Adverse Selection and Non-Exclusive Contracts[*]

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Abstract

This paper studies the Rothschild and Stiglitz (1976) adverse selection environment, relaxing the assumption of exclusivity of insurance contracts. Agents can engage in multiple insurance contracts simultaneously, and the terms of these contracts are not observed by other firms. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive conditions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with a lower probability of accident purchasing no insurance and agents with higher accident probability buying the actuarially-fair level of insurance. The equilibrium allocation also constitutes a linear price schedule for insurance. To sustain the equilibrium allocation, firms must offer latent contracts. These contracts are necessary to prevent deviations by other firms; in particular they can prevent cream-skimming strategies. As in Rothschild and Stiglitz (1976), pooling equilibrium still fails to exists.

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1 Introduction

What type of insurance contracts are available when insurance providers compete? This paper characterizes the equilibrium of an environment where the insured has private information regarding the risk probability and can sign, without being observed, multiple insurance contracts.

Insurance contracts are written to offset the risk associated with a wide variety of events. Examples of different types of insurance contracts include insurance against person-related events (medical, life, annuities), property events (car, home), and financial events (credit default swaps [CDS][1]). These insurance contracts share two common properties: that the realization of uncertainty can be verified, and that subscribers might have additional private information about the *probabilities* that an event realizes. However, due to different regulatory oversight, a feature that varies greatly amongst them is the ability of the insurer to enter into additional contracts with other insurance providers. This possibility of non-exclusive insurance holding, while rare in property insurance, is a definite possibility – for example, in the case of credit default swaps.[2]

Motivated by the above observations, in this paper we investigate the restrictions on the equilibrium insurance contracts that arise once we dispense with the exclusivity assumption. We extend the standard Rothschild and Stiglitz (1976) (RS henceforth) environment by allowing agents to engage simultaneously in multiple insurance contracts, with the terms of these contracts not being observed by other insurance providers. Insurance providers behave non-cooperatively and compete offering menus of insurance contracts from an unrestricted contract space. We derive parameter restrictions under which a separating equilibrium exists and fully characterize it. The unique equilibrium allocation consists of agents with a lower probability of accident purchasing no insurance and agents with higher accident probability buying the actuarially-fair level of insurance. The equilibrium allocation also constitutes a

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[1] For a review, refer to Duffie (1999).

[2] Until early 2009, CDS were issued in private bilateral trades without any intermediation by any clearing house. On March 10, 2009 ICE Trust$^{\text{TM}}$ began operating as a central counterparty clearing house for credit default swaps in North America.

linear price schedule for insurance; i.e., it can alternatively be implemented by firms offering a linear insurance contract with quantity restriction and with price equal to the actuarially-fair for the bad type.

A novel feature of this environment is the result that, in any equilibrium, latent contracts must be offered. These contracts are necessary to prevent cream-skimming deviations by entrants and also deviation of incumbents. This highlights the dual role that non-exclusivity plays in our environment. First, by allowing agents to be able to sign an additional insurance contract, it constitutes a constraint on what an insurance providers can offer. Second, it enables insurance providers to sustain equilibrium contracts, since entry can by prevented with the threat of having the agent signing on the entrant contract and a latent one offered by the incumbent. We illustrate this point further by comparing our results with the analysis in RS. We show that, contrary to RS, latent menus can prevent cream-skimming strategies; however, pooling equilibrium still fails to exist due to deviations of the incumbent. Finally, we show how, when the RS equilibrium is offered, there is a profitable deviation available to entrants that cannot be prevented with latent menus. Hence, the RS equilibrium does not survive the non-exclusivity restriction introduced in this paper.

Related Literature

This paper is related to the work of Wilson (1976). He extends the equilibrium concept used in RS beyond static Nash equilibrium by allowing insurance providers to take into account how a change in their policy offers might affect the set of policies offered by other insurance providers. In this paper, latent contracts play a similar role by enabling a reaction of insurance providers to deviations by other insurance providers.

This paper is also related to a series of papers that analyze the effect of non-exclusive contracting in the purchase of goods and insurance. Ales and Maziero (2008) study a dynamic environment with private information (but unlike in this paper, the realization of private information happens after agents sign the contract) where agents can engage in multiple non-exclusive contracts for both labor and credit relationships. We show that a unique equilibrium always exists and that latent contracts are necessary. As in this paper, the equilibrium can be implemented using linear contracts for wages and bonds. In a recent paper, Attar, Mariotti, and Salanie (2008) extend the environment of Akerlof (1970) to include non-exclusive contracting. They focus on the goods market (using a linear utility) and show, contrary to this paper, that a unique equilibrium always exists, involves linear prices, and is sustained by latent contracts. Also, Arnott and Stiglitz (1991) and Bisin and Guaitoli (2004) study static moral hazard environments. In particular, the latter shows that latent contracts are necessary to sustain the equilibrium and lead to positive profit for the insurance providers.

Finally, this paper relates to the literature that studies inter-firm communication in insurance settings, such as Jaynes (1978) and Hellwig (1988).[3] The first considers a static adverse selection economy and allows firms the choice to disclose or not information on who accepts the insurance contract. It shows that some firms will share information leading to a separating equilibrium (that always exists), while in the case where no information is shared, no equilibrium exists. Sharing information allows a firm to offer an insurance contract that is contingent on additional purchases of insurance an agent might accept. In our paper, firms gather information on insurance purchased by also offering latent contracts, which allows us, in contrast to Jaynes, to have an equilibrium even without any information being shared directly. Latent contracts have the same role as information-sharing, since they enable firms to react to deviation of incumbent firms. Hellwig (1988) highlights that the ability to react is the key to equilibrium existence rather than inter-firm communication. In particular, inter-firm communication enables firms to react only if the equilibrium concept considered in Jaynes (1978) is implicitly assuming a non-stationary expectation similar to Wilson (1976).

This paper is organized as follows: Section 2 describes the environment. Characterization and implementation are studied respectively in Sections 3 and 4, Section 5 compares directly the current paper and Rothschild and Stiglitz (1976), and Section 6 concludes.

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[3] For an extension to a moral hazard environment, also look at Hellwig (1983).

2 Environment

Consider an economy populated by a continuum of measure one of agents and I firms (insurance providers), where I is a natural number. Agents are ex ante heterogeneous: there is a fraction p_g of type G consumers (the good type) and a fraction p_b of type B (the bad type). The economy lasts for 1 period. Agents' utility u is defined over consumption c. Assume $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing, and a strictly concave function. At time 1, an agent of type $j = b, g$ receives endowment ω_H with probability π_j and ω_L with probability $1 - \pi_j$, with $\omega_H > \omega_L$. The realization of the endowment occurs at the end of the period and is publicly observed. Assume $\pi_g > \pi_b$ and that these probabilities are private information of the agent. Each firm $i \in \{1, ..., I\}$ offers contracts to agents to insure against the endowment shock realization. A contract prescribes consumption transfers conditional on the realization of the endowment.

An important feature of our environment is that agents can simultaneously sign contracts with more than one firm and that the terms of the contract between an agent and a firm i are not observed by other firms. We do not impose any restriction on the contracts offered by each firm. As described in Ales and Maziero (2008), due to the delegation principle, we can restrict the analysis to menu games. In a menu game, each firm offers a menu consisting of a collection of transfers pairs, $\tau^i = (\tau^i_L, \tau^i_H)$, conditional on a realization of ω_L or ω_H. A menu is a set C^i in $\mathcal{P}(\mathbb{R}^2)$ (the power set of $\mathbb{R}^2$).[4] In the presence of non-exclusivity of contracts, restricting menus to have the same number of elements as the exogenous type space might not be without loss of generality, and the necessity of latent contracts in equilibrium illustrates this.[5]

The problem of firm i is to choose its menu, taking as given the menus offered by other

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[4] We do not allow the use of random contracts.

[5] See Martimort and Stole (2002) and Peters (2001).

firms C^{-i} and the agents' optimal choices:

$$\begin{aligned} \Pi(C^i, C^{-i}) = & \max_{C^i \in \mathcal{P}(\mathbb{R}^2)} - \sum_{j=B,G} p_j[\pi_j \tau^i_{H,j*} + (1-\pi_j)\tau^i_{L,j*}] \\ s.t. \qquad & (\tau^i_{L,j*}, \tau^i_{H,j*}) \in C^i \end{aligned} \tag{1}$$

where $(\tau^i_{L,j*}, \tau^i_{H,j*})$ are the pair of transfers chosen by an agent of type j.

Let $U^j(C)$ be the expected utility of an agent of type $j = B, G$ given menus $C = C^1 \times \ldots \times C^I$, defined as:[6]

$$U^j(C) = \max_{(\tau_L, \tau_H) \in C} \left[\pi_j u \left(\omega_H + \sum_{i=1}^{I} \tau^i_H \right) + (1-\pi_j) u \left(\omega_L + \sum_{i=1}^{I} \tau^i_L \right) \right]. \tag{2}$$

where $(\tau_L, \tau_H) = (\tau^1_L, \tau^1_H) \times \ldots \times (\tau^I_L, \tau^I_H)$ are the transfers chosen from each menu.

Definition 1 (**Equilibrium of Menu Games**). *A pure strategy equilibrium of a menu game is a collection of menus C and agents' choices $(\tau^i_L, \tau^i_H) \in C^i \quad \forall i \in \{1, ..., I\}$, for each $j = B, G$ such that:*

1. *Agents' choices $(\tau^i_L, \tau^i_H) \in C^i \quad \forall i \in \{1, ..., I\}$ solve the agent problem (2).*
2. *For each $i \in \{1, ..., I\}$, C^i solves (1) taking as given C^{-i} chosen by firms $-i$ and the agents' choice $(\tau^i_L, \tau^i_H) \in C^i \quad \forall i \in \{1, ..., I\}$.*

Note that a menu might contain more alternatives than the number of types, implying that some alternatives are not chosen in equilibrium. We denote a contract as *latent* if it is offered in equilibrium by some firm and is not chosen in equilibrium by any agent.

In this environment, two distinct sources of uncertainty can be insured: the private realization of the type and the public realization of the endowment shock. If the private type is perfectly insured, both agents will receive identical contracts. Following the adverse selection literature, we call this allocation a *pooling* equilibrium. If different contracts are chosen by

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[6] To incorporate voluntary participation, we require that each menu C^I contains a no-transfer pair, $(0, 0)$.

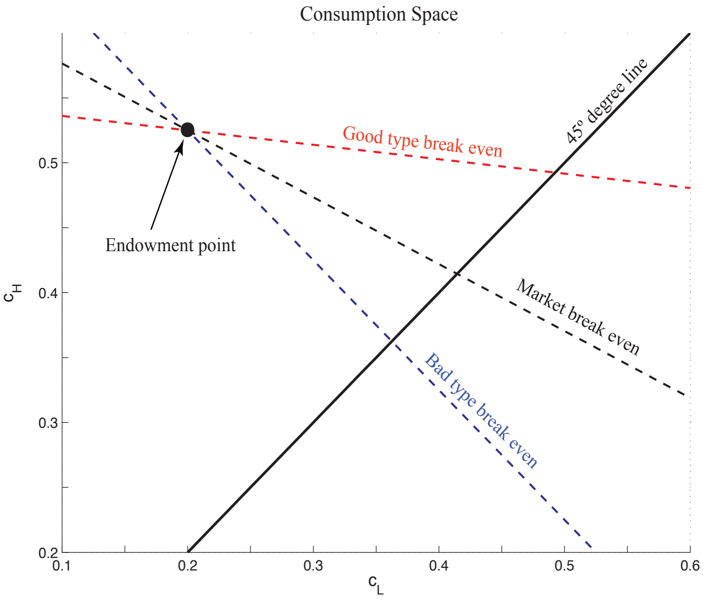

Figure 1: Representation in Consumption Space.

different types, we call the resulting allocation a *separating* equilibrium. In the characterization, we show that no pooling equilibrium allocation exists. The unique equilibrium is the following separating equilibrium: the bad type receives full insurance against the public realization of the endowment at his actuarially-fair price, and the good type receives no insurance.

3 Characterization of Equilibrium

In the following sections, to help explain the keys steps of our results, we make extensive use of graphical analysis. We represent the allocations in the consumption space, as in Figure 1. The figure displays in (c_L, c_H) space (consumption in the low and high state) the endowment point, the full insurance line (45 degree line), and the zero-profit lines if only good agents buy the insurance, only bad and if all agents purchase the contract.

We start the equilibrium characterization by providing the necessary conditions a pooling equilibrium must satisfy.[7]

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[7] All the proofs for this section are provided in Appendix A.

Lemma 1. *In any pooling equilibrium allocation* $c = (c_L, c_H)$*, the following conditions must hold:*

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} \leq \frac{1-\pi_b}{\pi_b}, \tag{3}$$

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} \geq \frac{1-\hat{p}_H}{\hat{p}_H}, \tag{4}$$

where $\hat{p}_H = p_g\pi_g + p_b\pi_b$*.*[8]

The first condition implies that the marginal rate of substitution between consumption in the two states for the B agent is less than or equal to the actuarially-fair price for the insurance only if B agents accept. If not, a firm can provide some additional insurance at a price slightly below that. Such deviation is profitable for the firm and increases expected utility for the bad agents. The second condition requires that the marginal rate of substitution between consumption in the two states for the G agent is greater than the price for insurance when all agents accept the contract (the actuarially-fair pooling price). Otherwise, entrants can profitably provide alternative insurance contracts at a slightly lower price. A direct implication of this lemma is that there is no pooling equilibrium, since there is no allocation that satisfies these two conditions at the same time.

Proposition 1. *There is no pooling equilibrium.*

Proof. Suppose there exists a pooling equilibrium $c = (c_L, c_H)$. This equilibrium must satisfy condition (3) and (4). This implies:

$$\frac{1-\pi_g}{\pi_g}\frac{\hat{p}_H}{1-\hat{p}_H} \geq 1 \quad \Rightarrow \quad \frac{1-\pi_g}{\pi_g} \geq \frac{1-\hat{p}_H}{\hat{p}_H},$$

which is a contraction since $\pi_b < \pi_g$. □

We now characterize the necessary conditions a separating equilibrium satisfies. In this equilibrium, each type receives a different allocation, which we denote as: $c^B = (c^B_L,$

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[8] Note that equation 3 implies that $c_L \geq c_H$.

$c_H^B)$, $c^G = (c_L^G, c_H^G)$, which is denoted by $C = \{c^B, c^G\}$. The first condition is that each agent prefers his own allocation:

$$U^B\left(c^B\right) \geq U^B\left(c^G\right), \qquad U^G\left(c^G\right) \geq U^G\left(c^B\right). \tag{5}$$

The equilibrium must also deliver non-negative profits: $\Pi(C) \geq 0$. The following lemma characterizes three necessary conditions an equilibrium must satisfy. The first two conditions refer to the allocation chosen by G agents. First, the indifference curve of the good agent, at the equilibrium allocation, must be less steep than the average zero-profit line. Second, the allocation for the G type must be on the under-insurance region. The last condition states that the equilibrium allocation for the bad type delivers full insurance at his actuarially-fair price, since consumption is the same in both states and the allocation lies on the zero-profits curve of B agents.

Lemma 2. *Any separating equilibrium allocation must satisfy:*

1. *For the G agent:*

$$\frac{1-\pi_g}{\pi_g}\frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} \leq \frac{1-\hat{p}_H}{\hat{p}_H}, \tag{6}$$

$$\frac{1-\pi_g}{\pi_g}\frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} \geq \frac{1-\pi_g}{\pi_g}, \tag{7}$$

2. *For the B agent:*

$$\begin{aligned} \frac{1-\pi_b}{\pi_b}\frac{u'\left(\omega_B\right)}{u'\left(\omega_B\right)} &= \frac{1-\pi_b}{\pi_b} \\ \textit{where } \omega_B &= \pi_b\omega_H + (1-\pi_b)\omega_L. \end{aligned} \tag{8}$$

This lemma completely characterizes the allocation for the bad type in any separating equilibrium: full insurance at his actuarially-fair price. Also, this allocation is independent of the allocation of the good agents. The key intuition is that an entrant, when designing a profitable contract for the bad type, cannot be induced in a region of negative profit (since

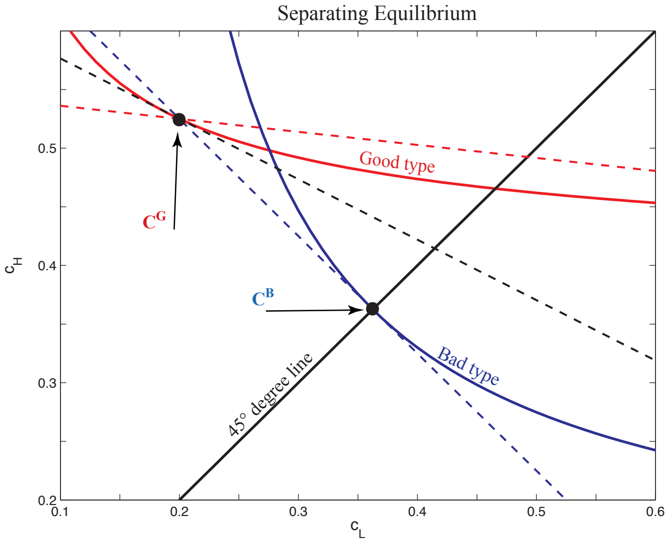

Figure 2: Separating Equilibrium. C^G: consumption for good type, C^B: consumption for bad type.

he is already insuring the group with highest risk). This induces the equilibrium allocation for the bad type to be on his zero-profit line.

Using the above restrictions, we now characterize the equilibrium and show it is unique. Let the candidate equilibrium be $c^B = (\omega^B, \omega^B)$, $c^G = (\omega_L, \omega_H)$, with ω_B defined above. Figure 2 illustrates this equilibrium. This equilibrium provides no insurance to the G agent and the actuarially-fair insurance to the B type. Note that this allocation satisfies conditions (5), (7), (8), and delivers zero profits. To satisfy (6), the following parameter restriction is needed:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(\omega_L)}{u'(\omega_H)} \leq \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{9}$$

This condition is satisfied if, for example, π_g is large relative to π_b or if the spread between ω_L and ω_H is sufficiently small. Note that there exists a non-empty set of parameter values for which it holds.[9] Under this condition, the good agent prefers autarky to the allocation

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[9] As in Rothschild and Stiglitz (1976), equilibrium might fail to exist for this environment if this assumption is not satisfied.

designed for the B agent: $\pi_g u(\omega_H) + (1-\pi_g)u(\omega_L) \geq u(\omega_B)$.[10]

Proposition 2. *Let $\{\pi_g, \pi_b, \omega_h, \omega_l, u\}$ satisfy condition (9); then any equilibrium allocation of a menu game satisfies:*

1. $c^B = (\omega^B, \omega^B)$, *where* $\omega_B = \pi_b\omega_H + (1-\pi_b)\omega_L$;
2. $c^G = (\omega_L, \omega_H)$.

A key step of this proof involves showing that the good type does not buy additional insurance. In particular, the equilibrium allocation of the good type cannot be in the green triangular area of figure 3(a). To illustrate this, we display in figure 3(b) how any allocation in this area (as $\hat{C}^G$) can be made unprofitable. This allocation is profitable if only good agents select it. However, when combined with a contract as $\hat{D}$ – which is always profitable – it is also chosen by the bad type, thus inducing negative profits to any firm that offers additional insurance to the good agents.

4 Implementation of Equilibrium

The following proposition shows that (c^B, c^G) can be implemented in equilibrium when there are a sufficiently high number of bad types; in addition, it directly shows that it is necessary to have more than one firm active in equilibrium, with each of these firms offering latent contracts.

Proposition 3. *Let $\{\pi_g, \pi_b, \omega_h, \omega_l, u, p_h, p_b\}$ satisfy condition (9) and*

$$\frac{1}{2} + \frac{\pi_g - \pi_b}{2\pi_g\pi_b} < p_b,$$

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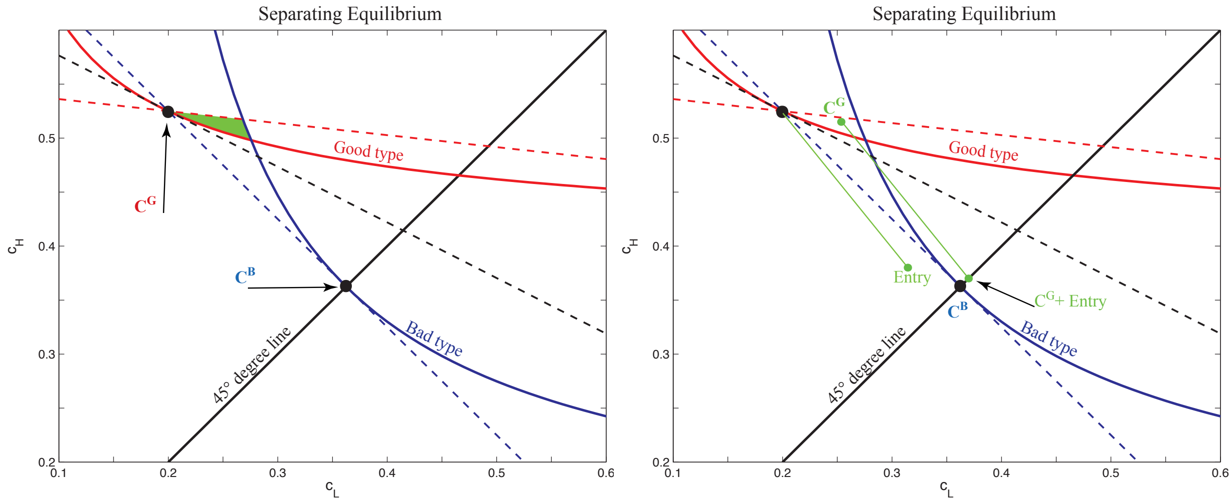
[10] Suppose that $\pi_g u(\omega_H) + (1-\pi_g)u(\omega_L) < u(\omega_B)$. This implies that in the consumption space (c_l, c_h), the indifference curve for the high type passing through the endowment point is below the point ω_B. In this space, we represent indifference curves as a one-dimensional function, denoted by $U^g_{aut}(c_{lt})$. Let c_{tb} be the level of consumption so that $U^{g\prime}_{aut}(c_{tb}) = -\frac{1-\pi_b}{\pi_b}$. By the contradicting assumption, this point must lie on the right of ω_L, which follows from the fact that, at c_{tb}, the indifference curve and the zero-profit line for the bad type are at the maximum distance. Denote by c_{tm} the value of consumption in the low state so that $U^{g\prime}_{aut}(c_{tm}) = -\frac{1-\hat{p}_H}{\hat{p}_H}$. Since the slope of the indifference curve is decreasing in the consumption of the low state (keeping the level of utility constant), we have that $\omega_L > c_{tb} > c_{tm}$. Since condition (9) requires $U^{g\prime}_{aut}(\omega_L) \leq -\frac{1-\hat{p}_H}{\hat{p}_H}$, we reach a contradiction.



(a) (b)

Figure 3: Sketch of the proof of Proposition 2.

then there exists an equilibrium of the menu game.

Proof. The following strategies for the firms implement the equilibrium allocation. Let firms $i = 1, 2$ offer the menu: $C^i = \left\{ \left(\frac{\tau_{L,B}}{2}, \frac{\tau_{H,B}}{2} \right), (x, y), (0, 0) \right\}$, where $\tau^B_L = \pi_b(\omega_H - \omega_L)$, $\tau^B_H = (1 - \pi_b)(\omega_L - \omega_H)$ and $x, y \geq 0$ such that $\pi_B(\omega_L - x) + (1 - \pi_B)(\omega_H - y) = 0$. The points (x, y) represent the zero-profit line if only B agents accept the contract. Let all remaining firms $i \neq 1, 2$ offer the null menu: $C^i = \{(0, 0)\}$. The agents' strategies, given these menus, are: type B chooses $\left(\frac{\tau_{L,B}}{2}, \frac{\tau_{H,B}}{2} \right)$ from firms 1 and 2; type G chooses $(0, 0)$ from all firms. In this equilibrium, all firms make zero profits, and agents B and G get allocations c^B and c^G respectively.

We now show that there are no profitable deviations by any firm (either entrant or incumbent). Given that there is always at least one firm offering a collection of menus that contains $(\tau_{L,B}, \tau_{H,B})$, there is no profitable deviation that an entrant or incumbent can make that makes the B agent better off, since the allocation c^B is the unique solution of maximizing the B agent's utility subject to non-negative profits if only B agents accept the contract.

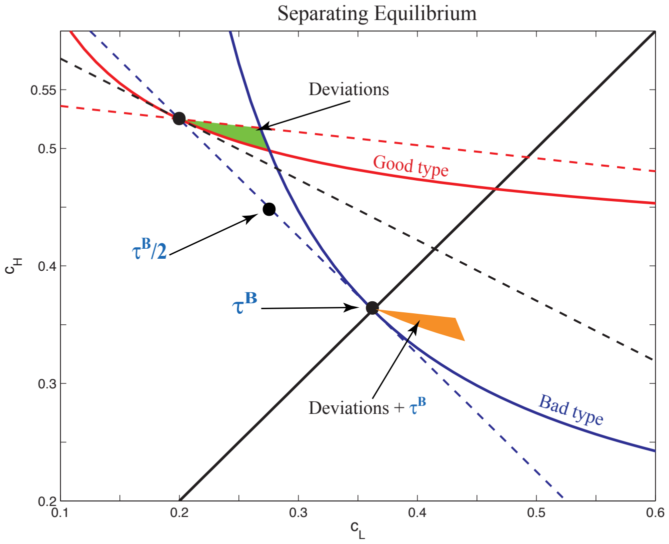


Figure 4: Proof of Proposition 3.

Any deviation that provides some insurance to G agents in order to attract these agents must increase their utility; it must also be profitable and such that it does not attract B agents. Graphically, this means that it must lie in the green area in figure 4. Any allocation in this area, represented by a contract $\tilde{C} = \{(\varepsilon, -\alpha\varepsilon)\}$ with $\varepsilon > 0$ and $\alpha > 0$, increases the utility of G agents and is profitable as long as only G agents accept it. However, given that the incumbent firms offer (τ_L^B, τ_H^B), if a firm (either one of the incumbents or an entrant) offers contract $\tilde{C}$, the B agent will choose the contract (τ_L^B, τ_H^B) together with $\tilde{C}$, since the slope of his indifference curve at c_B is $\frac{1-\pi_b}{\pi_b}$ and condition (9) holds. In this case, this firm will make negative profits since all agents will accept the deviation; hence, this deviation is not offered. This argument is displayed graphically in figure 4.

Using a similar argument, there is no profitable deviation that can increase agents' utility. Note that if there is only one active firm, this firm would deviate and offer additional insurance to the G agent as $\tilde{C}$.

The last step of the proof is to show that latent menus can prevent the entry of insurance providers offering a menu rather than a single contract. The additional complication is that the entrant can cross-subsidize the bad type with profits generated from the good type.[11] Let an entrant offer a menu $(\tilde{c}^G, \tilde{c}^B)$ consisting of consumption pairs respectively designed for the good and bad type. The contract $\tilde{c}^G$ must lie on the green area in figure 5. We show that for any such contract, any contract $\tilde{c}^B$ that is chosen by B agents leads to negative profits in the presence of latent menus. The reason for that is that the contract $\tilde{c}^G$ imposes a lower bound on the utility that $\tilde{c}^B$ must deliver, since B agents can choose the latent contract and the allocation offered by the entrant to the good type, leading to negative profits.

Consider the following choice from the bad type: choose $\tilde{c}^G$ together with additional insurance at his actuarially-fair price until the point he reaches full insurance (this is point c^* in figure 5).

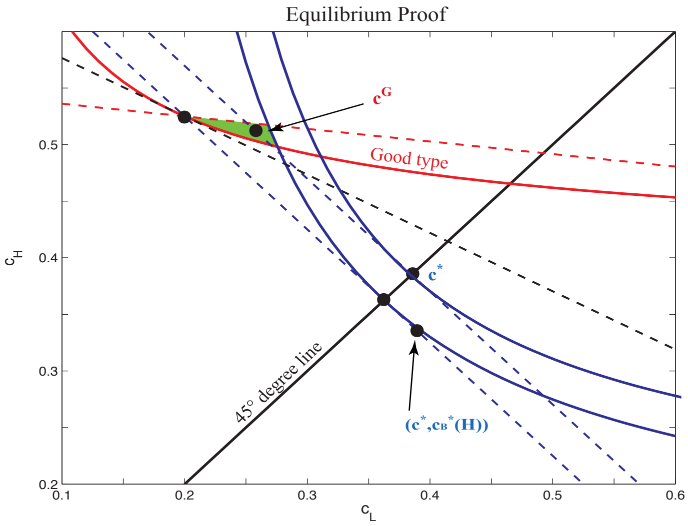


Figure 5: Preventing entry by menus.

Since this is the most preferred insurance contract using $\tilde{c}^G$ and the latent contract, if the agent were to pick $\tilde{c}^B$ it must deliver strictly higher utility than the combination just described, denoted by c^*. We show that under certain conditions the total loss from such $\tilde{c}^B$ is greater than the total profits received by $\tilde{c}^G$.

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[11] The opposite case, cross-subsidizing the good type with profits generated by the contract offered to the bad type, would not be accepted by B agents since it reduces their utility with respect to the equilibrium contract.

We begin by geometrically characterizing point c^*.[12] Let $\tilde{c}^G = (c_H, c_L)$. The point c^*, since it is on the 45 degree line, has both quantities for consumption and is given by

$$(c^* - c_H) = -\frac{1-\pi_b}{\pi_b}(c^* - c_L) \Rightarrow c^* = \pi_b c_H + (1-\pi_b)c_L.$$

Denote by c_B^* the point on the zero-profit line for the bad type with horizontal coordinate equal to c^*. The vertical component of this point, denoted by $c_B^*(H)$ is given by

$$c_B^*(H) = \omega_H - \frac{1-\pi_b}{\pi_b}(\pi_b c_H + (1-\pi_b)c_L - \omega_L).$$

This is the value of consumption in the high state that would make the contract $(c^*, c_B^*(H))$ generate zero profit. With this in mind, we calculate the loss incurred by offering the $\hat{c}_B$ contract as the additional consumption payed out in the high state. The loss from this contract when purchased by bad types is given by

$$\Pi_B = \pi_b c^* - \pi_b c_B^*(H) = \pi_b(c_H - \omega_H) + (1-\pi_b)(c_L - \omega_L). \tag{10}$$

The next step is to calculate the profits from the contract picked by the good type $\tilde{c}^G = (c_H, c_L)$. Similarly, let $c_G^*(H)$ be the point on the zero-profit line for the good type when the horizontal coordinate is equal to c_L

$$c_G^*(H) = \omega_H - \frac{1-\pi_g}{\pi_g}(c_L - \omega_L).$$

Profits when the good type choose c_G are then equal to

$$\Pi_G = \pi_g c_G^*(H) - \pi_g c_H = \pi_g(\omega_H - c_H) - (1-\pi_g)(c_L - \omega_L). \tag{11}$$

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[12] Recall that the equation of a line with slope m going through a given point (x_0, y_0) is given by:

$$(y - y_0) = m(x - x_0).$$

Total profits from the the menu $(\tilde{c}^G, \tilde{c}^B)$ are then less than or equal to the profits from (c_G, c_B^*) denoted by Π, where

$$\begin{aligned}
&\Pi = p_g \Pi_g - p_b \Pi_B = \\
&= (\omega_H - c_H)(p_g \pi_g + \pi_b \pi_b) + (c_L - \omega_L)(-p_b + p_b \pi_b - p_g + p_g \pi_g).
\end{aligned}$$

Rewriting the above, we get

$$\Pi = \frac{c_L - \omega_L}{\pi_b} \left[\pi_b \frac{\omega_H - c_H}{c_L - \omega_L} (p_g \pi_g + \pi_b \pi_b) + \pi_b (p_g(\pi_g - 1) + p_b(\pi_b - 1)) \right].$$

Since $\frac{\omega_H - c_H}{c_L - \omega_L} < \frac{1-\pi_b}{\pi_b}$ we have

$$\Pi < \frac{c_L - \omega_L}{\pi_b} \left[(1 - \pi_b)(p_g \pi_g + \pi_b \pi_b) + \pi_b (p_g(\pi_g - 1) + p_b(\pi_b - 1)) \right]$$

so that

$$\Pi < \frac{c_L - \omega_L}{\pi_b} \left[p_g \pi_g + p_b \pi_g - \pi_b^2 p_b - \pi_b \pi_g p_b + \pi_b \pi_g p_g - \pi_b p_g + p_b \pi_b^2 - \pi_b p_b \right]$$

$$\Pi < \frac{c_L - \omega_L}{\pi_b} \left[p_g(\pi_g - \pi_b) + p_b(\pi_g - \pi_b) + \pi_g \pi_b [p_g - p_b] \right].$$

If $\pi_g \pi_b [1 - 2p_b] < \pi_b - \pi_g$, this will imply that $\Pi < 0$. This will be the case if

$$\frac{1}{2} + \frac{\pi_g - \pi_b}{2\pi_g \pi_b} < p_b. \tag{12}$$

This relation is satisfied if there are a sufficient high number of bad types (p_b high) and if the difference between being a good and a bad type is sufficiently small (π_g close to π_b). Also note that condition (12) is a sufficient condition but not necessary; it can be the case (conditional on the curvature of the indifference curve) that additional latent contracts, at a price lower than the zero profit for the bad agent, might be needed. □

The proof shows that the allocation (c^B, c^G) is an equilibrium, and that to be implemented it is necessary that there be more than one active firm and latent contracts to prevent deviations by potential entrants and incumbents.

Corollary 1. *The allocation* (c^B, c^G) *can be implemented as an equilibrium only if latent contracts are offered and if there is more than one active firm.*

5 The Rothschild-Stiglitz Equilibrium

A natural benchmark to compare our results with is the standard case with exclusive contracts characterized in RS. In that environment, under certain parameter restrictions, there is a unique separating equilibrium. The allocation for the B type coincides with the equilibrium under non-exclusivity; however, the G agent receives an allocation different than autarky. The RS equilibrium is defined below and displayed in figure 6.

Definition 2. *The RS separating allocation is* $(\tilde{c}^B, \tilde{c}^G)$ *which satisfies* $\tilde{c}^B = (\omega^B, \omega^B)$ *and* $\tilde{c}^G = (c_L^G, c_H^G)$*, where* $\omega^B = \pi_b \omega_h + (1-\pi_b)\omega_l$ *and such that* $U^B(c^B) = U^B(c^G)$ *and* $\pi_g \left(\omega_H - c_H^G\right) + (1-\pi_g)\left(\omega_L - c_L^G\right) = 0$.

As shown in the previous section, this allocation is not an equilibrium when agents can sign non-exclusive contracts.[13]

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[13] We can also show directly that this allocation is not an equilibrium using the following argument. If $(\tilde{c}^B, \tilde{c}^G)$ were an equilibrium, an entrant can offer a small additional contract $\hat{c}$ that attracts B agents and that delivers strictly positive profits. This contract is such that B agents accept the G contract from the incumbent together with $\hat{c}$, delivering negative profits to the incumbent. Let the contract $\hat{c}$ be $\hat{c} = (\varepsilon, -\alpha\varepsilon)$ for some small ε and where α satisfies:

$$\frac{1-\pi_b}{\pi_b} \frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} > \alpha > \frac{1-\pi_b}{\pi_b}. \tag{13}$$

This deviation is preferred by B agents since

$$\begin{aligned} U^B\left(c^G + \hat{c}\right) &= \pi_b u\left(c_H^G - \alpha\varepsilon\right) + (1-\pi_b) u\left(c_L^G + \varepsilon\right) \\ &= \pi_b u\left(c_H^G\right) + (1-\pi_b) u\left(c_L^G\right) - \pi_b u'\left(c_H\right)\alpha + (1-\pi_b) u'\left(c_L\right) \\ &> U^B\left(c^G\right) - \pi_b u'\left(c_H\right) \frac{1-\pi_b}{\pi_b} \frac{u'\left(c_L\right)}{u'\left(c_H\right)} + (1-\pi_b) u'\left(c_L\right) \\ &> U^B\left(c^G\right) = U^B\left(c^B\right). \end{aligned}$$

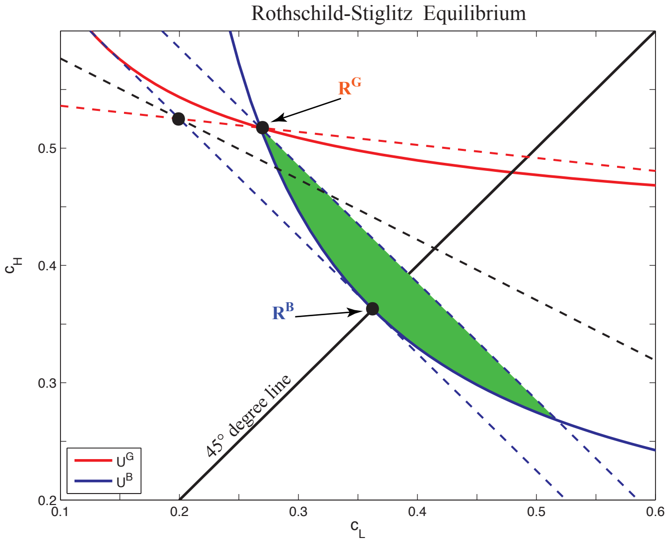

Figure 6: The Rothschild-Stiglitz Equilibrium: R^G consumption for good type, R^B consumption for the bad type.

RS also show that there is no pooling equilibrium when contracts are exclusive, because there is always an alternative contract that can be offered by an entrant that is profitable and attracts only good types (cream skimming). When agents can sign non-exclusive contracts, these cream-skimming deviations can be prevented by latent contracts. We illustrate this argument in Figure ??. The solid lines represent the indifference curves of B and G agents at the *best* pooling equilibrium.[14] Any contract in the green area in Figure ?? is profitable to a firm as long as only G agents accept it and it is preferred to the pooling equilibrium by G agents but not by B agents. When contracts are exclusive, this argument is enough to show that there are no pooling equilibria. Under non-exclusivity, a firm can offer a latent contract,

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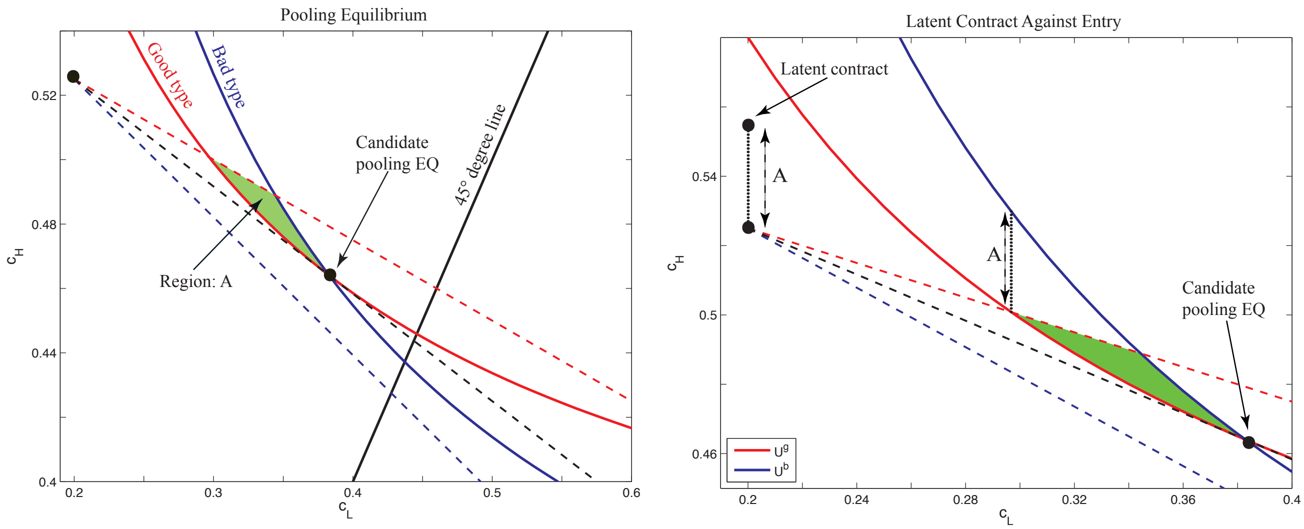
The deviation delivers strictly positive profits to the entrant even if only B agents accept it, since

$$\Pi^B\left(\hat{c}\right) = \pi_b \alpha \varepsilon - (1-\pi_b)\varepsilon \Rightarrow \frac{\Pi^B(\hat{c})}{\pi_b \varepsilon} = \alpha - \frac{1-\pi_b}{\pi_b} > 0,$$

with the last inequality following from (13). In this case, no latent contract can be used to prevent such deviation because it is profitable also in the worst case for the firm when only B agents accept it.

[14] The pooling equilibrium that delivers the highest expected utility when agents are weighted equally. The same argument holds for other pooling equilibria that deliver non-negative profits.

as point A in Figure ?? panel b, that makes any deviation in the green area unprofitable. If an incumbent firm offers such a contract, together with the pooling allocation, no other firm will find it profitable to offer a contract in the green area since it would be chosen by all agents: good types prefer it to the pooling allocation, and bad types prefer it to the pooling allocation once it is combined with the latent contract A. This implies that this deviation is no longer profitable. Hence, it would not be offered. This argument highlights an important feature of non-exclusivity of contracts: although competition reduces the contracts that can be offered, the presence of latent contracts allows firms to prevent some deviations, which is not possible when contracts are exclusive.



(a)

(b)

Figure 7: Equilibrium examples: in panel (a) pooling equilibrium; in panel (b) the role of latent contracts.

6 Conclusion

Before concluding, we discuss briefly the implications for the insurance provision in a natural extension of our environment when the economy is populated by agents with more than two types. Let $\Theta = \{\theta_1, ..., \theta_N\}$ be the possible types, ordered such that higher types have a higher probability of the good state happening, i.e., $\pi_{\theta_i} > \pi_{\theta_j}$ if $\theta_i < \theta_j$. Arguments in the previous sections can be extended to show that there is no pooling equilibrium with the worst type (type θ_1): that is, no cross-subsidization will occur with the worst type. Also, following the previous analysis, the unique equilibrium allocation for this agent type is full insurance under his actuarially-fair price. This limits significantly the insurance contracts that can be offered to the other types, since they must deliver less utility to the worst type than his own equilibrium contract. Finally, any contract, different than autarky, offered to the other types can be made unprofitable in the presence of latent contracts (as the line that delivers zero profit to the worst type). As in the two-types case, any contract that delivers positive insurance to a lower-risk type can be combined with a latent contract and be chosen by the high-risk type, inducing negative profits. Hence, the unique equilibrium –if it exists– in this case would feature full insurance to the high-risk type without subsidization from lower-risk types.

In this paper we characterize the equilibrium of a standard adverse selection economy in which agents can sign simultaneous insurance contracts with more than one firm. In this case, the amount of insurance provided is reduced when compared with the environment in which agents sign exclusive contracts. We show that there is no pooling equilibrium and that, under certain parameter restrictions, the bad type receives full insurance at his actuarially-fair price while the good type receives no insurance. An important message of this paper is that non-exclusivity of contracts imposes strong restrictions on the insurance contracts that are offered, reducing drastically the provision of insurance. In this sense, this friction can be interpreted as a rationale for the strong regulations on multiplicity of insurance contracts observed in several insurance markets, such as property and health insurance.

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Appendix

A Proofs of Section 3

Proof of Lemma 1

Proof. 1. Suppose (3) does not hold; this implies:

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} > \frac{1-\pi_b}{\pi_b}. \tag{14}$$

Consider the following menu offered by an entrant: $\hat{c} = (\varepsilon, -\alpha\varepsilon)$ for some small ε and where α satisfies:

$$\frac{1-\pi_b}{\pi_b}\frac{u'(c_L)}{u'(c_H)} > \alpha > \frac{1-\pi_b}{\pi_b}. \tag{15}$$

The parameter α can be interpreted as the slope of a line passing between the zero-profit line of the bad type and the slope of his indifference curve through c (the original pooling equilibrium). This deviation is chosen by the B agents since

$$\begin{aligned} U^B(c+\hat{c}) &= \pi_b u(c_H - \alpha\varepsilon) + (1-\pi_b)u(c_L+\varepsilon) \\ &= \pi_b u(c_H) + (1-\pi_b)u(c_L) - \pi_b u'(c_H)\alpha + (1-\pi_b)u'(c_L) \\ &> U^B(c) - \pi_b u'(c_H)\alpha + \pi_b u'(c_H)\alpha \\ U^B(c+\hat{c}) &> U^B(c) \end{aligned} \tag{16}$$

where the first inequality comes from (15).

The minimum profits for the entrant occur when only the bad types accept this contract, since this increases the probability of a positive transfer from the insurance provider to the agent.[15] This deviation delivers strictly positive profits to the entrant even if only B agents accept it:

$$\Pi^B(\hat{c}) = \pi_b\alpha\varepsilon - (1-\pi_b)\varepsilon \quad \Rightarrow \quad \frac{1}{\pi_b\varepsilon}\Pi^B(\hat{c}) = \alpha - \frac{(1-\pi_b)}{\pi_b}.$$

The first inequality of (15) then implies $\Pi^B(\hat{c}) > 0$. Hence, no equilibrium contract can prevent this deviation.

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[15] In general, given a contract of the type $\Gamma = (x\varepsilon, y\varepsilon)$, let $\pi^* = argmin_{\pi_a \in [\pi_b, \pi_g]}\{\pi_a \cdot x\varepsilon + (1-\pi_a)\cdot y\varepsilon\}$. The minimum profits are given when the type of agent has an accident probability equal to π^*; that is, $\pi^* = \pi_b$ if $x > y$ and $\pi^* = \pi_g$ if $x < y$.

2. Suppose (4) does not hold. This implies:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} < \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{17}$$

Consider the following deviating menu $\hat{c} = (c_L - \varepsilon, c_H + \alpha\varepsilon)$ where α satisfies:

$$\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} < \alpha < \frac{1-\hat{p}_H}{\hat{p}_H}. \tag{18}$$

Differently from the previous case, this deviation will be accepted as a substitute allocation by G agents rather than an additional allocation as in the previous case, since:

$$\begin{aligned} U^G(\hat{c}) &= \pi_g u(c_H + \alpha\varepsilon) + (1-\pi_g)\, u(c_L - \varepsilon) \\ &= \pi_g u(c_H) + (1-\pi_g)\, u(c_L) + \pi_g u'(c_H)\,\alpha - (1-\pi_g)\, u'(c_L) \\ &> U^G(c) + \pi_g u'(c_H)\frac{1-\pi_g}{\pi_g}\frac{u'(c_L)}{u'(c_H)} - (1-\pi_g)\, u'(c_L) > U^G(c). \end{aligned} \tag{19}$$

This deviation delivers strictly positive profits to the entrant if all agents accept it, since:

$$\begin{aligned} \Pi(\hat{c}) &= \hat{p}_H(\omega_H - c_H - \alpha\varepsilon) + (1-\hat{p}_H)(\omega_L - c_L + \varepsilon) \\ &= \Pi(c) - \hat{p}_H\alpha\varepsilon + (1-\hat{p}_H)\,\varepsilon > \Pi(c) \geq 0. \end{aligned}$$

The deviation also delivers strictly positive profits to the entrant if only G agents accept it, since:

$$\begin{aligned} \Pi^G(\hat{c}) &= \pi_g(\omega_H - c_H - \alpha\varepsilon) + (1-\pi_g)(\omega_L - c_L + \varepsilon) \\ &= \pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) - \varepsilon(\alpha\pi_g - (1-\pi_g)) \\ &> \pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) - \varepsilon\left(\frac{(1-\hat{p}_H)}{\hat{p}_H}\pi_g - (1-\pi_g)\right), \end{aligned}$$

so that

$$\frac{\Pi^G(\hat{c})}{\pi_g} > (\omega_H - c_H) + \frac{(1-\pi_g)}{\pi_g}(\omega_L - c_L) - \varepsilon\left(\frac{(1-\hat{p}_H)}{\hat{p}_H} - \frac{(1-\pi_g)}{\pi_g}\right).$$

Since $\frac{(1-\hat{p}_H)}{\hat{p}_H} - \frac{(1-\pi_g)}{\pi_g} > 0$, showing that $(\omega_H - c_H) + \frac{(1-\pi_g)}{\pi_g}(\omega_L - c_L) > 0$ is enough to prove that there exists ε small enough so that $\Pi(\hat{c}) > 0$.[16] Suppose by way of contradiction that $\pi_g(\omega_H - c_H) + (1-\pi_g)(\omega_L - c_L) \leq 0$. Equation (3) implies that

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[16] Since the first part of the Lemma implies that $c_L \geq c_H$ in any pooling equilibrium, the endowment point can never be a pooling equilibrium.

$c_L \geq c_H$, so:

$$\begin{aligned}
(\omega_H - c_H) &> (\omega_L - c_L) \\
\pi_g(\omega_H - c_H) + (1 - \pi_g)(\omega_L - c_L) &> \pi_b(\omega_H - c_H) + (1 - \pi_b)(\omega_L - c_L) \geq 0.
\end{aligned}$$

The last inequality comes from the fact that total profits under c must be non-negative, which under the contradicting assumption implies that $\pi_b\left(\omega_H - c_H\right) + \left(1 - \pi_b\right)\left(\omega_L - c_L\right) \geq 0$, reaching a contradiction. So it must be true that $\pi_g\left(\omega_H - c_H\right) + \left(1 - \pi_g\right)\left(\omega_L - c_L\right) > 0$, and consequently condition (4) must hold.

□

Proof of Lemma 2

Proof.
Part 1. Suppose that condition (6) is violated. If so, there exists an α so that

$$\frac{1 - \pi_g}{\pi_g}\frac{u'(c_L^G)}{u'(c_H^G)} > \alpha > \frac{1 - \hat{p}_H}{\hat{p}_H}. \tag{20}$$

Consider an entrant firm offering the menu $\hat{c} = (\varepsilon, -\alpha\varepsilon)$. This menu is profitable even if all agents accept it (and as will be shown next ,the G type always accepts it), since

$$\Pi(\hat{c}) = \hat{p}_H \alpha\varepsilon - (1 - \hat{p}_H)\varepsilon \Rightarrow \frac{\Pi(\hat{c})}{\varepsilon \hat{p}_H} = \alpha - \frac{(1 - \hat{p}_H)}{\hat{p}_H} > 0,$$

where the last inequality follows from (20). In addition, $\hat{C}$ is accepted by the good type, since

$$U^G(C+\hat{C}) = \pi_g u(c_H^G - \alpha\varepsilon) + (1-\pi_g)u(c_L^G + \varepsilon) = U^G(C) - \alpha\pi_g u'(c_H^G) + (1-\pi_g)u'(c_L^G) > U^G(C),$$

where the last inequality follows from (20): $(1 - \pi_g)u'(c_L^G) > \alpha\pi_g u'(c_H^G)$.

Suppose that condition (7) is violated. If so, there exists an α so that

$$\frac{1 - \pi_g}{\pi_g}\frac{u'(c_L^G)}{u'(c_H^G)} < \alpha < \frac{1 - \pi_g}{\pi_g}. \tag{21}$$

Consider an entrant firm offering the menu $\hat{c} = (-\varepsilon, \alpha\varepsilon)$. This menu is profitable even if all types accept it:

$$\Pi(\hat{c}) = -\hat{p}_H \alpha\varepsilon + (1 - \hat{p}_H)\varepsilon \Rightarrow \frac{\Pi(\hat{c})}{\varepsilon \hat{p}_H} = -\alpha + \frac{(1 - \hat{p}_H)}{\hat{p}_H} > 0,$$

where the last inequality follows from (21): $\alpha < \frac{1-\pi_g}{\pi_g} < \frac{(1-\hat{p}_H)}{\hat{p}_H}$. Also $u^G(c^G + \tilde{c}) > u^G(\tilde{c})$, so that the G agent is willing to accept it.

Part 2. The proof that the slope of the indifference curve for the B agent at his equilibrium allocation is less steep that the zero-profits line for B agents follows the proof of (3) in Lemma 1. So, $c^B \equiv (c_L^B, c_H^B)$ must be in the over-insurance region ($c_L^B \geq c_H^B$). First, the allocation for the B agents cannot generate positive profits for a firm if only B agents accept it $\left(\Pi^B(c^B) = \pi_b(\omega_H - c_H^B) + (1-\pi_b)(\omega_L - c_L^B) \leq 0\right)$. If so, an entrant can offer an alternative allocation that increases an agent's utility and delivers strictly positive profits independently of the agents that accept it.

Using a similar argument, we conclude that the allocation for the G agent cannot deliver positive profits if all agents accept it $\left(\Pi^{all}(c^G) = \hat{p}_H(\omega_H - c_H^G) + (1-\hat{p}_H)(\omega_L - c_L^G) < 0\right)$. We now show that the allocation c^B delivers 0 profits. Since it delivers non-positive profits, if it is such that $U^B(c^B) > U^B(c^G)$, there exists $\varepsilon > 0$ small enough such that the $U^B(c^B - \varepsilon) > U^B(c^G)$, and profits are higher. If it is such that $U^B(c^B) = U^B(c^G)$, we consider two cases. If $c_H^G = c_L^G$, the fact that c^B is in the same indifference curve and that it is on the over-insurance region implies that the total profits for both c^G and c^B are less than the profits from all agent picking c^G. A contradiction is then reached, since $\Pi^{all}(c^G) < 0$.

If $c_H^G > c_L^G$, an entrant can offer a contract $(\varepsilon, -\varepsilon\alpha)$ such that $\frac{1-\pi_b}{\pi_b}\frac{u'\left(c_L^G\right)}{u'\left(c_H^G\right)} > \alpha > \frac{1-\pi_b}{\pi_b}$. This contract, combined with c^G, increases the utility of B agent and delivers positive profits. This contradicts the original allocation being an equilibrium. Finally, given that c^B delivers 0 profits, it must be the allocation that maximizes agents' utility under this condition. If not, c^B is on the zero-profit line and $c_L^B > c_H^B$ (the opposite inequality is excluded by (8)), so there exists α such that $\frac{1-\pi_b}{\pi_b}\frac{u'(c_L^B)}{u'(c_H^B)} < \alpha < \frac{1-\pi_b}{\pi_b}$. An entrant can offer the contract $\tilde{c} = (c_L^B - \varepsilon, c_H^B + \alpha\varepsilon)$. This contract is preferred by B agents and delivers positive profits even if all agents accept it. Hence, the equilibrium c^B must be on the intersection of the zero-profit line for B agents and the 45° line: $c^B = (\omega^B, \omega^B)$.

□

Proof of Proposition 2

Proof. Lemma 2 fully characterizes the allocation for B agents. In equilibrium, the allocation for the G agent must satisfy $U^G(c^G) \geq U^G(c^B)$ and $U^G(c^G) \geq U^G(\omega_L, \omega_H)$. This implies that in the cartesian plane (c_L, c_H), c^G must lie in the triangle delimited by the endowment point, the intersection of the indifference curve of the B agent at c^B with the zero-profit line for the G agent, and the intersection of the indifference curve of the B agent at c^B, with the intersection of indifference curve of the G agent at the endowment.

Suppose c^G is in this triangle and $c^G \neq (\omega_L, \omega_H)$. An entrant can offer the following contract: $\tilde{c} = (c_L^B - c_L^G + \varepsilon, c_H^B - c_H^G)$ for a small $\varepsilon > 0$. This allocation will be chosen by the B type together with c^G since $\tilde{c} + c^G = c^B + (\varepsilon, 0)$. Also, this allocation is profitable for the entrant if any type chooses it. To see this, rewrite

$$\tilde{c} = (c_L^B - \omega_L + \omega_L - c_L^G + \varepsilon, c_H^B - \omega_H + \omega_H - c_H^G) = (t_L^B - t_L^G + \varepsilon, t_H^B - t_H^G),$$

where $t_L^B, t_L^G > 0$ and $t_H^B, t_H^G < 0$. Minimum profits occur when only the bad type accepts it. Suppose profits are negative: $-\pi_b(t_H^B - t_H^G) - (1-\pi_b)(t_L^B - t_L^G + \varepsilon) < 0$. Since $t_H^B = -\frac{1-\pi_b}{\pi_b} t_L^B$, $\pi_b t_H^G + (1-\pi_b)(t_L^G - \varepsilon) < 0$. For sufficiently small ε, this implies $-\frac{t_H^G}{t_L^G} > \frac{1-\pi_b}{\pi_b}$, since $t_L^G > 0$, $|\frac{t_H^G}{t_L^G}| > \frac{1-\pi_b}{\pi_b}$. This is a contradiction with $\left|\frac{\omega_H - c_H^G}{\omega_L - c_L^G}\right| \leq \frac{1-\hat{p_H}}{\hat{p_H}}$, since c^G lies above the average zero-profit line given the parameter restriction. $\square$