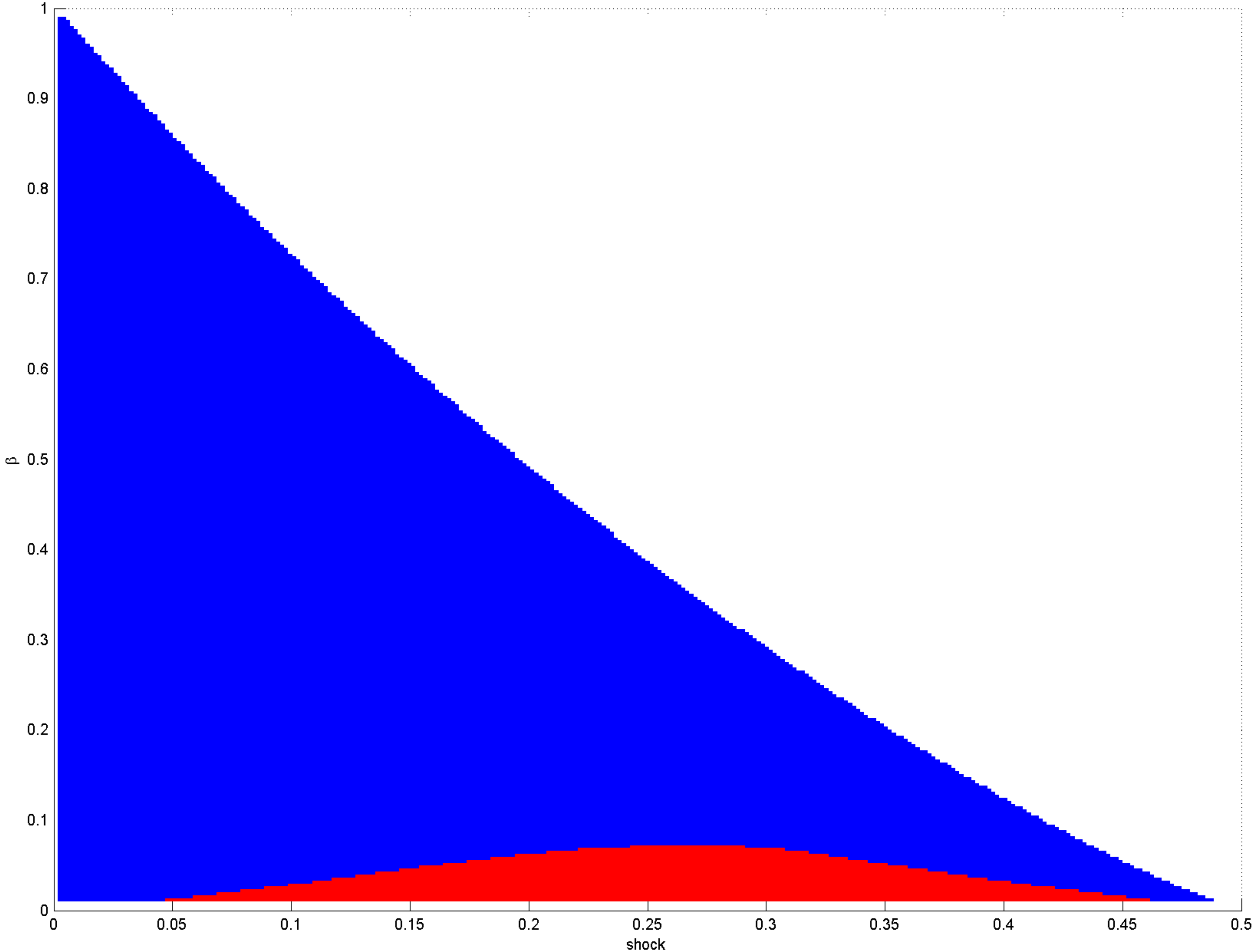
Online appendix: Numerical simulation

The figure below displays a numerical simulation for additive shocks. Recall that in this case, $\hat{X}^*(s_1^*) = c'$ (see Lemma 5 and preceding text). For the case in which the agent's period utility function is $u_t(c_t; \phi_t) = \ln(c_t + \phi_t)$, it is possible to obtain a closed-form solution to the differential equation (5) determining the contract $\hat{X}^*$: for $s_1 \in [s_1^{***}, s_1^*]$,

$$\hat{X}_2^*(s_1)+\theta_2 = \frac{1}{\beta}\left(\left(\beta\left(c_2' + \theta_2\right) - c_1' - \theta_1 + (1+\beta)\, s_1^*\right)\left(\frac{c_1' - s_1 + \theta_1}{c_1' - s_1^* + \theta_1}\right)^{\frac{1}{1-\beta}} + c_1' + \theta_1 - (1+\beta)\, s_1\right),$$

where s_1^{***}, the savings level below which the contract is constant, is determined by $\frac{d}{ds_1}\hat{X}_2^*(s_1) = 0$. For $s_1 \in [s_1^{***}, s_1^*]$, the date-3 component of the contract, $\hat{X}_3^*(s_1)$, is determined by the solution to $U^1(c;\theta) = U^1\left(s_1 + \left(c_1', \hat{X}_2^*(s_1), x_3\right); \theta\right)$.

The figure is drawn for the case of a total endowment of $W = 1$; a range of hyperbolic discount factors $\beta \in (0,1)$, displayed on the vertical axis; and a family of one-period ahead shocks (from Section 6, SPR is satisfied), $\theta = (0, -\chi, 0)$ for $\chi \in [0, 1/2]$, and $\theta' = (0,0,0)$, where χ is displayed on the horizontal axis. The figure displays three regions: no commitment problem (the upper-right white region); commitment is not possible (the lower hump-shaped red region); and commitment is possible (the large blue region). The figure is truncated at $\chi = 1/2$ because for $\chi > 1/2$ no commitment problem exists. The figure illustrates the conclusion of Proposition 6. Commitment is possible for all hyperbolic discount factors β close enough to the critical value β^* at which no commitment problem exists. As the figure shows, commitment is impossible only for very low values of β. i.e., below 0.1.



β
shock
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1