each vacancy opening, the firm fills $q(\theta_t)$ vacancies. Therefore, the marginal cost of hiring is the proportional cost of opening a vacancy divided by the vacancy-filling rate, and is in the denominator of equity return in (18). The numerator in (18) is the marginal benefit of hiring. The marginal benefit is the next period’s marginal cash flow of a job to the firm, namely, productivity net of wages. The firm also saves the cost of hiring in the next period unless the job is destroyed, which occurs at rate $s$.

To ensure the existence of a balanced growth path, we assume that $b_t = bZ_t$ and $\kappa_t = \kappa Z_t$. The linearity of $b_t$ and $\kappa_t$ in $Z_t$ allows us to normalize aggregate consumption, wages, as well as the value function of the representative household by aggregate productivity so that the problem becomes stationary and can be solved numerically.

Furthermore, the linearity of $\kappa_t$ and $b_t$ in $Z_t$ imply

$$R_{t+1} = e^{\mu + \kappa + d_{t+1} + d_{t+1} \zeta_{t+1}} \left[ \frac{1 - w_{t+1} + (1 - s) \frac{\kappa}{q(\theta_{t+1})}}{\frac{\kappa}{q(\theta_t)}} \right], \quad (19)$$

where $w_t$ is the wage level normalized by productivity and is given by

$$w_t = (1 - B)b + B(1 + \kappa((1 - \nu)\bar{\theta} + \nu \theta_t)). \quad (20)$$

Appendix E shows that labor market quantities and, as a result, the price-productivity ratio, are determined by disaster probability and current employment level. High disaster probability increases uncertainty in the economy, which leads to a decrease in investment in hiring and valuation.
In equilibrium, the representative household holds all equity shares of the representative firm. The wealth of the household equals the cum-dividend value of the firm. The government bill is in zero net supply.

The representative household consumes the output $Z_t N_t$ net of investment in hiring $\kappa_t V_t$, and the value of non-market activity $b_t(1 - N_t)$ achieved by the unemployed members:

$$C_t = Z_t N_t + b_t (1 - N_t) - \kappa_t V_t.$$  

(21)

The sources of household income from the firm are wages and dividends. The definition of dividends in (1) shows that the sum of wages and dividends amount to $Z_t N_t - \kappa_t V_t$. Furthermore, we interpret the value of non-market activity achieved by the unemployed members as a source of consumption. This interpretation is in line with the notion of home production. Using a rich set of data sources, Chodorow-Reich and Karabarbounis (2013) find that the opportunity cost of employment is volatile and pro-cyclical. Although unemployment benefits are counter-cyclical, $b_t$ has a large and pro-cyclical component including consumption and work differences between the employed and unemployed such as home production. While omitting the term $b_t (1 - N_t)$ in (21) has only minor effects on our quantitative results, we keep it in line with our interpretation of $b_t$.

### 3.6 Comparative Statics with Constant Disaster Probability

The labor market in the present model is isolated from realized productivity shocks, and the driving force is expectations about the productivity of new hires driven by the time-varying disaster probability. Although this view may seem extreme, the empirical analysis shows that a mechanism linking labor market volatility to stock market valuations rather than productivity shocks is plausible. We solve the model numerically and examine its quantitative properties in Section 4.

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17 We examine the quantitative implications of $\psi > 1$ for consumption in the next section.

18 See Bridgman (2013) and Aguiar, Hurst, and Karabarbounis (2013) for detailed discussions of home production.
In this section, we inspect the mechanism by examining comparative statics.

The proportionality assumptions on vacancy costs $\kappa_t$ and the flow value of unemployment $b_t$ in productivity $Z_t$ imply that we can write $c_t = \frac{C_t}{Z_t}$. A disaster realization at time $t$ results in the multiplication of aggregate consumption, output and investment in hiring by $e^{\zeta_t}$. The decrease in output, consumption and investment is permanent. Employment level does not change on impact.

The model has three state variables: disaster probability $\lambda_t$ and productivity $Z_t$ are the exogenous state variables, and employment level $N_t$ is the endogenous state variable. The homogeneity in consumption implies that we can normalize the household’s value function by productivity:\footnote{See Appendix A.1 for the homogeneity of the value function.}

$$J(Z_t, \lambda_t, N_t) = Z_t j(\lambda_t, N_t).$$ \hspace{1cm} (22)

The normalized value function is given by

$$j(\lambda_t, N_t) = \left[ c_t^{1-\frac{1}{\psi}} + \beta \left( \mathbb{E}_t \left[ e^{(1-\gamma)(\mu + c_{t+1} + d_{t+1} \zeta_{t+1})} j(\lambda_{t+1}, N_{t+1}) \right] \right)^{1-\frac{1}{\psi}} \right]^{1-\frac{1}{\psi}},$$ \hspace{1cm} (23)

which implies

$$j(\lambda_t, N_t)^{1-\frac{1}{\psi}} = c_t^{1-\frac{1}{\psi}} + \beta \left( \mathbb{E}_t \left[ e^{(1-\gamma)(\mu + c_{t+1} + d_{t+1} \zeta_{t+1})} j(\lambda_{t+1}, N_{t+1}) \right] \right)^{1-\frac{1}{\psi}}.$$ \hspace{1cm} (24)

**Result 2.** Assume that disaster probability is constant. This version of the model can be interpreted as a model without disaster risk, but with the time discount factor $\hat{\beta}$, where\footnote{This result is also shown in Gabaix (2011) and Gourio (2012).}

$$\hat{\beta}(\lambda) = \beta \left( \mathbb{E} \left[ e^{(1-\gamma)(d_{t+1} \zeta_{t+1})} \right] \right)^{1-\frac{1}{\psi}},$$ \hspace{1cm} (25)

The expectation in (25) is taken with respect to disaster probability $\lambda$ and the time-invariant distribution of disaster size $\zeta$. Then, $\hat{\beta}(\lambda)$ is decreasing in $\lambda$ if and only if EIS is greater than one, i.e. $\psi > 1$.

Appendix D provides a proof of Result 2. This static result is useful to investigate the response of labor market variables to changes in disaster probability.

We set productivity shocks $\epsilon$ to zero for now and examine dynamics when productivity is
growing at rate \( \mu \) in normal times. The equity return on the balanced growth path using the Nash-bargained wage becomes

\[
R_{t+1} = e^{\mu + dt_{t+1}/\zeta_{t+1}} \left[ 1 - s + \frac{1 - (1 - B)b - B(1 + \kappa \theta)}{\kappa} \xi \theta^{-\eta} \right],
\]

(26)

which is decreasing in \( \theta \).\(^{21}\) Due to the homogeneity of the value function in consumption, the stochastic discount factor becomes:

\[
M_{t+1} = \frac{\beta e^{-\frac{\theta}{\psi} - \gamma dt_{t+1}/\zeta_{t+1}}}{\mathbb{E}[e^{-(1 - \gamma)dt_{t+1}/\zeta_{t+1}}]^{\frac{\psi - \gamma}{1 - \gamma}}}. \tag{27}
\]

The numerator does not depend on disaster probability and is constant up to disaster realizations. The denominator is increasing in disaster probability if and only if \( \gamma > \frac{1}{\psi} \), namely, if the agent prefers early resolution of uncertainty. Therefore, the stochastic discount factor is constant up to disaster realizations and is decreasing in disaster probability in a comparative static sense.

Finally, the representative firm’s first order condition \( \mathbb{E}_t [M_{t+1} R_{t+1}] = 1 \) along with (26) and (27) implies

\[
\beta e^{\mu(1 - \frac{1}{\psi})} \frac{\mathbb{E}[e^{-(1 - \gamma)dt_{t+1}/\zeta_{t+1}}]}{(\mathbb{E}[e^{-(1 - \gamma)dt_{t+1}/\zeta_{t+1}}])^{\frac{\psi - \gamma}{1 - \gamma}}} = 1. \tag{28}
\]

Using the definition of \( \hat{\beta}(\lambda) \), we can write

\[
\hat{\beta}e^{\mu(1 - \frac{1}{\psi})} \left[ 1 - s + \frac{1 - (1 - B)b - B(1 + \kappa \theta)}{\kappa} \xi \theta^{-\eta} \right] = 1. \tag{29}
\]

**Result 3.** If \( \hat{\beta} \) is decreasing in \( \lambda \), labor market tightness \( \theta \) is decreasing in disaster probability \( \lambda \) in the balanced growth path.

Result 3 follows directly from Result 2 and (26). It conveys the intuition behind the alternative force that drives fluctuations in the labor market in the present model. Labor market tightness falls in times of high uncertainty if and only if EIS is greater than unity. The present value of a new hire is not only driven by the future productivity of jobs, but also by the discount rate firms apply to future cash flows from a new hire. In times of high uncertainty, discount rates rise

\(^{21}\)Tightness-insulated wage is a tool for dampening the quantitative effect of labor market tightness on wages. The qualitative results in this section are not affected by the tightness-insulation of wages.
(discount factors fall) and the present value of job creation falls. As we will show in the next section, this mechanism is helpful in quantitatively explaining historical fluctuations in the labor market that are difficult to explain using fluctuations in labor productivity.

4 Quantitative Results

4.1 Model Parameters

Table 1 describes model parameters for our benchmark calibration. We calibrate and simulate the model at a monthly frequency and calculate quarterly and annual values by aggregating monthly values.

We calibrate the labor productivity process to match the quarterly seasonally adjusted real average output per person in the non-farm business sector using data compiled by the Bureau of Labor Statistics (BLS) from Current Employment Statistics (CES). Accordingly, the monthly growth rate $\mu$ and standard deviation $\sigma_\epsilon$ of log productivity are set to 0.18% and 0.47%.

Labor market parameters determining wages and the aggregate law of motion for employment are calibrated following the literature. Separation rate $s$ is set to 3.5% following Shimer (2005) and Hall (2014). Higher values for $\eta$, the elasticity of the Cobb-Douglas matching function, imply that high volatility in the labor market translates to high volatility in returns, holding everything else equal. Petrongolo and Pissarides (2001) find that the range of appropriate estimates of $\eta$ is between 0.3 and 0.5, which is consistent with Yashiv (2000)’s finding that the elasticity of the matching function in the U.S. with respect to unemployment is lower than that with respect to vacancies. Hall and Milgrom (2008) and Hall (2014) take the value to be 0.5. We set $\eta$ to 0.35. Following Hall and Milgrom (2008) and Hall (2014), we set the bargaining power of workers $B$ in case of Nash bargaining to 0.5 and the flow value of unemployment $b$ to 0.76. The vacancy cost parameter $\kappa$, which corresponds to unit costs of vacancy opening normalized by labor productivity in our model, is set to 0.5, the average of values taken by Hall and Milgrom (2008) and Hagedorn
Tightness-insulation parameter $\nu$ is set to 0.05. Parameters $B$, $b$, $\kappa$ and $\nu$ jointly determine dynamics of wages given dynamics of labor market tightness and labor productivity. The tightness-insulation parameter $\nu$ is calibrated to match wage dynamics discussed in Section 4.4. Finally, we set the matching efficiency $\xi$ to 0.365, targeting a model population value for labor market tightness equal to the data value of 0.74.

In terms of preference parameters, we assume the EIS $\psi$ is equal to 2 and risk aversion $\gamma$ is equal to 5.7. Risk aversion is low in comparison to many asset pricing models (e.g. Bansal and Yaron (2004) and Petrosky-Nadeau, Zhang, and Kuehn (2013) who assume a risk aversion of 10). It is higher than some models with disaster risk, because our average disaster probability is substantially lower than other models assume. Section 3.6 shows that, as is standard in production models explaining asset pricing facts (e.g. Gourio (2012)), the EIS must be greater than 1 for the model to deliver qualitatively realistic predictions. Campbell (2003) and Hall (1988) show that empirical estimates of this parameter are close to zero. We can reconcile our results with these estimates, as shown in Section 4.5.

4.2 Size Distribution and Probability of Disasters

The distribution of productivity declines in disasters is taken directly from the data on GDP declines. In our model, the percentage decline in consumption and output in a disaster realization are equal. In line with the characteristics of a production economy, we use the data on GDP declines to characterize the distribution of the disaster term in productivity because employment level is not impacted by a disaster realization in the model. Barro and Ursua (2008) construct historical data on consumption per capita and GDP per capita from 42 countries (24 for consumption, 36 for GDP) from 1870 to 2006. Following Barro and Ursua (2008), we characterize a disaster by a 10% or higher decline in GDP. Figure 6 shows the distribution of GDP declines in a disaster, which corresponds to $1 - e^{-\xi}$ in the model.

We approximate the dynamics of monthly disaster probability $\lambda$ in (17) using a 12-state Markov

\footnote{Hagedorn and Manovskii (2008) find a constant and a pro-cyclical component in vacancy costs. We specify vacancy costs proportional to productivity for simplicity.}
process. The nodes, along with their stationary probabilities, are presented in Table 2. The stationary distribution of probabilities approximates a lognormal distribution with mean 0.20% and standard deviation 1.97%. Mean disaster probability implies a lower frequency of disasters compared to the historical annual value of 3.69% calculated by Barro and Ursua (2008) and used by Gourio (2012) and Wachter (2013). The stationary distribution of disaster probability is highly skewed. We choose the persistence and the volatility of the disaster probability process to match the autocorrelation and volatility of unemployment in postwar U.S. data and in model population.

Table 3 describes the properties of disaster probability. The fraction of simulations, including no disaster realization among 10,000 simulations with length 60 years, is 53%. The mean of average disaster probability across no-disaster samples is 0.05%, which roughly corresponds to an annual value of 0.60%. This value is 0.20% across all simulations and in population. Medians are below mean due to the skewness of the distribution. The distribution of disaster probability in samples with no disasters has a much lower mean and standard deviation compared to population values. This implies that our specification of the disaster probability process is conservative because we evaluate the model’s performance in explaining postwar U.S. data using paths with no disaster realizations. While we calibrate the disaster probability process to unemployment, the model’s performance in explaining the dynamics of vacancies as well as business cycle and financial moments confirms the accuracy of our mechanism.

4.3 What Happens when Disaster Probability Increases?

Figure 7 plots the response of macroeconomic variables to an increase in monthly disaster probability from 0.05% to 0.32%. This is close to a two standard deviation increase in a typical no-disaster path. Productivity does not change on impact because exogenous shocks in the model are independent. Following the increase in disaster risk, the optimal level of employment in the economy decreases. Therefore, the firm substantially lowers investment in hiring, namely, the number of vacancies. Employment falls until it reaches the optimal employment level. The num-

23Model values in the discussion of quantitative results refer to results from no-disaster simulations unless stated otherwise.
ber of vacancies per period converges to a value below the pre-shock period because the matching function is increasing both in unemployment and vacancies. Because unemployment is higher, a lower vacancy rate can maintain the optimal employment level. Within 20 months, employment falls by 6% and the price-productivity ratio and vacancy rate fall by 25%. The fluctuations in disaster probability generate high volatility in vacancy and unemployment rates as well as asset prices while keeping consumption volatility at reasonable levels.

Following the disaster probability shock, consumption rises slightly on impact, and falls below the pre-shock value after six months. The initial increase in consumption is a result of the high EIS, combined with the desire to lower investment in hiring. Although consumption rises on impact, high disaster probability states are still high marginal utility states as can be seen calculating the stochastic discount factor.

As Figure 8 shows, an increase in the disaster probability raises the equity premium and lowers the government bill rate.

### 4.4 Labor Market Moments

Table 4 describes labor market moments in the model and in the U.S. data from 1951 to 2013. In Panel A, U.S. data on unemployment $U$, vacancies $V$, vacancy-unemployment ratio $V/U$, labor productivity $Z$ and price-productivity ratio $P/Z$ are reported. We replicate the statistics reported in Shimer (2005) using more recent data. Log deviations of $V/U$ from a slow-moving trend have a quarterly volatility of 39%, twenty times higher than the volatility of labor productivity of 2%. The correlation between $Z$ and $V/U$ is 10%, whereas the correlation between $V/U$ and $P/Z$ is 47%, consistent with the analysis in Section 2.\footnote{This correlation is 22% in the subsample until 1985 and 72% in the subsample from 1986 to 2013.} This finding together with the more detailed analysis in Section 2 motivates the mechanism in this paper. Labor productivity is very smooth as in Shimer’s critique of the DMP model. In addition, the low correlation of labor productivity with labor market variables raises the question whether productivity should be the driving force of unemployment and vacancies while most studies following Shimer’s critique propose models where
labor market tightness is perfectly correlated with labor productivity. The analysis of recent data, specifically since 1980s, strengthens the argument that realized productivity seems less related to labor market outcomes while the price of the U.S. stock market normalized by productivity is closely aligned with the time series of unemployment and vacancies.

Panel B of Table 4 reports the results from no-disaster simulations. Our model generates a volatility of 33% in $V/U$. The correlation between $V$ and $U$ is -0.68 compared to the data value of -0.86. This shows that time-varying risk in productivity is a successful mechanism in explaining the volatility of the labor market. Labor market tightness and price-productivity ratio have a correlation of 0.99. Labor market models that operate through productivity shocks imply a perfect correlation between $V/U$ and $Z$, which is much lower in the data compared to the correlation between $V/U$ and $P/Z$. Note that this level of correlation between the labor market and the stock market arises because both are driven by a single state variable. However, the data analysis shows that this united mechanism of the stock market and the labor market is a better description of the data compared to models based on realized productivity, especially for the U.S. data from mid-1980s to today.

Figure 9 shows the Beveridge curve in the data and in the model. While the model values are concentrated along a downward sloping line, a wide range of values can be generated by the model, including data values at the lower right corner of the Beveridge curve observed during the Great Recession.

Figure 5 plots the time series of the vacancy-unemployment ratio in the data and implied by the model. We calculate model-implied vacancy openings by plugging in data on the unemployment rate and the price-productivity ratio in (6) and calculating the implied vacancy rate. The plot shows that the mechanism of our model, which links stock market valuation to the labor market, rests on solid ground. The model performs particularly well in the subsample from 1986 to 2013, as suggested in Section 2. The pattern and magnitude of model-implied values in Figure 5 confirms that our labor market parameter values are reasonable.

\footnote{The type of shock is crucial to generating the observed negative correlation between $V$ and $U$. E.g., Shimer (2005) shows that separation rate shocks generate a counterfactual positive correlation between $V$ and $U$.}
Table 5 describes the dynamics of wages in the data and in the model. These results should be viewed as a channel that disciplines the calibration of the parameters that determine the dynamics of wages. Specifically, the parameters $B$, $\kappa$ and $\nu$ jointly determine the behavior of wages. While we calibrate $B$ and $\kappa$ using conventional values in the literature, the value of $\nu$ is chosen to match the volatility of aggregate wages as well as the responsiveness of wages to fluctuations in labor market tightness. Following Hagedorn and Manovskii (2008), we calculate wages by multiplying the labor share by productivity. The wage specification in (14) follows Hall (2014) who suggests a tightness-insulated version of Nash-bargained wages that can result from the alternating offer model of Hall and Milgrom (2008). As Table 5 shows, the volatility and autocorrelation of log deviations of wages from trend in quarterly data are 1.77% and 0.91, respectively. Our model implies a volatility of 1.71% which is close to the data value, and we match the autocorrelation of wages exactly. Elasticity of wages to labor market tightness is zero in the data from 1951 to 2013 while it is 0.01 and statistically different from zero in the subsample until 1985. Our model implies a value of 0.01 while not being significantly different from zero. Elasticity of wages to labor productivity is 0.67 in the entire sample and more than one in the subsample after 1985. Our model implies an almost unit elasticity of wages to productivity because wages are equal to productivity multiplied by parameters and variables that do not depend on productivity. However, note that the influence of productivity on wages does not affect labor market variables. In the model solution, labor market variables can be calculated without knowledge of productivity realizations because investment in hiring is purely calculated by taking conditional expectations, which depend on disaster probability and current employment. Productivity growth is iid up to disaster risk and does not contribute to time-variation in labor investment. Finally, the elasticity of labor market tightness to productivity is zero with a wide range of possible realizations from -6.39 at the 5% quantile and 6.08 at the 95% quantile. This is consistent with the data value, which is not significantly different from zero in the whole sample while being significantly positive in the earlier subsample and significantly negative in the later subsample.

Panel C of Table 5 describes dynamics of the Nash-bargained wage without tightness insulation, namely $\nu = 1$. The volatility of wages is 2.26%, higher than in the data. Furthermore, the elasticity
of wages to labor market tightness is 0.13 and significantly different from zero, which contradicts the value in the data. This high elasticity sabotages the impact of aggregate shocks on labor market volatility. When disaster probability increases, labor market tightness decreases only moderately because wages strongly adjust downward which leads to a weak impact on job creation incentives.

4.5 Business Cycle and Financial Moments

Table 6 describes consumption, GDP, equity return and government bill rate moments in the data and in the model. Cyclicality of variables has two independent dimensions in the model, namely, comovement with labor productivity and with disaster risk. In the model, the effect of productivity shocks on consumption and output growth is identical. Consumption equals output by the firm, plus home production minus investment in hiring. Because both output and investment are pro-cyclical, the consumption response to disaster probability shocks is weaker than the output response, as shown in Figure 7. This creates a higher volatility in output growth compared to consumption growth, in line with the data. The model-implied volatility of consumption growth and output growth in a typical no-disaster path is 1.41% and 2.47%, respectively.

Table 6 reports a historical equity premium of 5.32% and a return volatility of 12.26%. These returns are calculated by adjusting net market returns for financial leverage, whereas the unadjusted values are 7.90% and 17.55%, respectively.26 Although our model does not incorporate financial leverage, the equity premium in no-disaster paths is 6.66% and return volatility is 19.78%. A general issue of asset pricing in production economies is the low riskiness of cash flows, making it difficult to generate the high volatility of equity returns and the high equity premium in the data. Financial leverage makes returns more volatile due to higher dividend payout volatility, but the additional volatility in dividends can be counterfactually high (Jermann (1998)). Our model readily incorporates two sources of uncertainty. First, the presence and time-variation in disaster risk renders productivity risky. Second, the search friction in the labor market together with realistic wage dynamics can be interpreted as operating leverage, making firm profits risky because labor

26Lemmon, Roberts, and Zender (2008) report an average market leverage ratio of 28% among U.S. firms from 1965 to 2003. Accordingly, the unlevered equity premium is calculated multiplying stock returns by 0.72.
costs respond only weakly to labor market conditions and because adjusting employment level is costly. The annual volatility of dividend payout divided by GDP is 10.32% in the model while the data value is 8.89%. As a result of these risk channels that make the stochastic discount factor volatile, the model is able generate high return volatility while keeping dividend volatility at a reasonable level.

Another reason for the model’s performance in matching the return volatility without financial leverage is that it is designed to match a very volatile type of investment. Equity return is equal to the return on investment in hiring. The annual volatility of log investment growth in hiring in the data is 23.15% and the disaster probability process is calibrated to match the high volatility of unemployment and vacancies as discussed in Section 4.1. Compared to the 7.46% that Jermann (2010) reports as the volatility of investment growth in physical capital in the postwar period, investment growth in hiring is more volatile, which translates to the realistic higher return volatility in the model compared to models focusing on physical capital such as Gourio (2012).

Following Barro (2006), we set the probability of government default conditional on disaster to 0.4. The mean government bill rate is increasing and volatility of government bill rate is decreasing in this value. The expected return and volatility of the government bill is 3.64% and 3.83%, respectively, in the model. This is significantly higher than the 1-month Treasury bill rate of 1.01% with 2.22% volatility in the data due to the specification of the disaster probability process. The government bill rate is decreasing in disaster probability and risk aversion. Our model implies a lower disaster probability and uses higher risk aversion compared to the disaster risk literature, as discussed in Section 4.2. The effect of lower disaster probability dominates the risk aversion effect, leading to a high government bill rate. Moreover, the volatility of the government bill rate is determined by the volatility of disaster probability, which is higher in the present paper compared to the disaster risk literature.

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27 Dividend payout $D_t$ can be negative in the model. Therefore, we cannot calculate dividend growth. We follow Jermann and Quadrini (2012) and calculate equity payout net of share repurchases and proprietor’s net investment of financial business.

28 We calculate investment in hiring as the product of labor productivity and vacancy openings consistent with the model.
While the theoretical value of EIS is 2 in our calibration, empirical estimates of this parameter are close to zero (Hall (1988), Campbell (2003)). Empirical estimation is conducted by an instrumental variables regression of consumption growth on asset returns. Bansal and Yaron (2004) argue that these estimates assume that consumption growth and returns are \textit{iid} and are therefore biased in an economy with time-varying uncertainty. Following Campbell (2003), we regress consumption growth on government bill rate in the model using twice lagged consumption growth, government bill rate and log price-productivity ratio as instruments. Among 10,000 paths, the mean estimate is 0.15 with a standard error of 0.19. In our model, government bill rate and consumption growth are heteroskedastic. As a result, the regression approach does not identify the assumed value of 2 for EIS.

4.6 Sources of Volatility and Risk Premia

We compare three alternative specifications to our benchmark model to highlight the sources of volatility and risk premia: a model with constant disaster probability, where disaster probability is set to 0.20%, the stationary mean in the benchmark model; a model with no disaster risk; and a model with Nash-bargained wages, namely, $\nu = 1$. In all cases, the productivity process up to disasters remains unchanged.

Table 7 describes labor market volatility in alternative specifications. If risk is not time-varying, labor market variables and $P/Z$ are constant. This confirms that the only source of fluctuation in the labor market and stock market valuation is disaster probability. Without tightness insulation, the sensitivity of labor market variables to aggregate shocks decreases. Specifically, Nash-bargained wages result in an 11% volatility in $V/U$ whereas the benchmark model can generate a volatility of 33%. This finding is in line with Hagedorn and Manovskii (2008) and Hall (2014). Table 5 describes wage dynamics in the Nash bargaining case, as discussed in Section 4.4. Nash-bargained wages imply a high elasticity of wages to vacancy-unemployment ratio inconsistent with the data.

Table 8 describes business cycle and financial moments. In the absence of time-varying risk,
consumption growth and output growth have the same standard deviation. The only source of variation in these variables is the productivity shock because labor market variables are constant in these economies. The absence of tightness insulation renders consumption growth and output growth statistics close because labor market volatility, which is the source of differences between consumption and output dynamics, as discussed in Section 4.5, is dampened, as shown in Table 7. A higher proportion of fluctuation is driven by the common productivity component.

The equity premium is extremely low without the tightness insulation of wages. In this case, investment in the firm becomes very safe because the firm has a cost structure that is highly sensitive to cyclical conditions in the economy. In times of low disaster probability, employment increases and wages increase substantially due to the high sensitivity of wages to labor market tightness. In contrast, when employment falls, wages adjust rapidly downward. Therefore, disaster probability shocks have little effect on job creation incentives of the firm. This leads to low volatility and extremely low risk premia because returns become more countercyclical than the government bill rate.

Panel C of Table 8 describes a version of the model with tightness-insulated wages, but without disaster risk. The equity premium is close to zero, and the risk-free rate is 5.12%. The only source of return volatility is labor productivity. This version of the model with realistic wage dynamics but without disaster risk is similar to a baseline real business cycle model with a labor share lower than one.

The investigation of the constant disaster risk model described in Panel B of Table 8 highlights the role of time-varying risk in risk premia and volatility. The equity premium in population is 13.34% in the benchmark model and 9.94% in the model with constant disaster risk. While these values might seem to imply that 3/4 of the equity premium comes from the presence of disaster risk, the median equity premium in the absence of time-variation in disaster risk paths is 10.27%, which is higher than 6.66% in the benchmark model with time-varying disaster risk. The reason is the strong discrepancy in disaster probability characteristics between paths with and without disasters, as illustrated in Table 3. The constant disaster risk model is calibrated using the stationary mean (0.20%) of the benchmark disaster probability process. However, the benchmark
model implies a much lower mean for disaster probability (0.05%) in no-disaster simulations. Finally, the only source of government bill rate volatility in the constant disaster risk model is disaster realizations, which leads to a positive volatility in population but to a constant rate in no-disaster paths.

5 Conclusion

This paper shows that a business cycle model with search and matching frictions in the labor market and a small and time-varying risk of an economic disaster can simultaneously explain labor market volatility, stock market volatility and the relation between unemployment and stock market valuations. While tractable, the model can generate high volatility in labor market tightness along with realistic aggregate wage dynamics. The findings suggest that time variation in aggregate uncertainty offers an important channel, through which the DMP model of labor market search and matching can operate. The mechanism rests on the ground that equity returns of a firm that uses labor as the only input for production are equal to returns to a new hire. Therefore, job creation incentives of firms and stock market valuations are tightly linked as the comovement of labor market tightness and stock market valuations in the data suggest. While the presence of disaster risk and realistic wage dynamics generate a high unlevered equity premium, the source of labor market volatility and stock market volatility is time variation in risk. Finally, the model is consistent with basic business cycle moments such as consumption growth and output growth.
Appendix

A Aggregation

A.1 Homogeneity and the Stochastic Discount Factor

In this section, we show that the recursive utility function homogenous in consumption following Van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012). We illustrate that homogeneity of the utility function implies that all households that receive a constant fraction of aggregate consumption have the same SDF as the representative household.

We generalize the notation to $C_t = C(Z_t, N_t)$ and write the representative household’s utility function $J(\lambda_t, C(Z_t, N_t))$ as

$$J(\lambda_t, C(Z_t, N_t)) = \left[C(\lambda_t, N_t)^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t\left[J(\lambda_{t+1}, C(Z_{t+1}, N_{t+1}))^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\right]^{1-\frac{1}{\psi}}.$$  

We want to show that

$$J(\lambda_t, \omega_i C(Z_t, N_t)) = \omega_i J(\lambda_t, C(Z_t, N_t)),$$  

where $\omega_i > 0$. If we find a function $\tilde{J}$ that is homogenous in consumption and that satisfies (A.1), then $J$ needs to be homogenous by uniqueness of the solution to the Bellman equation. Suppose

$$\tilde{J}(\lambda_t, \omega_i C(Z_t, N_t)) = \omega_i \tilde{J}(\lambda_t, C(Z_t, N_t)),$$

then we have

$$\tilde{J}(\lambda_t, C(Z_t, N_t)) = \left[C(\lambda_t, N_t)^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t\left[J(\lambda_{t+1}, \omega_i C(Z_{t+1}, N_{t+1}))^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}\right]^{1-\frac{1}{\psi}},$$

where the term after $\frac{1}{\omega_i}$ corresponds to $\tilde{J}(\lambda_t, \omega_i C(Z_t, N_t))$. In other words, we can multiply consumption by $\omega_i$ and still satisfy the Bellman equation which implies (A.2). In the paper, we used
this to normalize the value function by productivity:

\[
J(\lambda_t, \omega_i C(Z_t, N_t)) = Z_t J(\lambda_t, \omega_i C(1, N_t)) = Z_t j(\lambda_t, N_t), \quad (A.4)
\]

using the linearity of consumption in productivity.

Overall, a household that consumes a share \( \omega_i \) of aggregate consumption has the same utility function as the representative household up to the scaling factor \( \omega_i \).

Next, we consider the problem in our paper and the SDF \( M_{t+1}^i \) of a household that receives share \( \omega_i \) of aggregate consumption in each period. We showed that, if \( C_t^i = \omega_i C_t \), then \( J_t^i = J(\lambda_t, \omega_i C(Z_t, N_t)) = \omega_i J(\lambda_t, C(Z_t, N_t)) = \omega_i J_t \). The form of the SDF in (8) reveals that \( \omega_i \) cancels out in both the consumption growth term and the value function term, which implies

\[
M_{t+1}^i = M_{t+1}. \quad (A.5)
\]

The SDF of a households that receives a constant share of aggregate consumption in each period is identical to the SDF of the representative household.

### A.2 Households

We consider three cases of household structures that add up to the representative household. In all cases, the productivity of each household is identical and equal to \( Z_t \).

The first case is similar to Merz (1995). Let there be \( N_h \) households with measure \( \omega_i \), where \( \sum_{i=1}^{N_h} \omega_i = 1 \). Further, assume that the fraction \( N_t \) of the members in each household are at work and each households owns the fraction \( \omega_i \) of the representative firm shares. In this case, the condition \( C_t^i = \omega_i C_t \) is satisfied. As shown in Section A.1, the SDF of each household is the same as the representative household and can be used to solve the firm’s optimization problem.

Second, we consider the case of employed and unemployed households where all households hold shares of the firm. Let \( \omega_i^E (\omega_i^U) \) denote the measure of the employed (unemployed) household \( i \) at time \( t \). The measure of households is the same as their ownership share at the representative firm. Note that the measure of each family does not change over time, but the employment status
of a family may change in the future. However, we are interested in a period-by-period aggregation. Let \( N^E_t \) (\( N^U_t \)) denote the number of employed (unemployed) households. Then we have

\[
\sum_{i=1}^{N^E_t} \omega^E_i = N_t, \quad \sum_{i=1}^{N^U_t} \omega^U_i = 1 - N_t.
\]

We assume that there is a perfect consumption insurance for households. This translates into an insurance payment from employed to unemployed households that compensates for the wage loss of unemployed households. Specifically, we have

\[
C^E_{t,i} = \omega^E_i D_t + \frac{\omega^E_i}{N_t} W_t N_t - \frac{\omega^E_i}{N_t} I_t
\]

\[
C^U_{t,i} = \omega^U_i D_t + \frac{\omega^U_i}{1 - N_t} b_t (1 - N_t) + \frac{\omega^U_i}{1 - N_t} I_t,
\]

where \( C^E_{t,i} \) and \( C^U_{t,i} \) denote the consumption of the employed and unemployed households with weights \( \omega^E_i \) and \( \omega^U_i \), respectively. \( I_t \) is the aggregate insurance payment. Both types of households receive a share of the aggregate dividend. Employed households also receive their wage share while unemployed households are equipped with the value of non-market activity. The insurance payment that facilitates perfect consumption insurance satisfies

\[
C^E_{t,i} = C^U_{t,i} = \omega_i C_t \quad \text{if} \quad \omega^E_i = \omega^U_i = \omega_i.
\]

The aggregate insurance payment that satisfies this condition is

\[
I_t = (1 - N_t) W_t N_t - (1 - N_t) b_t N_t,
\]

which corresponds to a compensation for lost wages due to unemployment net of the amount that employed households need to receive from unemployment benefits to equate consumption in employment and unemployment. If all households internalize the structure of insurance payments, household consumption is determined by aggregate quantities and all shareholders have the same SDF as the representative agent.

Finally, we consider the case where only employed hold shares of the firm. Assume that unemployed households have no access to the stock market and only employed households own
shares of the firm. In this case, perfect consumption insurance is achieved with an aggregate insurance payment that also includes compensation for dividend income. In this case, we have

\[
C_t^{E,i} = \frac{\omega_t^E}{N_t} D_t + \frac{\omega_t^E}{N_t} W_t N_t - \frac{\omega_t^E}{N_t} I_t
\]

\[
C_t^{U,i} = \frac{\omega_t^U}{1 - N_t} b_t (1 - N_t) + \frac{\omega_t^U}{1 - N_t} I_t.
\]

(A.10)

The insurance payment that satisfies (A.8) is

\[
I_t = (1 - N_t) D_t + (1 - N_t) W_t N_t - (1 - N_t) b_t N_t.
\]

(A.11)

**A.3 Firms**

Suppose the stock market is populated by \(N^F\) firms. Each firm employs the fraction \(f_j\) of the employed labor force, namely, \(N^j_t = f_j N_t\), where \(\sum_{j=1}^{N^F} f_j = 1\). If we can show that an individual firm’s dividend, stock price and vacancies are also proportional to the corresponding aggregate quantities, namely,

\[
D^j_t = f_j D_t, \quad P^j_t = f_j P_t, \quad V^j_t = f_j V_t,
\]

(A.12)

we can argue that the ownership structure of firms does not matter for aggregate quantities, and firm value is only a function of aggregate quantities up to the scaling factor \(f_j\). In other words, different households described in Section A.1 can own different shares at different firms. As long as the shares of a household \(i\) add up to the household’s weight \(\omega_i\), each firm uses the unique SDF \(M_{t+1}\) to solve for optimal hiring and firms add up to the representative firm in the paper.

We assume that each firm solves the optimization problem taking the aggregate vacancy filling rate \(q(N_t, V_t)\) and wages \(W_t\) as given. In other words, all firms face identical labor market conditions. The problem of the firm is

\[
P_t^{i,c} = \max_{\{V^j_t, N^j_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} \left[ Z_{t+\tau} N^j_{t+\tau} - W_t N^j_{t+\tau} - \kappa_t V^j_t \right]
\]

(A.13)

subject to

\[
N^j_{t+1} = (1 - s) N^j_t + q(N_t, V_t) V^j_t.
\]

(A.14)
The first-order conditions w.r.t. vacancies and employment are

\[ 0 = -1 + l_t^j \frac{q(N_t, V_t)}{\kappa_t} \]

\[ l_t^j = \mathbb{E}_t \left[ M_{t+1}(Z_{t+1} - W_{t+1} + l_{t+1}^j(1 - s)) \right], \]

where \( l_t^j \) denotes the Lagrange multiplier on (A.14). Note that \( l_t^j \) is a function of aggregate quantities \( \kappa_t \) and \( q(N_t, V_t) \) only, and therefore identical for all firms. Using the recursive substitution of the first-order conditions in (C.2) as in the paper, the cum-dividend value of firm \( j \) can be written as

\[ P_{t,c}^j = Z_t N_t^j - W_t N_t^j + l_t^j (1 - s) N_t^j. \] (A.16)

Because we started with \( N_t^j = f_j N_t \), we have \( P_{t,c}^j = f_j P_t^c \). Furthermore, the law of motion for labor implies \( V_t^j = f_j V_t \), and \( D_t^j = f_j D_t \). As shown in the paper, (A.16) implies

\[ P_t^j = \frac{\kappa_t}{q(N_t, V_t)} N_{t+1}^j, \] (A.17)

which establishes \( P_t^j = f_j P_t \). The stock price of each firm is proportional to the fraction of the employed labor force it employs.

**B  The Nash Wage Bargain**

The following proof is adapted from Petrosky-Nadeau, Zhang, and Kuehn (2013) and shows that the canonical Nash-bargained wage formula of the DMP model holds in our setting. Let \( S_t \) denote the joint surplus from a match in terms of marginal benefits for the household and the firm:

\[ S_t = \frac{J_{N,t} - J_{U,t}}{J_{C,t}} + P_{N_t}^c - P_{V,t}^c, \] (B.1)

where subscripts denote partial derivatives and time and \( P_t^c \) is the cum-dividend value of the firm. We divide \( J_{N,t} - J_{U,t} \) by \( J_{C,t} \) to make the household and firm benefits in units of the consumption good. The bargaining power \( B \) of the household implies

\[ BS_t = \frac{J_{N,t} - J_{U,t}}{J_{C,t}}. \] (B.2)

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The household faces the following resource constraint:

\[ C_t = D_t + W_t N_t + b_t U_t - \kappa_t V_t, \] (B.3)

along with the following employment and unemployment dynamics

\[ N_{t+1} = (1 - s) N_t + f(\theta_t) U_t \] \hspace{1cm} (B.4)
\[ U_{t+1} = s N_t + (1 - f(\theta_t)) U_t. \]

Taking the partial derivative of \( J_t \) with respect to \( C_t \) and \( N_t \) we have

\[ J_{C,t} = C_t^{-\frac{1}{\psi}} J_t^{\frac{1}{\psi}} \]
\[ J_{N,t} = W_t J_{C,t} + \beta \left( \mathbb{E}_t \left[ J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\gamma}{\gamma}} \mathbb{E}_t \left[ (1 - \gamma) J_{t+1}^{-\gamma} ((1 - s) J_{N,t+1} + s J_{U,t+1}) \right], \] (B.5)

which results in the following marginal benefit of an employed member for the household

\[ \frac{J_{N,t}}{J_{C,t}} = W_t + \mathbb{E}_t \left[ M_{t+1} \left( (1 - s) \frac{J_{N,t+1}}{J_{C,t+1}} + s \frac{J_{U,t+1}}{J_{C,t+1}} \right) \right], \] (B.6)

Using the same procedure for the unemployment we get

\[ \frac{J_{U,t}}{J_{C,t}} = b_t + \mathbb{E}_t \left[ M_{t+1} \left( f(\theta_t) \frac{J_{N,t+1}}{J_{C,t+1}} + (1 - f(\theta_t)) \frac{J_{U,t+1}}{J_{C,t+1}} \right) \right]. \] (B.7)

Next we consider the firm’s value function with optimal policy \( V_t \):

\[ P^c_t = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^\infty} \mathbb{E}_t \left[ \sum_{\tau=0}^\infty M_{t+\tau} \left( (Z_{t+\tau} - W_{t+\tau}) N_{t+\tau} - \kappa_{t+\tau} V_{t+\tau} \right) \right] \] (B.8)

subject to

\[ N_{t+1} = (1 - s) N_t + q(\theta_t) V_t. \] (B.9)

Taking the partial derivative with respect to \( V_t \) we get

\[ P^c_{V,t} = -\kappa_t + \mathbb{E}_t \left[ M_{t+1} (Z_{t+1} - w_{t+1}) q(\theta_t) \right] \]
\[ = -\kappa_t + q(\theta_t) \mathbb{E}_t \left[ M_{t+1} F_{N,t+1} \right]. \] (B.10)

Due to free entry of firms into the labor market, firms open vacancies until the marginal benefit from vacancy openings is zero which implies \( P^c_{V,t} = 0 \). This term disappears in the surplus equation.
Furthermore it implies
\[ \frac{\kappa_t}{q(\theta_t)} = \mathbb{E}_t [M_{t+1} P_{N,t+1}^c]. \]  
\[ \text{(B.11)} \]

The marginal benefit of employment to the firm is
\[ P_{N,t}^c = Z_t - W_t + \mathbb{E}_t [M_{t+1}(Z_{t+1} - W_{t+1})(1 - s)] \]
\[ = Z_t - W_t + (1 - s)\mathbb{E}_t [M_{t+1} P_{N,t+1}^c]. \]  
\[ \text{(B.12)} \]

Using the marginal benefit equations above, we can write the total match surplus as
\[ S_t = \frac{J_{N,t} - J_{U,t}}{J_{C,t}} + P_{N,t}^e - P_{V,t}^e \]
\[ = W_t + \mathbb{E}_t \left[ M_{t+1} \left( 1 - s \frac{J_{N,t+1}}{J_{C,t+1}} + s \frac{J_{U,t+1}}{J_{C,t+1}} \right) \right] - b_t - \mathbb{E}_t \left[ M_{t+1} \left( f(\theta_t) \frac{J_{N,t+1}}{J_{C,t+1}} + (1 - f(\theta_t)) \frac{J_{U,t+1}}{J_{C,t+1}} \right) \right] + Z_t - W_t + (1 - s)\mathbb{E}_t [M_{t+1} P_{N,t+1}^c]. \]  
\[ \text{(B.13)} \]

Merging terms we have
\[ S_t = Z_t - b_t + (1 - s)\mathbb{E}_t \left[ M_{t+1} \left( \frac{J_{N,t+1} - J_{U,t+1}}{J_{C,t+1}} + P_{N,t+1}^c \right) \right] - f(\theta_t)\mathbb{E}_t \left[ M_{t+1} \frac{J_{N,t+1} - J_{U,t+1}}{J_{C,t+1}} \right]. \]  
\[ \text{(B.14)} \]

The surplus splitting rule implies
\[ S_t = Z_t - b_t + (1 - s)\mathbb{E}_t [M_{t+1} S_{t+1}] - f(\theta_t)B\mathbb{E}_t [M_{t+1} S_{t+1}]. \]  
\[ \text{(B.15)} \]

Recall the marginal value of employment to the firm is
\[ P_{N,t}^c = (1 - B)S_t. \]  
\[ \text{(B.16)} \]

Plugging (B.15) into (B.16) and using (B.12) we get the canonical wage equation from Nash bargaining
\[ W_t = (1 - B)b_t + B(Z_t + \kappa_t \theta_t), \]  
\[ \text{(B.17)} \]

which corresponds to the Nash-bargained wage in Pissarides (2000).
C Equity Return and Firm Value

The representative firm pays out as dividend what is left from output after subtracting wage costs and investment:

\[ D_t = Z_t N_t - W_t N_t - \kappa_t V_t. \] (C.1)

The firm takes wages and labor market tightness as given and solves

\[ P_t^c = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} \left[ Z_{t+\tau} N_{t+\tau} - W_{t+\tau} N_{t+\tau} - \kappa_{t+\tau} V_{t+\tau} \right] \] (C.2)

subject to

\[ N_{t+1} = (1 - s) N_t + q(\theta_t) V_t. \] (C.3)

The first order conditions with respect to \( V_{t+\tau} \) and \( N_{t+\tau+1} \) are given by

\[ 0 = -1 + l_t \frac{q(\theta_t)}{\kappa_t} \] (C.4)

\[ l_t = \mathbb{E}_t \left[ M_{t+1} (Z_{t+1} - W_{t+1} + l_{t+1} (1 - s)) \right], \] (C.5)

where \( l_t \) is the Lagrange multiplier on the aggregate law of motion for employment level. Combining the first order conditions we have \( \mathbb{E}_t [M_{t+1} R_{t+1}] = 1 \) where

\[ R_{t+1} = \frac{Z_{t+1} - W_{t+1} + (1 - s) \frac{\kappa_{t+1}}{q(\theta_{t+1})}}{\frac{\kappa_t}{q(\theta_t)}}. \] (C.6)

We can rewrite the equity return using \( \kappa_t = \kappa Z_t \) and \( b_t = b Z_t \):

\[ R_{t+1} = e^{\mu + \epsilon_{t+1} + d_{t+1} \kappa_{t+1}} \left[ \frac{1 - w_{t+1} + (1 - s) \frac{\kappa}{q(\theta_t)}}{\frac{\kappa}{q(\theta_t)}} \right]. \] (C.7)

Consider the ex-dividend value of equity \( P_t = P_t^c - D_t \). We can rewrite the value of the firm:

\[ P_t^c = P_t + Z_t N_t - W_t N_t - \kappa_t V_t \] (C.8)
Expanding (C.2) and recursively substituting the expressions obtained from the first-order conditions (C.4) and (C.5), we can use

\[
P^c_t = Z_t N_t - W_t N_t - \kappa_t V_t - l_t (N_{t+1} - (1-s)N_t) - \frac{q(\theta_t)}{\kappa_t} \kappa_t V_t
\]

\[
+ \mathbb{E}_t \left[ M_{t+1} \left[ Z_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1} - l_{t+1} \left( N_{t+2} - (1-s)N_{t+1} - \frac{q(\theta_{t+1})}{\kappa_{t+1}} \kappa_{t+1} V_{t+1} \right) \right] \right]
\]

+ \ldots

(C.9)

and verify that

\[
P^c_t = Z_t N_t - W_t N_t + l_t (1-s)N_t.
\]

(C.10)

Specifically the investment terms in the first line of (C.8) cancel out as a result of (C.4). Furthermore the term \( l_t N_{t+1} \) in the first line cancels out with next period’s zero-coupon equity value value up to the investment terms and \( l_{t+1} N_{t+2} \) and so on. Therefore, the ex-dividend value of the firm is given by

\[
P_t = Z_t N_t - W_t N_t + l_t (1-s)N_t - Z_t N_t + W_t N_t + \kappa_t V_t
\]

\[
= \kappa_t V_t + l_t (1-s)N_t
\]

\[
= \frac{\kappa_t}{q(\theta_t)} (N_{t+1} - (1-s)N_t) + \frac{\kappa_t}{q(\theta_t)} (1-s)N_t
\]

\[
= l_t N_{t+1}.
\]

(C.11)
Now we return to the basic definition of stock return and show that it is equivalent to the labor investment return:

\[
R_{t+1} = \frac{P_{t+1}^c}{P_t^c - D_t} = \frac{P_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t} = l_{t+1}N_{t+2} + Z_{t+1}N_{t+1} - w_{t+1}N_{t+1} - \kappa_{t+1}V_{t+1}
\]

\[
= l_{t+1}N_{t+1} + \frac{l_t}{\kappa_{t+1}V_{t+1}} N_{t+1} - W_{t+1} - \frac{\kappa_{t+1}V_{t+1}}{N_{t+1}}
\]

\[
= l_{t+1}1 - s + \frac{q(\theta_{t+1})}{\kappa_{t+1}V_{t+1}} N_{t+1} + Z_{t+1} - W_{t+1} - \frac{\kappa_{t+1}V_{t+1}}{N_{t+1}}
\]

\[
= Z_{t+1} - W_{t+1} + l_{t+1}(1 - s)
\]

\[
= \frac{Z_{t+1} - W_{t+1} + (1 - s)\frac{\kappa_{t+1}}{q(\theta_{t+1})}}{\frac{\kappa_{t}}{q(\theta_t)}}
\]

(C.12)

This shows the equivalence.

### D Static Results

This section shows that \(\hat{\beta}(\lambda)\) is decreasing in \(\lambda\) if and only if \(\psi > 1\). We can rewrite (25) as

\[
\hat{\beta}(\lambda) = \beta \left(1 - \lambda + \lambda \mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right]\right)^{\frac{1-\frac{\lambda}{\psi}}{1-\gamma}}.
\]

(D.1)

Note that \(\zeta\) takes only negative values and \(\mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right]\) is always positive. There are two cases we need to investigate: \(\gamma < 1\) and \(\gamma > 1\). Suppose first that \(\gamma < 1\). In this case, we have \((1 - \gamma)\zeta_{t+1} < 0\), and therefore, \(\mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right] < 1\). When \(\lambda\) goes up, \((1 - \lambda + \lambda \mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right]\)^{\frac{1}{1-\gamma}}\) goes down. As a result, \(\hat{\beta}(\lambda)\) goes down only if \(\psi > 1\). The second case is \(\gamma > 1\). In this case, \((1 - \gamma)\zeta_{t+1} > 0\) and \(\mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right] > 1\). When \(\lambda\) goes up, \(1 - \lambda + \lambda \mathbb{E} \left[e^{(1-\gamma)\zeta_{t+1}}\right]\) goes up as well. Because \(1 - \gamma < 0\), \(\hat{\beta}(\lambda)\) goes down only if \(\psi > 1\). The argument can be reversed since (25) is an equality.
E Equilibrium Solution

Let $x'$ denote the value of the variable $x$ in period $t+1$ and $x$ the value at $t$. We can rewrite the normalized value function of the household as

$$g(\lambda, N) = j(\lambda, N)^{1-\frac{1}{\psi}}. \quad (E.1)$$

The value function and policy functions are functions of the exogenous state variable $\lambda$ and the endogenous state variable $N$. The dynamics of the stochastic discount factor and returns are driven by four shocks: disaster probability $\lambda'$, normal times productivity shock $\epsilon'$, disaster indicator $d'$ and disaster size $\zeta'$. Let $E$ be the expectation operator over four shocks. In our numerical procedure, we solve for the consumption policy $c(\lambda, N)$ and the value function $g(\lambda, N)$. The market clearing condition allows us to compute the vacancy rate given the consumption policy.

The stochastic discount factor is characterized by

$$M(\lambda, N; \lambda', \epsilon', d', \zeta') = \beta e^{-\frac{\mu}{\psi} + \frac{1}{2}(1-\gamma)(\gamma-\frac{1}{\psi})\sigma^2 e^{-\gamma(\epsilon'+d')\zeta'}} \cdot \mathbb{E} \left[ e^{(1-\gamma)d'\zeta'} \frac{1-\gamma}{1-\gamma} \frac{c(\lambda', N')}{c(\lambda, N)} g(\lambda', N')^{\frac{1-\gamma}{1-\gamma}} \right]. \quad (E.2)$$

The equity return is given by

$$R(\lambda, N; \lambda', \epsilon', d', \zeta') = e^{\mu+\epsilon'+d'\zeta'} \left[ 1 - w(\lambda', N') + (1-s) \frac{\kappa}{q(\theta(\lambda', N'))} \right], \quad (E.3)$$

where

$$w(\lambda, N) = (1-B)b + B(1+\kappa((1-\nu)\bar{\theta} + \nu\theta(\lambda, N))) \quad (E.4)$$

and

$$\theta(\lambda, N) = \frac{N + b(1-N) - c(\lambda, N)}{\kappa(1-N)}, \quad (E.5)$$

which follows from (21).

The equilibrium conditions that $c(\lambda, N)$ and $g(\lambda, N)$ have to satisfy are

$$\mathbb{E} [M(\lambda, N; \lambda', \epsilon', d', \zeta')R(\lambda, N; \lambda', \epsilon', d', \zeta')] = 1 \quad (E.6)$$
\[ g(N, \lambda) = c(N, \lambda)^{1 - \frac{1}{\gamma}} + \beta e^{(1 - \frac{1}{\gamma})\mu + \frac{1}{2}(1 - \frac{1}{\gamma})(1 - \gamma)\sigma^2} \left( \mathbb{E} \left[ e^{(1 - \gamma)\zeta'} g(\lambda', N')^{\frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{\gamma}}} \right] \right)^{1 - \frac{1}{\gamma}}. \]  

(E.7)

We approximate the AR(1) process for log disaster probability by a 12-state Markov process and use the corresponding probability transition matrix to calculate expectations over \( \lambda' \). The expectations over \( \zeta' \) and \( e' \) can be taken directly since their distributions are iid.

We approximate the policy function and the value function by a polynomial of employment level \( N \) where the polynomial coefficients are estimated for each value of the disaster probability separately. We use \( n + 1 \) nodes for employment to conduct the approximation by an \( n' \)th order polynomial. As a result we have \( 24(n + 1) \) unknowns and equations resulting from the equilibrium conditions (E.6) and (E.7). We evaluate the equilibrium conditions at the nodes of the Chebyshev polynomial of order \( n \). Our quantitative results are not significantly different for polynomial approximations of order 3 or higher.

**F Data Sources**

We use data from 1951 to 2013 for all variables.

- \( Z \) is the seasonally adjusted quarterly real average output per person in the nonfarm business sector, constructed by the Bureau of Labor Statistics (BLS) from National Income and Product Accounts (NIPA) and the Current Employment Statistics (CES).


- \( P/Z \) is the price-productivity ratio scaled to have the same value as \( P/E \) in the first quarter of 1951.
• $U$ is the seasonally adjusted unemployment, constructed by the BLS from the Current Population Survey (CPS). Quarterly values are calculated averaging monthly data.

• $V$ is the help-wanted advertising index constructed by the Conference Board until June 2006. We use data on vacancy openings from Job Openings and Labor Turnover Survey (JOLTS) from 2000 to 2013. We extrapolate the help-advertising index until 2013 and observe that our extrapolation has a correlation of 0.96 in the period from 2000 to 2006 where both data sources are available. For data plots, we remove a downward sloping time trend in $\log V/U$. Quarterly values are calculated averaging monthly data.

• $W$ denotes wages measured as the product of labor productivity $Z$ and labor share from the BLS. Quarterly values are calculated averaging monthly data.

• $C$ is annual aggregate per capita consumption of nondurables and services from the Bureau of Economic Analysis (BEA). Real consumption growth is calculated using $C$ and inflation rate from the Center for Research in Security Prices (CRSP).

• $Y$ is annual real gross domestic product (GDP) per capita from the BEA.

• $R$ is the value weighted return market index return including distributions from CRSP. Real returns are calculated using inflation rate data from CRSP. Net returns are multiplied by 0.68 to adjust for financial leverage.

• $R_b$ is the 1-month Treasury bill rate from CRSP. Real rates are calculated using inflation rate data from CRSP.

• $\Delta c$ and $\Delta y$ denote log consumption and log output growth. Annual growth rates from monthly simulations that we compare to data values are calculated aggregating consumption and output levels over every year. Let $C_{t,h}$ denote the consumption level in year $t$ and month $h$. Annual log consumption growth in the model is calculated as

$$\Delta c_{t+1} = \log \left( \frac{\sum_{i=1}^{12} C_{t+1,i}}{\sum_{i=1}^{12} C_{t,i}} \right).$$ (F.1)
The same method is applied to output growth as well.
References


Figure 1: Valuation Ratios: 1951 - 2013

Notes: $P/Z$ denotes the price-productivity ratio defined as the real price of the S&P composite stock price index $P$ divided by labor productivity $Z$. $P/E$ is the cyclically adjusted price-earnings ratio of the S&P composite stock price index. $P/Z$ is scaled such that $P/Z$ and $P/E$ are equal in the first quarter of 1951.
Figure 2: Vacancy-Unemployment Ratio and Labor Productivity: 1951 - 2013

Notes: The solid line shows the vacancy-unemployment ratio, the dashed line labor productivity. Both variables are reported as log deviations from an HP trend with smoothing parameter $10^5$. Shaded periods are recessions defined by the National Bureau of Economic Research (NBER).
Notes: The solid line shows the vacancy-unemployment ratio, the dashed line the price-productivity ratio. Both variables are reported as log deviations from an HP trend with smoothing parameter $10^5$. Shaded periods are recessions defined by the National Bureau of Economic Research (NBER).
Notes: The solid line shows vacancies, the dashed line the price-productivity ratio. Both variables are reported as log deviations from an HP trend with smoothing parameter $10^5$. Shaded periods are recessions defined by the National Bureau of Economic Research (NBER).
Notes: The solid line and the dashed line show the vacancy-unemployment ratio in the data and in the model, respectively. Model implied vacancies are calculated by plugging observed price-productivity ratio and employment level into equation (6). All values are log deviations from an HP trend with smoothing parameter $10^5$. Shaded periods are recessions defined by the National Bureau of Economic Research (NBER).
Figure 6: Size Distribution of Disaster Realizations

Notes: Histogram shows the distribution of large declines in GDP per capita (in percentages). Data are from Barro and Ursua (2008). Values correspond to $1 - e^c$ in the model.
Notes: $Z$ denotes productivity, $C$ consumption, $N$ employment level, $V$ number of vacancies and $P/Z$ price-productivity ratio. In month zero, monthly disaster probability increases from 0.05% to 0.32% and stays at 0.32% in the remaining months.
Figure 8: Return Response to Increase in Disaster Probability

Notes: $R$ denotes the equity return, $R_b$ the government bill rate. Both are in monthly terms. In month zero, monthly disaster probability increases from 0.05% to 0.32% and stays there in the remaining months.
Notes: Data are quarterly from 1951 to 2013. Model implied curve is a quarterly sample with length 10,000 years from the stationary distribution. All values are log deviations from an HP trend with smoothing parameter $10^5$. 
Table 1: Parameters Values for Monthly Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time preference, $\beta$</td>
<td>0.997</td>
</tr>
<tr>
<td>Risk aversion, $\gamma$</td>
<td>5.7</td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution, $\psi$</td>
<td>2</td>
</tr>
<tr>
<td>Disaster distribution (GDP), $\zeta$</td>
<td>multinomial</td>
</tr>
<tr>
<td>Productivity growth, $\mu$</td>
<td>0.0018</td>
</tr>
<tr>
<td>Productivity volatility, $\sigma_{\epsilon}$</td>
<td>0.0047</td>
</tr>
<tr>
<td>Matching efficiency, $\xi$</td>
<td>0.365</td>
</tr>
<tr>
<td>Separation rate, $s$</td>
<td>0.035</td>
</tr>
<tr>
<td>Matching function parameter, $\eta$</td>
<td>0.35</td>
</tr>
<tr>
<td>Bargaining power, $B$</td>
<td>0.50</td>
</tr>
<tr>
<td>Value of non-market activity, $b$</td>
<td>0.76</td>
</tr>
<tr>
<td>Vacancy cost, $\kappa$</td>
<td>0.50</td>
</tr>
<tr>
<td>Tightness insulation, $\nu$</td>
<td>0.05</td>
</tr>
<tr>
<td>Government default probability, $q$</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Table 2: Monthly Disaster Probability

<table>
<thead>
<tr>
<th>Value</th>
<th>Stationary Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{-7}$</td>
<td>0.0005</td>
</tr>
<tr>
<td>$7 \times 10^{-7}$</td>
<td>0.0054</td>
</tr>
<tr>
<td>$4 \times 10^{-6}$</td>
<td>0.0269</td>
</tr>
<tr>
<td>$3 \times 10^{-5}$</td>
<td>0.0806</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.1611</td>
</tr>
<tr>
<td>0.0012</td>
<td>0.2256</td>
</tr>
<tr>
<td>0.0076</td>
<td>0.2256</td>
</tr>
<tr>
<td>0.0495</td>
<td>0.1611</td>
</tr>
<tr>
<td>0.3212</td>
<td>0.0806</td>
</tr>
<tr>
<td>2.0827</td>
<td>0.0269</td>
</tr>
<tr>
<td>13.5045</td>
<td>0.0054</td>
</tr>
<tr>
<td>87.5661</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Notes: Table lists the nodes of a 12-state Markov process which approximates an AR(1) process for log probabilities. Disaster probabilities are in percentage terms.
Table 3: Monthly Disaster Probability in Simulations

<table>
<thead>
<tr>
<th></th>
<th>No-Disaster</th>
<th>All Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Population</td>
<td>Mean</td>
</tr>
<tr>
<td>$E[\lambda]$</td>
<td>0.20</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma(\lambda)$</td>
<td>1.97</td>
<td>0.20</td>
</tr>
<tr>
<td>$\rho(\lambda)$</td>
<td>0.91</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Notes: $\sigma$ denotes volatility, $\rho$ monthly autocorrelation. Disaster probabilities are in percentage terms. Population is a sample of 100,000 years. We simulate 10,000 samples with length 60 years at monthly frequency and report statistics from all simulations as well as from 53% of simulations that include no disaster realization. All simulations are in monthly frequency.
Table 4: Labor Market Moments

<table>
<thead>
<tr>
<th></th>
<th>$U$</th>
<th>$V$</th>
<th>$V/U$</th>
<th>$Z$</th>
<th>$P/Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>0.19</td>
<td>0.21</td>
<td>0.39</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>AC</td>
<td>0.94</td>
<td>0.94</td>
<td>0.95</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.86</td>
<td>-0.96</td>
<td>-0.18</td>
<td>-0.44</td>
</tr>
<tr>
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<td></td>
<td>1</td>
<td>0.97</td>
<td>0.03</td>
<td>0.47</td>
</tr>
<tr>
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<td>0.10</td>
<td>0.47</td>
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<td>0.00</td>
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<tr>
<td></td>
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<td></td>
<td>1</td>
</tr>
<tr>
<td>Panel B: No-Disaster Simulations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>0.17</td>
<td>0.19</td>
<td>0.33</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>AC</td>
<td>0.95</td>
<td>0.76</td>
<td>0.90</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
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<td>-0.90</td>
<td>-0.06</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.93</td>
<td>-0.06</td>
<td>0.90</td>
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<td></td>
<td>1</td>
<td>0.00</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>0.01</td>
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<tr>
<td></td>
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<td></td>
<td>1</td>
</tr>
<tr>
<td>Panel C: Population</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>0.19</td>
<td>0.22</td>
<td>0.39</td>
<td>0.04</td>
<td>0.17</td>
</tr>
<tr>
<td>AC</td>
<td>0.95</td>
<td>0.76</td>
<td>0.90</td>
<td>0.93</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
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<td>-0.69</td>
<td>-0.91</td>
<td>-0.06</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
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<td>1</td>
<td>0.93</td>
<td>-0.06</td>
<td>0.90</td>
</tr>
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<td></td>
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<td>1</td>
<td>0.00</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
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<td>1</td>
<td>0.01</td>
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<td></td>
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</tr>
</tbody>
</table>

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Data are from 1951 to 2013. All data and model moments are in quarterly terms. $U$ is unemployment, $V$ vacancies, $Z$ labor productivity and $P/Z$ productivity ratio. We simulate 10,000 samples with length 60 years at monthly frequency and report means from 53% of simulations that include no disaster realization in Panel B. Standard errors across simulations are reported in parentheses. Population values in Panel C are from a path with length 100,000 years at monthly frequency. Standard deviations, autocorrelations and the correlation matrix are calculated using log deviations from an HP trend with smoothing parameter $10^5$. 

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Table 5: Properties of Aggregate Wages

<table>
<thead>
<tr>
<th></th>
<th>SD</th>
<th>AC</th>
<th>$\epsilon_{W,\theta}$</th>
<th>$\epsilon_{W,Z}$</th>
<th>$\epsilon_{\theta,Z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1951 - 2013</td>
<td>1.77</td>
<td>0.91</td>
<td>0.00</td>
<td>0.67</td>
<td>2.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.33)</td>
<td>(5.43)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.76)</td>
<td></td>
</tr>
<tr>
<td>1951 - 1985</td>
<td>1.21</td>
<td>0.91</td>
<td>0.01</td>
<td>0.35</td>
<td>11.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(2.75)</td>
<td>(3.04)</td>
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<td></td>
<td>(3.86)</td>
<td></td>
</tr>
<tr>
<td>1986 - 2013</td>
<td>2.29</td>
<td>0.91</td>
<td>-0.01</td>
<td>1.07</td>
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<tr>
<td></td>
<td></td>
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<td>(-1.15)</td>
<td>(6.79)</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(-2.37)</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: Benchmark model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>1.71</td>
<td>0.91</td>
<td>0.01</td>
<td>0.99</td>
<td>0.00</td>
</tr>
<tr>
<td>5%</td>
<td>1.33</td>
<td>0.87</td>
<td>-0.01</td>
<td>0.95</td>
<td>-6.39</td>
</tr>
<tr>
<td>95%</td>
<td>2.31</td>
<td>0.95</td>
<td>0.03</td>
<td>1.05</td>
<td>6.08</td>
</tr>
<tr>
<td><strong>Panel C: No tightness insulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>2.26</td>
<td>0.89</td>
<td>0.13</td>
<td>1.00</td>
<td>0.04</td>
</tr>
<tr>
<td>5%</td>
<td>1.80</td>
<td>0.83</td>
<td>0.08</td>
<td>0.74</td>
<td>-1.95</td>
</tr>
<tr>
<td>95%</td>
<td>2.89</td>
<td>0.93</td>
<td>0.18</td>
<td>1.27</td>
<td>1.93</td>
</tr>
</tbody>
</table>

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Z is labor productivity, $\theta$ labor market tightness. Data are from 1951 to 2013. All data and model moments are in quarterly terms. We simulate 10,000 samples with length 60 years at monthly frequency and report quantiles from 53% of simulations that include no disaster realization. $\epsilon_{x,y}$ is the elasticity of variable $x$ to $y$, namely, the regression coefficient of log $x$ on log $y$. Data t-statistics in parentheses are based on Newey-West standard errors. All variables are used in logs as deviations from an HP trend with smoothing parameter $10^5$. 

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Table 6: Business Cycle and Financial Moments

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[^\Delta c]$</th>
<th>$\mathbb{E}[^\Delta y]$</th>
<th>$\sigma(^\Delta c)$</th>
<th>$\sigma(^\Delta y)$</th>
<th>$\mathbb{E}[R - R_b]$</th>
<th>$\mathbb{E}[R_b]$</th>
<th>$\sigma(R)$</th>
<th>$\sigma(R_b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.97</td>
<td>1.90</td>
<td>1.25</td>
<td>2.17</td>
<td>5.32</td>
<td>1.01</td>
<td>12.26</td>
<td>2.22</td>
</tr>
<tr>
<td>Simulation 50%</td>
<td>2.16</td>
<td>2.16</td>
<td>1.41</td>
<td>2.47</td>
<td>6.66</td>
<td>3.64</td>
<td>19.78</td>
<td>3.83</td>
</tr>
<tr>
<td>Simulation 5%</td>
<td>1.80</td>
<td>1.79</td>
<td>1.19</td>
<td>1.73</td>
<td>-0.02</td>
<td>0.06</td>
<td>11.75</td>
<td>0.87</td>
</tr>
<tr>
<td>Simulation 95%</td>
<td>2.51</td>
<td>2.54</td>
<td>1.67</td>
<td>3.70</td>
<td>20.39</td>
<td>4.96</td>
<td>33.94</td>
<td>12.50</td>
</tr>
<tr>
<td>Population</td>
<td>1.63</td>
<td>1.63</td>
<td>5.88</td>
<td>6.89</td>
<td>13.32</td>
<td>1.22</td>
<td>38.97</td>
<td>12.19</td>
</tr>
</tbody>
</table>

Notes: $^\Delta c$ denotes log consumption growth, $^\Delta y$ log output growth, $R$ the unlevered equity return, $R_b$ the government bill rate. All data and model moments are in annual terms. We simulate 10,000 samples with length 60 years at monthly frequency and report quantiles from 53% of simulations that include no disaster realization. Population values are from a path with length 100,000 years. Returns and growth rates are aggregated to annual values.
### Table 7: Labor Market Volatility in Comparative Statics

<table>
<thead>
<tr>
<th></th>
<th>$U$</th>
<th>$V$</th>
<th>$V/U$</th>
<th>$Z$</th>
<th>$P/Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.19</td>
<td>0.21</td>
<td>0.39</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.17</td>
<td>0.19</td>
<td>0.33</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>Constant $\lambda$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>No disaster</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>No tightness insulation</td>
<td>0.06</td>
<td>0.06</td>
<td>0.11</td>
<td>0.02</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Table reports only standard deviations. $U$ is unemployment, $V$ vacancies, $Z$ labor productivity and $P/Z$ price-productivity ratio. Data are from 1951 to 2013. All data and model moments are in quarterly terms. We simulate 10,000 samples with length 60 years at monthly frequency and report means from 53% (24%) of simulations that include no disaster realization for the benchmark (constant $\lambda$) model. In the constant disaster probability model, we set disaster probability to 0.20%, the stationary mean of the disaster probability process used in the benchmark model. Standard deviations are calculated using log deviations from an HP trend with smoothing parameter $10^5$. 


Table 8: Business Cycle and Financial Moments in Comparative Statics

<table>
<thead>
<tr>
<th></th>
<th>$\mathbb{E}[\Delta c]$</th>
<th>$\mathbb{E}[\Delta y]$</th>
<th>$\sigma(\Delta c)$</th>
<th>$\sigma(\Delta y)$</th>
<th>$\mathbb{E}[R - R_b]$</th>
<th>$\mathbb{E}[R_b]$</th>
<th>$\sigma(R)$</th>
<th>$\sigma(R_b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>1.97</td>
<td>1.90</td>
<td>1.25</td>
<td>2.17</td>
<td>5.32</td>
<td>1.01</td>
<td>12.26</td>
<td>2.22</td>
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<tr>
<td>Panel A: Benchmark</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>50%</td>
<td>2.16</td>
<td>2.16</td>
<td>1.41</td>
<td>2.47</td>
<td>6.66</td>
<td>3.64</td>
<td>19.78</td>
<td>3.83</td>
</tr>
<tr>
<td>Population</td>
<td>1.63</td>
<td>1.63</td>
<td>5.88</td>
<td>6.89</td>
<td>13.32</td>
<td>1.22</td>
<td>38.97</td>
<td>12.19</td>
</tr>
<tr>
<td>Panel B: Constant $\lambda$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>50%</td>
<td>2.16</td>
<td>2.16</td>
<td>1.31</td>
<td>1.31</td>
<td>10.27</td>
<td>-3.48</td>
<td>1.73</td>
<td>0.00</td>
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<td>1.59</td>
<td>4.03</td>
<td>4.03</td>
<td>9.94</td>
<td>-3.66</td>
<td>3.49</td>
<td>2.16</td>
</tr>
<tr>
<td>Panel C: No Disaster</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50%</td>
<td>2.16</td>
<td>2.16</td>
<td>1.33</td>
<td>1.33</td>
<td>0.16</td>
<td>5.12</td>
<td>1.70</td>
<td>0.00</td>
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<tr>
<td>Population</td>
<td>2.16</td>
<td>2.16</td>
<td>1.33</td>
<td>1.33</td>
<td>0.16</td>
<td>5.12</td>
<td>1.71</td>
<td>0.00</td>
</tr>
<tr>
<td>Panel D: No Tightness Insulation</td>
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<td></td>
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</tr>
<tr>
<td>50%</td>
<td>2.16</td>
<td>2.16</td>
<td>1.34</td>
<td>1.52</td>
<td>-49.63</td>
<td>3.67</td>
<td>11.55</td>
<td>3.32</td>
</tr>
<tr>
<td>Population</td>
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<td>1.68</td>
<td>6.29</td>
<td>6.44</td>
<td>-47.76</td>
<td>1.53</td>
<td>20.32</td>
<td>11.27</td>
</tr>
</tbody>
</table>

Notes: $\Delta c$ denotes log consumption growth, $\Delta y$ log output growth, $R$ the unlevered equity return, $R_b$ the government bill rate. All data and model moments are in annual terms. We simulate 10,000 samples with length 60 years at monthly frequency and report the 50% quantile from 53% (24%) of simulations that include no disaster realization for the benchmark (constant $\lambda$) model. In the constant disaster probability model, we set disaster probability to 0.20%, the stationary mean of the disaster probability process used in the benchmark model. Population values are from a path with length 100,000 years. Returns and growth rates are aggregated to annual values.