# Internet Appendix for: <br> "Foreseen Risks" 

João F. Gomes*<br>Marco Grotteria ${ }^{\dagger}$<br>Jessica A. Wachter ${ }^{\ddagger}$

January 16, 2023

[^0]
## Contents

1 Analytical model solution and model extension to leverage choices ..... 2
1.1 Model without leverage choices ..... 2
1.2 Model with both portfolio and leverage choice ..... 9
2 Additional Results ..... 15

## 1 Analytical model solution and model extension to leverage choices

### 1.1 Model without leverage choices

Terminal period $N$ In the terminal period $N$ the bank has no continuation value and solves:

$$
\begin{align*}
V(N)=\max _{\varphi}[ & (1-p)\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}-R^{D}\right) \\
& \left.p(1-q)\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}-R^{D}\right) \mathbb{1}_{\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}>R^{D}}\right]  \tag{1}\\
& \text { s.t. } \varphi \leq 1 \tag{2}
\end{align*}
$$

We separate the solutions in two cases, i.e., a case in which the bank invests in such a way that it cannot default in state $m$ and a case in which the bank defaults also in state $m$. We then compare the maximum bank's value in those 2 cases.

Case 1: Bank decides in such a way that it avoids default if next period state $m$ realizes $\varphi \zeta_{L}+(1-\varphi) \zeta_{G m} \geq R^{D}$, or $\varphi \leq \frac{\zeta_{G m}-R^{D}}{\zeta_{G m}-\zeta_{L}}(=\bar{\varphi}) . \bar{\varphi}$ by the assumptions made is between 0 and 1.

Call $V_{\bar{\varphi}}$ the value of the bank when the allocation $\varphi$ is constrained to be lower than $\bar{\varphi}$.

$$
\begin{align*}
V_{\bar{\varphi}}(N)=\max _{\varphi}[ & (1-p)\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}-R^{D}\right) \\
& \left.p(1-q)\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}-R^{D}\right)\right]  \tag{3}\\
& \text { s.t. } \varphi \leq \bar{\varphi} \tag{4}
\end{align*}
$$

Using (2) and (3) in the main text,

$$
\begin{align*}
V_{\bar{\varphi}}(N)=\max _{\varphi}[ & {\left[1-p q \zeta_{G d}-(1-p q) R^{D}+\varphi p q\left(\zeta_{G d}-\zeta_{L}\right)\right] }  \tag{5}\\
& \text { s.t. } \varphi \leq \bar{\varphi} \tag{6}
\end{align*}
$$

Since this is linear and increasing in $\varphi, \varphi^{*}=\bar{\varphi}$ and thus

$$
\begin{equation*}
V_{\bar{\varphi}}^{*}(N)=\left[(1-p)\left(\bar{\varphi} \bar{R}^{L}+(1-\bar{\varphi}) \bar{R}^{G}-R^{D}\right)\right] \tag{7}
\end{equation*}
$$

Case 2: Bank decides in such a way that it will default if next period state $m$ realizes
$\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}<R^{D}$, or $\varphi>\bar{\varphi}$
Call $V_{1}$ the value of the bank when the allocation $\varphi$ is constrained to be lower than 1 .

$$
\begin{equation*}
V_{1}(N)=\left[(1-p)\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}-R^{D}\right)\right] \tag{8}
\end{equation*}
$$

Again this is increasing in $\varphi, \varphi^{*}=1$ and thus

$$
\begin{equation*}
V_{1}^{*}(N)=\left[(1-p)\left(\bar{R}^{L}-R^{D}\right)\right] \tag{9}
\end{equation*}
$$

Thus from (9) and (7), we get the optimal $\varphi$ in the last period as

$$
\begin{equation*}
\varphi^{*}(N)=1 \tag{10}
\end{equation*}
$$

and $V^{*}(N)=V_{1}^{*}(N)=(1-p)\left(\bar{R}^{L}-R^{D}\right)$
Thus the bank in the last period always chooses a $100 \%$ loan portfolio regardless of the probability $p$ because there is no continuation value to protect from default.

Period N-1 The bank has continuation value $(1-p)\left(\bar{R}^{L}-R^{D}\right)$ and solves:

$$
\begin{align*}
V(N-1)=\max _{\varphi} & {\left[(1-p)\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}-R^{D}+(1-p)\left(\bar{R}^{L}-R^{D}\right)\right)\right.} \\
& \left.p(1-q)\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}-R^{D}+(1-p)\left(\bar{R}^{L}-R^{D}\right)\right) \mathbb{1}_{\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}>R^{D}}\right] \tag{11}
\end{align*}
$$

$$
\begin{equation*}
\text { s.t. } \varphi \leq 1 \tag{12}
\end{equation*}
$$

Again, $\varphi$ does not affect the continuation value, but only whether state $m$ involves a default or not. So, compute the optimal values in the 2 cases which are now equal to

$$
\begin{align*}
& V_{\bar{\varphi}}^{*}(N-1)=(1-p)^{2}\left(\bar{R}^{L}-R^{D}\right)+(1-p)\left(\bar{R}^{G}-R^{D}+\bar{\varphi}\left(\bar{R}^{L}-\bar{R}^{G}\right)\right)+ \\
& \quad p(1-q)(1-p)\left(\bar{R}^{L}-R^{D}\right)  \tag{13}\\
& V_{1}^{*}(N-1)=\left((1-p)+(1-p)^{2}\right)\left(\bar{R}^{L}-R^{D}\right) \tag{14}
\end{align*}
$$

Both optimal values are decreasing function of $p$. The bank chooses to default if state $m$ realizes if

$$
\begin{equation*}
V_{1}^{*}(N-1)>V_{\bar{\varphi}}^{*}(N-1) \tag{15}
\end{equation*}
$$

or equivalently

$$
\begin{equation*}
1>p(1-q)+\frac{\bar{R}^{L} \zeta_{G m}-\bar{R}^{G} \zeta_{L}-R^{D}\left(\bar{R}^{L}+\zeta_{G m}-\bar{R}^{G}-\zeta_{L}\right)}{\left(\bar{R}^{L}-R^{D}\right)\left(\zeta_{G m}-\zeta_{L}\right)} \tag{16}
\end{equation*}
$$

By investing $\bar{\varphi}$ rather than 1 in loans, the bank gives up the larger returns in the state in which both the bond and the loans do not default, but does not default in state $m$. Two effects can be seen when $p$ changes. First, it becomes more valuable to retain the continuation value, i.e., investing $\bar{\varphi}$ rather than 1. Second, given the equilibrium response of $\bar{R}^{L}$ and $\bar{R}^{G}$
to $p$ the bank has more incentives to invest 1 rather than $\bar{\varphi}$.

Period $N-n$ We solve the problem for a bank in period $N-n$ where $n \leq N .{ }^{1}$ The structure of the problem allows for a simple solution for the allocation choice. Define $C V_{\varphi=1}$ to be the (potentially suboptimal) continuation value if in the last $n-1$ periods the bank always decides to fully invest in loans.

$$
\begin{equation*}
C V_{\varphi=1}=\sum_{j=1}^{n-1}(1-p)^{j}\left(\bar{R}^{L}-R^{D}\right)=\left(\bar{R}^{L}-R^{D}\right) \frac{(1-p)\left(1-(1-p)^{n-1}\right)}{p} \tag{17}
\end{equation*}
$$

This is an increasing function of the periods remaining, and (substituting for the equilibrium value of $\bar{R}^{L}$ ) a decreasing function of $p$.

Call the actual continuation value $C V$. Then $C V$ is always greater than or equal to $C V_{\varphi=1}$. For $C V$ to be greater than $C V_{\varphi=1}$, at some point in the last $n-1$ periods the bank has to choose $\bar{\varphi}$. Take a generic $n$ and assume $C V=C V_{\varphi=1}$. The condition for choosing $\bar{\varphi}$ rather than 1 is

$$
\begin{equation*}
V_{\bar{\varphi}}^{*}(N-n)>V_{1}^{*}(N-n) \tag{18}
\end{equation*}
$$

or equivalently

$$
\begin{equation*}
(1-q)\left(1-(1-p)^{n-1}\right)+\frac{\bar{R}^{G}-R^{D}+\bar{\varphi}\left(\bar{R}^{L}-\bar{R}^{G}\right)}{\bar{R}^{L}-R^{D}}>1 \tag{19}
\end{equation*}
$$

The LHS is increasing in $n$ implying that for a sufficiently large $n$ the bank can decide to invest $\bar{\varphi}$ rather than 1. More importantly, a) if the bank chooses $\bar{\varphi}$ at $N-n$, it will choose $\bar{\varphi}$ for all periods farther from the terminal period; b) if the bank decides to invest 1 at $N-n$ it will decide to invest 1 for all periods closer to the terminal period. This implies a switch in $\varphi$ as a function of $n$. But this also implies we can simplify the solution, and characterise this inequality when $n \rightarrow \infty$ (and so does $N$ ), exploiting the properties of the continuation value.

[^1]Given that the continuation term is an increasing function of $n$, if the bank chooses $\varphi=1$ when $n \rightarrow \infty$, it always does.

For $n \rightarrow \infty$, we solve

$$
\begin{equation*}
\frac{\bar{R}^{G}-R^{D}+\bar{\varphi}\left(\bar{R}^{L}-\bar{R}^{G}\right)}{\bar{R}^{L}-R^{D}}>q \tag{20}
\end{equation*}
$$

The equation can be rewritten as

$$
\begin{equation*}
\left(\zeta_{G m}-\zeta_{L}\right)(1-p)\left(\left(-1+R^{D}\right)\left(\zeta_{L}-\zeta_{G m}\right)(-1+q)+\left(-R^{D}+\zeta_{L}\right)\left(-\zeta_{G d}+\zeta_{L}\right) q p\right)<0 \tag{21}
\end{equation*}
$$

or

$$
\begin{equation*}
\left(1-R^{D}\right)\left(\zeta_{G m}-\zeta_{L}\right)(1-q)-\left(R^{D}-\zeta_{L}\right)\left(\zeta_{G d}-\zeta_{L}\right) q p>0 \tag{22}
\end{equation*}
$$

implying that for

$$
\begin{equation*}
p<\frac{\left(1-R^{D}\right)\left(\zeta_{G m}-\zeta_{L}\right)(1-q)}{\left(R^{D}-\zeta_{L}\right)\left(\zeta_{G d}-\zeta_{L}\right) q} \tag{23}
\end{equation*}
$$

the bank decides to invest in $\bar{\varphi}$ rather than 1 .
It's easy to see that for the solution to be sensible it necessary that $R^{D}<1$, so $p>0$ under our assumptions. More in general, if we had not normalized $R^{f}$ to 1 , the expression would require $R^{D}<R^{f}$ : rents in the model come from the bank paying subsidized debt, that is, we have assumed $R^{D}<R^{f}=1$.

If $\zeta_{G m}=R^{D}$, so for $\bar{\varphi}=0$, the solution simplifies to

$$
\begin{equation*}
p<\frac{\left(1-R^{D}\right)(1-q)}{\left(\zeta_{G d}-\zeta_{L}\right) q} \tag{24}
\end{equation*}
$$

Having established these results, we can now solve two separate fixed point problems, i.e., one for $\varphi=1$ and one for $\varphi=\bar{\varphi}$. When $n$ goes to infinity, if the bank chooses $\varphi=1$ in one period, it will always make the same choices in all periods.

When the bank always chooses $\varphi=1$

$$
\begin{equation*}
V_{1}=(1-p)\left(\bar{R}^{L}-R^{D}\right)+(1-p) V_{1} \tag{25}
\end{equation*}
$$

implies

$$
\begin{equation*}
V_{1}=\frac{1-p}{p}\left(\bar{R}^{L}-R^{D}\right) \tag{26}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial V_{1}}{\partial p}=\frac{R^{D}-1}{p^{2}}<0 . \tag{27}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial^{2} V_{1}}{\partial p^{2}}=\frac{2-2 R^{D}}{p^{3}}>0 \tag{28}
\end{equation*}
$$

In the presence of a bank paying subsidized debt as before, the equilibrium increase in $\bar{R}^{L}$ following an increase in $p$ does not fully compensate the bank for the loss of the continuation value. Moreover the function is convex.

Similarly when $\varphi^{*}=\bar{\varphi}$,

$$
\begin{equation*}
V_{\bar{\varphi}}=(1-p)\left(\bar{R}^{G}-R^{D}+\bar{\varphi}\left(\bar{R}^{L}-\bar{R}^{G}\right)\right)+(1-p q) V_{\bar{\varphi}} \tag{29}
\end{equation*}
$$

which simplifies to

$$
\begin{equation*}
V_{\bar{\varphi}}=\frac{(1-p)\left(\bar{R}^{G}-R^{D}+\bar{\varphi}\left(\bar{R}^{L}-\bar{R}^{G}\right)\right)}{p q} \tag{30}
\end{equation*}
$$

whereby

$$
\begin{equation*}
\frac{\partial V_{\bar{\varphi}}}{\partial p}=\frac{R^{D}-1}{q p^{2}}<0 \tag{31}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{\partial^{2} V_{\bar{\varphi}}}{\partial p^{2}}=\frac{2-2 R^{D}}{q p^{3}}>0 . \tag{32}
\end{equation*}
$$

If $\zeta_{G m}=R^{D}$

$$
\begin{equation*}
V_{\bar{\varphi}}=\frac{(1-p)\left(\bar{R}^{G}-R^{D}\right)}{p q} \tag{33}
\end{equation*}
$$

Please note solving the inequality $V_{\bar{\varphi}}>V_{1}$ one will obtain the same solution as (20).
We now present a numerical example. Assume $q=0.3, \zeta_{L}=0.1, \zeta_{G m}=0.90, \zeta_{G d}=0.80$, $R^{D}=0.85, N=40$. We plot the optimal allocation $\varphi^{*}(p, n)$ as a function of the disaster probability and time to the end.


Optimal allocation against $\bar{\varphi}$. The figure shows optimal allocation in the numerical example as a function of the time, where the bank lives at most 40 periods.


Optimal allocation against the level of $\bar{\varphi}$ that, for each level of $p$, guarantees no default in state $m$. The figure shows optimal allocation in the numerical example as a function of the disaster probability $p$ when there are $n=40$ periods until the end, i.e., the first period in which the bank exists.

### 1.2 Model with both portfolio and leverage choice

Now the bank makes simultaneously an investment and a leverage decision. We can solve the problem in a similar fashion as before.

Terminal period $N$ Now the value of the bank's equity is given as:

$$
\begin{align*}
V(N)=\max _{\varphi}[ & {\left[(1-p)\left(a\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}\right)-R^{D}\right)\right.} \\
& \left.p(1-q)\left(a\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}\right)-R^{D}\right) \mathbb{1}_{a\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}\right)>R^{D}}-\frac{\xi}{2}(a-\bar{a})^{2}\right]  \tag{34}\\
& \text { s.t. } \varphi \leq 1 \tag{35}
\end{align*}
$$

There is an adjustment cost associated with changing the bank's leverage, in absence of which banks can rake up $a$ instantly to increase the equity value.

Case 1: Bank decides in such a way that it avoids default if next period state $m$ realizes $a\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}\right) \geq R^{D}$, or $\varphi \leq \frac{\zeta_{G m}-R^{D} / a}{\zeta_{G m}-\zeta_{L}}(=\bar{\varphi}(a))$

$$
\begin{gather*}
V_{\bar{\varphi}}(N)=\max _{\varphi, a}\left[(1-p)\left(a\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}\right)-R^{D}\right)\right. \\
\left.p(1-q)\left(a\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}\right)-R^{D}\right)-\frac{\xi}{2}(a-\bar{a})^{2}\right]  \tag{36}\\
\text { s.t. } \varphi \leq \bar{\varphi}(a)  \tag{37}\\
V_{\bar{\varphi}}(N)=\max _{\varphi, a}\left[a\left(1-p q \zeta_{G d}\right)-(1-p q) R^{D}-\frac{\xi}{2}(a-\bar{a})^{2}\right. \\
\left.+a \varphi p q\left(\zeta_{G d}-\zeta_{L}\right)\right] \tag{38}
\end{gather*}
$$

Again, this is increasing in $\varphi$. Hence,

$$
\begin{gather*}
V_{\bar{\varphi}}(N)\left(a, \varphi^{*}\right)=\left[a\left(1-p q \zeta_{G d}\right)-(1-p q) R^{D}-\frac{\xi}{2}(a-\bar{a})^{2}\right. \\
\left.+a \bar{\varphi}(a) p q\left(\zeta_{G m}-\zeta_{L}\right)\right] \tag{39}
\end{gather*}
$$

F.O.C. wrt a pins down $a^{*}$

$$
\begin{equation*}
a_{\bar{\varphi}}^{*}=\bar{a}+\frac{1-p q \zeta_{G d}+\frac{\zeta_{G m}}{\zeta_{G m}-\zeta_{L}}\left(\zeta_{G d}-\zeta_{L}\right)}{\xi} \tag{40}
\end{equation*}
$$

which is decreasing in $p$.
The value is given by.

$$
\begin{equation*}
V\left(N, a_{\bar{\varphi}}^{*}, \varphi^{*}\right)=(1-p)\left(a_{\bar{\varphi}}^{*} \bar{\varphi} \bar{R}^{L}+a_{\bar{\varphi}}^{*}(1-\bar{\varphi}) \bar{R}^{G}-R^{D}\right)-\frac{\xi}{2}\left(a_{\bar{\varphi}}^{*}-\bar{a}\right)^{2} \tag{41}
\end{equation*}
$$

Case 2: Bank decides in such a way that it will default if next period state $m$ realizes $a\left(\varphi \zeta_{L}+(1-\varphi) \zeta_{G m}\right)<R^{D}$, or $\varphi>\bar{\varphi}(a)$

$$
\begin{gather*}
V_{1}(N)=\max _{\varphi, a}\left[(1-p)\left(a\left(\varphi \bar{R}^{L}+(1-\varphi) \bar{R}^{G}\right)-R^{D}\right)-\frac{\xi}{2}(a-\bar{a})^{2}\right]  \tag{42}\\
\text { s.t. } \varphi \leq 1 \tag{43}
\end{gather*}
$$

This is again increasing in $\varphi$, therefore

$$
\begin{equation*}
V_{1}\left(N, a, \varphi^{*}\right)=\left[(1-p)\left(a \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}(a-\bar{a})^{2}\right] \tag{44}
\end{equation*}
$$

Also,

$$
\begin{equation*}
a_{1}^{*}=\frac{(1-p) \bar{R}^{L}}{\xi}+\bar{a} \tag{45}
\end{equation*}
$$

which can be rewritten as

$$
\begin{equation*}
a_{1}^{*}=\frac{1-p \zeta_{L}}{\xi}+\bar{a} \tag{46}
\end{equation*}
$$

which is decreasing in $p$. Moreover, $a_{\bar{\varphi}}^{*}-a_{1}^{*}>0$ if

$$
\begin{equation*}
p<\frac{\zeta_{G m}\left(\zeta_{G d}-\zeta_{L}\right)}{\left(\zeta_{G m}-\zeta_{L}\right)\left(q \zeta_{G d}-\zeta_{L}\right)} \tag{47}
\end{equation*}
$$

Assuming $q \zeta_{G d}-\zeta_{L}>0$, this is always true. The value is

$$
\begin{equation*}
V\left(N, a_{1}^{*}, \varphi^{*}\right)=(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2} \tag{48}
\end{equation*}
$$

Comparing $V\left(N, a_{1}^{*}, \varphi^{*}\right)$ to $V\left(N, a_{\bar{\varphi}}^{*}, \varphi^{*}\right)$, it follows that the optimal $\varphi$ in the last period is

$$
\begin{equation*}
\varphi^{*}(N)=1 \tag{49}
\end{equation*}
$$

and $V^{*}(N)=V\left(N, a_{1}^{*}, \varphi^{*}\right)=(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}$.
Thus banks always chooses a $100 \%$ loan portfolio regardless of the probability $p$ because there is no continuation value to protect from default and profits are lower when when the bank tries to avoid default in state $m$.

Period N-1 The bank has continuation value $(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}$ and solves:

$$
\begin{align*}
& V(N-1)=\max _{\varphi, a}\left[(1-p)\left(\varphi a \bar{R}^{L}+(1-\varphi) a \bar{R}^{G}-R^{D}+(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}\right)\right. \\
& \left.p(1-q)\left(\varphi a \zeta_{L}+(1-\varphi) a \bar{R}^{G}-R^{D}+(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}\right) \mathbb{1}_{a\left(\varphi \zeta_{L}+(1-\varphi) \bar{R}^{G}\right)>R^{D}}\right] \\
& -\frac{\xi}{2}(a-\bar{a})^{2}  \tag{50}\\
& \text { s.t. } \varphi \leq 1 \tag{51}
\end{align*}
$$

Again compute the optimal value in the 2 cases

$$
\begin{gather*}
V_{\bar{\varphi}}^{*}(N-1)=(1-p)\left(a_{\bar{\varphi}}^{*}\left(\bar{\varphi} \bar{R}^{L}+(1-\bar{\varphi}) \bar{R}^{G}\right)-R^{D}\right)+(1-p)^{2}\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)+ \\
p(1-q)(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{\bar{\varphi}}^{*}-\bar{a}\right)^{2}-(1-p q) \frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}  \tag{52}\\
V_{1}^{*}(N-1)=\left((1-p)+(1-p)^{2}\right)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}(1+(1-p)) \tag{53}
\end{gather*}
$$

The bank chooses to default if state $m$ realizes if

$$
\begin{equation*}
V_{1}^{*}(N-1)>V_{\bar{\varphi}}^{*}(N-1) \tag{54}
\end{equation*}
$$

or equivalently

$$
\begin{array}{r}
(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-(1-p+p q) \frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}>(1-p)\left(a_{\bar{\varphi}}^{*}\left(\bar{\varphi} \bar{R}^{L}+(1-\bar{\varphi}) \bar{R}^{G}\right)-R^{D}\right) \\
+p(1-q)(1-p)\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)-\frac{\xi}{2}\left(a_{\bar{\varphi}}^{*}-\bar{a}\right)^{2} \tag{55}
\end{array}
$$

When $n \rightarrow \infty$ When $n \rightarrow \infty$, the bank chooses $\bar{\varphi}$ if
$(1-p)\left(a_{\bar{\varphi}}^{*}\left(\bar{\varphi} \bar{R}^{L}+(1-\bar{\varphi}) \bar{R}^{G}\right)-R^{D}\right)-\frac{\xi}{2}\left(a_{\bar{\varphi}}^{*}-\bar{a}\right)^{2}>q\left[\left(a_{1}^{*} \bar{R}^{L}-R^{D}\right)(1-p)-\frac{\xi}{2}\left(a_{1}^{*}-\bar{a}\right)^{2}\right]$

We now present a numerical example. Assume $q=0.3, \zeta_{L}=0.1, \zeta_{G m}=0.90, \zeta_{G d}=0.80$, $R^{D}=0.85, N=40, \bar{a}=0.97, \xi=20$. We plot the optimal allocation $\varphi^{*}(p, n)$ and the optimal investment/leverage $a^{*}(p, n)$ as a function of the disaster probability and time.


Optimal allocation. The figure shows optimal allocation in the numerical example as a function of disaster probability for $n=40$, i.e., the first period the bank exists.


Optimal investment/leverage. The figure shows optimal investment in the numerical example as a function of disaster probability for $n=40$, i.e., the first period the bank exists.


Optimal allocation against time. The figure shows optimal allocation in the numerical example as a function of the time, where the bank lives at most 40 periods.


Optimal investment/leverage against time. The figure shows optimal investment in the numerical example as a function of the time, where the bank lives at most 40 periods.

## 2 Additional Results



Fig. 1. Gallup Economic confidence index. The figure shows the Gallup's Economic Confidence Index, which is based on the combined responses to two questions, the first asking Americans to rate economic conditions in this country today, and second, whether they think economic conditions in the country as a whole are getting better or getting worse. Results are based on telephone interviews with approximately 3,500 national adults.


Fig. 2. Mortgage origination by credit score. The figure shows mortgage origination by credit scores. Data are from the quarterly report on household debt and credit, November 2022, from the Federal Reserve Bank of New York.

Time series of the average leverage


Fig. 3. Cross-sectional Average of bank leverage. The figure shows the cross-sectional average of bank leverage forcing $p$ to $1.84 \%$ and $\epsilon_{t+1}$ to 0 in the simulated sample.

Time series of the cross-sectional standard deviation of leverage


Fig. 4. Cross-sectional standard deviation of bank leverage. The figure shows the crosssectional standard deviation of bank leverage forcing $p$ to $1.84 \%$ and $\epsilon_{t+1}$ to 0 in the simulated sample.


Fig. 5. Simulated Bank Lending and Portfolio decision. The figure shows the behavior of a simulated bank over a sequence of 80 periods.


Fig. 6. Optimal Bank leverage for high $\omega$. The figure shows the optimal amount of bank assets (lending), scaled by deposits. Alternative levels of lagged asset-to-debt ratio $a_{t-1}=\left(\frac{A_{t-1}}{D_{t-1}}\right)$ are plotted on the x -axis. Different lines represent different crises probability $p_{t} . \omega_{t}$ is fixed to 0.04 .


Fig. 7. GDP and Household Debt growth. The top figure shows the empirical relationship between the (demeaned) GDP growth rate from year $t$ to $t+3$ and the growth rate of the household debt to GDP ratio from year $t-4$ to $t-1$. Data are from the Bank of International Settlements and cover 39 countries between 1961 and 2012. The bottom figure reproduces the same relationship in the model using however the growth rate of aggregate bank's loans (to household) from year $t-5$ to $t$. Results are from simulating the model with 10,000 banks for 10,000 periods. The solid line is the estimated regression line from

$$
\Delta_{3} y_{i, t+3}-\overline{\Delta_{3} y_{i}}=\vec{\alpha}_{i}^{1}+\beta_{H} \Delta_{3} d_{i, t-1}^{H H}+u_{i t},
$$

where $y$ is GDP and $d^{H H}$ is the measure of credit to households.


Fig. 8. Bank value for different levels of $\omega$. The figure shows the bank's value, scaled by deposits, $\widetilde{v}_{t}=\left(\frac{\widetilde{V}_{t}}{D_{t}}\right)$. Alternative levels of bank-specific determinant of loan performance $(\omega)$ are plotted on the x-axis. Different lines represent alternative levels of crises probability $p_{t}$. The lagged asset-to-debt ratio $a_{t-1}=\left(\frac{A_{t-1}}{D_{t-1}}\right)$ is fixed to 1.12.


Fig. 9. Optimal Bank portfolio allocation for different levels of $\omega$. T The figure shows the policy for portfolio allocation of an individual bank $\left(\varphi_{t}\right)$. Alternative levels of bank-specific determinant of loan performance $(\omega)$ are plotted on the x-axis. Different lines represent alternative levels of crises probability $p_{t}$. The lagged asset-to-debt ratio $a_{t-1}=\left(\frac{A_{t-1}}{D_{t-1}}\right)$ is fixed to $1.12 . \varphi$ equal to 1 represents investment in the portfolio of household loans, while $\varphi$ equal to 0 stands for investment in the government T-bill.


[^0]:    *The Wharton School, University of Pennsylvania. Email: gomesj@wharton.upenn.edu
    ${ }^{\dagger}$ London Business School. Email: mgrotteria@london.edu
    ${ }^{\ddagger}$ The Wharton School, University of Pennsylvania and the SEC. Email: jwachter@wharton.upenn.edu. This paper was initially released prior to the author joining the Commission. The Securities and Exchange Commission disclaims responsibility for any private publication or statement of any SEC employee or Commissioner. This article expresses the author's views and does not necessarily reflect those of the Commission, the Commissioners, or other members of the staff.

[^1]:    ${ }^{1}$ The initial period is period 0.

