

1. (10 pts) Suppose these are today's prices of these Treasury bonds (all mature on Feb 15, 2010, and Feb 05-10 indicates a bond that becomes callable at par on February 15, 2005. Don't worry about accrued interest here.)

	Coupon	Bid	Ask
Feb 10n	6.5%	120:12	120:13
Feb 05-10	11.75%	122:10	122:11
Feb 10 STRIP	0%	77:08	77:12

Do these prices represent an arbitrage opportunity? If so, what does the arbitrage argument depend on, and if not, why not?

*Construct a mimicking portfolio for the callable Feb 05-10 note, assuming that it is not called.*

$$\begin{aligned}
 \text{Fraction of Feb 10n} &= 11.75/6.5 = 1.8077 \\
 \text{Fraction of Feb 10 STRIP} &= 5.25/6.5 = -0.8077 \\
 \text{Cost of portfolio} &= 1.8077 * (120.40625) - 0.8077 * 77.25 \\
 &= \$155.2633
 \end{aligned}$$

One Solution:

*The cost of the call is less than the amount to synthesize the bond. The call price reflects the imbedded option on the note, and as such should sell for less than the mimicking portfolio. We do not have an arbitrage. Given the high interest rate, the market is assigning a high probability to the event of the note being called.*

Other Solution:

*We look at the payoffs of buying the callable note, and shorting the synthetic portfolio above on a case by case scenario.*

**a) The option is never called.**

*The mimicking portfolio will have the same payoffs as the callable note. In this scenario, we can make money by shorting the synthetic and buying the callable note.*

**b) The option is called sometime during its lifetime.**

*We will be left with a naked short position in the Feb 10n note, and a long position in the Feb 10 strip. We will receive \$100, and have to either liquidate the portfolio for its market price, or pay a stream of coupon payments until 2010. There is no guarantee we will be able to profit in this scenario*

*The above trade is risky. We are not making money for sure. It is not an arbitrage. Note: For a given trade to be an arbitrage, we must be sure of making money in some states, and not losing anything in other states.*

2. (10 pts) An entrepreneur has three potential projects, with the following payoffs in depression  $D$  and prosperity  $P$ , which each have probability  $\frac{1}{2}$  of occurring:

<i>Project</i>	<i>D</i>	<i>P</i>
<i>A</i>	0	190
<i>B</i>	50	180
<i>C</i>	70	140

Each project costs 100, and the entrepreneur would like to raise some of that 100 by selling a bond with face value 100. What the bond doesn't raise the entrepreneur must pay in himself. All investors are risk neutral and the discount rate is 0%. After they invest, bondholders will be able to see which project the entrepreneur chooses and will be able to liquidate immediately for 80. What happens?

*Firstly, observe that the bondholder has the right to liquidate the investment if she is not happy with the entrepreneur's choice of project. The expected payoffs to the bondholder are given below*

<i>Project</i>	<i>Depression</i>	<i>Prosperity</i>	<i>Expected Payoff</i>	<i>Liquidate</i>
<i>A</i>	0	100	50	<i>Yes</i>
<i>B</i>	50	100	75	<i>Yes</i>
<i>C</i>	70	100	85	<i>No</i>

*The entrepreneur receives nothing if the bondholder liquidates, therefore he will not undertake any project which the bondholder will liquidate. Projects A, B will not occur. Considering Project C, note that the bondholder will have an expected payoff of 85. As the bondholder will liquidate if projects A or B are taken a rational entrepreneur will not invest in either project. Knowing this, the bondholders are happy to buy the bonds at their expected value of \$85. The entrepreneur will need to provide the remaining \$15 out of his own funds. The entrepreneur will then get a payoff of 0 in depression, and \$40 in prosperity, which is \$20 in expectation. The entrepreneur will be able to make money with project C, and will therefore go ahead with the project with \$85 of funding from the bondholders.*

3. (10 pts) Which costs more: a 40-day Treasury Bill with a discount rate of 6.05%, or a 30-day Treasury Bill with a discount rate of 7.15%?

*For a 40-day Tbill with discount rate 6.05%, the discount per \$100 FV is  $6.05(40/360)=0.6722$*

*For a 30-day Tbill with a discount rate of 7.15%, the discount per \$100 FV is  $7.15(30/360)=0.5958$ . This has the smaller discount so it costs more.*

4. (10 pts) The argument that a company's bank debt should be senior to its bonds follows from
- A) Junior debt often recovers little value in bankruptcy
  - B) Absolute-priority is often violated in bankruptcy
  - C) Managers can choose projects that benefit equity at the expense of debt
  - D) Managers can retain earnings that should be returned to shareholders
  - E) Monitoring is easier for banks than for dispersed bondholders

Choose all that apply, and defend your answer.

A,C,E

- A: *Junior debt recovers very little in bankruptcy.  
If the bank debt is junior, then bank can't make a credible threat since they have little incentive to force bankruptcy*
- C: *Managers are willing to take projects that's risky to debt holders, but good for equity holders  
Risky projects will increase the volatility of company equity value thus increase the equity value  
When bank debt is senior to other debts, banks can make credible threat if managers are found to transfer value from debt to equity*
- E: *Banks have monitor ability that bank holders don't have  
Banks usually have more capacity than debt holders to monitor the company's financial situation/strategy decisions.  
In this sense, bank debt should be senior (safer) than other debts*

5. (10 pts) The headline yesterday at the open of the Repo market was "Current 10-Year Note at Lowest Rate of 0.75%," meaning that of all Treasury securities, this one had the lowest repo rate (the general-collateral rate was 1.75%). The last auction of a 10-year was August 15, and the next one will be auctioned in a month. What can explain this security's current repo rate (i.e., why is it so low, not why is it exactly 0.75%), how do you expect its repo rate to move over the next couple months, and why?

*This bond is on special with specialness = 100 bp, meaning that it's on-the-run. This is because the demand for shorting Treasuries focuses on on-the-run issues, due to their high liquidity, but the supply of on-the-run securities to borrow shrinks over the period between auctions. This is 2/3 of the way to the next auction, so it is substantially on special and will go further on special until shortly before the next auction in a month, when the next 10 year will start trading when-issued. At that point, specialness will shrink, eventually to zero.*

6. The convertible issued by eToys, which matures 12/1/04, becomes callable at par in six weeks, on December 1, 2002. The conversion ratio is 13.5323 shares of eToys per \$1000 face value. At the time of issuance, eToys was trading at \$62.625/share.
- (5 pts) What was the conversion premium at issuance?
  - (5 pts) eToys is now trading around \$74/share. Suppose JP Morgan says to eToys, "I see that you will probably want to call your bond in a few weeks, but you'll be worried investors won't convert. How about this: if they don't convert, then I'll buy the bonds at par, and I'll convert them." What option on eToys would Mr. Morgan's exposure to eToys resemble?

*Conversion ratio*

$$= 13.5323 / 1000 \text{ shares}$$

*Conversion price*

$$= 1000 / 13.5323$$

$$= 73.897$$

a.) *conversion premium*  

$$= (73.897 - 62.625) / 62.625$$
  

$$= 18.00\%$$

b.) *In the situation where investors do not convert, the share price  $S$  is below the conversion price of 73.897 (otherwise, they would have converted). If JP Morgan buys the bonds and converts them to shares then he is effectively buying shares worth  $S < 73.897$  for 73.897, so he makes a loss of  $73.897 - S$ . This is equivalent to being short a put option with strike price 73.897. Another way to put it is, JP Morgan is telling eToys, if you can't sell shares to the public for 73.897, you can sell them to me for 73.897.*

7. (10 pts) Last Friday, Moody's announced the following ratings of the debt of AES Corp.:
- Senior Unsecured: B3
  - Senior Subordinated: Caa1
  - Junior Subordinated: Caa2
- Does this support the view that bond ratings are estimates of the probability of default?

*The probability of default is the same for all the bonds. However, conditional on default, the expected payment of the Senior Unsecured is greater than that of the Senior Subordinated and the expected payment of the Senior Subordinated is greater than that of the Junior Subordinated. The difference in ratings reflects the difference in the recovery rates.*

8. (10 pts) Amazon currently sells for 18 and in a year it will be either 15 or 25, each with probability  $\frac{1}{2}$ , and you can borrow or lend for a year at 2%. What is the value of a European put on Amazon with strike price 22.5?

$$S = 18, S_{down} = 15, S_{up} = 25, r_F = 0.02, X = 22.5$$

$$C_{down} = \max\{0, S_{down} - X\} = 0, C_{up} = \max\{0, S_{up} - X\} = 2.5$$

$$n = \frac{C_{up} - C_{down}}{S_{up} - S_{down}} = \frac{2.5 - 0}{25 - 15} = 0.25$$

$$B = \frac{1}{1 + r_F} \left( \frac{C_{down} S_{up} - C_{up} S_{down}}{S_{up} - S_{down}} \right) = \frac{1}{1 + 0.02} \left( \frac{0 - 2.5 \times 15}{25 - 15} \right) = -3.6765$$

So, we would buy  $1/4$  of a share, and lend  $-3.6765$ , or in other words borrow 3.6765.

The price of a European call on Amazon:

$$C = nS + B = 0.25 \times 18 - 3.6765 = 0.8235$$

Using the Put-Call Parity, we can calculate the price of a European put on Amazon:

$$C + PV(X) = S + P$$

$$P = C + PV(X) - S$$

$$P = 0.8235 + \frac{22.5}{1.02} - 18 = 4.8823$$