Today:
I. Equity is a call on firm value
II. Senior Debt
III. Junior Debt
IV. Convertible Debt
V. Variance and Debt Capacity
VI. Dividends

I. Equity is a call on firm value

Imagine a firm with debt outstanding
• Face value $F$
• Maturing at time $T$
• No coupons

When time $T$ arrives, the company has to pay the $F$ to creditors, or else go bankrupt
• Assume that bankruptcy courts obey *absolute priority*
  • Junior claim gets nothing until Senior claim is paid in full
  • We’ll see later, this is only an approximation
• Also, no bankruptcy cost
  • Also an approximation

If the company’s value at $T$ is $V_T$, this implies that creditors will get
• $F$, if $V_T$ is at least $F$, and
• $V_T$ if $V_T$ is less than $F$

Equity gets the rest
• $V_T - F$ if $V_T$ is at least $F$, and
• 0 otherwise

Or in other words, equity gets $\max\{0, V_T - F\}$, so equity is effectively a call option on firm value, with expiration $T$ and strike price $F$
• Debt is the remainder: firm value *minus* a call with strike $F$ and expiration $T$
Equity is like a call on firm value, so we can invoke lessons from option pricing, especially these two:
- Other things constant, equity goes down when firm value goes down
- Other things constant, equity goes up when firm variance goes up

So what happens if firm value goes down by \( x \) and firm variance goes up by \( y \)? It depends on the face value of debt:
- If the face value is zero, equity just goes down by \( x \)
- As the face value goes up relative to firm value, the effect on equity improves and eventually becomes positive

Here’s a numerical example to illustrate. We can use the Black-Scholes option-pricing formula to calculate the price of a call option on a stock under two scenarios
- \( S=100 \) and \( \text{var} = 0.2 \) (high value, low variance scenario)
- \( S=98 \) and \( \text{var} = 0.25 \) (lower value, higher variance scenario)
- The effect on equity value of simultaneously reducing firm value by \( x \) and increasing firm variance by \( y \) is illustrated by the change in the call price from the first scenario to the second scenario

We will fix \( T=0.25 \) and \( r=5.5 \), and calculate the option value under each scenario for a range of strike prices
- As the strike price goes up, we see that the change in the call price goes from negative to positive
We’re not going to prove this mathematically, but the intuition is simple. Higher firm variance makes the firm’s upside better and its downside worse, and equity gets all the upside above the face value of debt but none of the downside below the face value of debt. The important implication is that distressed firms have a perverse incentive to ramp up risk

- Choose higher-risk, lower-expected-value investments
- They don’t want low expected value, but they aren’t put off by it because they aren’t the ones bearing the downside, Creditors bear the downside

An NHL analogy may be helpful. In a hockey game, a team that’s behind will often pull its goalie in the last minute in favor of an additional offensive player

- Chance of giving up another goal is higher than the chance of scoring one (how often does this ever pan out?)
- So, unconditionally, it isn’t a +NPV way to play hockey
  - Higher variance, lower expected value (measured in goals)
- But in the last minute, the losing side gets a big upside from scoring a goal, and hardly any downside from giving up another, so in that situation it makes sense

**The Big Picture:** Equity is a call on firm value, and equity controls the variance of firm value. This is a conflict of interest that is severe for firms in or near insolvency.
II. Senior Debt

We just showed, holding senior debt is like holding the firm value, and being short a call option on firm value, i.e. $V_T - \max\{0, V_T - F_S\}$, where $F_S$ is the face value of senior debt.

- By put-call parity, this is the same as holding a risk-free bond with face value $F_S$, and being short a put with strike price $F_S$.
- Remember, $S - C = PV(X) - P$

So the credit risk of a corporate bond is represented by the value of this put.

To some extent, you can price corporate bonds this way, that is:

- Price the bond as if it were risk-free (i.e. the price of a government bond with the same maturity and coupon, which you would get from STRIP prices).
- Calculate the put value with Black-Scholes:
  - Time to expiration = maturity
  - Variance = firm variance
- Subtract B from A

Caveat is, B-S presumes that you could buy and rebalance the replicating portfolio of the underlying security and the risk-free asset.

- Underlying security = firm value
- No way you’re going to hold a fraction of, let alone actively trade, the firm value
- Firm value is held by all its different claimants: banks, trade creditors, employees, other bondholders, etc. These are generally not traded securities
- You could maybe assemble a portfolio of traded claims that is roughly correlated with the total firm value

Option pricing helps us think about bond risk and incentive problems, but the precision is limited by what you can trade, and how often
III. Junior Debt

Suppose now that the firm has two debt issues outstanding
- Senior debt with face value $F_S$
- Junior debt with face value $F_J$
- Same maturity date $T$

It’s still true that Senior debt gets the value of the firm up to $F_S$
- Existence of a junior debt claim doesn’t change this
- Still like holding firm value, and being short a call with strike $F_S$

And equity is still like a call on firm value, but with a different strike
- Have to pay both issues, so strike is $F_S + F_J$

Junior debt is everything else, i.e. firm value minus
- Senior debt: Firm value minus call with strike $F_S$
- Equity: call with strike $F_S + F_J$

Which works out to
- Call with strike $F_S$ minus
- Call with strike $F_S + F_J$

So holding junior debt is like being long a call on firm value, with strike price equal to all debt it is junior to, and short another call with a higher strike price, where the difference is the face value of the junior debt

Both of these calls will increase in value when firm variance goes up
- But jr. debt is long one and short the other, so the net effect is not obvious
Intuition suggests that

- Junior debt benefits from higher variance when the firm is in big trouble
  - Suppose the firm owes 40 to sr. debt and 60 to jr. debt, and is currently worth 45
  - Jr. debt gets a lot of the upside, and almost none of the downside
  - From a financial perspective, jr. debt is the *de facto* equity of this company
  - Big incentive for increased variance

- Junior debt is hurt by higher variance when the firm is solvent
  - Suppose the same firm is worth 105
  - Jr. debt gets none of the upside, and much of the downside
  - Big incentive for decreased variance

If you do the math (we won’t), you’ll see that, as firm value increases relative to the face values of jr. and sr. debt, the effect of increasing variance on jr. debt is

- Positive for low firm value
- Negative for high firm value

as intuition suggests. Notice that jr. and sr. debt have conflicting agendas when the debtor is in financial distress.
**AN EXAMPLE USING BLACK-SCHOLES:**

Consider a firm with sr. debt of 40 and jr. debt of 60, so that jr. debt is
- Call with strike price 40, *minus*
- Call with strike price 40+60=100

Here’s the change in jr. debt value when variance goes from 0.2 to 0.25 (setting $T=0.25$ and $r=5.5\%$), for a range of firm values:

When firm value is low, jr. debt benefits from the higher variance, but when it is high, it is hurt
- For those who feel like doing math, what’s going on here is that the sensitivity of the call price w/r/t variance is highest when the call is near the money. The long option is near the money when firm value is near 40, and the short option is near the money when firm value is near 100.

**The Big Picture:** Junior debt is similar to Senior debt when the firm is far from default, and is more like equity when default is likely or inevitable. In a distress situation, where creditors try to bargain with each other and with management, this can drive a wedge between the goals of different creditor classes.
IV. Convertible Debt

A convertible bond is one that can be exchanged for the common stock of the issuer. Consider the January 17th issue by Theravance:

$172.5 million face value
Issued at par on 1/17/08, Maturing 1/15/15 (i.e. seven-year bond)
3% coupons, semiannually (just like a regular bond)

The interesting part is:
$1000 face value convertible into 38.6548 shares of THRX
• Note, $1000/38.6548 = $25.87, called the conversion price

When the deal was priced on 1/16, THR’s stock closed at $19.90/share
• $25.87/19.90 = 1.3; said to be a 30% conversion premium
• If the stock goes up by the conversion premium, then the shares the bond is convertible into are worth more than the bond’s face value
The bond is not callable at all for its first four years; this period is called the “call-protected” period
- At four years, the issuer acquires a *soft call*
- Starting 1/15/12, Theravance can call the bond at par *if the stock is trading at least 30% above the conversion price*
  - Technically, we may not redeem the notes prior to January 15, 2012. On or after January 15, 2012 and prior to the maturity date, we may redeem for cash all or part of the notes if the last reported sale price of our common stock has been greater than or equal to 130% of the conversion price then in effect for at least 20 trading days during any 30 consecutive trading day period prior to the date on which we provide notice of redemption. The redemption price will equal 100% of the principal amount of the notes to be redeemed, plus accrued and unpaid interest to but excluding the redemption date.
- A regular call would not have conditioned on the recent stock price

If Theravance calls the bond, investors can *either*
- Turn in their bonds for the call price, *or*
- Convert them into shares
Given that Theravance can call this bond only when the shares are worth 30% more than the call price, it is immediate that investors will take the shares, not the cash

We’ll get back to convertibles in a few classes. For now, just notice that
- Convertible holders get a piece of THRX’s upside if it goes above the conversion price during the call-protected period
  - If THRX goes up to $50, say, bondholders will be able to exchange for shares worth $50(38.6548) = $1932.74
- If the stock is far below the conversion price, the bond will trade like any other risky debt (i.e., the call will be a small part of the bond’s value)
  - Convertibles in this situation are often called *broken converts*
Holding a convertible is like being
• Long a risk-free bond
• Short a put and
• Long a call (conversion feature)

One way to look at converts: If the value of the firm goes up enough, e.g. above $X$, convertible holders will convert and get $m$ shares of the firm.

Suppose
• Convertible is the firm’s only debt, and has face value $F$
• Currently $n$ shares outstanding of the firm

There are three possible scenarios
• Firm value $V$ goes below $F$, so it goes bankrupt and bondholders get $V$
• Firm value is between $F$ and $X$, so bondholders don’t convert and get $F$
• Firm value goes above $X$, so bondholders convert and get $m$ shares
  • There were $n$ shares already, so now there are $n+m$
  • So convertible holders get $m/(n+m)$ times $V$
  • The old equity holders get the rest: $[n/(n+m)]V$
• Since $V=X$ is where convertible holders are indifferent between getting $[m/(n+m)]V$ and $F$, it follows that $[m/(n+m)]X=F$
Putting this together:

<table>
<thead>
<tr>
<th>Firm value</th>
<th>Convertible</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V&lt;F$</td>
<td>$V$</td>
<td>0</td>
</tr>
<tr>
<td>$F&lt;V&lt;X$</td>
<td>$F$</td>
<td>$V-F$</td>
</tr>
<tr>
<td>$X&lt;V$</td>
<td>$F + \left<a href="V-X">\frac{m}{n+m}\right</a>$</td>
<td>$V-F-\left<a href="V-X">\frac{m}{n+m}\right</a>$</td>
</tr>
</tbody>
</table>

Convince yourself this is right. It implies that, for a firm that sold a convertible, there is a number $X$ such that equity shares the upside of firm value with convertible investors, for values above $X$.

V. Variance and Debt Capacity
An entrepreneur has two projects $A$ and $B$ to choose between. They are risky; their payoffs in one year depend on whether there is depression ($D$) or prosperity ($P$). The probability of each outcome is $\frac{1}{2}$. Each project costs $800, and their payoffs look like this:

<table>
<thead>
<tr>
<th>Project</th>
<th>$D$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$500$</td>
<td>$1500$</td>
</tr>
<tr>
<td>$B$</td>
<td>$0$</td>
<td>$1551$</td>
</tr>
</tbody>
</table>

The numbers are chosen so that $A$ has a higher expected payoff ($1000 vs. $775.5) and a lower variance (more in the bad state, less in the good state).

The entrepreneur wants to raise money to invest in these projects by selling debt with face value $600, maturing in a year. Potential investors are risk-neutral, and discount next-year’s cash flows at 10%.
- They calculate the expected payment to debt, divide it by 1.10, and that’s what they’re willing to pay for the bonds

*The entrepreneur can not commit to choosing a particular project. Once the bonds are sold the entrepreneur can choose what he wants.*
So what happens?

Look at the problem from an investor’s point of view. He has to ask himself
- If the firm has this much leverage, which project will it choose?
- Given that it will choose that project, what are the bonds worth?

The entrepreneur’s choice:
With $600 face value of debt outstanding, the net payments to the entrepreneur in one year, after paying off debt, are

<table>
<thead>
<tr>
<th>Depression</th>
<th>Prosperity</th>
<th>Expected Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 0 ($500&lt;$600)</td>
<td>1500-600=900</td>
<td>½0 + ½900 = 450</td>
</tr>
<tr>
<td>B 0 ($0&lt;$600)</td>
<td>1551-600=951</td>
<td>½0 + ½951 = 475.5</td>
</tr>
</tbody>
</table>

Project B is the value-maximizing choice for the entrepreneur.

With $600 in debt outstanding,
- Neither project pays him anything in depression
- B pays more than A in prosperity
- So B is better

Back to the investor:
The investor computes that the entrepreneur will choose B, so the bonds will get the cash flows that bonds get from project B:
- Nothing in depression; B pays $0, there’s nothing to seize
- $600 in prosperity
- Expected payment is ½0 + ½600 = 300
- Discounted to the present is 300/1.10 = $272.73
So the investor computes the value of the bonds to be $272.73

Putting this all together:
If the entrepreneur floats the $600 in bonds, he will get $272.73 for them
- Needs to pay in $800-$272.73 = $527.27 of his own money
- Expected payment in a year is $475.50
- Present value is $475.50/1.1 = $432.27<527.27
- NPV is negative $95! He won’t do it!
Notice what’s happened here:

Project A is a good project
- Costs 800, expected payment in a year is 1000
- $1000/1.10 = $909.09, more than the 800 it costs

Project B is a bad project
- $775.5/1.10 = $705, $95 less than the 800 it costs

The reason the financing doesn’t work is that
- Investors recognize the bias and anticipate the choice of B
- Investors pay a price that gives them the market rate of return, so the entrepreneur ends up bearing the $95 expected loss the project creates.

The Lesson Here:

1. High leverage distorts management’s project choice, encouraging risk at the expense of value.
2. When creditors price a new bond, they will impute that distortion. If they perceive that the higher leverage will cause the manager to move value from debt to equity, they will not pay for that value in the first place.
3. So if a new debt issue encourages lower-value projects, then equity—and not debt—bears the cost of this value destruction.
4. In this situation, the firm faces a limit, known as its debt capacity, on its leverage. The cost of exceeding this limit is the prohibitive discount that potential creditors impose.

Make sure you are comfortable with this logic, because it is fundamental to debt contracts since it implies

5. The issuer can create value by credibly committing not to choose a value-reducing project that would otherwise be optimal after the issuance.

In the example, the issuer would be able to issue more debt if he could promise, in a way that investors could believe, that he would not choose project B. This is the logic behind bond covenants.
VI. Dividends

Another source of conflict between equity and debt is dividend policy
- Management controls payouts to shareholders – dividends and share repurchases
- Generally, payouts are good for equity and bad for debt

Another Thought Experiment: An all-equity company is worth $50M, then sells $100M of debt. Next, management liquidates the entire company for $120M, and pays out all the cash to equity
- Equity makes a big profit, and creditors are left with an empty shell of a company
- Notice, this strategy destroyed value ($120M<$150M) but it created value for equity by taking it from debt

Payouts to equity are bad for debt because they remove value from the reach of creditors

If you hold a call option on GE, you would prefer that GE not pay dividends
- You don’t get the dividends (you haven’t bought the shares yet) and they decrease the value of the shares you have an option to buy

But even though equity is like a call option on firm value, equityholders benefit from the firm paying dividends
- Difference is, equityholders get the dividends.

So dividend policy is another issue that equity and debt have to contract on in advance