Human capital investment and portfolio choice over the life-cycle

Preliminary

Nikolai Roussanov

The Wharton School, University of Pennsylvania, and NBER. *

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Abstract

I develop a model of life-cycle portfolio choice with non-tradeable idiosyncratic labor income where the agent has an option to invest in human capital, for example through education. The inability to borrow against her human capital depresses the agent's demand for equity, as she is concerned about being liquidity constrained when it is optimal to invest. The model has many predictions that are broadly consistent with the standard financial advice for people saving towards a target such as education. Specifically, wealthy agents display a hump-shaped pattern of lifetime risky asset holdings, as the young are more likely to exercise the option than the old. In contrast, the option generates local risk seeking behavior among less wealthy agents, who then show a declining risky asset profile over the life time, as their option is either exercised or becomes worthless. Finally, the model generates realistic life cycle labor income profiles jointly with the endogenous levels of educational attainment. Allowing for financial aid, which relaxes the borrowing constraint, does not produce a strong mitigating effect on the demand for liquidity as long as the agent has to forgo her labor income earnings while in school. This opportunity cost creates a liquidity demand that is akin to precautionary saving. Optimal investment rule depends on the marginal product of labor and therefore varies over the business cycle. This induces cyclical variation in the demand for liquidity, suggesting potential implications for asset pricing.

JEL Classification: G11, D91, D14. **Keywords:** Portfolio Choice, Household Finance, Human Capital, Optimal Investment, Education, Real Options, Liquidity Constraints.

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1 Introduction

This paper incorporates endogenous evolution of illiquid human wealth into the standard framework of optimal portfolio selection. The presence of human capital investment (rather than an exogenous stream of labor income) helps explain some patterns of the life-cycle asset allocation observed in the data, as well as generate new predictions.

Despite the conventional wisdom of financial planners as well as the normative prescriptions of portfolio theory, which suggest that the share of stocks in households' portfolio should decrease with age, young people hold very little stocks. This observation is the driving force behind the approach of Constantinides, Donaldson, and Mehra (2002) to addressing the equity premium puzzle. Moreover, conditional on participation, researchers have found that the share of equities in investors' portfolios display increasing or hump-shaped profiles over the life-cycle (e.g. Poterba and Samwick (1995) and Ameriks and Zeldes (2001)). Again, this is in stark contrast with the standard life-cycle portfolio advice that young households should allocate most of their portfolio to equities, and reduce this allocation as the present value of their future labor income decreases with age (eg. Jagannathan and Kocherlakota (1996)).

Introducing endogenous human capital accumulation[1] allows me to refine the predictions of portfolio theory about the effect of labor income on asset allocation in the different stages of the life cycle. In particular, the presence of an option to invest in human capital alters the optimal allocation to risky assets for a given life-time present value of human wealth. Moreover, it allows for endogenous heterogeneity in life-cycle labor income profiles, as well as in portfolio holdings.

There are several key features of human capital investment that yield interesting implications for asset allocation. First, educational investment is indivisible (one cannot get one quarter of an MBA). Second, besides the direct financial cost (e.g. tuition), education has large opportunity costs, since it is hard to both work and study at the same time (evening and part-time programs form an important exception). Most importantly for the life cycle considerations, given the finite life span, the value of the option to invest in human capital declines over time, since there are fewer years to collect the rewards. Finally, the crucial difference between human capital and most other types of investment is liquidity. Given that future labor income is non-tradeable for reasons of moral hazard and adverse selection, it may be hard to borrow in order to invest in education (although various financial aid programs exist in order to help individuals overcome this constraint). This restriction is even more severe for other forms of human capital investment (such as health).

[1]While my primary focus is on education, the present model of human capital can be interpreted quite broadly, including investment in one's health or in children.

The predictions of my model with respect to the stock holdings of the young investors are two-fold. I find that for the wealthier young investors the presence of the option to invest in human capital depresses risky asset holdings, which thus peak in the middle of the life-cycle, after the option has been exercised. On the contrary, for the poorer young investors the presence of such an option can increase their risky asset holdings (generating the usual pattern of declining equity shares over the life-cycle), thus potentially deepening the puzzle of low stock holding among the young.

The key assumptions that investment is illiquid and indivisible are what drives the model's predictions. The increased demand for safe assets by the young is achieved due to their concern that the liquidity constraint will bind when it becomes optimal to invest. This demand for liquidity creates a non-convexity in the value function around the optimal investment threshold. This non-convexity is also responsible for the locally risk-seeking behavior of the poor agents, whose only chance to increase their human capital is "winning the lottery."

Since the investment threshold rises over time as the value of the option to invest falls with investors' age, the importance of the liquidity constraint diminishes as well. Once the option is exercised, the demand for liquidity disappears and the agent's asset allocation problem becomes standard. Thus both the risk-aversion and the risk-seeking effects attenuate with investors' age, and eventually disappear.

There is another interesting implication of liquidity demand induced by human capital investment. Educational investment has an important business cycle component due to the strong complementarity between the two inputs: labor and financial capital. Adjustments to human capital are costly, particularly due to the opportunity cost of foregone wages (time is scarce). Thus an agent is more willing to commit time and money to augmenting her human capital when its productivity is currently low - i.e., in states of nature that can be identified with recessions. Thus, intuitively, financial assets are relatively less risky to the agent in good times, since she smooths consumption sufficiently well with labor income (which is not very volatile) than in bad times, when financial assets are necessary not only to support consumption, but also to pay for acquiring human capital. While these results are highly sensitive to the specification of exogenous income process, they can potentially justify the inclusion of endogenous human capital into equilibrium asset pricing models with the view towards addressing the size and time variation of equity risk premia.

The paper is structured as follows. The remainder of this section places my model in the broader context of the literature on portfolio choice, asset pricing and human capital. Section 2 presents a simple model of illiquid human capital investment and portfolio choice that clearly depicts the mechanism at work. Section 3 develops a more realistic life-cycle model of portfolio choice in the presence of human capital, which is solved numerically. Section 4 outlines the directions for future research and concludes.

Human capital has played an important role in the recent revival of academic interest in portfolio choice (see Campbell and Viceira (2001) for a survey of main results). The recognition that human wealth cannot be traded in financial markets suggests that the optimal portfolio of a constrained investor should differ from that prescribed by the classical portfolio theory, eg. Merton (1971). Indeed, Heaton and Lucas (2000) argue that in incomplete markets background risks can help explain empirically observed household portfolio allocations, in particular large bond holdings. However, labor income by itself does not contribute to resolving this puzzle. As shown, for example, by Heaton and Lucas (1997), as long as labor income is not highly correlated with stock returns and is bounded above zero, wages act as an implicit holding of a riskless asset, thus creating an even stronger demand for stocks than suggested by the traditional models.

Allowing for flexible labor supply only worsens the problem, since it provides households with an additional margin of adjustment that allows them to absorb financial shocks, as demonstrated by Bodie, Merton, and Samuelson (1992). While labor supply is determined endogenously in my model, this effect is not present here. This is because the absence of explicit preference for leisure makes labor highly inelastic (the agent will always work unless it is optimal to study).

In the life-cycle context, there is another important effect of human capital on asset allocation. Since human wealth declines as agents age (simply because there are fewer years of labor income left), so does the implicit holding of the near-riskless asset. Thus as agents near retirement, they should shift some of their assets out of stocks and into bonds/bills. Jagannathan and Kocherlakota (1996) show that declining present value of future labor income is the only economically sound justification for the widespread conventional wisdom that investors should reduce stock market exposure as they age. Cocco, Gomez, and Maenhout (2002) develop and calibrate a quantitative model that generates declining equity shares over time. Campbell, Cocco, Gomez, and Maenhout (2001) show that this pattern is present in the data for the older households. However, it is not consistent over the entire life-cycle. In fact, Poterba and Samwick (1995) and Ameriks and Zeldes (2001) find that the observed stock holding profile appears to be hump-shaped: the young households tend to stay out of the stock market or have minimal stock holdings, but as they get older their participation in the stock market increases. The stock holdings peak for the middle aged, and then decrease with age as agents approach retirement.[2] They also find that stock holdings rise with education and income levels.

Viceira (2001) calibrates a dynamic portfolio choice model with different retirement horizons and finds that it is possible to generate a hump-shaped pattern of risky asset holdings via a combination of a high discount rate and a hump-shaped pattern of labor income growth, which, he argues, is consistent

[2]These findings are subject to a caveat that it is impossible to completely disentangle the effects of age, birth cohort and time on portfolio allocation. Depending on whether cohort or time effects are assumed away, Ameriks and Zeldes (2001) find either a hump-shaped profile or a pattern of stock holdings that increases with age.

with the data. However, if one allows for endogenous human capital accumulation, it is unclear to what extent the steep labor income profiles are consistent with high rates of time preference required to match the observed portfolio holdings. My model provides a potential laboratory for addressing this question.

Davis, Kubler, and Willen (2002) consider a life-cycle model in which households, instead of facing a hard borrowing constraint as in the studies mentioned above, can borrow at a rate that significantly exceeds the riskless interest rate. Their model generates a number of predictions that allow to better reconcile portfolio theory with the empirical evidence of low equity market participation and high bond holdings. In particular, in their model stock ownership rates and equity shares increase with age. However, their model makes a counterfactual prediction that equity shares decrease with education. Combining their approach with my model of human capital investment provides an interesting venue for future research.

Cocco (2003) develops a model of portfolio choice in the presence of housing. He finds that the demand for housing and the presence of house price risk can crowd out investment in stocks for young and low-wealth individuals. Since these predictions regarding the demand for equity for different levels of wealth are in sharp contrast with those of this paper, the implications of the two models can be potentially distinguished empirically. (See Curcuru (2003) for some empirical evidence on the effect of housing on portfolio holdings.)

The explanation for the low levels of stock holdings among the young investors that is advocated in the present paper is based on the intuition that the young potentially have greater liquidity needs than the old. These might include a down-payment on a home, as well as a wide range of human capital related investments, such as education, starting up a private business, raising children, etc. The ability of liquidity demands created by the presence of lumpy personal projects and borrowing constraints to reduce risky asset holdings in households' financial portfolios has been explored by Faig and Shum (2002). However, these authors abstract from both the life-cycle and the business-cycle aspects by ignoring labor income. At the same time, the case for the illiquidity of human capital is perhaps stronger than it is for a range of other household investments that are more easily collateralized (such as housing and durable goods purchases).

The predictions of my model regarding the variation in labor supply, human capital investment, and asset allocation over the business cycle appear broadly consistent with the prior theoretical and empirical literature. Becker (1964) proposed that human capital investments should take place when the opportunity cost of not working is the lowest. Despite the fact that many aspects of human capital investment (broadly defined) are very hard to observe, there are studies concerning its various forms that are observable (e.g. college attendance, health improvement, skill acquisition, etc.) separately. Kahn (2003) finds that college students who graduate in recession years and, therefore, receive lower initial wage offers than those graduating in boom years, are also more likely to enroll in graduate or professional school. Ruhm (2003) finds that the decrease in mortality during recession years can be attributed largely to the increase in the amount of time devoted to health-improving lifestyle activities (e.g. exercise, walking, etc.), as well as to the decline in health-damaging activities that might be correlated with hours worked. DeJong and Ingram (2001) build a general equilibrium model that directly incorporates investment in both human and physical capital. By estimating their model they are able to infer distinctly countercyclical aggregate behavior of skill acquisition activities. Dellas and Sakellaris (2003) build a model of optimal investment in education over the business cycle. They find that unless borrowing constraints are binding, college enrollment should be countercyclical. They find that empirically this is indeed the case, despite the fact that the availability of financing tends to be procyclical. This suggests that while the liquidity constraints might not be tight enough to prevent investment in education in low income states, they might affect the portfolio allocations of students and their families. The mere possibility of such adverse effects of these potentially binding constraints can create a strong precautionary motive, which would tilt the portfolio allocation towards riskless assets. This paper makes a prediction that this is indeed the case, at least for the agents with low levels of financial wealth relative to labor income.

Some features of the labor supply and human capital investment decisions that drive my model have also been incorporated into the real business cycle models of macroeconomics. In particular, it is the assumption that time (i.e. labor) is required to produce human capital, and that its investment is lumpy (e.g. one cannot buy a fraction of a degree). Therefore, labor supply adjustments are made on an extensive instead of an intensive margin (e.g. to work or to study). Similar features are present in the indivisible labor models, such as Hansen (1985) and Rogerson (1988), as well as in the models of home production, such as Benhabib and Wright (1991) and Greenwood and Hercowitz (1991). The former two authors embed the indivisible household labor supply into general equilibrium and find that it is not inconsistent with high elasticity of aggregate labor supply. The latter two papers show that including household production improves the performance of real business cycle models. The main mechanism is that labor is substituted out of the market sector and into the household sector in bad times. A potentially important implication for finance is that in a model with home production the derived preferences over market goods only are time varying, suggesting a potential venue for explaining time varying risk aversion over financial wealth. Overall, this literature illustrates the importance of household production (which can be related to the production of human capital) and indivisible labor supply in the macroeconomic context as well as establishes the procyclical relation between labor supply and the business cycle.

In asset pricing, the importance of human capital has been recognized in the context of the market-portfolio based models (i.e. the CAPM and the conditional CAPM). Although human wealth, unlike financial wealth, is largely unobservable, using aggregate labor income as a proxy for the return on aggregate human capital can improve the empirical performance of these models. This was first empirically studied by Fama and Schwert (1977) and later established by Jagannathan and Wang (1996). While it is possible that the cyclical variation in labor income helps explain asset returns by being a proxy for some underlying macroeconomic factor, it is also possible that human capital is itself such a factor that drives predictability of excess returns (e.g. such as in Santos and Veronesi (2001)) A link between asset pricing and the business cycle literature on human capital is forged by Basak (1999), who shows that in a production economy with preferences for leisure that are consistent with Hansen (1985) and Rogerson (1988) it is possible to generate high volatility of stock returns while keeping consumption fairly smooth.

Showing that risky asset holdings can be strongly influenced by the presence of human capital investments in conjunction with the strong theoretical and empirical results on the cyclicality of such investments can contribute towards our understanding of the cyclical variation in financial risk premia. Wei (2003) is a first attempt to incorporate human capital investment into asset pricing. She calibrates a representative agent production model that is able to match the quantitative asset pricing facts. While the model is based on rather stylized assumptions, present paper provides support to the intuition behind it. The methodology developed here can provide micro-foundations for a more realistic general equilibrium model with incomplete markets and heterogeneous agents.

There is extensive literature that applies the methodology of portfolio choice to human capital investment. Early examples include Williams (1978) and Williams (1979), among others. More recently, Judd (2000) uses a similar framework to answer the question of Becker (1964) whether there is underinvestment in education. He concludes that the empirically observed returns on human capital investment are too large to be explained by its risk, i.e. there is a human capital premium puzzle. Saks and Shore (2003) show that wealthier agents endogenously choose riskier careers, which is consistent with portfolio theory. My approach is markedly different from this literature, since I study human capital investment in order to address issues of portfolio choice, and not the other way around. However, given that issues such as liquidity constraints are often seen as important in the study of human capital, my framework might be potentially used to address questions in that area (e.g. the human capital premium puzzle).

The contribution of this paper is twofold. First, by introducing illiquid human capital investment into a standard portfolio choice problem I can reproduce the empirically observed pattern of asset allocation over the life-cycle. In particular, I show that allocation to stocks increases over time for the young who have not yet exercised their option to invest in human capital. For the older agents, whether they have used their option or not, allocation to stocks decreases from its mid-life peak as they near retirement and the value of their future human wealth declines. Second, I show that portfolio allocations vary considerably over the business cycle, especially for the younger agents with low levels of financial wealth. It suggests that embedding this model into general equilibrium might help explain the time-variation in expected returns on the risky assets.

2 Simple model

Before proceeding to develop the fully dynamic life-cycle model of portfolio choice, I consider its simplest, stripped-down version, in order to highlight the main mechanism behind my results. The investor lives three periods, and is risk-neutral with no time preference. Hence she maximizes the expected total consumption $E\left[c_1 + c_2 + c_3\right]$ (in fact, she maximizes the final period consumption, as long as expected returns on financial and human capital are assumed greater than 1). While these assumptions on preferences are, in general, not very interesting within the portfolio choice context, they allow me to illustrate clearly the effect that human capital investment under liquidity constraints can have on investor's risk aversion over wealth. There is no loss of generality in assuming risk neutral preferences and I am doing so purely for the sake of analytical simplicity (intuitively one might expect the results to be the weakest in this case).

In the first period, the agent can either work for a wage w_1, which is randomly drawn at the beginning of the period from some distribution, or forego the wage and study, paying the education cost e. In the second period, the agent's wage w_2 is randomly drawn from the same distribution.

If the agent chose to work in the first period, she has the option to study in the second period. "Studying" takes one period, and upon completion her human capital grows by g_h, which means that her wage in subsequent periods t becomes $g_h w_t$. Note that the individual who has exercised the option to study in the first period, will have more human capital in both periods 2 and 3, whereas the individual who postpones it to the second period only can have augmented human capital in period 3. If she chooses not to study in period 2 either, her wage in period 3 is just w_3 (e.g., drawn from the same distribution, although in the risk neutral case we can set this wage to be constant without loss of generality).

The agent starts off with the endowment a_1 of financial assets, which she can invest in cash (earning riskless return r_f) or stock (with stochastic return R, $E(R) > r_f$). No riskless borrowing is allowed.

Obviously, the portfolio allocation problem becomes trivial in period 2, since the risk-neutral investor will put all of her wealth in stocks. In period 1, however, this is true only if she exercises the option to study immediately.

The agent's problem in the first period is given by

$$\max\left\{EV(w_2,a_2), E\left[g_h\left(w_2R+w_3\right)+\left(a_1-e\right)R^2\right]\right\},$$

where the first component is the continuation value of working in period 1 (with the second period assets $a_2=(w_1+\ a_1)\left[\alpha R+(1-\alpha)r_f\right]$), and the second component is the expected value of consumption in period 3 given that the agent exercises the option to study in period 1 (which is only possible if $a_1\geq e$). The period-2 value function V is given by

$$V(w_2,a_2)=\max\left\{E\left[\left(a_2+w_2\right)R+w_3\right], E\left[\left(a_2-e\right)R+g_hw_3\right]\right\},$$

where the first component is the expected value of foregoing the option to study, while the second component is the value of exercising the option. The optimal exercise boundary (period-2 reservation wage) $\bar{w}_2$ is given by

$$\begin{aligned}\left(a_2+\bar{w}_2\right)E(R)+E(w_3) &= \left(a_2-e\right)E(R)+g_hE(w_3),\\ \bar{w}_2 &= \frac{g_h-1}{E(R)}E(w_3)-e,\end{aligned}$$

provided, of course, that $a_2\geq e$. Thus we have

$$V(w_2,a_2)=\begin{cases}\left(a_2-e\right)E(R)+g_hE(w_3) & \text{if}\quad w_2\leq\bar{w}_2 \text{ and } a_2\geq e\\ \left(a_2+w_2\right)E(R)+E(w_3) & \text{otherwise}\end{cases}$$

The agent's preferences over period-2 wealth (given by $V(w_2,.)$ for a fixed w_2) have a discontinuity at $a_2=e$, which makes the objective non-linear and thus the portfolio choice problem nontrivial. Assuming that stock returns and wages are independently distributed, the period-1 portfolio allocation problem can be written as

$$\hat{V}\left(w_1+\ a_1\right)=\max_{\alpha}EV\left(w_2,\left(w_1+\ a_1\right)\left[\alpha R+(1-\alpha)r_f\right]\right),$$

where

$$\begin{aligned}
& EV\left(w_2,(w_1+\ a_1)\left[\alpha R+(1-\alpha)r_f\right]\right) \\
= \ & E\left[(w_1+\ a_1)\left(\alpha R+(1-\alpha)r_f\right)E(R)+w_2E(R)+E(w_3)\right]\Pr\left[w_2>\bar{w}_2 \text{ or } a_2<e\right] \\
& +E\left[(w_1+\ a_1)\left(\alpha R+(1-\alpha)r_f\right)E(R)-eE(R)+g_hE(w_3)\right]\Pr\left[w_2\leq\bar{w}_2 \text{ and } a_2\geq e\right] \\
= \ & (w_1+\ a_1)\left(\alpha E(R)+(1-\alpha)r_f\right)E(R)+E(w_2)E(R)+E(w_3) \\
& +\left(\left(g_h-1\right)E(w_3)-eE(R)\right)\Pr\left[w_2\leq\bar{w}_2\right]\Pr\left[(w_1+\ a_1)\left(\alpha R+(1-\alpha)r_f\right)\geq e\right].
\end{aligned}$$

Assuming that stock return distribution is continuously differentiable and letting

$$p\left(\alpha\right)=\Pr\left[(w_1+\ a_1)\left(\alpha R+(1-\alpha)r_f\right)\geq e\right],$$

the first order necessary condition for an interior solution is given by

$$0=(w_1+\ a_1)\left(E(R)-r_f\right)E(R)+p'\left(\alpha\right)\left(\left(g_h-1\right)E(w_3)-eE(R)\right)\Pr\left[w_2\leq\bar{w}_2\right]$$

Thus the existence of an interior solution to the portfolio problem depends on the sign (and magnitude) of the marginal probability $p'\left(\alpha\right)$. Obviously, if $p'\left(\alpha\right)>0$ (which happens if $(w_1+\ a_1)\,r_f<e$) the agent becomes risk-seeking, since investing in risky stocks gives her a positive probability of saving up enough to pay for education, whereas investing in cash only would render this impossible. However, if $(w_1+\ a_1)\,r_f\geq e$, then for stock return distributions whose support includes $\frac{e}{w_1+\ a_1}$ there exists such $\alpha^*>0$ that $p'\left(\alpha\right)<0$ for any $\alpha>\alpha^*$. Thus, for certain sets of parameter values there is a region of initial wealth levels such that the portfolio problem has a solution in $[0,1]$, i.e. a region where the agent exhibits risk-aversion over financial wealth. The key feature of the model is, of course, the borrowing constraint, creating the risk that the agent might not have enough liquid wealth at time 2 to self-finance her education. Since investment in human capital is assumed to have a sufficiently higher return than stocks, a risk-neutral investor would thus lower her investment in stocks in order to reduce this liquidity risk.

Finally, we can address the period-1 option exercise problem. Period-1 value function is given by

$$U\left(w_1,a_1\right)=\begin{cases}\max\left\{\hat{V}\left(w_1+\ a_1\right),E\left[g_h\left(w_2R+w_3\right)+\left(a_1-e\right)R^2\right]\right\} & if \quad a_1\geq e \\ \hat{V}\left(w_1+\ a_1\right) & otherwise\end{cases}$$

When the liquidity constraint is not binding, the option exercise problem reduces to finding the reser-

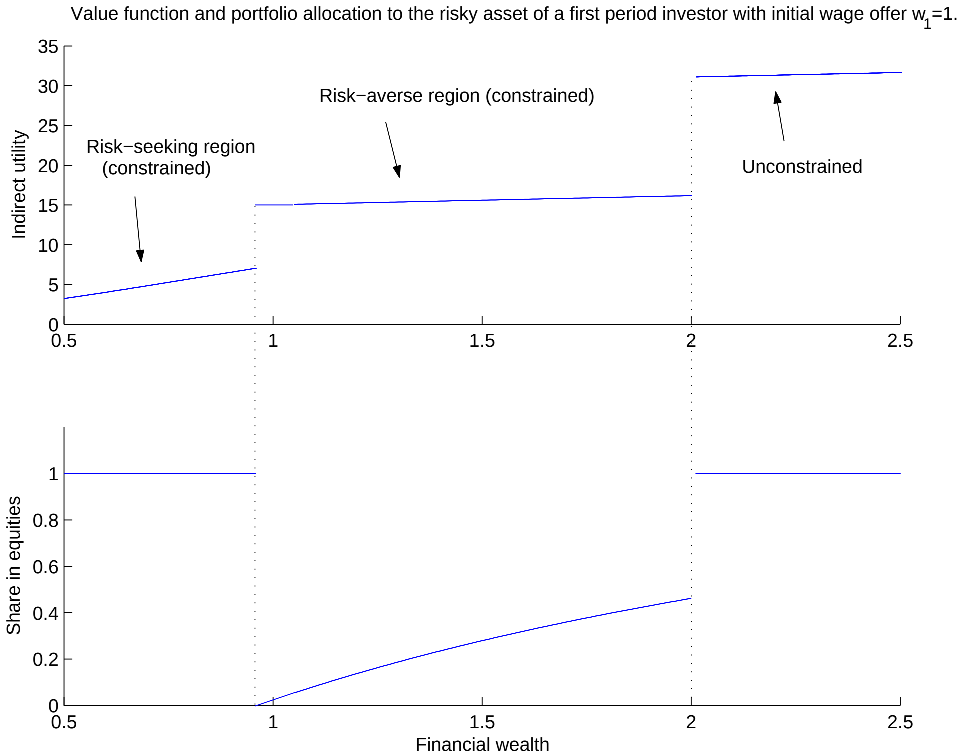

Figure 1: Value function and portfolio allocation for the first-period investor with an initial wage offer $w_1 = 1$ (example 1).

vation wage $\bar{w}_1$ such that

$$\hat{V}\left(\bar{w}_1 + \ a_1\right) = g_h \left(E(w_2)E(R) + E(w_3)\right) + \left(a_1 - e\right) E(R)^2.$$

When the constraint is binding, i.e. the agent's endowment is insufficient to fund education immediately, the option obviously cannot be exercised. Thus we can rewrite the period-1 value function as

$$U\left(w_1, a_1\right) = \begin{cases} E\left[g_h\left(w_2 R + w_3\right) + \left(a_1 - e\right) R^2\right] & if \quad w_1 \leq \bar{w}_1 \text{ and } a_1 \geq e \\ \hat{V}\left(w_1 + \ a_1\right) & otherwise \end{cases}$$

Example 1 *Consider the following calibration: education cost* $e = 2$*, human capital investment growth rate* $g_h = 2$*, risk-free rate* $r_f = 1.02$*, stock returns and wages are independent lognormally distributed variables (each i.i.d. over time, as assumed above), with* $E(R) = 1.07, Var(\ln R) = 0.0274, E(w) = 3.$

This parametrization implies $\bar{w}_2 = 0.8,$. The period-1 value function and portfolio allocation are shown in Figure 1. A liquidity constrained investor has to work in the first period (even despite a below-average wage offer) in order to save up for the period-2 human capital investment. The presence of liquidity risk creates a type of precautionary motive, hence the smaller allocation to the risky asset. This effect is akin to first order risk aversion, since the value function effectively has a kink at e. The discontinuity (and non-convexity) in the value function arises from the particular form of market incompleteness that we have considered. Notice that for the level of liquid assets below the cost of education all wealth is invested in stocks. This is the risk-seeking region. Ideally, the agent would like to take a bet that maximizes her probability of breaking the threshold e, even if such a strategy by itself has negative expected return. The fact that borrowing is not allowed limits the agent's ability in constructing such a strategy. While I consider this assumption innocuous in the portfolio choice context, in real life it might be possible that agents do convexify their preferences over wealth via lotteries, etc. See Kwang (1965) for a discussion of non-convex preferences in a similar context. For an unconstrained investor these effects disappear, since it is optimal to invest in the first period and thus the option value becomes irrelevant.

The example above demonstrates how the combination of an opportunity cost implicit in human capital investments together with liquidity constraints can induce a precautionary motive that effectively increases risk aversion over financial wealth in the region where liquidity risk is present.

A richer and more dynamic model is required to incorporate both the life-cycle and the business-cycle effects. The main life-cycle implication is that for the young the option to invest in human capital is higher, but their liquidity constraint is more likely to bind. The main business-cycle implication, intuitively, is that given some persistence in wages, recessions are the times when the precautionary motive is the strongest. This is the case because it is optimal to exercise the option to study if times are bad (i.e. the marginal product of labor is low), but this in turn makes it more likely that the liquidity constraint will bind in the near future (especially if the investment takes more than one period to mature). The following section explores these issues in greater detail.

3 Life-cycle model

3.1 Preferences and Technology

In order to conduct numerical evaluation and calibration, I develop a finite-horizon dynamic model of human capital investment and portfolio choice. Each period the agent maximizes her lifetime utility of consumption

$$U_\tau = E_\tau \left\{ \sum_{t=\tau}^{T-1} \beta^{t-\tau} \frac{c_t^{1-\gamma}}{1-\gamma} + \beta^{T-\tau} \kappa \frac{W_T^{1-\gamma}}{1-\gamma} \right\}.$$

For simplicity I assume that the agent has an option to invest in human capital only once. In the final period the agent does not receive any labor income. This final period can be considered as either "death" or "retirement" without loss of generality. Bellman's principle of optimality allows to truncate the problem this way since the i.i.d. stock returns (and possibly the timing of death) are the only source(s) of uncertainty after retirement (see Merton (1971)). Thus a judicious choice of parameter κ allows to capture a rich set of possibilities (see Ingersoll (1987) for the explicit calculation used for calibration).

The control variables are dollar amounts invested in stocks and bonds, s_t and b_t. Borrowing and short-selling is prohibited:

$$s_t \geq 0, b_t \geq 0,$$

which effectively imposes a non-negativity constraint on wealth.

Exogenous state variables are stock returns R_t^s and wage rate θ_t, summarized together in Z_t. In general this could be modelled as a vector autoregressive process, although in the numerical examples below I assume that the exogenous state variables follow a Markov chain for simplicity. Thus, I assume that the shocks to wages are purely transitory. This is sensible since the increase in labor income due to increase in human capital is permanent, but the wage rate in my model is per unit of human capital. After investment has been successfully completed, the agent's labor income per period equals the wage rate times g_h, the growth rate of human capital. Making an investment costs e units of consumption per period. I make an assumption that the human capital investment matures stochastically with a probability p in each period of investment. This specification of the human capital investment technology allows me to parsimoniously capture several important characteristics of human capital investment. First, it is risky, and thus is a non-trivial part of the portfolio choice problem. Second, by varying the probability of success one can model the *ex ante* heterogeneity in learning ability and generate endogenously heterogeneous life-time labor income profiles. Finally, this specification recognizes the lumpy nature of human capital investment - i.e., the fact that once investment is made, it might be necessary to make additional investment in the future for the project to mature. Therefore the liquidity demand described in the previous section arises not only in the periods preceding the investment, but also after the option has been exercised. In fact, if wage shocks are persistent, this precautionary motive might be stronger in the states of nature in which it is optimal to invest.

The timing of human capital investment is a non-trivial problem due to both the life-cycle and the business-cycle effects. The finite horizon of the life-cycle model implies that the value of the option to invest diminishes over time, since the total value of human capital decreases over time as there are fewer remaining years of labor income. On the other hand, there is an option value to waiting, since due to the liquidity constraint it is necessary to have a certain amount of cash on hand to finance the investment (i.e., e), as well as to support the intermediate consumption. Even though the option might be "in the money," one might prefer to wait and save enough wealth to last through the years of "schooling" rather than make the investment immediately but suffer meager quality of life in the meantime. This tension points at the likely states of nature when the option might be exercised - the states when the opportunity cost of withdrawing from the labor force (i.e. wage rate per unit of human capital) is the lowest. This introduces the "business cycle effect" - the option to invest in human capital is more likely to be exercised in the bad states than in the good states.

3.2 Decision Theory

The agent's optimization problem is given by a system of Bellman equations. For the agent who has not yet exercised the option the value function is given by

$$V_1(W_t, Z_t, t) = \begin{cases} \max_{s_t,b_t}\left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E V_1(W_{t+1}, Z_{t+1}, t+1), V_2(W_t, Z_t, t)\right] & \text{if} \quad W_t \geq e \\ \max_{s_t,b_t}\left[\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E V_1(W_{t+1}, Z_{t+1}, t+1)\right] & \text{otherwise} \end{cases},$$

with the budget constraint and the law of motion for the state variables

$$\begin{aligned} c_t &= W_t + y_t - s_t - b_t, \\ W_{t+1} &= s_t R^s_{t+1} + b_t r_f, \\ y_t &= \theta_t \end{aligned}$$

Here V_2 is the value function of an agent who has exercised the option and is currently not working, given by

$$V_2(W_t, Z_t, t) = \max_{s_t,b_t}\left\{\frac{c_t^{1-\gamma}}{1-\gamma} + \beta E\left[p V_3(W_{t+1}, Z_{t+1}, t+1) + (1-p) V_1(W_{t+1}, Z_{t+1}, t+1)\right]\right\}$$

with the budget constraint

$$\begin{aligned} c_t &= W_t - e - s_t - b_t, \\ W_{t+1} &= s_t R^s_{t+1} + b_t r_f \end{aligned}$$

Finally, the value function of an agent whose investment has 'matured' and who is working again is given by

$$V_3(W_t, Z_t, t) = \max_{s_t, b_t} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} + \beta E V_3(W_{t+1}, Z_{t+1}, t+1) \right\}$$

with the budget constraint

$$\begin{aligned} c_t &= W_t + y_t - s_t - b_t, \\ W_{t+1} &= s_t R_{t+1}^s + b_t r_f, \\ y_t &= g_h \theta_t \end{aligned}$$

Since in the final period there is no labor income, the value function is just the utility of consumption/bequest

$$V_1(W_T, Z_T, T) = V_2(W_T, Z_T, T) = V_3(W_T, Z_T, T) = \kappa \frac{W_T^{1-\gamma}}{1-\gamma}.$$

This allows us to calculate the rest of the value functions recursively.

3.3 Numerical Solution and Calibration

I solve the above system of Bellman equations numerically using backward induction. Since I assume that the exogenous state variables follow a Markov chain, I can use a discrete grid to approximate the endogenous state variable (liquid wealth).[3] I use piecewise cubic Hermite interpolating polynomials (PCHIP) to approximate the value functions at each iteration. It is common in the literature to use cubic spline interpolation, however it performs poorly in my setup. PCHIP interpolation has the advantage of being shape-preserving, unlike splines. This property turns out to be particularly important in the presence of nonconvexities, and also allows to keep approximation error fairly low without increasing the state space proportionally to the number of periods.[4]

Optimal policies are found via standard grid search. I allow the wage rate and the stock return to take only two values, thus producing a four-state Markov chain. Since I assume no correlation between stock returns and labor income innovations, it is enough to specify two transition probabilities, $p_\theta = \Pr[\theta_{t+1} = i | \theta_t = i]$, where $i = \{\theta^{High}, \theta^{Low}\}$, and $p_R = \Pr[R_{t+1} = j | R_t = j]$, where

[3] This is without loss of generality, since given a continuous state-space one can use a Markov chain approximation to solve the problem numerically following the Gaussian quadrature method outlined in Tauchen and Hussey (1991), as is frequently done in the literature. Alternatively, one can use the simulation technique of Brandt, Goyal, Santa-Clara, and Stroudd (2003).

[4] Linear interpolation, which is also shape-preserving, works well for low levels of wealth and small number of periods, but does not allow extrapolation outside the boundaries of the given state space grid. For a discussion of shape-preserving interpolation in the context of numerical dynamic programming, see Judd (1999). Piecewise cubic Hermite interpolation is introduced in Fritsch and Carlson (1980).

Table 1: **Baseline Calibration.**

Parameter		Value
Relative Risk Aversion	γ	3
Time Preference Factor	β	0.95
Risk-free Rate	r_f	2%
Equity Risk Premium	$E(R) - r_f$	3%
Stock Return Volatility	σ_R	25%
State-Contingent Stock Returns	R^{High}, R^{Low}	$\{1.3, .8\}$
Transition Probability for Stock	p_R	0.5
State-Contingent Wages	$\theta^{High}, \theta^{Low}$	$\{3, 1\}$
Transition Probability for Wage	p_θ	0.5
Cost of Education (Per Year)	e	2
Human Capital Growth Rate	g_h	1.5
Probability of Completion	p	0.4
Retirement Utility Weight	κ	40

$j = \left\{R^{High}, R^{Low}\right\}$. I assume that stock returns are i.i.d., in order to isolate the effect of the business cycle fluctuations on portfolio holdings. For the baseline calibration I assume that the wage rate is i.i.d. as well, which is sufficiently close to the business cycle calibration of Mehra and Prescott (1985). In an alternative calibration (to be reported in the subsequent drafts of the paper) I also consider a persistent wage process, following the parametrization in Heaton and Lucas (1997). Table 1 summarizes the parameter values of the baseline calibration.

The parameters of the stock return process in the baseline calibration may appear extreme, in that they substantially understate the equity premium puzzle. However, these parameter values allows me to use a relatively low level of risk aversion ($\gamma = 3$), which, in turn, makes the human capital investment problem interesting for a range of success probabilities.

I calculate the value function of an agent who lives for $T = 40$ years before retiring, where the κ coefficient of the retirement value function is calibrated to correspond approximately to the retirement life span of 15 years. Assuming the age of 21 as the starting point, this corresponds to the total life span of 75 years, which is approximately equal to the average life expectancy in the U.S.[5] I solve the model for the range of financial wealth of $[0, 15]$. Given computational limitations, this is a reasonable range, since median household labor income is almost twice as large as median household financial wealth (even though mean wealth is over five times as large as mean labor income), see e.g. Ameriks and Zeldes (2001).

[5]A higher value of κ, which would allow for longer life expectancy and, potentially, a'bequest motive, would be more plausible, in particular in light of the quantitative findings by Gourinchas and Parker (2002). However, a higher weight on the retirement value function induces the agents to accumulate large amounts of retirement wealth, which creates problems for interpolation. Simplifying the problem this way does not contaminate the findings, since it mainly effects the life-time patterns of consumption and saving, but has less of an impact on asset allocation.

3.3.1 Decision to invest

The pattern of investment in human capital over the life cycle is quite intuitive: the old do not undertake the investment (the latest period in which the option might be exercised for the wealth levels in the given range is 56, i.e. when there is still a quarter of working life left). Whether investment is undertaken or not in the first 56 periods depends both on the amount of financial wealth accumulated and on the state of nature (i.e. the wage rate). Since stock returns are i.i.d. throughout this example, there are effectively two states: "good" and "bad". As we might expect intuitively (and consistently with earlier models, such as Dellas and Sakellaris (2003)), the option is more likely to be exercised in the bad state than in the good state, however for most levels of wealth it is optimal to do so even in the good state. Figure 2 depicts the minimal levels of financial wealth at which the option is exercised across ages in the baseline case.

Allowing little persistence in the wage rate actually makes the 'business cycle effect' more pronounced, since it pays off to wait for a bad state if the agent is currently in the good state. With greater persistence it is optimal not to wait for a bad state since it is less likely to come soon, thus the minimum investment levels are not very different across states for the persistent case.

One would expect general trend of minimum wealth requirement increasing with age due to the simple trade-off between current and future consumption. Exercising the option early yields higher expected wages for more years and thus is worth a greater drop in consumption now. The older the agent is, the less inclined she is to sacrifice current consumption, since there are fewer years to reap the benefits of investment. Thus investment is made only if the agent has enough cash on hand to both finance the investment and guarantee a reasonable level of current consumption. Since return on human capital is set higher that the return on either financial asset[6], they are both relevant margins of substitution between current and future consumption. It might appear strange that the option to invest in human capital remains valuable until almost the end of the working life. In fact, as we can see by looking at the life time labor income profiles (Figure 4), most rich agents exercise the option early in their lifetime, except for a few 'unlucky' ones, while the poorer agents accumulate human capital much more slowly due to the liquidity constraints. Allowing human capital to depreciate over time (which is not presently done in my model) is likely to weaken this effect. Slightly puzzling is the decrease of the good state investment threshold in the mid-life. While not very robust to variations in parameter values, it can be explained by the greater incentive to accumulate financial assets early in life before investing in education, which stems from the agent's desire to smooth consumption over time.

[6]The human capital growth rate of 1.5 appears empirically plausible. For example, Graduate Management Admission Council reports median annual salaries of MBA students of $50000 and $75000 before and after attending business school, respectively (GMAC 2003).

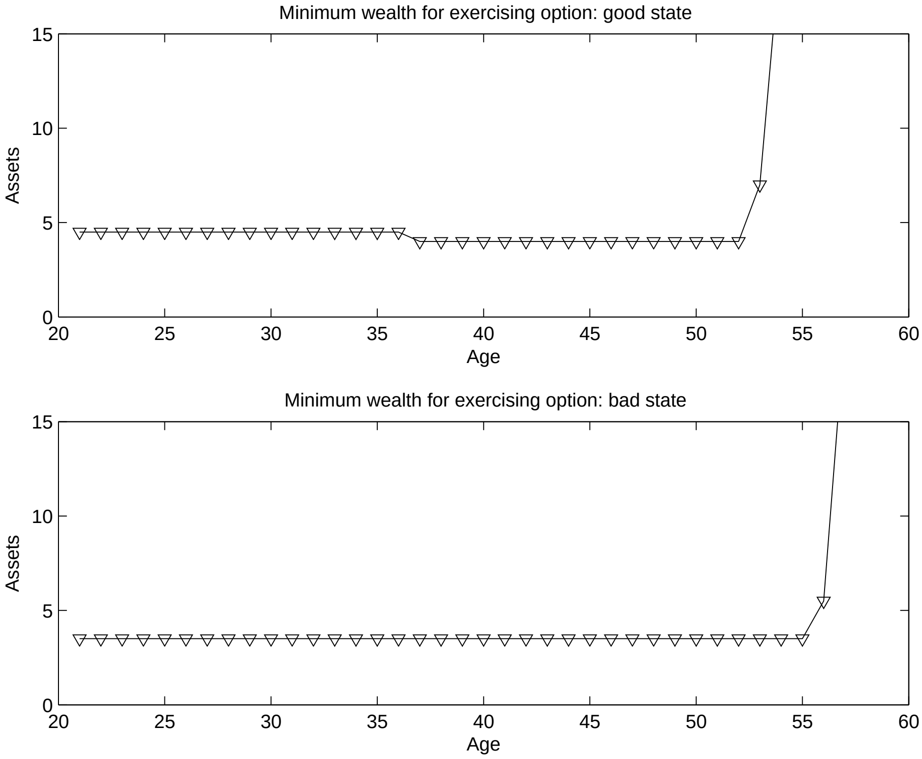

Figure 2: **Optimal investment threshold:** minimum level of financial wealth at which the human capital investment is made.

3.3.2 Portfolio allocation

The effect of the option to invest in human capital on portfolio allocation is most pronounced for the young investors (see Figures 4 and 5, as well as Table 2). The allocation to stocks is close to zero for levels of wealth near the minimum level of wealth required for investment, which we described above. This is precisely the liquidity effect described in the previous section. While the investor does not have enough cash on hand to both make the investment and consume, she puts all her wealth (including part of the current wages) into the riskless asset, so that she can surely make the investment in the next period if it still optimal to do so.

Portfolio allocations are non-monotonic, however, which is due to the non-convexities in the value function (Figure 3). The allocations trough between 2 and 5, i.e. in the region just below the threshold at which it becomes optimal to invest. But, as figure 5 shows, the agents whose financial wealth is close to zero allocate all their savings to stocks, just like the rich ones. This is the 'risk-seeking' effect described in Section 2, which arises due to the non-convexity in the value function (see Figure 6).

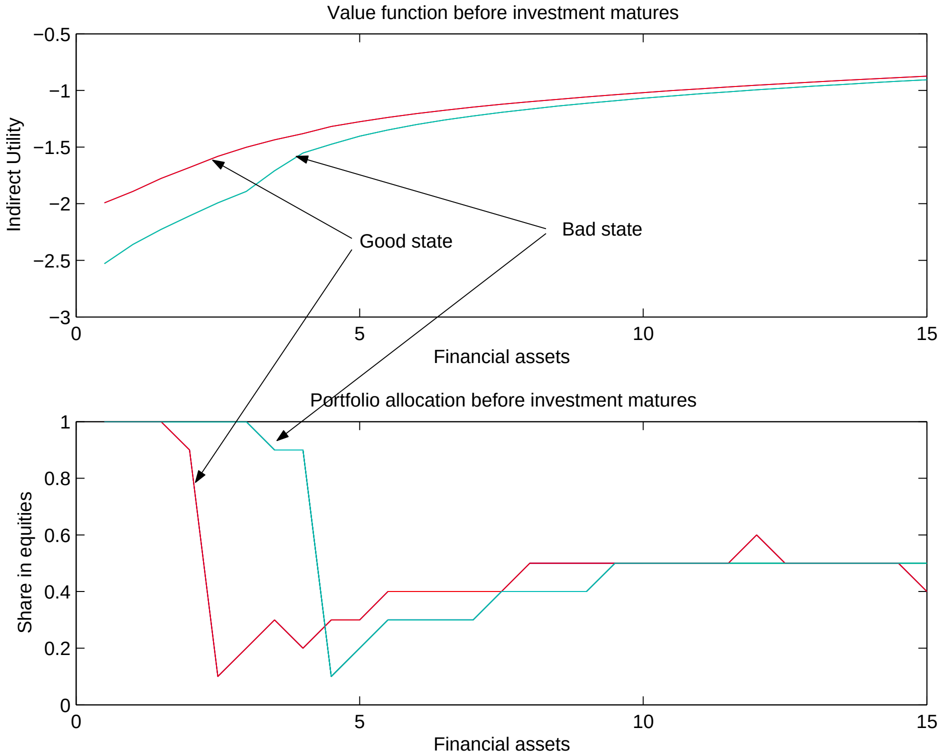

Figure 3: **Value functions and asset allocation in different states of nature (baseline case).** The value function before option is exercised exhibits the non-convexity described in Section 2. The non-convexity less pronounced in the 'good state' value function.

Investing everything in stocks maximizes the chance that the agent will have enough wealth to invest in the next period. This effect is not present in good states due to the persistence in the wage process. Being in the bad state implies that the agent must raise funds for her investment quickly in order to make it next period. In good states, the agent can wait several periods to accumulate financial capital for future investment, however, and is thus less willing to take the risk of losing the investment opportunity altogether.

Away from the regions most directly affected by liquidity constraints, allocation to stocks decreases both over time and with increasing financial wealth (the effect is most pronounced for the agents who have already exercised their option, which is equivalent to having an exogenously given labor income process.

The decrease in stock holdings over time is the familiar pattern consistent with the usual portfolio theory advice: as the amount of future human wealth shrinks, there is less of an implicit riskless holding which has to be substituted for by an explicit holding of bonds. The fact that risk aversion appears

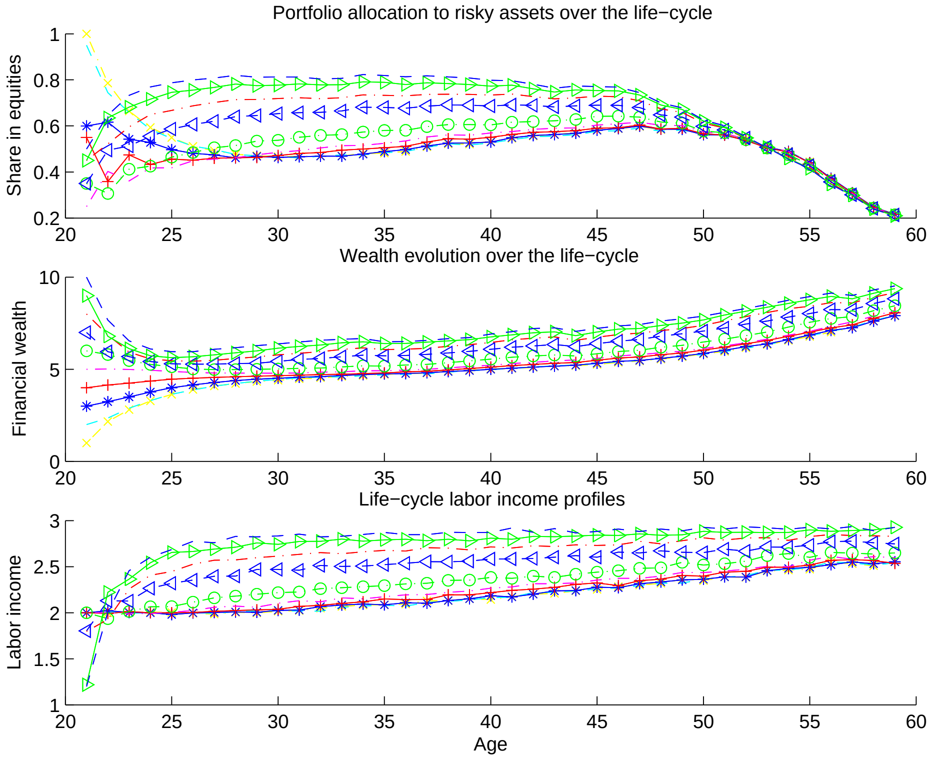

Figure 4: **Simulation results: life cycle profile of labor income, financial wealth and asset allocation.** Each path corresponds to one of the 10 initial wealth levels ($W_{21} = \{1, ..., 10\}$) and is an average over $N = 1000$ simulations. Each line in the top and the bottom graphs correspond to an identically marked line in the middle graph.

to be increasing with wealth, also should not be troubling: since total human wealth in this model is bounded, a greater level of financial wealth implies that the implicit holding of the risk-free asset (i.e. human wealth) constitutes a smaller share of total wealth. Thus, given constant relative risk aversion, the agent's allocation to the risky (riskless) assets should decrease (increase) with financial wealth. This particular intuition has nothing to do with the presence of the human capital investment option. In fact, in certain cases it might mask the effect of the latter, since it is hard to disentangle the two at higher wealth levels. However, for medium levels of wealth it also allows us to see the change in allocations over time.

The implication that portfolio allocations to stocks increase with age for young agents who have an option to invest in human capital comes from the fact that once the agent's wealth is sufficient to make an investment, with probability p the investment matures in the next period. After that, the investor's allocation to stocks goes up. However, since there is a probability that the investment will not mature in the next period, the agents save for the future continuing investment via riskless assets, keeping their allocation to stocks low. As they grow older and the option to invest goes "out of money" the precautionary motive weakens and allocation to stocks grows . Starting in the later-midlife period, the allocation to stocks decreases for both types of investors: those who have successfully exercised their option, and those who didn't (or whose investments did not mature on time).

I use the value functions and policy functions computed above to simulate the life-cycle asset allocation profiles of model agents. I simulate $N = 1000$ paths for each initial wealth level in $\{1, 2, \ldots, 8\}$ and compute the average path for each initial wealth[7]. The paths are plotted in Figure 4. This perhaps is the most dramatic illustration of the model's predictions for the life-cycle asset allocation. For the lowest wealth levels (under 3), we see an initial decrease in allocations due to the risk-seeking induced by the non-convexity in value function, where as for the higher wealth levels the share of equity holdings peaks earlier in the life-cycle, since these agents are able to exercise their option earlier. Thus, with the exception of the lowest wealth levels, the simulated profiles are consistent with the observed hump-shaped profile of risky asset holdings: they increase with age (for the first half of the life cycle), wealth (since the allocations of the wealthy peak earlier in the life cycle), and education (essentially by construction).

The prediction that for the agents on the lower end of the wealth distribution it is optimal to hold more stocks earlier in the life cycle than it is for the richer agents is interesting in several respects. Interestingly, it is consistent with the financial planners' advice based on the idea of "targeting," in the case when the "target" level of wealth is greater than the amount that can be attained by investing the agent's current wealth in the riskless asset. Note that the critique of targeting as a justifiable reason for higher stock holdings earlier in the life cycle by Jagannathan and Kocherlakota (1996) does not apply here, since their model specification does not allow for non-convexities.

While investigating whether the predictions of my model are borne out in the data is subject to further research, it might turn out that the implication of high equity holdings among poor young investors is counterfactual, thus deepening the life-cycle portfolio allocation puzzle. However, in my model the risk seeking motive is so strong for these investors that participating in non-portfolio related lotteries is more desirable than participating in the stock market, especially in the presence of borrowing constraints (that prevent agents from taking highly levered positions). While in my model there are no explicit costs to participating in the stock market, it can be argued that introducing a fixed cost in the manner of Cocco, Gomez, and Maenhout (2002) and Cocco (2003) can eliminate the counterfactual prediction that the poorest agents hold their wealth in stocks. It could also contribute to explaining the demand for lotteries by the poor. Arguably, even modest cost of participation in the stock market together with the presence of alternative opportunities, such as gambling and lotteries, would make the

[7] This procedure effectively "averages out" time and cohort affects but leaves the age and wealth effects

Table 2: **Optimal consumption, human capital investment, and portfolio choice over the life-cycle for different wealth levels.** Columns: "y/n" equals one when it is optimal to invest and zero when it is optimal to wait, $A_t = W_t + y_t$ is the amount of liquid assets available to the agent in a given period for consumption and portfolio investment, c_t is optimal consumption, α^{Bef} is the optimal share of the risky assets in the portfolio before the option is exercised, or before investment matures and α^{Aft} is the optimal portfolio share after investment matures.

		Good state					Bad state				
Age	W_t	"y/n"	A_t	c_t	α^{Bef}	α^{Aft}	"y/n"	A_t	c_t	α^{Bef}	α^{Aft}
21	1.0	0.0	4.0	1.8	1.0	0.9	0.0	2.0	1.5	1.0	1.0
21	5.0	1.0	3.0	1.0	0.3	0.9	1.0	3.0	0.8	0.2	1.0
21	7.5	1.0	5.5	1.3	0.4	0.8	1.0	5.5	1.3	0.4	0.9
21	10.0	1.0	8.0	1.5	0.5	0.8	1.0	8.0	1.5	0.5	0.9
25	1.0	0.0	4.0	1.8	1.0	0.9	0.0	2.0	1.5	1.0	1.0
25	5.0	1.0	3.0	1.0	0.3	0.9	1.0	3.0	0.8	0.2	1.0
25	7.5	1.0	5.5	1.3	0.4	0.8	1.0	5.5	1.3	0.4	0.9
25	10.0	1.0	8.0	1.5	0.5	0.8	1.0	8.0	1.5	0.5	0.9
30	1.0	0.0	4.0	1.8	1.0	0.9	0.0	2.0	1.5	1.0	1.0
30	5.0	1.0	3.0	1.0	0.3	0.9	1.0	3.0	0.8	0.2	1.0
30	7.5	1.0	5.5	1.3	0.4	0.9	1.0	5.5	1.3	0.4	0.9
30	10.0	1.0	8.0	1.5	0.5	0.8	1.0	8.0	1.5	0.5	0.9
35	1.0	0.0	4.0	1.8	1.0	0.9	0.0	2.0	1.5	1.0	1.0
35	5.0	1.0	3.0	1.0	0.3	0.9	1.0	3.0	0.8	0.2	1.0
35	7.5	1.0	5.5	1.5	0.5	0.8	1.0	5.5	1.3	0.4	0.9
35	10.0	1.0	8.0	1.5	0.5	0.7	1.0	8.0	1.5	0.5	0.9
40	1.0	0.0	4.0	1.5	1.0	1.0	0.0	2.0	1.5	1.0	1.0
40	5.0	1.0	3.0	1.0	0.3	0.9	1.0	3.0	0.8	0.2	0.9
40	7.5	1.0	5.5	1.5	0.5	0.8	1.0	5.5	1.3	0.4	0.9
40	10.0	1.0	8.0	1.8	0.6	0.7	1.0	8.0	1.5	0.5	0.8
45	1.0	0.0	4.0	1.5	1.0	1.0	0.0	2.0	1.5	1.0	1.0
45	5.0	1.0	3.0	1.3	0.4	0.9	1.0	3.0	0.8	0.2	1.0
45	7.5	1.0	5.5	1.5	0.5	0.7	1.0	5.5	1.3	0.4	1.0
45	10.0	1.0	8.0	1.8	0.6	0.6	1.0	8.0	1.5	0.5	0.8
50	1.0	0.0	4.0	1.5	1.0	1.0	0.0	2.0	1.5	1.0	1.0
50	5.0	1.0	3.0	1.5	0.5	0.7	1.0	3.0	0.8	0.2	0.9
50	10.0	1.0	8.0	1.5	0.5	0.5	1.0	8.0	1.5	0.5	0.6
60	1.0	0.0	4.0	1.0	0.2	0.2	0.0	2.0	0.5	0.2	0.2
60	5.0	0.0	8.0	1.8	0.2	0.2	0.0	6.0	1.5	0.2	0.2
60	10.0	0.0	13.0	3.0	0.2	0.2	0.0	11.0	2.5	0.2	0.2

poor agents stay out of the equity markets altogether. Thus it is unclear to what extent the prediction of high risky asset holdings for poor investors is so troubling in the context of portfolio choice.

3.3.3 Labor Income Profiles

The model makes interesting predictions regarding the endogenous life cycle labor income profiles. The evolution of financial wealth shows that in this model, since there is no ex ante heterogeneity in labor income, the option to invest in human capital acts as a "great equalizer", making wealth evolution paths for a range of initial wealth levels almost converge as the retirement date approaches. However, much of this convergence is due to the high consumption of the rich throughout their lifetime (after investing in human capital early in the life cycle, they do not need to save for retirement as aggressively as the poor). Most importantly, there are large differences in the life-time paths of human capital accumulation (Figure 6, bottom panel). Despite no differences in ability (in the given calibration), the rich accumulate human capital much faster than the poor, since they are less financially constrained.

4 Conclusions and directions for future research

I develop a life-cycle model of portfolio choice that includes optimal investment in human capital. If the investor is prevented from borrowing against her human capital, the model predicts that risky asset holdings increase with age early in the life-cycle for higher-wealth individuals, thus helping explain the hump-shaped pattern of stock holdings reported in empirical studies. I also show that the indivisibility of human capital investment induces locally risk seeking behavior among the low wealth individual. This implies a counterfactually high level of stock holdings for this group, although allowing for a fixed participation cost might be sufficient to overturn this prediction. I also show that the allocation to risky assets fluctuates over the business cycle, which suggests potential implications for asset pricing.

This paper makes sharp predictions about the joint evolution of labor earnings, financial wealth and portfolio holdings over the life cycle. Verifying the empirical validity of these implications is an obvious future extension of this work. There are significant challenges that need to be overcome for such study to be successful. Besides the notoriously poor data on household assets, the identification problem plagues much of empirical work on portfolio choice. Due to the fact that age equals time minus birth date, the three effects, i.e. the age, time, and cohort effects, can not be separately identified[8]. In much of literature cohort effects are ruled out, so that only age and time effects are identified. However, in the present context cohort effects are just as important as age and time effects, since the entire history

[8]See Ameriks and Zeldes (2001) for an extensive discussion of this issue.

of the agent's presence in the labor force and the pattern of human capital investments over the life cycle depends on the history of wage rate fluctuations, which is obviously different for different cohorts. Again, finding a dataset that would allow to track the same individuals over their life-cycle would be part of a solution to this problem.

References

Ameriks, John, and Stephen P. Zeldes, 2001, How do household portfolio shares vary with age?, unpublished paper.

Basak, Suleyman, 1999, On the fluctuations in consumption and market returns in the presence of labor and human capital: An equilibrium analysis, Journal of Economic Dynamics and Control 23, 1029.

Becker, Gary S., 1964, Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education. (Columbia University Press: New York).

Benhabib, J., Rogerson R., and R. Wright, 1991, Homework in macroeconomics: Household production and aggregate fluctuations, Journal of Political Economy 99, 1166 – 1187.

Bodie, Zvi, Robert C. Merton, and W. Samuelson, 1992, Labor supply flexibility and portfolio choice in a life-cycle model, Journal of Economic Dynamics and Control.

Brandt, Michael W., Amit Goyal, Pedro Santa-Clara, and Jonathan R. Stroudd, 2003, A simulation approach to dynamic portfolio choice with an application to learning about return predictability, unpublished paper.

Campbell, John Y., Joao F. Cocco, Francisco Gomez, and Pascal Maenhout, 2001, Investing retirement wealth: A life-cycle model, in John Y. Campbell, and Martin Feldstein, ed.: Risk Aspects of Investment-Based Social Security Reform (University of Chicago Press and NBER: Chicago).

Campbell, John Y., and Luis M. Viceira, 2001, Strategic Asset Allocation: Portfolio Choice for Long-Term Investors (Oxford University Press: New York).

Cocco, João F., 2003, Portfolio choice in the presence of housing, unpublished paper.

——— , Francisco Gomez, and Pascal Maenhout, 2002, Consumption and portfolio choice over the life-cycle, unpublished paper.

Constantinides, George M., John B. Donaldson, and Rajnish Mehra, 2002, Junior can't borrow: A new perspective on the equity premium puzzle, Quarterly Journal of Economics 117, 269 296.

Curcuru, Stephanie E., 2003, The determinants of real estate in the household asset portfolio, unpublished paper.

Davis, Stephen J., Felix Kubler, and Paul Willen, 2002, Borrowing costs and the demand for equity over the life cycle, NBER Working Paper No. 9331.

DeJong, David, and Beth Ingram, 2001, The cyclical behavior of skill acquisition, Review of Economic Dynamics 4, 536561.

Dellas, Harris, and Plutarchos Sakellaris, 2003, On the cyclicality of schooling: Theory and evidence, Oxford Economic Papers 55, 148–72.

Faig, Miquel, and Pauline Shum, 2002, Portfolio choice in the presence of personal illiquid projects, Journal of Finance 57, 303 – 328.

Fama, Eugene F., and G. William Schwert, 1977, Human capital and capital market equilibrium, Journal of Financial Economics 4, 115–146.

Fritsch, F. N., and R. E. Carlson, 1980, Monotone piecewise cubic interpolation, SIAM Journal on Numerical Analysis 17, 238–246.

GMAC, 2003, Global MBA graduate survey, Graduate Management Admission Council.

Gourinchas, Pierre-Olivier, and Jonathan A. Parker, 2002, Consumption over the life cycle, Econometrica 70, 47–89.

Greenwood, J., and Z. Hercowitz, 1991, The allocation of capital and time over the business cycle, Journal of Political Economy 99, 11881214.

Hansen, Gary, 1985, Indivisible labor and the business cycle, Journal of Monetary Economics 16, 309 327.

Heaton, John C., and Deborah J. Lucas, 1997, Market frictions, savings behavior, and portfolio choice, Macroeconomic Dynamics 1, 76–101.

——— , 2000, Portfolio choice in the presence of background risk, The Economic Journal 110, 1–26.

Ingersoll, Jonathan E., 1987, Theory of Financial Decision Making (Rowman & Littlefield: Totowa, NJ).

Jagannathan, Ravi, and Narayana Kocherlakota, 1996, Why should older people invest less in stocks than younger people?, Federal Reserve Bank of Minneapolis Quarterly Review 20, 1123.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, Journal of Finance 51, 3–53.

Judd, Kenneth, 1999, Numerical Methods in Economics (MIT Press: Cambridge, MA).

———, 2000, Is education as good as gold? A portfolio analysis of human capital investment, unpublished paper.

Kahn, Lisa Blau, 2003, The labor market consequences of graduating college in a bad economy, unpublished thesis, University of Chicago.

Kwang, Ng Yew, 1965, Why do people buy lottery tickets? Choices involving risk and indivisibility of expenditure., Journal of Political Economy 73, 530–535.

Mehra, Rajnish, and Edward Prescott, 1985, The equity premium: A puzzle, Journal of Monetary Economics 15, 145–161.

Merton, Robert C., 1971, Optimum consumption and portfolio rules in a continuous-time model, Journal of Economic Theory 3, 373–413.

Poterba, James M., and Andrew A. Samwick, 1995, Stock ownership patterns, stock market fluctuations, and consumption, Brookings Papers on Economic Activity 2, 295–372.

Rogerson, Richard, 1988, Indivisible labor, lotteries and equilibrium, Journal of Monetary Economics 21, 3 16.

Ruhm, Christopher J., 2003, Healthy living in hard times, NBER Working Paper No. w9468.

Saks, Raven E., and Stephen H. Shore, 2003, Risk and career choice, unpublished paper.

Santos, Jesus, and Pietro Veronesi, 2001, Labor income and predictable stock returns, University of Chicago Working Paper.

Tauchen, George, and Robert Hussey, 1991, Quadrature based methods for obtaining approximate solutions to nonlinear asset pricing models., Econometrica 59, 371–396.

Viceira, Luis M., 2001, Optimal portfolio choice for long-horizon investors with nontradable labor income, Journal of Finance 56, 433–470.

Wei, Min, 2003, Human capital, business cycles and asset pricing, unpublished paper.

Williams, Joseph T., 1978, Risk, human capital, and the investor's portfolio, Journal of Business 51, 65–89.

———, 1979, Uncertainty and the accumulation of human capital over the life cycle, Journal of Business 52, 521–548.