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Private Renegotiations and Government Interventions in Credit Chains

Vincent Glode
University of Pennsylvania, USA

Christian C. Opp
University of Rochester and National Bureau of Economic Research, USA

We propose a model of strategic renegotiation in which businesses are sequentially interconnected through their liabilities. This financing structure, which we refer to as a credit chain, gives rise to externalities, as each lender's willingness to provide concessions to its borrower depends on how this lender's own liabilities are expected to be renegotiated. We highlight how government interventions aimed at preventing default waves should account for private renegotiation incentives and interlinkages. In particular, we contrast the consequences of targeted subsidy and debt reduction programs following economic shocks, such as pandemics and financial crises. (*JEL* G21, G32, G33, G38)

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The COVID-19 pandemic imposed unprecedented hardships on businesses worldwide. During the second quarter of 2020, more than 20% of U.S. small businesses either permanently or temporarily shut down (see Casselman 2020). In response to these events, private parties and governments implemented measures aimed at preventing large-scale default waves. Whereas many businesses renegotiated their credit relationships (see Cherry et al. 2021), governments enacted policies providing subsidies (e.g., the CARES Act passed by the U.S. Congress) and amending private contracts (e.g., the Eviction Moratorium imposed by the Centers for Disease and Control [CDC]).[1]

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[1] See U.S. Treasury Department (2020) and CDC (2020) for more details about these policies.

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In this paper, we argue that the effectiveness of such private and public efforts is crucially influenced by the fact that businesses tend to be sequentially interconnected through their liabilities, a financing structure we refer to as a *credit chain*. For example, a small business like a restaurant might owe rent payments to its landlord. The landlord, in turn, might have a mortgage loan outstanding with a local credit union, which has financial obligations to a large national bank. Perhaps this large national bank is partly financed with bonds held by a pension fund that owes retirement benefits to workers, etc.

Practitioners involved in renegotiations recognize the importance of these interlinkages, as highlighted by the following depiction of commercial lease renegotiations in the midst of the COVID-19 pandemic:

> *I don't want to say that even a nice landlord or having a good relationship with the landlord, in my experience, has ensured a better lease deal (...) especially if the landlord has a mortgage on the property or is otherwise leveraged or otherwise has their own liquidity or cashflow issues. Some of those folks, their hands are just tied. They're like, 'We've talked to the bank, the bank is only going to give us as much. We can't give you the kind of discount you're asking for.' (...) a good relationship or a nice landlord, isn't even the end-all-be-all, it depends on what the surrounding situations are for that specific landlord.*[2]

Correspondingly, it is crucial that governments take these interlinkages into account when designing interventions. But as noted by the *Washington Post*, policy-making then becomes a complex undertaking:

> *The problem for the broader U.S. economy is that when businesses like Ross Stores and T.J. Maxx stop paying rent, it sets off an alarming chain reaction. Landlords are now at risk of bankruptcy, too. (...) and cash-strapped city and local governments are realizing the property taxes they usually rely on from business properties are unlikely to be paid this summer and fall. (...) Many small companies are asking landlords for a break, but commercial properties often have a complex chain of owners. (...) Lawmakers are trying to figure out how to prevent businesses — as well as their landlords — from going out of business, but government leaders are struggling to figure out how to help.*[3]

Motivated by these challenges, we develop a model of strategic renegotiation in credit chains. In this setting, each lender decides whether to reduce

[2] See Kludt (2020).

[3] See Long (2020).

the nominal liability of its borrower in order to lower the likelihood that inefficiencies associated with default are incurred. Our model accounts for two key frictions affecting renegotiation in practice. First, borrowers' ability to pay their liabilities is uncertain to lenders at the time of renegotiation. This uncertainty from a lender's perspective can arise because of the arrival of new information between renegotiation and payment dates or because of information asymmetries at a given point in time (for related evidence, see Chava and Roberts 2008; Adelino, Gerardi, and Willen 2013; Roberts 2015). Second, bargaining between a borrower and its lender is bilateral, giving rise to the possibility that an agent's bargaining power impedes efficient renegotiation not only in one credit relationship but also in the whole chain (for related evidence, see Chava and Roberts 2008; Roberts and Sufi 2009; Denis and Wang 2014).

Our analysis reveals how private renegotiation decisions are interrelated in a credit chain: a lender's willingness to provide concessions to its borrower depends on its own liabilities and how they are expected to be renegotiated (for related evidence, see Murfin 2012; Chodorow-Reich and Falato 2022). Going back to our earlier example, suppose a credit union has lent to a landlord who in turn is owed a payment by a restaurant. Receiving a concession from the credit union makes the landlord more likely to stay afloat when the restaurant does not make its rent payment. Anticipating this lower probability of using its limited liability, the landlord expects to internalize more of the losses incurred when the restaurant defaults on its payment. This increased exposure to losses can, in turn, make it optimal for the landlord to be more lenient with the restaurant in renegotiations. On the other hand, a business whose liability is not renegotiated by its lender and that remains deeply indebted may find it in turn suboptimal to reduce its borrower's liabilities. When its mortgage loan from the credit union remains large, the landlord is more likely to default on it when the restaurant fails to make its promised payment. The possibility of these types of "knock-on defaults" in our model implies that the costs of a borrower's default are not fully internalized by the lender, who is protected by limited liability. Instead, part of the losses are internalized by the lender's lender, or by agents even further "upstream" in the chain. A tough but socially inefficient renegotiation strategy may therefore be privately optimal for an indebted lender, imposing negative externalities on renegotiation efforts elsewhere in the chain. In particular, an unaccommodating renegotiation strategy in a given credit relationship may trigger tough renegotiations and higher default risk among "downstream" agents (whose debt payments are expected to flow upstream).

We then analyze potential government interventions aimed at reducing default losses in the economy. As an initial benchmark, we study stylized mechanism design problems that abstract from real-world bilateral renegotiation frictions by stipulating a centralized mechanism for renegotiation. Next, we consider policies that are constrained by the decentralized nature of renegotiations in practice. In particular, we analyze how targeted government interventions that were prevalent during recent economic crises affect, and potentially complement, private renegotiation efforts throughout a credit chain.

First, we show how subsidies targeting downstream borrowers like the restaurant can be particularly effective in eliminating default waves (see, e.g., the 2020 CARES Act passed by the U.S. Congress). A subsidy does not need to fully cover the potential balance-sheet shortfall of its recipient to prevent its default. Less may be needed because a subsidy to a downstream borrower also increases the stakes for the lender, making it more beneficial for the lender to make a lenient renegotiation offer and avoid default. That is, government subsidies and private renegotiation tend to act as complements. Moreover, providing subsidies to a downstream borrower like the restaurant may strengthen *all* upstream lenders' incentives to renegotiate their borrowers' liabilities and reduce default risk throughout the whole chain. A subsidy to the restaurant can first lead the landlord, then the local credit union, and then the large national bank to more efficiently renegotiate with their respective borrowers. As a result of the interlinkages of credit-chain members' optimal renegotiation decisions, awarding a subsidy to a downstream borrower can by itself prevent default waves whereas awarding the same subsidy to an upstream borrower would not.

However, the effectiveness of such subsidies generally depends also on their magnitude and the distributions of borrowers' assets. A generic feature of credit chains is that agents' asset value distributions can be multimodal and feature discrete jumps because of the possibility of various downstream defaults on debt claims. As a result, if a subsidy to a borrower is relatively small, a lender may find it optimal to adjust the renegotiated face value one-for-one with any subsidies paid to the borrower. Subsidies cause a borrower's asset value distribution to be shifted to the right, but in the presence of jumps in this distribution and associated local corner solutions, the optimal renegotiation offer may also move by exactly that amount. As a result of these endogenous adjustments in renegotiation offers, a borrower's default risk may not be affected by a subsidy at all. Our results show that private renegotiation is an important factor determining the magnitudes of government subsidies needed to prevent a borrower's default, and reveal under which economic conditions subsidies can be effective.

Second, we show how government interventions affecting the allocation of bargaining power in private renegotiations can help prevent default waves (see, e.g., the 2020 Eviction Moratorium imposed by the CDC). In particular, forcing an upstream agent to be lenient with its borrower can incentivize downstream agents to voluntarily renegotiate their respective borrowers' liabilities as well. For example, reducing how much the local credit union owes to the large national bank may first lead the credit union, and then the landlord to more efficiently renegotiate with their respective borrowers. If poorly designed, this type of intervention can, however, also backfire. For instance, mandating a reduction of the restaurant's rent owed to its landlord would reduce how much the credit union could collect from efficiently renegotiating the landlord's mortgage. Such an intervention could thereby result in the credit union toughening its renegotiation strategy with the landlord and increasing default risk in the credit chain.

A key friction impeding efficient renegotiation in our environment is that, at the time of renegotiation, each lender does not have all the information that its borrower will use at the time of its payment/default decision. If a lender had perfect foresight when renegotiating, it would be suboptimal to ask for more (or less) than what its borrower can actually pay and inefficient default would never occur. In contrast, if uncertain about its borrower's future ability to pay its debt, a lender faces a generic trade-off when renegotiating with its borrower. On the one hand, lowering how much the borrower owes increases the probability of repayment and reduces expected default losses. On the other hand, doing so also reduces the payoff to the lender whenever the borrower happens to be able to fully make its payment. The uncertainty a lender faces about its borrower's financial condition as well as expectations about renegotiation outcomes elsewhere in the chain determine a lender's renegotiation trade-off. How much the landlord knows about the restaurant's ability to pay its debt and whether it expects its own loan to be renegotiated by the credit union jointly determine how the landlord will decide to renegotiate the restaurant's liabilities.

Finally, as a third policy experiment, we examine how the timing of the renegotiation process can be an important determinant of inefficiencies, shedding light on policy-makers' efforts to affect this margin (see, e.g., the 2009 Home Affordable Modification Program run by the U.S. Treasury). In particular, we show how inefficient default can be fully eliminated only if renegotiation occurs before agents have obtained all information they use to make default decisions. As a result, government policies facilitating *early* renegotiation following a large shock can be helpful in facilitating more efficient renegotiation throughout a credit chain.[4]

Our paper sheds light on renegotiation decisions in credit chains and how they are affected by government interventions. We contribute to the existing literature on renegotiation that abstracts from credit chains and the associated externalities of each renegotiation decision. Riddiough and Wyatt (1994a) study the dynamic decision whether to reorganize a single distressed firm, and Riddiough and Wyatt (1994b) study the reputational effects of renegotiation when a lender has several loans that mature sequentially. Bolton and Scharfstein (1996) show how creditor dispersion

[4] More generally, the benefits of early renegotiation uncovered by our analysis shed light on the fact that renegotiation indeed tends to occur early in the life of most loans (see Roberts and Sufi 2009).

can impede the efficient renegotiation of debt (see also He and Xiong 2012; Brunnermeier and Oehmke 2013; Zhong 2021; Donaldson et al. 2022). Gârleanu and Zwiebel (2009) analyze the design and renegotiation of debt covenants, showing that adverse selection problems lead to the allocation of greater ex ante decision rights to the creditor.

Our paper is related to models of sequential strategic interactions in financial and nonfinancial markets. In an unpublished working paper, Kiyotaki and Moore (1997) analyze how economic shocks propagate in a supply chain context in which term credit is provided. They show that postponing unpaid debt may be bilaterally efficient, but socially worse than liquidation since postponement does not inject liquidity in the supply chain. In contrast, our analysis reveals how unaccommodating renegotiation in one credit relationship can lead to tougher renegotiation in downstream credit relationships, and how prevalent government interventions that target specific borrowers can complement private renegotiation incentives. Di Maggio and Tahbaz-Salehi (2015) study sequential lending relationships, but unlike us, they focus on the use of collateral in origination decisions, rather than on debt renegotiation (see also Park and Kahn 2019). They show how the allocation of collateral affects an intermediation chain's ability to shepherd liquidity toward a good investment opportunity. Relatedly, shedding light on the benefits of intermediation chains, Glode and Opp (2016) show that trading through moderately informed intermediaries can improve the efficiency of asset allocations in over-the-counter markets. Doepke and Schneider (2017) highlight the benefits of a dominant unit of account (e.g., a specific currency) when agents can be both suppliers and customers in sequential, bilateral interactions subject to random matching. Our focus on chains of bilateral renegotiations also differentiates our paper from the sequential principals literature, where multiple principals deal sequentially with a single agent (see, e.g., Bizer and DeMarzo 1992; Kahn and Mookherjee 1998).

Our paper contributes to the theoretical literature studying the effects of debt and limited liability on firm decisions. This literature shows how outstanding debt can affect firms' incentives to invest (see Myers 1977), take risks (see Jensen and Meckling 1976; Inderst and Mueller 2008), and charge high prices for their products (see Brander and Lewis 1986). The existing literature also highlights the role of specific types of debt contracts (e.g., demand deposits or junior debt) and the number of debt holders in providing commitment for either tough or weak renegotiations with other claimants of the same firm. For example, in Diamond and Rajan (2001), issuing demand deposits to multiple unskilled lenders commits a bank not to use its special asset liquidation skills as a bargaining chip to extract concessions in renegotiations with its lenders.[5]

[5] Their model also features a second commitment problem, but this problem is addressed by the bank's special liquidation skills. Without this skill, a bank's borrower would potentially threaten to quit its project at an interim stage to extract more surplus.

In contrast, in Perotti and Spier (1993), junior debt creating a debt overhang problem is used as a bargaining tool to force tougher renegotiation of senior claims (in particular, liabilities to risk-averse workers).[6] In our setting, a firm's debt is not used as a commitment device in negotiations with other claimants of the same firm. Rather, our analysis of credit chains shows how credit interlinkages affect renegotiations and socially inefficient defaults, in particular when lenders do not have all the information that their borrowers will be able to use when deciding whether to default. In a credit chain, an agent's indebtedness has implications for how it renegotiates with its *borrower* (asset side of the balance sheet) rather than with other lenders or claimants (liability side of the balance sheet). Providing concessions to a struggling borrower reduces the probability that this borrower defaults. Because of limited liability, the social benefits of renegotiation are, however, not fully internalized by a lender at risk of defaulting on its own debt.

Finally, our analysis complements insights from the literature on cascades and contagion in financial networks, which abstracts from the strategic renegotiation of liabilities. We show how tough renegotiations in upstream credit relationships can promote tough renegotiations in downstream credit relationships, contrasting with a typical default cascade which propagates from the final borrower's balance sheet to that of the initial lender (i.e., from downstream to upstream agents). Allen and Gale (2000), Elliott, Golub, and Jackson (2014), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) study different channels through which small economic shocks can spread and expand through networks of firms connected by their financial obligations. Allen, Babus, and Carletti (2012) study the interaction between asset commonality and funding maturity in generating this type of contagion. Babus and Hu (2017) study how agents' incentives to default on their financial obligations can be weakened by a star network, in which a central intermediary keeps track of all agents' behavior. Taschereau-Dumouchel (2022) studies how firms' failure to produce inputs can lead to a cascade of firm shutdowns.

1. The Environment

In this section, we introduce our model of private renegotiations in credit chains.

Agents and asset endowments We consider an environment with $N \geq 3$ agents. At date $t=1$, each agent j owns an endowment asset that takes a random value v_j at date $t=2$. Based on the public information available at $t=1$, the cumulative distribution function (CDF) of v_j is denoted as $F_j(v_j)$ and its probability density function (PDF) is denoted as $f_j(v_j)$. We write the lower

[6] For related empirical evidence, see Matsa (2010).

and upper bounds of the support of v_j as $\underline{v}_j$ and $\overline{v}_j$, respectively. Asset values v_j are assumed to be independently distributed across agents as of date $t=1$ to reflect the notion that agents face heterogeneous financial conditions, yet it is still possible that aggregate shocks occurred prior to $t=1$ and shaped the distributions $F_j(v_j)$ as of $t=1$ (we revisit this specification in Section 4). For example, a large aggregate shock like COVID-19 hitting before $t=1$ would have caused all distributions $F_j(v_j)$ to be shifted to the left. Yet, conditional on that aggregate shock, the remaining idiosyncratic uncertainty about each agent's future asset values is captured by the distributions $F_j(v_j)$. All agents value future payments using a discount rate of zero.

The endowment asset value realizations in excess of their lower bounds, $(v_j - \underline{v}_j)$, are observable at $t=2$, but not verifiable, in the sense of Grossman and Hart (1986). Subject to the constraint that its assets have a verifiable component $\underline{v}_j$, an agent j can underreport its value realization at $t=2$ in order to reduce payments to security holders and divert the additional residual value. A special feature of debt contracts in this environment is that they provide security holders with a foreclosure right, that is, they allow the lender to seize its borrower's assets in case of default. This feature of debt contracts mitigates borrowers' strategic incentives to underreport their asset value realizations and provides a rationale for the prevalence of these contracts in our environment (see also Bolton and Scharfstein 1990; Bolton and Scharfstein 1996; Hart and Moore 1998).

Existing liabilities The N agents in the economy are linked through existing debt obligations (e.g., personal or commercial loans, accounts payable to suppliers, or rent payments owed to landlords). In particular, at $t=1$, each agent $j \geq 2$ owes agent $(j-1)$ a payment $\overline{d}_j$ that is due at $t=2$. This credit chain structure succinctly captures crucial asymmetries across agents based on their relative positions (upstream vs. downstream) in a network of credit relationships (we discuss alternative network structures in Section 4).

We consider a setting in which the initial face values $\overline{d}_j$ specified in each debt contract were chosen at a prior date (e.g., at an unmodeled date $t=0$) based on the information available at that time. Our paper's focus on the renegotiation of existing contracts (rather than the process of establishing a liability in the first place) is motivated by the relevance of such phenomena after an economy is hit by a large negative shock, such as the recent worldwide pandemic, which was essentially unanticipated prior to the end of 2019.[7]

[7] For example, Adam Silver, the commissioner of the National Basketball Association (which generates over \$8B of worldwide revenues per year) explained the need to renegotiate the league's collective bargaining agreement (CBA) with the players in the midst of the COVID-19 pandemic as "This CBA was not built for an extended pandemic (...) There's not a mechanism in it that works to properly set the cap when you've got so much uncertainty, when our revenue could be \$10 billion or it could be \$6 billion. Or less" (Wojnarowski 2020).

Debt contract settlement and default costs Debt contracts are sequentially settled at $t=2$ starting with agent N's liability, then agent $(N-1)$'s liability, and so on until we reach agent 2's liability to agent 1.[8] If at $t=2$ agent j defaults on the payment of its (potentially renegotiated) face value d_j, its lender (i.e., agent $(j-1)$) seizes the assets that agent j owns, which generally consist of the endowment asset with value v_j at $t=2$ and the funds agent j collected from agent $(j+1)$. However, only a fraction $(1-\rho)$ of agent j's assets ends up being transferred to the lender, where $\rho\ (>0)$ captures proportional deadweight losses associated with liquidating productive assets, going through the bankruptcy process, and losing customers, employees, and suppliers.[9] These default costs, which are standard in the corporate finance literature, are the key source of surplus destruction in our model (see Section 4 for an alternative specification of default costs).

This specification of default costs implies that if two neighboring agents were to default, say agents 3 and 4, then agent 2 would collect only a fraction $(1-\rho)^2$ of agent 4's assets, that is, inefficiencies accumulate in the case of sequential defaults. This specification thus captures the notion that directly connected agents tend to be closer in terms of their business operations and expertise (e.g., in the context of a supply chain), and a lender is likely to be more efficient in extracting value from its direct borrower's assets in default. While we take agents' existing debt obligations as given in our analysis, these differences in asset redeployability provide a rationale for the initial formation of credit chains (before $t=1$) in a setting like ours: a directly connected agent has a competitive advantage in providing credit to a borrower, since it obtains higher recovery rates in default than other potential lenders would.

Private information and the timing of renegotiations At $t=1$, each agent j may obtain a private signal $s_j \in \Omega_s$ that is informative about the future realization of its endowment asset value v_j. The distribution $F_j(v_j|s_j)$ denotes the conditional CDF of v_j as perceived by agent j. The private signals ensure that we can meaningfully analyze how the timing of renegotiations (early vs. late) affects equilibrium outcomes. In particular, when considering the latest possible renegotiation, that is, the case in which dates $t=1$ (renegotiation) and $t=2$ (payment due) coincide, each agent is fully informed about its own date-2 endowment asset value at the time of renegotiation. In that case, if

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[8] This assumption increases the tractability of our model by ensuring that each agent j collects the realized value of its debt claim to agent $(j+1)$ (i.e., the face value payment or the recovery value) before deciding whether to default on what it owes to agent $(j-1)$. If contracts were settled in the reverse order, agents could not rely on payments from the debt claims they own to fulfill their financial obligations. In this case, a firm could consider issuing additional securities to bridge a temporary shortfall caused by the delayed settlement of the debt claim it owns. However, such issuance would generally involve a security design decision and associated signaling concerns that would complicate the model and obfuscate its main insights.

[9] Andrade and Kaplan (1998), Almeida and Philippon (2007), Korteweg (2010), Davydenko, Strebulaev, and Zhao (2012), Glover (2016), Greenwood, Iverson, and Thesmar (2020), and Dou et al. (2021) provide evidence of the magnitude of these costs.

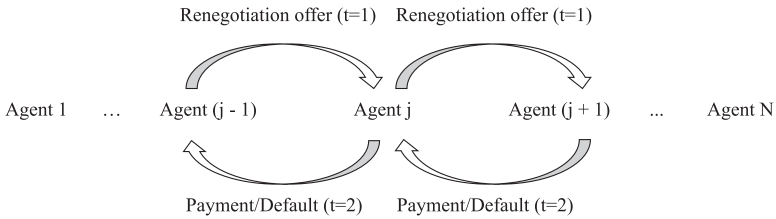

Figure 1
The figure illustrates a chain of renegotiation offers, payments, and default decisions.

that information were not private, the renegotiation problem would become degenerate, since a lender could simply set the new face value equal to the minimum of the original liability $\bar{d}_j$ and the realized assets of the borrower. However, since several of our main results do not focus on the timing of renegotiations, we abstract from private information (i.e., $F_j(v_j|s_j)=F_j(v_j)$) whenever it simplifies the exposition.

Renegotiation At $t=1$, agents can renegotiate their debt contracts. Specifically, agent $(j-1)$ chooses whether to lower the face value of agent j's debt to any level $d_j \le \bar{d}_j$. Formally, agent $(j-1)$ proposes a new face value through a take-it-or-leave-it offer to agent j. It is then a dominant strategy for agent j to accept any offered face value that is weakly lower than the initial face value $\bar{d}_j$. However, at $t=2$, agent j can use its limited liability and default on this renegotiated face value if it is privately optimal. Renegotiation offers and outcomes are not publicly observable at $t=1$. Figure 1 gives an overview of the environment by illustrating the chain of renegotiation offers, payments, and default decisions.

Since we show below that debt securities are optimal in our environment, it is without loss of generality to focus on renegotiations that adjust existing debt contracts rather than introducing other types of securities. Moreover, while renegotiation in our model pertains to adjusting the face value of debt, considering other contractual features and renegotiation margins that also change the *present value* of payments promised by a borrower (e.g., payment delays or adjustments to coupon payments) would yield similar economic insights.

Timeline In summary, the timeline of the model is as follows.

- Date $t=1$: Renegotiation
    - (i) Each agent j obtains a signal s_j that may be informative about its future endowment asset value v_j.
    - (ii) Each agent $j=1,...,(N-1)$ simultaneously makes a take-it-or-leave-it offer to its borrower $(j+1)$, specifying a new face value d_{j+1}.

(iii) Each agent $j=2,...,N$ decides whether to accept the newly proposed face value d_j.

- Date $t=2$: Payment/Default

(i) Each agent j observes its endowment asset value v_j.

(ii) Debt contracts are settled (through payment or default) sequentially, starting with the contract owed by agent N, then the contract owed by agent $(N-1)$, and so on.

Throughout the analysis, we characterize Perfect Bayesian equilibria of this game. Even in the presence of private information, signaling concerns do not arise in our environment. When making a renegotiation offer, a lender is not using private information that could be of use to its borrower. Moreover, since lenders make their offers simultaneously, each one of them makes decisions based on prior beliefs about the offer it will receive from its respective lender. In equilibrium, these beliefs need to be consistent with actual offers.

2. Equilibrium Renegotiation and Default

In this section, we characterize agents' optimization problems as borrowers and as lenders in our most general environment and emphasize the generic trade-off associated with a lender's decision to marginally adjust its borrower's face value. We then derive conditions for the existence of default-free equilibria when payoff distributions $F_j(v_j)$ satisfy a standard regularity condition. Finally, we analyze the optimal renegotiation strategies and associated levels of default risk in a chain with $N=3$ agents for two types of endowment asset value distributions.

As a first step, we establish the optimality of debt contracts in our environment (proofs of our formal results are all relegated to the appendix).

Lemma 1. For any lender making a renegotiation offer it is optimal to propose a new debt contract with a face value $d_j \leq \overline{d}_j$.

Consistent with insights from the existing literature, debt is an optimal contract in our environment due to the nonverifiability of asset values and the foreclosure right that debt bestows. Borrowers would like to report the lowest possible asset value realizations (which is the only verifiable component), unless they face a debt contract, in which case doing so can trigger default and the seizure of their assets. Anticipating this strategic behavior by borrowers, lenders find it optimal to propose a new debt contract when making a renegotiation offer.[10]

[10] In practice, other frictions, such as adverse selection and moral hazard, may also contribute to the use of debt contracts (see, e.g., Nachman and Noe 1994; DeMarzo and Duffie 1999; Biais and Mariotti 2005; Hébert 2018; Yang 2020). More broadly, the literature has also shown circumstances under which other types of securities are optimal (see, e.g., Boot and Thakor 1993; Fulghieri and Lukin 2001; Inderst and Mueller 2006).

2.1 Renegotiation and equity values

Agents choose their renegotiation strategies to maximize their equity value at $t=1$. Each agent's equity value depends on the agent's location in the chain and on other agents' renegotiation strategies.

Agent N Agent N is special in that it does not hold a claim against any other agent in the chain. At $t=1$, it is a dominant strategy for agent N to accept any renegotiation offer below the preexisting face value, $d_N < \overline{d}_N$.

Agent (j – 1) Suppose agent $(j-1)$ expects to owe a given face value $\hat{d}_{j-1}$ to its lender and is considering whether to renegotiate agent j's debt contract by lowering its face value to $d_j < \overline{d}_j$. To forecast agent j's future wealth and default behavior, agent $(j-1)$ must conjecture how much agent j will collect from its credit relationship with agent $(j+1)$.[11] We write the *stochastic* transfer from agent $(j+1)$ to agent j as δ_{j+1}. We also write agent j's stochastic total-asset value as $a_j \equiv v_j + \delta_{j+1}$ and its associated CDF as $G_j(\cdot)$ and PDF as $g_j(\cdot)$.

When considering reducing its borrower's debt level to d_j, agent $(j-1)$ anticipates that agent j will make a full payment $\delta_j = d_j$ whenever:

$$d_j \leq a_j = v_j + \delta_{j+1}. \tag{1}$$

Thus, when agent $(j-1)$ proposes a new face value d_j at $t=1$, it is with the expectation that all date-2 borrower types with assets a_j greater or equal to d_j will be *included* by this offer, in the sense that they will not default on the new face value. On the other hand, all date-2 borrower types below d_j will be *excluded* in the sense that they will default at $t=2$ and, as a result, the lender will only collect $\delta_j = (1-\rho)a_j$. The deadweight losses introduced in the event of default help to capture the prevalent view that default waves are undesirable outcomes.[12] In our model, welfare decreases when agents find it optimal to take tough renegotiation stances and keep their borrowers' liabilities elevated at $t=1$, thereby triggering correspondingly higher default probabilities for these borrowers at $t=2$.

We can write agent $(j-1)$'s optimization problem as choosing a renegotiation offer d_j to maximize its expected equity value given its signal s_{j-1} and its

[11] Agent $(N-1)$ differs from agents 1 to $(N-2)$ in that its borrower, agent N, does not have a debt claim to another agent's assets. Everything we derive here follows if one simply sets $\overline{d}_{N+1} = 0$.

[12] Even when borrowers use their debt issuance proceeds to fund negative-NPV projects, renegotiating their liabilities at $t=1$ can be efficient, provided that their investment decisions were already made prior to $t=1$ and liquidation is not optimal at that point. Moreover, a default that appears to be efficient from a partial equilibrium perspective (i.e., for a given credit relationship) might, in bad economic times, trigger default waves elsewhere in the credit chain that are harmful to the whole economy. Overall, we view the deadweight losses from excessive defaults as a first-order concern after large economic shocks, such as the COVID-19 pandemic (see, e.g., Becker and Oehmke 2021).

own anticipated post-renegotiation debt level $\hat{d}_{j-1}$:

$$\begin{aligned}\Pi_{j-1}(d_j) &\equiv \mathbb{E}\left[(a_{j-1}-\hat{d}_{j-1})^{+} \,\middle|\, s_{j-1}\right]\\ &= \Pr[a_j \ge d_j]\cdot\mathbb{E}\left[(v_{j-1}+d_j-\hat{d}_{j-1})^{+} \,\middle|\, s_{j-1}\right]\\ &\quad + \Pr[a_j < d_j]\cdot\mathbb{E}\left[(v_{j-1}+(1-\rho)a_j-\hat{d}_{j-1})^{+} \,\middle|\, s_{j-1}, a_j < d_j\right], \end{aligned} \tag{2}$$

where we define the operator $(\cdot)^{+} \equiv \max\{\cdot, 0\}$. Equation (2) splits the lender's expected equity value at $t=1$ into two terms that respectively capture the possibility that its borrower makes a full payment d_j and the possibility that its borrower defaults. The maximum operators in Equation (2) reflect agent $(j-1)$'s own limited liability: whenever the agent's total payoff would be negative after paying off its own debt, it prefers to default and collect a payoff of zero. Moreover, we can see that agent $(j-1)$'s renegotiation offer generally depends on its private information about v_{j-1}, as represented by the signal s_{j-1}. The extent to which the agent anticipates using limited liability depends on its information about the future value of its endowment asset and on the expected renegotiation offer from agent $(j-2)$.

Agent 1 The first agent in the chain is special in that it does not owe a payment to another agent. Thus, the expected equity value from observing a signal s_1 and making a renegotiation offer d_2 simplifies to

$$\Pi_1(d_2) = \mathbb{E}[v_1 \mid s_1] + \Pr[a_2 \ge d_2]\cdot d_2 + \Pr[a_2 < d_2]\cdot(1-\rho)\mathbb{E}[a_2 \mid a_2 < d_2]. \tag{3}$$

2.2 Optimal renegotiation

To illustrate the trade-offs associated with renegotiations in a credit chain, consider a generic agent $(j-1)$ choosing its renegotiation offer to its borrower, agent j. The expected net benefit of marginally increasing the renegotiated face value $d_j < \overline{d}_j$ can be written as

$$\begin{aligned}\Pi'_{j-1}(d_j) = &\underbrace{[1-G_j(d_j)]}_{\text{Inframarginal types } j} \cdot \underbrace{[1-F_{j-1}(\hat{d}_{j-1}-d_j \mid s_{j-1})]}_{(j-1)\text{'s survival given inframarginal types } j} - \underbrace{g_j(d_j)}_{\text{Marginal type } j}\\ &\cdot \mathbb{E}\left[\min\left\{\underbrace{\rho d_j}_{\text{Losses if } j \text{ defaults}}, \underbrace{\left(v_{j-1}+d_j-\hat{d}_{j-1}\right)^{+}}_{(j-1)\text{'s equity value}}\right\} \,\middle|\, s_{j-1}\right]. \end{aligned} \tag{4}$$

The first term on the right-hand side of Equation (4) represents the lender's marginal benefit of being tougher with its borrower. The lender collects a higher face value from all inframarginal borrower types ($a_j > d_j$), provided that the lender itself survives, that is, if it pays its own renegotiated debt with expected face value $\hat{d}_{j-1}$. Thus, the lender's own default risk may limit the benefits from taking a tough stance with its borrower. The second term represents the lender's incremental cost from causing the marginal borrower type $a_j = d_j$ to default as a result of a tougher renegotiation stance. This cost is equal to the likelihood of facing the marginal borrower type $a_j = d_j$, written as $g_j(d_j)$, multiplied by the minimum of the total default costs, ρd_j, and the lender's own equity value when that type does not default, $(v_{j-1} + d_j - \hat{d}_{j-1})^+$. The minimum of these two quantities is the relevant object here since the lender internalizes the losses associated with its borrower defaulting only until its own equity value reaches zero. At that point, the borrower's default triggers a knock-on default by the lender, implying that incremental losses from the borrower's (i.e., agent j) default are now internalized by the lender's lender (i.e., agent $(j-2)$). This force is an essential part of our analysis of optimal renegotiation in credit chains: a lender's indebtedness limits the extent to which it internalizes deadweight losses from its borrower's default. In sum, a lender's indebtedness enters the marginal trade-off featured above on both the benefit and cost sides.

2.2.1 Discussion: The irregularity of lenders' optimization problems

When the marginal benefit of being tough is lower than its marginal cost, that is when $\Pi'_{j-1}(d_j) < 0$, agent $(j-1)$ finds it optimal to marginally decrease agent j's debt level d_j. While Equation (4) captures the generic marginal trade-off agent $(j-1)$ faces when renegotiating, the marginal net-benefit function $\Pi'_{j-1}(\cdot)$ is generally not well behaved in that it does not cross zero from above only once if any of the downstream agents $k = j+1, j+2, ..., N$ is at risk of defaulting in equilibrium. As such, marginal optimality conditions are generally not sufficient conditions for a global optimum. The reason for this analytical irregularity is the simple fact that, in a credit chain, a borrower's assets generically include a debt claim to another agent. When that debt claim features default risk, the borrower's total asset value distribution is the convolution of a potentially well-behaved endowment asset value distribution, and a distribution of the debt claim that is discontinuous (due to default). This generic feature leads to jumps and nonmonotonicities in a lender's marginal net-profit function $\Pi'_{j-1}(\cdot)$ and, correspondingly, to the possibility of multiple local optima. In contrast, when a credit chain is default-free along the equilibrium path, the debt held by a borrower is safe, and thus the randomness of a borrower's total assets is fully described by the distribution of its endowment asset value, which is well behaved under standard regularity assumptions. As a result, first-order conditions are sufficient conditions in that case.

In light of these issues, our remaining analysis of optimal renegotiation proceeds as follows. First, we maintain our general environment and characterize conditions on distributions and parameters for the existence of a default-free equilibrium. These conditions can be derived analytically for a wide range of distributions and they emphasize key insights of our analysis, including the forces that impede agents' willingness to renegotiate efficiently. Thereafter, we analyze equilibrium behavior in the presence of default on the equilibrium path. These analyses necessarily have to condition on specific distributions and consider three agents to limit the plethora of cases that may arise. In these analyses, we characterize the downstream pass-through effects of renegotiations in the case of trinomially distributed endowment asset values and provide an equilibrium solution for an example with uniformly distributed asset values. Given the specific distributions and the limited number of agents, key insights can be clearly illustrated in these settings. Nonetheless, these analyses vividly highlight the technical irregularities that emerge when a borrower's borrower is expected to default with positive probability in equilibrium.

2.3 General conditions for default-free equilibria

We now characterize conditions under which default-free equilibria exist in general environments (i.e., for any $N \geq 3$, a broad class of distributions, and the possibility of informative signals). For a default-free equilibrium to occur, all agents must find it optimal to renegotiate their debt contracts to levels that are low enough to guarantee full repayment by borrowers at $t=2$, rather than opting for tougher renegotiation strategies that result in positive default risk. Thus, the conditions we derive center on lenders' incentives to deviate to strategies that trigger defaults and highlight the role played by knock-on defaults. The analysis remains tractable since the above-described irregularities associated with default-state compounding do not arise in default-free equilibria.[13]

The term $\underline{d}_j$ denotes the level of debt that borrower j is guaranteed to be able to pay. Conditional on its information at $t=1$, we assume that an agent j's endowment asset delivers at least a value $\underline{v}_j > 0$ at $t=2$. In addition to its endowment asset value, agent j collects a transfer from its borrower, which in a conjectured default-free equilibrium must satisfy $\delta_{j+1} = d_{j+1}$ with probability 1. As a result, the total value of agent j's assets at $t=2$ is bounded from below by $(\underline{v}_j + d_{j+1})$. Note that this amount is not per se the lowest possible value of an agent's total assets; rather it is the lowest possible value conditional on its information at $t=1$ *and* on being in an equilibrium in which agents $(j+1)$ through N do not default. In contrast, if some default did occur on the equilibrium path among agents $(j+1)\ldots N$, then agent j would possibly end up having assets worth less than $(\underline{v}_j + d_{j+1})$, as we will emphasize later. The new

[13] Consistent with many papers in the economics and finance literature, we focus on equilibria that maximize total surplus (see, e.g., Grossman and Hart 1980; Albuquerque and Hopenhayn 2004; Acemoglu, Golosov, and Tsyvinski 2008; Aguiar, Amador, and Gopinath 2009). When it exists, a default-free equilibrium maximizes the social surplus in our environment.

face values proposed by lenders in a default-free equilibrium correspondingly satisfy the recursive relation:

$$\underline{d}_j \equiv \underline{v}_j + d_{j+1}, \tag{5}$$

provided that the initial face value $\overline{d}_j$ exceeds this value, that is, $\overline{d}_j \geq \underline{d}_j$ (otherwise, the face value remains at its initial level). Moreover, if $\overline{d}_j \geq \underline{d}_j$ for all j, the recursive relation (5) yields the explicit formulae:

$$\underline{d}_j = \sum_{i=j}^{N} \underline{v}_i. \tag{6}$$

Whereas Equation (6) indicates that the default-free renegotiated face values represent the accumulated lower bounds of the endowment asset values, higher renegotiated face values would apply if we introduced additional default costs that are internalized by the debtors, such as a reputation cost from defaulting (see Section 4 for details).[14]

Suppose that the density function $f_j(v_j)$ of each endowment asset takes strictly positive and finite values everywhere on the support $v_j \in [\underline{v}_j, \overline{v}_j]$. We now impose the standard regularity condition that the hazard rate $\frac{f_j(v_j)}{1-F_j(v_j)}$ is increasing on this support $[\underline{v}_j, \overline{v}_j]$. This condition ensures that (local) first-order conditions are sufficient for global optimality conditional on a lender anticipating that no default will occur outside of its own bilateral credit relationship with its borrower (in particular, no default by the borrower's borrower). Moreover, agent j obtains a signal $s_j \in \Omega_s = [\underline{s}_j, \overline{s}_j]$ at $t=1$ that implies that the conditional density of its endowment asset value at $t=2$ is given by $f_j(v_j|s_j)$. We assume that this conditional density takes finite values everywhere on the support $[\underline{v}_j, \overline{v}_j]$ for all possible signal realizations $s_j \in \Omega_s$. Further, let $f_j(v_j, s_j)$ denote the joint density of v_j and s_j. We can now present the conditions for the existence of a default-free (i.e., efficient) equilibrium.

Proposition 1. When the hazard rates $\frac{f_j(v_j)}{1-F_j(v_j)}$ are increasing on their respective support $[\underline{v}_j, \overline{v}_j]$, private renegotiation leads to a default-free credit chain on the equilibrium path whenever the following conditions hold:

$$\Pi_1'(\underline{d}_2) = 1 - f_2(\underline{v}_2) \cdot \rho \underline{d}_2 \leq 0, \tag{7}$$

and for $j=3,...,N$:

$$\Pi_{j-1}'(\underline{d}_j) = 1 - f_j(\underline{v}_j) \cdot \mathbb{E}\left[\min\left\{\rho \underline{d}_j\ ,\ v_{j-1} - \underline{v}_{j-1}\right\} \middle|\ s_{j-1}\right] \leq 0 \quad \forall s_{j-1} \in \Omega_s. \tag{8}$$

[14] Note that conceptually, the endowment assets in our setup represent firms' tangible and intangible assets *gross* of default costs and gross of any liabilities that a firm might have to suppliers, customers, debt holders, or landlords, etc. Whereas *equity* values can naturally turn zero in our model and in practice, this is not the case for these *asset* values. Empirically, even *net* of default costs, the combined recovery value for all agents holding the liabilities of a defaulting firm is virtually always positive (see, e.g., Dou et al. [2021] who estimate both the potential liquidation proceeds and the reorganization values of a large sample of bankrupt firms).

In contrast to our earlier derivations that allowed for default risk, the equilibrium conditions above are simplified by the fact that when a borrower's borrower (say agent $(j+1)$) does not default in equilibrium, the only uncertainty a lender (agent $(j-1)$) faces at the time of renegotiation relates to its own endowment asset value (i.e., v_{j-1}) and that of its borrower (i.e., v_j). Moreover, the regularity conditions for asset value distributions now ensure that first-order conditions are sufficient conditions for global optimality in each lender's problem.

Consistent with our discussion of the generic trade-off of renegotiation, conditions (7)–(8) subtract the marginal cost of following a tougher renegotiation strategy from the marginal benefit. Condition (8) now accounts for the fact that in a default-free equilibrium, each lender $(j-1)$ anticipates not to default after collecting $\underline{d}_j$ from its borrower, implying that the marginal benefit of increasing the borrower's face value above $\underline{d}_j$ and collecting that face value from all inframarginal borrowers is fully internalized by this lender. Moreover, the minimum operator in (8) highlights that the marginal cost of following a tougher renegotiation strategy is affected by the same two channels discussed earlier. First, default losses are incurred through a tough renegotiation strategy. Second, an indebted lender internalizes losses only when it has sufficient equity value to absorb them. Thus, the possibility of knock-on defaults limits the extent to which agents internalize the inefficiencies caused by their own tough renegotiation stances. In a default-free equilibrium, agent $(j-1)$'s equity value at $t=2$ is given by $(v_{j-1}-\underline{v}_{j-1})$ since its lender, agent $(j-2)$, chooses a new face value that allows agent $(j-1)$ to avoid default even if it is the lowest type $a_{j-1}=\underline{d}_{j-1}\equiv\underline{v}_{j-1}+\underline{d}_j$. As a result, when agent $(j-1)$'s actual type exceeds this lowest type, the equity value at $t=2$ is strictly positive. A higher equity value and the associated skin-in-the-game, in turn, discourage agent $(j-1)$ from choosing a tough renegotiation strategy for its own borrower, agent j.

2.3.1 Discussion: The role of private information The magnitude of this skin-in-the-game effect depends on the private signal s_{j-1} that agent $(j-1)$ obtains at $t=1$. The worse the signal, the less likely it is that agent $(j-1)$ will have an endowment asset worth more than the lower bound $\underline{v}_{j-1}$ at $t=2$. Thus, agent $(j-1)$ is less willing to renegotiate down its borrower's liabilities after receiving a bad interim signal about v_{j-1}. In Section 3, we further investigate how default risk in a credit chain is affected by policies affecting the timing of renegotiation, relative to the arrival of private information.

2.4 The downstream pass-through effect of debt renegotiation

In this subsection, we move beyond default-free equilibria to analytically characterize how the renegotiation offer an upstream agent is anticipated to make affects downstream agents' optimal renegotiation offers. To do so, we consider a particular setting that yields a tractable analysis of these externalities. Specifically, suppose that $N=3$ and that for each agent j the endowment asset takes values v_j in the set $\{0, v_j^L, v_j^H\}$. All agents assign the same probability $f_j(v_j)$ (now defined as a discrete probability) to a given realization v_j. For reasons that will become clear later, introducing the state $v_j = 0$ yields a setting in which default occurs with positive probability, yet analyzing lenders' optimal renegotiation problems remains relatively simple.

We study how the level of debt that agent 2 anticipates to owe to agent 1, $\hat{d}_2$, affects agent 2's willingness to renegotiate its borrower's debt $\overline{d}_3$. The following proposition summarizes the optimal renegotiation behavior of agent 2.

Proposition 2. In the trinomial case with $N = 3$ agents, $v_j \in \{0, v_j^L, v_j^H\}$, and no private information, if $\overline{d}_3 > v_3^L$, then agent 2 renegotiates agent 3's liabilities down to $d_3 = v_3^L$ whenever:

$$f_3(v_3^L) \cdot \mathbb{E}\left[\min\left\{\rho v_3^L, (v_2 + v_3^L - \hat{d}_2)^+\right\}\right]$$
$$\geq f_3(v_3^H) \cdot \mathbb{E}[(v_2 + \min\{\overline{d}_3, v_3^H\} - \hat{d}_2)^+ - (v_2 + v_3^L - \hat{d}_2)^+]; \tag{9}$$

otherwise, agent 2 sets $d_3 = \min\{\overline{d}_3, v_3^H\}$. If instead $\overline{d}_3 \leq v_3^L$, agent 2 never renegotiates agent 3's liabilities.

Proposition 2 identifies key forces determining how agent 2 optimally renegotiates with its borrower. Consistent with our interpretation of the marginal net benefit of renegotiation captured by Equation (4), the left-hand side of condition (9) represents the default losses internalized by agent 2 when agent 3 defaults on a high debt level. In contrast, the right-hand side represents the benefit of receiving a higher payment when agent 3 is not defaulting. Unlike the marginal debt-level adjustment featured in Equation (4), the difference in considered debt levels (and associated payments) is now a discrete amount: $\min\{\overline{d}_3, v_3^H\} - v_3^L$. Yet, the economic trade-off between the costs and benefits of renegotiation is fundamentally the same and echoed in all environments we analyze.

When condition (9) is satisfied and agent 2 is willing to reduce its borrower's liabilities, the probability that agent 3 defaults on its liabilities drops from $[f_3(0) + f_3(v_3^L)]$ to $f_3(0)$. The following corollary formalizes the impact of agent 2's own liabilities on its incentives to renegotiate its borrower's liabilities.

Corollary 1. In the trinomial case with $N = 3$ agents, $v_j \in \{0, v_j^L, v_j^H\}$, and no private information, marginally increasing agent 2's anticipated liabilities to agent 1 (i.e., $\hat{d}_2$) shrinks the set of parameters where agent 2 finds it optimal to renegotiate agent 3's liabilities to $d_3 = \min\{\overline{d}_3, v_3^H\}$ rather than to $d_3 = v_3^L$ whenever:

$$f_3(v_3^H) \cdot \Pr(\hat{d}_2 - \min\{\overline{d}_3, v_3^H\} \leq v_2 < \hat{d}_2 - v_3^L)$$
$$< f_3(v_3^L) \cdot \Pr(\hat{d}_2 - v_3^L \leq v_2 < \hat{d}_2 - (1-\rho) v_3^L). \tag{10}$$

The left-hand side of condition (10) is the probability that agent 3 could pay $d_3 = \min\{\overline{d}_3, v_3^H\}$ times the increase in agent 2's default probability when collecting $d_3 = v_3^L$ instead of $d_3 = \min\{\overline{d}_3, v_3^H\}$. The right-hand side is the probability that agent 3 could pay $d_3 = v_3^L$ but would default on $d_3 = \min\{\overline{d}_3, v_3^H\}$ times the increase in agent 2's default probability when seizing assets worth $(1-\rho)v_3^L$ instead of collecting the renegotiated level $d_3 = v_3^L$. Thus, the impact of an increase in $\hat{d}_2$ on agent 2's renegotiation incentives depends on how agent 2's default risk varies with the debt payment collected from agent 3. For example, if agent 2 is guaranteed to have enough assets to make its anticipated debt payment $\hat{d}_2$ whenever it collects at least v_3^L, then the left-hand side of condition (10) is equal to zero and an increase in $\hat{d}_2$ makes the renegotiation condition (9) weakly harder to satisfy. On the other hand, if agent 2 is guaranteed to default on its anticipated debt payment $\hat{d}_2$ whenever it collects v_3^L, then the right-hand side of condition (10) is equal to zero and an increase in $\hat{d}_2$ makes the renegotiation condition (9) weakly easier to satisfy.

These analytical results emphasize the downstream pass-through effects of debt renegotiations, that is, agent 1's anticipated renegotiation offer affects how agent 2 optimally renegotiates with agent 3. However, the trinomial setting does not lend itself to a tractable analysis of agent 1's optimal renegotiation decisions, since absent further parameter restrictions, a plethora of cases arises (because of all the potential combinations of defaults by agents 2 and 3). To shed light on agent 1's optimal behavior, we instead turn our attention to a case in which endowment asset values are parameterized to be uniformly distributed. This environment allows to fully characterize the equilibrium behavior of all agents, despite featuring the irregular optimization problems associated with downstream default risk.

2.5 Default risk of a borrower's borrower

We now consider a setting in which each agent's endowment asset value follows a uniform distribution $U[1,2]$. As in the trinomial case above, we set $N = 3$ and abstract from private information. For added tractability, we focus our attention on $\rho = 1$ (which will be relaxed later). Finally, we assume that the original face values $\overline{d}_2$ and $\overline{d}_3$ are sufficiently high to not constrain the optimal renegotiation offers d_2 and d_3. This analysis allows for a characterization of all agents, including agent 1 who must account for the default risk of its borrower as well as its borrower's borrower.

By backward induction, we first solve for agent 2's renegotiation strategy. Depending on the level of d_2 that agent 2 anticipates, a variety of cases can obtain, including interior solutions and corner solutions. If an interior global optimum obtains, that is, if $d_3 \in (1,2)$, the local condition $\Pi_2'(d_3) = 0$ holds. Otherwise, in case of a corner solution, one of the two conditions, $\Pi_2'(\underline{v}_3) < 0$ or $\Pi_2'(\overline{v}_3) > 0$, holds. Agent 2's expected net benefit from marginally changing

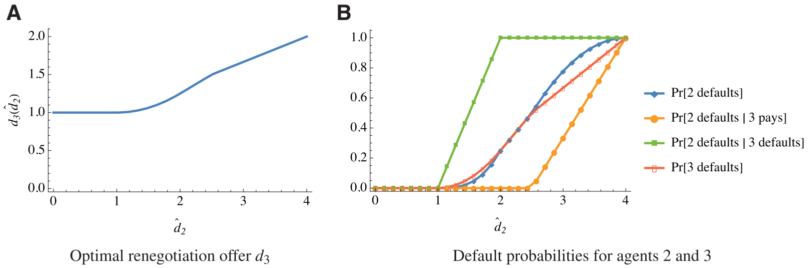

Figure 2
Panel A illustrates agent 2's optimal renegotiation offer d_3 as a function of agent 2's belief about how much it will owe to agent 1, denoted as $\hat{d}_2$. Panel B illustrates the associated default probabilities for agents 2 and 3, as well as agent 2's default probability conditional on whether or not agent 3 fully pays its debt. The figure illustrates the baseline case detailed in subsection 2.5, where agent 2's and agent 3's endowment asset values are each uniformly distributed on the interval $[1,2]$ and the default loss parameter is $\rho = 1$.

d_3 when anticipating to owe $\hat{d}_2$ is given by

$$\Pi_2'(d_3) = [1 - G_3(d_3)] \cdot [1 - F_2(\hat{d}_2 - d_3)] \\ - g_3(d_3) \cdot \ \mathbb{E}\left[\min\left\{d_3, \ \left(v_2 + d_3 - \hat{d}_2\right)^+\right\}\right], \tag{11}$$

which is a parameterized version of the general expression (4).

Agent 2's optimal renegotiation offer to agent 3 can then be expressed as a piece-wise-defined function of the anticipated debt level $\hat{d}_2$:

$$d_3(\hat{d}_2) = \begin{cases} 1 & 0 \leq \hat{d}_2 \leq 1 \\ \frac{1}{4}\left[2\hat{d}_2 + 1 + (\hat{d}_2 - 2)^2\right] & 1 < \hat{d}_2 \leq 2 \\ \frac{1}{4}(2\hat{d}_2 + 1) & 2 < \hat{d}_2 \leq \frac{5}{2} \\ \frac{1}{3}(\hat{d}_2 + 2) & \frac{5}{2} < \hat{d}_2 \leq 4. \end{cases} \tag{12}$$

Panel A of Figure 2 plots this solution and illustrates once again that if an upstream agent (here, agent 1) is expected to follow an unaccommodating renegotiation strategy with its borrower (i.e., a high renegotiation offer d_2 to agent 2), it can cause this borrower (agent 2) to take a tougher stance with its own borrower (agent 3). That is, agent 2 will also ask for a higher repayment d_3.

Panel B of Figure 2 plots the resultant probabilities with which agents 2 and 3 default. It also plots the probability with which agent 2 defaults, conditional on whether or not agent 3 makes a full debt payment. The graph helps clarify the economics underlying the various cases indicated in the piece-wise-defined solution (12), which depend on the renegotiated face value agent 2 anticipates to owe.

For $\hat{d}_2 \leq 1$, agent 2 faces so little leverage that it can never default: even after collecting nothing from agent 3, agent 2 has an endowment asset that is sufficient to always make the full debt payment. Thus, at the margin, agent 2

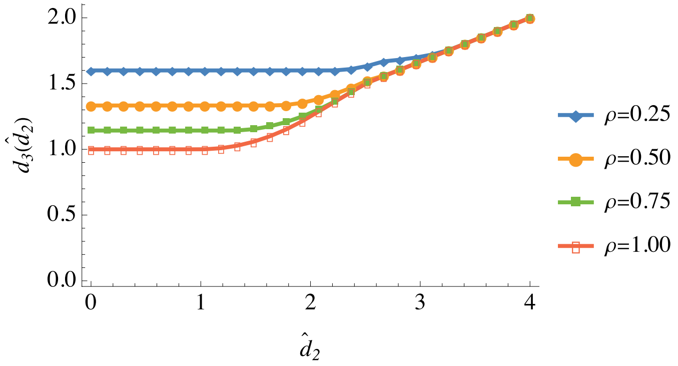

Figure 3
The figure illustrates agent 2's optimal renegotiation offer d_3 as a function of agent 2's belief about the offer it will obtain from agent 1, denoted as $\hat{d}_2$. We vary the default-cost parameter ρ as indicated in the legend. All other parameters are identical to the baseline case detailed in subsection 2.5, where agent 2's and agent 3's endowment asset values are each uniformly distributed on the interval $[1,2]$.

fully internalizes the costs and benefits associated with pursuing a tougher renegotiation strategy. Agent 2 then optimally pursues an accommodating renegotiation strategy by going to the corner solution $d_3 = \underline{v}_3 = 1$, which ensures that agent 3 never defaults. In contrast, for $\hat{d}_2 > 1$, agent 2 defaults with positive probability and finds it optimal to pick a renegotiated face value $d_3 > 1$, causing agent 3 to also default with positive probability. For $1 < \hat{d}_2 \leq 2.5$, agent 2 risks defaulting only as a result of agent 3 defaulting. In fact, for $\hat{d}_2 > 2$, agent 2 is forced to default whenever agent 3 defaults. Finally, for $\hat{d}_2 > 2.5$, agent 2 is so deeply indebted that it risks defaulting even after receiving a full payment from agent 3.

We show the robustness of these effects with respect to changes in the default-cost parameter ρ in Figure 3. As default becomes less costly (for lower levels of ρ), agent 2 becomes more aggressive and insists on higher renegotiated face values from agent 3. Yet, a key property of agent 2's behavior is maintained: agent 2 takes a tougher renegotiation stance with agent 3 when expecting to owe a higher level of debt d_2.

We now return to our baseline case with $\rho = 1$ and analyze agent 1's optimal renegotiation strategy. Note that agent 2 makes its renegotiation offer d_3 to agent 3 conditional on its belief $\hat{d}_2$ about the actual renegotiation offer d_2 it will receive from agent 1. Given this belief, agent 1 chooses the actual offer d_2. In equilibrium, agent 1's actual offer and agent 2's belief about it must coincide, that is, $\hat{d}_2 = d_2$.

Figure 4 plots four different functions that are of relevance for agent 1's optimal renegotiation strategy, for the case in which agent 2 anticipates that agent 1's renegotiation offer will be $\hat{d}_2 = 2.5$ (consistent with the equilibrium in this example). When agent 1 decides how to renegotiate agent 2's debt, it trades off the upside of reducing the probability of costly default with the

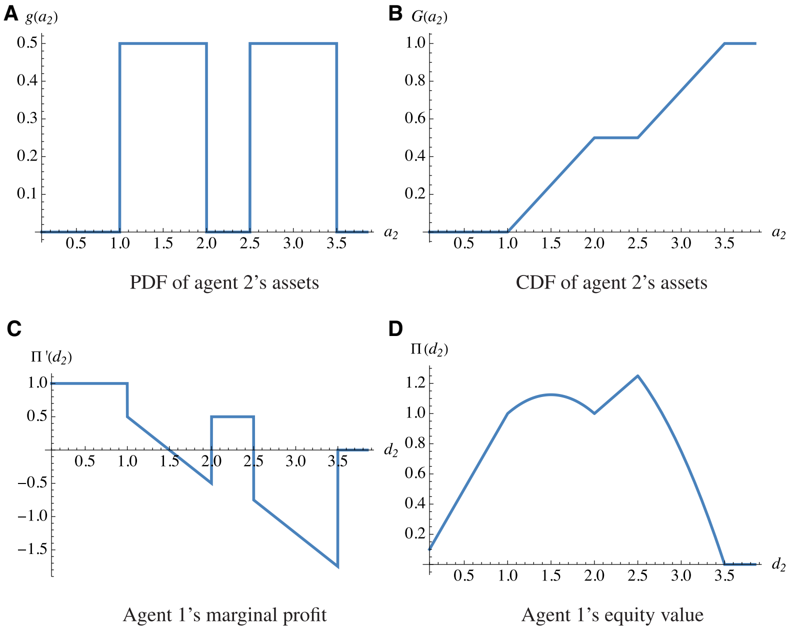

Figure 4
Panels A and B plot the PDF and CDF of the total value of agent 2's assets, denoted as $a_2 = v_2 + \delta_3$. Panels C and D plot agent 1's marginal profit function and expected profit from renegotiating agent 2's debt to a face value of d_2, conditional on agent 2 anticipating an offer $\hat{d}_2 = 2.5$. The optimal face value set by agent 1 is indeed $d_2 = 2.5$, which is a corner solution. The figure illustrates the baseline case detailed in subsection 2.5, where agent 2's and agent 3's endowment asset values are each uniformly distributed on the interval $[1,2]$ and the default loss parameter is $\rho = 1$.

downside of collecting a lower amount in case of full payment. This trade-off crucially depends on the distribution of agent 2's total asset value. Panels A and B therefore plot the PDF and CDF of a_2, which consists of agent 2's endowment asset value v_2 and the post-renegotiation stochastic debt payment δ_3 collected from agent 3. At the margin, agent 1's trade-off is then again characterized by the marginal profit function (4), which now simplifies to

$$\Pi_1'(d_2) = [1 - G_2(d_2)] - g_2(d_2) \cdot d_2, \tag{13}$$

since agent 1 is not indebted. In the existing literature on bilateral bargaining, agents' asset value distributions are typically assumed to be exogenous and to satisfy regularity conditions (e.g., monotone hazard rate conditions) that ensure that marginal profit functions are weakly decreasing. However, in the economic environment we consider, agents' asset value distributions are endogenous (because of renegotiation) and generically multimodal (because of potential downstream defaults). As a result, marginal profit functions are not well behaved in that they do not cross zero only once from above, as illustrated in Panel C. This generic irregularity leads to multiple local optima, as shown in Panel D, which plots agent 1's expected profit as a function of its choice of the renegotiated face value for agent 2.

In the considered setting, agent 1's renegotiation problem features a corner solution. Agent 1's optimal renegotiation offer to agent 2 is $d_2 = 2.5$ and, expecting this offer, agent 2's optimal renegotiation offer to agent 3 is $d_3 = 1.5$, consistent with the solution described in (12). Thus, a default by agent 3 always results in a knock-on default by agent 2. Yet, conditional on agent 3 making its full payment $d_3 = 1.5$, agent 2 always makes it full payment $d_2 = 2.5$, that is, regardless of its endowment asset value realization $v_2 \in [1, 2]$. In equilibrium, agents 2 and 3's defaults are perfectly correlated and occur with a 50% probability.

3. Government Interventions

In this section, we investigate the impact of various types of government interventions on renegotiation and default throughout the credit chain. We start by developing benchmark solutions to stylized mechanism design problems that abstract from the bilateral renegotiation frictions present in practice and featured in our model. A clear takeaway from this analysis is the importance of recognizing decentralized renegotiation processes when designing government interventions. Thereafter, we evaluate how government interventions implemented during recent economic crises affect agents' optimal renegotiation strategies everywhere in a chain. In particular, we contrast the renegotiation consequences of two types of interventions aimed at reducing the shortfall between a targeted borrower's assets and liabilities: subsidies (see, e.g., the 2020 CARES Act passed by the U.S. Congress) and mandated debt reductions (see, e.g., the 2020 Eviction Moratorium imposed by the CDC). We also show that policies that incentivize early renegotiation by lenders (see, e.g., the 2009 Home Affordable Modification Program run by the U.S. Treasury) may reduce the negative impact of private information on the social efficiency of privately optimal renegotiation.

3.1 Mechanism design approaches

To provide a conceptual benchmark, we start by analyzing how a planner would design mechanisms that maximize social surplus. By abstracting from the decentralized nature of bilateral debt renegotiations, these approaches isolate the role of bargaining frictions in our baseline model. Conversely, when policy-makers cannot eliminate bilateral renegotiations in practice, they have to employ other approaches than those stipulated by these mechanisms. Nonetheless, these optimal (centralized) mechanisms contribute to shedding light on the potential benefits of various more realistic and empirically prevalent interventions, such as government subsidies and mandated debt reductions.

3.1.1 Mechanism with transfers Suppose agents can choose the renegotiation offers they make to their borrowers but a centralized planner can commit to transfers that depend on these offers. Specifically, the planner proposes new debt levels $d_2^P,...,d_N^P$ and commits to taxing agents' $t=2$ net cash flows at a rate of τ_j^{NP} if these agents do not accept the planner's proposal. The planner aims to maximize agents' aggregate surplus:

$$\sum_{j=1}^{N} \Pi_j(d_j^P, d_{j+1}^P, ..., d_N^P), \tag{14}$$

subject to all lenders' participation constraints:

$$\Pi_j(d_j^P, d_{j+1}^P, ..., d_N^P) \geq (1-\tau_j^{NP}) \cdot \Pi_j(d_j^P, d_{j+1}^A, ..., d_N^P) \quad \text{for } j=1,...,N-1, \tag{15}$$

where d_{j+1}^A denotes the renegotiation offer to which agent j would deviate when not choosing the planner's proposed debt level d_{j+1}^P.

By imposing a tax rate of $\tau^{NP}=100\%$ for agents not choosing $d_j=d_j^P$, the planner can incentivize all agents to participate. Further, to maximize aggregate surplus, the planner must propose new debt levels that avoid default. The largest face values that still achieve a default-free equilibrium are $\underline{d}_2,...,\underline{d}_N$, but the planner could also simply propose that all debt levels be set to zero (which would effectively nullify all debt contracts in the economy).

3.1.2 Mechanism without transfers As shown above, centralization and taxation are powerful tools that, when combined, allow the planner to achieve the first-best level of social surplus. However, even absent the authority to enforce transfers like taxes, centralization by itself represents an important deviation from the decentralized economy we study. Consider the case in which a planner cannot tax or subsidize agents based on their participation in a centralized renegotiation agreement. Instead, the planner proposes a contract that is to be agreed to ex ante by all participating agents and this contract commits *participating* agents to choosing specific renegotiated debt levels in case other agents do *not* participate. It is now the threat of being quoted high debt levels by their lenders that incentivizes all agents to participate in the contract. As a result, the debt offers associated with nonparticipation do not arise in equilibrium.

Specifically, the planner proposes debt levels $d_2^P,...,d_N^P$ that apply when all agents participate in the contract and debt levels $d_j^{NP:j}$, $d_{j+2}^{NP:j}$,..., $d_N^{NP:j}$ that apply when an agent j refuses to offer its borrower the debt level d_{j+1}^P desired by the planner. The planner again maximizes agents' aggregate surplus:

$$\sum_{j=1}^{N} \Pi_j(d_j^P, d_{j+1}^P, ..., d_N^P), \tag{16}$$

subject to all lenders' participation constraints:

$$\Pi_j(d_j^P, d_{j+1}^P, ..., d_N^P) \geq \Pi_j(d_j^{NP:j}, d_{j+1}^A, ..., d_N^{NP:j}) \quad \text{for } j = 1, ..., N-1, \quad (17)$$

where d_{j+1}^A again denotes the renegotiation offer agent j would deviate to when not offering the planner's desired debt level d_{j+1}^P.

In this setting, we can derive the conditions for the implementability of equilibria that yield the first-best level of aggregate surplus. The maximum punishment for nonparticipation by an agent j is to set its liabilities at $d_j^{NP:j} = \overline{d}_j$ (i.e., to provide no debt forgiveness) and to minimize agent j's borrower's assets by setting its borrower's borrower's liabilities at $d_{j+2}^{NP:j} = 0$. This approach maximizes agent j's liabilities while minimizing its assets (by minimizing agent $(j+1)$'s assets).

The right-hand side of inequality (17) is generally not zero, implying that the planner cannot always implement the first-best level of aggregate surplus associated with no default throughout the chain. In the appendix, we derive the necessary and sufficient condition under which the planner can achieve this outcome. This condition is not available in closed form; it depends on the specific distributional assumptions for v_j and v_{j+1} and on the endogenous (optimal) value for d_{j+1}^A. Yet, our analysis highlights what is missing from agents' optimal bilateral bargaining outcomes relative to centralized planner solutions. Centralized renegotiation among all agents imposes discipline by threatening nonaccommodating agents with a renegotiation plan that harms them on both the asset and the liability sides. Whereas their own debt levels are not reduced, their borrowers' claims are written down, minimizing the borrowers' assets. In contrast, with bilateral renegotiation, a lender's unaccommodating offer to a borrower is not penalized by the surrounding agents in the chain. As a result, accommodating renegotiation is harder to sustain in a realistic setting with bilateral renegotiations.

3.2 Prevalent Government Interventions

In light of the important role played by decentralized bargaining in practice, we now analyze how policies that U.S. government agencies used during recent economic crises affect agents' privately optimal renegotiation strategies in a credit chain. Specifically, we examine the endogenous responses by all credit-chain members to policies that target a subset of agents.

3.2.1 Balance-sheet-targeting interventions In a credit chain, agents are linked through their balance sheets: agent j's liabilities are part of agent $(j-1)$'s assets. To reduce the likelihood of a balance-sheet shortfall resulting in socially costly default, a government might target either side of an agent's balance sheet. In fact, both types of interventions were used by U.S. government agencies in recent economic crises: subsidies aimed at supporting the asset-side of agents' balance sheets were part of the 2020 CARES Act passed by the U.S. Congress whereas mandated debt reductions aimed at reducing the liability-side of agents' balance sheets were part of the 2020 Eviction Moratorium imposed by the CDC. We now analyze and contrast the renegotiation consequences of subsidies and mandated debt reductions in a chain.

We revisit the expected equity value associated with agent $(j-1)$ offering a renegotiated face value $d_j < \overline{d}_j$ to its borrower, but now assuming that agent $(j-1)$'s anticipated level of liabilities $\hat{d}_{j-1}$ is the result of a mandated debt reduction by the government and that agent $(j-1)$'s assets are supported by government subsidies $u_k \geq 0$ offered to agents $k \geq j-1$ (subsidies offered to agents $k < j-1$ do not matter to agent $(j-1)$ beyond their impact on d_{j-1}). As before, agent $(j-1)$'s equity value takes the form:

$$\Pi_{j-1}(d_j) = \mathbb{E}\left[(a^g_{j-1} - \hat{d}_{j-1})^+ \,\middle|\, s_{j-1}\right], \tag{18}$$

where a^g_{j-1} denotes agent $(j-1)$'s total asset value as a result of the government intervention. Similarly denoting by δ^g_{j-1} the stochastic debt transfer that agent $(j-1)$ collects from agent j, we can write agent $(j-1)$'s equity value as

$$\begin{aligned}\Pi_{j-1}(d_j) =& \Pr[v_j + u_j + \delta^g_j \geq d_j] \cdot \mathbb{E}\left[(v_{j-1} + u_{j-1} + d_j - \hat{d}_{j-1})^+ \,\middle|\, s_{j-1}\right] \\ &+ \Pr[v_j + u_j + \delta^g_j < d_j] \\ &\cdot \mathbb{E}\left[(v_{j-1} + u_{j-1} + (1-\rho)(v_j + u_j + \delta^g_j) - \hat{d}_{j-1})^+ \,\middle|\, s_{j-1}, a^g_j < d_j\right].\end{aligned} \tag{19}$$

As discussed in Section 2, reducing agent $(j-1)$'s liabilities implies that this specific agent is more likely to make its own debt payment and has a greater propensity to internalize the losses incurred when its borrower defaults. A government subsidy to agent $(j-1)$ has a similar impact on this agent's renegotiation incentives.

However, the expression above also reveals that how much agent j collects from its borrower, δ^g_j, affects agent $(j-1)$'s decision to renegotiate its debt. By reducing how much agent j might receive from agent $(j+1)$, mandated debt reductions for downstream agents increase the default risk of agent j as the default probability increases and the recovery values are lowered. In contrast, by increasing how much agent j might receive from agent $(j+1)$, subsidies to downstream agents decrease the default risk of agent j as the default probability decreases and the recovery values increase. Thus, while providing a subsidy or mandating a debt reduction might similarly reduce the default risk within one isolated credit relationship, these two tools have different implications for how default propagates in a credit chain with endogenous renegotiation.

To illustrate the various impacts of balance-sheet targeting interventions on private renegotiations, we revisit our earlier setting with $N = 3$ agents and

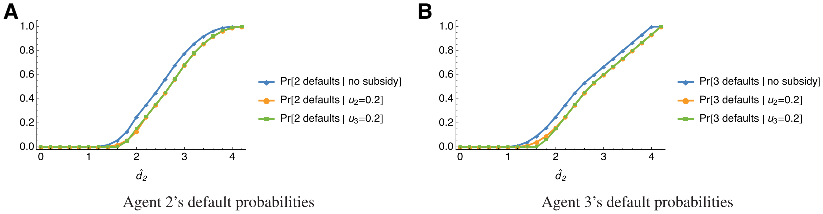

Figure 5
Panels A and B illustrate agent 2's and agent 3's default probabilities conditional on agent 2's belief about how much it will owe to agent 1, denoted as $\hat{d}_2$. We consider subsidies to agent 2 and agent 3 as indicated in the legend. All other parameters are identical to the baseline case detailed in subsection 2.5, where agent 2's and agent 3's endowment asset values are each uniformly distributed on the interval $[1,2]$, and the default loss parameter is $\rho=1$.

uniformly distributed asset values. First, we analyze policies that provide a subsidy to either agent 2 or agent 3, while considering a specific debt level for agent 2. This debt level d_2 could be the result of a mandated debt reduction or could be due to an original debt level $\overline{d}_2$ that constrains higher renegotiation offers.

Figure 5 follows the familiar format of plotting outcomes as functions of the anticipated debt level $\hat{d}_2$. Panels A and B plot agent 2's and agent 3's default probabilities for three different cases: (a) no subsidies, (b) a subsidy exclusively to agent 2, and (c) a subsidy exclusively to agent 3. These comparative statics illustrate the differential effects of the location of a subsidy recipient within the chain.

The figure shows that each of the two subsidies reduces default risk for *both* agents. The fact that a subsidy directly boosting agent 2's asset value is effective in reducing this agent's default risk is not surprising, especially since d_2 is considered to be fixed here (compare the blue and orange lines in Panel A). Yet, supporting agent 2 with a subsidy also reduces the default risk of its borrower, agent 3 (compare the blue and orange lines in Panel B). This latter result obtains due to the previously highlighted force that a better-capitalized agent 2 internalizes more of the default losses associated with high face values for its borrower and therefore finds it optimal to be more lenient when choosing d_3. That is, there is a positive *downstream* pass-through effect of a subsidy to agent 2 due to the endogenous renegotiation in our setting.

For a substantial range of values for d_2, supporting agent 2's assets is in fact equally effective in reducing agent 3's default risk as giving the same subsidy to agent 3 directly (compare the orange and green lines in Panel B). A priori, how effective a subsidy to agent 3 would be is not obvious. This is because anticipating the subsidy, agent 2 might simply charge a higher value d_3, thereby offsetting the boost in agent 3's assets. However, when choosing its renegotiation offer, agent 2 realizes that more continuation value is lost in default once agent 3 receives subsidies from the government. Finally, giving a subsidy to agent 3 is as effective in reducing agent 2's default risk as giving this subsidy directly to agent 2 would be (compare the orange and blue lines in Panel A). This effect stems from the fact that a subsidy to agent 3 makes agent 2's debt collection safer and more valuable, after accounting for renegotiation adjustments in d_3, thereby reducing the risk of agent 2's assets. That is, there is also a positive *upstream* pass-through effect of a subsidy to agent 3.

One may wonder how these outcomes differ once agent 1 is not constrained by either the original debt level $\overline{d}_2$ or a government mandate on the level of debt. In this case, awarding a subsidy to either agent 2 or agent 3 leads to offsetting adjustments in renegotiated face values that can leave the overall default risk of all parties *unchanged*. Specifically, when the government subsidizes agent 2 with $u_2=0.01$, agent 1 increases the face value d_2 by that same amount, going from 2.50 to 2.51. As a result, agent 2's default risk is unchanged and so is its offer to agent 3 (it remains at $d_3=1.5$). Moreover, when the government allocates a subsidy $u_3=0.01$ to agent 3, agent 2 responds by increasing its renegotition offer by 0.01 to $d_3=1.51$ and agent 1 also increases its offer by 0.01 to $d_2=2.51$. In sum, overall default risk is unchanged in either case, as the subsidy benefits are fully appropriated by agent 1 (who has no debt on its own).

This pattern of perfectly offsetting adjustments is, however, not a general result. Rather, it obtains in this case because the example considered in Figure 4 features a local corner solution in agent 1's problem, implying that this solution is locally insensitive to shifts in asset value distributions induced by subsidies (the solution shifts one-for-one with a shift in the distribution). In contrast, when we adjust parameters and consider a setting in which default is associated with lower deadweight losses (e.g., $\rho=0.25$), the same subsidies reduce default risk in the chain. In that case, renegotiated debt levels are not increased one-for-one with asset-value gains from subsidies. For example, awarding a subsidy $u_2=0.01$ to agent 2 incentivizes agent 1 to increase d_2 only by 0.005, thereby reducing agent 2's default risk (agent 2 is not adjusting its offer to agent 3). Instead, awarding a subsidy $u_3=0.01$ to agent 3 incentivizes agent 2 to increase d_3 by 0.008, leading to a reduction in agent 3's default risk. In response, agent 1 increases its renegotiation offer d_2 only by 0.0028. In sum, these results highlight how the generic nonregularity of agents' optimization problems in credit chains has important implications for the effectiveness of government subsidies. In fact, the results suggest that using subsidies and mandated debt reductions jointly is generally a more robust tool for policy-makers, as it controls the endogenous renegotiation responses by upstream agents.

To further highlight how the benefits of balance-sheet-targeting interventions propagate throughout the chain, we now revisit general environments in which downstream defaults are not expected to occur. For situations where default risk is small or the government aims to achieve first-best levels of surplus, we can use the conditions for the existence of default-free equilibria to shed light on each agent's incentives to renegotiate its borrower debt and avoid its borrower's default and, in some cases, its own default. Thus, we now emphasize the renegotiation externalities of government interventions by formally analyzing the endogenous responses of all credit-chain members to policies that only target one credit relationship. Our key results are then obtained through corollaries to Proposition 1.

We first characterize how providing a subsidy to a borrower not only improves the recipient's ability to make its payments but also incentivizes upstream lenders to renegotiate the debt owed to them to default-free levels. The following corollary formalizes the benefits of targeting downstream agents in the generalized environment where $f_j(v_j)$ is continuous and its hazard rate is increasing.

Corollary 2. Let Ψ denote the set of joint distribution functions $f_j(v_j, s_j)$ for $j=1,...,N$ associated with default-free credit chains of N agents for a given default loss parameter ρ and absent government interventions. Further, let Ψ^{u_k} denote the corresponding set if the government provides a subsidy $u_k = u > 0$ to agent k, where $u < \overline{d}_j - \underline{d}_j$ for all j. Providing the subsidy to agent $k=N$ is most effective in expanding the set of default-free credit chains, that is, $\Psi^{u_k=u} \subset \Psi^{u_N=u}$ for any $k<N$.

When a lender anticipates that its borrower will collect a subsidy, it generally responds by increasing the renegotiated face value relative to the counterfactual where no subsidy is paid. In particular, if the borrower's renegotiated debt would already be risk-free absent a subsidy, then the renegotiated debt rises one-for-one with the subsidy amount. However, a different effect arises when default risk would be present absent government interventions. As the value of a borrower's assets increases due to the collection of a subsidy (or even just the anticipation of a future subsidy payment), the lender recognizes that the stakes associated with risking that borrower's default are greater. Thus, the lender takes a more cautious stance when the borrower's assets are subsidized. Moreover, since there is pass-through of resources in a credit chain, giving a subsidy to the last borrower implies that every other upstream lender recognizes that its respective borrower's assets are more valuable and worth conserving, thereby providing heightened incentives for lenient private renegotiation throughout the whole chain.

Because of this effect, awarding a subsidy to a downstream borrower can expand the set of chains that become default-free more than awarding the same subsidy to an upstream borrower would. Specifically, this is the case when each credit relationship is a bottleneck in the sense that it would exhibit a nonzero probability of default. Otherwise, the positive spillover effects on upstream renegotiations still apply but specifically to the set of sequentially connected agents that are not unambiguously solvent. That is, the most downstream agent with positive default risk should then be the recipient of subsidies in order to maximize the set of default-free credit chains.

Note that if a subsidy was expected to be paid by the government after $t=2$, a lender would recognize that risking that borrower's default could eliminate fully this potential boost in the value of the borrower's asset (rather than just eliminating a fraction ρ of it). Anticipating the subsidy, a borrower's equityholders would optimally inject additional funds at $t=2$ to keep the firm afloat only if the firm's asset value including the anticipated subsidy exceeds the renegotiated face value. On the other hand, if a subsidy was paid only conditional on a borrower defaulting, it would not avoid the deadweight losses associated with default. Worse, these conditional subsidies would encourage tough renegotiations by lenders, since, in the renegotiation trade-off, the cost of being harsh would be reduced.[15]

Further, the optimal renegotiation channel featured throughout our paper implies that a government subsidy can prevent default even when the amount injected is not large enough to make up for a borrower's maximum possible shortfall. For example, consider the impact a subsidy has for the first borrower, agent 2, in the case in which all other downstream lending relationships are already efficiently renegotiated in equilibrium. Absent renegotiation between agents 1 and 2, the maximum shortfall that agent 2 may experience is then equal to $\overline{d}_2-\underline{d}_2$. Yet, the government does not necessarily need to provide a subsidy of this magnitude to ensure that agent 2 does not default in equilibrium. Rather, it suffices to provide a subsidy of:

$$u_{min}=\min\left\{\frac{1}{\rho\cdot f_2(\underline{v}_2)}-\underline{d}_2,\overline{d}_2-\underline{d}_2\right\}, \tag{20}$$

which is the amount that causes agent 1's efficient renegotiation condition to hold with equality.

While an equivalent closed-form expression is not available for the renegotiation decisions involving agents $j\geq 3$, we can define u_{min} in those cases as follows:

$$\begin{aligned} u_{min}=\min\{u\geq 0:1-f_j(\underline{v}_j)\cdot\mathbb{E}[\min\{\rho\cdot(u+\underline{d}_j)\ ,\ v_{j-1}-\underline{v}_{j-1}\}\mid s_{j-1}]\leq 0 \\ \forall s_{j-1}\in\Omega_s,\ \text{or}\ u\geq\overline{d}_j-\underline{d}_j\}. \end{aligned} \tag{21}$$

An increase in the subsidy is less effective in relaxing the efficient renegotiation constraints associated with downstream lenders than for agent 1, because downstream agents' existing liabilities reduce the extent to which they internalize inefficiencies. Moreover, ceteris paribus (assuming identical $f_j(\cdot)$ functions for all agents), downstream constraints are more binding since downstream borrowers collect less (i.e., $\underline{d}_{j+1}<\underline{d}_j$). Thus, in that case, the

15 See DeMarzo, Krishnamurthy, and Rauh (2020) for a discussion of the potential ex post benefits of such subsidies.

downstream constraints are more likely to be binding and require larger subsidies to ensure a default-free credit chain.

We now turn our attention to government interventions that target the liability-side of the balance sheet, and thereby a lender's ability to choose its renegotiation strategy. We show how eliminating a lender's bargaining power by mandating a debt reduction can incentivize downstream lenders to renegotiate their borrowers' liabilities to default-free levels. Consistent with earlier insights, if the government were to forgive some of the debt an agent owes, it would relax this agent's efficient-renegotiation conditions as a lender.

Formally, consider a mandated debt reduction that is generous enough to ensure that agent $(j-1)$ can make a full repayment even after renegotiating agent j's debt to a default-free level, that is,

$$\hat{d}_{j-1} \leq \underline{v}_{j-1} + \underline{d}_j. \tag{22}$$

For the generalized environment where $f_j(v_j)$ is continuous and its hazard rate is increasing, we can obtain the following corollary revealing how the efficient-renegotiation conditions depend on $\hat{d}_{j-1}$.

Corollary 3. When expecting efficient renegotiation by downstream lenders (i.e., $d_i = \underline{d}_i$ for all $i > j$), agent $(j-1)$ optimally renegotiates agent j's debt to a default-free level if:

$$\Pi'_{j-1}(\underline{d}_j) = 1 - f_j(\underline{v}_j) \cdot \mathbb{E}\left[\min\left\{ \rho \cdot \sum_{i=j}^{N} \underline{v}_i \ , \ v_{j-1} + \sum_{i=j}^{N} \underline{v}_i - \hat{d}_{j-1} \right\} \middle| \ s_{j-1} \right] \leq 0 \quad \forall s_{j-1} \in \Omega_s. \tag{23}$$

Corollary 3 reveals how multiple efficient-renegotiation conditions can be relaxed by a reduction of agent $(j-1)$'s liability $\hat{d}_{j-1}$. Forgiving agent $(j-1)$'s debt to agent $(j-2)$ might incentivize agent $(j-1)$ to renegotiate agent j's debt and avoid default, which then incentivizes agent j to do the same with agent $(j+1)$'s debt and so on.

How this type of intervention affects upstream lenders, however, depends on what happens to agent $(j-2)$. If the government solely reduces the amount that is transferred from agent $(j-1)$ to agent $(j-2)$, this intervention also reduces agent $(j-3)$'s incentives to efficiently renegotiate with agent $(j-2)$. By reducing how much agent $(j-2)$ collects from agent $(j-1)$, the government effectively lowers how much agent $(j-2)$ and all upstream agents can pay without defaulting. It thus makes efficient renegotiation less attractive to upstream lenders. A poorly designed intervention can therefore lead to higher default risk in the credit chain. As a result, debt reduction policies that do not involve subsidies for the lenders become more effective if the targeted liabilities are owed to lenders that are still expected to have their own (upstream) liabilities renegotiated after the intervention, or lenders that have low levels of liabilities (like agent 1, who has none). If on the other hand, the government forgives agent $(j-1)$'s debt to agent $(j-2)$ but makes up for it by giving agent $(j-2)$ the difference between the renegotiated debt amount without intervention and the new debt amount, then the efficient-renegotiation conditions of upstream lenders are unchanged by the intervention.[16] This intervention relaxes downstream lenders' efficient-renegotiation conditions without affecting upstream lenders'.

As shown above, a government can reduce default risk in a credit chain by providing subsidies to a subset of borrowers or by mandating that their liabilities be reduced. These two policies might look similar at first as both policies reduce the gap between a targeted borrower's assets and liabilities. Our analysis, however, shows that these policies affect renegotiation outcomes differently when the targeted credit relationship is part of a credit chain. First, mandated debt reductions differ from subsidies in how they affect renegotiation within a given credit relationship. A subsidy increases the costs a lender faces when its borrower defaults whereas a mandated debt reduction increases the equity value that a lender might lose by defaulting on its own liabilities. Second, subsidies can relax the efficient-renegotiation conditions of upstream lenders, whereas mandated debt reductions can relax the efficient-renegotiation conditions of downstream lenders. These differences in how targeted subsidies and debt reductions affect the private renegotiation process throughout a chain thus inform the choice and design of government interventions.

3.2.2 Information-targeting interventions We now show that policies that incentivize early renegotiation by lenders (see, e.g., the 2009 Home Affordable Modification Program run by the U.S. Treasury) may reduce the negative impact of private information on the social efficiency of privately optimal renegotiation. To do so, we revisit the general environment that allows for analytical characterizations of no-default equilibrium outcomes despite the presence of privately informative signals.

As pointed out in subsection 2.3, a lender's information at the time of renegotiation affects its incentives to efficiently renegotiate its borrower's liabilities. In particular, if at the renegotiation stage an agent receives a signal indicating that its future asset value will be low, then lenient renegotiation with the borrower generates little equity value. Thus, the agent might be better off gambling for a positive profit by keeping its borrower's debt at a higher level. In contrast, if each agent in the chain has only imprecise information at the time of renegotiation, beliefs are more optimistic than they would be under the worst possible ex post scenario. As a result, relative to this agent-specific worst-case scenario, each agent has stronger incentives to follow a lenient

16 One way of compensating a lender for a debt reduction is through a tax credit provided by the government, as Greenwood and Thesmar (2020) proposed at the onset of the COVID-19 pandemic.

renegotiation strategy with its own borrower. Since incentives to be lenient are nonlinear in an agent's asset value, obtaining efficient renegotiation outcomes in *all* states of the world (i.e., a default-free equilibrium) is easier to achieve when renegotiation occurs before agents have had the opportunity to acquire precise information about their individual asset values.

An immediate implication of this channel is that the timing of the renegotiation process is an important determinant of inefficiencies, as it affects the quality of information available to agents at the time of renegotiation. Formally, we obtain the following result highlighting how late renegotiation and associated precise information can in fact eliminate default-free equilibria altogether.

Corollary 4. Suppose $\overline{d}_j > \underline{d}_j$ for some $j \in \{2, ..., N\}$. If the renegotiation date ($t=1$) is immediately followed by the payment date ($t=2$) such that agents have perfect information about the realizations of their own endowment asset value v_j at the time of renegotiation ($t=1$), a default-free equilibrium does not exist.

When knowing that $\underline{v}_{j-1}$ will be realized, agent $(j-1)$ knows that a renegotiation strategy that avoids its borrower's default would result in an equity value of zero for this lender. Thus, agent $(j-1)$ is always better off taking a tougher stance by keeping agent j's debt at its initial level $\overline{d}_j$. In contrast, if renegotiation occurs sufficiently early, so that no agent already knows with certainty that it received the lowest possible asset value, default-free equilibria can exist.

In practice, agents are likely to know less about how an economic shock will affect their financial conditions if the renegotiation takes place right after the shock hits. As a result, a government policy that promotes early renegotiations after a large economic shock can facilitate private parties' efforts to curb inefficient default waves.[17]

4. Robustness

We now discuss the robustness of our key insights to alternative types of default costs, asset value dependence, borrower bargaining power, and more complex network structures, such as debt trees. For tractability, we compare analytically derived conditions for the existence of a default-free equilibrium within these alternative environments to those presented in Proposition 1.

4.1 Borrower-specific default costs

In our baseline model, we assumed that the only inefficiency associated with default emanates from losses that reduce the value of the assets a lender

[17] See Agarwal et al. (2017) for a description of how the 2009 Home Affordable Modification Program incentivized debt renegotiation during the most recent financial crisis.

recovers from its borrower. The parameter ρ captured proportional deadweight costs associated with default. It is however plausible that borrowers also internalize a subset of the inefficiencies triggered by default. For example, a defaulting borrower might experience a worsened reputation, affecting its future labor market outcomes and limiting its access to capital markets for future projects.

Formally, suppose that each borrower internalized a nonpecuniary fixed cost equal to $\phi > 0$ upon default. In this case, borrower j would agree to pay its debt if $d_j \leq v_j + d_{j+1} + \phi$, that is, the introduction of borrower-specific costs would make defaulting less attractive for the borrower. In fact, the owner of the firm would potentially find it optimal to inject additional funds just to avert default. Relative to our baseline model, the lender would choose a higher debt level without triggering default. Borrowers' default costs would then increase the default-free debt level for each credit relationship, which would be given by

$$\underline{d}_j \equiv \sum_{i=j}^{N} \underline{v}_i + (N+1-j)\phi. \tag{24}$$

The conditions ensuring that the renegotiation offers by agents $(j-1) = 1, \ldots, (N-1)$ yield a default-free equilibrium outcome for the whole credit chain would become:

$$1 - f_j(\underline{v}_j) \cdot \mathbb{E}\left[\min\left\{ \rho \cdot \left(\sum_{i=j}^{N} \underline{v}_i + (N-j)\phi \right) + \phi \ , \ v_{j-1} - \underline{v}_{j-1} \right\} \middle| \ s_{j-1} \right] \leq 0. \tag{25}$$

As to be expected, this condition would reduce to our previous condition (8) when $\phi = 0$.

Whereas the costs considered here would increase the renegotiated face values, the default costs internalized by the creditor and captured by the parameter ρ in our baseline model do not. The reason for this difference is that in a default-free equilibrium, an agent j's borrower, agent $(j+1)$, is collecting the full face value from its borrower, agent $(j+2)$, so default costs are not incurred in equilibrium. Yet, the marginal borrower type (and the associated renegotiated face value) is increased when default costs are internalized by borrowers, as these borrowers are then willing to pay more to avoid defaulting. These results highlight how borrower-specific default costs increase the default-free debt levels and loosen lenders' efficient-renegotiation conditions, yet do so without qualitatively affecting our key insights.

4.2 Asset value dependence

In our baseline model, endowment asset values were independently distributed across agents as of $t = 1$ (while still allowing for aggregate shocks that occurred prior to $t = 1$ and shaped the distributions $F_j(v_j)$). Thus, at that time, agent $(j-1)$ did not use its signal realization s_{j-1} to update the distribution of agent j's asset value, $F_j(v_j)$. In contrast, if agent $(j-1)$'s signal was also informative about agent j's asset value, due to a dependence between asset values, the distribution $F_j(v_j)$ would be replaced by the updated distribution $F_j(v_j|s_{j-1})$. Moreover, if the lower bound of the support of v_j was still $\underline{v}_j$ under this updated distribution, then the default-free debt level $\underline{d}_j$ would stay the same as in the baseline model, and agent $(j-1)$'s efficient-renegotiation condition would be

$$1-f_j(\underline{v}_j|s_{j-1})\cdot\mathbb{E}\left[\min\left\{\rho\cdot\sum_{i=j}^{N}\underline{v}_i\ ,\ v_{j-1}-\underline{v}_{j-1}\right\}\ \middle|\ s_{j-1}, v_j=\underline{v}_j\right]\leq 0. \quad (26)$$

This result implies that positively correlated asset values would partially mitigate the effect that bad signals have on a lender's renegotiation trade-off. Whereas a bad signal s_{j-1} reduces the rents agent $(j-1)$ expects to earn on the default-free path as pointed out in our baseline analysis, it also increases the probability that agent j defaults whenever $d_j>\underline{d}_j$. In sum, while introducing dependence between asset values would enrich the role of signals in our model, it would not alter the main takeaways of our baseline analysis.

4.3 Borrowers with bargaining power

In our baseline model, lenders were assumed to have bargaining power when renegotiating their borrowers' liabilities in order to capture the empirical fact that incumbent lenders often wield their renegotiation power and influence borrowing firms' decisions (see, e.g., Chava and Roberts 2008; Roberts and Sufi 2009; Denis and Wang 2014). Further, from a technical perspective, lenders making ultimatum offers ensured that, even in the presence of private information, signaling concerns did not arise in our environment. If we instead assumed that the borrower was making the renegotiation offer, a lender would use this offer as a signal. Multiple equilibria would arise within each credit relationship, which would significantly reduce the tractability of our analysis of credit chains.

Setting aside complications linked to private information, the renegotiation protocol in our model yields the natural decisional trade-off of a lender collecting a high payment conditional on borrower survival but risking a high cost of default. The optimal renegotiation strategy depended on the uncertainty a lender faces regarding its borrower's ability to pay. If instead a borrower j made an ultimatum offer to the lender about its own liabilities, it would propose to reduce the debt level d_j as much as possible, subject to the restriction that the lender's equity value associated with the renegotiated debt level weakly exceeds the lender's equity value under the original contract. This allocation of bargaining power would result in weakly lower renegotiated debt levels than in our baseline analysis where the lender picks its profit-maximizing renegotiation offer. For example, absent private information, the conditions for the existence of a default-free equilibrium would then be given by

$$\Pi_{j-1}(\underline{d}_j) \geq \Pi_{j-1}(\overline{d}_j) \;\; \forall j \in \{2,...,N\}. \tag{27}$$

Thus, endowing borrowers with all the bargaining power would eliminate a force that tends to exacerbate inefficiencies in our environment. Yet, by comparing the lender's expected profit function $\Pi_{j-1}(d_j)$ for different levels of d_j, the borrower's decision would still center on the same channels discussed in our baseline analysis: lowering the debt level would result in a lower amount collected by the lender in case of payment but also lower default costs.

4.4 Other network structures

In our baseline model, we analyzed the renegotiation behavior of agents that are part of a credit chain, in which each lender has one borrower (and vice versa). A credit chain is the most streamlined network that features downstream and upstream agents and their sequential interconnectedness in a meaningful way. More generally, the network of credit relationships might feature some lenders that renegotiate with multiple borrowers (e.g., a "credit tree") and/or some borrowers that have multiple lenders. We now discuss additional forces that more complex network structures would introduce.

First, consider a credit tree in which agent $(j-1)$ would owe an amount $\underline{d}_{j-1}$ to another agent and would choose how to renegotiate the liabilities of its M borrowers, agents $j_m \in \{j_1, j_2, ... j_M\}$. Specifically, agent $(j-1)$ would choose whether to renegotiate each borrower j_m's liabilities to a lower level $d_{j_m} < \overline{d}_{j_m}$. In contrast to our baseline model, agent $(j-1)$ would now have to make M renegotiation decisions, which require comparing agent $(j-1)$'s equity value for every possible combination of renegotiation strategies with its borrowers $j_m \in \{j_1, j_2, ... j_M\}$. To explore the economic forces and trade-offs in this setting, let us maintain the assumption of endowment asset value independence and zoom in on the decision to renegotiate j_m's liabilities when every other liability in the network, including those of agents $j_{m'}$ where $m' \neq m$, are expected to be renegotiated to their respective default-free levels. Extending our notation, we then can show that agent $(j-1)$ prefers renegotiating all of its borrowers' liabilities to a default-free level over renegotiating those of all agents except j_m as long as

$$1 - f_{j_m}(\underline{v}_{j_m}) \cdot \mathbb{E}[\min\{\rho \cdot \underline{d}_{j_m} \;,\; v_{j-1} - \underline{v}_{j-1}\} \mid s_{j-1}] \leq 0 \;\; \forall s_{j-1} \in \Omega_s. \tag{28}$$

Comparing this condition to the efficient-renegotiation condition in credit chains derived in Proposition 1 reveals that the strategic decision whether to renegotiate with one specific borrower in order to avoid default in a credit tree (or in many other types of networks) features economic forces and trade-offs that are consistent with those from our baseline environment.

While in a credit tree each agent has several borrowers, agents may also have multiple lenders in practice.[18] In our baseline model, an agent's optimal renegotiation behavior depended on the amount of debt it expects to owe; whether this amount is owed to one or more lenders is irrelevant to the borrower. However, with multiple lenders, each lender's incentives to renegotiate down this agent's debt would be weakened by a free-rider problem. The additional social surplus obtained from an individual lender's lenient renegotiation stance would be partially internalized by other lenders. Given this externality, each lender would prefer the other lenders to provide concessions (see, e.g., Sachs 1990). The resultant free-rider problem would then complement the knock-on default effects we highlighted in our analysis, which also limit lenders' incentives to renegotiate their borrowers' debt and reduce socially inefficient default costs.

5. Conclusion

To analyze the effectiveness of private and public interventions aimed at avoiding large-scale default waves after negative economic shocks, we develop a model of strategic renegotiation in credit chains. Our model shows how private renegotiation decisions are interrelated: a lender's willingness to provide concessions to its borrower depends on how it expects its own liabilities to be renegotiated. Whereas a tough renegotiation strategy may be privately optimal for the lender, it can create negative externalities for renegotiation efforts elsewhere in the chain. In fact, an unaccommodating renegotiation strategy by one lender may trigger tough renegotiations and increased default probabilities throughout the whole chain.

Our policy analysis reveals how government subsidies to downstream borrowers not only mechanically improve their recipients' ability to make their original payments but also, importantly, may incentivize upstream lenders to privately renegotiate their borrowers' liabilities. Accounting for the interlinkages between the optimal renegotiation decisions of agents in a credit chain, we show that awarding relatively small subsidies to downstream borrowers can be more effective in preventing default waves than awarding the same subsidies to upstream borrowers. We also examine how forgiving a struggling borrower's debt or backing it to prevent default can further incentivize downstream lenders to efficiently renegotiate the debt of their borrowers. Finally, we highlight that facilitating early debt renegotiations after a large shock tends to increase incentives for providing concessions, thus reducing default risk. In sum, our analysis not only sheds light on the implications of different types of government interventions but also reveals

[18] See Bolton and Scharfstein (1996), He and Xiong (2012), Brunnermeier and Oehmke (2013), Zhong (2021), and Donaldson et al. (2022), among others, for analyses focusing on the effects of creditor dispersion on renegotiation of one borrower's liabilities.

how the targeting of specific members of a credit chain optimally complements private renegotiation efforts.

Appendix: Proofs and Derivations

Proof of Lemma 1: As is standard in the security design literature, we consider securities satisfying limited liability and monotonicity (see, e.g., Harris and Raviv 1989; Innes 1990; Nachman and Noe 1994). A special feature of debt in our environment with nonverifiability is that it provides a lender with a foreclosure right, that is, a lender can seize the assets of a borrower who defaults to pay the specified face value. To simplify the exposition, suppose that the total asset value of borrower j takes the random value $a_j \geq \underline{a}_j$ at $t=2$, where a_j reflects both the endowment asset value and any debt collection coming from agent $(j+1)$. Since the uncertain component of asset values is nonverifiable and securities are monotone, a borrower always finds it optimal to report the lowest possible value of its total assets $\underline{a}_j$, unless doing so triggers default (in which case the borrower's assets are seized by the lender). Default, in turn, can only be triggered in the case of a debt contract. As a result, the lender anticipates that any security that is not a debt contract will yield a payoff of at most $\underline{a}_j$ (given limited liability, conditional on reporting $\underline{a}_j$, the contract cannot pay more than $\underline{a}_j$).

The only way a lender can obtain a higher expected payoff than $\underline{a}_j$ is to propose a new debt contract with face value d_j where $\underline{a}_j < d_j \leq \overline{d}_j$ (provided that the face value of the original debt contract satisfies $\overline{d}_j > \underline{a}_j$, which is necessary for there to be any scope for renegotiation in the first place). Conditional on that debt contract, a borrower with total assets worth a_j at $t=2$ will optimally pay d_j when $a_j \geq d_j$. On the other hand, default occurs when $a_j < d_j$, resulting in the lender collecting only $(1-\rho)a_j$ from seizing the assets. A lender then optimally weighs these potential outcomes when choosing the new face value, as described in our main analysis of Section 2. Using a debt contract, a lender can at a minimum replicate the payoff of all other securities, $\underline{a}_j$, by choosing $d_j = \underline{a}_j$. Yet, debt with an optimally chosen face value generally allows a lender to achieve a higher expected payoff. As a result, offering a new debt security is always weakly optimal, and potentially strictly optimal. This result allows us to restrict attention to renegotiation offers that maintain a liability taking the form of a debt contract. □

Proof of Proposition 1: Using Equation (4), we know that if agent $(j-1)$ expects the debt of all agents $k \neq j$ to be renegotiated to $\underline{d}_k$, the marginal net benefit of increasing d_j is

$$\Pi'_{j-1}(d_j) = [1-G_j(d_j)] \cdot \; [1-F_{j-1}(\underline{d}_{j-1}-d_j \mid s_{j-1})] - g_j(d_j) \cdot \; \mathbb{E}\Big[\min\Big\{\rho d_j \,,\, \Big(v_{j-1}+d_j-\underline{d}_{j-1}\Big)^+\Big\} \;\Big|\; s_{j-1}\Big]. \tag{A1}$$

The necessary and sufficient condition for an equilibrium in which agent $(j-1)$ chooses a face value that ensures that agent j does not default is

$$\Pi'_{j-1}(\underline{a}_j) \leq 0, \tag{A2}$$

where $\underline{a}_j \equiv \underline{v}_j + \underline{d}_{j+1}$. A default-free equilibrium requires this condition to hold for all possible signal realizations $s_{j-1} \in \Omega_s$ that agent $(j-1)$ might observe. Plugging in the relation for default-free debt levels from (5) and recognizing that $g_j(v_j + \underline{d}_{j+1}) = f_j(v_j)$ and $F_j(v_j + \underline{d}_{j+1}) = F_j(v_j)$ along the default-free path, we obtain condition (8) for a generic agent $(j-1)$ and condition (7) for agent 1 (who does not owe debt to another agent). □

Proof of Proposition 2: Let us first consider the case in which $\overline{d}_3 > v_3^H$. Since agent 3 would default on the original debt level $\overline{d}_3$ with probability 1, agent 2 is strictly better off asking for $d_3 = v_3^H$ than for $\overline{d}_3$ as setting $d_3 = v_3^H$ delivers v_3^H rather than $(1-\rho)v_3^H$ when $v_3 = v_3^H$ and has identical payoffs otherwise. Thus, we can restrict our attention to maximizing d_3 over the set $\{0, \min[v_3^L, \overline{d}_3], \min[v_3^H, \overline{d}_3]\}$. Suppose for now that $d_3 \geq v_3^L$. Then we compare the following 3 equity values.

When agent 3's renegotiation debt level is $d_3 = \min\{\overline{d}_3, v_3^H\}$, agent 2's equity value is

$$\Pi_2(v_3^H) = f_3(v_3^H) \cdot \mathbb{E}[(v_2 + \min\{\overline{d}_3, v_3^H\} - \hat{d}_2)^+] + f_3(v_3^L) \cdot \mathbb{E}[(v_2 + (1-\rho)v_3^L - \hat{d}_2)^+]$$
$$+ f_3(0) \cdot \mathbb{E}[(v_2 - \hat{d}_2)^+]. \tag{A3}$$

By renegotiating agent 3's debt down to $d_3 = v_3^L$, agent 2 can expect to collect

$$\Pi_2(v_3^L) = [1 - f_3(0)] \cdot \mathbb{E}[(v_2 + v_3^L - \hat{d}_2)^+] + f_3(0) \cdot \mathbb{E}[(v_2 - \hat{d}_2)^+]. \tag{A4}$$

By renegotiating agent 3's debt further down to $d_3 = 0$, agent 2 can expect to collect

$$\Pi_2(0) = \mathbb{E}[(v_2 - \hat{d}_2)^+]. \tag{A5}$$

This last strategy is always weakly dominated, and often strictly dominated, by either one of the strategies involving a higher debt level. Thus, we can focus our attention on the decision whether to set the debt level at $d_3 = \min\{\overline{d}_3, v_3^H\}$ or at $d_3 = v_3^L$.

Comparing (A3) and (A4) implies that lowering the debt from $d_3 = \min\{\overline{d}_3, v_3^H\}$ to $d_3 = v_3^L$ is preferable whenever

$$f_3(v_3^L) \cdot \mathbb{E}\left[\min\left\{\rho v_3^L, \left(v_2 + v_3^L - \hat{d}_2\right)^+\right\}\right] - f_3(v_3^H)$$
$$\cdot \mathbb{E}\left[\left(v_2 + \min\left\{\overline{d}_3, v_3^H\right\} - \hat{d}_2\right)^+ - \left(v_2 + v_3^L - \hat{d}_2\right)^+\right] \geq 0. \tag{A6}$$

Finally, let's consider the case in which $\overline{d}_3 < v_3^L$. Since renegotiating down the debt does not reduce default risk yet lowers the payment collected, agent 2 finds it optimal to keep $d_3 = \overline{d}_3$. □

Proof of Corollary 1: Taking the partial derivative of the left-hand side of the renegotiation condition (A6) with respect to $\hat{d}_2$ yields

$$- f_3(v_3^L) \cdot \Pr(0 \leq v_2 + v_3^L - \hat{d}_2 < \rho v_3^L)$$
$$- f_3(v_3^H) \cdot [-\Pr(v_2 + \min\{\overline{d}_3, v_3^H\} - \hat{d}_2 \geq 0) + \Pr(v_2 + v_3^L - \hat{d}_2 \geq 0)], \tag{A7}$$

which simplifies to

$$f_3(v_3^H) \cdot \Pr(\hat{d}_2 - \min\{\overline{d}_3, v_3^H\} \leq v_2 < \hat{d}_2 - v_3^L) - f_3(v_3^L) \cdot \Pr(\hat{d}_2 - v_3^L \leq v_2 < \hat{d}_2 - (1-\rho)v_3^L). \tag{A8}$$

□

Proof of Corollary 2: First, note that providing a subsidy to agent 1 has no effect on renegotiation outcomes. Next, suppose the government provides a subsidy $u_k = u$ to an agent $k \geq 2$. Agent k then effectively obtains an endowment cash flow of $(v_k + u)$ instead of v_k absent subsidies and we can rewrite the efficient-renegotiation conditions provided in Proposition 1 as follows:

$$1 - f_2(\underline{v}_2) \cdot \rho \cdot (u + \underline{d}_2) \leq 0, \tag{A9}$$

$$1 - f_j(\underline{v}_j) \cdot \mathbb{E}[\min\{\rho \cdot (u + \underline{d}_j),\ v_{j-1} - \underline{v}_{j-1}\} \mid s_{j-1}] \leq 0 \quad \text{for } j = 3, \ldots, k \text{ and } \forall s_{j-1} \in \Omega_s, \tag{A10}$$

$$1 - f_j(\underline{v}_j) \cdot \mathbb{E}[\min\{\rho \cdot \underline{d}_j,\ v_{j-1} - \underline{v}_{j-1}\} \mid s_{j-1}] \leq 0 \quad \text{for } j = (k+1), \ldots, N \text{ and } \forall s_{j-1} \in \Omega_s. \tag{A11}$$

Conditions (A9)-(A10) are relaxed by the subsidy term u, whereas conditions (A11) are not. Moreover, condition (A9) is relaxed the same way by a subsidy u regardless of which agent

$k \geq 2$ receives the subsidy. Similarly, no matter which agent $k \geq 3$ receives the subsidy u, condition (A10) is relaxed the same way. The condition that $u < \overline{d}_j - \underline{d}_j$ for all j implies that absent private renegotiation, credit relationships would exhibit a nonzero probability of default even in the presence of the government subsidy. However, we show that the pass-through of resources in a credit chain implies that a subsidy provided to agent k affects the efficient renegotiation condition for agent $(k-2)$ in just the same way that a subsidy to the direct borrower $(k-1)$ would. Yet, by providing the subsidy to agent $k = N$, all conditions are relaxed by the maximum amount attainable with a given subsidy, thereby providing the maximum expansion of the default-free set Ψ. □

Proof of Corollary 3: From Equation (4), if agent $(j-1)$ expects to owe $\hat{d}_{j-1} \leq \underline{v}_{j-1} + \underline{d}_j$ and expects the debt of any other agent k except j to be renegotiated to $\underline{d}_k$, the necessary and sufficient condition for an equilibrium in which agent $(j-1)$ chooses a face value $\underline{d}_j$ that ensures that agent j does not default is

$$[1 - F_{j-1}(\hat{d}_{j-1} - \underline{d}_j \mid s_{j-1})] - f_j(\underline{v}_j) \cdot \mathbb{E}\Big[\min\Big\{\rho \underline{d}_j \ , \ \Big(v_{j-1} + \underline{d}_j - \hat{d}_{j-1}\Big)^+\Big\} \ \Big| \ s_{j-1}\Big] \leq 0. \quad \text{(A12)}$$

Plugging $\underline{d}_j = \sum_{i=j}^N \underline{v}_i$ into this inequality and recognizing that $\hat{d}_{j-1} \leq \underline{v}_{j-1} + \underline{d}_j$ implies that $F_{j-1}(\hat{d}_{j-1} - \underline{d}_j \mid s_{j-1}) = 0$ allow to simplify this condition as (23).

We now can take the derivative of the left-hand side of condition (23) with respect to $\hat{d}_{j-1}$ and get

$$f_j(\underline{v}_j) \cdot \Pr\left[v_{j-1} + \sum_{i=j}^{N} \underline{v}_i - \hat{d}_{j-1} < \rho \cdot \sum_{i=j}^{N} \underline{v}_i \ \middle| \ s_{j-1} \right]$$

$$= f_j(\underline{v}_j) \cdot \Pr\Big[v_{j-1} < \hat{d}_{j-1} - (1-\rho)\underline{d}_i \ \Big| \ s_{j-1} \Big]. \quad \text{(A13)}$$

Note that we had assumed to begin with that

$$\underline{v}_{j-1} \geq \hat{d}_{j-1} - \underline{d}_j. \quad \text{(A14)}$$

Thus, as long as

$$\hat{d}_{j-1} \in (\underline{v}_{j-1} + (1-\rho)\underline{d}_i \ , \ \underline{v}_{j-1} + \underline{d}_i], \quad \text{(A15)}$$

a *decrease* in $\hat{d}_{j-1}$ strictly loosens the condition for agent $(j-1)$ to pick a renegotiated debt level that leads to no default in equilibrium. □

Proof of Corollary 4: Under the conditions laid out in Corollary 4, an agent $(j-1)$ may already know during the renegotiation process (at $t = 1$) that the realization of its asset value is the worst-possible outcome, $v_{j-1} = \underline{v}_{j-1}$, implying that cost of marginally increasing the renegotiation offer featured in condition (8) is equal to zero. Thus, this condition cannot be satisfied in all states of the world, ruling out the existence of an equilibrium where default does not occur at all on the equilibrium path. □

Derivation of condition for first-best level of surplus under mechanisms without transfers: Given the punishment for nonparticipation described in subsection 3.1 for the case of mechanisms without transfers, agent $(j+1)$'s assets consist of only its endowment asset value and, as a result, d_{j+1}^A maximizes agent j's objective:

$$\Pi_j(d_j^{NP:j}, d_{j+1}^A, ..., d_N^{NP:j}) = [1 - F_{j+1}(d_{j+1}^A)] \cdot \mathbb{E}\Big[(v_j + d_{j+1}^A - \overline{d}_j)^+ \ \Big| \ s_j\Big]$$

$$+ F_{j+1}(d_{j+1}^A) \cdot \mathbb{E}\Big[(v_j + (1-\rho)v_{j+1} - \overline{d}_j)^+ \ \Big| \ s_j, v_{j+1} < d_{j+1}^A\Big], \quad \text{(A16)}$$

where we use the fact that $G_{j+1}(d^A_{j+1})=F_{j+1}(d^A_{j+1})$ since $a_{j+1}=v_{j+1}$ in this punishment scheme. Agent j's marginal profit from increasing d^A_{j+1} is then given by

$$\Pi'_j(d^A_{j+1}) = [1-F_{j+1}(d^A_{j+1})]\cdot[1-F_j(\overline{d}_j - d^A_{j+1} \mid s_j)] \\ - f_{j+1}(d^A_{j+1})\cdot\mathbb{E}\Big[\min\Big\{\rho d^A_{j+1}\ ,\ \Big(v_j + d^A_{j+1} - \overline{d}_j\Big)^+\Big\}\ \Big|\ s_j\Big]. \qquad \text{(A17)}$$

In case of an interior optimum, d^A_{j+1} solves $\Pi'_j(d^A_{j+1})=0$, or equivalently:

$$\frac{f_{j+1}(d^A_{j+1})}{1-F_{j+1}(d^A_{j+1})} = \frac{1-F_j(\overline{d}_j - d^A_{j+1} \mid s_j)}{\mathbb{E}\Big[\min\Big\{\rho d^A_{j+1}\ ,\ \Big(v_j + d^A_{j+1} - \overline{d}_j\Big)^+\Big\}\ \Big|\ s_j\Big]}. \qquad \text{(A18)}$$

For these first-order conditions to be sufficient optimality conditions, we need more than a standard monotone hazard rate assumption. When $\overline{d}_j$ is high enough, then $F_j(\overline{d}_j - d^A_{j+1} \mid s_j)=1$ and the right-hand side of Equation (A18) takes the value zero. Yet, agent j's objective is also zero.

To achieve the first-best outcome, the planner must prevent all default and the highest debt levels that still ensure a default-free chain are given by $\underline{d}_2$, ...,$\underline{d}_N$. As before, we maintain the assumption that $\overline{d}_j > \underline{d}_j \equiv \sum_{i=j}^N \underline{v}_i$. If these debt levels are proposed by the planner, the left-hand side of inequality (17) becomes

$$\Pi_j(\underline{d}_j,\underline{d}_{j+1},...,\underline{d}_N)=\mathbb{E}\big[v_j|s_j\big]+\underline{d}_{j+1}-\underline{d}_j=\mathbb{E}\big[v_j|s_j\big]+\sum_{i=j+1}^{N}\underline{v}_i-\sum_{i=j}^{N}\underline{v}_i=\mathbb{E}\big[v_j|s_j\big]-\underline{v}_j. \qquad \text{(A19)}$$

Thus, given the participation constraint (17), a no-default outcome can be achieved when the following condition holds:

$$\mathbb{E}\big[v_j|s_j\big]-\underline{v}_j \ \geq\ [1-F_{j+1}(d^A_{j+1})]\cdot\mathbb{E}\Big[(v_j+d^A_{j+1}-\overline{d}_j)^+\ \Big|\ s_j\Big] \\ +F_{j+1}(d^A_{j+1})\cdot\mathbb{E}\Big[(v_j+(1-\rho)v_{j+1}-\overline{d}_j)^+\ \Big|\ s_j, v_{j+1} < d^A_{j+1}\Big]. \qquad \text{(A20)}$$

This necessary and sufficient condition for the first-best level of surplus depends on the specific distributional assumptions for v_j and v_{j+1} and on the endogenous (optimal) value for d^A_{j+1} that solves Equation (A18) in case of an interior optimum.

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