Finance 100 Problem Set
Capital Structure (Alternative Solutions)

Note: Where appropriate, the “final answer” for each problem is given in bold italics for those not interested in the discussion of the solution.

I. Formulas
This section contains the formulas you might need for this homework set:

1. The Weighted Average Cost of Capital (WACC):

\[ r_A = r_E \left( \frac{E}{V} \right) + r_D \left( \frac{D}{V} \right) \]  

where \( r_A \) is the return on assets, \( r_E \) is the return on equity, \( r_D \) is the return on debt, \( E \) is the value of equity, \( D \) is the value of debt and \( V \) is the total value of the firm \((D + E)\).

2. The Weighted Average of Debt and Equity Betas:

\[ \beta_A = \beta_E \left( \frac{E}{V} \right) + \beta_D \left( \frac{D}{V} \right) \]  

where \( \beta_A \) is the beta on assets, \( \beta_E \) is the equity beta, \( \beta_D \) is the debt beta, \( E \) is the value of equity, \( D \) is the value of debt and \( V \) is the total value of the firm \((D + E)\).

3. The CAPM:

\[ \mathbb{E}(r_i) = r_f + \beta_i (\mathbb{E}(r_M) - r_f) \]  

1
where $E(r_i)$ is the expected return on any asset $i$, $r_f$ is the risk-free return, $E(r_M)$ is the expected return on the market and $\beta_i$ is the asset’s beta, also equal to:

$$\beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}$$  \hspace{1cm} (4)

*Often we drop the expectations and simply used realized rates of return.*

4. Present Value of an Annuity Formula:

$$A_0 = \frac{a}{(1 + i)} + \frac{a}{(1 + i)^2} + \ldots + \frac{a}{(1 + i)^{N-1}} + \frac{a}{(1 + i)^N}$$

$$= a \cdot \frac{1 - (1 + i)^{-N}}{i}$$ \hspace{1cm} (5)

where $a$ is the amount of the annuity payment, $i$ is the periodic interest rate and $N$ is the total number of periods.

### II. Problems

1. The CAPM (equation (3)) implies (on average):

$$r_E = r_f + \beta_E (r_M - r_f)$$

$$r_D = r_f + \beta_D (r_M - r_f)$$

Substituting in for the unknowns in both equations yields:

$$r_E = 0.10 + 1.5 (0.18 - 0.10) = 0.22$$

$$0.12 = 0.10 + \beta_D (0.18 - 0.10) \implies \beta_D = 0.25$$

Since $D/E = 1$, we have $E/V = D/V = 0.5$. Thus, the WACC implies

$$r_A = r_E \left(\frac{E}{V}\right) + r_D \left(\frac{D}{V}\right)$$

$$= 0.22 \times 0.5 + 0.12 \times 0.5 = 0.17$$

Similarly, the equation (2) implies

$$\beta_A = \beta_E \left(\frac{E}{V}\right) + \beta_D \left(\frac{D}{V}\right)$$

$$= 1.5 \times 0.5 + 0.25 \times 0.5 = 0.875$$
2.

2.a
The debt beta is zero since the return to debt is equal to the risk-free return. The asset beta follows from equation (2):

\[ \beta_A = \beta_E \left( \frac{E}{V} \right) + \beta_D \left( \frac{D}{V} \right) \]
\[ = 0.8 \times 0.90 + 0 \times 0.10 \]
\[ = 0.72 \]

The cost of capital follows from the CAPM relation:

\[ E(r_A) = r_f + \beta_A (E(r_M) - r_f) \]
\[ = 0.05 + 0.72(0.13 - 0.05) \]
\[ = 0.1076 \]

The risk premium of the stock market is the difference between the return on the market and the risk-free asset, or 13% - 5% = 8%.

2.b
The company’s equity is worth $360 million and is 90% of the total value of the company implying the total value of the company is $400 million.

The total amount of new debt outstanding is 60%, or 0.6 \times 400 = $240 million. Hence the company has to issue $200 million of debt to pay a dividend of $200 million.

2.c
The debt beta after refinancing follows from the CAPM:

\[ E(r_D) = 0.05 + \beta_D (0.08) \]

Solving for \( \beta_D \) yields 0.125.

Equation (2) can be solved for \( \beta_E \) as:

\[ \beta_E = \beta_A + \frac{D}{E} (\beta_A - \beta_D) \]
\[ = 0.72 + \frac{0.6}{0.4} (0.72 - 0.125) \]
\[ = 1.6125 \]
The required return on equity follows from the CAPM:

\[ E(r_E) = 0.05 + 1.6125 \times (0.08) = 0.179 \]

2.d

The wealth of the shareholders as not changed as a result of the refinancing. Before the refinancing, total equity was worth $360 million. After the refinancing, they owned shares worth $160 million but received $200 million in cash, equaling the original $360 million. This result makes sense since in perfect capital markets (i.e. the assumptions of the Modigliani and Miller theorem are correct), any financial restructuring undertaken by the firm can be “undone” by investors.

2.e

The shareholders owning the position in part 2.d have a beta of:

\[ \beta_E' = \frac{160}{360} \times 1.6125 + \frac{200}{360} \times 0 = 0.717 \]

where \( \beta_E' \) is the beta of the shareholders after the refinancing. This follows indirectly from equation (2). The new shareholders have a portfolio of equity with \( \beta_E = 1.6125 \) and cash (which is risk-free) with \( \beta_C = 0 \).

This is lower than before because the shareholders have not invested in the risky debt. In order to rebalance their portfolio they need to invest \( x \) dollars in the stock market so that:

\[ \beta_E' = \frac{160}{360} \times 1.6125 + \frac{x}{360} \times 1 = 0.8 \]

(remember the beta for the market is 1). Solving for \( x \) yields 30. Thus, by investing $30 million (out of the $200 million dividend) in the stock market, they will have restored their original portfolio risk.

3.

3.a

We can calculate the asset beta for Upstart by unlevering each company’s beta (i.e. finding their asset beta) and averaging. We are given debt-equity
ratios which may be used to find the relative weights on debt and equity. For example, company A’s debt-equity ratio is 20%, implying

\[
\frac{D}{E} = \frac{D}{V} = \frac{E}{V} = 20\%
\]

Since \( D/V = 1 - E/V \), we have:

\[
\frac{D}{V} = 0.20 \times \left( 1 - \frac{D}{V} \right) = 0.1667
\]

implying that \( E/V = 0.8333 \). Similar calculations for companies B and C reveal:

*Company B*: \( \frac{D}{V} = 0.2; \frac{E}{V} = 0.8 \)
*Company C*: \( \frac{D}{V} = 0.333; \frac{E}{V} = 0.667 \)

We can now determine each company’s asset beta using equation (2).

\[
\beta_A = 0.1667 \times 0 + 0.8333 \times 1.2 = 1.0
\]

\[
\beta_B = 0.2000 \times 0.1 + 0.8000 \times 1.0 = 0.82
\]

\[
\beta_C = 0.3333 \times 0.3 + 0.6667 \times 1.8 = 1.3
\]

The average of these three asset betas is 1.04. Plugging this number into the CAPM relation produces an estimate of *Upstart’s cost of capital*:

\[
r_{Upstart} = 0.05 + 1.04(0.08) = 0.1332
\]

**3.b**

Before using the CAPM, we must determine Upstart’s equity beta by using equation (2). Solving equation (2) for \( \beta_E \) yields

\[
\begin{align*}
\beta_E &= \beta_A + \frac{D}{E} (\beta_A - \beta_D) \\
&= 1.04 + 0.25 (1.04 - 0.1) \\
&= 1.275
\end{align*}
\]

Plugging numbers into the CAPM yields:

\[
\begin{align*}
r_E &= r_f + \beta_E (r_M - r_f) \\
&= 0.05 + 1.275(0.08) \\
&= 0.152
\end{align*}
\]
4.

4.a
Recall the WACC (equation (1)):

\[ r_A = r_E \left( \frac{E}{V} \right) + r_D \left( \frac{D}{V} \right) \]

We need to find numbers for \( r_E \), \( r_D \) and \( \frac{D}{V} \) (\( \frac{E}{V} \) follows from \( 1 - \frac{D}{V} \)).

Since the debt is risk-free, \( r_D = 0.05 \), the risk-free rate. The CAPM enables us to find \( r_E \):

\[ r_E = r_f + \beta_E (r_M - r_f) \]
\[ = 0.05 + 0.75(0.08) \]
\[ = 0.11 \]

Finally, since the debt-equity ratio = 0.25,

\[ \frac{D}{E} = \frac{\frac{D}{V}}{\frac{E}{V}} = 0.25 \implies \frac{D}{V} = 0.25(1 - \frac{D}{V}) \]

Solving for \( D/V \) yields 0.2, implying \( E/V = 0.8 \).

We now have all of the information needed to use the WACC formula.

\[ r_A = 0.11 \times 0.8 + 0.05 \times 0.2 = 0.098 \]

4.b
Since equity is expected to return 11% (by our CAPM calculation above), the stock price next period should be \((1.11) \times 50 = 55.5\) per share cum-dividend (i.e. including the dividend payment). Thus the total dollar gain per share is $5.50. However, of this amount, $3.75 is a dividend, implying that the ex-dividend (i.e. excluding the dividend) price is $51.75.

4.c
Under this analysts scenario, the total gain including the dividend is $3.00 + $3.75 = $6.75. This is a return of 13.5%, which is greater than the CAPM predicted return found in part 1.a. Thus, the security lies above the security market line and is therefore currently undervalued.\(^1\) In order to be consistent

\(^1\)The abnormally high return implies that the price must be too low today and hence undervalued.
with the CAPM prediction, the current price today must adjust so that the return is 11%. Since the price today is equal to tomorrow’s price plus the dividend divided by the expected return, we have:

\[
P_0 = \frac{P_1 + D_1}{(1 + r)} = \frac{$53 + $3.75}{(1 + 0.11)} = $51.13
\]

Thus, the today’s price must increase from $50 to $51.13.

4.d
The growth rate according to the dividend growth mode must satisfy:

\[
P_0 = \frac{D_1}{(r - g)}
\]

which implies

\[
g = r - \frac{D_1}{P_0} = 0.11 - \frac{3.75}{50} = 0.035
\]

Now we are looking at the end of the period and must determine \( P_1 \). The ex-dividend price is $50 one year from today.

\[
P_1 = \frac{3.75}{(0.11 - 0.035)} = $50
\]

Immediately before this, the stock price was expected to be $51.75 implying that the stock dropped \((51.75 - 50)/50 = 3.5\%\). In cum-dividend terms, the stock price dropped from $3.75+$51.75=$55.00 to $50+$2.50=$52.50, a drop of 5%.

4.e
In ex-dividend terms, the price now is:

\[
P_1 = \frac{D_2}{(r - g)} = \frac{(1.035 \times $2.50)}{0.11 - 0.035} = $34.50
\]

Hence the price drops from $51.75 to $34.50 or by 33.33%.
5.

5.a
We can use the CAPM to determine the required rate of return on equity as:

\[ \mathbb{E}(r_E) = r_f + \beta_E (\mathbb{E}(r_M) - r_f) \]
\[ = 0.05 + 1.5(0.15 - 0.05) \]
\[ = 0.20 \]

5.b
We can use the CAPM to determine the required rate of return on debt as:

\[ \mathbb{E}(r_D) = r_f + \beta_D (\mathbb{E}(r_M) - r_f) \]
\[ = 0.05 \]

(or we simply could have recognized the 0 beta for debt as implying that it is risk-free)

The total value of the company is the sum of equity and all debt: $120 + $60 + $20 = $200 million. Thus, D/V = 0.4 implying that E/V = 0.6. We can now use the WACC to show:

\[ r_A = 0.4 \times 0.05 + 0.6 \times 0.20 = 0.14 \]

Alternatively, we could have used the CAPM after recovering the unlevered (i.e. asset) beta for the company.

5.c
Recall that the NPV of the project is simply the present value of all cash flows to the project. Thus,

\[ NPV = -2 + 0.45 \times \frac{1 - (1 + 0.14)^{-10}}{0.14} - \frac{1}{(1 + 0.14)^{10}} = 0.0775 \]

where all dollar amounts are in millions and we have use equation (5). Since the project has a positive NPV, the firm should undertake the project.
5.d
Since the lease payments are essentially guaranteed (i.e. risk-free), we may
discount them at the risk-free rate. The cash flows from the project, however,
have a typical uncertainty associated with the company’s assets and thus
should be discounted at the company’s cost of capital.

The present value of cash flows from the project are:

\[ PV = -0.5 + 0.45 \times \frac{(1 - (1 + 0.14)^{-10})}{0.14} - \frac{1}{(1 + 0.14)^{10}} = 1.5775 \]

where all dollar amounts are in millions.

The present value of the lease payments is:

\[ PV = 0.2 \times \frac{1 - (1 - 0.05)^{-10}}{0.05} = 1.5443 \]

Subtracting 1.5443 from 1.5775 yields the NPV of the project, $0.0332. Since this figure is less than the NPV from purchasing the machine, **we should buy the machine.**

6.

6.a
The share price is the present value of the end of period dividend divided by
the number of shares (liquidating dividend meaning the company is dissolving
itself of all value and paying it out to the shareholders). Thus,

\[ V_0 = \frac{D_1}{(1 + r_E)} = \frac{55}{(1 + 0.10)} = 50 \]

Thus, **the per share price is $50 / 2.5 = $20.**

6.b
The total value of Z is equal to the value of its debt and equity or $42 + $10
$52 million.

6.c
By the Modigliani and Miller theorem, these two companies should have the
same value. Thus, comparing our answers from parts 1.a and 1.b, company
Z is overvalued relative to company X (or company X is undervalued relative to company Z). We should be able to establish an arbitrage strategy to exploit this mispricing, by selling the overvalued security and buying the replicating portfolio. The arbitrage strategy is presented in Table 1.

Table 1:

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Today</td>
</tr>
<tr>
<td>Sell Z’s stock</td>
<td>42</td>
</tr>
<tr>
<td>Replicating Portfolio</td>
<td></td>
</tr>
<tr>
<td>Buy X’s stock</td>
<td>-50</td>
</tr>
<tr>
<td>Borrow</td>
<td>10</td>
</tr>
<tr>
<td>Total Cash Flows</td>
<td>2</td>
</tr>
</tbody>
</table>

where $V_1$ is the value of both companies in the next period.

7.

7.a
The risk premium in the market is $r_M - r_f = 0.08$. The equity beta follows from the CAPM relation which implies:

$$\beta_E = \frac{(r_E - r_f)}{(E(r_M) - r_f)} = \frac{(0.16 - 0.06)}{(0.14 - 0.06)} = 1.25$$

The debt beta is zero since the rate of return is equal to the risk-free rate.

7.b
We can use the WACC to compute the cost of capital.

$$r_A = r_E\left(\frac{E}{V}\right) + r_D\left(\frac{D}{V}\right) = 0.16 \times 0.875 + 0.06 \times 0.125 = 0.1475$$
Unlevering betas yields:

\[ \beta_A = \beta_E \left( \frac{E}{V} \right) + \beta_D \left( \frac{D}{V} \right) \]
\[ = 1.25 \times 0.875 + 0 \times 0.125 \]
\[ = 1.094 \]

The cost of capital follows from the CAPM:

\[ E(r_A) = r_f + \beta_A (E(r_M) - r_f) \]
\[ = 0.06 + 1.094(0.08) \]
\[ = 0.1475 \]

We get the same answer as expected.

**7.c**

In order to achieve a return of 20% under the assumptions of the CAPM, the company’s equity beta has to satisfy

\[ \beta_E = \frac{(r_E - r_f)}{(E(r_M) - r_f)} \]
\[ = \frac{(0.20 - 0.06)}{(0.14 - 0.06)} \]
\[ = 1.75 \]

We can use equation (2) to solve for the debt-value ratio:

\[ \frac{D}{V} = \frac{\beta_A - \beta_E}{\beta_D - \beta_E} \]
\[ = \frac{1.09 - 1.75}{0 - 1.75} \]
\[ = 0.375 \]

implying that \( E/V \) equals 0.625.

Since the market capitalization (i.e. equity value) of the company is $180 million, the value of the company as a whole is:

\[ V = \frac{180}{(1 - 0.125)} = $205.71 \text{ million} \]
The total value of debt outstanding is:

\[ V = V \times \frac{D}{V} = 77.14 \text{ million} \]

Thus, the firm should issue an additional amount of debt equal to $51.43 million implying an ex-dividend share price drop equal to $12.86 per share.

†

\[ \text{These solutions are produced by Michael R. Roberts. Thanks go to Jen Rother for her excellent assistance, and to an anonymous TA. Any remaining errors are mine.} \]