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LONG-HORIZON RETURNS AND SHORT-HORIZON MODELS

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#### ABSTRACT

First-order vector autoregressive models for monthly time series appear to describe the documented behavior of expected stock returns over long holding periods. We examine the implied autocorrelations of long-horizon returns and the implied R-squared in regressions of long-horizon returns on predetermined variables. A simple model is estimated using monthly data, and the estimated parameter values imply patterns for the autocorrelation and the R-squared that coincide well with the patterns of the actual sample values. The framework is also used to analyze other properties of expected returns for various horizons.

#### 1. Introduction

Empirical evidence indicates that expected returns on stocks and bonds vary through time. Much of this evidence is characterized either by autocorrelations of returns or by regressions of returns on various predetermined variables. 1 The length of the holding period over which a return is computed, or the return "horizon," seems to affect the nature of this evidence in significant ways. For example, the estimated autocorrelations of returns on indexes of NYSE stocks are positive and in the range of 0.1 to 0.2 for one-month horizons [e.g., Fama and Schwert (1977)], but sample autocorrelations for horizons of four years are negative and in the range of -0.2 to -0.5 [e.g., Fama and French (1987a)]. Regressions of one-month stock returns on predetermined variables often produce R-squared values less than 0.02 [e.g., Keim and Stambaugh (1986)], whereas regressions of longer horizon returns (several years) on similar predetermined variables often produce R-squared values in excess of 0.30 [e.g., Fama and French (1987c)].

Civen the apparent sensitivity of evidence about time-varying expected returns to the length of the return horizon, it would be useful to have a single parsimonious framework capable of integrating the existing empirical evidence. This study proposes and investigates such a framework. Using monthly data, we estimate a simple autoregressive model for predicting one-month-ahead returns given a set of predetermined variables. We then use the estimated parameters to compute, for various return horizons, the implied autocorrelation in returns and the implied R-squared for a regression of returns on the predetermined variables. The behaviors of these implied values

<sup>&</sup>lt;sup>1</sup>A partial list of the studies reporting such evidence includes Fama and Schwert (1977), Hall (1981), Huizinga and Mishkin (1984), Fama (1984), Rozeff (1984), Keim and Stambaugh (1986), Campbell (1987), Fama and Bliss (1987), Fama and French (1987a, 1987b, 1987c), Lo and MacKinlay (1987), and Poterba and Summers (1987).

across various return horizons coincide well with the results reported in previous studies.

Although the statistical framework investigated here appears to capture reasonably well the existing evidence about changing expected returns, we do not explain why expected returns change. The framework presented here simply provides a unifying framework in which to analyze the existing evidence. In other words, efforts to pursue economic exlanations for the behavior of expected returns for various horizons can be directed toward understanding the parameter values in a single model. Given such a model, analyzing long-horizon returns is unlikely to produce information about the behavior of expected returns that is not already implied by the monthly time-series model. The choices of variables and the precise forms of the models considered are necessarily arbitrary. We suggest that this draft be viewed primarily as an exercise illustrating the promise of this unifying approach.

#### 2. The Models

Let  $r_{t,N}$  denote the continuously compounded return on an asset for the N-month period starting at the beginning of month t, and let  $x_t$  denote a K×1 vector of state variables observed at the end of month t. (Month t is the month beginning at time t-1 and ending at time t.) To illustrate the model initially, we let  $r_{t,1}$  be the monthly return on the equally weighted NYSE index, and we let  $x_t$  contain four variables:  $\frac{2}{t}$ 

 $\Delta y_{\text{Baa,t}}$ : the change from the end of month t-1 to the end of month t in Moody's average yield on bonds rated Baa.

The last three variables are essentially the same as those used by Fama and French (1987c). Similar variables are also used by Keim and Stambaugh (1986).

- (y<sub>Baa</sub> y<sub>Aaa</sub>)<sub>t</sub> : the difference at the end of month t between Moody's average yield on bonds rated Baa and bonds rated Aaa.
- $(y_{Aaa} y_{TB})_t$ : the difference at the end of month t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.
- $\left(D/P\right)_{t}$ : for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

The yield-related variables are stated as percent per month. Let

$$y_{t} = \begin{bmatrix} x_{t} \\ r_{t,1} \end{bmatrix}$$

and define the first-order vector autoregression [VAR(1)],

$$y_t = g_0 + G y_{t-1} + \nu_t$$
 , (1)

where  $E\{\nu_t|y_{t-1}\}=0$  and  $E\{\nu_t\nu_{t-s}'\}=0$  for  $s\neq 0$ . The eigenvalues of G are assumed to be less than unity in absolute value. Table 1 reports descriptive statistics for the five variables in  $y_t$ .

A special case of (1), in which  $r_{t,1}$  is Granger caused by  $x_{t-1}$ , is given by the set of equations

$$r_{t,1} = \alpha_0 + \alpha' x_{t-1} + u_{t,1}$$
 , and (2a)

$$x_t = a_0 + Ax_{t-1} + \eta_t$$
 (2b)

where  $u_{t,1}$  and  $\eta_t$  have the same properties as the disturbances in (1). The correlation between  $u_{t,1}$  and the elements in  $\eta_t$  is represented by the auxiliary regression

$$u_{t} - b'\eta_{t} + \varepsilon_{t}$$
 , (2c)

where  $E\{\varepsilon_t|\eta_t\}=0$ . We will denote the model represented in (2a)-(2c) as the "restricted" model. This specification essentially assumes that  $x_t$  captures all changes in expected returns on the asset (since  $u_{t,1}$  is serially uncorrelated). The inclusion of lagged return as a predictive variable in (1) can capture autocorrelated components in expected returns that are not accounted for by  $x_t$ . We will refer to the more general VAR(1) model in (1) as the "unrestricted" model (although the assumed first-order autoregressive nature of the process clearly represents a restriction within a more general class of models).

Table 2 reports ordinary least-squares estimates of the parameters in the unrestricted VAR(1) model, and table 3 reports estimates of the parameters in the restricted version of the model. The estimation is based on monthly data for the period from January 1927 through December 1985 (708 observations). We do not, in the present draft, conduct tests of the adequacy of these VAR(1) models for representing the behavior of the monthly time series used here. Our initial objective is to investigate the extent to which simple models can capture the behavior of returns for various horizons. If this basic approach appears to be useful, then future research should consider alternative timeseries models.

## 3. Implications for Expected Returns over Various Horizons

#### 3.1 An Initial Look

Although the models above are formulated in terms of one-month-ahead forecasts, the parameters of these models can be used to obtain implications about the behavior of expected returns for longer horizons. To illustrate this point, consider the expected return for a five-year horizon. The estimated parameters for the above models, reported in tables 1 through 3, can be used to obtain implied values for the coefficients in a regression of the five-year return  $\mathbf{r}_{t'60}$  on  $\mathbf{y}_{t-1}$ . Figure 1 plots the implied expected returns for five-year horizons (stated on a per-month basis) obtained from the unrestricted VAR(1) model. Also shown are the fitted expected returns obtained by regressing directly the five-year return  $\mathbf{r}_{t,60}$  on  $\mathbf{y}_{t-1}$ . The expected returns implied by the VAR(1) model coincide rather well with the expected returns estimated directly in the regression.

We discuss below several other ways in which the above framework yields implications about the behavior of expected long-horizon returns. In order that the reader not be burdened unnecessarily with algebraic manipulation, we simply present the results of our analyses graphically and omit the underlying formulas. All of the computations follow directly, however, from the estimated parameters of the VAR(1) models reported in tables 1 through 3.

#### 3.2 Autocorrelations of Returns

We consider next the first-order autocorrelation for returns over N-month horizons,  $\operatorname{corr}(r_{t,N}, r_{t-N,N})$ . The VAR models imply values for this autocorrelation. Fama and French (1987a) report the first-order autocorrelations of returns with horizons up to ten years. Figure 2 plots, for horizons of one month through ten years, the autocorrelations implied by both the restricted and the unrestricted versions of the VAR(1) model. Also

shown are the sample estimates of these autocorrelations. Both versions of the model imply properties for corr(r<sub>t,N</sub>, r<sub>t-N,N</sub>) that correspond to properties of the sample estimates reported in previous studies. For short horizons (N small), the implied autocorrelations are positive, and this implication is consistent with previous evidence indicating positive autocorrelation in short-horizon stock-market returns [e.g., Fama and Schwert (1977) and Lo and MacKinlay (1987)]. Given that the unrestricted VAR(1) model includes the lagged one-month return as a predictive variable, it is not surprising that this model yields an implied autocorrelation for one-month returns that closely resembles the sample estimates. More noteworthy is the fact that the restricted version of the model, which does not include lagged return as a predictive variable, also implies positive autocorrelation for short horizons.

The autocorrelations of long-run returns that are implied by the VAR(1) models also exhibit properties similar to those documented by previous studies. As the return horizon increases, the implied first-order autocorrelations decrease up to horizons of approximately three to four years. In the restricted model, for example, the implied autocorrelation begins at 0.03 for 1-month returns, becomes negative at a 4-month horizon, declines to -0.28 at a 44-month horizon, and then moves back toward zero for longer horizons. Although the implied long-horizon autocorrelation does not reach as low a value as the actual sample autocorrelation, the overall pattern of the implied autocorrelations is similar to that of the sample autocorrelations.

None of the values shown are adjusted for finite-sample bias. Finite sample bias is present in the sample autocorrelations as well as in the

These estimates are obtained using overlapping data in the same manner as Fama and French (1987a), and the estimates are not adjusted for small-sample bias.

estimates of the model parameters reported in tables 2 and 3. It may be the case that such biases have different effects on the implied autocorrelations and the sample autocorrelations, so comparisons based on figure 2 should be viewed cautiously. We plan to incorporate the effects of finite-sample bias in future work.

### 3.3 Regression R-squared

Another striking characteristic of the results reported by previous studies is the behavior of the R-squared in regressions of returns on predetermined variables. Although the R-squared statistic is generally viewed as, at best, a useful descriptive statistic, the magnitudes obtained in recent studies using long-horizon returns have been objects of significant attention by researchers in finance. We demonstrate here that the simple VAR(1) models introduced above yield implications about R-squared that are similar to the characteristics observed by previous studies.

Regressions of monthly stock returns on predetermined variables generally produce R-squared less than 0.05 [e.g, Fama and Schwert (1977) and Keim and Stambaugh (1986)]. This characteristic is shared by the above VAR(1) models, in that the R-squared values in the monthly-return regression equations are approximately 0.03 in the unrestricted version and 0.02 in the restricted version. Fama and French (1987b, 1987c) report regressions of returns on predetermined variables with return horizons from one month to four years, and the R-squared values in their regressions increase with return horizon. For

There are exceptions to this statement. Campbell (1987) reports R-squared values between 0.11 and 0.23 for regressions of monthly excess stock returns on variables related to short-term interest-rates. Keim and Stambaugh (1986) find R-squared values in excess of 0.30 for regressions of monthly excess stock returns on predetermined variables when the regressions are limited to January observations.

example, Fama and French (1987c) report that, when excess returns on the equally weighted NYSE portfolio are regressed on  $(D/P)_{t}$  and  $(y_{Aaa} - y_{TB})_{t}$ , the R-squared is 0.01 for a return horizon of one month but the R-squared is 0.34 for a four-year return horizon.

Figure 3 displays the value of the "true" R-squared implied by the VAR(1) models for regressions of returns on the four predetermined variables in  $\mathbf{x}_{t-1}$ . That is, in the regression

$$r_{t,N} = \alpha_{0N} + \alpha'_{N} x_{t-1} + u_{t,N}$$
 (3)

the R-squared is the value of  $var(\alpha_N^x x_{t-1})/var(r_{t,N})$  implied by the VAR(1) model. This R-squared value is computed for return horizons (N) ranging from one month to four years. Also shown in figure 3 is the actual (unadjusted) sample R-squared value obtained by regressing  $r_{t,N}$  on  $x_{t-1}$  using overlapping observations.

The R-squared values implied by both versions of the VAR(1) model exhibit properties similar to those of the sample values. For example, the values implied by the restricted version of the model begin at 0.02 for a 1-month horizon and increase to 0.19 for a 32-month horizon. As in the analysis of autocorrelations, the sample R-squared value is not adjusted for any finite sample bias and, therefore, provides an upward biased estimate of the true R-squared value, due primarily to the autocorrelation in the residuals caused by the use of overlapping observations. In future work we plan to account for this effect and provide a more appropriate comparison of the implied values and the actual values.

 $<sup>^{5}</sup>$  The R-squared values correspond to those computed by Fama and French (1987c).

## 3.4 Impulse Response Functions

One potential benefit of the VAR model is that it permits a deeper analysis of the manner in which changes in the predetermined variables impact expected future returns for various horizons. One framework that appears to be especially useful in this analysis is that proposed by Sims (1980, 1981). This approach computes the response of a given variable, in this case  $r_{t+n,1}$ , to a set of orthogonal shocks in each variable in the system.

The VAR model allows  $r_{t+n.1}$  (the return in month t+n) to be written as

$$r_{t+n,1} = \mu + \sum_{j=0}^{\infty} \theta'_{j} \xi_{t+n-j}$$
 (4)

where  $\xi_s$  is a vector containing five elements, with  $\mathrm{E}(\xi_s)=0$ ,  $\mathrm{var}(\xi_s)=\mathrm{I}$ , and  $\mathrm{cov}(\xi_s,\,\xi_{s-j})=0$  for all  $j\neq 0$ . The elements of  $\xi_t$  are constructed as follows. The first element  $\xi_{1t}$  is the shock in period t to the first variable in  $\mathbf{x}_t$ . The second element  $\xi_{2t}$  is the shock to the second variable in  $\mathbf{x}_t$  that is uncorrelated with  $\xi_{1t}$ ; the third element  $\xi_{3t}$  is the shock to the third variable that is uncorrelated with  $\xi_{1t}$  and  $\xi_{2t}$ , etc. Thus, the construction of  $\xi_t$  depends on the ordering of variables. We order the variables as in the vector  $\mathbf{y}_t$  defined above, so that the fifth element of  $\xi_t$  is the shock to  $\mathbf{r}_t$ , that is uncorrelated with shocks to any of the variables in  $\mathbf{x}_t$ .

The response of  $r_{t+n,1}$  to the shocks (or "impulses") in the variables in period t is represented by the parameter vector  $\boldsymbol{\theta}_n$ . That is,  $\boldsymbol{\theta}_{1n}$  is the response of  $r_{t+n,1}$  to a one-standard-deviation shock at period t in the first variable,  $\boldsymbol{\theta}_{2n}$  is the response to the orthogonalized one-standard-deviation shock in the second variable, etc. The values of  $\boldsymbol{\theta}_{1n}$  for various n represent the "impulse response function," the responses of returns in various future

months to a one-standard-deviation (orthogonalized) shock to variable i at time t.

Figure 4 plots the responses of monthly returns to shocks in each of the four variables in  $x_t$  in the restricted version of the VAR(1) model. Responses of  $r_{t+n,1}$  to shocks in the second, third, and fourth variables  $[(y_{Baa} - y_{Aaa})_t, (y_{Aaa} - y_{TB})_t, \text{ and } (D/P)_t] \text{ are positive for all n.}$  The responses to shocks in the two yield spreads increase for several months and then decline monotonically toward zero. The responses to shocks in the dividend-price ratio decline more slowly and in a nearly monotonic fashion. The responses of  $r_{t+n,1}$  to shocks in the first variable,  $\Delta y_{Baa,t}$ , are negative for small n and positive for larger n.

## 3.5 Decomposing the Autocorrelation

The infinite moving average representation in (4) can also be used to analyze the contribution of each of the predictive variables to the autocorrelation of  $r_{t,N}$ , the return for an N-month horizon. First note that, since

$$r_{t,N} = \sum_{j=0}^{N-1} r_{t+j,1}$$
 (5)

the N-horizon return can be written as

$$r_{t,N} - N\mu + \sum_{j=1}^{\infty} \gamma'_{j} \xi_{t+N-j}$$
 (6)

Equation (6) can be rewritten as

$$r_{t,N} - N\mu + \sum_{i=1}^{5} \zeta_{i,t+N-1}$$
 (7)

where

$$\varsigma_{i,t+N-1} = \sum_{j=1}^{\infty} \gamma_{i,j} \xi_{i,t+N-j}$$
 (8)

Since the elements of  $\xi_s$  are mutually uncorrelated by construction, equation (7) expresses the N-month return as a sum of five orthogonal components. By decomposing the return in this fashion, the autocovariance of  $r_{t,N}$  can be similarly decomposed:

$$cov(r_{t,N}, r_{t-N,N}) = \sum_{i=1}^{5} cov(r_{t,N}, s_{i,t-1})$$
 (9)

Each term on the right-hand side of equation (9) represents the portion of the autocovariance of  $r_{t,N}$  that can be attributed to the covariance between  $r_{t,N}$  and past (orthogonalized) shocks to a given variable in the system. Dividing each side of (9) by  $var(r_{t,N})$  gives the autocorrelation as a sum of five components.

Figure 5 displays, for various return horizons (N), the five components of the autocorrelation in  $\mathbf{r}_{t,N}$ . The solid line represents the autocorrelation of  $\mathbf{r}_{t,N}$  implied by the unrestricted VAR(1) model, which was displayed previously in figure 1. The other five curves in figure 5 sum to this total autocorrelation. Perhaps the most striking result of this exercise is that shocks to the dividend-price ratio appear to make the most important contribution to both the positive autocorrelation in short-horizon returns as well as to the negative autocorrelation for the longer horizons.

## 4. Conclusions and Directions for Future Research

Vector autoregressive models for monthly stock returns and four additional monthly time series of financial variables contain implications about the behavior of expected returns for longer horizons. Estimates of the parameters of such models using monthly data imply values of statistics for long-horizon returns that appear to correspond reasonably well to actual sample values of those statistics. The statistics examined here include the first-order autocorrelation on long-horizon returns and the R-squared value obtained in regressions of long-horizon returns on a set of predictive variables. The framework analyzed here also allows the researcher to understand better the manner in which shocks to various financial variables impact expected returns for future periods.

Several directions for future research are suggested by this preliminary investigation. Expected returns on other assets can be investigated within this framework. For example, previous studies report evidence indicating that the type of predictive variables used here can also predict returns on bonds of various default risks and maturities for both short horizons [Keim and Stambaugh (1986)] and long horizons [Fama and French (1987c)]. It would be useful to know whether the models considered in this study can also capture the behavior of expected bond returns for various horizons.

If the investigation of additional assets suggests that a parsimonious model can describe expected returns of various assets for both short and long horizons, then additional cross-sectional analyses could be pursued. For example, such a model could allow a cross-sectional pricing restriction on expected returns for a given horizon to be used to obtain implications about cross-sectional relations for expected returns over other horizons.

The first-order autoregression used here to model the behavior of the

monthly time series should be viewed as tentative. We use these models because they provide a simple framework in which to investigate the relations among expected returns for various horizons. Alternative time-series models, although possibly more complex, might improve the ability of this overall approach to capture the behavior of long-horizon expected returns.

We have confined our attention in this study to variables that are functions of prices and returns on financial assets. The framework used here could also be expanded to include macroeconomic variables. For example, Litterman and Weiss (1985) apply the impulse-response framework to analyze responses of various macroeconomic variables to shocks in the nominal interest rate. It would be interesting to investigate whether shocks in the financial variables used in this study appear to impact future macroeconomic variables in patterns similar to the ways in which they impact future expected returns.

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Table 1
Sample Means, Standard Deviations, and Correlations

			Correlations							
Variable	Mean	Standard Deviation	y <sub>Baa</sub> -y <sub>Aaa</sub>	y <sub>Aaa</sub> -y <sub>TB</sub>	D/P	r				
Δy <sub>Baa</sub>	0.0007	0.0203	0.00	-0.25	0.17	-0.42				
y <sub>Baa</sub> -y <sub>Aaa</sub>	0.1007	0.0676		0.58	0.08	0.01				
y <sub>Aaa</sub> -y <sub>TB</sub>	0.1839	0.1077			-0.04	0.09				
D/P	0.0402	0.0133				-0.20				
r	0.0104	0.0753								

Note: The variables are defined as follows.

 $\Delta y_{Baa,t}$ : the change from the end of month t-1 to the end of month t in Moody's average yield on bonds rated Baa.

 $(y_{Baa} - y_{Aaa})_{t}$ : the difference at the end of month t between Moody's average yield on bonds rated Baa and bonds rated Aaa.

(yAaa - yTB)t : the difference at the end of month t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

 $(D/P)_{t}$ : for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

 $\mathbf{r}_{\mathsf{t}}$  : the return in month t on the equally weighted portfolio of NYSE stocks.

Table 2
Estimates of the Unrestricted VAR(1) Model's Parameters.

Dependent Variable		Independent variables (lagged one month)b							Residual Autocorrelations					
	Intercept		y <sub>Baa</sub> -y <sub>Aaa</sub>		D/P	r	adj. R <sup>2</sup>	x <sup>2(c)</sup>	P <sub>1</sub>	ρ <sub>2</sub>	P <sub>3</sub>	ρ4	P <sub>5</sub>	<sup>6</sup> 6
<sup>ly</sup> Baa	0.0101	0.1798 (0.0874)	-0.0185 (0.0363)	-0.0262 (0.0120)	-0.0478 (0.0515)	-0.0916 (0.0195)	0.24	47.1 (0.00)	0.07	-0.02	-0.25	-0.09	0.14	0.09
Baa <sup>-y</sup> Aaa	0.0009	0.0739 (0.0702)	0.9645 (0.0306)	0.0120 (0.0088)	0.0319 (0.0434)	-0.0760 (0.0169)	0.96	4040.9 (0.00)	0.04	-0.07	-0.26	-0.12	0.12	0.09
'Aaa <sup>-y</sup> TB	0.0081	-0.0704 (0.1292)	0.1332 (0.0395)	0.8704 (0.0345)	0.0825 (0.0810)	-0.0641 (0.0223)	0.85	3427.7 (0.00)	-0.10	0.05	-0.06	-0.02	-0.04	0.09
)/P	0.0024	0.0119 (0.0127)	-0.0077 (0.0054)	-0.0004 (0.0015)	0.9623 (0.0128)	-0.0028 (0.0028)	0.93	9145.1 (0.00)	0.04	-0.01	-0.07	0.03	0.13	0.05
r	-0.0188 (0.0135	-0.1132 ) (0.2770)	0.0876 (0.1076)	0.0206 (0.0331)	0.3770 (0.2494)	0.1506 (0.0769)	0.03	11.1 (0.05)	0.00	0.02	-0.08	-0.01	0.07	-0.02

The variables are defined as follows.

ΔyBaa,t : the change from the end of month t-1 to the end of month t in Moody's average yield on bonds rated Baa.

(yBan - yAan); the difference at the end of month t between Moody's average yield on bonds rated Ban and bonds rated Ann.

(yAaa - yTB) : the difference at the end of month t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

(D/P) : for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

 $\mathbf{r}_{\mathbf{t}}$  : the return in month t on the equally weighted portfolio of NYSE stocks.

bThe coefficients are estimated using ordinary least squares and the standard errors (in parentheses) are based on the heteroskedasticity-consistent estimator of the covariance matrix of White (1980) and Hsieh (1983).

The statistic reported is asymptotically distributed as  $\chi^2$  with five degrees of freedom under the null hypothesis that all of the coefficients on the independent variables (excluding the intercept) are equal to zero. The p-value is shown in parentheses.

Table 3

Estimates of the Restricted VAR(1) Model's Parameters.

ependent Variable	Independent variables (lagged one month) <sup>b</sup>						Residual Autocorrelations						
ASLISDIA	Intercept		yBaa-yAaa		D/P	adj. R <sup>2</sup>	x <sup>2(c)</sup>	<b>P</b> 1	ρ2	P <sub>3</sub>	ρ4	P 5	P <sub>6</sub>
by <sub>Baa</sub>	0.0062	0.3193 (0.0915)	-0.0241 (0.0403)	-0.0230 (0.0124)	0.0229 (0.0595)	0.15	33.5 (0.00)	0.04	-0.01	-0.29	-0.07	0.10	0.08
y <sub>Baa</sub> -y <sub>Aaa</sub>	-0.0024 (0.0031)	0.1895 (0.0756)	0.9598 (0.0341)	0.0147 (0.0094)	0.0905 (0.0490)	0.95	2503.7 (0.00)	0.08	-0.03	-0.30	-0.13	0.07	0.09
y <sub>Aaa</sub> -y <sub>TB</sub>	0.0054 (0.0058)	0.0272 (0.1233)	0.1293 (0.0393)	0.8726 (0.0346)	0.1320 (0.0843)	0.84	3182.1 (0.00)	-0.10	0.06	-0.06	-0.02	-0.04	0.09
D/P	0.0023	0.0161 (0.0125)	-0.0079 (0.0054)	-0.0003 (0.0015)	0.9644 (0.0131)	0.93	7921.8 (0.00)	0.08	-0.02	-0.07	0.02	0.13	0.05
r	-0.0124 (0.0129)	-0.3425 (0.2603)	0.0969 (0.1146)	0.0152 (0.0342)	0.2607 (0.2566)	0.02	7.1 (0.13)	0.13	0.01	-0.08	-0.02	0.06	-0.02

Coefficients (and standard errors) in the auxiliary regression of the estimated residuals for the first-stage return  $(r_t)$  regression on the estimated residuals from the other four first-stage regressions :

-0.1866 -0.2932 0.0211 -18.7863 (0.1056) (0.1828) (0.0199) (0.7773)

Estimated residual variance of the auxiliary regression: 0.00106

The variables are defined as follows.

ΔyBaa,t : the change from the end of month t-1 to the end of month t in Moody's average yield on bonds rated Baa.

<sup>(</sup>y<sub>Baa</sub> - y<sub>Aaa</sub>)<sub>t</sub> : the difference at the end of month t between Moody's average yield on bonds rated Baa and bonds rated Aaa.

<sup>(</sup>yAaa - yTB)t : the difference at the end of month t between the Aaa yield and the yield on a U.S. Treasury Bill with maturity closest to one month.

<sup>(</sup>D/P) : for the equally weighted portfolio of NYSE stocks, the ratio of dividends paid for the twelve months ending at t to the price at the end of month t.

 $<sup>\</sup>mathbf{r}_{\mathbf{t}}$  : the return in month t on the equally weighted portfolio of NYSE stocks.

bThe coefficients are estimated using ordinary least squares and the standard errors (in parentheses) are based on the heteroakedasticity-consistent estimator of the covariance matrix of White (1980) and Hsieh (1983).

The statistic reported is asymptotically distributed as  $\chi^2$  with four degrees of freedom under the null hypothesis that all of the coefficients on the independent variables (excluding the intercept) are equal to zero. The p-value is shown in parentheses.

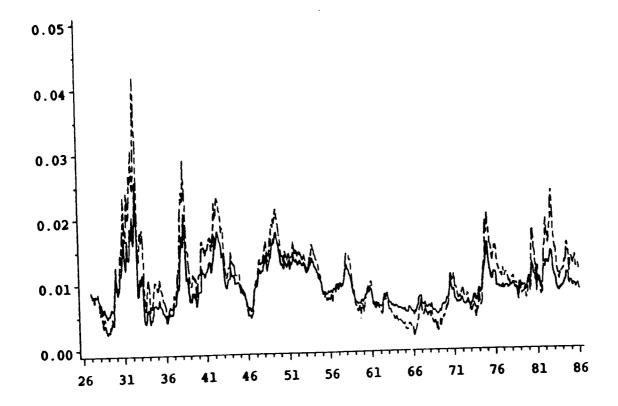


Figure 1. Estimated expected returns on the equally-weighted NYSE portfolio for five-year horizons. The value plotted corresponds to the expected monthly return for the five-year horizon beginning on the given date. The solid line represents expected five-year returns implied by the unrestricted VAR(1) model, and the dashed line represents expected returns estimated directly in a regression with five-year returns as the dependent variable.

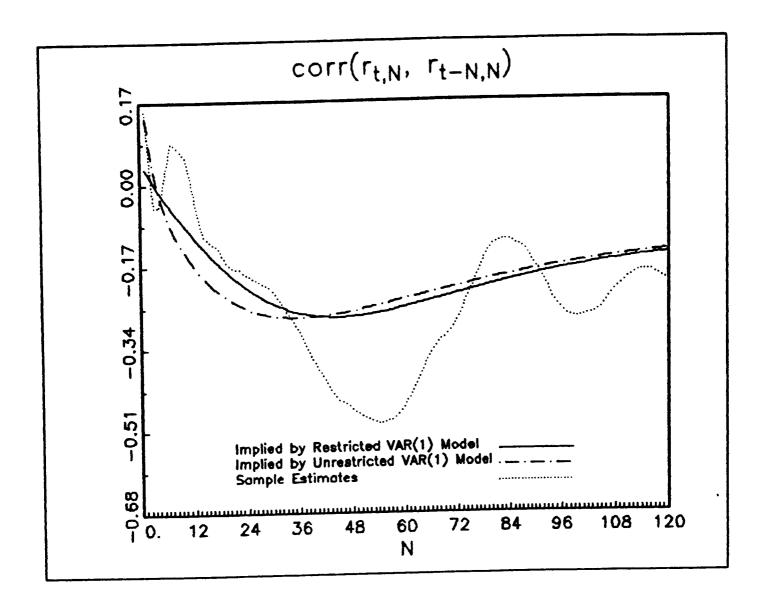


Figure 2. First-order autocorrelations of N-month returns on the equally weighted NYSE portfolio implied by the restricted VAR(1) model (solid line) and the unrestricted VAR(1) model (dashed line). The dotted line displays sample estimates obtained by regressing the N-month return on its lagged value, using monthly observations with overlapping return horizons.

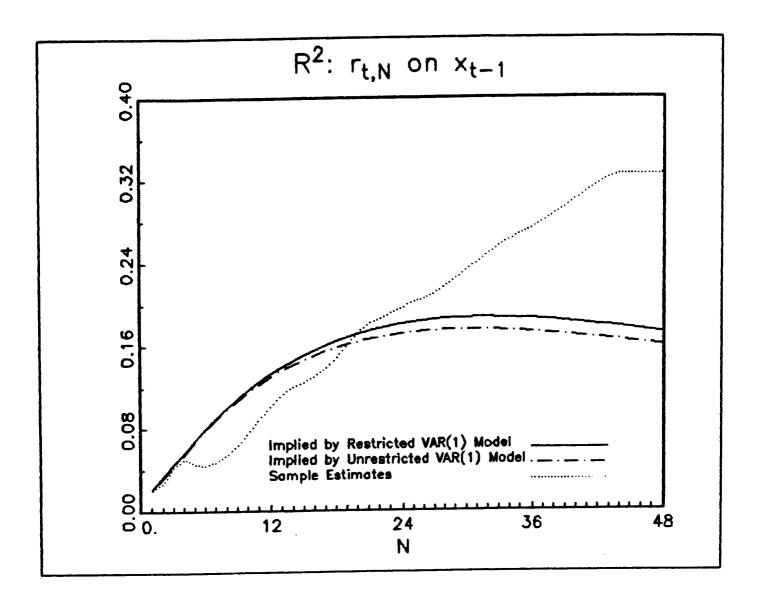


Figure 3. R-squared values in regressions of N-month returns on the equally weighted NYSE portfolio on four predictive variables (the change in the Baa yield, the Baa yield minus the Aaa yield, the Aaa yield minus the T-Bill yield, and the dividend-price ratio). The values implied by the VAR(1) models are equal to the ratio of the implied variance of the expected N-month return to the implied variance of the total N-month return. The sample value is obtained in a regression of the N-month return on the four variables, using monthly observations with overlapping return horizons.

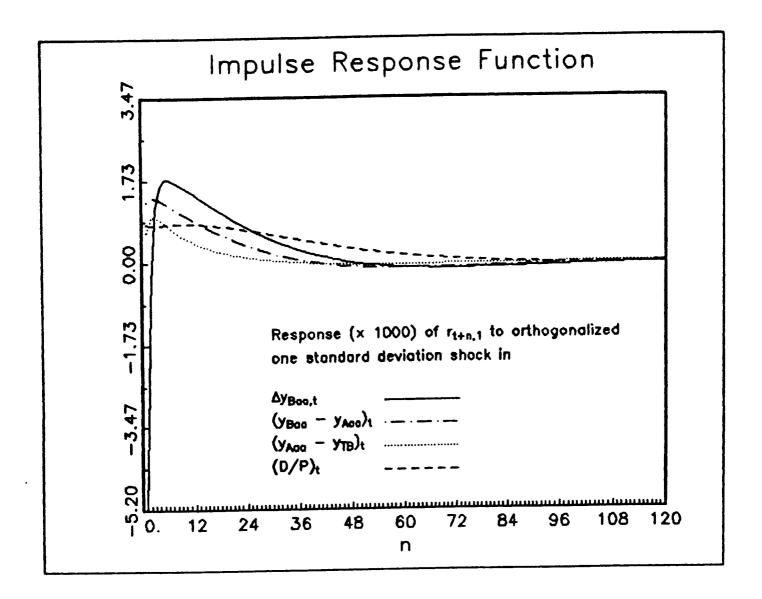


Figure 4. Responses of one-month returns on the equally weighted NYSE portfolio in month t+n to shocks in the predictive variables in month t, as implied by the restricted VAR(1) model.

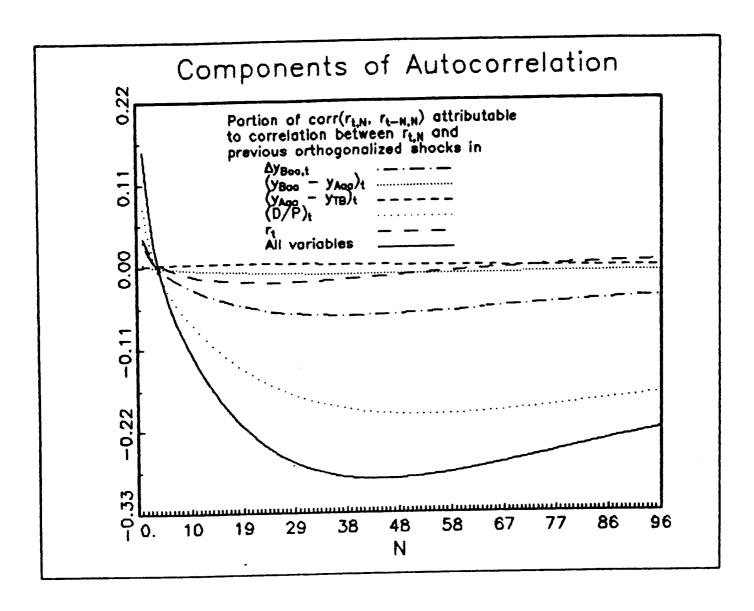


Figure 5. Components of the first-order autocorrelation of N-month returns on the equally weighted NYSE portfolio, as implied by the unrestricted VAR(1) model.