1. Consider a three-period problem. Suppose the stock’s price today is 50, and will change three times, each time to one of two possible values:

\[
\begin{array}{c}
50 \\
45 \\
41 \\
55 \\
62 \\
70 \\
55 \\
45 \\
41 \\
38 \\
\end{array}
\]

In each of the three periods, the risk-free return is 3%. What is the price today of a call on this stock with strike price 60? What is the price of a put with the same strike price?

2. Here’s another direction to take the same logic. Suppose there are three securities, x, y and z, which cost 0.6, 0.7 and 0.8, respectively. Suppose also there are exactly three possible future states of the world, call them a, b and c, respectively. Suppose finally that x pays 1 in states a and b, and 0 in c; y pays 1 in states a and c, and 0 in state b; and z pays 1 in states b and c, and 0 in state a. What is the cost of a portfolio that pays 1 in every state? (Note that now you have 3 equations and 3 unknowns, rather than 2 equations and 2 unknowns.) Suppose the risk-free return is 3% (i.e., for $1 you can buy a security that pays $1.03 in all future states); what trade does this suggest?

3. Here’s a simple example of how option-pricing techniques are applied to any security with embedded options. Suppose GE issues a note today which matures in two years but is callable at par (i.e. 100) in one year, and which has a 7% coupon, paid annually. Suppose everyone knows that in one year, the one-year rate will be either 6% or 8% (i.e., in one year, the value of $1 paid a year later will be either 1/1.06 or 1/1.08). Suppose finally that there are two non-callable GE notes outstanding: a 1-year and a 2-year, both paying 7% coupons, and both trading for 100. From this information, you can price the callable bond, as follows:

a. In one year, holders of the callable will get the coupon of 7, and in addition to that, will get 100 if the note is called, and will have a note paying 107 in a year if it is not called. Given this, what value will they get if the one-year rate is 6% in one year, and what value will they get if the one-year rate is 8% in one year?
b. In one year, holders of the 1-year *non*-callable will get 107, and holders of the 2-year non-callable will get the coupon of 7, and will have a note paying 107 in one year. Given this, what value will holders of the non-callable two-year note get if the one-year rate is 6% in one year, and what value will they get if the one-year rate is 8% in one year?

c. What portfolio of the one-year and two-year non-callable notes is worth the same as the callable note in one year if the one-year rate is 6% and if it is 8%? Since this portfolio is worth exactly the same as the callable in both future states, it must have the same price today. So what is the price of the callable today?