Risk, Unemployment, and the Stock Market: A Rare-Event-Based Explanation of Labor Market Volatility *

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Abstract

What is the driving force behind the cyclical behavior of unemployment and vacancies? What is the relation between job-creation incentives of firms and stock market valuations? We answer these questions in a model with time-varying risk, modeled as a small and variable probability of an economic disaster. A high probability implies greater risk and lower future growth, lowering the incentives of firms to invest in hiring. During periods of high risk, stock market valuations are low and unemployment rises. The model thus explains volatility in equity and labor markets, and the relation between the two.

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1 Introduction

The Diamond-Mortensen-Pissarides (DMP) model of search and matching offers an intriguing theory of labor market fluctuations based on the job creation incentives of employers (Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994)). When the contribution of a new hire to firm value decreases, employers reduce investment in hiring, decreasing the number of vacancies and, in turn, increasing unemployment. Due to the glut of jobseekers in the labor market, vacancies become easier for employers to fill. Therefore, unemployment stabilizes at a higher level and the number of vacancies at a lower level. That is, labor market tightness (defined as the ratio of vacancies to unemployment) decreases until the payoff to hiring changes again.

While the mechanism of the DMP model is intuitive, it fails to answer a fundamental question: what causes job-creation incentives, and hence unemployment, to vary? The canonical DMP model and numerous successor models suggest that the driving force is labor productivity. However, explaining labor market volatility based on productivity fluctuations is difficult, because unemployment and vacancies are much more volatile than labor productivity (Shimer (2005)). Furthermore, unemployment does not track the movements of labor productivity, as is particularly apparent in the last three recessions. Rather, these recent data suggest a link between unemployment and stock market valuations (Hall (2015)).

In this paper, we make use of the DMP mechanism to explain the cyclical behavior of unemployment. However, rather than linking labor market tightness to productivity itself, we propose an equilibrium model in which fluctuations in labor market tightness arise from a small and time-varying probability of an economic disaster. Even if current labor productivity remains constant, disaster fears lower the job-creation incentives of firms. The labor market equilibrium shifts to a lower point on the vacancy-unemployment locus (the Beveridge curve), with higher unemployment and lower vacancy openings. At the same time, stock market valuations decline.

Our model generates a high volatility in unemployment and vacancies, along with a strong negative correlation between the two. This pattern of results accurately describes post-war U.S. data. We calibrate wage dynamics to match the behavior of the labor share in the data and find that matching the observed low response of wages to labor market conditions is crucial for both labor market volatility and realistic behavior of financial markets. Furthermore, the search and matching friction in the labor market and time-varying disaster risk result in a realistic equity premium and stock return volatility. Because the labor market and the stock market are driven by the same force, the price of the aggregate stock market and labor market tightness are highly correlated, while the correlation between labor productivity and tightness is realistically low.

Our paper is related to three strands of literature. First, since Shimer (2005) showed that the DMP model with standard parameter values implies small movements in unemployment and vacancies, a strand of literature has further developed the model to generate large responses of unemployment to aggregate shocks. In these papers, the aggregate shock driving the labor market is labor productivity. Hagedorn and Manovskii (2008) argue that a calibration of the model combining low bargaining power of workers with a high opportunity cost of employment can reconcile unemployment volatility in the DMP model with the data. Other papers suggest alternatives to the Nash bargaining assumption for wages (Hall (2005), Hall and Milgrom (2008), Gertler and Trigari (2009)). Compared with Nash bargaining, these alternatives render wages less responsive to productivity shocks. Thus a productivity shock can have a larger effect on job-creation incentives. Our paper departs from these in that we do not rely on time-varying labor market productivity as a driver of labor market tightness, which leads to a counterfactually high correlation between these variables. Furthermore, we also derive implications for the stock market, and explain the equity premium and volatility puzzles.[1]

Second, the present work relates to ones that embed the DMP model into the real business cycle framework, with a representative risk averse household that makes investment and consumption decisions. In the standard real business cycle (RBC) model (Kydland and Prescott (1982)),

[1]Other recent work connects time-variation in discount rates to unemployment. Eckstein, Setty, and Weiss (2015) solve a DMP model with risk-neutral investors and exogenous discount rates where labor and capital are complements. They show that volatility in corporate discount rates can account for volatility in unemployment. Hall (2015) conjectures that a DMP model in which discount rates that rise in recessions can explain unemployment, and shows that, when an exogenous stochastic discount factor is estimated using the aggregate stock market, the resulting time series of unemployment tracks that in the data. Neither paper provides a general equilibrium model. Our results show that the connection between discount rates, recessions, and unemployment in a general equilibrium DMP model is more subtle than one might think (see Section 3.4).

employment is driven by the marginal rate of substitution between consumption and leisure, and, because the labor market is frictionless, no vacancies go unfilled. Merz (1995) and Andolfatto (1996) observe that this model has counterfactual predictions for the correlation of productivity and employment, and build models that incorporate RBC features and search frictions in the labor market. These models capture the lead-lag relation between employment and productivity while having more realistic implications for wages and unemployment compared to the baseline RBC model. In this paper, we also document the lead-lag relation between productivity and employment in the period that this literature analyzes (1959 - 1988). However, our empirical analysis shows that this lead-lag relation is absent in more recent data. These papers do not study asset pricing implications.

Third, our paper is related to the literature on asset prices in dynamic production economies. In these models, as in the RBC framework described above, consumption and dividend dynamics are endogenously determined by the optimal equilibrium policy of a representative firm. This contrasts with the more standard asset-pricing approach of assuming an endowment economy, in which consumption and dividends are taken as given. The main difficulty in production economies is endogenous consumption smoothing (Kaltenbrunner and Lochstoer (2010), Lettau and Uhlig (2000)). While higher risk aversion raises the equity premium in an endowment economy, this leads to even smoother consumption in production economies resulting in very little fluctuation in marginal utility. One way of overcoming this problem is to assume alternative preferences, for example, habit formation as in Boldrin, Christiano, and Fisher (2001) and Jermann (1998), though these can lead to highly volatile riskfree rates. Another approach is to allow for rare disasters. Barro (2006) and Rietz (1988) demonstrate that allowing for rare disasters in an endowment economy can explain the equity premium puzzle. Building on this work, Gourio (2012) studies the implications of time-varying disaster risk modeled as large drops in productivity and destruction of physical capital in a business cycle model with recursive preferences and capital adjustment costs. Gourio's model can explain the observed co-movement between investment and risk premia. However, unlevered equity returns have little volatility, and thus the premium on unlevered equity is low. This model can be reconciled with the observed equity premium by adding financial leverage, but the leverage ratio must be high in comparison with the data. Also, as in RBC models with frictionless labor markets, Gourio's model does not explain unemployment. Petrosky-Nadeau, Zhang, and Kuehn (2013) build a model where rare disasters arise endogenously through a series of negative productivity shocks. Like our paper, they make use of the DMP model, but with a very different aim and implementation. Their paper incorporates a calibration of Nash-bargained wages similar to Hagedorn and Manovskii (2008), leading to wages that are high and rigid. Moreover, their specification of marginal vacancy opening costs includes a fixed component, implying that it costs more to post a vacancy when labor conditions are slack and thus when output is low. Finally, they assume that workers separate from their jobs at a rate that is high compared with the data. The combination of a high separation rate, fixed marginal costs of vacancy openings and high and inelastic wages amplifies negative shocks to productivity and produces a negatively skewed output and consumption distribution. Like other DMP-based models described above, their model implies that labor market tightness is driven by productivity. Furthermore, while their model can match the equity premium, the fact that their simulations contain consumption disasters make it unclear whether the model can match the high stock market volatility and low consumption volatility that characterize the U.S. postwar data.

The paper is organized as follows. Section 2 provides empirical evidence about the relation between the labor market, labor productivity and the stock market. Section 3 presents the model and illustrates the mechanism in a simplified version. Section 4 discusses the quantitative results from the benchmark calibration and alternative calibrations. Section 5 concludes.

2 Labor Market, Labor Productivity and Stock Market Valuations

In the literature succeeding the canonical DMP model, labor productivity serves as the driving force behind volatility in unemployment and vacancies. Recent empirical work, however, has challenged this approach on the grounds that labor productivity is too stable compared with unemployment and vacancies, and that the variables are at best weakly correlated. In this section we summarize evidence on the interplay between unemployment, productivity and the stock market.

In Figure 1, we plot the time series of labor productivity Z and of the vacancy-unemployment ratio V/U, the variable that summarizes the behavior of the labor market in the DMP model.[2] Both variables are shown as log deviations from an HP trend.[3] Figure 1 shows the disconnect between the volatility of V/U and of productivity: labor productivity Z never deviates by more than 5 percent from trend, while, in contrast, V/U is highly volatile and deviates up to a full log point from trend. The lack of volatility in productivity as compared with labor market tightness is one challenge facing models that seek to explain unemployment using fluctuations in productivity.

Another challenge arises from the co-movement in these variables. Figure 1 shows that tightness and productivity did track each other in the recessions of the early 1960s and 1980s. However, this contemporaneous correlation disappears in the later part of the sample. A striking example of this disconnect is the aftermath of the Great Recession, which simultaneously features a small productivity boom along with a labor-market collapse. Overall, the contemporaneous correlation between the variables is 0.10 as measured over the full sample, 0.47 until 1985 and -0.36 afterwards. There is some evidence that Z leads V/U; the maximum correlation between V/U and lagged Z occurs with a lag length of one year. However, this relation also does not persist in the second subsample; while the correlation over the full sample is 0.31, it is 0.62 in the subsample before 1985 and -0.09 after 1985.

While the data display little relation between unemployment and productivity, there is a relation between unemployment and the stock market.[4] We will focus on the ratio of stock market valuation P to labor productivity (output per person in the non-farm business sector) Z because P/Z has a clean counterpart in our model.[5] Figure 3 shows a consistently positive correlation

[2] All variables are measured in real terms. See Appendix D for a description of the data.

[3] Following Shimer (2005) we use a low-frequency HP filter with smoothing parameter 10^5 throughout to capture business cycle fluctuations. All results are robust to using an HP filter with smoothing parameter 1,600.

[4] Our study focuses on the time-series relation between hiring and the stock market. Belo, Lin, and Bazdresch (2014) demonstrates a cross-sectional relation between required rates of return and hiring: firms that hire more appear to have lower risk premia. The same mechanism that we employ to explain the time series patterns can also account for this evidence.

[5] P/Z closely tracks Robert Shiller's cyclically adjusted price-earnings ratio (P/E), as shown in Figure 2. The correlation between the quarterly observations of these series is 0.97 for the period from 1951 to 2013.

between labor market tightness V/U and valuation P/Z. The correlation over the full sample is 0.47. In the period from 1986 to 2013, the correlation is 0.71. Moreover, like V/U, P/Z is volatile, with deviations up to 0.5 log points below trend. Figure 4 shows that vacancies V follow a similar pattern to V/U.

Why might labor markets be tightly connected with stock market valuations, but not with current productivity? In the sections that follow, we offer a model to answer this question.

3 Model

In Section 3.1 we review the DMP model of the labor market with search frictions. In Section 3.2, we use the DMP model with minimal additional assumptions to demonstrate a link between equity market valuations and labor market quantities. We confirm that this link holds in the data. In Section 3.3 we present a general equilibrium model that explains labor market and stock market volatility in terms of time-varying disaster risk (we will examine the quantitative implications of this model in Section 4). In Section 3.4 we give closed-form solutions in a special case of the model in which disaster risk is a constant. This special case gives intuition for how disaster risk affects labor market quantities and prices in financial markets.

3.1 Search frictions

The labor market is characterized by the DMP model of search and matching. The representative firm posts a number of job vacancies $V_t \geq 0$. The hiring flow is determined according to the matching function $m(N_t, V_t)$, where N_t is employment in the economy and lies between 0 and 1. We assume that the matching function takes the following Cobb-Douglas form:

$$m(N_t, V_t) = \xi(1 - N_t)^\eta V_t^{1-\eta}, \tag{1}$$

where ξ is matching efficiency and η is the unemployment elasticity of the hiring flow. As a result, the aggregate law of motion for employment is given by

$$N_{t+1} = (1-s)N_t + m(N_t, V_t), \tag{2}$$

where s is the separation rate.[6] Define labor market tightness as follows:

$$\theta_t = \frac{V_t}{U_t}.$$

The unemployment rate in the economy is given by $U_t = 1 - N_t$. Thus the probability of finding a job for an unemployed worker is $m(N_t, V_t)/U_t = \xi\theta_t^{1-\eta}$. Accordingly, we define the job-finding rate $f(\theta_t)$ to be

$$f(\theta_t) = \xi\theta_t^{1-\eta}. \tag{3}$$

Analogously, the probability of filling a vacancy posted by the representative firm is $m(N_t, V_t)/V_t = \xi\theta_t^{-\eta}$ which corresponds to the vacancy-filling rate $q(\theta_t)$ in the economy:

$$q(\theta_t) = \xi\theta_t^{-\eta}. \tag{4}$$

The functional form of f and q provide useful insights about the mechanism of the DMP model. The job-finding rate is increasing, and the vacancy-filling rate is decreasing in the vacancy-unemployment ratio. In times of high labor market tightness, namely, when the vacancy rate is high and/or the unemployment rate is low, the probability of finding a job per unit time increases, whereas filling a vacancy takes more time.

Finally, the representative firm incurs costs κ_t per vacancy opening. As a result, aggregate investment in hiring is $\kappa_t V_t$.

3.2 Equity Valuation and the Labor Market

In this section we consider a partial-equilibrium model of stock market valuation, using the framework discussed in Section 3.1 but with minimal additional assumptions. We show that a link

[6] The assumption of $V_t > 0$ implies that the maximum drop in employment level is s.

between the stock market and the labor market prevails under these very general conditions.

Let M_{t+1} denote the representative household's stochastic discount factor. Consider a representative firm which produces output given by

$$Y_t = Z_t N_t, \tag{5}$$

where Z_t is the non-negative level of aggregate labor productivity. Assume that labor productivity follows the process

$$\log Z_{t+1} = \log Z_t + \mu + x_{t+1}, \tag{6}$$

where, for now, we leave x_{t+1} unspecified; it can be any stationary process. Let $W_t = W(Z_t, N_t, V_t)$ denote the aggregate wage rate. The firm pays out dividends D_t, which is what remains from output after paying wages and investing in hiring:

$$D_t = Z_t N_t - W_t N_t - \kappa_t V_t. \tag{7}$$

The firm then maximizes the present value of current and future dividends

$$\max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \tag{8}$$

subject to

$$N_{t+1} = (1-s)N_t + q(\theta_t)V_t, \tag{9}$$

where $q(\theta_t)$ is given by (4). The firm takes θ_t and W_t as given in solving (8). The economy is therefore subject to a congestion externality. By posting more vacancies, firms raise the aggregate V_t, therefore increasing θ_t and lowering the probability that any one firm will be able to hire.

The following result establishes a general relation between the stock market and the labor market.

Theorem 1. *Assume the production function (5) and that the firm solves (8). Then the ex-dividend value of the firm is given by*

$$P_t = \frac{\kappa_t}{q(\theta_t)} N_{t+1}, \tag{10}$$

and the equity return equals

$$R_{t+1} = \frac{(1-s)\frac{\kappa_{t+1}}{q(\theta_{t+1})} + Z_{t+1} - W_{t+1}}{\frac{\kappa_t}{q(\theta_t)}}. \tag{11}$$

Furthermore, if $\kappa_t = \kappa Z_t$ for fixed κ, then

$$\frac{P_t}{Z_t} = \frac{\kappa}{q(\theta_t)} N_{t+1}. \tag{12}$$

Proof. See Appendix A. □

Some notation is helpful in understanding this theorem. Let l_t denote the Lagrange multiplier on the firm's hiring constraint (9). We can think of l_t as the value of a worker inside the firm at time $t+1$. In deciding how many vacancies to post at time t, the firm equates the marginal benefit of an additional worker with marginal cost. Because the probability of filling a vacancy with a worker is $q(\theta_t)$ (see Section 3.1), the marginal benefit is $l_t q(\theta_t)$ while the marginal cost is simply the cost of opening a vacancy, κ_t. Thus a condition for optimality is:

$$\kappa_t = l_t q(\theta_t). \tag{13}$$

It follows that $l_t = \kappa_t / q(\theta_t)$, and furthermore, that the value of the firm equals the number of workers employed multiplied by the value of each worker. This is what is shown in (10).

Equation 11 has a related interpretation. The $t+1$ return on the investment of hiring a worker is the value of the worker employed in the firm at time $t+2$ (multiplied by the probability that the worker remains with the firm), plus productivity minus the wage, all divided by the value of the worker at time $t+1$. Note that the previous discussion implies that the value of the worker employed at $t+1$ is $\frac{\kappa_t}{q(\theta_t)}$.

Equation 12 follows directly from (10) and from the assumption that the cost of posting a vacancy is proportional to productivity (given our assumption of a nonstationary component to productivity, this implies a balanced growth path). We can evaluate (12) empirically. We take the historical time series of the price-productivity ratio and of N_{t+1} (equal to one minus the unemployment rate). Given standard parameters for the matching function (discussed further below), this implies, by way of (12), a time series for the vacancy-unemployment ratio θ_t. Figure 5 shows that the resulting ratio of vacancies to unemployment lines up closely with its counterpart in the data.

3.3 General equilibrium

In this section, we extend our previous results to general equilibrium. Theorem 1 still holds, but the general equilibrium model allows us to model the underlying source of employment and stock price fluctuations.

3.3.1 The Representative Household

Following Merz (1995) and Gertler and Trigari (2009), we assume that the representative household is a continuum of members who provide one another with perfect consumption insurance. We normalize the size of the labor force to one.[7] The household maximizes utility over consumption, characterized by the recursive utility function introduced by Kreps and Porteus (1978) and Epstein and Zin (1989):

$$J_t = \left[C_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[J_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{14}$$

where β is the time discount factor, γ is relative risk aversion and ψ is the elasticity of intertemporal substitution (EIS). In case of $\gamma = 1/\psi$, recursive preferences collapse to power utility.

The recursive utility function implies that, assuming optimal consumption, the stochastic discount factor takes the following form:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{\mathbb{E}_t \left[J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}. \tag{15}$$

[7] This assumption implies that our model focuses on the transition between employment and unemployment rather than between in and out of labor force.

3.3.2 Wages

The canonical DMP model assumes that wages are determined by Nash bargaining between the employer and the jobseeker. Both parties observe the surplus of job creation; the fraction received by the jobseeker is determined by his bargaining power. Pissarides (2000) shows that the Nash-bargained wage, W_t^N, is given by

$$W_t^N = (1 - B)b_t + B(Z_t + \kappa_t \theta_t), \tag{16}$$

where $0 \leq B \leq 1$ represents the worker's bargaining power and b_t is the flow value of unemployment.[8] The Nash-bargained wage is a weighted average of two components: the opportunity cost of employment and the contribution of the worker to the firm's profits. If the bargaining power of the worker is high, the firm has to pay a higher fraction of the output the worker produces as wage, as well as the foregone costs from not having to hire.

The Nash-bargained wage is a useful benchmark. However, it implies wages that are unrealistically responsive to changes in labor market conditions (see Section 2). This is a well-known problem in the literature on labor market search. Hall (2005) proposes a rule that partially insulates wages from tightness in the labor market. Let

$$W_t = \nu W_t^N + (1 - \nu) W_t^I, \tag{17}$$

where

$$W_t^I = (1 - B)b_t + B(Z_t + \kappa_t \bar{\theta}). \tag{18}$$

The parameter ν controls the degree of tightness insulation.[9] With $\nu = 1$, we are back in the Nash bargaining case. With $\nu = 0$, wages do not respond to labor market tightness. The resulting wage remains sensitive to productivity but loses some of its sensitivity to tightness. Furthermore, this formulation allows a direct comparison between versions of the model with and without tightness

[8]The canonical Nash-bargained wage equation holds in our model. See Petrosky-Nadeau, Zhang, and Kuehn (2013) for the proof in a similar setting.

[9]Hall (2005) specifies W_t^I as constant and productivity as stationary. In our setting with non-stationary productivity, W_t^I must be proportional to Z_t to allow for balanced growth. In Section 2, we show that, in the data, wages are responsive to Z_t but not to θ_t.

insulated wages. Hall and Milgrom (2008) provides a microfoundation for (17).

To have a balanced-growth path, we will assume $b_t = bZ_t$ (recall that $\kappa_t = \kappa Z_t$, see Section 3.2). Besides being necessary from a modeling perspective, it is also realistic to link unemployment benefits (broadly defined) with productivity: as Chodorow-Reich and Karabarbounis (2015) show using micro data, the time benefits of unemployment are an empirically large fraction of total unemployment benefits. The importance of these time benefits imply that, in the data, total benefits to unemployment are procyclical.

3.3.3 Technology and the Representative Firm

The representative firm produces output Y_t with technology $Z_t N_t$ given in (5). In normal times, $\log Z_t$ follows a random walk with drift. In every period, there is a small and time-varying probability of a disaster.[10] Thus,

$$\log Z_{t+1} = \log Z_t + \mu + \epsilon_{t+1} + d_{t+1}\zeta_{t+1}, \tag{19}$$

where $\epsilon_t \overset{iid}{\sim} N(0, \sigma_\epsilon^2)$,

$$d_{t+1} = \begin{cases} 1 & \text{with probability } \lambda_t \\ 0 & \text{with probability } 1 - \lambda_t. \end{cases}$$

and where $\zeta_t < 0$ gives the decline in log productivity, should a disaster occur.[11] We assume the log of the disaster probability λ_t follows an autoregressive process which (for convenience) is independent of the shocks to productivity. That is,

$$\log \lambda_t = \rho_\lambda \log \lambda_{t-1} + (1 - \rho_\lambda) \log \bar{\lambda} + \epsilon_t^\lambda, \tag{20}$$

where $\bar{\lambda}$ is the mean log probability, ρ_λ is the persistence, and $\epsilon_t^\lambda \overset{iid}{\sim} N(0, \sigma_\lambda^2)$. In solving the model, we approximate this process using a finite-state Markov chain with all nodes smaller than one (see Table 3).

Following the literature on disasters and asset pricing (e.g. Barro (2006), Gourio (2012)) we

[10] See Gabaix (2012), Gourio (2012) and Wachter (2013).

[11] The distribution of ζ_t is time-invariant and therefore independent of all other shocks.

interpret a disaster broadly as any event that results in a large drop in GDP and consumption. Major wars, for example, lead to a large destruction in the capital stock, rendering existing workers less productive. A disruption in the financial system, or a major change in economic institutions could also lead to sharply lower output per worker.

3.3.4 Equilibrium

In equilibrium, the representative household holds all equity shares of the representative firm. The representative household consumes the output $Z_t N_t$ net of investment in hiring $\kappa_t V_t$, and the value of non-market activity $b_t(1 - N_t)$ achieved by the unemployed members:

$$C_t = Z_t N_t + b_t(1 - N_t) - \kappa_t V_t. \tag{21}$$

Note that consumption includes firm wages and dividends; the definition of dividends in (7) shows that the sum of wages and dividends amounts to $Z_t N_t - \kappa_t V_t$. The household also consumes the flow value of unemployment. This implies that we are treating this flow value primarily as home production as opposed to unemployment benefits (which would be a transfer that would net to zero).[12] To summarize, households maximize (14), subject to the budget constraint (21) and the law of motion for N_t (9), where θ is taken as given. The fact that the household owns all equity shares implies that the optimal investment in hiring is also that which solves the firm's problem.

The proportionality assumptions on vacancy costs κ_t and the flow value of unemployment b_t in productivity Z_t imply that we can write:

$$C_t = Z_t N_t + b Z_t(1 - N_t) - \kappa Z_t V_t. \tag{22}$$

Therefore, we can define consumption normalized by productivity, $c_t = C_t/Z_t$, as

$$c_t = N_t + b(1 - N_t) - \kappa V_t. \tag{23}$$

[12] This is consistent with the results of Chodorow-Reich and Karabarbounis (2015) as discussed in Section 3.3.2. Changing to the alternative assumption that these benefits net to zero, however, does not impact our results. To ensure that our model-data comparison is valid, when quantitatively assessing the model we report the model-implied dynamics of consumption from dividends and wages, namely, $Z_t N_t - \kappa_t V_t$, as this is what is measured in consumption data.

In equilibrium, the value function J_t is determined by productivity, the disaster probability and the employment level. That is, $J_t = J(Z_t, \lambda_t, N_t)$. Given our assumptions on productivity and the homogeneity of utility, the value function takes the form

$$J(Z_t, \lambda_t, N_t) = Z_t j(\lambda_t, N_t), \tag{24}$$

where we refer to $j(\lambda_t, N_t)$ as the *normalized value function*. The normalized value function solves

$$j(\lambda_t, N_t) = \max_{c_t, V_t} \left[c_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_t \left[e^{(1-\gamma)(\mu+\epsilon_{t+1}+d_{t+1}\zeta_{t+1})} j(\lambda_{t+1}, N_{t+1})^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \tag{25}$$

subject to (23) and (9). This normalization implies that we can solve for all quantities of interest as functions of two stationary state variables, λ_t and N_t.

3.4 Comparative Statics in a Model with Labor Search and Constant Disaster Probability

Before exploring the quantitative implications of our full model in Section 4, we consider the simpler case of constant disaster probability. We show that the economy is isomorphic to one without disasters but with a different time discount factor. When the EIS is greater than one, the effect of disasters is to make the agent less patient and lead him to invest less in hiring. An analogous isomorphism is present in the models of Gabaix (2011) and Gourio (2012). Furthermore, stock prices are decreasing, and unemployment increasing as a function of the disaster probability, provided that the EIS is greater than one. The closed-form solutions allow us to give intuition for these results, which will carry over to the dynamic results in Section 4.

To derive closed-form solutions, we replace the random variable $d_{t+1}\zeta_{t+1}$ with a compound Poisson process with intensity $\tilde{\lambda}$. At our parameter values, the difference between the probability of a disaster λ and the intensity $\tilde{\lambda}$ is negligible, and we continue to refer to $\tilde{\lambda}$ as the disaster probability. Unless otherwise stated, proofs are contained in Appendix B.

Theorem 2. *Assume that disaster risk is constant. The value function in a model with labor search and disasters is the same as the value function in a model without disasters but with a* *different time-discount factor. That is, the normalized value function solves*

$$j(\tilde{\lambda}, N_t)^{1-\frac{1}{\psi}} = c_t^{1-\frac{1}{\psi}} + \hat{\beta}(\tilde{\lambda}) \left(\mathbb{E}_t \left[e^{(1-\gamma)(\mu+\epsilon_{t+1})} j(\tilde{\lambda}, N_{t+1})^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}, \tag{26}$$

with the time-discount factor $\hat{\beta}(\tilde{\lambda})$ defined by

$$\log \hat{\beta}(\tilde{\lambda}) = \log \beta + \frac{1-\frac{1}{\psi}}{1-\gamma} \left(\mathbb{E}\left[e^{(1-\gamma)\zeta} \right] - 1 \right) \tilde{\lambda}, \tag{27}$$

Moreover, $\hat{\beta}(\tilde{\lambda})$ is decreasing in $\tilde{\lambda}$ if and only if $\psi > 1$.

Note that (26) recursively defines the normalized value function in an economy without disaster risk. Theorem 2 shows that an economy with disasters is equivalent to one without, but with a less patient agent when the EIS $\psi > 1$ and a more patient agent when $\psi < 1$. As this statement suggests, the change to the time-discount factor due to disasters reflects a trade-off between an income and a substitution effect. On the one hand, the presence of disasters lead the agent to want to save (the income effect). But the mechanism that the agent has to shift consumption, namely, investing in hiring, becomes less attractive because there is a greater chance that the workers will not be productive (the substitution effect). When $\psi > 1$, the substitution effect dominates, and the agent, in effect, becomes less patient.

We can also see the effect of the probability of disaster on the riskfree rate and on the equity premium. In the case with constant λ_t, these equations turn out to be the same as in an endowment economy model (Tsai and Wachter (2015)).

Lemma 1. *Assume in a model with labor search that the disaster risk is constant and the labor market is at its steady state. The log risk-free rate is given by*

$$\log R_f = -\log \beta + \frac{1}{\psi}\left(\mu + \frac{1}{2}\sigma_\epsilon^2\right) - \frac{1}{2}\left(\gamma + \frac{\gamma}{\psi}\right)\sigma_\epsilon^2 + \left(\frac{\frac{1}{\psi} - \gamma}{1-\gamma} \mathbb{E}\left[e^{(1-\gamma)\zeta} - 1 \right] - \mathbb{E}\left[e^{-\gamma\zeta} - 1 \right] \right) \tilde{\lambda}. \tag{28}$$

The riskfree rate is decreasing in $\tilde{\lambda}$.

The risk of a rare disaster increases agents' desire to save, which drives down the riskfree rate. In contrast to Theorem 2, this result holds regardless of the value of ψ.

Lemma 2. *Assume in a model with labor search that disaster risk is constant and the labor market is at its steady state. The equity premium is given by*

$$\log\left(\frac{\mathbb{E}_t[R_{t+1}]}{R_f}\right) = \gamma\sigma_\epsilon^2 + \tilde{\lambda}\mathbb{E}\left[\left(e^{-\gamma\zeta}-1\right)\left(1-e^{\zeta}\right)\right]. \tag{29}$$

The equity premium is increasing in $\tilde{\lambda}$.

The first term in the equity premium represents the normal-times risk in production. Given the low volatility in productivity and consumption, this first term will be very small in our calibrated model. The second term represents the effect of rare disasters. A rare disaster causes an increase in marginal utility, represented by the term $e^{-\gamma\zeta}-1$, at the same time as it causes a decrease in the value of the representative firm, as represented by $e^{\zeta}-1$. Because the representative firm declines in value at exactly the wrong time, its equity carries a risk premium. This also implies that the equity premium is unambiguously increasing in the probability of a disaster.

How are the risk premium and the riskfree rate connected to the effective time-discount factor and to firm valuations? We now answer this question. Consider a transformation of the price-dividend ratio:

$$h(\tilde{\lambda}) = -\log\left(1+\frac{D_t}{P_t}\right). \tag{30}$$

Then $h(0)$ is the price-dividend ratio when there is no disaster risk:

$$h(0) = \log\beta + \left(1-\frac{1}{\psi}\right)\left(\mu+\frac{1}{2}(1-\gamma)\sigma_\epsilon^2\right). \tag{31}$$

Note that in this iid economy where quantities are at their steady-state values, P_t/D_t is a constant that depends on $\tilde{\lambda}$. There is a tight connection between the price-dividend ratio and the effective time-discount factor.

Theorem 3. *Assume in a model with labor search that disaster risk is constant and the labor market is at its steady state. Define $\hat{\beta}(\tilde{\lambda})$ as in Theorem 2. Define $h(\tilde{\lambda})$ as in (30). Then*

$$h(\tilde{\lambda}) - h(0) = \log\hat{\beta}(\tilde{\lambda}) - \log\beta. \tag{32}$$

Thus the price-dividend ratio is decreasing in $\tilde{\lambda}$ if and only if $\psi > 1$.

Applying (28) and (29), we see that the effect of disaster risk on $h(\tilde{\lambda})$ can be decomposed into a discount rate effect (which in turn can be decomposed into a risk premium and riskfree rate effect) and an expected growth effect, as in Campbell and Shiller (1988):

$$h(\tilde{\lambda}) - h(0) = -\underbrace{\left(\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\left(\mathbb{E}\left[e^{(1-\gamma)\zeta}\right] - 1\right) - \left(\mathbb{E}\left[e^{-\gamma\zeta}\right] - 1\right)\right)\tilde{\lambda}}_{\text{risk-free rate effect}}$$
$$+ \underbrace{\mathbb{E}\left[\left(e^{-\gamma\zeta} - 1\right)\left(e^{\zeta} - 1\right)\right]\tilde{\lambda}}_{\text{risk premium effect}} + \underbrace{\left(\mathbb{E}\left[e^{\zeta}\right] - 1\right)\tilde{\lambda}}_{\text{expected cash-flow effect}}. \quad (33)$$

The decomposition (33) provides additional intuition for the effect of changes in the disaster probability on the economy. On the one hand, an increase in the risk of a disaster drives down the riskfree rate. This will raise valuations, all else equal. However, it also increases the risk premium and lowers expected cash flows. When $\psi > 1$, the risk premium and cash flow effects dominate the riskfree rate effect and an increase in the disaster probability lowers valuations.

We now explicitly connect these results to the labor market. First, as suggested by the result in Section 3.2, the greater are valuations, the greater is labor market tightness (see Appendix A for a rigorous proof). Because an increase in the probability lowers valuations, it lowers labor market tightness, provided that the EIS is greater than 1.

Corollary 1. *Assume in a model with labor search that disaster risk is constant and the labor market is at its steady state.*

1. *The price-dividend ratio is increasing as a function of labor market tightness.*

2. *Labor market tightness is decreasing in the probability of a disaster if and only if $\psi > 1$.*

When firms are faced with a higher risk of an economy-wide disaster, they have an incentive to reduce hiring. This decreases equilibrium tightness θ to the point where firms are indifferent between hiring and not. Thus higher disaster risk results in higher unemployment, lower vacancies, and lower firm valuations.

The previous discussion separates the effects of the risk premium and the riskfree rate on the price-dividend ratio and hence on firm incentives. What about the discount rate overall? Hall

(2015) conjectures that a model that produces higher discount rates in recessions can drive co-movement of unemployment and the stock market. The analysis in this section shows that it is not discount rates per se that matter, but the combination of discount rates and growth expectations (it is also not necessary for these to be related to recessions driven by lower current productivity). For higher discount rates to be associated with lower unemployment, EIS greater than 1 is a necessary but not sufficient condition:

Corollary 2. *Assume in a model with labor search that disaster risk is constant and the labor market is at its steady state. The expected return is increasing in $\tilde{\lambda}$ if and only if*

$$1 - \mathbb{E}\left[e^{\zeta}\right] < \frac{1-\frac{1}{\psi}}{1-\gamma}\left(1 - \mathbb{E}\left[e^{(1-\gamma)\zeta}\right]\right). \tag{34}$$

The analysis in this section sheds light on the tight link between the valuation mechanism and the labor market. As we will show in the next section, this mechanism is helpful in quantitatively explaining historical fluctuations in the labor market.

4 Quantitative Results

Below, we compare statistics in our model to those in the data. Section 4.1 describes the calibration of parameters for preferences, labor market variables, and productivity in normal times. Section 4.2 describes assumptions on the disaster distribution. Given these assumptions, Section 4.3 shows what happens to labor market, business cycle, and financial moments when a disaster occurs or when the disaster probability increases. We then simulate repeated samples of length 60 years from our model. Section 4.4 describes statistics of labor market moments in simulated data. Section 4.5 describes statistics for business cycle and financial moments. Section 4.6 makes use of alternative calibrations to highlight the main mechanisms behind our results.

4.1 Model Parameters

Table 1 describes model parameters for our benchmark calibration. Unless otherwise stated, parameters are given in monthly terms. Labor productivity in normal times is calibrated to the labor productivity process from the postwar data (see Appendix D for data description). This implies a monthly growth rate μ of 0.18% and standard deviation σ_ϵ of 0.47%. We calibrate the separation rate to 3.5% as estimated by Shimer (2005). We calibrate the Cobb-Douglas elasticity η to 0.35, consistent with empirical estimates in Petrongolo and Pissarides (2001) and Yashiv (2000). The parameter κ, corresponding to unit costs of vacancy openings normalized by labor productivity, is set to 0.5, the average of estimates from Hall and Milgrom (2008) and Hagedorn and Manovskii (2008).[13] For the bargaining power of workers (B) and the flow value of unemployment (b), we use values from Hall and Milgrom (2008); these are 0.5 and 0.76 respectively. We set the matching efficiency ξ to 0.365, targeting a model population value for unemployment equal to 10%.

We calibrate the tightness-insulation parameter ν to match wage dynamics in the data.[14] Table 2 shows the standard deviation and autocorrelation of wages in the data, as well as the elasticity of wages with respect to labor market tightness and productivity. Also shown is the elasticity of labor market tightness to productivity. The elasticity of wages to labor market tightness is low throughout the sample, while the elasticity of wages to labor productivity ranges from 0.67 in the full sample, to close to unity in the sample after 1985. We consider two versions of the model, one that insulates wages from labor market tightness (our benchmark specification), and one with no tightness insulation (the Nash bargaining solution). For each case, we simulate 10,000 sample paths of 60 years of data and report the median, and the 5th and 95th percentile of each statistic. Tightness insulation allows the model to match the standard deviation of wages to that of the data; without tightness insulation, wages are too volatile. Tightness insulation is also consistent with other aspects of the data: it implies wages with unit elasticity with respect to productivity, but

[13] Hagedorn and Manovskii (2008) find a constant and a pro-cyclical component in vacancy costs. We specify vacancy costs proportional to productivity for simplicity.

[14] Following Hagedorn and Manovskii (2008), we calculate wages by multiplying the labor share by productivity.

near zero elasticity with respect to labor market tightness. Under the Nash-bargaining solution, however, wages are unrealistically elastic with respect to labor market tightness.

We assume the EIS ψ is equal to 2 and risk aversion γ is equal to 5.7. As is standard in production-based models with recursive utility, an EIS greater than one is necessary for the model to deliver qualitatively realistic predictions for stock prices (see Section 3.4). An important question is whether this level of the EIS is consistent with other aspects of the data. Using instrumental variable estimation of consumption growth on interest rates, Hall (1988) and Campbell (2003) estimate this parameter to be close to zero. However, as noted by Bansal and Yaron (2004), this parameter estimate may be biased in models with time-varying second (or higher-order) moments. To gauge the impact of the mis-specification, we repeat the instrumental-variable regressions of consumption growth on government bill rates in data simulated from our model.[15] We find a mean estimate of 0.15, consistent with the data. Thus, despite the assumption of an EIS greater than 1, our model replicates the weak relation between contemporaneous consumption growth and interest rates.

4.2 Size Distribution and Probability of Disasters

The distribution for the disaster impact ζ_t is taken from historical data on GDP declines in 36 countries over the last century (Barro and Ursua (2008)). Following Barro and Ursua, we characterize a disaster by a 10% or higher cumulative decline in GDP. The resulting distribution for $1 - e^{\zeta}$ is shown in Figure 6. We assume that, if a disaster occurs, there is a 40% probability of default on government debt (Barro (2006)).

We approximate the dynamics of the disaster probability λ_t in (20) using a 12-state Markov chain. The nodes and corresponding stationary probabilities are given in Table 3. The stationary distribution of monthly probabilities is approximately lognormal with a mean of 0.20% and standard deviation 1.97%. In comparison, the 10% criterion for a disaster implies that the annual frequency of disasters in the data is 3.7%, indicating that our assumption on the disaster frequency

[15] The instruments are twice-lagged consumption growth, the government bill rate, and the log price-productivity ratio.

is conservative. We choose the persistence and the volatility of the disaster probability process to match the autocorrelation and volatility of unemployment in U.S. data.

Table 4 describes properties of the disaster probability distribution. Because this distribution is not available in closed form, we simulate 10,000 sample paths of length 60 years. We find that 53% of these sample paths do not have a disaster; thus the post-war period was not unusual from the point of view of our model. Because the distribution for the disaster probability is highly skewed, the average λ_t is much lower in samples that, ex post, have no disasters than it is in population. Below, we report statistics from these simulated data for unemployment, vacancies, and business cycle and financial moments. Unless otherwise stated, the model statistics are computed from the no-disaster paths.

4.3 The effect of disasters and disaster probabilities

To highlight the implications of time-varying disaster probability, our model assumes a simplified view of the disaster itself. As described in Section 3.3, a disaster is a one-time, permanent drop in labor productivity. Because consumption, dividends and wages scale with productivity, these variables all fall by equal percentages in a disaster; if for example productivity drops by 15% in a disaster, they also drop by 15%. While this view of a disaster is stylized, results in the literature (e.g. Nakamura, Steinsson, Barro, and Ursua (2013) and Tsai and Wachter (2015)) suggest that introducing more complicated dynamics are unlikely to alter the implications for non-disaster states, which are the main focus of our analysis.

Figure 7 shows what happens to the labor market and to the business cycle in the months following an increase in the disaster probability. We assume an increase in the (monthly) probability from 0.05% to 0.32%, representing an approximately two-standard deviation increase along a typical no-disaster path. This increased probability of a disaster reduces the optimal employment level because, even though current productivity is unchanged, future productivity is more risky. Firms substantially reduce vacancies when the shock hits; vacancies then slowly rise to a new steady state which is lower than before. During this time, unemployment steadily rises as well.

Vacancies and unemployment take about two years to converge to their new steady states. This two-standard deviation increase in the disaster probability leads to an approximately 6% decrease in employment and a 25% decrease in vacancies at the end of the two-year period. As Figure 7 shows, the increase in unemployment coincides with a decline in stock valuations.

While vacancies and unemployment respond substantially to an increase in disaster probability, consumption does not. In the very short term, an increase in the disaster probability slightly increases consumption because investment in hiring falls. In the longer term, consumption falls because the lower level of employment implies lower output.[16]

Figure 8 shows what happens to financial markets following a two-standard deviation increase in the disaster probability. Equity returns fall dramatically because of the sharp decline in stock prices described above. However, in the months following the increase, equity returns are slightly higher because of the greater risk premium needed to compensate investors for bearing the risk of a disaster. At the same time, the government bill rate falls because the greater degree of risk in the economy leads investors to want to save.

4.4 Labor Market Moments

Table 5 describes labor market moments in the model and in the U.S. data from 1951 to 2013. Panel A reports U.S. data on unemployment U, vacancies V, the vacancy-unemployment ratio V/U, labor productivity Z, and the price-productivity ratio P/Z. The labor market results replicate those reported by Shimer (2005) using more recent data. The vacancy-unemployment ratio has a quarterly volatility of 39%, twenty times higher than the volatility of labor productivity of 2%. The correlation between Z and V/U is 10%, whereas the correlation between P/Z and V/U is 47%, consistent with the findings in Section 2.[17] The correlation is lower in the pre-1985 sample, and higher in the post-1985 sample. These findings, together with the more detailed analysis in Section 2, motivate the mechanism in this paper.

[16] Bloom (2009) solves a model in which time-varying uncertainty leads to lower consumption and output; our model is consistent with the data he reports.

[17] As noted in Section 2, we follow Shimer (2005) in using a low-frequency HP filter with smoothing parameter 10^5. We report volatilities of log deviations from trend.

Panel B of Table 5 reports the statistics calculated from sample paths simulated from the model. We simulate 10,000 sample paths of length 60 years. We report means from the 53% of simulations that contain no disaster. Our model is calibrated to match the volatility of unemployment. However, the model can also explain the volatility of vacancies, and the high volatility of the vacancy-unemployment ratio. The model also correctly generates a large negative correlation between vacancies and unemployment. Other possible mechanisms, such as shocks to the separation rate, generate a counterfactual positive correlation between V and U (Shimer (2005)). In addition, our model captures the low correlation between the labor market and productivity and the relatively high correlation between the labor market and stock prices; it overstates the latter correlation because a single state variable drive both. However, a united mechanism for both stock market and labor market volatility is a better description of the data compared to models based on realized productivity, especially for the U.S. data from mid-1980s to the present.

Figure 9 shows the Beveridge curve (namely, the locus of vacancies and unemployment) in the data and in the model. The position of the economy along the historically downward sloping Beveridge curve is an important business cycle indicator (Blanchard, Diamond, Hall, and Yellen (1989)). The time-varying risk mechanism in our model is able to generate such negative correlation, and as a result, the model values are concentrated along a downward sloping line. In our model, an increase in risk and a decrease in expected growth leads to downward movement along the Beveridge curve. Following an increase in disaster probability, the economy converges to the new optimal level of employment which is lower than before. Because the matching function is increasing in both vacancies and unemployment, a lower level for vacancies is needed to maintain the employment level. The model is able to generate a wide range of values on the vacancy-unemployment locus, including data values at the lower right corner of the Beveridge curve observed during the Great Recession which correspond to high values for the disaster probability.

4.5 Business Cycle and Financial Moments

We now turn to the model's implications for consumption, output, and for financial market variables. Table 6 shows that the model produces a low volatility of consumption and output, just as in the data. The volatility for consumption (2.3%) is slightly lower than for output (2.5%), reflecting the consumption-smoothing motives of the agent.

There are two independent dimensions to cyclicality in the model, namely, comovement with labor productivity and with disaster risk. In the model, the effect of productivity shocks on consumption and output growth is identical. This is not the case for disaster risk, however. Consumption equals output by the firm, plus home production, minus investment in hiring. Because both output and investment are pro-cyclical with respect to disaster probability, the consumption response to disaster probability shocks is weaker than the output response, as shown in Figure 7. This creates a higher volatility in output growth compared to consumption growth, in line with the data.[18] The volatility of consumption and output is substantially higher in population than in samples without rare disasters, which are comparable to the post-war period.

Table 6 also shows that the model produces a realistically low average return and volatility for government bills; these are 3.6% and 3.8%. respectively. While somewhat higher than in the data postwar, these are very low compared with the values for equity returns (see below), and lower than in many models of production. The data fall well within the confidence bands implied by the model. Average returns on government bills are low in the model because of the precautionary savings motives arising from the risk of a disaster (Section 3.4).

Even though output can have long periods with small shocks, there remains the possibility of a large disaster. Because firms' cash flows are exposed to this disaster, in equilibrium, investors require a high premium to hold equity. Indeed, in samples without disasters generated from the model, the median equity premium is 6.7%. Because our model does not include financial leverage, we follow common practice (see, e.g. Nakamura, Steinsson, Barro, and Ursua (2013)) and report

[18]Note that our definition of measured consumption does not include the flow value of unemployment as described in Section 3.3.4, and is therefore directly comparable to consumption expenditures in the data. Model-implied consumption volatility including the flow value of unemployment, $b_t(1 - N_t)$, is 1.4%.

data values that are adjusted for leverage in the table.[19] The equity premium generated by our model is in fact higher than the adjusted value in the data, 5.3%, and is not far from the unadjusted value of 7.9%.[20]

Besides matching the equity premium, our model can also generate high levels of return volatility. We can see this already in Figure 8 from the large return response in the event of an increase in the disaster probability. Table 6 shows, indeed, that return volatility implied by the model is 19.8% per annum, above the unlevered value in the data and close to the unadjusted value of 17.6%. While iid models such as Barro (2009) and Gabaix (2011) can explain why there is an equity premium in the context of production, it is harder to explain why returns are volatile even in periods when no disasters take place.[21] In our model, return volatility comes about through time variation in the probability of the disaster. When this probability rises, future prospects for growth dim, and more importantly, the discount rate for this future growth increases. Embedded in the value of a firm is the value of a worker who is in place. When firm values fall, so too do the incentives for hiring. Thus our model produces high equity volatility, even though volatility of output is low.

A problem often faced by dynamic models with production is low riskiness of firm cash flows. Firms respond to bad news about future productivity (concerning its mean, its riskiness, or both) by cutting investment, and increasing dividends. This makes firm equity a hedge and decreases both the equity premium and return volatility. To produce reasonable values, models that focus on investment assume counterfactually high leverage (Gourio (2012)), or assume that stocks are something other than the dividend claim (Croce (2014)). Our model is also one of investment; posting a vacancy implies an investment in hiring. However, we are able to match the equity premium and return volatility without the use of leverage. One reason for this is the relative insensitivity of wages to labor market conditions. Another reason is that our model is one where

[19] Lemmon, Roberts, and Zender (2008) report an average market leverage ratio of 28% among U.S. firms from 1965 to 2003. Accordingly, the unlevered equity premium is calculated multiplying stock returns by 0.72.

[20] The population equity premium generated by the model is even higher: 13.3%. This higher value reflects the fact that samples that contain disasters have higher disaster probabilities, and hence higher risk premia. Because of the noise induced by disasters, this value is difficult to compare to any one historical sample.

[21] In periods with disasters, returns will be volatile because cash flows are volatile. In our model, the population value for return volatility is 40% per annum.

unemployment and stock returns share an underlying process, and unemployment is highly volatile. We discuss these mechanisms further in the next section.

4.6 Sources of Volatility and Risk Premia

We compare three alternative specifications to our benchmark model to highlight the sources of volatility and risk premia: a model with constant disaster probability, where disaster probability is set to 0.20%, the stationary mean in the benchmark model; a model with no disaster risk; and a model with Nash-bargained wages, namely, $\nu = 1$. In all cases, we follow the same simulation strategy as before, namely simulating 10,000 samples with length 60 years. We report results from samples without disasters, which is 53% of samples in the time-varying model and 24% in the constant disaster risk model.[22] When relevant, we also report population values.

Table 7 reports labor market volatility in the alternative specifications. If risk is not time-varying, labor market variables and P/Z are constant. This confirms that the only source of fluctuation in the labor market is disaster probability. The case without any disasters yields the same volatility as the case with constant disasters (recall that we are reporting results from no-disaster samples). The case without tightness insulation (but with time-varying risk) does produce some volatility in unemployment, vacancies, and in the vacancy-unemployment ratio, but much less than in our benchmark case. In this case, the risk of future productivity declines (as represented by low tightness) is passed on to workers in the form of lower wages. Thus firms maintain hiring when risk goes up, and unemployment as well as prices fluctuate much less than in the data. The resulting wage process also differs sharply from its empirical counterpart, as shown in Table 2.

Table 8 reports business cycle and financial moments. We first describe the volatility of consumption and output. In the absence of time-varying risk, consumption growth and output growth have the same volatility. Moreover, this volatility is lower as compared to our benchmark case with time-varying risk. Thus time-varying disaster risk causes some fluctuations in consumption

[22]There are fewer disasters in the model with time-varying λ as opposed to constant λ because the process is highly skewed; most of the time λ takes on values consistent with few disasters.

and output due to firm's optimal investment decisions. Constant λ and zero λ (no disaster risk) have the same implications for consumption and output in samples without disasters. Allowing for time-varying λ, but eliminating the tightness insulation from wages, has similar macroeconomic implications as setting λ to be a constant. Without tightness insulation, time-varying risk has only a small impact on firms' investment in hiring for the reasons described above: firms can pass greater risk of a disaster on to their employees in the form of lower wages. However, we do not see this in the data.

We now turn to the financial moments. The model without disaster risk delivers a negligible equity premium and equity volatility, as well as an unrealistically high riskfree rate. This is in spite of the fact that the model is not the benchmark real-business-cycle model; rather it still is a DMP model with tightness-insulation. The reason is that output and thus firm cash flows remain smooth in this model. The model with constant disaster risk has a high equity premium, however equity volatility is still negligible in periods without disasters.[23]

Interestingly, the case with time-varying λ and no tightness insulation has implications for equity returns that are dramatically different than the case with tightness insulation. In this case, investment in the firm becomes very safe because the firm has a cost structure that is highly sensitive to cyclical conditions in the economy. In times of low disaster probability, employment increases and wages increase substantially due to the high sensitivity of wages to labor market tightness. In contrast, when employment falls, wages adjust rapidly downward. Thus investment in the firm forms a hedge against the main risks in the economy, and, in equilibrium, risk premia are negative.

While these results point to the importance of tightness-insulation for wages, it is also the case that tightness-insulation alone does not lead to an equity premium, high stock return volatility, or for that matter, volatile unemployment, as illustrated by our case with constant disaster risk,

[23]In the model with constant λ, samples without disasters have higher average excess returns than in population; this is the effect of the Peso problem described in Jorion and Goetzmann (1999). In the model with time-varying λ, somewhat surprisingly, the opposite effect holds and the samples without disasters have lower average excess returns. The reason is that samples that, ex post, have no disasters are also those that, ex post, have lower disaster probabilities, and hence lower equity premia. The time-varying λ case has higher population risk premia for the reasons given in Wachter (2013).

or no disaster risk. In these cases, equity volatility is indeed higher than consumption and output volatility, but the difference is slight: 1.7% versus 1.3% per year. Unlike in models with wage rigidities (Uhlig (2007), Favilukis and Lin (2014)), or high operating leverage induced by high and stable wages (Petrosky-Nadeau, Zhang, and Kuehn (2013)), wages fluctuate fully in response to changes in productivity in our model; it is their response to labor market conditions that is dampened (see Table 2). It is time-varying risk premia arising from the risk of a disaster that generates equity volatility.

5 Conclusion

This paper shows that a business cycle model with search and matching frictions in the labor market and a small and time-varying risk of an economic disaster can simultaneously explain labor market volatility, stock market volatility and the relation between unemployment and stock market valuations. While tractable, the model can generate high volatility in labor market tightness along with realistic aggregate wage dynamics. The findings suggest that time variation in aggregate uncertainty offers an important channel, through which the DMP model of labor market search and matching can operate. The model provides a mechanism through which job creation incentives of firms and stock market valuations are tightly linked, as the comovement of labor market tightness and stock market valuations in the data suggest. While the presence of disaster risk and realistic wage dynamics generate a high unlevered equity premium, the source of labor market volatility and stock market volatility is time variation in risk. Finally, the model is consistent with basic business cycle moments such as consumption growth and output growth.

Appendix

A Proofs of results for a general stochastic discount factor

The results in this section do not depend on our assumptions on M_{t+1} or Z_{t+1}.

Proof of Theorem 1 The representative firm pays out as dividend what is left from output after subtracting wage costs and investment in hiring:

$$D_t = Z_t N_t - W_t N_t - \kappa_t V_t. \tag{A.1}$$

The firm takes wages W_t and labor market tightness θ_t as given and maximizes the cum-dividend value

$$P_t^c = \max_{\{V_{t+\tau}, N_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} \left[Z_{t+\tau} N_{t+\tau} - W_{t+\tau} N_{t+\tau} - \kappa_{t+\tau} V_{t+\tau} \right], \tag{A.2}$$

subject to the law of motion for employment

$$N_{t+1} = (1-s) N_t + q(\theta_t) V_t. \tag{A.3}$$

The first order conditions with respect to V_t and N_{t+1} are given by

$$0 = -1 + l_t \frac{q(\theta_t)}{\kappa_t} \tag{A.4}$$

$$l_t = \mathbb{E}_t \left[M_{t+1} (Z_{t+1} - W_{t+1} + l_{t+1}(1-s)) \right], \tag{A.5}$$

where l_t is the Lagrange multiplier on the aggregate law of motion for employment level. Note that (A.5) can be interpreted as an Euler equation with l_t as the value of a worker inside the firm.

We expand (A.2), adding to each term in the summation an expression that, by (A.3), is equal to zero:

$$\begin{aligned} P_t^c = {} & Z_t N_t - W_t N_t - \kappa_t V_t - l_t \left(N_{t+1} - (1-s) N_t - \frac{q(\theta_t)}{\kappa_t} \kappa_t V_t \right) \\ & + \mathbb{E}_t \left[M_{t+1} \left[Z_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1} - l_{t+1} \left(N_{t+2} - (1-s) N_{t+1} - \frac{q(\theta_{t+1})}{\kappa_{t+1}} \kappa_{t+1} V_{t+1} \right) \right] \right] \\ & + \ldots \end{aligned} \tag{A.6}$$

The terms $-\kappa_t V_t$ and $l_t \frac{q(\theta_t)}{\kappa_t} \kappa_t V_t$ cancel out for all t as a result of (A.4). Furthermore, $l_t N_{t+1}$ cancels out with $\mathbb{E}_t \left[Z_{t+1} N_{t+1} - W_{t+1} N_{t+1} + l_{t+1}(1-s) N_{t+1} \right]$ for all t as a result of (A.5). It follows that

$$P_t^c = Z_t N_t - W_t N_t + l_t(1-s) N_t. \tag{A.7}$$

Consider the ex-dividend value of equity $P_t = P_t^c - D_t$. Equation A.7 and the definition of dividends implies

$$\begin{aligned} P_t &= Z_t N_t - W_t N_t + l_t(1-s) N_t - Z_t N_t + W_t N_t + \kappa_t V_t \\ &= \kappa_t V_t + l_t(1-s) N_t \\ &= \frac{\kappa_t}{q(\theta_t)} (N_{t+1} - (1-s) N_t) + \frac{\kappa_t}{q(\theta_t)} (1-s) N_t \\ &= l_t N_{t+1}. \end{aligned} \tag{A.8}$$

Combining (A.8) with (A.4) results in (10).

We now show (11). From (10) and the definition of dividends, it follows that

$$\begin{aligned} R_{t+1} &\equiv \frac{P_{t+1} + D_{t+1}}{P_t} \\ &= \frac{l_{t+1} N_{t+2} + Z_{t+1} N_{t+1} - W_{t+1} N_{t+1} - \kappa_{t+1} V_{t+1}}{l_t N_{t+1}} \\ &= \frac{l_{t+1} \frac{N_{t+2}}{N_{t+1}} + Z_{t+1} - W_{t+1} - \frac{\kappa_{t+1} V_{t+1}}{N_{t+1}}}{l_t} \\ &= \frac{l_{t+1} \left[1 - s + \frac{q(\theta_{t+1})}{\kappa t+1} \frac{\kappa_{t+1} V_{t+1}}{N_{t+1}} \right] + Z_{t+1} - W_{t+1} - \frac{\kappa_{t+1} V_{t+1}}{N_{t+1}}}{l_t} \\ &= \frac{Z_{t+1} - W_{t+1} + l_{t+1}(1-s)}{l_t} \\ &= \frac{Z_{t+1} - W_{t+1} + (1-s) \frac{\kappa_{t+1}}{q(\theta_{t+1})}}{\frac{\kappa_t}{q(\theta_t)}} \end{aligned} \tag{A.9}$$

Using this result, we provide characterizations of returns and prices that will be useful in what follows.

Lemma A.1. *Under the assumptions $\kappa_t = Z_t \kappa$ and $b_t = Z_t b$, the equity return equals*

$$R_{t+1} = \frac{(1-s) \frac{\kappa}{q(\theta_{t+1})} + 1 - w(\theta_{t+1})}{\frac{\kappa}{q(\theta_t)}} \frac{Z_{t+1}}{Z_t}, \tag{A.10}$$

where $w(\theta_t)$ is the wage normalized by productivity:

$$w(\theta_t) = (1 - B)b + B(1 + \kappa(\nu\theta_t + (1 - \nu)\bar{\theta})). \tag{A.11}$$

The result follows directly from Theorem 1, Equation 11.

Given l_t as the value of a worker inside the firm, the Euler equation (A.5) suggests a notion of a payout of a worker inside the firm:

$$D_t^l = Z_t - W_t - sl_t. \tag{A.12}$$

Lemma A.2. *Under the assumptions $\kappa_t = Z_t\kappa$ and $b_t = Z_t b$, the payout ratio of a worker employed in a firm is given by*

$$\frac{D_t^l}{l_t} = \frac{Z_t - W_t - sl_t}{l_t} \tag{A.13}$$

$$= \frac{1 - w(\theta_t) - s\frac{\kappa}{q(\theta_t)}}{\frac{\kappa}{q(\theta_t)}}. \tag{A.14}$$

Proof Equation A.14 follows directly from (A.12) and the assumptions.

How does this notion of payout ratio relate to the more traditional dividend-price ratio?

Lemma A.3. *Consider the dividend-price ratio for the firm, D_t/P_t. Then,*

$$1 + \frac{D_t}{P_t} = \left(1 + \frac{D_t^l}{l_t}\right)\frac{N_t}{N_{t+1}} \tag{A.15}$$

Thus, if the labor market is in a steady state (defined as $N_t = N_{t+1}$), $D_t/P_t = D_t^l/l_t$.

Proof It follows from (10), the definition of dividends (7), and the law of motion for N_t (9) that

$$\begin{aligned} P_t + D_t &= l_t N_{t+1} + Z_t N_t - W_t N_t - \kappa_t V_t \\ &= (Z_t - W_t + l_t(1 - s))N_t \end{aligned}$$

Thus

$$
\begin{aligned}
1 + \frac{D_t}{P_t} &= \frac{P_t + D_t}{P_t} \\
&= \frac{Z_t - W_t + l_t(1-s)}{l_t} \frac{N_t}{N_{t+1}} \\
&= \left(1 + \frac{D_t^l}{l_t}\right) \frac{N_t}{N_{t+1}}
\end{aligned}
$$

where the last line follows from (A.13).

The following lemma gives a comparative static result on the price-dividend ratio. It is strictly applicable in the case of iid productivity growth (in our specification, constant disaster probability) because it relies on constant labor market tightness θ. When λ_t is constant, the economy converges deterministically to a steady state where θ is constant.

Lemma A.4. *When the labor market is in a steady state ($\theta_{t+1} = \theta_t$), the price-dividend ratio is increasing in θ.*

Proof. It follows from (A.14) that

$$
\frac{D_t^l}{l_t} = \frac{1 - w(\theta)}{\frac{\kappa}{q(\theta)}} - s
$$

Because of (4), the first term is proportional to $(1 - w(\theta))\theta^{-\eta}$. It follows from (A.11) that $w(\theta)$ is increasing in θ (intuitively, wages are increasing in tightness). It is also necessary that $1 - w(\theta)$ is positive; otherwise, in this iid economy the firm would operate continually at a loss. Therefore $(1 - w(\theta))\theta^{-\eta}$ is decreasing in θ, and, by the second statement in Lemma A.3, the price-dividend ratio is increasing in θ. $\square$

B Constant Disaster Risk Model

Appendix B.1 describes the compound Poisson process that is useful in the constant disaster risk case. Appendix B.2 provides proofs for this case. When disaster risk is constant, labor market variables N_t, V_t and θ_t are deterministic. We assume that the economy has run for long enough that it has reached its steady state, with $N_t = N_{t+1}$, and similarly for V_t and θ_t.

B.1 Compound Poisson Process

The algebraic rules for compound Poisson processes illustrated in this section are adapted from Cont and Tankov (2004). Drechsler and Yaron (2011) model jumps in expected growth and volatility using compound Poisson processes. Let $Q_{t,t+1}$ be a compound Poisson process with intensity $\tilde{\lambda}$. Specifically, $\tilde{\lambda}$ represents the expected number of jumps in the time period $(t, t+1]$. Agents in the model view the jumps in $(t, t+1]$ as occurring at $t+1$. Then, $Q_{t,t+1}$ is given by

$$Q_{t,t+1} = \begin{cases} \sum_{i=1}^{\mathcal{N}_{t+1}-\mathcal{N}_t} \zeta_i & \text{if } \mathcal{N}_{t+1} - \mathcal{N}_t > 0 \\ 0 & \text{if } \mathcal{N}_{t+1} - \mathcal{N}_t = 0, \end{cases}$$

where $\mathcal{N}_t$ is a Poisson counting process and $\mathcal{N}_{t+1} - \mathcal{N}_t$ is the number of jumps in the time interval $(t, t+1]$. Jump size ζ is *iid*. We can take conditional expectations with $Q_{t,t+1}$ using

$$\mathbb{E}_t\left[e^{uQ_{t+1}}\right] = e^{\tilde{\lambda}\left(\mathbb{E}\left[e^{u\zeta}\right]-1\right)}, \tag{B.1}$$

where log of the right-hand side is the cumulant-generating function of $Q_{t,t+1}$. More precisely, the probability of observing k jumps over the course one period $(t, t+1]$ is equal to $e^{\tilde{\lambda}}\frac{\tilde{\lambda}^k}{k!}$. We take the t to be in units of months in our quantitative assessment of the model.

B.2 Proof for the constant disaster risk case

We first prove the equation and comparative statics for the effective time discount factor.

Proof of Theorem 2 Consider the normalized value function in (25) and replace the disaster term with the compound Poisson process $Q_{t,t+1}$ with constant intensity $\tilde{\lambda}$:

$$j(\tilde{\lambda}, N_t) = \left[c_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t\left[e^{(1-\gamma)(\mu+\epsilon_{t+1}+Q_{t,t+1})} j(\tilde{\lambda}, N_{t+1})^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}. \tag{B.2}$$

Conditional on time-t information, the realizations of ϵ_{t+1}, $Q_{t,t+1}$ and N_{t+1} are independent. Therefore, we can write (26) with

$$\hat{\beta}(\tilde{\lambda}) = \beta \mathbb{E}_t \left[e^{(1-\gamma)Q_{t,t+1}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{B.3}$$

Taking the expectation using the algebra introduced in Appendix B.1, we compute the log of the effective time discount factor:

$$\log \hat{\beta}(\tilde{\lambda}) = \log \beta + \frac{1-\frac{1}{\psi}}{1-\gamma} \left(\mathbb{E}\left[e^{(1-\gamma)\zeta} \right] - 1 \right) \tilde{\lambda}. \tag{B.4}$$

Note that ζ takes only negative values. For $\gamma > 1$ and $\gamma < 1$ we have

$$\frac{\mathbb{E}\left[e^{(1-\gamma)\zeta} \right] - 1}{1-\gamma} < 0. \tag{B.5}$$

Therefore, $\log \hat{\beta}(\tilde{\lambda})$ is decreasing in $\tilde{\lambda}$ if and only if $1 - \frac{1}{\psi} > 0$ which is equivalent to $\psi > 1$.

Next we derive the equation for the riskfree rate:

Proof of Lemma 1 Because λ_t is constant and the economy is at its steady state, the stochastic discount factor (15) becomes:

$$M_{t+1} = \frac{\beta e^{-\frac{\mu}{\psi} - \gamma(\epsilon_{t+1} + Q_{t+1})}}{\mathbb{E}_t \left[e^{(1-\gamma)(\epsilon_{t+1} + Q_{t+1})} \right]^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}}. \tag{B.6}$$

Here we have used (24) to substitute in for the value function. Taking the expectation in the denominator, the log stochastic discount factor becomes

$$\begin{aligned} \log M_{t+1} = {} & \log \beta - \frac{\mu}{\psi} - \gamma(\epsilon_{t+1} + Q_{t+1}) \\ & - \frac{1}{2} \left(\frac{1}{\psi} - \gamma \right) (1-\gamma)\, \sigma_\epsilon^2 - \frac{\frac{1}{\psi} - \gamma}{1-\gamma} \left(\mathbb{E}\left[e^{(1-\gamma)\zeta} \right] \right) \tilde{\lambda}. \end{aligned} \tag{B.7}$$

It follows that the log risk-free rate $\log R_f = -\log \mathbb{E}[M_{t+1}]$ is given by:

$$\begin{aligned} \log R_f = {} & -\log \beta + \frac{\mu}{\psi} + \frac{1}{2} \left(\frac{1}{\psi} - \frac{\gamma}{\psi} - \gamma \right) \sigma_\epsilon^2 \\ & + \left[\frac{\frac{1}{\psi} - \gamma}{1-\gamma} \left(\mathbb{E}\left[e^{(1-\gamma)\zeta} \right] - 1 \right) - \left(\mathbb{E}\left[e^{-\gamma\zeta} \right] - 1 \right) \right] \tilde{\lambda}. \end{aligned} \tag{B.8}$$

Note that the term $\frac{\frac{1}{\psi}-\gamma}{1-\gamma}$ is bounded above by $\gamma/(\gamma-1)$. The properties of the exponential implies $\frac{1}{\gamma}\mathbb{E}\left(\left[e^{-\gamma\zeta}\right]-1\right) > \frac{1}{\gamma-1}\mathbb{E}\left(\left[e^{-\gamma\zeta}\right]-1\right)$, which, together with the fact that ζ takes only negative values, implies that the risk-free rate is decreasing in disaster intensity (Tsai and Wachter (2015)).

The following Lemma recharacterizes the Euler equation in terms of model primitives:

Lemma B.1. *The first-order conditions of the firm imply*

$$\hat{\beta}(\tilde{\lambda})e^{\mu\left(1-\frac{1}{\psi}\right)+\frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\sigma_\epsilon^2}\left[\frac{1-w(\theta)+(1-s)\frac{\kappa}{q(\theta)}}{\frac{\kappa}{q(\theta)}}\right]=1, \tag{B.9}$$

where $w(\theta)$ is the wage normalized by productivity defined by (A.11) and $\hat{\beta}(\tilde{\lambda})$ is defined as in (B.3).

Proof We rewrite the Euler equation (A.5) in a more familiar form

$$E_t\left[M_{t+1}R_{t+1}\right]=1 \tag{B.10}$$

where we have divided through by l_t and used the characterization of returns in (11). The result follows from substituting (A.10) and (B.6) into (B.10) and solving the expectation using the definition of $\hat{\beta}(\tilde{\lambda})$ in (B.3).

Lemma B.2. *The log expected equity return is given by*

$$\begin{aligned}\log\mathbb{E}_t\left[R_{t+1}\right]=&-\log(\beta)+\frac{\mu}{\psi}+\frac{1}{2}\left(\frac{1}{\psi}-\frac{\gamma}{\psi}+\gamma\right)\\&+\underbrace{\left(\mathbb{E}\left[e^{\zeta}\right]-1\right)}_{\textit{Productivity growth}}\tilde{\lambda}\\&-\underbrace{\left(\frac{1-\frac{1}{\psi}}{1-\gamma}\left(\mathbb{E}\left[e^{(1-\gamma)\zeta}\right]-1\right)\right)}_{\textit{Labor market}}\tilde{\lambda}.\end{aligned} \tag{B.11}$$

Proof. (B.9) implies

$$\frac{1-w(\theta)+(1-s)\frac{\kappa}{q(\theta)}}{\frac{\kappa}{q(\theta)}}=\frac{1}{\hat{\beta}(\tilde{\lambda})e^{\mu\left(1-\frac{1}{\psi}\right)+\frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\sigma_\epsilon^2}}. \tag{B.12}$$

Therefore, by (A.10)

$$R_{t+1} = \frac{e^{\mu+\epsilon_{t+1}+Q_{t+1}}}{\hat{\beta}(\tilde{\lambda})e^{\mu\left(1-\frac{1}{\psi}\right)+\frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\sigma_\epsilon^2}}, \tag{B.13}$$

Equation (B.11) follows from taking the expectation of (B.13) using rules introduced in Section B.1.

□

Lemma 2 follows from (B.11) and the equation for the riskfree rate given in (B.8). Corollary 2 follows from inspection of the terms multiplying $\tilde{\lambda}$ in (B.11).

Finally we establish comparative statics for the price-dividend ratio.

Proof of Theorem 3 It follows from Lemma B.1 (Equation B.9) that

$$-\log\left(1-s+\frac{1-w(\theta)}{\kappa}\xi\theta^{-\eta}\right) = \log\hat{\beta}(\tilde{\lambda}) + \mu\left(1-\frac{1}{\psi}\right) + \frac{1}{2}(1-\gamma)\left(1-\frac{1}{\psi}\right)\sigma_\epsilon^2, \tag{B.14}$$

where, from Theorem 2

$$\log\hat{\beta}(\tilde{\lambda}) = \log\beta + \frac{1-\frac{1}{\psi}}{1-\gamma}\left(\mathbb{E}\left[e^{(1-\gamma)\zeta}\right]-1\right)\tilde{\lambda}.$$

Define

$$h(\tilde{\lambda}) \equiv -\log\left(1+\frac{D_t^l}{l_t}\right) = -\log\left(1+\frac{D_t}{P_t}\right) \tag{B.15}$$

The second equality follows from Lemma A.3. The result then follows from adding 1 to (A.14) and taking the negative of the log, then substituting the result into the left hand side of (B.14).

C Equilibrium Solution

Let x' denote the value of the variable x in period $t+1$ and x the value at t. We can rewrite the normalized value function (25) as

$$g(\lambda, N) = j(\lambda, N)^{1-\frac{1}{\psi}}. \tag{C.1}$$

The value function and policy functions are functions of the exogenous state variable λ and the endogenous state variable N. The dynamics of the stochastic discount factor and returns are driven by four shocks: disaster probability λ', normal times productivity shock ϵ', disaster indicator d' and disaster size ζ'. Let $\mathbb{E}$ be the expectation operator over four shocks. In our numerical procedure, we solve for the consumption policy $c(\lambda, N)$ and the value function $g(\lambda, N)$. The market clearing condition allows us to compute the vacancy rate given the consumption policy.

It follows from (15) and (24) that the stochastic discount factor can be written as

$$\begin{aligned} M(\lambda, N; \lambda', \epsilon', d', \zeta') =& \beta e^{-\frac{\mu}{\psi}+\frac{1}{2}(1-\gamma)\left(\gamma-\frac{1}{\psi}\right)\sigma_\epsilon^2} e^{-\gamma(\epsilon'+d'\zeta')} \\ & \cdot \mathbb{E}\left[e^{(1-\gamma)d'\zeta'} g(\lambda', N')^{\frac{1-\gamma}{1-\frac{1}{\psi}}}\right]^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} \left(\frac{c(\lambda', N')}{c(\lambda, N)}\right)^{-\frac{1}{\psi}} g(\lambda', N')^{\frac{\frac{1}{\psi}-\gamma}{1-\frac{1}{\psi}}}. \end{aligned} \tag{C.2}$$

The equity return is given by

$$R(\lambda, N; \lambda', \epsilon', d', \zeta') = e^{\mu+\epsilon'+d'\zeta'} \left[\frac{1 - w(\lambda', N') + (1-s)\frac{\kappa}{q(\theta(\lambda', N'))}}{\frac{\kappa}{q(\theta(\lambda, N))}}\right], \tag{C.3}$$

where

$$w(\lambda, N) = (1-B)b + B(1 + \kappa((1-\nu)\bar{\theta} + \nu\theta(\lambda, N))) \tag{C.4}$$

and

$$\theta(\lambda, N) = \frac{N + b(1-N) - c(\lambda, N)}{\kappa(1-N)}, \tag{C.5}$$

which follows from (21).

The equilibrium conditions that $c(\lambda, N)$ and $g(\lambda, N)$ have to satisfy are

$$\mathbb{E}\left[M(\lambda, N; \lambda', \epsilon', d', \zeta') R(\lambda, N; \lambda', \epsilon', d', \zeta')\right] = 1 \tag{C.6}$$

and

$$g(N, \lambda) = c(N, \lambda)^{1-\frac{1}{\psi}} + \beta e^{\left(1-\frac{1}{\psi}\right)\mu + \frac{1}{2}\left(1-\frac{1}{\psi}\right)(1-\gamma)\sigma_\epsilon^2} \left(\mathbb{E}\left[e^{(1-\gamma)d'\zeta'} g(\lambda', N')^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}. \tag{C.7}$$

We approximate the AR(1) process for log disaster probability by a 12-state Markov process and use the corresponding probability transition matrix to calculate expectations over λ'. The expectations over ζ' and ϵ' can be taken directly since their distributions are *iid*.

We approximate the policy function and the value function by a polynomial of employment level N where the polynomial coefficients are estimated for each value of the disaster probability separately. We use $n+1$ nodes for employment to conduct the approximation by an n'th order polynomial. As a result we have $24(n+1)$ unknowns and equations resulting from the equilibrium conditions (C.6) and (C.7). We evaluate the equilibrium conditions at the nodes of the Chebyshev polynomial of order n. Our quantitative results are not significantly different for polynomial approximations of order 3 or higher.

D Data Sources

We use data from 1951 to 2013 for all variables.

- Z is the seasonally adjusted quarterly real average output per person in the nonfarm business sector, constructed by the Bureau of Labor Statistics (BLS) from National Income and Product Accounts (NIPA) and the Current Employment Statistics (CES).

- P is the real price of the S&P composite stock price index, downloaded from Robert Shiller's website (www.econ.yale.edu/ shiller/data.htm).

- P/E is the cyclically adjusted price-earnings ratio, downloaded from Robert Shiller's website (www.econ.yale.edu/ shiller/data.htm).

- P/Z is the price-productivity ratio scaled to have the same value as P/E in the first quarter of 1951.

- U is the seasonally adjusted unemployment, constructed by the BLS from the Current Population Survey (CPS). Quarterly values are calculated averaging monthly data.

- V is the help-wanted advertising index constructed by the Conference Board until June 2006. We use data on vacancy openings from Job Openings and Labor Turnover Survey (JOLTS) from 2000 to 2013. We extrapolate the help-advertising index until 2013 and observe that our extrapolation has a correlation of 0.96 in the period from 2000 to 2006 where both data sources are available. For data plots, we remove a downward sloping time trend in log V/U. Quarterly values are calculated averaging monthly data.

- W denotes wages measured as the product of labor productivity Z and labor share from the BLS.
- C is annual real personal consumption expenditures per capita from the BEA.
- Y is annual real gross domestic product (GDP) per capita from the BEA.
- R is the value weighted return market index return including distributions from CRSP. Real returns are calculated using inflation rate data from CRSP. Net returns are multiplied by 0.68 to adjust for financial leverage.
- R_b is the 1-month Treasury bill rate from CRSP. Real rates are calculated using inflation rate data from CRSP.
- Δc and Δy denote log consumption and log output growth. Annual growth rates from monthly simulations that we compare to data values are calculated aggregating consumption and output levels over every year. Let $C_{t,h}$ denote the consumption level in year t and month h. Annual log consumption growth in the model is calculated as

$$\Delta c_{t+1} = \log \left(\frac{\sum_{i=1}^{12} C_{t+1,i}}{\sum_{i=1}^{12} C_{t,i}} \right). \tag{D.1}$$

  The same method is applied to output growth as well.

References

Andolfatto, David, 1996, Business Cycles and Labor-Market Search, *American Economic Review* 86, 112–32.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Barro, Robert J, 2006, Rare Disasters and Asset Markets in the Twentieth Century, *The Quarterly Journal of Economics* 121, 823–866.

Barro, Robert J., 2009, Rare disasters, asset prices, and welfare costs, *American Economic Review* 99, 243–264.

Barro, Robert J., and Jose F. Ursua, 2008, Macroeconomic crises since 1870, *Brookings Papers on Economic Activity* 1, 255–350.

Belo, Frederico, Xiaoji Lin, and Santiago Bazdresch, 2014, Labor hiring, investment, and stock return predictability in the cross section, *Journal of Political Economy* 122, 129–177.

Blanchard, Olivier Jean, Peter Diamond, Robert E Hall, and Janet Yellen, 1989, The beveridge curve, *Brookings papers on economic activity* pp. 1–76.

Bloom, Nicholas, 2009, The impact of uncertainty shocks, *Econometrica* 77, 623–685.

Boldrin, Michele, Lawrence J Christiano, and Jonas DM Fisher, 2001, Habit persistence, asset returns, and the business cycle, *American Economic Review* 91, 149–166.

Campbell, John Y, 2003, Consumption-based asset pricing, in G. Constantinides, M. Harris, and R. Stulz, eds.: *Handbook of the Economics of Finance,* (Elsevier Science, North-Holland).

Campbell, John Y, and Robert J Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Chodorow-Reich, Gabriel, and Loukas Karabarbounis, 2015, The cyclicality of the opportunity cost of employment, Working paper, National Bureau of Economic Research.

Cont, Rama, and Peter Tankov, 2004, *Financial modelling with jump processes* vol. 2. (CRC press).

Croce, Mariano Massimiliano, 2014, Long-run productivity risk: A new hope for production-based asset pricing?, *Journal of Monetary Economics* 66, 13–31.

Diamond, Peter A, 1982, Wage determination and efficiency in search equilibrium, *The Review of Economic Studies* 49, 217–227.

Drechsler, Itamar, and Amir Yaron, 2011, What's vol got to do with it, *Review of Financial Studies* 24, 1–45.

Eckstein, Zvi, Ofer Setty, and David Weiss, 2015, Financial Risk and Unemployment, Working paper, CEPR Discussion Papers.

Epstein, Larry, and Stan Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Favilukis, Jack, and Xiaoji Lin, 2014, Wage rigidity: A quantitative solution to several asset pricing puzzles, *Fisher College of Business Working Paper*.

Gabaix, Xavier, 2011, Disasterization: A Simple Way to Fix the Asset Pricing Properties of Macroeconomic Models, *The American Economic Review* 101, 406–409.

Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, *Quarterly Journal of Economics* 127, 645–700.

Gertler, Mark, and Antonella Trigari, 2009, Unemployment fluctuations with staggered Nash wage bargaining, *Journal of Political Economy* 117, 38–86.

Gourio, François, 2012, Disaster Risk and Business Cycles, *American Economic Review* 102, 2734–2766.

Hagedorn, Marcus, and Iourii Manovskii, 2008, The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited, *American Economic Review* 98, 1692–1706.

Hall, Robert E, 1988, Intertemporal Substitution in Consumption, *The Journal of Political Economy* 96, 339–357.

Hall, Robert E, 2005, Employment Fluctuations with Equilibrium Wage Stickiness, *American Economic Review* 95, 50–65.

Hall, Robert E, 2015, High Discounts and High Unemployment, Working paper, National Bureau of Economic Research.

Hall, Robert E, and Paul R Milgrom, 2008, The Limited Influence of Unemployment on the Wage Bargain, *The American Economic Review* 98, 1653–1674.

Jermann, Urban J, 1998, Asset pricing in production economies, *Journal of Monetary Economics* 41, 257–275.

Jorion, Philippe, and William N. Goetzmann, 1999, Global Stock Markets in the Twentieth Century, *The Journal of Finance* 54, 953–980.

Kaltenbrunner, Georg, and Lars A Lochstoer, 2010, Long-run risk through consumption smoothing, *Review of Financial Studies* 23, 3190–3224.

Kreps, D., and E. Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.

Kydland, Finn E, and Edward C Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica* pp. 1345–1370.

Lemmon, Michael L, Michael R Roberts, and Jaime F Zender, 2008, Back to the beginning: persistence and the cross-section of corporate capital structure, *The Journal of Finance* 63, 1575–1608.

Lettau, Martin, and Harald Uhlig, 2000, Can habit formation be reconciled with business cycle facts?, *Review of Economic Dynamics* 3, 79–99.

Merz, Monika, 1995, Search in the labor market and the real business cycle, *Journal of Monetary Economics* 36, 269–300.

Mortensen, Dale T, and Christopher A Pissarides, 1994, Job creation and job destruction in the theory of unemployment, *The Review of Economic Studies* 61, 397–415.

Nakamura, Emi, Jon Steinsson, Robert Barro, and Jose Ursua, 2013, Crises and Recoveries in an Empirical Model of Consumption Disasters, *American Economic Journal: Macroeconomics* 5, 35–74.

Petrongolo, Barbara, and Christopher A Pissarides, 2001, Looking into the Black Box: A Survey of the Matching Function, *Journal of Economic Literature* 39, 390–431.

Petrosky-Nadeau, Nicolas, Lu Zhang, and Lars-Alexander Kuehn, 2013, Endogenous economic disasters and asset prices, *Fisher College of Business Working Paper*.

Pissarides, Christopher, 1985, Short-run equilibrium dynamics of unemployment, vacancies, and real wages, *American Economic Review* 85, 676–690.

Pissarides, Christopher A, 2000, *Equilibrium unemployment theory*. (MIT press).

Rietz, Thomas A, 1988, The equity risk premium a solution, *Journal of Monetary Economics* 22, 117–131.

Shimer, Robert, 2005, The Cyclical Behavior of Equilibrium Unemployment and Vacancies, *American Economic Review* 95, 25–49.

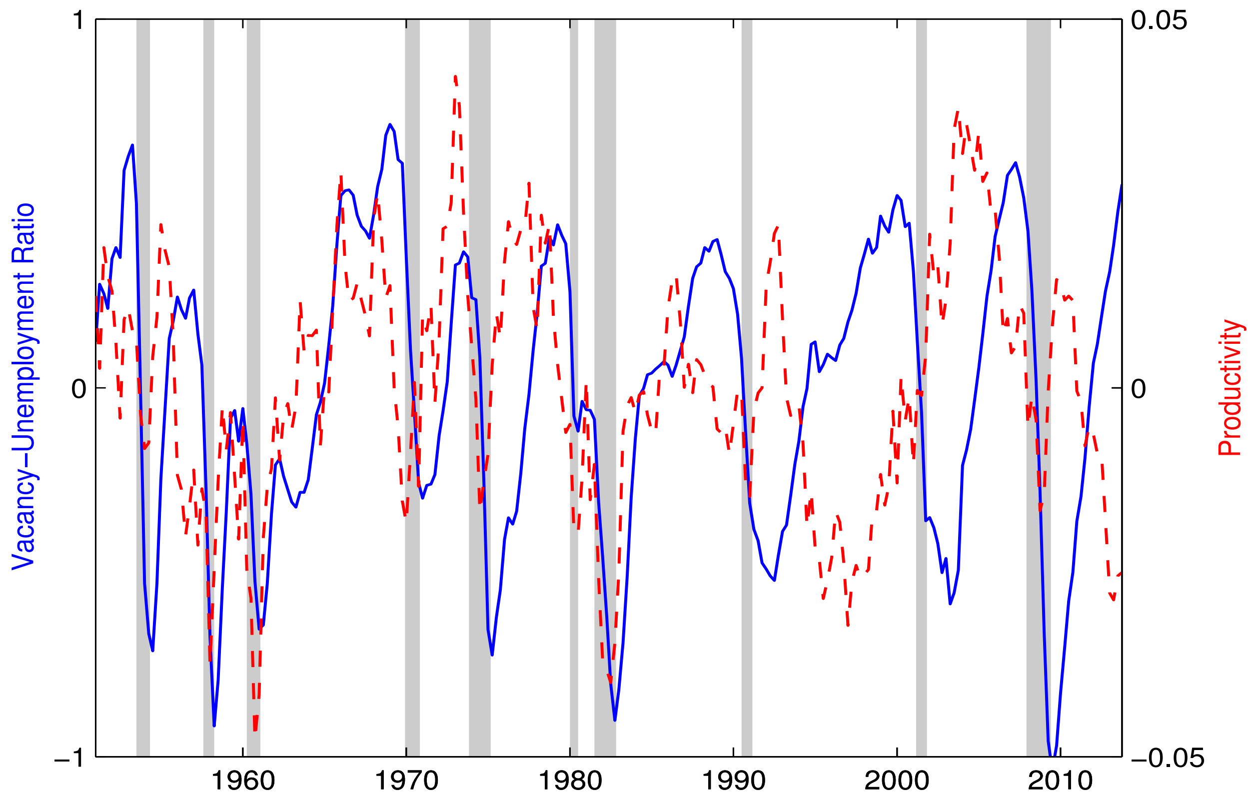
Tsai, Jerry, and Jessica A Wachter, 2015, Disaster Risk and Its Implications for Asset Pricing, *Annual Review of Financial Economics* 7.

Uhlig, Harald, 2007, Explaining asset prices with external habits and wage rigidities in a DSGE model, *American Economic Review* 97, 239–243.

Wachter, Jessica A, 2013, Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?, *The Journal of Finance* 68, 987–1035.

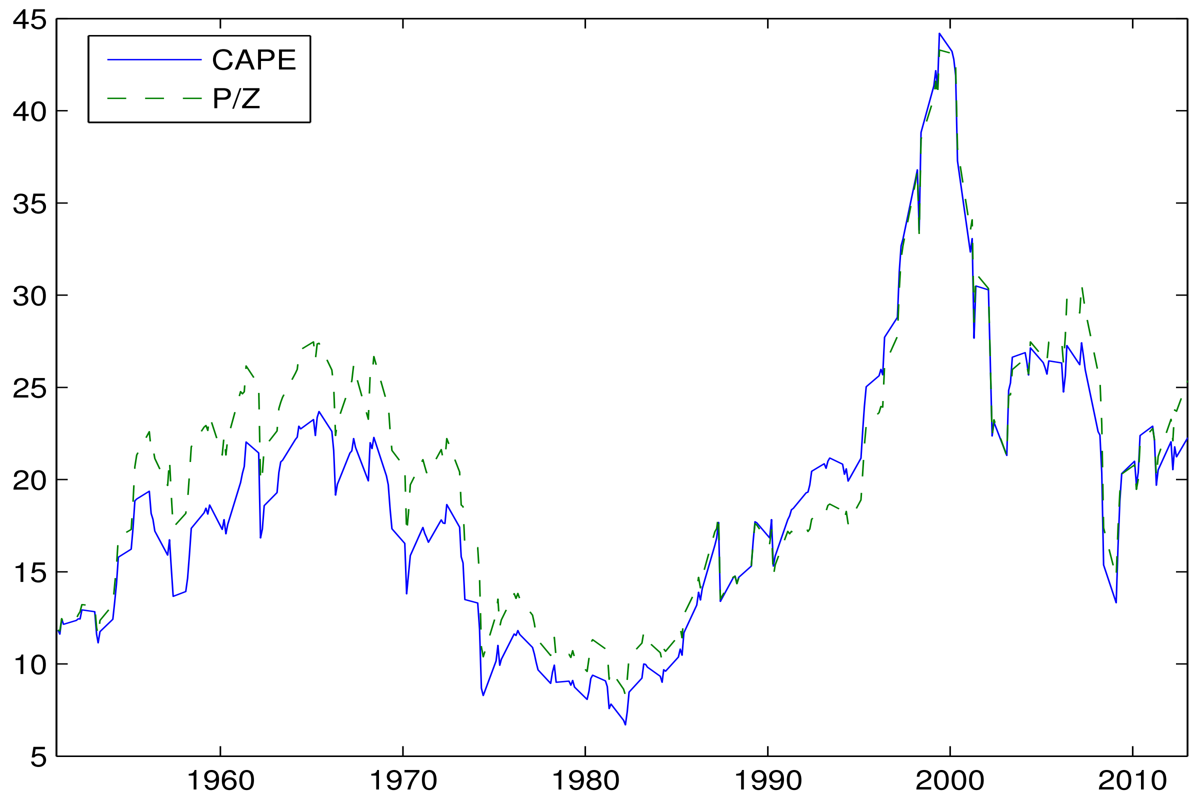
Yashiv, Eran, 2000, The Determinants of Equilibrium Unemployment, *American Economic Review* 90, 1297–1322.

Figure 1: Vacancy-Unemployment Ratio and Labor Productivity: 1951 - 2013



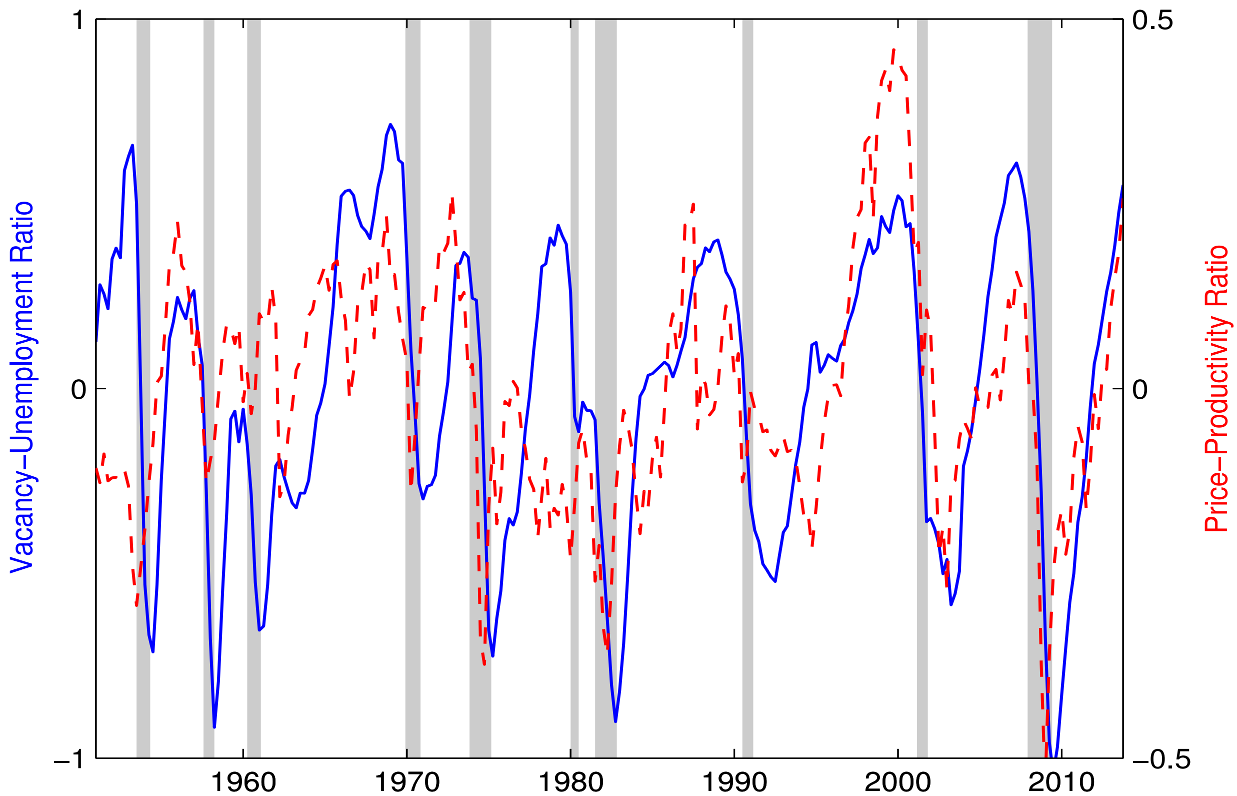
Notes: The solid line shows the vacancy-unemployment ratio, the dashed line labor productivity. Both variables are reported as log deviations from an HP trend with smoothing parameter 10^5. Shaded periods are NBER recessions.

Figure 2: Valuation Ratios: 1951 - 2013



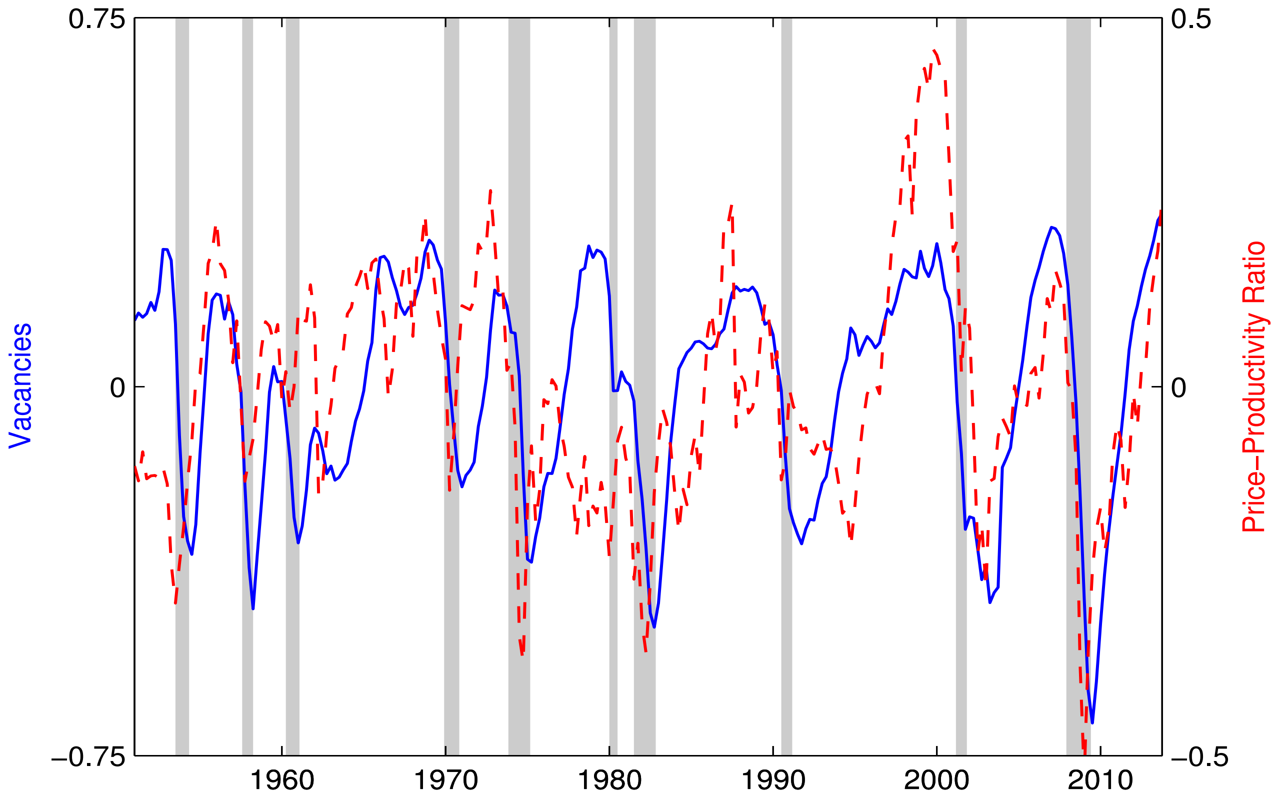
Notes: P/Z denotes the price-productivity ratio defined as the real price of the S&P composite stock price index P divided by labor productivity Z. P/E is the cyclically adjusted price-earnings ratio of the S&P composite stock price index. P/Z is scaled such that P/Z and P/E are equal in the first quarter of 1951.

Figure 3: Vacancy-Unemployment Ratio and Price-Productivity Ratio: 1951 - 2013



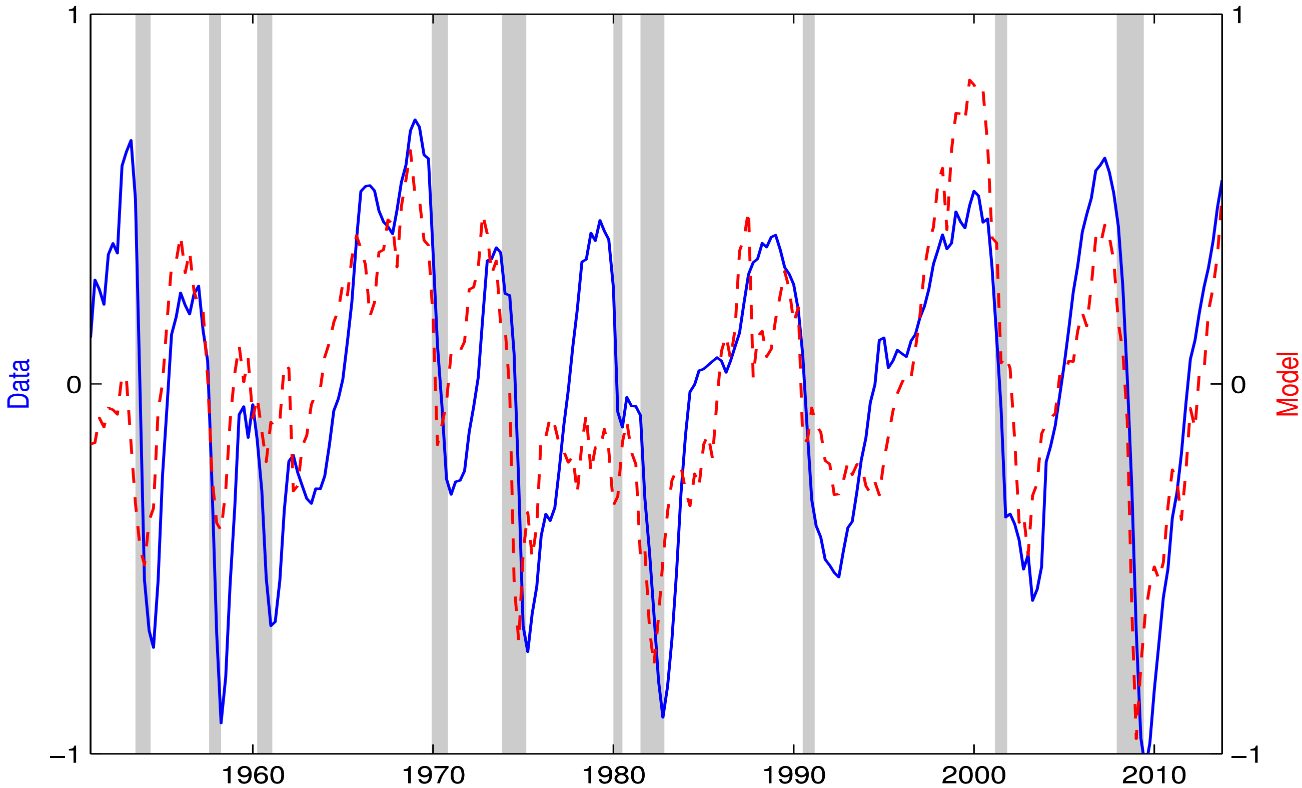
Notes: The solid line shows the vacancy-unemployment ratio, the dashed line the price-productivity ratio. Both variables are reported as log deviations from an HP trend with smoothing parameter 10^5. Shaded periods are NBER recessions.

Figure 4: Vacancy Openings and Price-Productivity Ratio: 1951 - 2013



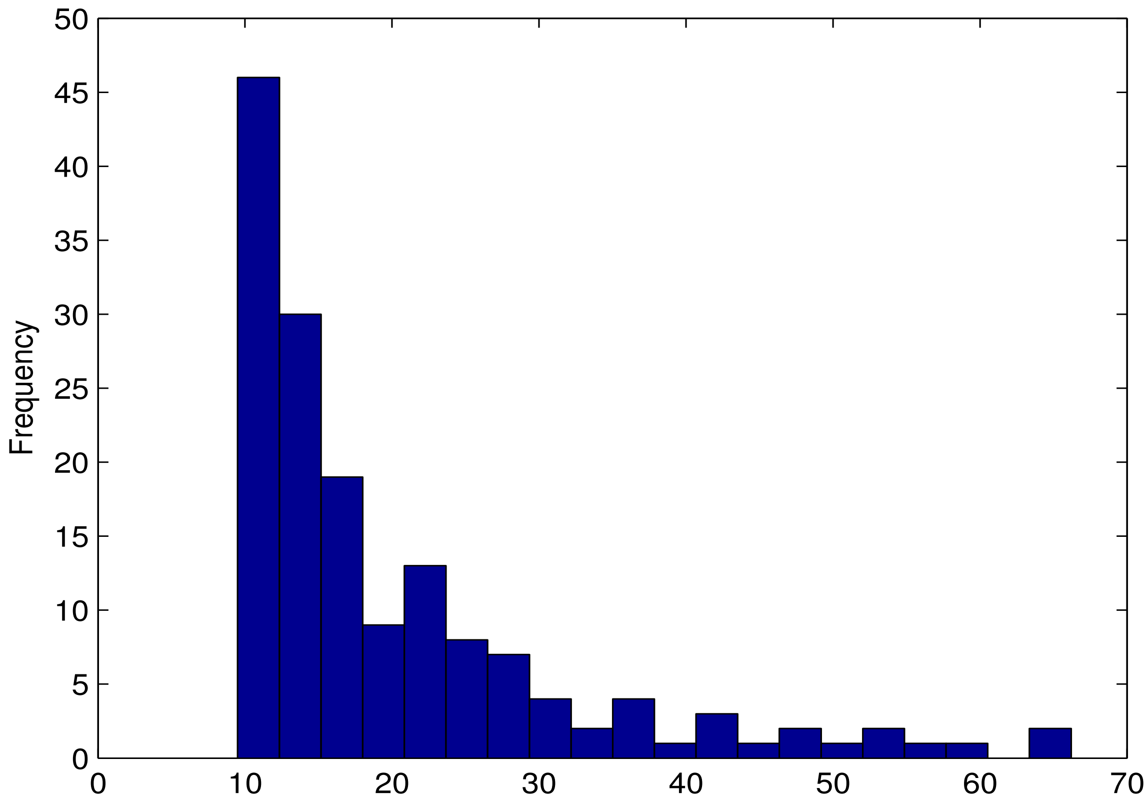
Notes: The solid line shows vacancies, the dashed line the price-productivity ratio. Both variables are reported as log deviations from an HP trend with smoothing parameter 10^5. Shaded periods are NBER recessions.

Figure 5: Vacancy-Unemployment Ratio: Data vs. Model



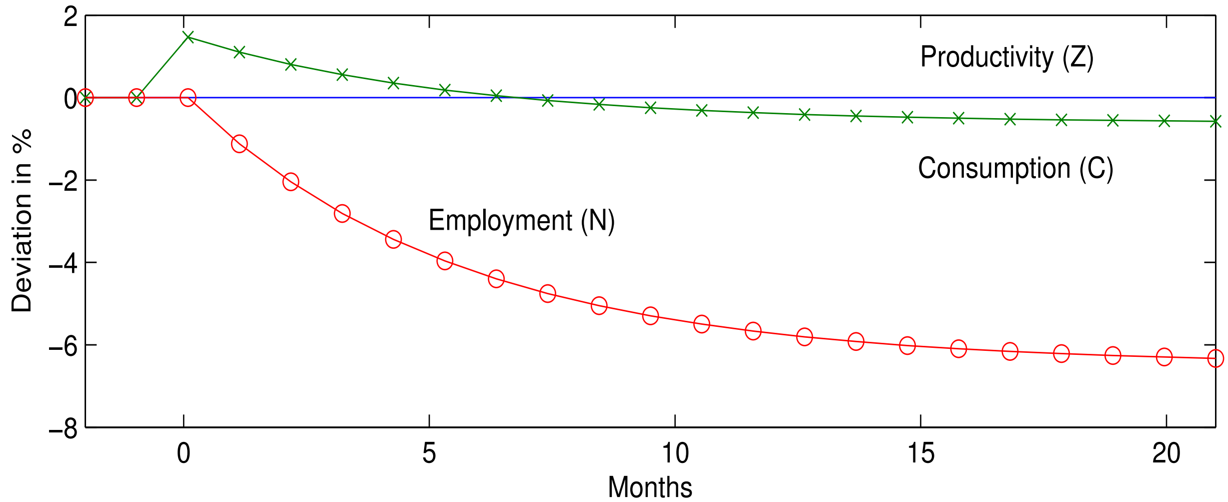
Notes: The solid line and the dashed line show the vacancy-unemployment ratio in the data and in the model, respectively. Model-implied vacancies are calculated by substituting the price-productivity ratio and employment level from the data into equation (12), assuming labor-market parameters given in Table 1. Values are log deviations from an HP trend with smoothing parameter 10^5. Shaded periods are NBER recessions.

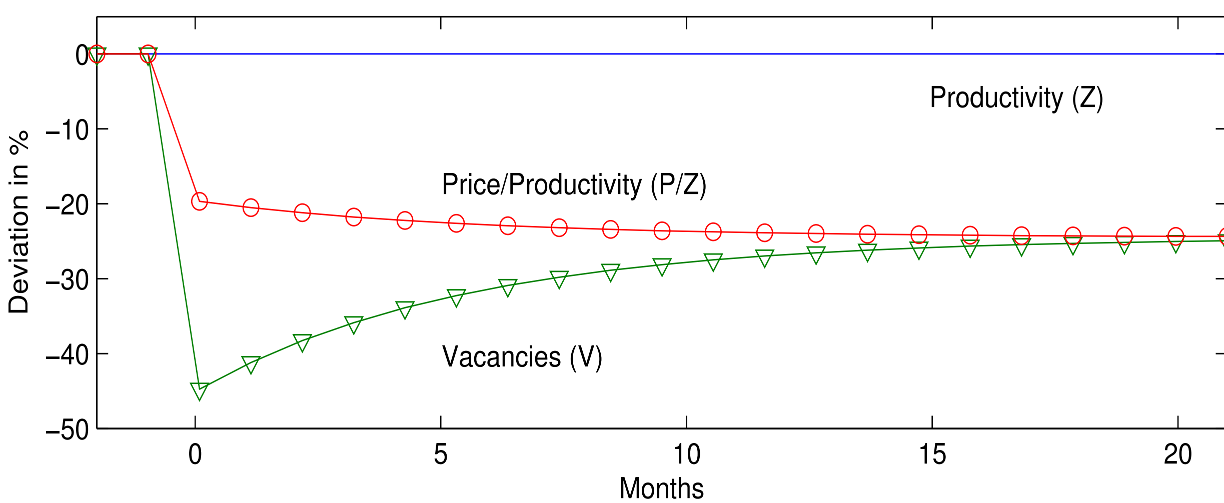
Figure 6: Size Distribution of Disaster Realizations



Notes: Histogram shows the distribution of large declines in GDP per capita (in percentages). Data are from Barro and Ursua (2008). Values correspond to $1 - e^{\zeta}$ in the model.

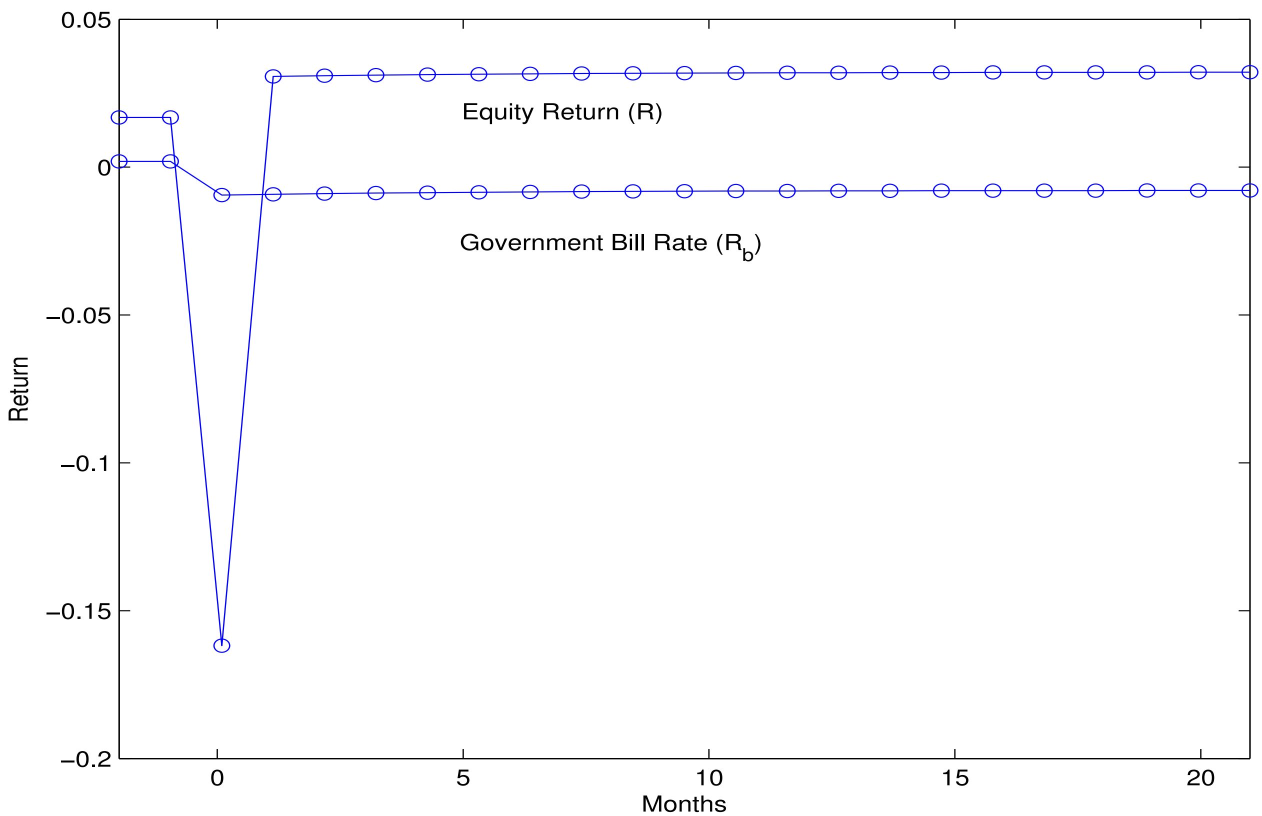
Figure 7: Macroeconomic Response to Increase in Disaster Probability





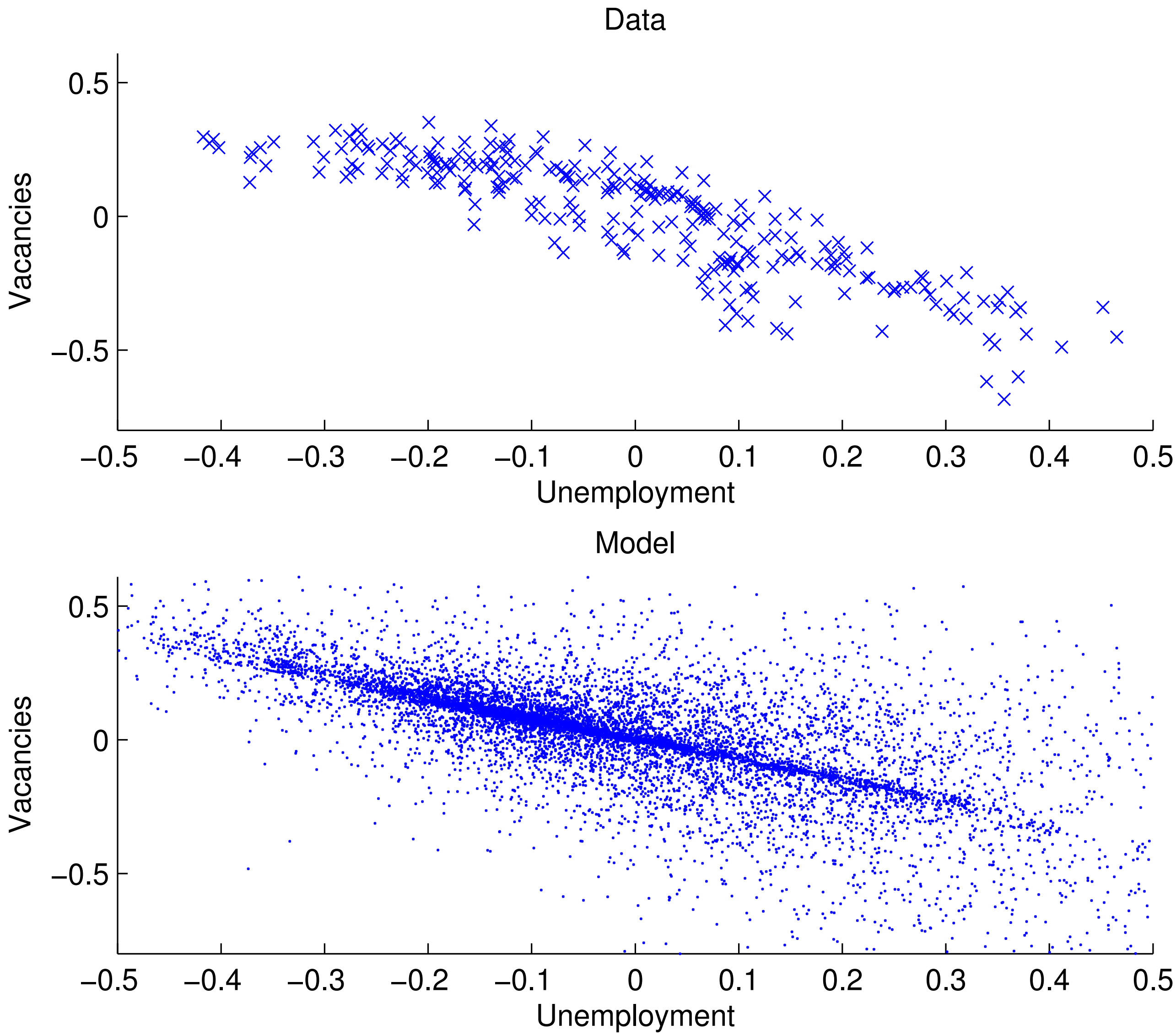
Notes: In month zero, monthly disaster probability increases from 0.05% to 0.32% and stays at 0.32% in the remaining months.

Figure 8: Return Response to Increase in Disaster Probability



Notes: In month zero, monthly disaster probability increases from 0.05% to 0.32% and stays there in the remaining months.

Figure 9: Beveridge Curve



Notes: Data are quarterly from 1951 to 2013. Model implied curve is a quarterly sample with length 10,000 years from the stationary distribution. All values are log deviations from an HP trend with smoothing parameter 10^5.

Table 1: Parameters Values for Monthly Benchmark Calibration

Parameter	Value
Time preference, β	0.997
Risk aversion, γ	5.7
Elasticity of intertemporal substitution, ψ	2
Disaster distribution (GDP), ζ	multinomial
Productivity growth, μ	0.0018
Productivity volatility, σ_ϵ	0.0047
Matching efficiency, ξ	0.365
Separation rate, s	0.035
Matching function parameter, η	0.35
Bargaining power, B	0.50
Value of non-market activity, b	0.76
Vacancy cost, κ	0.50
Tightness insulation, ν	0.05
Government default probability, q	0.40

Table 2: Properties of Aggregate Wages

	SD	AC	$\epsilon_{W,\theta}$	$\epsilon_{W,Z}$	$\epsilon_{\theta,Z}$
Panel A: Data					
1951 - 2013	1.77	0.91	0.00	0.67	2.46
	—	—	[0.33]	[5.43]	[0.76]
1951 - 1985	1.21	0.91	0.01	0.35	11.22
	—	—	[2.75]	[3.04]	[3.86]
1986 - 2013	2.29	0.91	-0.01	1.07	-8.49
	—	—	[-1.15]	[6.79]	[-2.37]
Panel B: Benchmark model					
50%	1.71	0.91	0.01	0.99	0.00
5%	1.33	0.87	-0.01	0.95	-6.39
95%	2.31	0.95	0.03	1.05	6.08
Panel C: No tightness insulation					
50%	2.26	0.89	0.13	1.00	0.04
5%	1.80	0.83	0.08	0.74	-1.95
95%	2.89	0.93	0.18	1.27	1.93

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Z is labor productivity, θ labor market tightness. Data are from 1951 to 2013. All data and model moments are in quarterly terms. We simulate 10,000 samples with length 60 years at monthly frequency and report quantiles from 53% of simulations that include no disaster realization. $\epsilon_{x,y}$ is the elasticity of variable x to y, namely, the regression coefficient of log x on log y. Data t-statistics in brackets are based on Newey-West standard errors. All variables are used in logs as deviations from an HP trend with smoothing parameter 10^5.

Table 3: Monthly Disaster Probability

Value	Stationary Probability
1×10^{-7}	0.0005
7×10^{-7}	0.0054
4×10^{-6}	0.0269
3×10^{-5}	0.0806
0.0002	0.1611
0.0012	0.2256
0.0076	0.2256
0.0495	0.1611
0.3212	0.0806
2.0827	0.0269
13.5045	0.0054
87.5661	0.0005

Notes: Table lists the nodes of a 12-state Markov process which approximates an AR(1) process for log probabilities. Disaster probabilities are in percentage terms.

Table 4: Monthly Disaster Probability in Simulations

		No-Disaster				All Simulations			
	Population	Mean	5%	50%	95%	Mean	5%	50%	95%
$\mathbb{E}[\lambda]$	0.20	0.05	0.01	0.03	0.16	0.20	0.01	0.07	0.75
$\sigma(\lambda)$	1.97	0.20	0.01	0.11	0.58	0.84	0.02	0.27	2.81
$\rho(\lambda)$	0.91	0.86	0.65	0.89	0.96	0.87	0.66	0.90	0.96

Notes: σ denotes volatility, ρ monthly autocorrelation. Disaster probabilities are in percentage terms. Population is a sample of 100,000 years. We simulate 10,000 samples with length 60 years at monthly frequency and report statistics from all simulations as well as from 53% of simulations that include no disaster realization. All simulations are in monthly frequency.

Table 5: Labor Market Moments

	U	V	V/U	Z	P/Z	
Panel A: Data						
SD	0.19	0.21	0.39	0.02	0.16	
AC	0.94	0.94	0.95	0.88	0.89	
	1	-0.86	-0.96	-0.18	-0.44	U
	—	1	0.97	0.03	0.47	V
	—	—	1	0.10	0.47	V/U
	—	—	—	1	0.00	Z
	—	—	—	—	1	P/Z
Panel B: No-Disaster Simulations						
SD	0.17	0.19	0.33	0.02	0.14	
	(0.04)	(0.05)	(0.07)	(0.01)	(0.03)	
AC	0.95	0.76	0.90	0.93	0.91	
	(0.01)	(0.04)	(0.02)	(0.03)	(0.02)	
	1	-0.68	-0.90	-0.06	-0.92	U
	—	1	0.93	-0.06	0.90	V
	—	—	1	0.00	0.99	V/U
	—	—	—	1	0.01	Z
	—	—	—	—	1	P/Z
Panel C: Population						
SD	0.19	0.22	0.39	0.04	0.17	
AC	0.95	0.76	0.90	0.93	0.91	
	1	-0.69	-0.91	-0.06	-0.92	U
	—	1	0.93	-0.06	0.90	V
	—	—	1	0.00	0.99	V/U
	—	—	—	1	0.01	Z
	—	—	—	—	1	P/Z

Notes: SD denotes standard deviation, AC quarterly autocorrelation. Data are from 1951 to 2013. All data and model moments are in quarterly terms. U is unemployment, V vacancies, Z labor productivity and P/Z price-productivity ratio. We simulate 10,000 samples with length 60 years at monthly frequency and report means from 53% of simulations that include no disaster realization in Panel B. Standard errors across simulations are reported in parentheses. Population values in Panel C are from a path with length 100,000 years at monthly frequency. Standard deviations, autocorrelations and the correlation matrix are calculated using log deviations from an HP trend with smoothing parameter 10^5.

Table 6: Business Cycle and Financial Moments

	$\mathbb{E}[\Delta c]$	$\mathbb{E}[\Delta y]$	$\sigma(\Delta c)$	$\sigma(\Delta y)$	$\mathbb{E}[R - R_b]$	$\mathbb{E}[R_b]$	$\sigma(R)$	$\sigma(R_b)$
Data	1.97	1.90	1.78	2.29	5.32	1.01	12.26	2.22
Simulation 50%	2.16	2.16	2.28	2.47	6.66	3.64	19.78	3.83
Simulation 5%	1.80	1.79	1.59	1.71	-0.02	0.06	11.75	0.87
Simulation 95%	2.51	2.54	3.44	3.72	20.39	4.96	33.94	12.50
Population	1.63	1.63	6.85	6.89	13.32	1.22	38.97	12.19

Notes: The table reports means and volatilities of log consumption growth (Δc), log output growth (Δy), the government bill rate (R_b) and the unlevered equity return R in historical data and in data simulated from the model. All data and model moments are in annual terms. Historical data are from 1951-2013. We simulate 10,000 samples with length 60 years from the model and report quantiles from 53% of simulations that include no disaster realization. Population values are from a path with length 100,000 years. In the data, net equity returns are multiplied by 0.72 to adjust for leverage. Raw equity returns in the data have a premium of 7.90% and volatility of 17.55% over this period.

Table 7: Comparative Statics for Labor Market Volatility

	U	V	V/U	Z	P/Z
Data	0.19	0.21	0.39	0.02	0.16
Benchmark	0.17	0.19	0.33	0.02	0.14
Constant λ	0.00	0.00	0.00	0.02	0.00
No disaster	0.00	0.00	0.00	0.02	0.00
No tightness insulation	0.06	0.06	0.11	0.02	0.05

Notes: Standard deviations (in log deviations from an HP trend) for unemployment (U), vacancies (V), labor productivity (Z) and the price-productivity ratio (P/Z) in the data and in four versions of the model. Data are from 1951 to 2013. All data and model moments are in quarterly terms. Model values are calculated by simulating 10,000 samples with length 60 years at a monthly frequency. We report means from simulations that include no disaster realizations. In the constant disaster probability model, we set disaster probability to 0.20%, the stationary mean.

Table 8: Comparative Statics for Business Cycle and Financial Moments

	$\mathbb{E}[\Delta c]$	$\mathbb{E}[\Delta y]$	$\sigma(\Delta c)$	$\sigma(\Delta y)$	$\mathbb{E}[R - R_b]$	$\mathbb{E}[R_b]$	$\sigma(R)$	$\sigma(R_b)$
Data	1.97	1.90	1.78	2.29	5.32	1.01	12.26	2.22
Panel A: Benchmark								
50%	2.16	2.16	2.28	2.47	6.66	3.64	19.78	3.83
Population	1.63	1.63	6.85	6.89	13.32	1.22	38.97	12.19
Panel B: Constant λ								
50%	2.16	2.16	1.31	1.31	10.27	-3.48	1.73	0.00
Population	1.59	1.59	4.03	4.03	9.94	-3.66	3.49	2.16
Panel C: No Disaster Risk								
50%	2.16	2.16	1.32	1.32	0.16	5.12	1.70	0.00
Population	2.16	2.16	1.32	1.32	0.16	5.12	1.71	0.00
Panel D: No Tightness Insulation								
50%	2.16	2.16	1.47	1.52	-49.63	3.67	11.55	3.32
Population	1.68	1.68	6.46	6.44	-47.76	1.53	20.32	11.27

Notes: Δc denotes log consumption growth, Δy log output growth, R the unlevered equity return, R_b the government bill rate. All data and model moments are in annual terms. We simulate 10,000 samples with length 60 years at monthly frequency and report the median from samples that contain no disasters. In the constant disaster probability model, we set disaster probability to 0.20%, the stationary mean of the disaster probability process used in the benchmark model. Population values are from a path with length 100,000 years. Returns and growth rates are aggregated to annual values.