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Two-Class Voting: A Mechanism for Conflict Resolution?

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Two-Class Voting: A Mechanism for Conflict Resolution?[1]

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Two-Class Voting: A Mechanism for Conflict Resolution?

Abstract

This paper discusses the merits of two-class voting procedures where voters are separated into classes that vote separately. A prominent example are Chapter 11 bankruptcy proceedings, where claim-holders who decide on a workout proposal are divided into classes, and the approval of the proposal needs a majority of the votes in each class. Some political mechanisms employ a similar process. We analyze two-class voting in a context where voters have conflicts of interest as well as differences of opinion regarding a proposal. We investigate how voting mechanisms aggregate information dispersed among voters. We find that two-class voting provides a significant improvement over single-class voting in all those situations where voters have significant conflicts of interests, and where the electorate is relatively evenly divided between different interest groups. Then voting in homogeneous groups provides voters with protection against expropriation and allows them to reveal their information through voting. However, two-class voting is inefficient for relatively homogenous electorates.

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1. Introduction

In many situations voting contests are resolved by grouping voters into separate classes, so that the proposal put to a vote is only passed if it receives a majority from both classes of voters:

- Under Chapter 11 of the Federal Bankruptcy Reform Act of 1978, creditors and shareholders are divided into classes following a reorganization proposal. The proposal is only accepted if it receives a majority from both classes of claim-holders.

- In some countries constitutional amendments need separate approval from different bodies: Belgium requires that the Walloon and the Flemish region of the country approve certain laws separately.[2]

- Covenants of preferred stock sometimes require that mergers are agreed separately by common stock and preferred stock holders.

The purpose of this paper is to study two-class voting as a mechanism to resolve differences of opinion and conflicts of interest. Consider the creditors of the firm in Chapter 11 bankruptcy who have to determine whether a proposal for a workout should be accepted, or whether the firm should be liquidated. Each creditor has some information and therefore an opinion on the question which alternative would maximize the value of the firm and make *all* claim-holders collectively better off. However, suppose senior creditors have to give up more of their claims than junior creditors, creating a conflict of interest between creditors of different classes.

[2] This "strengthened special majority"–procedure applies only to some laws that affect the power of the regions. These require that (i) 50% of each language group should be present, (ii) 50% of each language group votes in favor and (iii) the total of yes votes in both groups exceeds 2/3 of all votes cast. We are grateful to Marc Zenner for providing us with this information.

Then senior creditors may vote against a workout proposal even though they have received private information indicating that accepting the workout would increase aggregate value.[3] Similarly, junior creditors may vote in favor of an inefficient workout because they receive sufficient transfers. In short, individuals are called to vote on a subject where two things matter: the information they have on the proposal, and specific interests that set them apart from other voters. If the conflict of interest between different groups of voters is sufficiently strong, then they will vote according to their preferences and ignore the information they have.[4] The resulting allocation is inefficient.

We analyze the efficiency of voting mechanisms in this context. We show how two-class voting may significantly improve the outcome, where improvements are measured with respect to the outcome chosen by a social planner who is only interested in increasing aggregate welfare and ignores distributional consequences of the decision.[5] The reason is that two-class voting induces voters to vote predominantly based on their information, so that a larger amount of valuable information is reflected in the final decision. Reconsider the example of junior and senior creditors in chapter 11, and suppose that 40% of the votes are held by senior creditors, and the remaining 60% by junior creditors. One-class voting with a simple majority leads to a situation where the votes of the junior creditors carry more weight for the

[3] Similarly, a referendum on a change in the tax regime may have different implications for different geographical regions, and a merger proposal in a company may have different implications for holders of different securities.

[4] The tension between information-based and partisan voting under one-class voting was analyzed by Feddersen and Pesendorfer (1997).

[5] Note that two-class voting is very different from weighted voting, where holders of different securities receive different numbers of votes. Then the information or the interests of certain voters simply receive a higher weight, but the procedure is fundamentally a one-class voting procedure.

final outcome. They can therefore expropriate senior creditors by voting in favor of a workout proposal that favors junior creditors. This vulnerability to expropriation may lead senior creditors to oppose the workout even if they expect acceptance would increase the outcome for all creditors as a whole. In this case two-class voting protects the interests of senior creditors, since then the workout proposal needs a majority of all senior creditors separately. By removing the vulnerability of senior creditors to expropriation, they are now more likely to vote in favor of the workout proposal if they receive favorable information. Hence, two-class voting removes voter's focus on their interests by protecting them, and therefore allows them to vote more according to the information they receive. The result is a lower incidence of incorrect decisions and an improvement of overall welfare.

We conclude that in a large number of circumstances it is beneficial to seek the approval of each class of voters (creditors of the same priority) separately.[6] This is true especially if voters are relatively evenly distributed among interest groups. Otherwise it may be socially efficient - though not necessarily equitable - to ignore the interests of the smaller group altogether. Then one-class voting is more effective, since a majority can only be obtained by convincing a significant proportion of the larger interest group. Also, a mechanism that eliminates conflicts of interest by mandating compensating side payments would be more effective than two-class voting. We can establish some of the efficiency and comparative static properties of the model only numerically, but our conclusions appear robust across a large range of parameter values.

We also analyze two-class voting when preferences are homogeneous and no conflicts of interest exist. It is well known that one-class voting can induce in-

[6] Mechanisms may refine this simple requirement by imposing additional constraints, for example that each group has to approve with a specified majority, and the number of votes as a whole has to pass a separate majority.

formative behavior for the entire electorate if preferences are homogenous.[7] In this case one-class voting dominates two-class voting. We analyze the majority requirements that induce voters to reveal their information fully. Interestingly, the resulting majority requirements with two-class voting pass the proposal with fewer votes than the corresponding majority rule with single-class voting.

We discuss the literature in the next section 2. Section 3 introduces the model. Section 4 presents a comparison between single-class and two-class voting without conflicts of interest. Section 5 analyzes the model with conflicts of interest. The numerical analysis in the following section 6 illustrates the argument the comparative static properties of the model. Section 7 concludes. All proofs are collected in Appendix A.

2. Related Literature

Information aggregation through voting goes back to the Condorcet Jury Theorem which states that a majority based rule is less likely to make a "mistake" than any single voter. Several proofs have been offered for variations and extensions of this claim.[8] One shortcoming of these papers is that they implicitly assume that each voter behaves sincerely.[9] Recently, a second generation of papers showed that "naive" voting is often irrational.[10]Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) investigate the features of strategic voting with homogenous preferences. From their analysis we know that strategic voting can aggregate the information perfectly if the correct majority rule is used. On the other hand,

[7]See Austin-Smith and Banks (1996) and Feddersen and Pesendorfer (1998).

[8]See Klevorick, Rothschild and Winship (1984), Ladha (1992), Miller (1986), Young (1988).

[9]Sincere voting means that a voter behaves as if his vote alone determines the outcome.

[10]See Austin-Smith and Banks (1996); Feddersen and Pesendorfer (1996a, 1996b, 1997, 1998); Myerson (1994,1997a, 1997b) and McLennan (1996).

Feddersen and Pesendorfer (1996a, 1998) study the asymptotic features of information aggregation in models with heterogenous preferences. They show that the probability of electing the wrong candidate converges to zero in an arbitrarily large electorate. Our modelling framework is closer to Feddersen and Pesendorfer (1998) and Austen-Smith and Banks (1996). However, our approach differs from them in two ways. First, we allow for conflicts of interest among the electorate and focus on non-asymptotic features of information aggregation. Consequently, we show that single class voting wastes large amounts of private information. More importantly, we discuss the possibility to enhance information aggregation through a bicameral system. In a more recent paper, Chwe (1999) shows that the optimal majority rule may move by one person in favor of the minority if there is some heterogeneity among voters. However, Chwe (1999) does not discuss two class voting.

3. The Model

Assume there are $M_J + M_S$ voters who jointly decide on a proposal that affects all of their preferences, where voters can be of one of two types $\tau \in \{J, S\}$, where 'S' refers to "senior" and 'J' refers to junior. The impact of the proposal on each voter's welfare depends on a state of nature $s \in \{l, h\}$, where both states are equally probable. Let D be a dummy variable that indicates acceptance of the proposal ($D = 1$ if the proposal is accepted, $D = 0$ if it is rejected). Voters' utility depends on the state of nature s, their type τ, and acceptance of the proposal D. Utility is denoted by $U_\tau(s, D)$. Then the M_J junior voters are characterized by payoffs that satisfy:

$$\begin{aligned} U_J(h,1) - U_J(h,0) &= G + A_J \\ U_J(l,1) - U_J(l,0) &= -L + A_J \end{aligned}$$

Similarly, for the senior voters:

$$\begin{aligned} U_S(h,1) - U_S(h,0) &= G + A_S \\ U_S(l,1) - U_S(l,0) &= -L + A_S \end{aligned}$$

We assume without loss of generality that:

$$M_J * A_J + M_S * A_S = 0 \tag{3.1}$$

Hence, the voters have to reach a decision on a proposal that has two effects: accepting the proposal yields a gain $G > 0$ for all voters if the state is h, and a loss $-L < 0$ for all voters if the state is l. In addition to this, accepting the proposal implies a transfer (positive or negative) from the junior to the senior voters. The size of A_J and A_S is a measure for the conflict of interest between senior and junior voters. If $A_J = A_S = 0$, then there is no conflict of interest at all. If $A_J > 0$, then $A_S = -\frac{M_S}{M_J}A_J < 0$ and the junior voters have a relative advantage from the proposal at the expense of the senior voters, and vice versa if $A_J < 0$ and $A_S > 0$. We also assume that $G + A_\tau > 0 > -L + A_\tau$ so that voters do not choose entirely on the basis of their preferences.

We consider two types of voting rules. If voting proceeds in one-class, then all voters have one vote, and acceptance of the proposal requires that at least a voters vote in favor of the proposal. This would be implied by requiring a fraction of $\frac{a}{M_J+M_S}$ voters voting in favor. We refer to this mechanism as one-class or single-class voting. Alternatively, voters are grouped in classes according to their type, with the junior voters in one and the senior voters in the other class. Then acceptance of the proposal requires that at least a_J junior and a_S senior voters vote in favor of the proposal. We refer to this mechanism as two-class voting. Clearly, this assumes that voters' type is associated with some observable characteristic.

Each voter observes a signal $\sigma_i \in \{g, b\}$ with the properties that:

$$\Pr(\sigma_i = g \,|s = h) = \Pr(\sigma_i = b \,|s = l) = 1 - \varepsilon \qquad 0 < \varepsilon < 1/2 \tag{3.2}$$

Let β denote voters' beliefs of being in the "good" state $s = h$. Conditional on observing a good signal $\sigma = g$ (a bad signal $\sigma = b$), each voter has beliefs $\beta = 1-\varepsilon$ (respectively, $\beta = \varepsilon$) of being in state $\tau = h$.

We will analyze the symmetric equilibria of this game. Every symmetric equilibrium is completely characterized by two pairs of randomizing probabilities, where ω_J^g (ω_J^b) is the probability of a junior voter to vote 'yes' subsequent to observing a good (bad) signal, and similarly ω_S^g (ω_S^b) for senior voters. Denote by $\pi_J(s)$ the probability of a junior voter to vote 'yes' if the state of nature is s. We have:

$$\begin{aligned} \pi_\tau(h) &= (1-\varepsilon)\,\omega_\tau^g + \varepsilon\omega_\tau^b \\ \pi_\tau(l) &= \varepsilon\omega_\tau^g + (1-\varepsilon)\,\omega_\tau^b \end{aligned} \tag{3.3}$$

In all voting models there are trivial equilibria where each shareholder votes for the same candidate irrespective of his private information. Then no one can influence the outcome, so this outcome is always an equilibrium. These equilibria are not very interesting and trivial to analyze, so we will restrict attention to equilibria in which each voter's decision is a function of her private information. Then we have the following definition:

Definition 3.1. *We say type τ is responsive if $\pi_\tau(h) \neq \pi_\tau(l)$.*

We should note that this definition implies that $\pi_\tau(h), \pi_\tau(l)$ are strictly inside the unit interval, and that $\omega_\tau^g \neq \omega_\tau^b$. As an equilibrium concept we use responsive symmetric Nash equilibrium. This rules out those equilibria where either $\pi(h) = \pi(l) = 0$ or $\pi(h) = \pi(l) = 1$, i.e., all voters vote either against or in favor of the proposal. We also distinguish pure responsive strategy from others:

Definition 3.2. *We say type τ is fully responsive if $\omega_\tau^g = 1$ and $\omega_\tau^b = 0$.*

Each voter's voting decision affects her utility only if she is pivotal. Denote the beliefs of voters of type τ conditional on (1) the signal they have observed and (2) conditional on being pivotal by β_τ^σ. Junior voters vote in favor of the proposal if and only if:

$$\beta_J^\sigma (G + A_J) + (1 - \beta_J^\sigma)(-L + A_J) \geq 0 \Rightarrow \beta_J^\sigma \geq \frac{L - A_J}{G + L} \equiv \beta_{0J} \tag{3.4}$$

otherwise they reject the proposal. We define β_{0S} analogously.

3.1. The social planner's choice: a benchmark for optimality

Our criterion for optimality is the aggregate welfare of all voters:

$$V(s, D(s)) \equiv M_J * U_J(s, D(s)) + M_S * U_S(s, D(s)) \tag{3.5}$$

This is the objective that would be maximized by a social planner who could implement a system of compensating transfers, where each senior voter receives $-A_S$ and each junior voter receives $-A_J$ if the proposal is implemented. Here the notation $D(s)$ emphasizes that the decision to implement the proposal is a function of the state s. Hence, the optimal decision rule maximizes $E[V(s, D(s))]$. Define by $P(s)$ the common component of the proposal: $P(h) = G$, $P(l) = -L$. Then we have:

$$\begin{aligned} V(s, D(s)) &= M_J * U_J(s, 0) + M_S * U_S(s, 0) \\ &\quad + (M_J + M_S) * P(s) * D(s) \end{aligned}$$

Hence, from the point of view of all voters as a whole, maximizing $E[V(s, D(s))]$ is equivalent to maximizing:

$$E[P(s) * D(s)] \tag{3.6}$$

As a collective, all voters share the objective of maximizing (3.6). Now, suppose a social planner could observe all $M_J + M_S$ signals. Then the planner's beliefs are a function of the number a of good signals, where $a \in \{0, 1, ..., M_J + M_S\}$:

$$\beta(a, M_J + M_S) = \frac{1}{1 + \left(\frac{1-\varepsilon}{\varepsilon}\right)^{M_J + M_S - 2a}} \tag{3.7}$$

which is increasing in a. Note that $\beta(k/2, k) = \frac{1}{2}$ for all k, hence observing the same number of good as bad signals confirms the prior probability of the good state. If the planner maximizes the collective objective of all shareholders she chooses to accept the project whenever:

$$\begin{aligned} & \beta(a, .)\, G - (1 - \beta(a, .))\, L \geq 0 \\ \Rightarrow \quad & \beta(a, .) \geq \frac{L}{G + L} \equiv \beta_0 \end{aligned} \tag{3.8}$$

We can describe the tension between the social planner's choice and voters' individual choices also in terms the hurdle values β_{0J} and β_{0S}. (see (3.4)) Suppose the proposal favors junior voters, so $A_J > 0 > A_S$. Then (3.4) and the equivalent expression for β_{0S} imply that $\beta_{0S} > \beta_0 > \beta_{0J}$. Clearly, $A_J < 0 < A_S$ implies the reverse ordering. The social planner's hurdle rate β_0 is a weighted average of the hurdle rates of individual voters:

$$\frac{M_J}{M_J + M_S}\beta_{0J} + \frac{M_S}{M_J + M_S}\beta_{0S} = \beta_0 \tag{3.9}$$

so the average hurdle for accepting the proposal is equal to the cutoff applied by the social planner.

We can now derive:

Proposition 3.3. ***(Social optimum):*** *If the social planner observes all information available to voters, then she accepts the proposal if and only if the number of* *good signals a exceeds a^*, where:*

$$a^* = \frac{M_J + M_S}{2} - \frac{\ln G/L}{2\ln(1-\varepsilon)/\varepsilon} \tag{3.10}$$

and rejects the proposal otherwise.

Hence, if the social planner observes at least a^* good signals and at most $M_J + M_S - a^*$ bad signals, then she accepts the project, otherwise she rejects it. We can rephrase this by defining the social planner's problem as a minimization of a loss function. Denote type I errors (accept a proposal that is expected to reduce social welfare) and type II errors (reject a proposal that increases welfare) by e_I and e_{II} respectively. Hence:

$$e_I = \Pr(l) \sum_{a=\overline{a}}^{M_J+M_S} \Pr(a\,|l) \qquad e_{II} = \Pr(h) \sum_{a=0}^{a=\overline{a}-1} \Pr(a\,|h) \tag{3.11}$$

where

$$\Pr(a|h) = \binom{M_J + M_S}{a} (1-\varepsilon)^a \varepsilon^{M_J+M_S-a} \tag{3.12}$$

$$\Pr(a|l) = \binom{M_J + M_S}{a} (1-\varepsilon)^{M_J+M_S-a} \varepsilon^a \tag{3.13}$$

The social planner's objective is then to minimize:

$$E_{Social} = \frac{L}{G+L} e_I + \frac{G}{G+L} e_{II} \text{ i.e., } a^* = \underset{\overline{a}\in[0,M_J+M_S]}{\arg\min} E_{Social} \tag{3.14}$$

Here E_{Social} is the expected loss from making a mistake for a given decision rule a.

4. No Conflicts of Interest

In this section we will compare information aggregation with single-class voting and two-class voting. The main objective is to derive the majority requirements that support sincere voting and to prove that information aggregation is superior in one-class voting as long as there is no significant conflict of interest among voters.

4.1. One class of voters

Assume there is one-class of voters, so that all voters vote on the proposal simultaneously.[11] The voter randomizes with $0 < \omega_\tau^\sigma < 1$ only if (3.4) is satisfied as an equality. Since $\beta^b < \beta^g$, voters will only randomize conditional on one realization of the signal, but not the other one.

If the conflict of interest is not in some sense "too large," then $\beta_0 \approx \beta_{0J} \approx \beta_{0S}$, and there is a simple majority requirement so that all voters vote 'yes' if and only if they observe a good signal:

Proposition 4.1. *If A_J and A_S are sufficiently small, then:*

(i) there is a majority requirement so that all voters vote sincerely with probability one ($\omega_J^g = \omega_S^g = 1$, $\omega_J^b = \omega_S^b = 0$) in the unique symmetric responsive equilibrium.

(ii) This majority requirement is equal to the rule of the social planner a^* and the information is aggregated perfectly, E_{Social} is minimized and:

(iii) The likelihood of making a mistake (type I and type II) converges to zero as $M_J + M_S$ becomes large for any majority rule (represented by fraction) α such that $0 < \alpha < 1$.[12]

Hence, single-class voting is optimal in the sense that all information is incorporated in the same way as the social planner would use it as long as the optimal social choice rule is employed. The optimal social choice rule is given by (3.10) and can be a submajority, simple majority or a supermajority rule depending on the other parameters of the model. Moreover, even if the social choice rule is not

[11] This case is well known from the analysis of Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998).

[12] Yılmaz (1998) shows that the probability of making a mistake converges to zero even for $\alpha \in \{0, 1\}$ in a similar model with a continuous signal space.

optimal, the likelihood of accepting bad or rejecting good proposals decreases to zero as the electorate becomes sufficiently large. One feature of voting without conflicts of interest is that all voters agree on the outcome ex post. Put differently, once the voting result is announced and the information contained in the vote becomes public, then all voters who voted in favor of a defeated proposal (or who voted against a successful proposal) change their minds and agree with the final outcome. This feature depends critically on the homogeneity of preferences across all voters.

4.2. Two classes of voters

There are several types of voting rules that can be applied (in order to accept a proposal) if voters vote in classes:

1. Both classes need to agree (supermajority or submajority in both classes).
2. One class agrees (supermajority or submajority in one-class).
3. Contingent majorities: the majority requirement in one-class depends on the outcome in the other class (e. g., accept if both classes have voted 1/2 in favor, or if one-class has voted 3/4 in favor).
4. Both classes need to agree, and all voters together must agree.[13]

In this paper we will restrict attention to the first case given its frequent use. Evidently, the second case has already been covered in the previous section. For example, Chapter 11 restructuring proposals are voted this way. A variation of this method is used by US Congress in which voting is sequential rather than simultaneous.

[13] See the remarks on the Belgian constitution in the introduction.

We consider the case of junior voters first, the analysis is completely analogous for senior voters. A junior voter is pivotal whenever two conditions are met: (1) the senior voters have agreed to the proposal, and (2) exactly $a_J - 1$ junior voters have voted 'yes', and all others have voted no. Let y_τ be the number of yes votes in class τ. It is convenient (but not necessary) to analyze the updating process in two steps. The first step updates the junior voters beliefs conditional on the fact that the majority of all senior voters has agreed. This happens whenever at least a_S senior voters have voted in favor of the proposal, and this implies beliefs of $\sigma = h$ or:

$$\beta_S = \frac{\Pr\left(y_S \geq a_S \left| h\right.\right)}{\Pr\left(y_S \geq a_S \left| h\right.\right) + \Pr\left(y_S \geq a_S \left| l\right.\right)} \tag{4.1}$$

$$where :$$

$$\Pr\left(y_S \geq a_S \left| s\right.\right) = \sum_{i=a_S}^{M_S} \begin{pmatrix} N \\ i \end{pmatrix} \pi_S\left(s\right)^i \left[1 - \pi_S\left(s\right)\right]^{M_S - i} \tag{4.2}$$

Then, conditional on being pivotal a voter of type τ has beliefs:

$$\widetilde{\beta}(a_\tau, a_{\tau'}) = \frac{\beta_{\tau'}}{\beta_{\tau'} + (1 - \beta_{\tau'})\left(\frac{\pi_\tau(l)}{\pi_\tau(h)}\right)^{a_\tau - 1}\left(\frac{1-\pi_\tau(l)}{1-\pi_\tau(h)}\right)^{M_\tau - a_\tau}} \tag{4.3}$$

We prove in Lemma 1 (in the appendix) that the probability of voting "yes" is higher following a good signal, i.e., $\omega^g > \omega^b$. Then (3.3) implies that $\pi\left(h\right) > \pi\left(l\right)$, so that $\beta_S > \frac{1}{2}$ from (4.1) above. (stochastic dominance).

The important question is how much information aggregation is possible with two-class voting. Specifically, is it possible to aggregate all the information? A necessary condition for full information aggregation is that every voter has to be fully responsive.

The following Proposition characterizes the majority requirements to achieve this objective.

Proposition 4.2. *(i) The smallest integer a_τ^* greater than*[14]

$$\frac{M_\tau}{2}-\frac{1}{2\ln(1-\varepsilon)/\varepsilon}\ln\frac{\beta_{\tau'}G}{(1-\beta_{\tau'})L} \tag{4.4}$$

is the majority requirement which induces informative voting for class τ.

(ii) The cutoff a^* (from (3.10)) chosen by a social planner is the upper bound for the sum of the optimal majority requirements: $a^* \geq a_J^* + a_S^*$.[15]

To see the intuition of the second part of the proposition, suppose we would set $a = a_J^* + a_S^*$. Then, whenever voters vote in separate classes, their priors are more optimistic conditional on their class being pivotal since they already know that their vote only matters if the voters in the other class have received a critical amount of favorable information regarding the proposal (see above: $\beta_S > \frac{1}{2}$). Hence, the majority requirement for this case is lower from (4.4): a higher prior $\beta_{\tau'}$ results in a lower value for a_τ^* since less additional evidence is needed in support of the proposal if the prior is more favorable. Two-class voting rules that require a majority of all voters *in addition to* a majority of each class reflect a concern that two-class voting with a simple majority requirement for each class passes proposals too easily.[16] Proposition 4.2 contradicts this notion by showing that two-class voting leads to a lower rather than a higher majority requirement compared to one-class voting: the two-class mechanism itself establishes an additional hurdle the proposal needs to take.

[14] Here, we assume that $\frac{M_\tau}{2}-\frac{1}{2\ln(1-\varepsilon)/\varepsilon}\ln\frac{\beta_{\tau'}G}{(1-\beta_{\tau'})L}$ is not an integer. Otherwise, both $\frac{M_\tau}{2}-\frac{1}{2\ln(1-\varepsilon)/\varepsilon}\ln\frac{\beta_{\tau'}G}{(1-\beta_{\tau'})L}$ and $\frac{M_\tau}{2}-\frac{1}{2\ln(1-\varepsilon)/\varepsilon}\ln\frac{\beta_{\tau'}G}{(1-\beta_{\tau'})L}+1$ may induce informative voting.

[15] As we will see in numerical examples this inequality is almost always strict. The equality $a^* = a_J^* + a_S^*$ may hold only due to the rounding up the numbers a_J^* and a_S^* to an integer. For real numbers $a^* = \frac{M_J+M_S}{2}-\frac{\ln G/L}{2\ln(1-\varepsilon)/\varepsilon}$ and $a_\tau^* = \frac{M_\tau}{2}-\frac{1}{2\ln(1-\varepsilon)/\varepsilon}\ln\frac{\beta_{\tau'}G}{(1-\beta_{\tau'})L}$ the inequality becomes strict: $a^* > a_J^* + a_S^*$.

[16] See point 4 in the list above and the Belgium constitution noted in the introduction as an example.

Equation (4.4) expresses a^*_τ as a function of $a_{\tau'}$ and vice versa, since $\beta_{\tau'}$ depends on $a_{\tau'}$. An analytic expression of (a^*_J, a^*_S) purely as a function of the parameters of the problem $(\varepsilon, M_J, M_S, \beta_{0J}, \beta_{0S})$ can therefore not be given. However, we can explore its properties numerically.[17] Figure 4.1 shows that a^*_τ is falling in $a_{\tau'}$. This is intuitive, since a^*_τ is always lower if $\beta_{\tau'}$ is higher: the $\tau-$voters are pivotal only if the $\tau'-$voters have accepted the proposal, and their posteriors conditional on this event are measured by $\beta_{\tau'}$. However, $\beta_{\tau'}$ increases in $a_{\tau'}$, which explains the shape of the a^*-function.[18] The intersection between the two integer-valued functions always represents a pure strategy equilibrium. This may not be unique, as demonstrated by the example in Figure 4.1.[19] Note also that for both equilibria in Figure 4.1 we have $a^*_J + a^*_S = 23 < a^* = 25$, demonstrating the second part of proposition 4.2.

Proposition 4.3. *There is no set of majority rules* $(a_S,\ a_J)$ *such that voting in two classes aggregates the information perfectly, and the likelihood of a mistake is always strictly larger than that attained by the social planner.*

The idea of the proof is illustrated in Figure 4.2. The horizontal axis has the number g_J of good signals received by junior voters, the vertical axis the number g_S of those received by senior voters. The diagonal line indicates the location of all $(g_J, g_S)-$tuples so that $g_J + g_S = a^*$. (see (3.10)). The proposal should only be accepted if (g_J, g_S) lies to the right and above the line.[20] Two class voting implies

[17] We conduct a more extensive numerical analysis of this model in Section 6 below.

[18] Note that for any $(a_J, a_S) \neq (a^*_J, a^*_S)$, voters would not follow the pure strategies assumed in the computation of β_τ. The assumed strategies are equilibrium strategies only if $(a_J, a_S) = (a^*_J, a^*_S)$.

[19] In this particular case there are two $(a^*_J, a^*_S)-$*tuples* that support a pure strategy equilibrium: $(a^*_J, a^*_S) = (11, 12)$ and $(a^*_J, a^*_S) = (12, 11)$.

[20] Also, perfect information aggregation with two-class voting would require that $a^*_J + a^*_S = a^*$ which is generally not the case, see the example in figure 4.1.

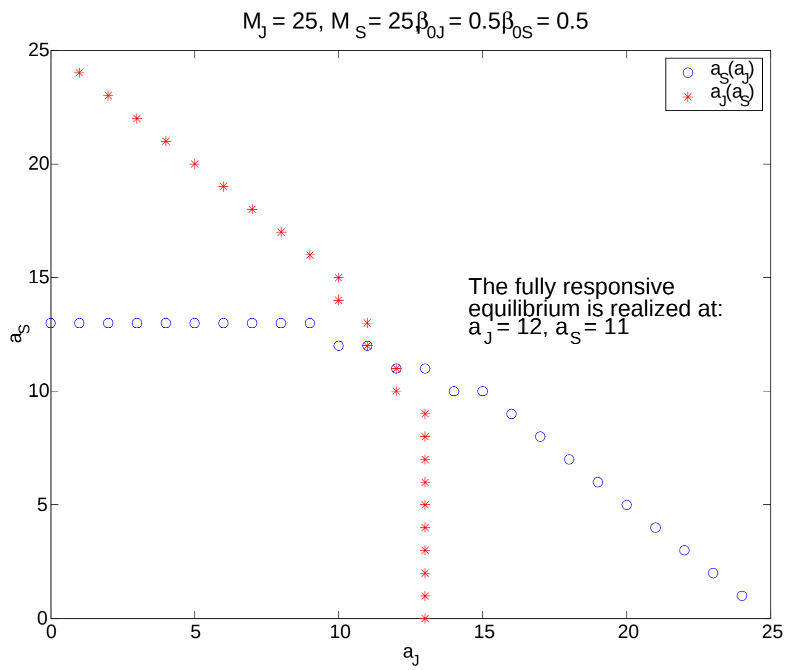


Figure 4.1:

that the proposal is accepted only if the proposal lies to the right of a_J and to the left of a_S, and the shaded areas I and III indicate those where the proposal is incorrectly rejected, whereas II indicates those where the proposal is incorrectly accepted. In some cases two-class voting rules require also a minimum fraction of the total vote. In Figure 4.2 this would be represented by a diagonal line parallel to the line where $g_J + g_S = a^*$. Clearly, such a requirement reduces the probability of an incorrect acceptance of the proposal. (it reduces the size of region III).

We know from the previous subsection that single-class voting can aggregate all the information. Therefore, we have the following theorem immediately:

Theorem 4.4. *If there is no significant conflict of interest between voters, then single-class voting strictly dominates two-class voting.*

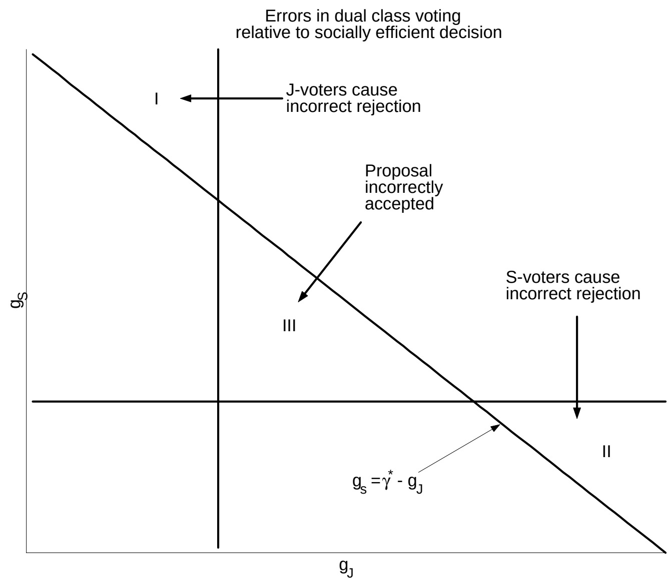


Figure 4.2:

Recall that single-class voting contests without conflicts of interest have the property that all voters agree with the outcome ex post. However, with two-class voting contests this is no longer the case. If the proposal is accepted marginally, then every voter revises her beliefs and wishes that the proposal had been rejected. The reason is simple. Every voter in class τ conditions her beliefs on acceptance of the proposal in class τ'. However, this implies that she gives a probability weighted average to all outcomes from $g_\tau = a_\tau$ to $g_\tau = M_\tau$. If she learns ex post that the realization of g_τ was at the lower end of her beliefs, she realizes that she has conditioned on beliefs that were too optimistic.

5. Conflicts of Interest

In the previous section we prove that if the conflict of interest is sufficiently small, then one-class voting aggregates information perfectly. Therefore, our focus was on fully responsive equilibria, i.e., responsive pure strategy equilibria. Now we drop the assumption that conflicts of interest are small. We will show that perfect information aggregation is no longer possible since a responsive pure strategy equilibrium fails to exist if conflicts of interest are sufficiently high. In the next subsection we analyze one-class voting with heterogenous preferences and establish that this mechanism is informationally inefficient. The subsequent subsection analyzes two-class voting.

5.1. One class of voters

A voter of type τ votes in favor only if his posterior belief, β_τ^σ, is greater than or equal to his hurdle value $\beta_{0\tau}$. Similarly, he votes against a proposal only if his posterior belief is less than or equal to his hurdle value $\beta_{0\tau}$. In other words, a voter votes responsively only if his hurdle rate is in the interval $[\beta_\tau^b, \beta_\tau^g]$. With a high level of conflict of interest, these hurdle values are quite different for both types. On the other hand, being pivotal carries the same information for both types in one-class voting. Therefore, two voters with different preferences but identical signals have the same posterior belief. Consequently, the interval $[\beta_\tau^b, \beta_\tau^g]$ is identical for both types. Hence, if the conflict of interest is large enough, then it is not possible for both types to have their hurdle rate in this interval. We can now state the major implication of conflicts of interest:

Proposition 5.1. *(i) For any electorate (M_S, M_J), proposal (G, L) and information error ε there are constants $\hat{A}_J$, $\hat{A}_S$ such that there is no equilibrium where both types of voters vote responsively if $|A_\tau| > |\hat{A}_\tau|$, $\tau \in \{S, J\}$. (ii) Assume*

$A_\tau > 0 > A_{\tau'}$. *Then, if* $a < M_\tau$, *the voters of type* τ *vote responsively and the voters of type* τ' *vote against. If* $a > M_\tau$, *the voters of type* τ *vote always in favor and those of type* τ' *vote responsively.*[21]

Hence, if conflicts of interest are too strong, then one-class voting can only elicit the information from one class, and the voting behavior of voters in the other class is completely determined by their preferences. Proposition 5.1 leads us to the following definition:

Definition 5.2. *We define a conflict of interest as significant whenever the constants* A_J, A_S *are sufficiently large so that only one group of voters can vote responsively in equilibrium.*

The majority rule determines which of the two classes votes responsively. If the majority requirement is low, then only the advantaged voters vote responsively, and the disadvantaged vote always against. Conversely, if the majority rule is high, then the disadvantaged voters vote responsively, and the advantaged voters vote always in favor. Then there are two candidate majority rules that can induce informative behavior in one of the classes, one for senior voters, and another for junior voters.

6. Two-Class Voting

So far we have shown that absent conflicts of interest single-class voting achieves the first best and hence aggregates information better than two-class voting. However,

[21] In an earlier version, Maug and Yılmaz (1999), we prove a stronger version of this proposition in a model with a richer state space: for any conflict of interest (i.e., $|A_\tau| > 0$) there exists large enough M_S and M_J such that there is no equilibrium where both classes of voters vote responsively.

we have also shown that the assumption of homogeneous preferences is critical, and single-class voting can no longer aggregate information perfectly if voters have conflicting interests. Then one-class voting can never aggregate the information of at least one-class of voters. In this case two-class voting can be more effective. The following proposition shows that two-class voting can be at least as efficient as single-class voting with conflicts interest:

Proposition 6.1. *If there is a significant conflict of interest, then two-class voting weakly dominates single-class voting.*

The intuition for this result is simple. If conflicts of interest are sufficiently strong, then the information of one group of voters is lost, and only the information of the other class is aggregated. Two-class voting can always do at least as well, because we can always design a two-class voting mechanism where class τ has no say on the outcome by setting $a_\tau = 0$. However, two-class voting can do potentially better by choosing a_τ optimally. At this point analytic methods cannot lead us much further since the equilibrium cannot be derived in closed form. However, we wish to show a stronger result and demonstrate under what circumstances two-class voting strictly dominates one-class voting, and also try to evaluate the quantitative dimension of the superiority of one voting mechanism over another. We therefore use numerical analysis to support our conclusions.

Figure 6.1 shows why two-class voting may be superior. It is constructed like Figure 4.2. We assume for the purpose of this discussion that the majority requirements are chosen so as to support a fully responsive pure strategy equilibrium.[22] The left panel of Figure 6.1 shows the errors that can occur if the $J-$voters have

[22] By superimposing one-class and two-class voting in one figure we ignore the fact that if we move from one-class to two-class voting, we must change the majority requirement somewhat in order to maintain a pure strategy equilibrium.

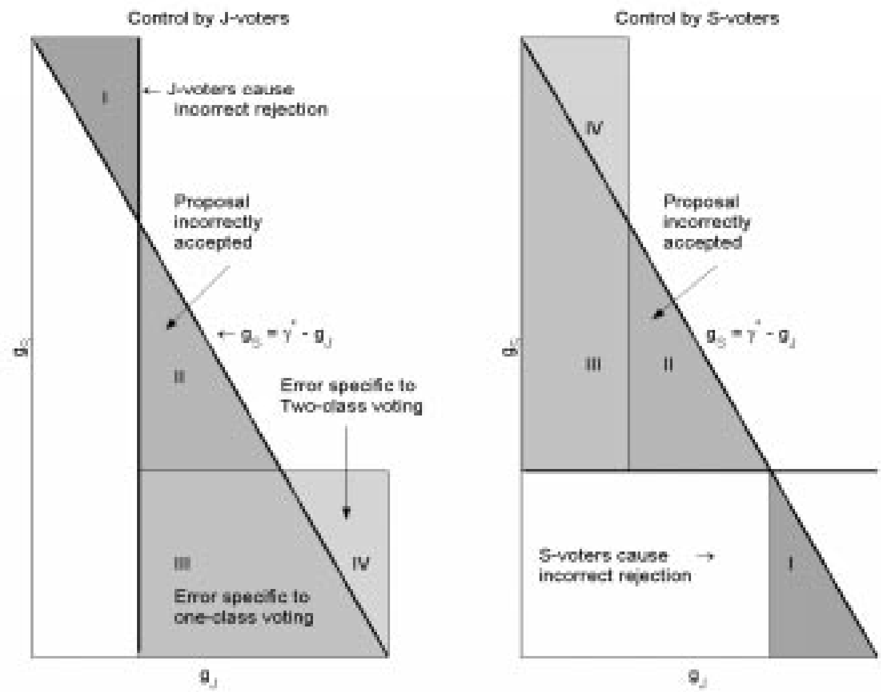


Figure 6.1:

voting control.[23] The errors represented by regions I and II are common to two and one-class voting. Region III shows the area where the proposal is incorrectly accepted with one-class voting, but two-class voting would lead to the correct decision. Conversely, region IV shows the area where the proposal would be incorrectly rejected by two-class voting, but is accepted with one-class voting. Hence, the superiority of dual-class voting can obtain only if region IV is small in a probabilistic sense relative to region III.

[23]The discussion of control by $S-$voters (right panel) is analogous.

6.1. Assumptions for Numerical Analysis[24]

For the numerical analysis we start with the following parameter values:

$$\varepsilon = 0.4 \tag{6.1}$$

$$G = L = 5 \tag{6.2}$$

$$M_\tau \in \{20, 50\}, \ \tau \in \{J, S\} \tag{6.3}$$

Later we will consider larger sets of parameters and perform comparative static analysis in order to check the robustness of our results.

In addition to ε,L, G and M, we also need to parameterize conflicts of interest and specify $(\beta_{0J}, \ \beta_{0S})$. Since we wish to make statements about varying precisions of the signal and compositions of the electorate, holding everything else constant, we also need to specify what we mean by holding conflicts of interest constant across varying electorates. This can have potentially different meanings, namely (1) holding transfers per voter A_J or A_S constant (either one, but not both, since condition (3.1) needs to be satisfied), (2) holding total transfers $M_J A_J = -M_S A_S$ constant, and (3) holding the difference in hurdle rates $\beta_{0J} - \beta_{0S}$ constant. The first definition has the disadvantage that a given transfer to all $J-$voters implies that whenever we increase the number of $J-$voters, the transfer from the $S-$voters has to increase, which contradicts the notion that conflicts of interest stay constant. Moreover, the resulting hurdle rates $\beta_{0\tau}$ can lie outside the unit interval if A_τ becomes very large, rendering the problem meaningless since the behavior of voters of type τ can never be responsive if $\beta_{0\tau}$ lies outside the unit interval. The second definition implies that conflicts of interest become irrelevant for large electorates, which is unattractive since the conclusion would be simply forced by the assumption that conflicts of interest are small for large electorates. We therefore adopt

[24] A more detailed description of the numerical approach and the code used for this analysis is availble from the authors upon request.

the third approach and define conflicts of interest as constant whenever $\beta_{0S} - \beta_{0J}$ is constant across different electorates, and we choose $\beta_{0S} - \beta_{0J} = 0.25$.[25]

6.2. Mixed Strategy Equilibria

In order to compute mixed strategy equilibria we employ an algorithm that is presented in more detail in Appendix B. In order to present the equilibria graphically we summarize the vector of randomizing probabilities $\left(\omega_\tau^g, \omega_\tau^b\right)$ by simply adding the randomizing probabilities:

$$\omega^\tau = \omega_\tau^g + \omega_\tau^b \tag{6.4}$$

This way of presenting the randomizing probabilities is more convenient, and there is no loss of information from adding the probabilities since from Lemma 8.1 we can have $\omega^b > 0$ only if $\omega^g = 1$, and $\omega^g < 1$ only if $\omega^b = 0$. Hence, it is always possible to recover each element in $\left(\omega_\tau^g, \omega_\tau^b\right)$ from ω^τ as:

$$\begin{aligned} \omega_\tau^g &= \min\left\{\omega^\tau, 1\right\} \\ \omega_\tau^b &= \max\left\{\omega^\tau - 1, 0\right\} \end{aligned}$$

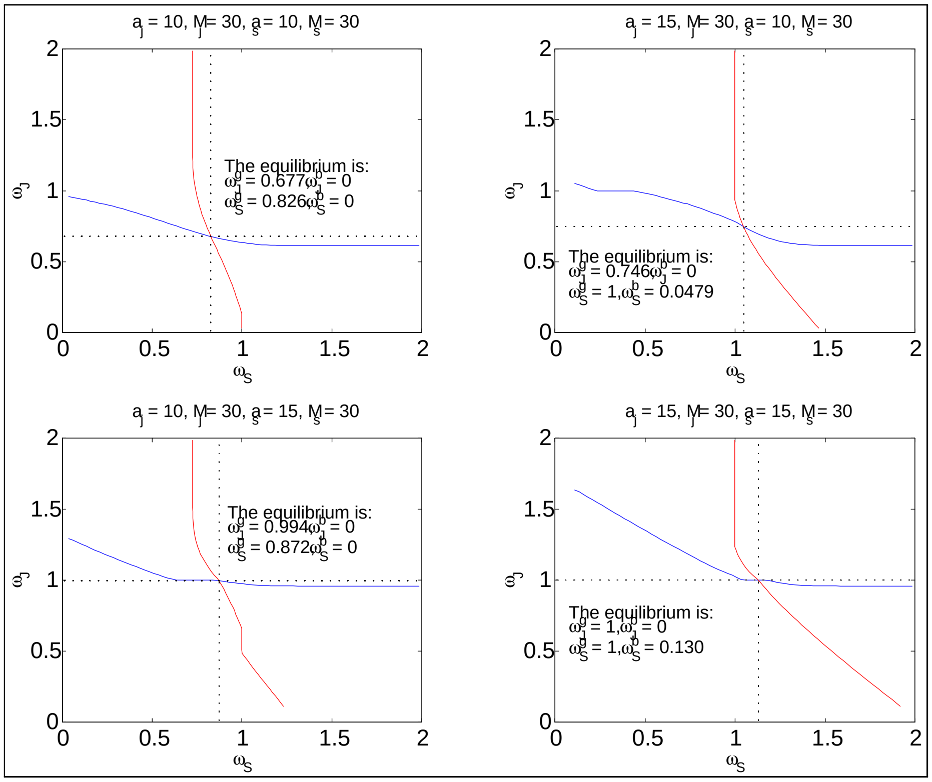
Clearly, $\omega^\tau \in [0;\ 2]$, and for any given parameter vector $[\varepsilon, M_J, M_S, a_J, a_S, \beta_{0J}, \beta_{0S}]$, we can construct a mapping

$$\begin{aligned} \omega^J &\rightarrow \omega^S \\ \omega^S &\rightarrow \omega^J \end{aligned} \tag{6.5}$$

which is represented in Figure 6.2 for different parameters. Mapping (6.5) maps $[0;\ 2]^2$ onto itself, and the mixed strategy equilibrium is simply the intersection

[25] Again, we cannot choose β_{0J} and β_{0S} independently since we lose one degree of freedom through condition (3.1).

Figure 6.2:



of the two lines in Figure 6.2. In all parameter combinations we have tried the mixed strategy equilibrium is unique.[26] Figure 6.2 displays the behavior we would expect. If the majority requirements $(a_J,\ a_S)$ are both below those that support a pure strategy equilibrium, then equilibrium strategies have all voters voting 'yes' with lower probability.[27] (Note that a pure strategy equilibrium has $\omega^\tau = 1$ for both classes) Similarly, if both majority requirements are higher than (a_J^*, a_S^*), then

[26] The only other equilibria are those where $\omega^\tau = 0$ (always vote no) or $\omega^\tau = 2$ (always vote against).

[27] For the parameters in Figure 6.2 the pure strategy equlilibrium has $a_J^* = 13$, $a_S^* = 15$.

voters vote in favor of the proposal more frequently.

6.3. Efficiency Comparisons

Now we address the fundamental question of efficiency differences between one-class and two-class voting. For this purpose we consider the errors of incorrectly accepting a bad or rejecting a good proposal, where we take the social welfare function from (3.14). For our parameterization ($G = L = 5$), this simplifies to

$$E_{Social} = \frac{1}{2}\left(e_I + e_{II}\right) \tag{6.6}$$

We compute E_{Social} for the case of one-class voting and two-class voting. Note

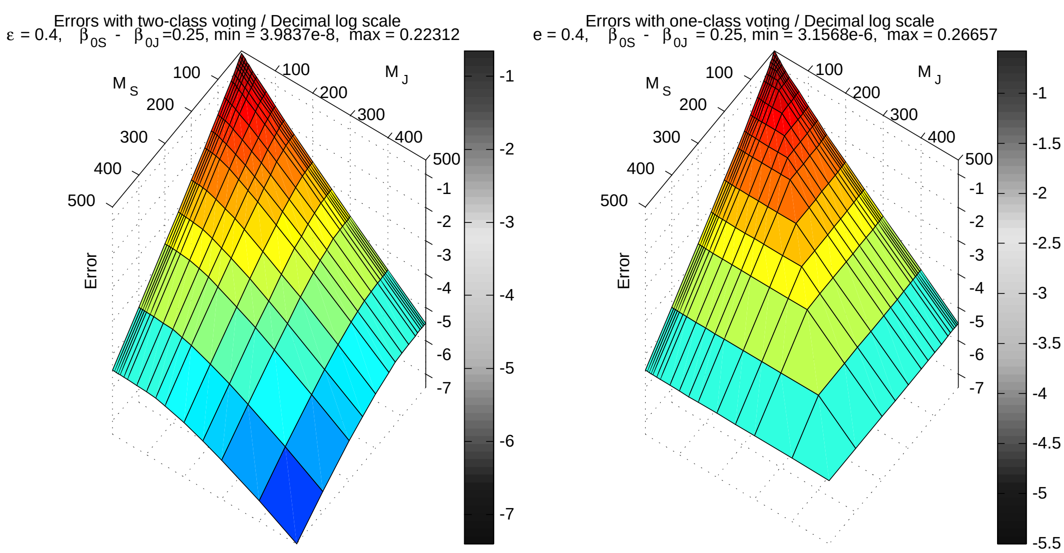


Figure 6.3:

that we suppress any reference to the majority requirements $(a_J,\ a_S)$ here. Clearly, E_{Social} takes on a different value for any $(a_J,\ a_S)$ −tuple and the resulting equilibrium. We compute all the different values of E_{Social} for alternative $(a_J,\ a_S)$ −tuples, and then assign that value which gives the lowest total error to E_{Social}.[28] We do not explicitly analyze the characteristics of these $(a_J,\ a_S)-$tuples here. However, we do note that the most efficient combination of majority requirements $(a_J,\ a_S)$ is generally one associated with a mixed-strategy equilibrium. This is true for one-class as well as for two-class voting, and distinguishes this situation from one-class voting with homogenous preferences. Figure 6.3 displays E_{Social} for selected parameter values. On the $x-$axis and $y-$axis we have different values for M_J and M_S ranging from 10 to 500, where the vertical axis corresponds to the log of E_{Social}. It can be seen clearly from the graph that the error for two-class voting drops significantly towards the main diagonal where $M_J = M_S$.

Generally, for any given size of the electorate, two-class voting performs best if the two classes are equal in size. The intuition for this result follows from our discussion of Figure 6.1 above. Starting from one-class voting, consider how the error changes from adding a second class. Conditioning the outcome on the vote of the second class adds information through the signals of the second class, as well as noise, since the decision of the second class is subject to some error.[29] However, the error of the second class decreases in the number of voters in this class. Hence, if the outcome is conditioned on the vote of a small second class, then two-class voting adds noise for a relatively small amount of additional information. If the outcome is conditioned on a large second class, then two-class voting adds more information for the price of adding very little noise. This intuition is fundamental

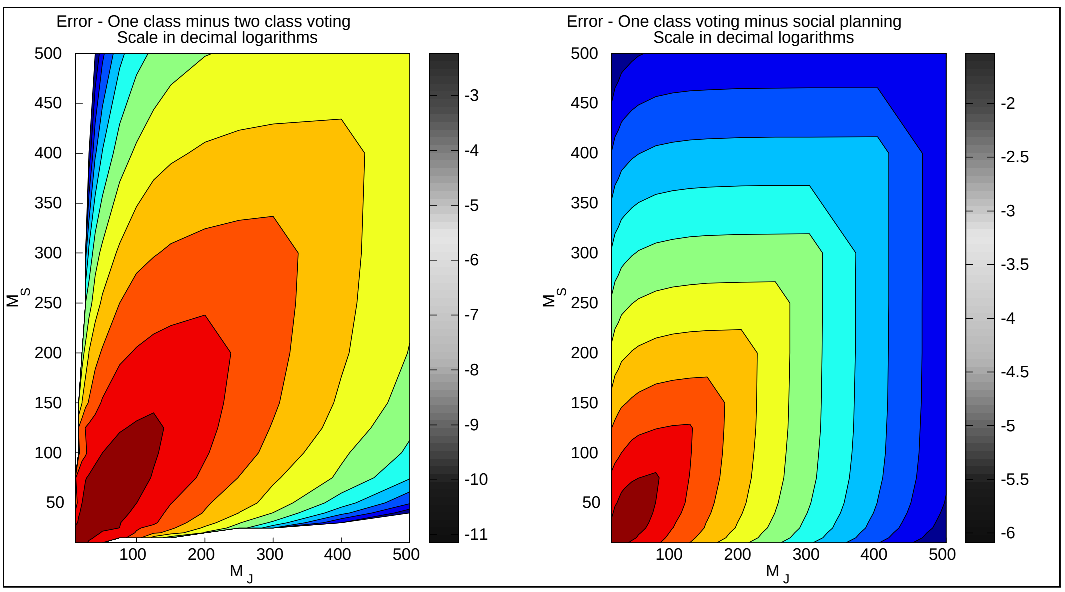
[28] More accurately, we observe that for all parameter values we have analyzed, the optimal $(a_J,\ a_S)-$tuple $(a_J,\ a_S)^*$ is close to the $(a_J,\ a_S)-$tuples that support a pure strategy equilibrium. Moving away from $(a_J,\ a_S)^*$ increases the error E_{Social} quickly. Therefore, we often analyze only a grid around the $(a_J,\ a_S)-$tuple that supports a pure strategy equilibrium and search for $(a_J,\ a_S)^*$ locally rather searching all possible combinations of a_J and a_S.

[29] In terms of Figure 6.1, adding the second class of voters adds the errors represented by region IV and removes the errors in region III.

for the subsequent results.

In order to compare two-class and single-class voting we compute a number of statistics. The first set of statistics compares absolute errors, i. e., the difference in E_{Social} for one and two-class voting.[30] Figure (6.4) displays the *absolute* difference

Figure 6.4:



in errors between one-class versus two-class voting (left panel), and one-class voting versus the error with perfect information aggregation (right panel). The contour chart (left panel of Figure 6.4) clearly demonstrates that two-class voting makes the largest difference when (1) the electorate is more or less evenly split ($M_J = M_S$), and (2) the electorate is small. In this case the difference in errors are between

[30] We compute the errors for one class voting by establishing the error where all voting rights are with the junior and senior voters respectively, and then selecting the minimum of the two. Hence our notion of one-class voting assumes that the majority rule for one-class voting is chosen so as to minimize the loss function of the social planner.

4% - 5%. It is also clear from the right hand panel that these are precisely the cases where one-class voting deviates most strongly from the social optimum. The chart suggests that both differences are of about the same order of magnitude. We investigate this further by looking at the *relative* differences. For this we compute two statistics. The first is simply the relative improvement from two-class voting:

$$Criterion\ I = \frac{E_{Social}\,(One\ Class)}{E_{Social}\,(Two\ Class)} \tag{6.7}$$

For our parameter values this statistic ranges from 1 (for very unevenly split electorates) to 79.2 for $M_J = M_S = 500$. The second statistic normalizes the absolute improvement by relating it to the gap between one-class voting and the social optimum:

$$Criterion\ II = \frac{E_{Social}(One\ Class) - E_{Social}(Two\ Class)}{E_{Social}(One\ Class) - E_{Social}(Social\ Optimum)} \tag{6.8}$$

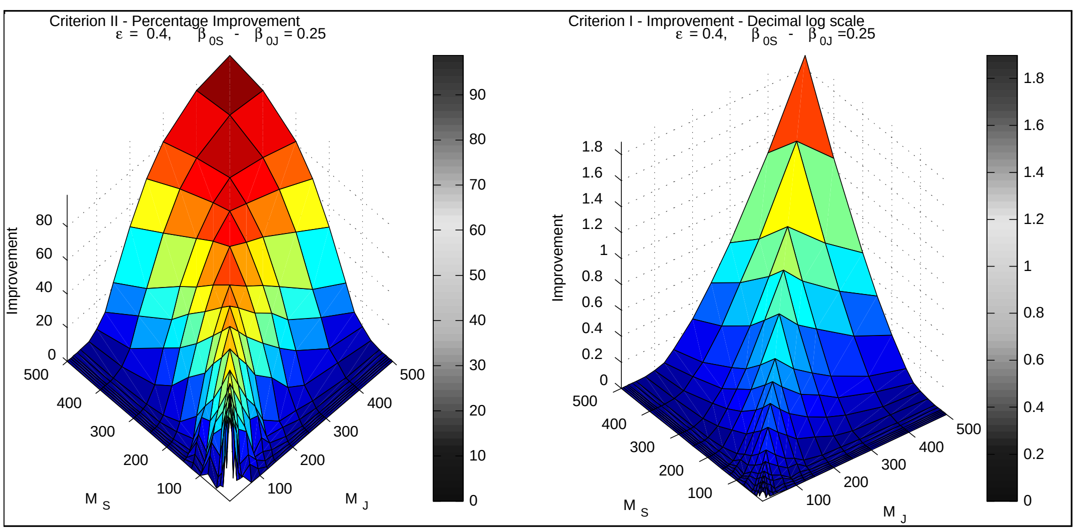


Figure 6.5:

The motivation behind this statistic is that it is normalized to lie between zero and one. If two-class voting achieves the social optimum, then Criterion II equals 1. If two-class voting is ineffective and leads to the same error as one-class voting, then Criterion II equals zero. We can interpret Criterion II as measuring the extent to which two-class voting helps to close the gap between one-class voting and the social optimum. Figure 6.5 displays both criteria. Both panels of Figure 6.5 show that the relative improvement is larger if the electorate is large and evenly split between the two classes. Criterion II approaches 1 as the electorate becomes large: for our parameters it becomes .984 for $M_S = M_J = 500$. This criterion converges fast for evenly split electorates, and more slowly for unevenly split electorates.

The errors $E_{Social}\ (One\ Class)$, $E_{Social}\ (Two\ Class)$, and $E_{Social}(Social\ Optimum)$ all approach zero as the electorate becomes sufficiently large. Criterion II allows us to look at the relative speeds of convergence, and shows that for small electorates the relative advantage of two-class over one-class voting is small, since both mechanisms lead to a significantly higher incidence of errors relative to the social optimum. For large electorates, the two-class voting mechanism behaves like the social planner: the difference in the incidence of errors between the social planner and two-class voting becomes negligible relative to the incidence of errors with one-class voting.

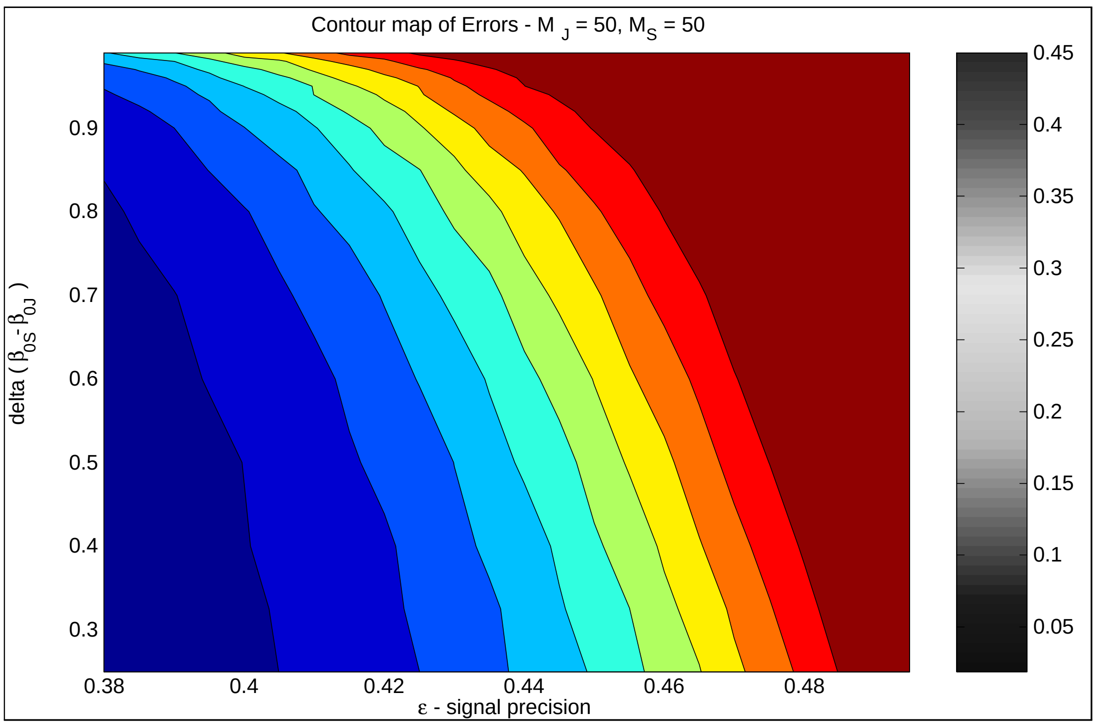
6.4. Comparative Static Analysis

We have performed the analysis in the previous section for different parameters.[31] Now we perform some comparative static analysis in order to check the robustness of our results. We fix $M_J = M_S = 50$ and vary ε between 0.38 and 0.495. We

[31] We repeated the analysis for $\varepsilon = 0.38,\ 0.42,\ 0.44,\ 0.46$, and 0.48. The salient results stay the same in all cases, and the numbers change as described by the comparative static analysis of this section.

do not consider smaller values of the error ε, since for smaller errors (holding the conflict of interest parameter constant at $\beta_{0J} - \beta_{0S}$), we can obtain parameter constellations where efficient information aggregation is again feasible. We also vary $\beta_{0S} - \beta_{0J}$ between 0.25 and 0.99. Figure 6.6 is a contour-plot of E_{Social} for

Figure 6.6:



these parameters and shows that two-class voting generates a larger error if (1) conflicts of interest are larger, and (2) if the signal error is larger. Both results are intuitive. However, the more important question is how the relative efficiency of two-class voting to one-class voting depends on exogenous parameters.

Figure 6.7 is a contour-plot of Criterion II and shows that a stronger conflict of interest and a larger error reduce the relative advantage of two-class voting.

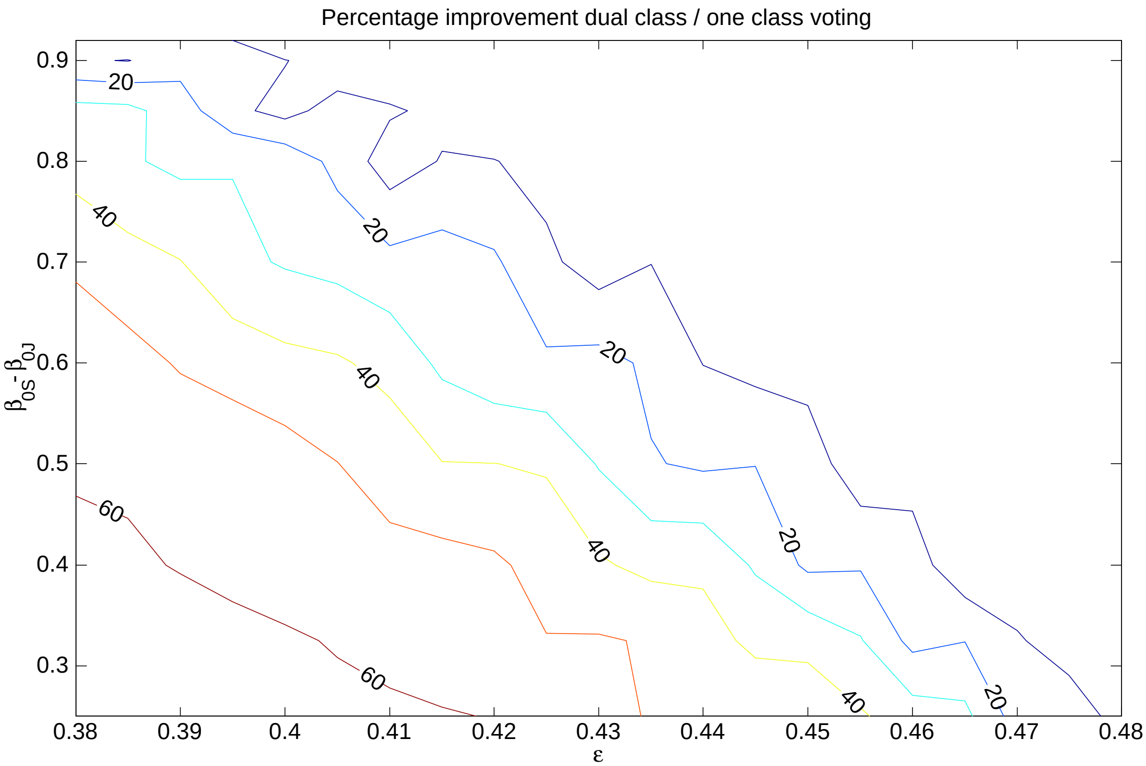


Figure 6.7:

To understand the first part of this result, recall that relative efficiency is always defined with respect to the social planner's solution. Also, recall that we define an increase in the conflict of interest as an increase in $\beta_{0S} - \beta_{0J}$. However, an increase in $\beta_{0S} - \beta_{0J}$ automatically implies that $\beta_{0S} - \beta_0$ and $\beta_0 - \beta_{0J}$ increase, so each group's objective moves away from the social welfare criterion: for stronger conflicts of interest, each group of voters is less representative of the population of voters as a whole, hence their voting behavior moves further out of step with the requirements of the social welfare criterion, and information aggregation deteriorates relative to the social optimum.

Information aggregation also deteriorates as the precision of the signals decreases (ε increases). The intuition follows from the discussion of figure 4.2 and figure 6.1 above. Consider the base case where only one class of voters, for example the $J-$voters, makes decisions and the $S-$voters are ignored completely. Then, moving from this situation to a mechanism with two classes increases information (avoid the mistake indicated by region III in figure 6.1) as well as noise (introducing the mistake indicated by region IV in figure 6.1). Now, observe that increasing the precision of the signal is similar to adding a larger number of voters with signals of a given precision, since the total amount of information is increased in either case. If the number of $S-$voters is small, then they add a large amount of noise relative to the information they add, and the advantage of two-class voting is small. For a large number of $S-$voters, they add a lot of information, but very little noise, and two-class voting offers a significant advantage. Hence, as the precision of the signal increases, the advantage of two-class voting increases, since the second class adds more to the probability of making correct decisions than it adds to the probability of making additional mistakes (region III is more likely than region IV).

7. Discussion and Conclusion

This paper has analyzed the comparative advantages of one-class voting and multi-class voting of Chapter 11 bankruptcy procedures in a situation where individuals do not only have differential information, but also different interests with respect to the proposal decided by a vote. The general finding is that in a large number of cases it is useful to segregate voters into different classes which are then more homogeneous with respect to voters' preferences. This also lends support to some institutions found in practice in addition to Chapter 11 bankruptcy proceedings, like bicameral parliamentary systems, and some constitutional assemblies.

At this point we need to emphasize that we have not established that two-class voting is in any sense the optimal mechanism. Clearly, a superior mechanism can always be found where voters agree on the proposal in conjunction with a scheme of offsetting transfers. Effectively, transfers would compensate for the distributional consequences of a proposal and amount to proposals on which voters do not have conflicting interests. We do not see examples of transfers of this kind, probably because eliminating conflicting interests is not generally possible.

Another limitation is the sequential nature of many voting processes, since, for example, the different chambers of parliaments vote often sequentially rather than simultaneously. Dekel and Piccione (1999) show that simultaneous and sequential voting games produce identical symmetric equilibria within a single-class framework. In single-class voting, observing prior votes does not give useful information in addition to being pivotal. However, this result does not extend to two-class voting games since observing the first class' vote gives useful information for the second class in addition to being pivotal. Hence, it is an open issue how the election outcomes and efficiency differ in a sequential two-class voting mechanism.

8. Appendix A: Proofs

Proof of Proposition 3.3:

Clearly:

$$\begin{aligned} \beta\left(a, M_J + M_S\right) &\geq \frac{L}{G+L} \\ \Leftrightarrow \quad \left(\frac{1-\varepsilon}{\varepsilon}\right)^{M_J+M_S-2a} &\geq \frac{G}{L} \end{aligned}$$

Taking logs on both sides and solving for a gives the result. ■

Lemma 8.1. *In all symmetric equilibria in which both types vote responsively we have that (i) either $\beta_\tau^g > \beta_{0\tau} > \beta_\tau^b$ and all voters of type τ play pure strategies ($\omega_\tau^g = 1$, $\omega_\tau^b = 0$), or (ii) $\beta_\tau^g = \beta_{0\tau}$ and $\omega_\tau^g < 1$, $\omega_\tau^b = 0$, or (iii) $\beta_\tau^b = \beta_{0\tau}$ and $\omega_\tau^g = 1$, $\omega_\tau^b > 0$. There is no equilibrium where $0 < \omega_\tau^g < 1$ and $0 < \omega_\tau^b < 1$.*

Proof of Lemma 8.1:

Let $\Pr\left(a-1\,|s\right)$ be the probability of having exactly $a-1$ yes votes out of $M_J + M_S - 1$ voters in state s where a is the social choice rule in a single-class voting. Then, conditional on being pivotal a voter has beliefs:

$$\bar{\beta}(a) = \frac{\Pr\left(a-1\,|h\right)}{\Pr\left(a-1\,|h\right) + \Pr\left(a-1\,|l\right)} \tag{8.1}$$

Then conditional on observing a good or bad signal, the voter concludes beliefs β^σ:

$$\beta^g = \frac{\bar{\beta}(a)}{\bar{\beta}(a) + \left[1-\bar{\beta}(a)\right]\frac{\varepsilon}{1-\varepsilon}} \tag{8.2}$$

$$\beta^b = \frac{\bar{\beta}(a)}{\bar{\beta}(a) + \left[1-\bar{\beta}(a)\right]\frac{1-\varepsilon}{\varepsilon}} \tag{8.3}$$

Responsive equilibrium implies that $\bar{\beta}(a) \in (0,1)$. Therefore, it is clear from $\varepsilon < \frac{1}{2}$ that $\beta_\tau^b < \bar{\beta}(a) < \beta_\tau^g$. Randomization if the signal is σ requires $\beta_\tau^\sigma = \beta_{0\tau}$.

Moreover, $\omega_\tau^\sigma = 1$ if $\beta_\tau^\sigma > \beta_{0t}$ and $\omega_\tau^\sigma = 0$ if $\beta_\tau^\sigma < \beta_{0\tau}$. Hence, the result follows for single class voting.

Similarly, in a two-class voting conditional on being pivotal a voter of type τ has beliefs:

$$\widetilde{\beta}(a_\tau, a_{\tau'}) = \frac{\beta_{\tau'}}{\beta_{\tau'} + (1-\beta_{\tau'})\left(\frac{\pi_\tau(l)}{\pi_\tau(h)}\right)^{a_\tau - 1}\left(\frac{1-\pi_\tau(l)}{1-\pi_\tau(h)}\right)^{M_\tau - a_\tau}} \tag{8.4}$$

Then, conditional on observing a good or bad signal, a voter of type τ concludes beliefs:

$$\beta_\tau^g = \frac{\widetilde{\beta}(a_\tau, a_{\tau'})}{\widetilde{\beta}(a_\tau, a_{\tau'}) + \left[1 - \widetilde{\beta}(a_\tau, a_{\tau'})\right]\frac{\varepsilon}{1-\varepsilon}} \tag{8.5}$$

$$\beta_\tau^b = \frac{\widetilde{\beta}(a_\tau, a_{\tau'})}{\widetilde{\beta}(a_\tau, a_{\tau'}) + \left[1 - \widetilde{\beta}(a_\tau, a_{\tau'})\right]\frac{1-\varepsilon}{\varepsilon}} \tag{8.6}$$

The rest of the argument is identical to that of single class voting. ■

Proof of Proposition 4.2:

(i) We have for voters of type τ that $\beta_\tau^g \geq \beta_{0\tau} \geq \beta_\tau^b$ where:

$$\beta_\tau^g = \frac{\beta_{\tau'}}{\beta_{\tau'} + (1-\beta_{\tau'})\left(\frac{\varepsilon}{1-\varepsilon}\right)^{2a_\tau^* - M_\tau}}$$

$$\beta_\tau^b = \frac{\beta_{\tau'}}{\beta_{\tau'} + (1-\beta_{\tau'})\left(\frac{\varepsilon}{1-\varepsilon}\right)^{2(a_\tau^* - 1) - M_\tau}}$$

a_τ^* is the lowest integer such that $\beta_\tau^g \geq \beta_{0\tau}$, hence:

$$\frac{1-\beta_{\tau'}^g}{\beta_{\tau'}^g}\left(\frac{1-\varepsilon}{\varepsilon}\right)^{2a_\tau^* - M} \leq \frac{G}{L}$$

Taking logs on both sides and solving for a_τ^* gives the result.

(ii) By definition (a_J^*, a_S^*) results informative behavior in both classes. In addition to being pivotal if each voter in class S conditions her decision on having $y_J = a_J^*$, then $a_S = a^* - a_J^*$ results in informative behavior in class S. Hence, if each voter in class S conditions her decision on her class being pivotal (i.e., $y_J \geq a_J^*$,rather than $y_J = a_J^*$) then $a_S \leq a^* - a_J^*$ results in informative behavior. Therefore, we have $a^* \geq a_J^* + a_S^*$. ■

Proof of Proposition 4.3:

(i) If the set of majority rules (a_S, a_J) does not induce informative behavior for each voter, the information cannot be aggregated perfectly. So we restrict attention to set of majority rules which induce such behavior. If $a^* > a_J^* + a_S^*$ then there is a possibility of approving the proposal even if the number of good signals is less than a^*. This creates a socially inefficient outcome. Therefore, we are left to prove the proposition for $a^* = a_J^* + a_S^*$. One should note that there is a possibility for uneven distribution of signals. For example, total number of good signals may exceed a^* although one of the classes receive less than a_τ^*. Hence, the there is an inefficiency compared to the socially optimal outcome. ■

Proof of Proposition 5.1:

Direct computation gives:

$$1 - \left(\beta^g - \beta^b\right) = \frac{\bar{\beta}^2 + \left(1-\bar{\beta}\right)^2 + \bar{\beta}\left(1-\bar{\beta}\right)\frac{\varepsilon}{1-\varepsilon}}{\bar{\beta}^2 + \left(1-\bar{\beta}\right)^2 + \bar{\beta}\left(1-\bar{\beta}\right)\frac{\varepsilon^2+(1-\varepsilon)^2}{\varepsilon(1-\varepsilon)}} > 0$$

So that $\beta^g - \beta^b < 1$. Furthermore, a responsive equilibrium has always $\beta_\tau^g \geq \beta_{0\tau}$ and $\beta_\tau^b \leq \beta_{0\tau}$, hence $\beta^g - \beta^b \geq |\beta_{0J} - \beta_{0S}|$. Hence, an equilibrium is responsive only if

$$1 > \beta^g - \beta^b \geq |\beta_{0J} - \beta_{0S}|$$

which is violated for $|A_J|$, $|A_S|$ too large.

(ii) From the assumption it follows that $\beta_{0\tau} < \beta_{0\tau'}$. Hence $\omega_{\tau}^{\sigma} \geq \omega_{\tau'}^{\sigma}$, $\sigma \in \{b, g\}$. Then it follows from (i) that $\omega_{\tau'}^{b} < 1 \Rightarrow \omega_{\tau}^{b} = \omega_{\tau}^{g} = 1$ and $\omega_{\tau}^{b} < 1 \Rightarrow \omega_{\tau'}^{b} = \omega_{\tau'}^{g} = 0$ ■

Proof of Proposition 6.1:

Without loss of generality, assume that each junior type ignores her information in single-class voting. Then we can improve on the outcome weakly by setting $a_J = 0$ and choose a_S optimally. ■

9. Appendix B: The $\omega-$Mapping

We use the following notation for the junior voters and for convenience omit the $J-$subscript whenever there is no ambiguity. The probabilities of voting 'yes' in the good and the bad state, respectively, are given by (3.3). We will repeatedly need the following ratios:

$$Ry = \frac{\pi(l)}{\pi(h)} \tag{9.1}$$

$$Rn = \frac{1-\pi(l)}{1-\pi(h)} \tag{9.2}$$

Then the beliefs of a junior voter conditional on being pivotal can be written as:[32]

$$\beta^G = \frac{\beta_S \pi(h)^{a_J-1}(1-\pi(h))^{M_J-a_J}(1-\varepsilon)}{(1-\varepsilon)\beta_S \pi(h)^{a_J-1}(1-\pi(h))^{M_J-a_J} \atop ...+\varepsilon(1-\beta_S)\pi(l)^{a_J-1}(1-\pi(l))^{M_J-a_J}} \tag{9.3}$$

Dividing through by the numerator gives:

$$\beta^G = \frac{1}{1+\frac{1-\beta_S}{\beta_S}\frac{\varepsilon}{1-\varepsilon}Ry^{a_J-1}Rn^{M_J-a_J}}$$

[32]Note that the denominator of this expression had to be broken into two lines.

Similarly, we find:

$$\beta^B = \frac{1}{1 + \frac{1-\beta_S}{\beta_S}\frac{1-\varepsilon}{\varepsilon} Ry^{a_J - 1} Rn^{M_J - a_J}} \tag{9.4}$$

Equilibrium conditions can be written as $\beta^G = \beta_{0J}$ if voters randomize subsequent to observing a good signal. Then $\beta^B < \beta_{0J}$ and $\omega_J^b = 0$. Conversely, if they randomize subsequent to observing a bad signal, we have $\beta^B = \beta_{0J}$, in which case $\beta^G > \beta_{0J}$ and $\omega_J^g = 1$. (See Lemma 8.1). Then we can rewrite the first condition ($\beta^G = \beta_{0J}$) as:

$$Ry^{a_J - 1} Rn^{M_J - a_J} = \frac{1-\beta_{0J}}{\beta_{0J}} \frac{1-\varepsilon}{\varepsilon} \frac{\beta_S}{1-\beta_S} \tag{9.5}$$

Similarly, the second condition ($\beta^B = \beta_{0J}$) becomes:

$$Ry^{a_J - 1} Rn^{M_J - a_J} = \frac{1-\beta_{0J}}{\beta_{0J}} \frac{\varepsilon}{1-\varepsilon} \frac{\beta_S}{1-\beta_S} \tag{9.6}$$

The next step is to determine the posterior odds after observing that the senior voters have accepted the proposal. These odds are $\beta_S / (1-\beta_S)$ From the definition of β_S (4.1) we have:

$$\frac{\beta_S}{1-\beta_S} = \frac{\Pr(y_S \geq a_S | h)}{\Pr(y_S \geq a_S | l)} \tag{9.7}$$

The next step is to determine the randomizing probabilities for given values of Rn and Ry. From (3.3), (9.1) and (9.2) above we have directly:

$$Ry = \frac{\varepsilon\omega^g + (1-\varepsilon)\omega^b}{(1-\varepsilon)\omega^g + \varepsilon\omega^b} \tag{9.8}$$

$$Rn = \frac{\varepsilon(1-\omega^g) + (1-\varepsilon)\left(1-\omega^b\right)}{(1-\varepsilon)(1-\omega^g) + \varepsilon(1-\omega^b)} \tag{9.9}$$

We have already established that the equilibrium can have randomizing after the bad signal or randomizing after the good signal, but not both. (Lemma 8.1). Hence we have to distinguish two cases:

Case 1: Randomizing after the good signal

In this case we have $\omega^g \geq 0$ and $\omega^b = 0$. Then:

$$\begin{aligned} Ry &= \frac{\varepsilon}{1-\varepsilon} \\ Rn &= \frac{1-\varepsilon\omega^g}{1-(1-\varepsilon)\,\omega^g} \end{aligned}$$

Then we can solve:

$$\omega^g = \frac{1-Rn}{\varepsilon\,(1+Rn)-Rn} \tag{9.10}$$

Case 2: Randomizing after the bad signal

In this case we have $\omega^b \geq 0$ and $\omega^g = 1$. Then:

$$\begin{aligned} Ry &= \frac{\varepsilon+(1-\varepsilon)\,\omega^b}{1-\varepsilon\,(1-\omega^b)} \\ Rn &= \frac{1-\varepsilon}{\varepsilon} \end{aligned}$$

Then we can solve:

$$\omega^b = \frac{\varepsilon\,(1+Ry)-Ry}{\varepsilon\,(1+Ry)-1} \tag{9.11}$$

The last step is to determine the R-values for Ry and Rn in equilibrium. We obtain for the case where $\omega^b = 0$, using (9.5):

$$\left(\frac{\varepsilon}{1-\varepsilon}\right)^{a_J-1} Rn^{M_J-a_J} = \frac{1-\beta_{0J}}{\beta_{0J}}\frac{1-\varepsilon}{\varepsilon}\frac{\beta_S}{1-\beta_S} \tag{9.12}$$

Solving for Rn:

$$Rn = \left(\frac{1-\beta_{0J}}{\beta_{0J}}\frac{\beta_S}{1-\beta_S}\left(\frac{1-\varepsilon}{\varepsilon}\right)^{a_J}\right)^{\frac{1}{M_J-a_J}} \tag{9.13}$$

Similarly, if $\omega^g = 1$, from (9.6):

$$Ry^{a_J-1}\left(\frac{1-\varepsilon}{\varepsilon}\right)^{M_J-a_J} = \frac{1-\beta_{0J}}{\beta_{0J}}\frac{\varepsilon}{1-\varepsilon}\frac{\beta_S}{1-\beta_S} \tag{9.14}$$

Solving for Ry:

$$Ry = \left(\frac{1-\beta_{0J}}{\beta_{0J}}\frac{\beta_S}{1-\beta_S}\left(\frac{\varepsilon}{1-\varepsilon}\right)^{M_J-a_J+1}\right)^{\frac{1}{a_J-1}} \tag{9.15}$$

Then equations (9.10) and (9.11) together with (9.13) and (9.15) and (4.1) gives us a mapping of $\left(\omega_S^g, \omega_S^b\right) \rightarrow \left(\omega_S^g, \omega_S^b\right)$ for any given parameter vector $(\varepsilon, a_J, a_S, M_J, M_S, \beta_{0J}, \beta_{0S})$.

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