BOND PRICING

Goals:

1. To describe coupon paying and zero-coupon bonds.

2. To explain bond price / yield relationship.

3. Highlight factors affecting bond prices.

Buzz Words:

Coupon rate, principal, zero-coupon bonds, reinvestment rate.

Coupon Paying Bonds

Price a 3-year, 5 percent coupon paying bond.

Step 1: Determine the cash flows:

Coupon payments = c * principal

$50 $50 $50

$1000=face value

Step 2: Determine the discount rate.

Step 3: Find the PV of future cash flows.

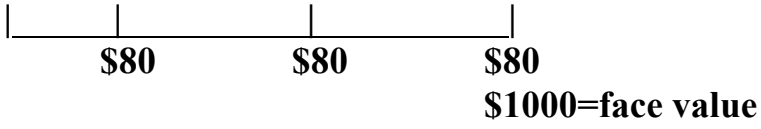
$$\textbf{Price} = 50/(1+r) + 50/(1+r)^2 + 50/(1+r)^3 + 1000/(1+r)^3$$

Price = PV of coupon payments + PV of principal.

$$\textbf{Price} = 50/(1+.05) + 50/(1+.05)^2 + 50/(1+.05)^3 + 1000/(1+.05)^3$$

Price = $1000.

Price a 3-year, 8 percent coupon-paying bond.

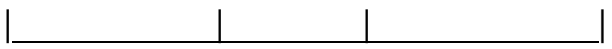


Price = 80 /(1+.05) + 80/(1+.05)^2 + 80/(1+.05)^3 + 1000/(1+.05)^3

Price = $1081.70

Which bond would you buy, the 5% coupon-paying or the 8 % coupon-paying bond?

Target Accumulation



50----------52.5----------55.125

50----------52.5

50

1000

(1000) **1157.625**

Megacept 2

1. Payments made by different bonds at the same point in time should be discounted at the same rate.

2. Payments made by the same bond at different points in time may be discounted at different rates.

Bond Price / Yield relationship

Consider a 20-year 10% coupon bond with a par value of $1000. Coupon payments are made semi-annually. Required yield on this bond is 11%. Determine the maximum price you are willing to pay for this bond.

Step 1: Cash flows are 40 semiannual coupon payments of $50.
$1000 to be received 40 six-month periods from now.

Step 2: Discount rate is 11% annual, 5.5% semiannual.

Step 3: Present value calculation.

Bond price = PV of coupon payments + PV of principal.

PV of coupon payments:

$$\$50\ [(1/.055) - (1/.055)(1/(1.055)^{40}) = \$802.31$$

PV of principal:

$$\$1000\ (1/(1.055)^{40}) = \$117.46$$

Bond price = 802.31 + 117.46 = 919.77

Now consider, instead of a 11%, the required yield is 6.8%.

Bond price = 1084.51 + 262.53 = $1347.04

Now consider the required yield to be equal to the coupon rate 10%.

Bond price = 857.95 + 142.05 = $1000.

Remember, when the required yield is assumed to be 11%,

Bond price = $919.77

Megacept 3

Fundamental property of a bond is that its price changes in the opposite direction from the change in the required yield. As required yields increase, PV of future cash flows decrease, hence, bond prices decline.

What does the shape of a price-yield relation of a non-callable bond look like?

This shape is referred as CONVEX.

2. Relationship between coupon rate, required yield and price.

Bond sells at par;

Bond sells at a discount:

Bond sells at a premium.

Crucial assumptions we made (so far) in Pricing Bonds

1. Coupon payments are exactly six months apart (or annually).

2. The cash flows are known.

3. The required yield is known.

4. One rate is used to discount all cash flows.

Are these assumptions realistic?

Why do bond prices change?

A. Required yields may change

Required yield = risk-free rate + risk premium

a. Risk - free rate may change

b. Required risk premium may change. Credit quality of the issuing firm may change.

B. Passage of time.

BOND YIELDS

Goals:

1. Describe yield to maturity (YTM).

2. Demonstrate the weaknesses of YTM in comparing bonds.

Buzz Words:

Yield-to-maturity, redemption yield.

Yield to Maturity

The yield to maturity of a bond is the expected rate of return to be earned per year over the life of the bond, if the bond is purchased today and hold till maturity.

The yield to maturity of a bond is that discount rate which equates the present value of future cash flows (coupon and principal) with the current price.

Using second definition, the YTM solves the following equation:

Bond price $= C_1/(1+y) + C_2/(1+y)^2 + \ldots\ldots + (C_t + P)/(1+y)^t$

where C_t **= coupon in period t**
P **= principal payment**
y **= yield to maturity**

Sometimes yield to maturity is called the internal rate of return or the redemption yield.

Application

Consider a three-year bond with an annual coupon of 10% per year. The face value of the bond is $100. The current price for this bond is $98.

To calculate the yield to maturity for this bond, we need to solve the following equation for y:

$$98 = 10/(1+y) + 10/(1+y)^2 + 110/(1+y)^3$$

You will discover that if you set y equal to 0.1082 (or 10.82%) in the above equation the equality holds.

Comparing Three Bonds

Consider three 10-year bonds. All bonds mature on the same date. All bonds pay annual coupons at the same point in time.

The coupons and current prices for the three bonds are:

BOND	COUPON	PRICE
A	8	100.00
B	6	87.60
C	4	74.40

Compare these bonds using yield to maturity.

Bond Comparisons Based on Yield to Maturity

BOND	COUPON	PRICE	YTM(%)
A	8	100.00	8.00
B	6	87.60	7.83
C	4	74.40	7.78

What is the appropriate reinvestment rate for the coupons?

Yields Are Not Reliable Guides for Investment Decisions

Year	d_t	Bond A	Bond B
0	1.00	-116	-108
1	.95	10	8
2	.91	10	8
3	.87	10	108
4	.80	10	-
5	.73	110	-
YTM		6.18%	5.06%

Calculation of the YTM for Bond B

$$-108 + 8/(1+y) + 8/(1+y)^2 + 108/(1+y)^3 = 0$$

Calculation of the YTM for Bond A

$$-116 + 10/(1+y) + 10/(1+y)^2 + 10/(1+y)^3 + 10/(1+y)^4 + 110/(1+y)^5 = 0$$

Calculation of the net value for Bond B:

$$-108 + 8(.95) + 8(.91) + 108(.87) = +0.84.$$

Calculation of the net value for Bond A:

$$-116 + 10(.95) + 10(.91) + 10(.87) + 10(.80) + 110(.73) = -0.40.$$

Recall our MEGACEPT 3

1. Payments made by different bonds at the same point in time should be discounted at the same rate.

2. Payments made by the same bond at different points in time may be discounted at different rates.

Precisely the opposite is true with yields to maturity.

Summary of the Problems Associated with Yields to Maturity

1. Difficult to calculate. This is especially true since

a. multiple solutions may exist or

b. no solution may exist.

2. Yield on a portfolio of bonds need not equal the average of the yields on the individual bonds in the portfolio.

3. Yields to maturity are not good guides for determining if a bond is under or over valued. (For example, the yield on a correctly priced bond need not correspond to actual spot rates of interest.)

4. You need to know the current value of the bond before you can calculate the yield. However, you need the value to be output and not input.

Term Structure of Interest Rates.

Goals:

1. Describe spot rates and forward rates.

2. Build a yield curve and highlight its uses.

Buzz words:

Spot rates, forward rate, and yield curve.

Spot Rates

Actual rate of interest at time t on an n-period loan. Spot rates are YTMs of zero-coupon bonds.

	Price	1	2	3
Bond1	**1036.36**	**1140**		
Bond2	**1016.68**		**1140**	
Bond3	**1000.00**			**1140**

Calculate the YTMs of these bonds:

YTM of Bond1 $1036.36 = 1140/(1 + y_1)$

YTM of Bond2 $1016.68 = 1140/(1 + y_2)^2$

YTM of Bond3 $1000.00 = 1140/(1 + y_3)^3$

YTM1 =

YTM2 =

YTM3 =

Zero Coupon Bonds

A zero-coupon bond maturing at time t is a bond that pays its face value at that time and no coupon prior. These are traded with names such as "zeros'," "CATS," "TIGRs," and "STRIPS."

These zero-coupon bonds sell at substantial discounts from their face value, with the discount representing interest earned on the investment through its life.

In August 1982, due to demand for zero-coupon instruments, Merrill Lynch and Solomon Brothers created synthetic zero-coupon Treasury Receipts.

CATS : Certificate of Accrual on Treasury Securities (Merrill Lynch)

TIGRs : Treasury Income Growth Receipts (Solomon Brothers)

LIONs , TBRs, ETRs, GATORs, COUGARs, and DOGS (Dibs on Government Securities).

Coupon Stripping

1. Purchase Treasury bonds and deposit them in a bank custody account.

2. Issue receipts representing an ownership interest in each coupon payment on the underlying Treasury bond in the account and a receipt for ownership of the underlying Treasury bond's maturity value.

This is called *coupon stripping*. Note that U.S. government does not issue the receipts.

A common problem was that settlement required physical delivery, which was cumbersome.

In February 1985, the Treasury announced the STRIPS program to facilitate the stripping of designated Treasury securities.

STRIPS : Separate Trading of Registered Interest and Principal of Securities.

These STRIPS are direct obligations of the U.S. government and these securities clear through the Federal Reserve's book-entry system. Creation of STRIPS ended the origination of coupon stripping by investment banking firms.

Treasury STRIPS Quotation

January 31, 1996

	Bid	Ask	Ask Yld.
Feb, 97	**95:05**	**95:06**	**4.83**

Bid price; price you will receive if you have this bond and wish to sell.

Ask price: price you will have to pay if you wish to purchase this price.

Spread: The difference between ask and bid price.

Colons in bid and ask prices represent 32nds. 101:01 means $101 1/32.

ci : stripped coupon interest.

np: Treasury note, stripped principal.

bp: Treasury bond, stripped principal.

Ask Yld.: YTM of the bond using ask prices.

Verification of Ask Yields in the WSJ

Ask price: 95:06 = 95 6/32 = $95.1875

Reported ask yield = 4.83%

Jan 31, 1996 **Feb 1997**

|_____|

$95.1875 **$100**

What is the YTM of this bond?

$95.1875 = 100/(1+y)

y = (100/95.1875) -1 = 5.056% vs. 4.83 ????

Ask Yield Formulation in the WSJ

$$P = \frac{100}{(1+\frac{r}{2})^{T+\frac{SN}{\mathrm{LN}}}}$$

$$r = 2[(\frac{100}{P})^{\frac{1}{T}+\frac{SN}{\mathrm{LN}}} \; 1]$$

T : the number of six-month periods from next coupon to maturity.

SN : the actual number of days from settlement to next coupon.

LN : the actual number of days from last coupon to next coupon.

|_____| 1 day |_____|_____.....|

last coupon trading day settlement day next coupon maturity

Yield Curve

r_1 = one year spot rate.

r_2 = two-year spot rate.

r_3 = three-year spot rate.

Calculation of spot rates from coupon paying bonds

Suppose the following bonds are available for investment:

	Bond Price	(1)	(2)	(3)
1 year Bond	**95.45**	**105**		
2 year Bond	**91.48**	**6**	**106**	
3 year Bond	**94.11**	**9**	**9**	**109**

a. What is the 1-year spot rate?

b. Given the 1-year rate in part (a) what is the 2 year rate?

c. Given the 1-year and 2-year spot rates in parts (a) and (b) what is the 3-year spot rate?

Forward Rates

Assume $r_1 = 5\%$ and $r_2 = 7\%$.

What is the total return over two years for a two year bond?

$$(1.07)^2 - 1 = 14.5\%$$

What is the extra return for lending for two periods rather than one?

$$(1.05)\,(1 + F) = (1.07)^2$$

$$F = 9.04\%$$

F represents the forward rate. i.e. forward rate is the marginal rate for extending the length of the loan's maturity.

Definition of a Forward Rate

The interest rate on money to be loaned at some future date when the contract for this future loan is made now is called the *forward rate*. This contract is termed as the *forward contract.* Even though the actual transaction will occur later, all the terms of a forward contract are specified in advance.

Calculation of Forward Rates

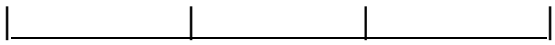
Forward rates are implied by today's spot rates structure.

WSJ - January 31, 1996

r_1 = 4.83

r_2 = 4.95

r_3 = 5.07.



Forward Contracts Revisited

The interest rate on money to be loaned at some future date when the contract for this future loan is made now is called the *forward rate*. This contract is termed as the *forward contract.* Even though the actual transaction will occur later, all the terms of a forward contract are specified in advance.

What if an explicit contract is not available?

An example of an Implicit Forward Contract

Assume that you will receive $1000 in year 1 from your rich uncle as a gift. When you receive the money, you plan to invest it in a one year bank CD.

Assume in year 0 you can borrow from the bank for one year at 25%, and the rate on a 2-year CD is 20%.

What is the implied forward rate?

You wish to lock in this rate. Would the bank write a forward contract?

Consider the following scenario as a way to lock in the interest rate on money that you wish to invest for one year, a year from now.

1. Borrow the present value of $1000 today and promise to repay $1000 in one year when the money becomes available.

2. Invest $800 in a 2-year CD, which offers an annualized rate of 20%. Note that since the amount of borrowing equals the amount of lending, today's cash flow is 0.

In one year you will receive (hopefully) the $1000. However, instead of investing the amount, you repay your loan. Hence, the net cash outflow in year 1 is $1000.

How much would you receive in year 2?

What rate of return have you earned on your $1000 investment?

Thus, individuals can create forward contracts even though there are no explicit contracts traded.