Adverse Selection and Intermediation Chains

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We propose a parsimonious model of over-the-counter trading with asymmetric information to shed light on the prevalence of intermediation chains that stand between buyers and sellers of assets in many decentralized markets. Trading an asset through moderately informed intermediaries can facilitate efficient trade by layering an information asymmetry over several sequential transactions. Intermediation then reduces the adverse selection problems each trader faces and limits the incentives to quote aggressive prices that jeopardize the gains to trade. Long chains of intermediaries can thus promote socially optimal trading behavior in the presence of adverse selection. (JEL D82, D85, G23, L10)

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1 Introduction

Transactions in decentralized markets often feature the successive involvement of several intermediaries.[1] In this paper, we propose a parsimonious model of over the counter (OTC) trading to study the implications of these trading arrangements in the presence of adverse selection. We show that chains of heterogeneously informed agents can fulfill an important economic role in intermediating trade by layering information asymmetries over multiple sequential transactions.

Our model considers two asymmetrically informed agents who wish to trade an asset over the counter in order to realize gains to trade. One agent is assumed to be an expert who is well informed about the value of the asset, whereas the other agent is uninformed. A standard result in models like ours is that trade breaks down between agents when the potential gains to trade are small relative to the degree of information asymmetry about the asset's value. Yet, we show in our model that involving a moderately informed intermediary — whose information quality ranks between those of the buyer and seller — can improve trade efficiency in some of these cases. Furthermore, in contrast to other theories that highlight the benefits of a single intermediary, our analysis also provides a rationale for why trading may go through long chains of intermediaries rather than through simpler networks centered around one dominant broker.

The intuition behind our main results can be explained by considering a trader who chooses which price to quote to a better informed counterparty. This "proposer" needs to offer price concessions to his better informed counterparty to ensure his quote is accepted with high probability. The magnitude of these price concessions then depends on the informational advantage of the better informed counterparty. When the informational advantage is large, the proposer finds it privately optimal to quote an aggressive price that reduces the price concessions and therefore his counterparty's information rent, even though doing so also lowers the probability of trade and jeopardizes the surplus from trade. Our paper shows that this inefficient behavior may be eliminated by the involvement of heterogeneously informed intermediaries who trade the asset sequentially, as part of an intermediation chain in which each trader's information set is similar, although not identical, to those of his direct counterparties. As the conditional distributions that characterize the information asymmetry between each pair of sequential traders are altered by the involvement of these intermediaries, so are traders' incentives to quote prices that sustain efficient trade.

In an intermediation chain, each proposer maximizes the difference between the total expected gains to trade and the cumulative information rents shared with subsequent traders. When a proposer and his immediate counterparty are similarly informed, the adverse selection problem this proposer faces is reduced and aggressive trading strategies that lower the probability of trade are

[1] In particular, we refer later in the introduction to several empirical papers that document the existence of intermediation chains in various financial markets.

less effective in reducing the information rents that need to be shared with subsequent traders. However, any decrease in the probability of trade jeopardizes the trade surplus just as much as without intermediaries, provided that trade is efficient in subsequent transactions. Thus, in these situations intermediation chains reduce each trader's benefit of quoting an inefficiently aggressive price while also maintaining the cost of doing so, therefore tilting the trade-off toward efficient behavior for all traders.

For large adverse selection problems, a high number of intermediaries may be needed to reduce information asymmetries enough to sustain efficient trade in each transaction. Greater information asymmetries may therefore be better bridged by longer intermediation chains that involve many sequential transactions, contrasting with the conventional wisdom that asymmetric information should be associated with low trading volume (as it is the case in the seminal model of Akerlof 1970, for example). We highlight, however, that in some of the cases where an intermediation chain fails to sustain efficient trade its presence may instead harm efficiency, for example due to problems of double marginalization, which are well known from settings with sequential layers of monopolists (e.g., Spengler 1950).

Related Literature. Intermediation is known to facilitate trade, either by minimizing transaction costs (Townsend 1978), by concentrating monitoring incentives (Diamond 1984), or by alleviating search frictions (Rubinstein and Wolinsky 1987, Yavaş 1994, Duffie, Gârleanu, and Pedersen 2005, Neklyudov 2013). Our paper, however, specifically speaks to how intermediaries may help alleviate inefficiencies due to asymmetric information. We know from Myerson and Satterthwaite (1983) that an uninformed third party who subsidizes transactions can help eliminate these problems in bilateral trade. Trade efficiency can also be improved by the involvement of fully informed middlemen who care about their reputation (Biglaiser 1993) or who worry that informed buyers could force them to hold on to low-quality goods (Li 1998). Contrary to these models, our setup considers the possibility that an intermediary's information set differs from that of the other agents involved in a transaction. In fact, in our static model without subsidies, warranties, or reputational concerns the involvement of an intermediary who is either fully informed or totally uninformed does not improve trade efficiency. Thus, the insight that moderately informed intermediaries can reduce trade inefficiencies simply by layering an information asymmetry over several sequential transactions fundamentally differentiates our paper from these earlier papers.

Our analysis of intermediation chains, which are observed in many financial markets, also distinguishes our paper from many market microstructure models that feature heterogeneously informed traders but no trade among intermediaries (e.g., Glosten and Milgrom 1985, Kyle 1985, Hagerty and McDonald 1995, Jovanovic and Menkveld 2012). The potential benefit of involving multiple intermediaries also distinguishes our paper from Babus (2012) who endogenizes OTC trading networks when agents meet sporadically and have incomplete information about other traders' past behaviors. In Babus (2012) a central intermediary becomes involved in all trades in equilibrium and penalizes anyone defaulting on prior obligations.[2]

Gofman (2011) allows for non-informational bargaining frictions and shows that socially optimal trading outcomes are easier to achieve if an OTC network is sufficiently dense (although the relationship is not necessarily monotonic). In his model, an exogenous surplus splitting rule implies that agents prefer to trade through short intermediation chains rather than long ones, even when it prevents the asset from reaching the most efficient buyer present in the network. In our model, each heterogeneously informed intermediary uses his monopoly power to extract surplus, potentially harming end traders. However, longer intermediation chains that layer an information asymmetry over several sequential transactions may still facilitate efficient trade.[3]

Although intermediation chains can be observed in various types of decentralized markets, we rely on the literature that documents the empirical importance of inter-dealer trading and intermediation chains in OTC financial markets to contextualize our theory. According to the Bank of International Settlements (2013), inter-dealer trading accounts for 35% of the $\$2.3$ trillion in daily transaction volume for OTC interest-rate derivatives. Goldstein and Hotchkiss (2012) find that roughly one third of transaction volume in secondary markets of newly issued corporate bonds is among dealers. For municipal bonds, Li and Schürhoff (2014) show that 13% of intermediated round-trip trades involve a chain of 2 intermediaries and an additional 10% of trades involve 3 or more intermediaries. Hollifield, Neklyudov, and Spatt (2014) also find evidence of intermediation chains in the market for securitized products: for example, intermediated round-trip trades of non-agency collateralized mortgage obligations (CMOs) involve 1.76 dealers on average and in some instances the chain includes up to 10 dealers.

Furthermore, several empirical regularities are hard to reconcile with alternative explanations for the existence of intermediation chains, at least in some markets. For example, Di Maggio, Kermani, and Song (2015) find evidence in the corporate bond market of high persistence in dealers' trading relationships while Li and Schürhoff (2014) estimate the probability that a given directional trade (buy vs. sell) between two municipal bond dealers is repeated in the following month to be 62%, compared to a probability of 1.4% if counterparties were meeting randomly. This evidence of high persistence in trading interactions is hard to reconcile with search-based theories where trading interactions are randomly determined (e.g., Wright and Wong 2014). Weller (2013) proposes

[2] See also Farboodi (2014) who shows that a centralized trading network is socially optimal when banks must establish credit relationships prior to learning about the allocation of investment projects in the economy.

[3] Our paper also relates to Malamud and Rostek (2013) who study the concurrent existence of multiple exchanges in decentralized markets. Creating a new private exchange may improve the liquidity in incumbent exchanges by reducing the price impact that strategic traders impart when simultaneously trading the same asset at different prices on multiple exchanges. This particular mechanism plays no role in our model as trading is bilateral, occurs sequentially among intermediaries, and entails a fixed transaction size.

an explanation for the high-frequency trading chains in metals futures markets that relies on heterogeneity in dealers' technological ability to quickly transact in a centralized trading venue. Many other securities that are traded through chains (e.g., securitized products, municipal and corporate bonds) are, however, traded in decentralized markets, thus hindering the applicability of his results in these settings. In Ho and Stoll (1983) inter-dealer trading is motivated by dealers' attempts to share inventory risk; yet such risk sharing would not per se require trade to occur sequentially, through a chain. Looking at futures markets, Manaster and Mann (1996) find a positive relationship between trader inventories and transaction prices, which violates the predictions of inventory control models. Manaster and Mann (1996, p.973) conclude that the intermediaries they study are "active profit-seeking individuals with heterogeneous levels of information and/or trading skill", elements usually absent from inventory-based theories. More recently, Goldstein and Hotchkiss (2012) find that dealers in the corporate bond market face little inventory risk, even during crises, yet inter-dealer trading is economically important in this market.

Viswanathan and Wang (2004) propose a model that highlights that a security issuer may prefer that a set of dealers with heterogeneous inventory levels trade the security sequentially rather than participate in a centralized auction where the supply of the security is split among the dealers. As with inventory control models, dealers resell only a portion of their acquired position to their respective counterparty in this setting. In their empirical study, Li and Schürhoff (2014), however, identify a chain only when a dealer buys and then sells the same quantity of a security to another dealer, thus excluding inter-dealer trading aimed at dispersing inventory through the network.

Moreover, related findings lend support to our information-based theory of intermediation chains. Intermediaries in our model are still averse to holding inventories (i.e., non-zero positions) since they are not the efficient holders of assets. Yet information asymmetries may prevent them from offloading assets to potential buyers, consistent with evidence in Jiang and Sun (2015) suggesting that the corporate bond market is affected by significant asymmetric information problems.[4] Consistent with the mechanism at play in our model, Li and Schürhoff (2014) show that municipal bonds without credit ratings and bonds with speculative ratings are more likely to be traded through long intermediation chains than municipal bonds with investment-grade ratings, which arguably are less likely to be associated with large adverse selection problems. Further, Di Maggio, Kermani, and Song (2015) find that average chain length in the corporate bond market increased following Lehman Brothers' collapse, a time during which uncertainty and the potential for information asymmetries spiked.

In the next section, we model a fairly standard adverse selection problem between two traders.

[4] Madhavan and Smidt (1993) also combine asymmetric information and inventory management motives, but their model remains silent about the empirical phenomenon of intermediation chains; their model features centralized trading, rather than OTC trading, and does not allow for multiple intermediaries.

We analyze in Section 3 how involving an intermediary affects this adverse selection problem. In Section 4, we extend our analysis to chains of intermediaries. In Section 5, we discuss the implementation of this type of intermediation, the robustness of our results to alternative information structures, and their relationship to other mechanisms aimed at solving informational problems. The last section concludes. Unless stated otherwise, proofs are relegated to Appendix A.

2 Direct Bilateral Trade

We initially consider two risk-neutral agents who can trade one unit of an asset over the counter: the current owner who values the asset ex post at $v \in [0,1]$ and a potential buyer who values it at $B(v)$. The function $B(v)$ is strictly increasing, continuous and twice differentiable. It also satisfies $B(v) > v$ for all $v \in [0,1]$, implying that trade is efficient if it occurs with probability 1. The probability density function (PDF) of the value v, denoted by $f(v)$, is strictly positive on the support $[0,1]$, and the cumulative distribution function (CDF), denoted by $F(v)$, is continuous and twice differentiable. The functions $B(v)$ and $F(v)$ are common knowledge, but v is uncertain and traders are asymmetrically informed about v at the time of the trade.

Although the role that intermediation will later play in our model is relatively simple, multi-layered bargaining problems with asymmetric information are usually complex to analyze given the potential for multiple equilibria. We therefore make a few stylized assumptions that will allow us to keep the model tractable, even when we consider in Section 4 multiple sequential transactions occurring among a large number of heterogeneously informed traders.

First, we assume that, in any transaction, the current holder of the asset makes an ultimatum offer (i.e., quotes an ask price) to his counterparty. Focusing on ultimatum offers simplifies the analysis of equilibrium bidding strategies and is consistent with the characterization of inter-dealer trading in financial markets by Viswanathan and Wang (2004, p.3) as "very quick interactions". Ultimatum offers are also consistent with how Duffie (2012, p.2) describes the typical negotiation process in OTC markets and the notion that each OTC dealer tries to maintain "a reputation for standing firm on its original quotes."

Second, we assume that prior to trading the seller is uninformed about the realization of v, whereas the buyer is an expert who observes v. Note that for many financial products endowing a "buyer" with the informational advantage rather than the "seller" is an unrestrictive assumption; for example, a firm could be viewed as the buyer of an insurance policy, or, alternatively, as the seller of a risk exposure. In Section 5, we consider a few alternative information structures, including one with two-sided asymmetric information, and show how our main results can survive outside of our baseline setting.

Third, agents know how well informed their counterparties are. Although traders in our setting are asymmetrically informed about v, all traders know the quality of the information available to their counterparties. Seppi (1990) lends support to this assumption arguing that agents knowing the identity of their trading counterparties is an important distinction between OTC trading and centralized/exchange trading.[5]

Together, these assumptions create a situation where an uninformed monopolist makes a proposal to a better informed responder. Thus, signaling concerns do not arise and we obtain a unique subgame-perfect Nash equilibrium under direct trade. Hence, we are able to characterize many objects of interest that would otherwise not be uniquely pinned down. Below we derive the condition that needs to be satisfied to sustain efficient trade when the seller and buyer trade directly with each other.

Analysis. If the seller quotes a price p, the buyer accepts the offer whenever $B(v) \geq p$, which occurs with probability $\Pr[v \geq B^{-1}(p)]$. Equivalently, we can express the seller's price quote in terms of the marginal buyer type, who observes a realization of v equal to $w \equiv B^{-1}(p)$ and who is then indifferent between accepting and rejecting to a price quote of p. We can write the seller's expected payoff as a function of $w \in [0, 1]$:

$$\Pi(w) \equiv [1 - F(w)]B(w) + F(w)E[v|v < w]. \tag{1}$$

The marginal benefit of increasing w is then given by:

$$\begin{aligned}\Pi'(w) =&[1 - F(w)]B'(w) - f(w)B(w) + f(w)E[v|v < w] \\ &+ F(w)\frac{\partial}{\partial w}E[v|v < w],\end{aligned} \tag{2}$$

which simplifies to:

$$\Pi'(w) = [1 - F(w)]B'(w) - f(w)[B(w) - w]. \tag{3}$$

The first term on the right-hand side of equation (3) represents the benefit of collecting a higher price when the buyer accepts the offer. The second term represents the cost from reducing the probability of trade and jeopardizing trade surplus. We can rewrite the marginal benefit of increasing w as:

$$\Pi'(w) = [1 - F(w)]B'(w)[1 - H(w)], \tag{4}$$

where $H(w) \equiv \left(\frac{f(w)}{1-F(w)}\right)\left(\frac{B(w)-w}{B'(w)}\right)$ summarizes the ratio of the above-mentioned cost and benefit of increasing w. The seller finds it optimal to quote a higher, less efficient price than $p = B(w)$

[5] Morris and Shin (2012) relax the common-knowledge assumption in a bilateral trading setup similar to the one in this section and show how the resulting coordination problems can magnify the effect adverse selection has on trade efficiency.

whenever $H(w) < 1$. If on the other hand $H(w) \geq 1$, the benefit of collecting a higher price when the buyer accepts to trade is dominated by the cost of reducing the acceptance probability and jeopardizing trade surplus.

We impose the following regularity condition on $H(w)$ to guarantee that the marginal profit $\Pi'(w)$ crosses zero (from above) at most once and that we have a unique subgame perfect Nash equilibrium under direct trade:

Assumption 1 *$H(v)$ is strictly increasing in v for $v \in [0, 1]$.*

For trade to occur efficiently, the seller needs to quote a price that is accepted with probability 1. Since $f(v)$ is strictly positive everywhere on the support $[0, 1]$, the maximum price the seller can quote while maintaining efficient trade is $p = B(0)$, which corresponds to choosing a marginal buyer type $w = 0$. As a result, direct trade is efficient if and only if

$$\Pi'(0) \leq 0, \tag{5}$$

or equivalently

$$H(0) \geq 1. \tag{6}$$

Efficient trade may be sustained in our setting despite the asymmetric information problem, because the gains to trade are assumed to always be positive (see, e.g., D'Aspremont and Gérard-Varet 1979, Myerson and Satterthwaite 1983, Samuelson 1984, for general analyses of the conditions required to have first-best allocations being implementable under asymmetric information). This assumption will allow for a tractable analysis of the impact of involving multiple intermediaries on the efficiency of trade (see Section 4).

When the gains to trade are independent of v, that is, when $B(v) = v + \Delta$ with $\Delta > 0$, the function $H(v)$ simplifies to $H(v) = \left(\frac{f(v)}{1-F(v)}\right)\Delta$ and Assumption 1 is satisfied if and only if the hazard rate $h(v) \equiv \frac{f(v)}{1-F(v)}$ is strictly increasing. It is easy to verify that this last condition is satisfied by the uniform distribution and by a range of (truncated) parameterizations of the normal distribution, the Chi squared distribution, and the Gamma distribution, just to name a few.

Assumption 1 is also reminiscent of the definition of a strictly regular environment by Fuchs and Skrzypacz (2015) and is closely related to a standard assumption in auction theory that bidders' virtual valuation functions are strictly increasing (Myerson 1981). To see this, we define the function $\varphi(w)$ as the derivative of the seller's expected payoff with respect to the probability of trade (rather than w.r.t. w) when quoting a price $p = B(w)$:

$$\varphi(w) \equiv \Pi'(w)\frac{dw}{d(1-F(w))} = B(w) - w - \frac{B'(w)}{f(w)}(1 - F(w)). \tag{7}$$

The function $\varphi(w)$ represents the difference between the buyer's virtual valuation and the seller's marginal valuation when $v = w$.[6] It also provides an intuitive interpretation of the seller's trade-off when quoting a price $p = B(w)$, but this time based on the probability of trade. The incremental price concession necessary to achieve a marginal increase in the probability of trade is $\frac{B'(w)}{f(w)}$ and trade occurs with probability $(1 - F(w))$. The product of these two terms can thus be interpreted as the additional information rent the seller has to share with the buyer, in expectation, in order to achieve a marginal increase in the probability of trade. The function $\varphi(w)$ then represents the *difference* between the gains to trade, $B(w) - w$, and the marginal information rent going to the buyer, $\frac{B'(w)}{f(w)}(1 - F(w))$. Computing the *ratio* of gains to trade to the marginal information rent yields $H(w)$. If we assume constant gains to trade, a strictly increasing $\varphi(v)$ simplifies to a strictly increasing hazard rate $h(v)$ and is thus equivalent to Assumption 1. With more general definitions of $B(v)$, these two conditions are mathematically different, yet they yield the same results in our model for the case of direct trade. As we further discuss in the next section, imposing Assumption 1 will, however, yield an additional useful property when we introduce intermediaries and analyze their impact on trade efficiency.

3 Intermediated Trade

In this section, we consider the involvement of an intermediary who is *moderately informed*, in the sense that he receives an imperfect signal about v, making him more informed than the seller but less informed than the buyer. Specifically, we assume that the intermediary receives one of N possible signal realizations, each associated with the conditional distributions $F_i(v)$ for v, where $i \in \{1, 2, ...N\}$. The probability of collecting each signal i is denoted $\pi_i > 0$. We also assume that $F_{i+1}(v)$ first-order stochastically dominates $F_i(v)$ and define $\underline{v}_i \equiv \inf\{v \in [0, 1] : F_i(v) > 0\}$ and $\overline{v}_i \equiv \sup\{v \in [0, 1] : F_i(v) < 1\}$. Just like the seller, this intermediary privately values the asset at v and thus cannot help realize gains to trade unless he resells the asset to the buyer and thereby facilitates a more efficient allocation. Moreover, this intermediary does not bring new information to the table, as his information set is nested by that of the expert buyer. However, as we show below, intermediation through this moderately informed trader can improve the efficiency of trade by layering the information asymmetry over two sequential transactions.

Consider a simple trading network in which the uninformed agent offers to sell the asset to

[6] The CDF of the buyer's valuation satisfies: $G(B(v)) = F(v)$, which implies that the PDF satisfies: $g(B(v)) = \frac{f(v)}{B'(v)}$. The quantity $\frac{B'(v)}{f(v)}(1 - F(v))$ is thus equal to $\frac{1-G(B(v))}{g(B(v))}$ and we can write $\varphi(w) = B(w) - \frac{1}{h_b(B(w))} - w$, where h_b is the hazard rate function for the buyer's valuation. The expression $B(w) - \frac{B'(w)}{f(w)}(1 - F(w))$ can thus be interpreted as the buyer's virtual valuation of the asset when $v = w$. If we considered, as is often the case in the auction literature, that the seller's valuation of the asset is always zero rather than v, $\varphi(w)$ would become the buyer's virtual valuation function.

the intermediary. If trade occurs, the intermediary offers to sell the asset to the next trader in the network, in this case, the expert buyer. Further, consistent with how we modeled trading without an intermediary, we assume that whoever owns the asset and tries to sell it makes an ultimatum offer to his counterparty.

To sustain efficient trade the intermediary has to quote a price, after any signal i, that the buyer accepts with probability 1. The intermediary must therefore find it optimal to quote $B(\underline{v}_i)$ to the buyer after receiving signal i (since $B(\underline{v}_i)$ then represents the buyer's lowest possible valuation of the asset). Using the same algebra as above, we know that a necessary condition for efficient trade is that a marginal deviation from this efficient price reduces the intermediary's conditional expected payoff $\Pi_i(w)$. That is, we need:

$$\Pi_i'(\underline{v}_i) = [1 - F_i(\underline{v}_i)]B'(\underline{v}_i)[1 - H_i(\underline{v}_i)] \leq 0 \qquad \text{for} \quad i \in \{1, 2, ..., N\}, \tag{8}$$

where we define $H_i(v) \equiv \left(\frac{f_i(v)}{1-F_i(v)}\right)\left(\frac{B(v)-v}{B'(v)}\right)$. Moreover, if it is the case (as we show below) that all functions $H_i(v)$ are strictly increasing in v on their respective domains $[\underline{v}_i, \overline{v}_i]$, these N inequalities become sufficient conditions for the efficiency of trade between the intermediary and the buyer.

Since the $B(v)$ function does not change with the involvement of an intermediary, it is the shape of the N conditional distributions $F_i(v)$ that will dictate whether efficient trade can be sustained with an intermediary. If for some signal i, the conditional PDF $f_i(v)$ is small enough at the minimum value $\underline{v}_i$, then the buyer's information rents at the margin, $B'(\underline{v}_i)/f_i(\underline{v}_i)$, are so large that the intermediary prefers to quote a price $p > B(\underline{v}_i)$, which leads to trade breaking down with positive probability. However, if the involvement of a moderately informed intermediary implies a set of $f_i(\underline{v}_i)$ that are high enough to make $H_i(\underline{v}_i) > H(0)$ for all signals i, then the intermediary may be able to sustain efficient trade in situations where direct trade would be inefficient (i.e., $H(0) < 1$). Intuitively, a higher value for the conditional PDF $f_i(\underline{v}_i)$ reflects the notion that the intermediary's information concentrates probability mass in specific regions of the unconditional support $[0, 1]$. As the intermediary knows the buyer's valuation with more precision, the information rents, or price concessions, that are required to increase the probability of trade are reduced. For a given level of gains to trade $B(v) - v$, the intermediary then has stronger incentives to trade efficiently with the buyer.

For intermediated trade to be efficient the seller also has to quote a price that the intermediary accepts with probability 1. Since stochastic dominance implies that $\underline{v}_1 = 0$, the seller has to prefer quoting a price of $B(0)$ to the intermediary rather than the intermediary's maximum willingness to pay after receiving any of the better signals $i \geq 2$, which is $B(\underline{v}_i)$, the price at which the intermediary will trade with the buyer, provided that trade is efficient.

Overall, the involvement of an intermediary leads to the replacement of the condition for efficient trade under direct trade (see equation (5)) by a set of $(N+1)$ conditions. Whether involving the intermediary increases the efficiency of trade greatly depends on the shape of the conditional distributions $F_i(v)$. As we show below, a particular type of conditional distributions that strictly weakens the condition for efficient trade are partitions. In this regard, it is useful to establish the following Lemma.

Lemma 1 *If Assumption 1 is satisfied under a distribution $F(v)$, it is also satisfied under any truncated version of that distribution.*

Lemma 1 is the reason why in Section 2 we imposed a regularity condition on $H(v)$ rather than on $\varphi(v)$. Unlike with a strictly increasing $H(v)$ function, a strictly increasing $\varphi(v)$ function does not guarantee that an analogous property holds for the truncated version of $F(v)$. As in the case with direct trade, Assumption 1 guarantees that the marginal profit function for the intermediary crosses zero (from above) at most once when quoting a price to the buyer and that we have, generically, a unique subgame perfect Nash equilibrium under intermediated trade.[7] We can now establish our first main result, which is followed by detailed explanations of the intuition with the help of a parameterized example.

Proposition 1 *Let $\Omega^1(F)$ and $\Omega^0(F)$ respectively denote the set of functions $B(v)$ that are associated with efficient trade with and without an intermediary when the CDF of v is given by $F(v)$. If the intermediary's signal partitions the support of v into $N \geq 2$ subintervals, then the set of functions $B(v)$ that allow for efficient trade is strictly larger with the intermediary, i.e., $\Omega^0(F) \subset \Omega^1(F)$.*

Proposition 1 shows that with the involvement of a specific type of intermediary efficient trade can be obtained more easily than without one. When holding the asset, the seller's expected payoff from quoting a price to his better informed counterparty is the difference between the total expected surplus from trade and the information rents to be appropriated by subsequent traders in the network (i.e., the intermediary, when present, and the buyer). Similarly, when holding the asset, the intermediary's expected payoff is the difference between the conditional expected surplus from trade and the buyer's conditional information rent. Just as under direct trade (see, e.g., equation (7)), each potential proposer thus trades off the positive impact of offering price concessions on the probability of trade, and thus trade surplus, against the cost from increased information rents

[7] As under direct trade, a strictly increasing $H_i(v)$ implies that the intermediary's optimal price quote to the buyer is unique. By backward induction, the seller can anticipate the intermediary's uniquely optimal response to any price quote, which implies a unique subgame perfect Nash equilibrium, except in knife-edge parameterizations for which the seller is indifferent between two prices.

going to other traders. The elasticity of those information rents with respect to the probability of trade is in turn affected by the responder's informational advantage, and therefore changes with the involvement of a moderately informed intermediary.

As the information advantage of the counterparty he faces is reduced, a proposer's ability to quote aggressive prices becomes a less powerful tool to reduce the information rents of subsequent traders. In the limiting case where his immediate counterparty is just as informed as the proposer, the probability of trade drops discretely from one to zero when the proposer quotes a price that just exceeds the maximum efficient price. While such an inefficient price eradicates all information rents going to subsequent traders it also eliminates any surplus from trade that the proposer could appropriate and is therefore a suboptimal trading strategy. When his immediate counterparty is slightly better informed than the proposer, deviating from efficient trade by quoting an aggressive price that reduces the probability of trade is associated only with a small reduction in subsequent traders' information rents. On the other hand, intermediation allows the full gains to trade to remain at stake in each transaction, provided that trade is expected to be efficient in subsequent transactions. In these cases intermediation reduces each proposer's benefit of deviating from efficient price quotes but maintains the cost, thus tilting the trade-off toward efficient behavior.

To illustrate more concretely how the involvement of a moderately informed intermediary reshapes the adverse selection problem between two asymmetrically informed traders, we analyze a simple parameterized example that satisfies Assumption 1. We first illustrate how intermediated trade may be efficient when direct trade is not. Then, we analyze cases where efficient trade cannot be achieved with an intermediary and show that, depending on parameter values, intermediated trade can either improve or worsen the efficiency of trade, relative to direct trade.

Parameterized Example. Suppose the asset is worth $v \sim U[0,1]$ to the seller and $v + \Delta$ to the buyer. Without an intermediary, the seller may consider quoting any price $p \in [\Delta, 1 + \Delta]$, which is then accepted by the expert buyer with probability $\Pr[v + \Delta \geq p] = 1 - (p - \Delta)$. Defining $w \equiv p - \Delta$, the seller's optimization problem can be written as:

$$\max_{w \in [0,1]} \Pi(w) = (1 - w)(w + \Delta) + w \cdot \frac{w}{2}. \tag{9}$$

For efficient trade to occur without the intermediary, we need $\Pi'(0) \leq 0$, or equivalently, $H(0) \geq 1$. Thus, efficient trade is only possible if $\Delta \geq 1$, since in this example:

$$H(w) = \left(\frac{1}{1 - w}\right) \Delta. \tag{10}$$

If $\Delta < 1$ instead, the seller sets w such that $\Pi'(w) = 0$, or equivalently, $H(w) = 1$. The price quoted is then $p = 1$ and the probability of trade is Δ.

Sustaining efficient trade with intermediation. With the involvement of an intermediary who knows whether $v \in [0, c)$ or $v \in [c, 1]$ trade may be efficient even if $\Delta < 1$. When holding the asset and knowing that $v < c$, the intermediary's optimization problem can be written as:

$$\max_{w \in [0,c]} \left(1 - \frac{w}{c}\right)(w + \Delta) + \frac{w}{c} \cdot \frac{w}{2}, \tag{11}$$

and we need $\Delta \geq c$ for trade to occur with probability 1 in this scenario. Similarly, when holding the asset and knowing that $v \geq c$, the intermediary's optimization problem can be written as:

$$\max_{w \in [c,1]} \left(1 - \frac{w - c}{1 - c}\right)(w + \Delta) + \left(\frac{w - c}{1 - c}\right)\left(\frac{c + w}{2}\right), \tag{12}$$

and we need $\Delta \geq 1 - c$ for trade to occur with probability 1 in this scenario.

Given that, the seller chooses between quoting a price Δ, which is accepted by the intermediary with probability 1, and a price $c+\Delta$, which is accepted by the intermediary with probability $(1-c)$. The seller is in expectation wealthier when quoting Δ rather than $c + \Delta$ if and only if:

$$\Delta \geq (1 - c)(c + \Delta) + c \cdot \frac{c}{2}, \tag{13}$$

which simplifies to $\Delta \geq 1 - \frac{c}{2}$. Since $c \in [0, 1]$, this last restriction is weakly more restrictive than the earlier restriction that $\Delta \geq 1 - c$.

For trade to be efficient with the intermediary, we thus need $\Delta \geq \max\{c, 1 - \frac{c}{2}\}$, which is a strictly weaker condition than $\Delta \geq 1$ whenever $c \in (0, 1)$. As highlighted above, it is useful to interpret these results in light of the trade-off between sustaining a high surplus from trade on the one hand and reducing the information rents appropriated by counterparties on the other. In particular, consider a seller who contemplates deviating from quoting the efficient price Δ. Under direct trade, the buyer's marginal information rent is $\frac{1}{h(0)} = 1$. When the intermediary is the one trading with the buyer, this information rent drops to either c or $(1 - c)$, depending on the intermediary's signal realization. When trading with the intermediary, the seller is limited in how he can adjust the probability of trade and must choose among trade probabilities of 1, $(1 - c)$, or 0. The information rent reduction associated with a deviation to the inefficient price quote of $c + \Delta$ is $(1 - \frac{c}{2})$ (per probability unit), which again is smaller than under direct trade. Overall, this reduced sensitivity of better-informed traders' rents to the prices quoted promotes efficient behavior by proposers.

When $\max\{c, 1 - \frac{c}{2}\} \leq \Delta < 1$, intermediation increases the surplus from trade from Δ^2 to Δ. The intermediary's expected surplus increases from 0 to $c(1 - c)$. The buyer's expected surplus goes from $\frac{\Delta^2}{2}$ to $\frac{1}{2} - c(1-c)$, which for large or small values of c implies an increase in the buyer's expected surplus. The seller's expected surplus, on the other hand, decreases from $\frac{\Delta^2}{2}$ to $\Delta - \frac{1}{2}$. When an intermediary is involved, the difference in information quality between counterparties is small enough in both transactions to allow for efficient trade throughout the network. However, this involvement comes at the cost of adding a strategic agent who uses his monopoly power to capture a share of the surplus, making the uninformed seller worse off.

As a consequence, if allowed, the seller would prefer to bypass the intermediary and make an ultimatum offer to the buyer. This deviation would, however, lead to a lower social surplus. The trading network achieving a higher social surplus thus centers around a moderately informed intermediary and is *sparse*, in the sense that the seller cannot contact the buyer himself. In many decentralized markets, it is often impossible for retail or unsophisticated traders to contact the most sophisticated traders directly and bypass the usual middlemen. In fact, Li and Schürhoff (2014) estimate that for municipal bonds only 2.4% of all possible directed links are formed in reality, highlighting that sparse intermediated networks are sensible features of our analysis. Moreover, we will highlight in Section 5 the beneficial role that ex ante transfers, such as payments for order flow, can play in ensuring that the socially efficient trading network is implemented in equilibrium.

Intermediation when trade is inefficient. Intermediation may have ambiguous effects when it fails to sustain efficient trade. When trade is expected to break down with positive probability between the intermediary and the buyer, the seller knows that the total surplus from trade is smaller, reducing his benefits from trading efficiently. Intermediation thus not only affects the information rents required to sustain a high probability of trade, as illustrated above, but also the expected trade surplus at stake. Inefficiencies due to double marginalization may thus also arise in our setting where multiple monopolists are trading sequentially (Spengler 1950).

Consider first the case where $c = 0.9$ and $\Delta = 0.8$. Under direct trade the probability of trade is $\Delta = 0.8$. If trade is intermediated instead and the intermediary trades with the buyer, trade only occurs with probability $8/9$ when $v < c$ (since $\Delta < c$), but trade occurs with probability 1 when $v \geq c$ (since $\Delta > 1 - c$). While some of the expected surplus from trade is destroyed, the seller still prefers to quote the intermediary a price that is accepted with probability 1. Intermediated trade is then more efficient than direct trade, since the buyer obtains the asset with probability $1 - c \cdot \frac{1}{9} = 0.9$ when the intermediary is involved compared to a probability 0.8 when he is not.

Yet the situation changes if the surplus from trade Δ is 0.3. Here, trade between the intermediary and the buyer only occurs with probability $1/3$ when $v < c$ (since $\Delta < c$), whereas trade occurs with probability 1 when $v \geq c$ (since $\Delta > 1 - c$). The expected trade surplus available is now so small that the seller opts to reduce the information rents shared with the intermediary and the buyer at the cost of destroying even more surplus from trade. While quoting a price that the intermediary accepts with probability 1 would yield a payoff of 0.5 to the seller, quoting a price that the intermediary only accepts with probability 0.1 yields 0.525 in expectation. Since trade now breaks down in both transactions, the buyer only obtains the asset with probability 0.1, which is much lower than under direct trade.

Above, we discussed that the seller effectively has a coarser strategy space with respect to the probability of trade in the presence of an intermediary who observes a signal from a discrete, finite set. This coarseness can also contribute toward lowering the efficiency of trade. When facing an intermediary who obtains two possible signals as in the example here, the seller can only discretely adjust the probability of trade. When the intermediary's information rent is large enough that the seller optimally deviates to an inefficient price (i.e., when $1 - \frac{c}{2} > \Delta$), the jump in the probability of trade may exacerbate efficiency losses relative to direct trade, where the seller can effectively choose any probability of trade between 0 and 1. Suppose that $\frac{1}{2} < \Delta < \frac{3}{4}$, such that, under direct trade the probability of trade is Δ and that we involve an intermediary with $c = \frac{1}{2}$. From the above derivations we know that trade will be efficient once the asset reaches the intermediary, who quotes a price of Δ when he knows that $v < c$ and $\frac{1}{2} + \Delta$ when he knows that $v \geq c$. However, the seller still finds it optimal to quote the inefficient price $\frac{1}{2} + \Delta$, which is accepted with probability $\frac{1}{2}$. Thus, the asset reaches the intermediary with probability $\frac{1}{2}$, which is lower than Δ, the probability of trade without the intermediary.

Thus, while trading through one moderately informed intermediary may improve the efficiency of trade in some cases, it may also lead to less efficient outcomes when adverse selection is too severe, a result that might help us understand the role that intermediation might have played in the recent crisis, when information asymmetries were likely large, as suggested by Adrian and Shin (2010). However, we show in the next section that in those cases where efficient trade cannot be sustained by a given chain of intermediaries, lengthening that chain by adding extra intermediaries may be sufficient to eliminate inefficiencies caused by adverse selection.

4 Intermediation Chains

This section extends our earlier results and shows how long chains of intermediaries may sustain efficient trade in cases where shorter chains do not. In contrast with most models of endogenous intermediation where the optimal trading network is centralized around a unique intermediary and trading among intermediaries plays no role, our model can help us understand the prevalence of intermediation chains in many decentralized markets (Goldstein and Hotchkiss 2012, Bank of International Settlements 2013, Hollifield, Neklyudov, and Spatt 2014, Li and Schürhoff 2014, Di Maggio, Kermani, and Song 2015).

Suppose there are M intermediaries, indexed by m based on their position in the network.

(To simplify the notation, we label the seller as trader 0 and the buyer as trader $M+1$.) Each intermediary m observes a signal that partitions the domain $[0,1]$ into sub-intervals. The main mechanism that makes intermediation chains valuable in our model is similar to that featured in Section 3 and is best highlighted by assuming that the information set intermediary m observes before trading is nested by the information set of intermediary $(m+1)$, that is, intermediary $(m+1)$ observes a signal that creates a strictly finer conditional partition than intermediary m's signal. Formally, we make the following assumption:

Assumption 2 *If intermediary $m < M$ knows that $v \in [\underline{v}_i^m, \overline{v}_i^m)$ then intermediary $(m+1)'s$ information partitions $[\underline{v}_i^m, \overline{v}_i^m)$ into at least three sub-intervals.*

Nesting sequential traders' information sets in this fashion eliminates signaling concerns and implies a generically unique subgame perfect Nash equilibrium in our model, even though there are $(M+1)$ bargaining problems among $(M+2)$ heterogeneously informed agents. Assuming that there are at least three sub-intervals that separate each pair of counterparties guarantees that we are able to insert a "moderately informed" intermediary between each pair of counterparties m and $(m+1)$, if needed. This particular structure will allow us to extend some of our earlier results and show that long intermediation chains can preserve the efficiency of trade in situations where surplus would be destroyed with fewer intermediaries. As will become clear soon, what ultimately contributes to sustaining efficient trade in equilibrium is that the chain reduces the informational distance between counterparties, although information sets would not necessarily have to be nested for our mechanism to work.

The proposition below formalizes our main result regarding intermediation chains and is followed by the analysis of a parameterized example. Since the logic of this proof resembles that of Proposition 3, we relegate it to our Supplementary Appendix B.

Proposition 2 *Let $\Omega^M(F)$ denote the set of functions $B(v)$ that are associated with efficient trade in a chain of M intermediaries with information sets consistent with Assumption 2 when the CDF of v is given by $F(v)$. There exists a set of $\tilde{M} \geq 1$ intermediaries who can be added to the chain such that the set of functions $B(v)$ associated with efficient trade is strictly enlarged, that is, $\Omega^M(F) \subset \Omega^{M+\tilde{M}}(F)$.*

As before, when holding the asset a proposer's expected payoff is the difference between the total surplus from trade and the cumulative information rents going to all subsequent traders in the chain. If the responder has a small informational advantage over the proposer, deviating from efficient trade by quoting aggressive prices is not as effective in reducing the cumulative information rents going to subsequent, better informed traders. Yet, provided that trade is efficient in subsequent transactions, the surplus from trade that can be destroyed in each transaction by quoting aggressive prices stays the same. When anticipating efficient trade at later stages in the chain, strategies aimed at quoting inefficient prices to counterparties are thus discouraged in a long intermediation chain. It follows that an important implication of our analysis is that, in order to achieve efficient trade, intermediaries must be located within the trading network such that each trader's information set is similar, but not identical, to those of nearby traders. In these cases, it is socially optimal to have, for example, the least sophisticated intermediaries trading directly with the uninformed seller and the most sophisticated intermediaries trading directly with the expert buyer.

Proposition 2 also implies that, if the functions $B(v)$ and $F(v)$ change in ways that slightly worsen the adverse selection problem and lead to inefficiencies in a given network, a higher number of intermediaries may be needed to bridge the information asymmetries and sustain efficient trade. This prediction may help us understand why Li and Schürhoff (2014) find that municipal bonds with no credit rating and bonds with a speculative rating are typically traded through longer intermediation chains than municipal bonds with an investment-grade rating, which arguably are less likely to be associated with large adverse selection problems. Further, Di Maggio, Kermani, and Song (2015) document a lengthening of the chains for the corporate bond market following Lehman Brothers' collapse, a time during which uncertainty and the potential for information asymmetries spiked.

Below we revisit our earlier parameterized example to illustrate that, as the adverse selection problem between the ultimate buyer and seller worsens, it takes more intermediaries to sustain efficient trade.

Parameterized Example. Suppose the asset is worth $v \sim U[0,1]$ to the seller and $v + \Delta$ to the buyer. As part of the earlier example in Section 3, we showed that direct trade between the seller and the buyer can be efficient whenever $\Delta \geq 1$. We then showed that if an agent who knows whether $v \in [0, c)$ or $v \in [c, 1]$ acts as an intermediary between the seller and the buyer, trade is efficient whenever $\Delta \geq \max\{c, 1 - \frac{c}{2}\}$, which is a strictly weaker condition than $\Delta \geq 1$ whenever $c \in (0, 1)$.

Now, consider a case where $c = 0.75$. Trade through such an intermediary can only be efficient if $\Delta \geq 0.75$. However, if we involve a second intermediary who buys from the first intermediary and sells to the buyer, we might be able to further expand the region of Δ for which efficient trade is possible. Suppose this second intermediary observes a signal that informs him whether $v \in [0, 0.5)$, $v \in [0.5, 0.75)$, or $v \in [0.75, 1]$.

Using the same logic as before we know that, when holding the asset and knowing that $v < 0.5$, the second intermediary finds it optimal to quote the efficient price Δ to the buyer if $\Delta \geq 0.5$. Similarly, when holding the asset and knowing that $v \in [0.5, 0.75)$, the second intermediary finds it optimal to quote the efficient price $0.5 + \Delta$ to the buyer if $\Delta \geq 0.25$. Finally, when holding the asset and knowing that $v \geq 0.75$, the second intermediary finds it optimal to quote the efficient price $0.75 + \Delta$ to the buyer if $\Delta \geq 0.25$.

If these conditions are all satisfied, the first intermediary must decide what price to quote the second intermediary. When $v \geq 0.75$, the second intermediary does not have an informational advantage over the first intermediary, hence trade occurs with probability 1 at a price $0.75 + \Delta$. When $v < 0.75$, the first intermediary chooses between quoting a price Δ, which the second intermediary accepts with probability 1, or a price $0.5 + \Delta$, which the second intermediary accepts only with probability $\frac{1}{3}$. The first intermediary finds it optimal to quote the efficient price Δ whenever:

$$\Delta \geq \frac{1}{3}(0.5 + \Delta) + \frac{2}{3} \cdot 0.25, \tag{14}$$

which simplifies to $\Delta \geq 0.5$.

Finally, when he expects trade to be efficient in subsequent transactions, the seller decides between quoting a price Δ, which the first intermediary accepts with probability 1, or a price $0.75 + \Delta$, which the first intermediary accepts only with probability 0.25. The seller finds it optimal to quote the efficient price Δ whenever:

$$\Delta \geq 0.25\,(0.75 + \Delta) + 0.75 \cdot 0.375, \tag{15}$$

which simplifies to $\Delta \geq 0.625$.

Overall, trade is efficient with these two intermediaries whenever $\Delta \geq 0.625$, whereas it was efficient with only one intermediary (whose $c = 0.75$) whenever $\Delta \geq 0.75$. Moreover, it is easy to verify that the partition cutoff c that makes the condition for efficient trade least restrictive with one intermediary is $c = \frac{2}{3}$. In this case, efficient trade requires that $\Delta \geq \frac{2}{3}$. The condition for efficient trade with the two intermediaries above is thus weaker than the condition with one intermediary. These results show that as adverse selection worsens and the incentives to quote inefficient prices increase (e.g., when Δ decreases), a greater number of intermediaries are needed to sustain efficient trade.

These results also highlight that the optimality of specific trading networks greatly depends on the trading frictions that are most relevant in a given context. When efficient trade is impeded by an information asymmetry related to the value of the asset, our model shows that multiple heterogeneously informed intermediaries may help sustain the social efficiency of trade. When the information asymmetry instead relates to traders' past behavior, Babus (2012) suggests that optimal trading networks should be centered around a single intermediary. Further, Gofman (2011) shows that in the presence of non-informational bargaining frictions, socially efficient outcomes may be easier to achieve when networks are dense (although the relationship between density and efficiency is not always monotonic). In contrast, in our model a trading network needs to be sufficiently sparse to sustain efficient trade; otherwise, uninformed parties might privately benefit from trading relationships that reduce social efficiency. (We also discuss in the next section the role that payments for order flow might play in alleviating this problem.) Given that various trading frictions are more relevant in some situations than in others, our results and those derived in the related papers above can help us understand the types of network we observe in different contexts.

5 Discussion

In this section, we first discuss how order-flow agreements can be used to ensure that a socially optimal intermediation chain becomes privately optimal for all traders involved. We then lay out how our results would survive under alternative information structures. Finally, we relate our results to other mechanisms that might be considered to solve adverse selection problems.

Implementation. We showed in earlier sections that if our goal is to maximize the social surplus generated by trade between an uninformed seller and an expert buyer, it might help if the uninformed seller trades with a slightly better informed intermediary, who then trades with another slightly better informed intermediary, and so on until the asset reaches the expert buyer. Here, we characterize order-flow agreements that traders, who are endowed with different information sets, commit to *ex ante* (i.e., before information is obtained and trading occurs) and that ensure that no trader involved in an intermediation chain that sustains efficient trade will be tempted to form an alternative trading network.

As a result, these order-flow agreements can help implement socially optimal trading networks in our model, shedding light on potential downsides of recent proposals by regulatory agency and stock exchange officials to ban related practices in financial markets.[8]

Definition 1 (Order-flow agreement) *Consider an economy with a set of traders $\mathbb{T}$. An order-flow agreement Σ between a subset of traders $\mathbb{C} \subseteq \mathbb{T}$ specifies the following objects:*

1. *A collection of directed network links: each trader $m \in \mathbb{C}$ is exclusively connected to a unique counterparty $m' \in \{\mathbb{C} \setminus m\}$ to which trader m quotes an ultimatum offer whenever he wishes to sell.*

[8] See, for example, the comments made by Jeffrey Sprecher, CEO of IntercontinentalExchange (which owns the New York Stock Exchange), reported in "ICE CEO Sprecher wants regulators to look at 'maker-taker' trading" by Christine Stebbins on Reuters.com (January 26, 2014), the document titled "Guidance on the practice of 'Payment for Order Flow' " prepared by the Financial Services Authority (May 2012), and the comments made by Harvey Pitt, former Securities and Exchange Commission Chairman, reported in "Options Payment for Order Flow Ripped" by Isabelle Clary in Securities Technology Monitor (May 3, 2004).

2. *A collection of ex ante transfers between the traders in $\mathbb{C}$.*

A key component of these order-flow agreements are ex ante transfers that incentivize traders to commit to transacting with specific counterparties. In financial markets, these transfers may come in the form of explicit agreements involving cash payments for order flow or soft dollars, or they may be implicit arrangements involving profitable IPO allocations or subsidies on the various other services that intermediaries provide. In fact, there is ample empirical evidence that "perks" are commonly used by financial intermediaries to compensate traders for their business (see, e.g., Blume 1993, Chordia and Subrahmanyam 1995, Reuter 2006, Nimalendran, Ritter, and Zhang 2007). Further, for many types of securities, order-flow agreements are required to be disclosed in advance in Rule 606 reports. Thus, just like in our definition above, transfers linked with order-flow agreements do not vary based on transaction-specific information (i.e., a particular realization of v), but they may vary based on the expertise of the traders involved (Easley, Kiefer, and O'Hara 1996). This characterization distinguishes these ex ante transfers from the transfers that occur later as part of the trading process, i.e., the transaction prices.

Definition 2 (Equilibrium) *An order-flow agreement Σ between a set of traders $\mathbb{C} \subseteq \mathbb{T}$ constitutes an equilibrium if there is no coalition of traders $\mathbb{C}' \subseteq \mathbb{T}$ that can block the agreement, that is, there does not exist an order-flow agreement Σ' that only includes traders in $\mathbb{C}'$ and that makes every trader in $\mathbb{C}'$ weakly better off and at least one trader in $\mathbb{C}'$ strictly better off.*

Consistent with our previous analysis, we are interested in the cases for which intermediation chains can help sustain efficient trade. Below we characterize the existence of equilibrium order-flow agreements that support the type of intermediation chains we introduced in Section 4.

Proposition 3 (Equilibrium order-flow agreements) *If the set $\mathbb{T}$ contains traders endowed with information sets consistent with Assumption 2 that can form a chain that sustains efficient trade:*

1. *Any order-flow agreement that does not lead to efficient trade is not an equilibrium.*
2. *For any intermediation chain that allows for efficient trade there exists a corresponding order-flow agreement that constitutes an equilibrium.*

In our model, deal-flow is valuable to any intermediary included in an efficient trading network, since his informational advantage over his counterparty allows him to extract a fraction of the gains to trade $E[B(v) - v]$. Hence, intermediaries are willing to offer cash payments, or subsidized services, to the ultimate buyer and seller of the asset if these are required concessions for being involved in the trading network.

Alternative Information Structures. A few stylized assumptions about traders' information sets kept the analysis of our baseline model tractable. We now discuss how the intuition developed so far can be extended to more complex information structures. In our baseline model, it is the seller's ability to potentially appropriate additional rents by charging higher prices that creates the social inefficiency that intermediation chains can help alleviate. Unsurprisingly, this type of inefficient behavior may also arise under alternative information structures. In other settings, some transactions could, however, feature a proposer who has private information not known to the responder, implying that the setup would not have a unique subgame perfect Nash equilibrium. The goal of the discussion below is to highlight that, under various circumstances, there exists at least one type of equilibrium for which intermediation chains expand the parameter region in which efficient trade is attainable. Detailed derivations for the parametric example with a uniform distribution and constant gains to trade are provided in our Supplementary Appendix B.

First, it is possible to generate results that are almost identical to those in our baseline setting when the seller is informed and the buyer is uninformed. As before, we can eliminate signaling concerns by retaining the property that the less informed party makes an offer to his better informed party, in each transaction. Specifically, if the intermediary is allowed to sell the asset short, we can set up the network such that the uninformed trader makes an offer to a slightly better informed counterparty who then makes an offer to a fully informed counterparty. The mechanics of intermediation with short selling are slightly different than what we had in our baseline model, but we still have an uninformed trader choosing between an efficient trading strategy that requires price concessions and more aggressive strategy that destroys part of the social surplus. Thus, we can show that moderately informed intermediaries can potentially improve the efficiency of trade.

A similar adverse selection problem can also arise under two-sided asymmetric information. Suppose that the buyer and the seller have different pieces of private information about the value of the asset. In this case, the proposer still faces the following choice when setting a price: he can choose an aggressive price to appropriate additional rents or he can make price concessions to ensure that trade occurs for any realization of the responder's private information. As long as it stays optimal for the proposer to offer a pooling price that does not depend on his own private information, given the responder's off-equilibrium beliefs, trading behavior is similar to what we observe in our baseline model and moderately informed intermediaries can improve the efficiency of trade.

Other Mechanisms. In this paper, we show that trading through chains of heterogenously informed intermediaries might improve the efficiency of trade. As explained throughout the paper, the uninformed agent's bargaining power plays an important role in this result — it is the seller's ability to potentially appropriate additional rents by charging higher prices that creates the social inefficiency that intermediation chains might help alleviate. If the seller had no ability to seek additional rents in the first place (e.g., if there were multiple sellers making simultaneous offers to a unique buyer), this inefficiency would be assumed away.

The mechanism we propose differs from other interventions that increase competition for the seller's asset, which presumably can be good for the efficiency of trade. The fact that intermediaries help in some cases and not in others implies that our result is not just a matter of "competition" being better than a monopoly. In fact, the intermediaries we involve in the network are each endowed with monopoly power once they obtain the asset, potentially creating problems of double marginalization (Spengler 1950). Moreover, if instead of adding heterogenously informed monopolists, we added several monopolists who are either uninformed like the seller or perfectly informed like the buyer, intermediation chains would not improve the efficiency of trade relative to direct trade. In this case, most pairs of counterparties would be trading without an information asymmetry but whenever an uninformed trader would have to quote a price to a perfectly informed counterparty, trade would still break down, just as under direct trade.

To improve trading efficiency via the involvement of *homogeneously informed* traders, traders would need to compete simultaneously for the asset, not sequentially like in our setup. This form of mechanism would then rely on different forces than the mechanism we propose in this paper. For example, suppose that instead of one moderately informed intermediary as assumed in Section 3, we had two identical, moderately informed intermediaries who compete in a Bertrand fashion, that is, *simultaneously*, for the seller's asset and then trade with the expert buyer, once one of them obtains the asset. In this setting the seller would no longer make a take-it-or-leave-it offer, but rather decide whether to accept offers made by the two intermediaries. In equilibrium, the seller would always accept one of the intermediaries' offers.[9] The only condition for efficient trade in this alternative setting would come from the interaction between the intermediary holding the asset and the expert buyer, which is strictly less restrictive than the condition for direct trade to be efficient (see Proposition 1). Involving intermediaries who compete simultaneously for the asset would then allow the seller to appropriate the information rents that a unique intermediary would have collected if he had some monopoly power, as considered in our setup.

More generally, if we allowed for any mechanism, it would be trivial to show that adding informed agents could help the seller extract more surplus, using a Cremer and McLean (1988)-type mechanism for example. In particular, if multiple informed traders were to bid simultaneously for the seller's asset, the seller would use competition to effectively extract information from these bidders, leaving less information rents to these agents. This competition effect is, however, absent

[9]Let $\bar{v}_I$ denote the expected payoff an intermediary obtains if he currently owns the asset and quotes a privately optimal price quote to the buyer. This expected payoff accounts for the potential for trade breaking down, in which case the intermediary just collects v. Due to Bertrand competition among identical intermediaries, the seller obtains a price equal to $\bar{v}_I$ from the intermediaries and optimally accepts this price.

in our setting where trade is bilateral and the asset moves through each trader sequentially. The seller does not extract any information from competing bidders, but rather faces a single intermediary who is less informed than the expert. This smaller information gap between the seller and his counterparty can strengthen the seller's incentives to quote an efficient price. Overall, our solution to an adverse selection problem features decentralized, sequential trading among heterogeneously informed agents and is thus different from these other types of mechanisms.

6 Conclusion

This paper illustrates how chains of moderately informed intermediaries may help alleviate inefficiencies associated with adverse selection problems. Layering information asymmetries over multiple transactions can weaken traders' incentives to quote prices that jeopardize gains from trade, as such inefficient behavior becomes less effective in curtailing other traders' information rents. The severity of adverse selection problems in a trading network may thus exhibit convexities in counterparties' informational distances. Greater information asymmetries may thus be better bridged by longer intermediation chains. However, in cases where efficient trade is not sustained, intermediation chains may exhibit a higher degree of fragility than direct trade as problems of double marginalization may arise.

More broadly, our paper sheds light on the phenomenon highlighted by Adrian and Shin (2010, p.604) that the whole U.S. financial system has shifted in recent decades from its traditional, centralized model of financial intermediation to a more complex, market-based model characterized by "the long chain of financial intermediaries involved in channeling funds" (see also Kroszner and Melick 2009, Cetorelli, Mandel, and Mollineaux 2012, Pozsar et al. 2013, for similar characterizations).

References

Adrian, Tobias, and Hyun Song Shin, 2010, "The Changing Nature of Financial Intermediation and the Financial Crisis of 2007-2009," *Annual Review of Economics* 2, 603-618.

Akerlof, George A., 1970, "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics* 84, 488-500.

Babus, Ana, 2012, "Endogenous Intermediation in Over-the-Counter Markets," Working Paper, Imperial College London.

Bank of International Settlements, 2013, "BIS Quarterly Review," International Banking and Financial Market Developments, Basel, Switzerland.

Biglaiser, Gary, 1993, "Middlemen as Experts," *RAND Journal of Economics* 24, 212-223.

Blume, Marshall E., 1993, "Soft Dollars and the Brokerage Industry," *Financial Analysts Journal* 49, No. 2, 36-44.

Cetorelli, Nicola, Benjamin H. Mandel, and Lindsay Mollineaux, 2012, "The Evolution of Banks and Financial Intermediation: Framing the Analysis," *FRBNY Economic Policy Review*, July Issue, 1-12.

Cho, In-Koo, and David M. Kreps, 1987, "Signalling Games and Stable Equilibria," *Quarterly Journal of Economics* 102, 381-413.

Chordia, Tarun, and Avanidhar Subrahmanyam, 1995, "Market Making, the Tick Size, and Payment-for-Order Flow: Theory and Evidence," *Journal of Business* 68, 543-575.

Crémer, Jacques, and Richard P. McLean, 1988, "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica* 56, 1247-1257.

D'Aspremont, Claude, and Louis-André Gérard-Varet, 1979, "Incentives and Incomplete Information," *Journal of Public Economics* 11, 25-45.

Diamond, Douglas W., 1984, "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies* 51, 393-414.

Di Maggio, Marco, Amir Kermani, and Zhaogang Song, 2015, "The Value of Trading Relationships in Turbulent Times," Working Paper, Columbia University.

Duffie, Darrell, 2012, *Dark Markets: Asset Pricing and Information Transmission in Over-the-Counter Markets*, Princeton: Princeton University Press.

Duffie, Darrell, Nicolae Gârleanu, and Lasse Heje Pedersen, 2005, "Over-the-Counter Markets," *Econometrica* 73, 1815-1847.

Easley, David, Nicholas M. Kiefer, and Maureen O'Hara, 1996, "Cream-Skimming or Profit-Sharing? The Curious Role of Purchased Order Flow," *Journal of Finance* 51, 811-833.

Farboodi, Maryam, 2014, "Intermediation and Voluntary Exposure to Counterparty Risk," Working Paper, University of Chicago.

Fuchs, William, and Andrzej Skrzypacz, 2015, "Government Interventions in a Dynamic Market with Adverse Selection," *Journal of Economic Theory*, forthcoming.

Glosten, Lawrence R., and Paul R. Milgrom, 1985, "Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics* 14, 71-100.

Gofman, Michael, 2011, "A Network-Based Analysis of Over-the-Counter Markets," Working Paper, University of Wisconsin - Madison.

Goldstein, Michael A., and Edith S. Hotchkiss, 2012, "Dealer Behavior and the Trading of Newly Issued Corporate Bonds," Working Paper, Boston College.

Green, Richard C., Burton Hollifield, and Norman Schürhoff, 2007, "Financial Intermediation and the Costs of Trading in an Opaque Market," *Review of Financial Studies* 20, 275-314.

Hagerty, Kathleen, and Robert L. McDonald, 1995, "Brokerage, Market Fragmentation, and Securities Market Regulation," in *The Industrial Organization and Regulation of the Securities Industry*, edited by Andrew W. Lo, The University of Chicago Press, 35-62.

Ho, Thomas, and Hans R. Stoll, 1983, "The Dynamics of Dealer Markets under Competition," *Journal of Finance* 38, 1053-1074.

Hollifield, Burton, Artem Neklyudov, and Chester Spatt, 2014, "Bid-Ask Spreads and the Pricing of Securitizations: 144a vs. Registered Securitizations," Working Paper, Carnegie Mellon University.

Jiang, Hao, and Zheng Sun, 2015, "News and Corporate Bond Liquidity," Working Paper, University of California, Irvine.

Jovanovic, Boyan, and Albert J. Menkveld, 2012, "Middlemen in Limit-Order Markets," Working Paper, New York University.

Kroszner, Randall S., and William Melick, 2009, “The Response of the Federal Reserve to the Recent Banking and Financial Crisis,” Working Paper, University of Chicago.

Kyle, Albert S., 1985, “Continuous Auctions and Insider Trading,” *Econometrica* 53, 1315-1335.

Li, Yiting, 1998, “Middlemen and Private Information,” *Journal of Monetary Economics* 42, 131-159.

Li, Dan, and Norman Schürhoff, 2014, “Dealer Networks,” Working Paper, Swiss Finance Institute.

Lyons, Richard K., 1996, “Foreign Exchange Volume: Sound and Fury Signifying Nothing?,” in *The Microstructure of Foreign Exchange Markets*, National Bureau of Economic Research, 183-208.

Madhavan, Ananth, and Seymour Smidt, 1993, “An Analysis of Changes in Specialist Inventories and Quotations,” *Journal of Finance* 48, 1595-1628.

Malamud, Semyon, and Marzena Rostek, 2013, “Decentralized Exchanges,” Working Paper, Swiss Finance Institute.

Manaster, Steven, and Steven C. Mann, 1996, “Life in the Pits: Competitive Market Making and Inventory Control,” *Review of Financial Studies* 9, 953-975.

Morris, Stephen, and Hyun Song Shin, 2012, “Contagious Adverse Selection,” *American Economic Journal: Macroeconomics* 4, 1-21.

Myerson, Roger, 1981, “Optimal Auction Design,” *Mathematics of Operations Research* 6(1):58-73.

Myerson, Roger B., and Mark A. Satterthwaite, 1983, “Efficient Mechanisms for Bilateral Trading,” *Journal of Economic Theory* 29, 265-281.

Neklyudov, Artem V., 2013, “Bid-Ask Spreads and the Decentralized Interdealer Markets: Core and Peripheral Dealers,” Working Paper, University of Lausanne.

Nimalendran, M., Jay R. Ritter, and Donghang Zhang, 2007, “Do Today’s Trades Affect Tomorrow’s IPO Allocations?,” *Journal of Financial Economics* 84, 87-109.

Pozsar, Zoltan, Tobias Adrian, Adam Ashcraft, and Hayley Boesky, 2013, “Shadow Banking,” *FRBNY Economic Policy Review*, December Issue, 1-16.

Reuter, Jonathan, 2006, “Are IPO Allocations for Sale? Evidence from Mutual Funds,” *Journal of Finance* 61, 2289-2324.

Rubinstein, Ariel, and Asher Wolinsky, 1987, “Middlemen,” *Quarterly Journal of Economics* 102, 581-594.

Samuelson, William, 1984, “Bargaining under Asymmetric Information,” *Econometrica* 52, 995-1005.

Seppi, Duane J., 1990, “Equilibrium Block Trading and Asymmetric Information,” *Journal of Finance* 45, 73-94.

Spengler, Joseph, 1950, “Vertical Integration and Anti-trust Policy,” *Journal of Political Economy* 58, 347-352.

Townsend, Robert M., 1978, “Intermediation with Costly Bilateral Exchange,” *Review of Economic Studies* 45, 417-425.

Viswanathan, S., and James J.D. Wang, 2004, “Inter-Dealer Trading in Financial Markets,” *Journal of Business* 77, 1-54.

Weller, Brian, 2013, “Liquidity and High Frequency Trading,” Working Paper, University of Chicago.

Wright, Randall, and Yuet-Yee Wong, 2014, “Buyers, Sellers, and Middlemen: Variations on Search-Theoretic Themes,” *International Economic Review* 55, 375-397.

Yavaş, Abdullah, 1994, “Middlemen in Bilateral Search Markets,” *Journal of Labor Economics* 12, 406-429.

Zhu, Haoxiang, 2012, “Finding a Good Price in Opaque Over-the-Counter Markets,” *Review of Financial Studies* 25, 1255-1285.

Appendix A: Proofs of Selected Results

Proof of Lemma 1: Assumption 1 implies that:

$$\left(\frac{f(x)}{1-F(x)}\right)\left(\frac{B(x)-x}{B'(x)}\right) > \left(\frac{f(x')}{1-F(x')}\right)\left(\frac{B(x')-x'}{B'(x')}\right) \tag{A1}$$

for any $x > x'$. We want to show that any truncated distribution on the support $[a, b] \subseteq [0, 1]$ with PDF $g(x) = \frac{f(x)}{F(b)-F(a)}$ and CDF $G(x) = \frac{F(x)-F(a)}{F(b)-F(a)}$ also satisfies:

$$\left(\frac{g(x)}{1-G(x)}\right)\left(\frac{B(x)-x}{B'(x)}\right) > \left(\frac{g(x')}{1-G(x')}\right)\left(\frac{B(x')-x'}{B'(x')}\right). \tag{A2}$$

Substituting the definitions of g and G we rewrite this inequality as:

$$\begin{aligned}
\left(\frac{\frac{f(x)}{F(b)-F(a)}}{\frac{F(b)-F(a)-F(x)+F(a)}{F(b)-F(a)}}\right)\left(\frac{B(x)-x}{B'(x)}\right) &> \left(\frac{\frac{f(x')}{F(b)-F(a)}}{\frac{F(b)-F(a)-F(x')+F(a)}{F(b)-F(a)}}\right)\left(\frac{B(x')-x'}{B'(x')}\right) \\
\Leftrightarrow \left(\frac{f(x)}{F(b)-F(x)}\right)\left(\frac{B(x)-x}{B'(x)}\right) &> \left(\frac{f(x')}{F(b)-F(x')}\right)\left(\frac{B(x')-x'}{B'(x')}\right) \\
\Leftrightarrow \frac{F(b)-F(x')}{F(b)-F(x)} &> \left(\frac{f(x')}{f(x)}\right)\left(\frac{B(x')-x'}{B(x)-x}\right)\left(\frac{B'(x)}{B'(x')}\right).
\end{aligned} \tag{A3}$$

By Assumption 1, we know that the following inequality holds:

$$\frac{1-F(x')}{1-F(x)} > \left(\frac{f(x')}{f(x)}\right)\left(\frac{B(x')-x'}{B(x)-x}\right)\left(\frac{B'(x)}{B'(x')}\right). \tag{A4}$$

It is therefore sufficient to verify that:

$$\frac{F(b)-F(x')}{F(b)-F(x)} \geq \frac{1-F(x')}{1-F(x)}. \tag{A5}$$

Recall that $F(x) > F(x')$ since $x > x'$ and because $f(x)$ is strictly positive on the support $[0, 1]$. If we set $z = F(b)$, our result simply follows from:

$$\frac{\partial}{\partial z}\left(\frac{z-F(x')}{z-F(x)}\right) = \frac{z-F(x)-(z-F(x'))}{(z-F(x))^2} = \frac{F(x')-F(x)}{(z-F(x))^2} < 0. \tag{A6}$$

∎

Proof of Proposition 1: Let $\Psi(F)$ denote the set of functions $B(v)$ that are consistent with our regularity condition, Assumption 1, for a given CDF $F(v)$. We can thus write the set $\Omega^0(F)$ as:

$$\Omega^0(F) = \{B \in \Psi(F) : \Pi'(0) \leq 0\}. \tag{A7}$$

In order to know whether a function $B(v) \in \Omega^0(F)$ also satisfies $B(v) \in \Omega^1(F)$, we must first verify that trade occurs with probability 1 once the intermediary quotes a price to the buyer. Suppose the intermediary holds the asset and has received a signal i that implies that $v \in [\underline{v}_i, \overline{v}_i)$. The intermediary picks a price $p = B(w)$ to maximize his expected payoff:

$$\Pi_i(w) = (1 - F_i(w))\, B(w) + F_i(w) E[v | v < w, i]. \tag{A8}$$

By Lemma 1, $H_i(v)$ is strictly increasing on $[\underline{v}_i, \overline{v}_i)$ and the condition for efficient trade between the intermediary and the buyer when the intermediary receives a signal i depend on the profitability of a marginal deviation from the lower bound of the conditional support:

$$\Pi_i'(\underline{v}_i) = (1 - F_i(\underline{v}_i) B'(\underline{v}_i)[1 - H_i(\underline{v}_i)] \leq 0. \tag{A9}$$

By definition, all $B(v) \in \Omega^0(F)$ satisfy $\Pi'(0) \leq 0$, or equivalently, $H(0) \geq 1$. This last inequality implies that $H_i(\underline{v}_i) > 1$ for all i, because when $\underline{v}_i > 0$:

$$\begin{aligned} H_i(\underline{v}_i) &= \left(\frac{f(\underline{v}_i)}{F(\overline{v}_i) - F(\underline{v}_i)}\right)\left(\frac{B(\underline{v}_i) - \underline{v}_i}{B'(\underline{v}_i)}\right) \\ &\geq \left(\frac{f(\underline{v}_i)}{1 - F(\underline{v}_i)}\right)\left(\frac{B(\underline{v}_i) - \underline{v}_i}{B'(\underline{v}_i)}\right) = H(\underline{v}_i) \\ &> H(0), \end{aligned} \tag{A10}$$

and when $\overline{v}_i < 1$:

$$\begin{aligned} H_i(\underline{v}_i) &= \left(\frac{f(\underline{v}_i)}{F(\overline{v}_i) - F(\underline{v}_i)}\right)\left(\frac{B(\underline{v}_i) - \underline{v}_i}{B'(\underline{v}_i)}\right) \\ &> \left(\frac{f(\underline{v}_i)}{1 - F(\underline{v}_i)}\right)\left(\frac{B(\underline{v}_i) - \underline{v}_i}{B'(\underline{v}_i)}\right) = H(\underline{v}_i) \\ &\geq H(0). \end{aligned} \tag{A11}$$

Thus, $\Pi'(0) \leq 0 \Rightarrow \Pi_i'(\underline{v}_i) < 0$.

Now, suppose that the N conditions for efficient trade between the intermediary and the buyer are satisfied. For $B(v) \in \Omega^1(F)$, the seller must find it optimal to quote the intermediary a price of $B(0)$ rather than any of the prices $B(\underline{v}_i)$ for $i \in \{2, 3, ..., N+1\}$, where we define $\underline{v}_{N+1} = 1$:

$$B(0) \geq (1 - F(\underline{v}_i))B(\underline{v}_i) + F(\underline{v}_i)E[v|v < \underline{v}_i] \qquad \text{for} \quad i \in \{2, ..., N+1\}. \tag{A12}$$

At these particular cutoffs $\underline{v}_i$, we can use the function $\Pi(w)$ introduced in equation (1) and rewrite the above condition as:

$$\Pi(\underline{v}_i) - \Pi(0) \leq 0 \qquad \text{for} \quad i \in \{2, ..., N+1\}. \tag{A13}$$

Note that we can further write:

$$\Pi(\underline{v}_i) - \Pi(0) = \int_0^{\underline{v}_i} \Pi'(w)dw = \int_0^{\underline{v}_i} [1 - F(w)]B'(w)[1 - H(w)]dw. \tag{A14}$$

Since $H(w)$ is strictly increasing in w it follows immediately that if $\Pi'(0) \leq 0$, and thus, $H(0) \geq 1$, it must be that $H(w) > 1$ for all $w > 0$. It follows from equation (A14) that $\Pi'(0) \leq 0 \Rightarrow \Pi(\underline{v}_i) - \Pi(0) < 0$ for all $i \neq 1$. Moreover, if $\Pi(\underline{v}_1) - \Pi(0) \leq 0$, then it must be that $H(\underline{v}_1) \geq 1$ and therefore $\Pi(\underline{v}_i) - \Pi(0) < 0$ for all $i \neq 1$.

Overall, we can therefore characterize the set $\Omega^1(F)$ as:

$$\Omega^1(F) = \{B \in \Psi(F) : \Pi(\underline{v}_1) - \Pi(0) \leq 0, \{\Pi_i'(\underline{v}_i) \leq 0\}_{\forall\, i \in N}\}, \tag{A15}$$

and we have shown above that $\Omega^0(F) \subseteq \Omega^1(F)$.

By continuity of the set $\Psi(F)$, there exists a subset of functions $B(v) \in \Omega^0(F)$ for which the condition for efficient trade between the seller and the buyer holds with equality, that is, $\Pi'(0) = 0$. Consider replacing any one of these functions $B(v)$ by a perturbed function $\tilde{B}(v, \epsilon) \equiv B(v) - \epsilon$, where $\epsilon \geq 0$ is bounded from above to ensure that $\tilde{B}(v, \epsilon) - v > 0$ for all $v \in [0, 1]$. Define the overall slack across all efficiency conditions in the presence of the intermediary under the function $\tilde{B}(v, \epsilon)$ as:

$$\tilde{\rho}(\epsilon) \equiv -\max\{\Pi(\underline{v}_1) - \Pi(0), \{\Pi_i'(\underline{v}_i)\}_{\forall\, i \in N}\}. \tag{A16}$$

Note that all $\Pi_i'(\underline{v}_i)$ are continuous and strictly increasing in ϵ and so is the difference $\Pi(\underline{v}_1) - \Pi(0)$. Moreover, applying the maximum operator to a set of continuous and strictly increasing functions yields a continuous and strictly increasing function. Thus, $\tilde{\rho}(\epsilon)$ is continuous and strictly decreasing in ϵ. Based on the above derivations, when the condition for efficient trade holds with equality without an intermediary, introducing an intermediary allows all conditions for efficient trade to hold with strict inequality, i.e., $\tilde{\rho}(0) > 0$. By continuity of $\tilde{\rho}(\epsilon)$ there exist strictly positive values for ϵ such that $\tilde{\rho}(\epsilon) \geq 0$, meaning that $\tilde{B}(v, \epsilon) \in \Omega^1(F)$. Yet, since we started with a function

$B(v) = \tilde{B}(v, 0)$ for which $\Pi'(0) = 0$, these perturbed functions $\tilde{B}(v, \epsilon)$ will have $\Pi'(0) > 0$ for these values of ϵ and therefore these $\tilde{B}(v, \epsilon) \notin \Omega^0(F)$. It thus follows that the set of functions $B(v)$ that satisfy efficient trade with the intermediary is strictly larger than without the intermediary, i.e., $\Omega^0(F) \subset \Omega^1(F)$. ∎

Proof of Proposition 3: [Part 1] Suppose there exists a set of traders $\mathbb{C} \subseteq \mathbb{T}$ and an order-flow agreement Σ for which trade breaks down with strictly positive probability and the total surplus across all traders in $\mathbb{C}$ is less than $E[B(v) - v]$. Further, consider that every trader in $\mathbb{C}$ obtains an ex ante surplus, net of transfers, that is weakly positive (otherwise equilibrium conditions are immediately violated, as every trader with negative surplus strictly prefers to exit the agreement). Order-flow agreement Σ can be blocked by a coalition of traders $\mathbb{C}' \subseteq \mathbb{T}$: based on the condition stated in the proposition, there exists an order-flow agreement Σ' associated with an intermediation chain that sustains efficient trade and preserves the full surplus $E[B(v) - v]$. Since the total surplus is greater under agreement Σ' and any trader not involved in Σ collects zero surplus, ex ante transfers can be chosen such that every trader in $\mathbb{C}'$ is strictly better off.

[Part 2] An intermediation chain that allows for efficient trade yields a total surplus of $E[B(v) - v]$ across all traders. To prove the existence of an order-flow agreement that constitutes an equilibrium and supports the efficient intermediation chain, we consider an order-flow agreement Σ that specifies a set of transfers that imply that all intermediaries involved in agreement Σ obtain zero ex ante surplus (net of transfers), and the ultimate buyer and seller split the total surplus of $E[B(v) - v]$. Any coalition of traders $\mathbb{C}'$ that attempts to block this order-flow agreement would need to include the ultimate buyer and seller, since they are needed to generate a positive surplus from trade. A blocking order-flow agreement Σ' would thus need to make both of these ultimate traders weakly better off and at least one agent in coalition $\mathbb{C}'$ strictly better off, which is impossible since the ultimate buyer and seller already split the maximum surplus of $E[B(v) - v]$ under agreement Σ and no intermediary would be willing to participate in the blocking order-flow agreement if promised a negative expected surplus. ∎

Appendix B: Not for Publication

Proofs Omitted from Paper

Proof of Proposition 2: In the following we will use the subscript m to identify trader m's information set. For example, $F_m(v)$ is the CDF of v given trader m's information set, $E_m[v]$ is the expectation of v given trader m's information set, and

$$\Pi_m(w) \equiv (1 - F_m(w))\, B(w) + F_m(w) E_m[v|v < w]. \tag{B1}$$

If trader $m < M$ knows that $v \in [\underline{v}_i^m, \overline{v}_i^m)$ then trader $(m+1)$ knows that v is in one of the $K(m, m+1, i) \geq 3$ non-overlapping sub-intervals associated with the boundaries $w_j(m, m+1, i)$, where $\underline{v}_i^m = w_0(m, m+1, i) < w_1(m, m+1, i) < ... < w_K(m, m+1, i) = \overline{v}_i^m$. Thus, $w_j(m, m+1, i)$ denotes the j-th partition cutoff of trader $(m+1)$'s information set if trader m observes signal $i \in N_m$. For a given $F(v)$ and a given chain with M intermediaries we define $\Omega^M(F)$ as the set of functions $B(v)$ that satisfy all conditions for efficient trade along the chain:

$$\begin{aligned}
&\Omega^M(F) = \{B \in \Psi(F) : \{\Pi'_M(\underline{v}_i^M) \leq 0\}_{\forall\, i \in N_M}, \\
&\{\Pi_m(w_1(m, m+1, i)) - \Pi_m(w_0(m, m+1, i)) \leq 0\ \}_{\forall\, i \in N_m} \forall m < M\},
\end{aligned} \tag{B2}$$

where $\Psi(F)$ denotes the set of functions $B(v)$ that satisfy Assumption 1 given the distribution function $F(v)$, just as it did in the proof of Proposition 1.

Analogously to the Proof of Proposition 1 we will use the following relations:

$$\Pi'_m(w) = (1 - F_m(w)) B'(w) [1 - H_m(w)], \tag{B3}$$

and

$$\Pi_m(w_1(m, m+1, i)) - \Pi_m(w_0(m, m+1, i)) = \int_{w_0(m,m+1,i)}^{w_1(m,m+1,i)} \Pi'_m(z) dz. \tag{B4}$$

To ensure efficient trade, we can focus on the profitability of a trader m's deviation from a marginal type $w_0(m, m+1, i)$ to $w_1(m, m+1, i)$ due to the fact that $H_m(w)$ is strictly increasing, that is, if trader m does not want to deviate to $w_1(m, m+1, i)$ he will also not want to deviate to any other higher marginal type.

By continuity of the set $\Psi(F)$, there exist functions $B(v) \in \Omega^M(F)$ such that for some transaction between traders m and $(m+1)$ the condition for efficient trade is holding with equality after some signal i, that is, either:

$$\Pi_m(w_1(m,m+1,i)) - \Pi_m(w_0(m,m+1,i)) = 0 \qquad \text{if} \quad m < M \tag{B5}$$

or

$$\Pi'_M(\underline{v}_i^M) = 0 \qquad \text{if} \quad m = M. \tag{B6}$$

We will show next that this condition for efficient trade holds with strict inequality if we introduce an intermediary $\tilde{m}$ between traders m and $(m+1)$. This intermediary $\tilde{m}$ knows that v is in one of $\tilde{K} \in \{2, ..., K(m,m+1,i)-1\}$ non-overlapping sub-intervals associated with the boundaries $\tilde{w}_i$, where $\underline{v}_i^m = w_0(m,m+1,i) = \tilde{w}_0 < \tilde{w}_1 < ... < \tilde{w}_{\tilde{K}} = w_K(m,m+1,i) = \overline{v}_i^m$ and $\tilde{w}_i \in \{w_2(m,m+1,i), ..., w_K(m,m+1,i)\}$ for all $i \in \{1,2,...,\tilde{K}\}$. Trader $\tilde{m}$'s partition is thus a strict refinement of trader m's partition, and trader $(m+1)$'s partition is a strict refinement of trader $\tilde{m}$'s partition. For notational simplicity, from now on we will omit indices and simply write w_j when referring to the cutoffs of trader $(m+1)$'s partition: $w_j(m,m+1,i)$.

Trade between intermediary $\tilde{m}$ and trader $(m+1)$ if trader $(m+1)$ is not the buyer.

Trader $\tilde{m}$ observes that $v \in [\tilde{w}_j, \tilde{w}_{j+1})$, where $j \in \{0,1,...,\tilde{K}-1\}$. Consistent with the derivations above, the condition for efficient trade between traders $\tilde{m}$ and $(m+1)$ can be written as:

$$\Pi_{\tilde{m}}(w_k) - \Pi_{\tilde{m}}(\tilde{w}_j) = \int_{\tilde{w}_j}^{w_k} (1 - F_{\tilde{m}}(z))B'(z)[1 - H_{\tilde{m}}(z)]dz \leq 0, \tag{B7}$$

where $k = \min\{s \in \{1,2,...,K(m,m+1,i)\} : w_s > \tilde{w}_j\}$. We now wish to show that this condition is satisfied with strict inequality whenever $\Pi_m(w_1) - \Pi_m(w_0) = 0$.

Let's first consider the case where $\tilde{w}_j > w_0$. When $\Pi_m(w_1) - \Pi_m(w_0) = 0$, it follows from equations (B3) and (B4) that $H_m(w_1) > 1$ since $H_m(v)$ is strictly increasing in v. As in the proof of Proposition 1, we can then use

$$f_{\tilde{m}}(v) = \frac{f_m(v)}{F_m(\tilde{w}_{j+1}) - F_m(\tilde{w}_j)} \tag{B8}$$

and

$$1 - F_{\tilde{m}}(v) = \frac{F_m(\tilde{w}_{j+1}) - F_m(v)}{F_m(\tilde{w}_{j+1}) - F_m(\tilde{w}_j)} \tag{B9}$$

to show that, whenever $\tilde{w}_{j+1} < w_K$ (that is, unless trader $\tilde{m}$ gets the highest possible signal), the following strict inequality holds:

$$H_{\tilde{m}}(v) = \left(\frac{f_m(v)}{F_m(\tilde{w}_{j+1}) - F_m(v)}\right)\left(\frac{B(v)-v}{B'(v)}\right)$$
$$> \left(\frac{f_m(v)}{1-F_m(v)}\right)\left(\frac{B(v)-v}{B'(v)}\right) = H_m(v). \tag{B10}$$

And since $H_{\tilde{m}}(w_1) \geq H_m(w_1) > 1$, it follows from equation (B7) that $\Pi_{\tilde{m}}(w_k) - \Pi_{\tilde{m}}(\tilde{w}_j) < 0$ whenever $\tilde{w}_j > w_0$. As for the case where $\tilde{w}_j = w_0$, we can write:

$$\begin{aligned}
&\Pi_{\tilde{m}}(w_k) - \Pi_{\tilde{m}}(\tilde{w}_0)\\
=&\int_{w_0}^{w_1}(1-F_{\tilde{m}}(z))B'(z)[1-H_{\tilde{m}}(z)]dz\\
=&\int_{w_0}^{w_1}\frac{F_m(\tilde{w}_1)-F_m(z)}{F_m(\tilde{w}_1)-F_m(w_0)}B'(z)\left[1-\left(\frac{f_m(z)}{F_m(\tilde{w}_1)-F_m(z)}\right)\left(\frac{B(z)-z}{B'(z)}\right)\right]dz\\
=&\frac{\int_{w_0}^{w_1}(1-F_m(z))B'(z)\left[\frac{F_m(\tilde{w}_1)-F_m(z)}{1-F_m(z)}-\left(\frac{f_m(z)}{1-F_m(z)}\right)\left(\frac{B(z)-z}{B'(z)}\right)\right]dz}{F_m(\tilde{w}_1)-F_m(w_0)}\\
=&\frac{\int_{w_0}^{w_1}(1-F_m(z))B'(z)\left[1-H_m(z)-\left(1-\frac{F_m(\tilde{w}_1)-F_m(z)}{1-F_m(z)}\right)\right]dz}{F_m(\tilde{w}_1)-F_m(w_0)}\\
=&\frac{\int_{w_0}^{w_1}(1-F_m(z))B'(z)\left[1-H_m(z)-\frac{1-F_m(\tilde{w}_1)}{1-F_m(z)}\right]dz}{F_m(\tilde{w}_1)-F_m(w_0)}\\
=&\frac{\Pi_m(w_1)-\Pi_m(w_0)-\int_{w_0}^{w_1}(1-F_m(w))B'(w)\left[\frac{1-F_m(\tilde{w}_1)}{1-F_m(w)}\right]dw}{F_m(\tilde{w}_1)-F_m(w_0)},
\end{aligned} \tag{B11}$$

which means that $\Pi_m(w_1) - \Pi_m(w_0) = 0 \Rightarrow \Pi_{\tilde{m}}(w_k) - \Pi_{\tilde{m}}(\tilde{w}_0) < 0$, since

$$-\frac{\int_{w_0}^{w_1}(1-F_m(z))B'(z)\left[\frac{1-F_m(\tilde{w}_1)}{1-F_m(z)}\right]dz}{F_m(\tilde{w}_1)-F_m(w_0)} < 0. \tag{B12}$$

Trade between intermediary $\tilde{m}$ and trader $(m+1)$ if trader $(m+1)$ is the buyer.

When trader $(m+1)$ is the buyer, the condition for efficient trade is given by:

$$\Pi'_{\tilde{m}}(\tilde{w}_j) = (1-F_{\tilde{m}}(\tilde{w}_j))B'(\tilde{w}_j)[1-H_{\tilde{m}}(\tilde{w}_j)] \leq 0 \tag{B13}$$

We want to show that this condition is satisfied with strict inequality whenever $\Pi'_m(w_0) = 0$. In that case, we know that $H_m(w_0) = 1$, which, according to the above derivations, implies $H_{\tilde{m}}(w_0) > 1$

and $H_{\tilde{m}}(\tilde{w}_j) > 1$. Thus, $\Pi'_m(w_0) = 0 \Rightarrow \Pi'_{\tilde{m}}(\tilde{w}_j) < 0$.

Trade between trader m and intermediary $\tilde{m}$.

Here, the condition for efficient trade is given by:

$$\Pi_m(\tilde{w}_1) - \Pi_m(w_0) \leq 0. \tag{B14}$$

We want to show that this condition is satisfied with strict inequality whenever $\Pi_m(w_1) - \Pi_m(w_0) = 0$ if trader $(m+1)$ is not the buyer or whenever $\Pi'_m(w_0) = 0$ if trader $(m+1)$ is the buyer.

We can write:

$$\begin{aligned} &\Pi_m(\tilde{w}_1) - \Pi_m(w_0) \\ =&\int_{w_0}^{\tilde{w}_1} (1 - F_m(z))B'(z)[1 - H_m(z)]dz \\ =&\int_{w_0}^{w_1} (1 - F_m(z))B'(z)[1 - H_m(z)]dz + \int_{w_1}^{\tilde{w}_1} (1 - F_m(z))B'(z)[1 - H_m(z)]dz \\ =&\Pi_m(w_1) - \Pi_m(w_0) + \int_{w_1}^{\tilde{w}_1} (1 - F_m(z))B'(z)[1 - H_m(z)]dz. \end{aligned} \tag{B15}$$

Since $H_m(v)$ is increasing, if trader $(m+1)$ is not the buyer and $\Pi_m(w_1) - \Pi_m(w_0) = 0$, then $H_m(w) > 1$ for $w \geq w_1$. Thus, $\Pi_m(w_1) - \Pi_m(w_0) = 0$ implies that:

$$\int_{w_1}^{\tilde{w}_1} (1 - F_m(w))B'(w)[1 - H_m(w)]dw < 0, \tag{B16}$$

which then implies that $\Pi_m(\tilde{w}_1) - \Pi_m(w_0) < 0$. Now, if trader $(m+1)$ is the buyer and $\Pi'_m(w_0) = 0$, then we also know that $\Pi_m(w_1) - \Pi_m(w_0) < 0$, thus $\Pi_m(\tilde{w}_1) - \Pi_m(w_0) < 0$.

Adding $\tilde{M} \geq 1$ intermediaries to the chain.

Suppose that for a given function $B(v)$ and a chain with M intermediaries, there are $\tilde{M}$ transaction(s) in the chain where the condition for efficient trade holds with equality (for at least one possible signal). Introducing $\tilde{M}$ new traders, with information sets that satisfy the conditions described above, to intermediate these $\tilde{M}$ transactions ensures that all conditions in the chain with $(M + \tilde{M})$ intermediaries hold with strict inequality.

Consider replacing any one of these functions $B(v)$ by a perturbed function $\tilde{B}(v, \epsilon) \equiv B(v) - \epsilon$, where $\epsilon \geq 0$ is bounded from above to ensure that $\tilde{B}(v, \epsilon) - v > 0$ for all $v \in [0, 1]$. Define the overall slack across all efficiency conditions in the new chain with $(M + \tilde{M})$ intermediaries under the function $\tilde{B}(v,\epsilon)$ as:

$$\tilde{\rho}(\epsilon) \equiv -\max\{\{\Pi_m(w_1(m,m+1,i)) - \Pi_m(w_0(m,m+1,i))\}_{\forall\, i\in N_m}\forall m < M, \\ \{\Pi'_M(\underline{v}_i^M)\}_{\forall\, i\in N_M}\}. \tag{B17}$$

Note that the functions $\Pi'_m(w)$ are continuous and strictly increasing in ϵ and so are the differences $\Pi_m(w_1) - \Pi_m(w_0)$. Moreover, applying the maximum operator to a set of continuous and strictly increasing functions yields a continuous and strictly increasing function. Thus, $\tilde{\rho}$ is continuous and strictly decreasing function in ϵ. As we have shown above, once we introduce the $\tilde{M}$ new intermediaries of the type described above wherever in the network the condition for efficient trade used to hold with equality, then all the new conditions for efficient trade are slack under $B(v)$, which implies that $\tilde{\rho}(0) > 0$. By continuity of $\tilde{\rho}(\epsilon)$ there exists strictly positive values for ϵ such that $\tilde{\rho}(\epsilon) \geq 0$, meaning that $\tilde{B}(v,\epsilon) \in \Omega^{M+\tilde{M}}(F)$. Yet, since we started with a function $B(v) = \tilde{B}(v,0)$ for which some of the conditions for efficient trade were holding with equality with the original M intermediaries, it follows that these same conditions are violated for these perturbed functions $\tilde{B}(v,\epsilon)$ with $\epsilon > 0$ and therefore these $B(v,\epsilon) \notin \Omega^M(F)$. It thus follows that the set of functions $B(v)$ that satisfy efficient trade becomes strictly larger once we add these $\tilde{M}$ intermediaries, that is, $\Omega^M(F) \subset \Omega^{M+\tilde{M}}(F)$. ∎

Derivations under Alternative Information Structures

Our main result that moderately informed intermediaries can facilitate efficient trade was made tractable in our baseline model thanks to a few stylized assumptions about traders' information structures. In this Appendix, we revisit the parameterized example with a uniform distribution and constant gains to trade (which satisfies Assumption 1) and show how the intuition developed in our baseline model can be extended to more complex informational settings.

In some of these settings transactions will feature a proposer who has private information not known to a responder, giving rise to signaling concerns and multiple equilibria. The goal of the analysis below is to show, under various circumstances, the existence of at least one type of equilibrium for which intermediation chains expand the parameter region in which efficient trade is attainable. To ensure that our results are not driven by the multiplicity of equilibria that off-equilibrium beliefs trigger in signaling games, we will first fix off-equilibrium beliefs and then compare the efficiency of trade across various trading networks given those beliefs. We will show that, for specific beliefs that strike us as reasonable, our result that intermediation chains facilitate efficient trade can survive these variations in the information structure.

Throughout, we will assume that transaction prices quoted in earlier rounds of trade are not observable to traders who were not involved in those transactions. In the context of decentralized markets price opacity appears more suitable than price transparency (Green, Hollifield, and Schürhoff 2007, Duffie 2012, Zhu 2012). This assumption will streamline our analysis, since an off-equilibrium price quote in one round of trade will trigger belief adjustments for only one trader (that is, the responder in that round of trade).

Informed Seller

Results similar to those derived in Section 3 can be obtained when the seller is informed and the buyer is uninformed. In fact, if intermediaries are allowed to short sell the asset, those results are obtained without the complications that arise in signaling games.

As before, suppose the asset is worth $v \sim U[0,1]$ to the seller and $v + \Delta$ to the buyer. To eliminate signaling concerns and remain consistent with the analysis from Section 3, the uninformed buyer is now assumed to make the ultimatum offer p to the seller. Without an intermediary, the buyer's optimization problem can be written as:

$$\max_{p \in [0,1]} \Pi(p) = p\left(\frac{p}{2} + \Delta - p\right). \tag{B18}$$

For efficient trade to occur, we need $\Pi'(1) \geq 0$, which is satisfied whenever $\Delta \geq 1$. This condition is identical to the one we had in our example in Section 3.

However, if an agent who knows whether $v \in [0, c]$ or $v \in (c, 1]$, where $c \in (0, 1)$, acts as an intermediary between the seller and the buyer, trade can be efficient even when $\Delta < 1$. Here, we allow the intermediary to sell the asset short, that is, he can accept to sell the asset to the buyer at the offered price as long as he later buys the asset from the seller. Consistent with the nested information sets assumed in Section 3, the uninformed buyer first makes an offer to purchase the asset from the intermediary who then makes an offer to the seller.

When trying to buy the asset from the seller, the intermediary must offer a price 1 if he knows that $v > c$ and a price c if he knows that $v \leq c$ in order to ensure he will get the asset and cover his short position with probability one. Since the buyer makes an ultimatum offer to the intermediary and the intermediary can only accept it if he commits to buy the asset from the seller, trade is efficient as long as the buyer prefers to offer a price of 1, which is always accepted by the intermediary, rather than a price c, which is only accepted with probability c:

$$\frac{1}{2} + \Delta - 1 \geq c\left(\frac{c}{2} + \Delta - c\right), \tag{B19}$$

which simplifies to $\Delta \geq \frac{1+c}{2}$. This condition for intermediated trade to be efficient is strictly weaker than $\Delta \geq 1$ whenever $c \in (0, 1)$. Although the mechanics of intermediation with short selling are slightly different than in our baseline model, there still exists a region for Δ for which trade can only be efficient if a moderately informed intermediary is involved.

Two-Sided Asymmetric Information

In Section 2, we introduced an information asymmetry between a buyer and a seller that was one sided. We now show that the intuition developed in our baseline model extends to situations where both of these traders have private information about the value of the asset. We first fix the off-equilibrium beliefs of the responder and prove the existence of perfect Bayesian equilibria in which heterogeneously informed intermediaries can improve trade efficiency, just as they did in our baseline model.

Before solving for the conditions for efficient trade in a given trading network, we introduce the following lemma:

Lemma 2 (Efficient trade and pooling equilibria) *The only equilibria in which efficient trade occurs are pooling equilibria in which the proposer does not alter his price quote based on his private information and this price quote is always accepted by the responder.*

Proof. Suppose there is an equilibrium in which the proposer alters his price quote based on his private information. In such an equilibrium, for trade to be efficient the responder needs to accept all of the proposer's offers. If the proposer anticipates such a response, he should quote the most profitable of these equilibrium prices, regardless of his information, contradicting the initial claim. ■

Consider, as previously, a parameterized example where the asset is worth $v \sim U[0,1]$ to the seller and $v + \Delta$ to the buyer. The seller knows whether $v \in \left[0, \frac{1}{3}\right)$ or $v \left[\frac{1}{3}, 1\right]$ whereas the buyer knows whether $v \in \left[0, \frac{2}{3}\right)$ or $v \left[\frac{2}{3}, 1\right]$. Both of these traders are thus partially informed about the value of the asset and the trader who makes the ultimatum offer now possesses information his counterparty does not possess. It will greatly simplify our analysis to restrict the off-equilibrium beliefs the responder (buyer) uses to update his expectation of v when quoted by the proposer any price higher than the equilibrium price quote. A natural choice for these off-equilibrium beliefs is that any deviation to a higher price quote (relative to the equilibrium price quote) is uninformative about the proposer's private information. These beliefs imply that the considered equilibrium satisfies the Intuitive Criterion of Cho and Kreps (1987). In our context, the Intuitive Criterion requires that a buyer ascribes zero probability to any seller type who would be worse off by quoting a higher price regardless of the buyer's actions. Clearly, all seller types would be better off with a higher price should the buyer accept. However, many other off-equilibrium beliefs would allow our results to survive qualitatively, although the region of Δ over which intermediation helps sustain efficient trade would differ.

We know from Lemma 2 that without an intermediary, efficient trade is possible if and only if there exists a pooling price that is always accepted by the buyer. We denote the highest pooling price that a buyer always accepts by $\bar{p} = E\left[v|v < \frac{2}{3}\right] + \Delta = \frac{1}{3} + \Delta$. This price is also the pooling price best able to sustain efficient trade in equilibrium. Since the buyer believes that any price quote $p > \bar{p}$ coming from the seller is uninformative about his signal, we only need to verify whether the seller prefers to quote $\bar{p}$, which is always accepted, rather than $E\left[v|v \geq \frac{2}{3}\right] + \Delta = \frac{5}{6} + \Delta$, the highest price that is accepted when the buyer observes a good signal. If the seller knows that $v \geq \frac{1}{3}$, he finds it optimal to quote the pooling price $\bar{p}$ if and only if:

$$\frac{1}{3} + \Delta \geq \frac{1}{2}\left(\frac{5}{6} + \Delta\right) + \frac{1}{2} \cdot \frac{1}{2}, \tag{B20}$$

which simplifies to $\Delta \geq \frac{2}{3}$. If the seller knows instead that $v < \frac{1}{3}$, he also knows that any price above $\bar{p}$ will be rejected with probability one by the buyer. Hence, no deviation can be profitable. Overall, direct trade is inefficient if $\Delta < \frac{2}{3}$.

In this setting with two-sided asymmetric information an uninformed intermediary is effectively moderately informed: his involvement splits an information asymmetry into two transactions that each involve less information asymmetry. Conjecturing that efficient trade occurred in the first transaction, the uninformed intermediary prefers to quote the buyer $\bar{p}$ rather than $\frac{5}{6} + \Delta$ if and only if:

$$\frac{1}{3} + \Delta \geq \frac{1}{3}\left(\frac{5}{6} + \Delta\right) + \frac{2}{3} \cdot \frac{1}{3}, \tag{B21}$$

which simplifies to $\Delta \geq \frac{1}{4}$. Given this inequality is satisfied, the highest pooling price the uninformed intermediary accepts to pay to the seller is also $\bar{p}$. Any higher price quote would be rejected by the intermediary, given his off-equilibrium beliefs. The seller then prefers to quote $\bar{p}$ rather than holding on to the asset, even after receiving a good signal, if and only if:

$$\frac{1}{3} + \Delta \geq \frac{2}{3}, \tag{B22}$$

which simplifies to $\Delta \geq \frac{1}{3}$. Hence, as in the baseline model, there exists a region for Δ where intermediated trade is efficient but direct trade is not.