FLEXIBLE (S,s) BANDS, UNCERTAINTY AND AGGREGATE CONSUMER DURABLES

by

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1 Introduction

Recent work in macroeconomics has suggested that (S,s) rules are a useful characterization of household purchases of durable goods, and that they provide a basis for expenditure dynamics consistent with those observed in aggregate data. The purpose of this paper is to determine whether a flexible (S,s) rule, that is, an inaction range that varies over time, provides a more complete characterization of the aggregate data. The results suggest that not only does a flexible rule bring additional explanatory power, but the manner in which the rule implicitly changes over time is highly cyclical and correlated with changes in financial volatility over the business cycle. These results provide an economic structure for the observation that durables purchases contract when consumer uncertainty increases.

Comparative statics show that in an (S,s) model with fixed bands, the width between the trigger and target points plays two important roles.[1] In steady state, it determines the average number of agents adjusting each period, and therefore in the case of durable goods, the average level of expenditure. Band width also affects how aggregate expenditure responds to shocks. For example, if the bands are very broad, aggregate expenditure will be smoother and take longer to fully incorporate disturbances than if the bands are narrow.

These effects of band width occur *across steady states,* but because threshold models exhibit path dependence, the transition from a narrow band regime to a wide band regime (for example) is an important consideration. In such a case, the effects on aggregate expenditure can be more dramatic. In the example just noted, for a given cross section density broadening the bands moves the trigger point *further away* from the position of any given agent, reducing the number hitting the trigger. The number of agents adjusting, and therefore expenditure, falls in this case. Narrowing the bands, of course, has a positive effect on expenditure.

Earlier work by Bertola and Caballero (1990) and Caballero (1990b) fit (S,s) models to aggregate purchases of durable goods. They hypothesize that the economy is composed of households adjusting their durables stocks according to an (S,s) rule which is common across all households and fixed over time. They then calculate the width of the (S,s) bands that

[1]These statements are made holding other parameters of the problem constant.

2 Estimating Band Widths

The estimation method used is in the spirit of Bertola and Caballero (1990), but with modifications suggested by the theoretical foundations I propose and the desire to estimate *time varying bands.* I begin briefly with the structure of the model derived by Grossman and Laroque (1990); for a complete description of the model, I refer the reader to their paper. More details of the estimation procedure are included in Appendix A.

2.1 Estimation Procedure

The intuition for the estimation is straightforward. In steady state, aggregate durable stocks would be held as an optimal share of wealth, conditional on the presence of transactions costs. Deviations from this steady state program implicitly define the threshold used by households in their purchase decisions. The following procedure estimates the steady state program in the first step, and in the second step uses the deviations from this program to estimate the thresholds.

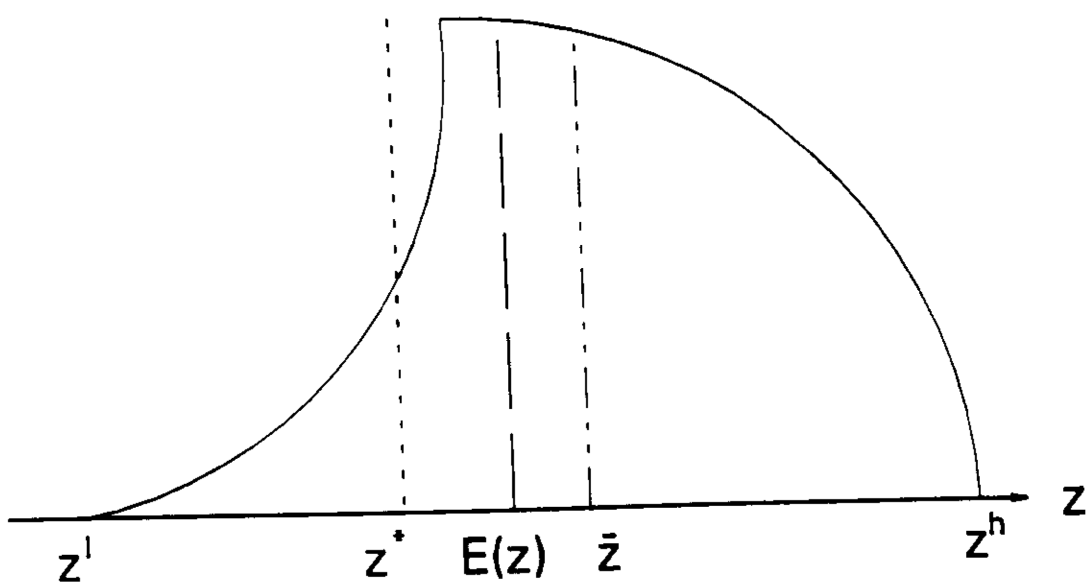
Households are assumed to derive utility from the services of a durable good, K, which depreciates at a constant rate. Their wealth, W, is stochastic and may be held in either the durable good or in assets that provide no consumption value. There is both a riskless and a risky asset. Households pay a transactions cost when they change their durables stock; they do so by selling their entire old stock and paying a proportional cost.[2]

Under these conditions, Grossman and Laroque show that optimal household behavior is to update the durables stock according to a two-sided (S,s) rule. The bands are fixed in terms of W/K after subtracting off the transactions cost; that is, when W becomes critically large relative to K, the household should pay the transactions cost and increase K, bringing W/K back down to the target value. Likewise, when W/K becomes critically low, the household should adjust K downward to bring W/K back up to its target value. The distance between

[2]The critical features of this specification are twofold. First, since the cost is proportional to the stock sold, it grows over time and therefore does not eventually become trivial to a household with increasing wealth. Second, the cost is not a function of the amount bought, and therefore behaves like a fixed cost, inducing threshold behavior.

Figure 1

DISTRIBUTION OF z SKEWED RIGHT



In household data, this distribution can be explicitly calculated as in Eberly (1991a), but in aggregate data some information can be inferred. This requires more structure, but the intuition comes from equation 1, the definition of the log state variable, z_i. This equation can be integrated over individuals as in equation 3 to obtain means of z,w, and k. If w and k are observable, then by taking the difference an estimate of the economy-wide mean of z is obtained. Again taking the example above where many households are near z^h, observed aggregate k would be low relative to observed w, so it would be possible to infer that the distribution of z was skewed to the right, again as shown in Figure 1. We can therefore obtain a measure of skewness of the actual distribution of z.

$$\bar{z} \equiv \int_{z^l}^{z^h} zf(z)dz = \int_{z^l}^{z^h} wf(z)dz - \int_{z^l}^{z^h} kf(z)dz \equiv \overline{w} - \overline{k} \quad (3)$$

Though they do not invoke this foundation for their estimation strategy, Bertola and Caballero (1990) implicitly exploit the distinction between this model's steady state and out-of-steady state predictions. When households are distributed between z^l and z^h according to the ergodic distribution derived in Eberly (1991b) and shown in Figure 2, the model has the same aggregate behavior as the frictionless model examined by Mankiw (1981). Deviations from this distribution likewise produce deviations from the behavior of the frictionless model. In their work on the aggregate data, therefore, Bertola and Caballero estimate the frictionless model and then use the (S,s) model to explain the residuals.

In the optimal (S,s) framework above, the mean of the ergodic distribution, E(z), represents the average value of w-k. In principle, this value can be estimated, and aggregate deviations from it attributed to deviations of the cross section distribution from its ergodic counterpart. The desired level of w-k depends on the portfolio decision of the household - in particular, how much of their wealth they choose to hold in durables. This decision depends on the variables that Bertola and Caballero include in their permanent income estimation (relative prices and user cost), as in equation 4 below. Estimation of this equation produces an estimate of how much of their wealth consumers desire to hold in durable goods. Deviations from this prediction provide estimates of the deviation of z from its mean over time.

model. This series can then be used to estimate an (S,s) model; the estimation method is described in detail in Appendix A, but the intuition is sketched here.

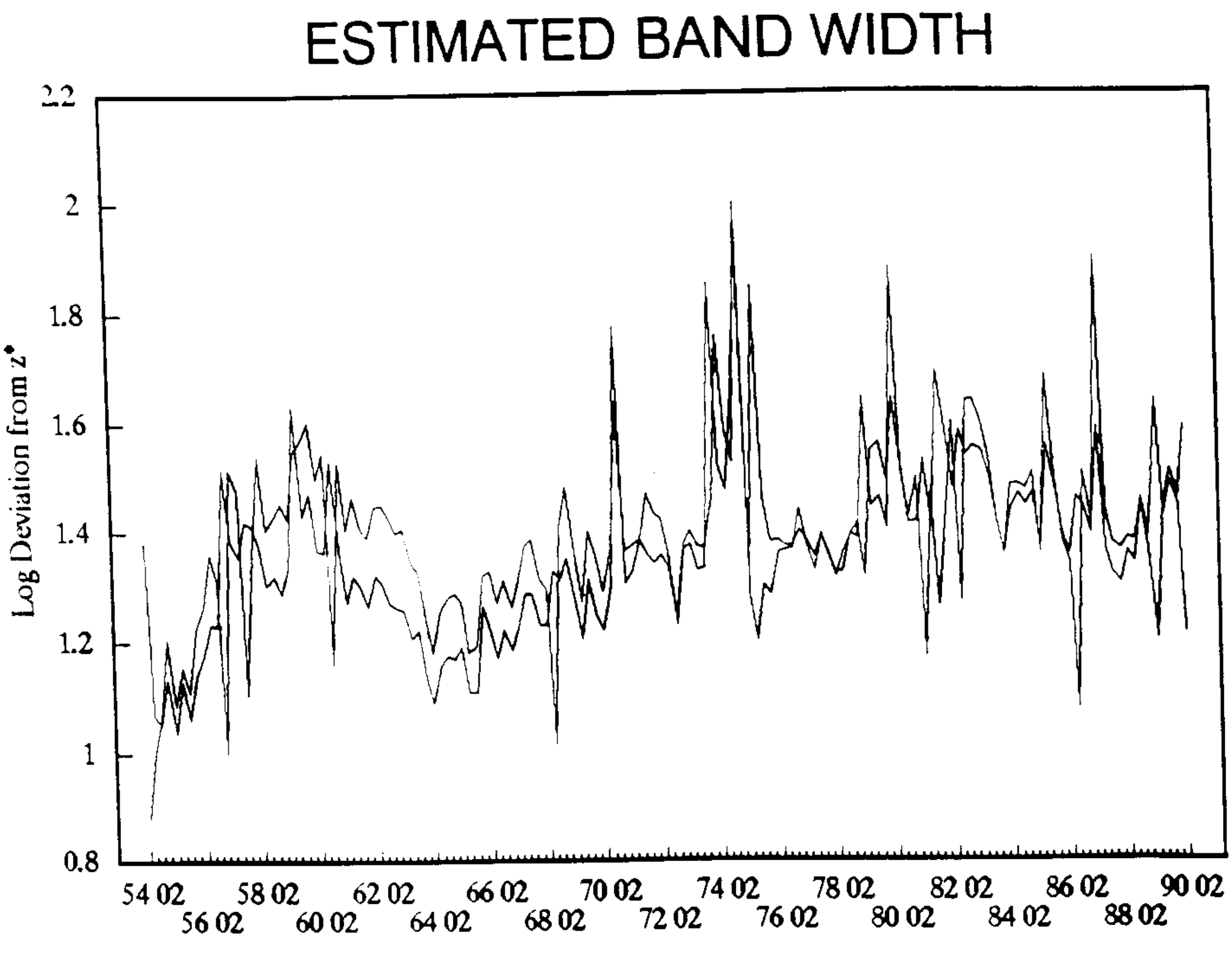
The residual series $\hat{z}$ represents the difference in the mean of the distribution of z across households, $\overline{z}$, from the mean of the ergodic distribution, E(z). Each period there is both idiosyncratic (with standard deviation σ_I) and aggregate uncertainty (with standard deviation σ_A). In the first estimations I assume that the ratio σ_A/σ_I is 0.06, but I will compare these results to those with σ_A/σ_I equal to 0.12.[7] The aggregate shocks are calculated from the shocks to the predicted value of $(\overline{w}-\overline{k})$ and are by construction orthogonal to the estimated $\hat{z}$ series. Each period the distribution is subjected to the estimated aggregate shock, as well as the random idiosyncratic uncertainty. The resulting deviation of the mean of the distribution, $\overline{z}$, from the frictionless mean, E(z), is then compared to the estimated deviation, $\hat{z}_t$, and bands are chosen so that $\overline{z_t} - E(z) = \hat{z}_t$.

Two sets of identifying assumptions must be made to identify the bands. The first is the choice of the initial cross section distribution. I use the ergodic distribution for the initial distribution and then disregard the first 15 estimation periods. The second assumption identifies the location of the bands. Here I assume that only the upper band is active, and therefore control is only exercised by households increasing their durables stocks. This is consistent with evidence in Eberly (1991a) that only 4 percent of households adjusted their durables stocks downwards.[8]

[7]Bertola and Caballero estimate this parameter endogenously to be between 0.05 and 0.063. Holding the bands fixed over the entire sample period gives them an additional degree of freedom that they use to estimate σ_A/σ_I.

[8]The second possibility is to assume that the bands instead remain in the same proportions found in the initial ergodic distribution. This is similar to the symmetry assumption made by Bertola and Caballero. Estimates in Eberly (1991b) required that the ratio $(z^* - z^l)/(z^h - z^*)$ be a constant, where Bertola and Caballero fix it at unity. The results are substantially the same as those with single sided control.

Figure 3



ESTIMATED BAND WIDTH
Log Deviation from z*
2.2
2
1.8
1.6
1.4
1.2
1
0.8
54 02
56 02
58 02
60 02
62 02
64 02
66 02
68 02
70 02
72 02
74 02
76 02
78 02
80 02
82 02
84 02
86 02
88 02
90 02

3.1 Cyclicality in Band Width

Figure 5 overlays the band width series with the NBER peaks and troughs. The peaks in band width noted above coincide with cyclical downturns, while declining band widths are associated with the subsequent recoveries. Over the typical cycle, band width changes by 0.5 (in logs). This implies that for a given level of wealth, the durable is allowed to fall 60 percent more at a trough than it would be if the purchase occured at a peak.[10] Previous work has found that households' choices of bands are sensitive to measures of uncertainty. If these measures rise around cyclical troughs, then households would broaden their (S,s) bands, consistent with the finding here.[11] I return to this interpretation below.

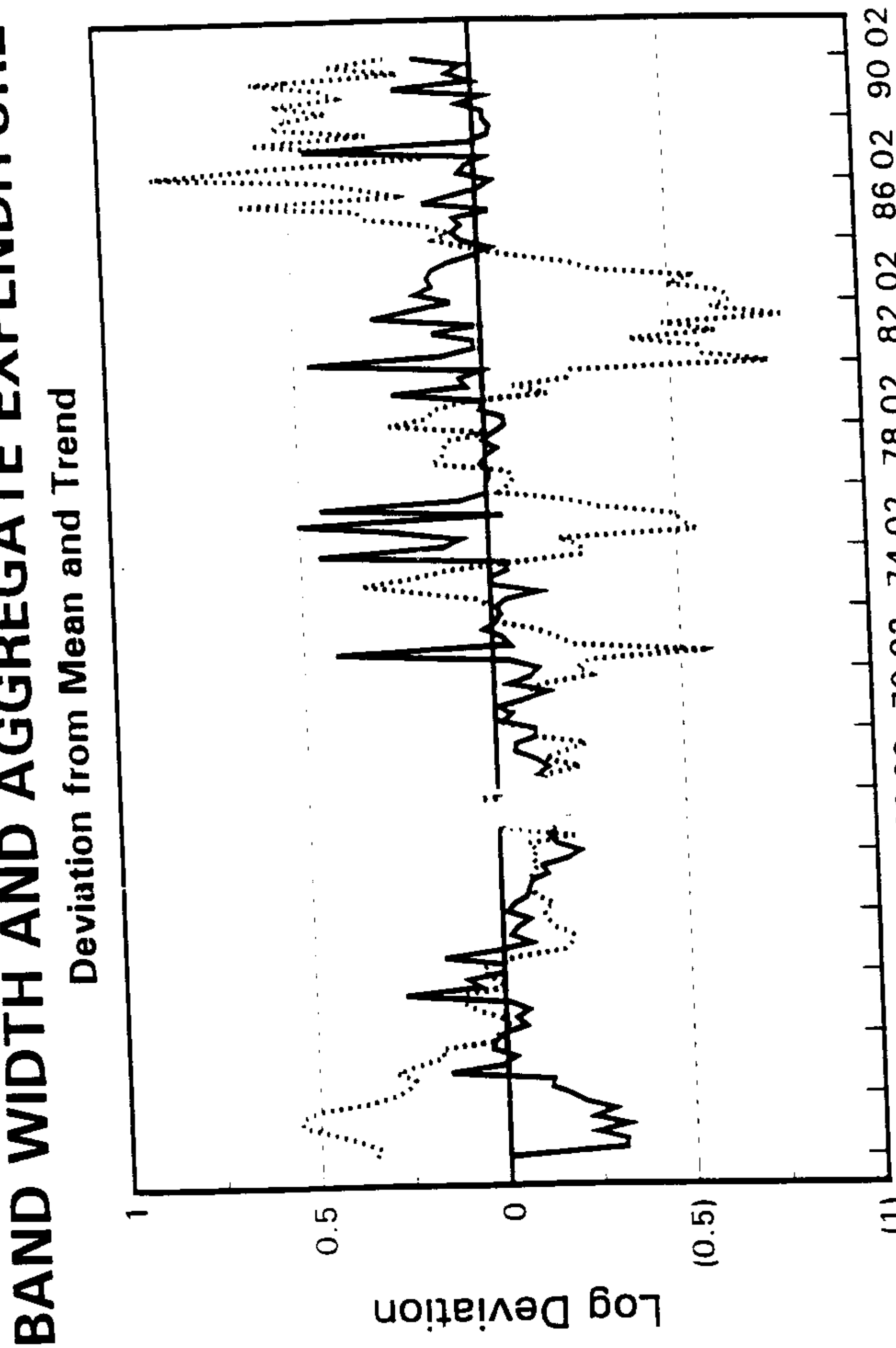
Recall from the example noted in the introduction that countercyclical band widths are consistent with procyclical expenditure on durable goods. Expenditure is determined jointly by the density of households, the trigger points for adjustment, and the shocks to the position of the households. For a given cross section distribution of households, broadening the bands takes the trigger point *further away* from the households. Therefore, households who previously might have hit the triggers are now *out of reach* and expenditure declines.

Theoretically, we therefore predict a negative correlation between band width and expenditure. Figure 6 charts the deviation of band widths from their mean against detrended real expenditure on automobiles. The negative correlation ($\rho = -.47$) is evident, particularly at cyclical peaks and troughs. This is consistent with the notion that when households broaden their (S,s) bands countercyclically, they effectively postpone durables purchases during downturns.

[10] If wealth falls at the trough, the change would be larger.

[11] The same result would also obtain if households become more risk averse during downturns.

Figure 6


BAND WIDTH AND AGGREGATE EXPENDITURE
Deviation from Mean and Trend
Log Deviation
1
0.5
0
(0.5)
(1)
54 02
56 02
58 02
60 02
62 02
64 02
66 02
68 02
70 02
72 02
74 02
76 02
78 02
80 02
82 02
84 02
86 02
88 02
90 02

bands accounts for much of the variation in durables stocks left unexplained by a fixed band model. Of course, fixed bands do much better than a frictionless model in explaining short run dynamics, but cyclical changes in band width are apparently an important part of the story at business cycle frequencies.

3.2 Determinants of Band Width

Earlier work in household data found that household uncertainty, as measured by the variability of income, had a significant positive effect on the inaction range. In this section I look for similar effects across time in the aggregate data. Since the extreme changes in the inaction range in the last section occur at business cycle frequencies, this section focusses on determinants of such cyclicality.

The theory suggests that the width of the bands should be affected by four variables. The most obvious is the transactions cost, which tends to broaden the bands as it rises. I do not consider this as a source of cyclical fluctuations in band width. The second possible source is the rate of drift through the bands, representing the depreciation rate and the rate of growth of lifetime wealth. Again I do not expect these to vary cyclically.[14] The final two measures are risk aversion and uncertainty. These have similar effects, and I focus on the latter since measurement is more direct. In practice I rely on two measures. The first is implied volatilities calculated from the Chicago Board Option Exchange index.[15] This series is the same as that used by Poterba and Summers (1986), updated to 1990. Implied volatilities provide a market measure of economic volatility and therefore should have a negative effect on the bands. A shortcoming of this series is that it begins in 1980, so its effect on band width can only be calculated for the 1980s. The second measure utilized is an *ex post* measure of uncertainty calculated from stock market variability. I take the squared first-differences of the log of the S&P 500 at monthly frequencies. I then sum these monthly observations to a quarterly series which represents the within quarter volatility of

[14] To the extent that the rate of growth of lifetime wealth (or permanent income) is procyclical, it would produce *procyclical* bands.

[15] The index is described in the CBOE publication Chi (1988).

Table 1 BAND WIDTH AS A FUNCTION OF VOLATILITY $width_t = \hat{\alpha}_0 + \hat{\alpha}_2\sigma_t^2 + \epsilon_t$		
Volatility Measure = Implied 1980-89		
Independent Variable	Coefficient	Standard Error
Constant	1.21	(0.13)
ImpliedVolatility	25.73	(12.12)
AR1 correction, $\hat{\rho}$=.06 (0.23) $\overline{R}^2 = .62$, $N = 31$		
Volatility Measure = Ex Post 1980-89		
Independent Variable	Coefficient	Standard Error
Constant	1.41	(0.06)
ExpostVolatility	13.56	(13.25)
AR1 correction, $\hat{\rho}$=-.02 (0.22) $\overline{R}^2 = 0.02$, $N = 31$		
Volatility Measure = Ex Post 1955-89		
Independent Variable	Coefficient	Standard Error
Constant	1.24	(0.04)
ExpostVolatility	36.73	(8.89)
AR1 correction, $\hat{\rho}$=0.40 (0.08) $\overline{R}^2 = 0.32$, $N = 136$		

The magnitude of the coefficient implies that when the implied volatility doubles, band width increases by 1.6 standard deviations. Furthermore, the $\overline{R}^2$ of the regression is 0.62, suggesting that much of the variation in the bands can be explained by the implied volatilities. This is a nontrivial result, since the 1980s did not represent a particularly smooth period for the estimated bands (recall Figure 4), nor does the AR(1) correction enter significantly.

The second panel of Table 1 reports the results of the same regression for the 1980s using *ex post* volatilities as the independent variable. As expected, this measure is less successful than the implied volatility measure. The coefficient is still large and positive, but is not

4 Conclusions

I find that the (S,s) bands sufficient to explain movements in aggregate automobile stocks vary significantly over time, particularly at business cycle frequencies. The estimated bands tend to rise preceding a cyclical peak, and then fall coincident with the subsequent recovery. This suggests that households postpone purchases of durables during downturns, promoting a durables boom when the economy recovers. Consistent with this interpretation, durables expenditures are negatively correlated with estimated band width. Further, estimated band width responds positively to stock market variability, both *ex ante* and *ex post,* suggesting that the bands widen in response to uncertainty. This is consistent with the theoretical predictions of an (S,s) model and provides an economic structure for the observation that durables expenditures fall when consumer uncertainty increases.

A Appendix: Estimation Procedure

The estimation implemented in Section 2 is conceptually in the spirit of Bertola and Caballero (1990). The procedure consists of two parts. The first uses a cointegrating regression to separate the durables stock into a part predicted by the steady state model and a residual, while the second uses an (S,s) model to explain this residual. Here, instead of identifying the fixed bands which best explain this residual as in Bertola and Caballero, I instead choose the bands period by period to exactly fit the data. In order to implement this procedure I assume that although the bands can change, they must be stationary (so that changes in their width are not permanent).

The first step estimates the equation found in the text, where lower case letters indicate natural logarithms,

$$\overline{k_t} = \hat{\beta}_0 + \hat{\beta}_1 \overline{w_t} + \hat{\beta}_2 p_t + \hat{\beta}_3 r_t - \hat{z}_t \tag{A1}$$

The data series used are as follows. $\overline{k_t}$ is constructed by accumulating expenditure on Motor Vehicles from the NIPA data. The initial durables stock is calculated by averaging the first 10 quarters of the data series and dividing by the depreciation rate, assumed to be 10 percent annually. The subsequent years are derived by applying the depreciation rate and adding period by period aggregate expenditure. Human wealth is estimated by the accumulated innovations of an AR1(1,1) process on the natural logarithm of real personal disposable income. These are both consistent with the procedures of Bertola and Caballero. Net worth is taken from the Flow of Funds data and deflated by the CPI. p_t is the (log) relative price of durables, calculated as the log ratio of the implicit price deflators for the durable (motor vehicles) and for non-durables, respectively. The interest rate is that on three month Treasury Bills.

Estimating this equation by Stock and Watson (1989) dynamic OLS, I cannot reject that $\beta_1 = 1$, so I impose this restriction in what follows, defining w =(net worth + human wealth). The results of the estimation are shown in Table A1.

In the second part of the estimation procedure, I estimate the band widths consistent proportion throughout. The results are very similar to those reported in the text, so I do not analyze them separately.

With these identifying assumptions, the estimation proceeds straightforwardly. Each period, the cross section distribution receives idiosyncratic shocks (assumed drawn from a normal distribution) and a common aggregate shock. This is calculated from the predicted value of the regression above as $\Delta(w/k^*)/\Delta t$, where k^* is the predicted value of k. Intuitively, positive aggregate shocks to w tend to increase the mean value of z, moving the distribution to the right. Changes in k^* move the target point, z^*. These shocks are added to the distribution known at the end of the previous period. The only parameter left undetermined so far is the variance of the aggregate shocks relative to the idiosyncratic. Since free choice of band width can perfectly match $\hat{z}_t$ each period, this parameter is not identified. I choose one value ($\sigma_A/\sigma_I = 0.06$) from within the bounds of Bertola and Caballero's estimates and another ($\sigma_A/\sigma_I = 0.12$) greater by three standard deviations and report the estimated band width for each.

Table A1
COINTEGRATING REGRESSION

$$\overline{k_t} = \hat{\beta}_0 + \hat{\beta}_1\overline{w_t} + \hat{\beta}_2 p_t + \hat{\beta}_3 r_t - \hat{z}_t$$

Independent Variable	Coefficient	Standard Error
Constant	-1.45	(0.004)
$\overline{w_t}$	1.00	imposed
p_t	-.40	(0.02)
r_t	-.02	(0.01)

$R^2 = 0.90$, $N = 146$

Stock-Watson Dynamic Least Squares