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INFORMATION AND EFFICIENCY IN TENDER OFFERS

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INFORMATION AND EFFICIENCY IN TENDER OFFERS

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We analyze tender offers where privately informed shareholders are uncertain about the raider's ability to improve firm value. The raider suffers a "lemons problem" in that, for any price offered, only shareholders who are relatively pessimistic about the value of the firm tender their shares. Consequently, the raider finds it too costly to induce shareholders to tender when their information is positive. In the limit as the number of shareholders gets arbitrarily large, when private benefits are relatively low, the tender offer is unsuccessful if the takeover has the potential to create value. The takeover market is therefore inefficient. In contrast, when private benefits of control are high, the tender offer allocates the firm to any value-increasing raider, but may also allow inefficient takeovers to occur. Unlike the case where all information is symmetric, shareholders cannot always extract the entire surplus from the acquisition.

KEYWORDS: Tender offers, shareholder information, efficiency.

1. INTRODUCTION

THIS PAPER EXPLORES THE ROLE of shareholder information in takeover contests. The analysis of tender offers has been much studied in the literature, with particular emphasis on the problems associated with the free-rider problem in takeover bidding.[2] Grossman and Hart (1980) established that costly takeovers may not be feasible for widely held firms since infinitesimal shareholders have an incentive to free-ride on other shareholders' tendering decisions. Bagnoli and Lipman (1988) studied the equilibrium behavior for finitely many shareholders and then allowed the number of shareholders to get arbitrarily large.[3] In a symmetric information setting, where both shareholders and the raider know the true post-takeover value of the firm, they showed that expected profits converge to zero for all possible prices when the raider has no private benefits but can increase firm value. Hence, payoff considerations for the raider get small as the number of shareholders increases.

Starting from this symmetric information benchmark, we add an element of private information to the model by assuming that shareholders are privately informed about the post-takeover value of the firm. Specifically, with some

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[2]There is also a literature on competitive takeover bidding that largely abstracts from the free-rider problem. See, for instance, Fishman (1988).

[3]The finite shareholder case has also been analyzed by Holmstrom and Nalebuff (1992). Cornelli and Li (2002) analyzed a setting with finitely many risk arbitrageurs who participate in the tendering game.

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probability, the state is "bad" so that the value-added of the takeover is zero or negative, and with complementary probability, the state is "good" and the value-added is positive. Each shareholder observes a private noisy signal about the value created by the takeover. The raider submits an unconditional offer for the equity shares of the firm by specifying a price per share, and shareholders either accept the offer and tender their shares or reject the offer and keep their shares.

We first characterize the tendering decisions of privately informed shareholders. We show that the free-rider problem associated with dispersed ownership gives rise to a "lemons problem" in that for any price offered, only those shareholders whose signals are suggestive of a relatively low post-acquisition firm value will be willing to tender. This introduces an important difference relative to the baseline model where information is symmetric since it reduces the likelihood that value-increasing takeovers will be successful: shareholders are more likely to retain their shares in the good state, but will unload them in the bad state. Hence, not only does the raider not get any surplus when the takeover adds value, he is left paying a premium for the shares when in the bad state. Moreover, this negative impact on the raider's profit persists even as the number of shareholders increases since an increase in the number of shareholders does not help eliminate the lemons problem.

We next extend the model to allow the raider to have some alternative motive for acquiring the firm, such as a private control benefit.[4] The existence of a private benefit changes the implications for bidding behavior by the raider. Now, we find that, even in instances when the raider might fail to increase firm value, a positive private benefit creates an incentive for the raider to bid.

Two distinct results arise as a function of the size of the private control benefit. For low values of the private benefit, the raider recognizes that with a relatively low price the offer will succeed only if the value of the shares after the takeover is expected to be low since many shareholders have observed a low signal. But then it is optimal for the raider to offer a price only slightly higher than the value of the shares, as the loss to the raider can be made arbitrarily small by reducing the price. As a consequence, a small benefit is enough to compensate the raider for the infinitesimally small loss, although it leads him to bid in such a way that the takeover is successful if and only if it adds no value. Therefore, the lemons problem that arises from shareholders' free-riding incentives prevents the firm from being allocated in an efficient manner. Moreover, a further inefficiency arises when we allow for the possibility

[4]One source can be "empire-building" incentives. A private benefit can also be obtained through oligopolistic competition, where the drive to expropriate a competing firm may produce an incentive to purchase a target firm. Since some of this expropriation accrues only to the acquirer, shareholders of the target firm that hold out are unable to reap all of the benefits associated with the takeover. Grossman and Hart (1980) discussed the relevance of private benefits of control and its impact on the market for corporate control.

that the raider may destroy value in the bad state, since then value-destroying takeovers will be more likely to be successful than those that increase value.

When the private benefit of control is high, however, the raider finds it optimal to bid a higher price and induce tendering more often. Here, we find that the limiting price converges to the lowest price necessary to guarantee that the offer always succeeds. Assuming there is no value destruction upon takeover, we now have allocative efficiency, since the tender offer is always successful. Moreover, although the price converges to a value greater than the value of the shares being purchased, this price is strictly lower than the post-takeover value in the good state. While shareholders benefit from their information by allowing them to extract rents ex ante, uncertainty about firm value prevents shareholders from capturing the entire surplus generated by a value-increasing raider.

As a final point, we note that we have described a rather stark dichotomy between the low and high private benefit cases in terms of the raider's optimal bidding strategy. This is an important consequence of the limiting behavior of the probability of having a successful takeover as the number of shareholders increases, since when these get sufficiently large, this probability essentially becomes a deterministic function of the offered price. These results, for both the low and the high private benefit cases, are unlike the symmetric information setting analyzed in Bagnoli and Lipman (1988), where adding any private benefit of control leads all takeover bids to be successful at prices that fully reflect the post-takeover value and allocate all surplus to shareholders. Private information therefore affects not only the efficiency of the takeover process, but also the allocation of surplus between the raider and the shareholders.

The key innovation in our work is that we allow for shareholders to be privately informed about the firm's value. Indeed, there is a broad literature on how corporate insiders may learn about the firm's fundamentals from outsiders (see, e.g., Holmstrom and Tirole (1993)). By analogy, target shareholders may well have information unavailable to a potential raider. For instance, while a raider may have a good estimate of the operating strategy to be developed as a result of the takeover, he may have little knowledge of how well the target firm's corporate culture is likely to fit with that of the raider. The effect of this private shareholder information is to bias acquisitions toward those likely to create the least, if any, value, and may in fact prevent takeover bids from being made at all. Moreover, as the analysis below makes clear, while private information held by shareholders clearly drives our results, whether information is dispersed (i.e., the distribution of information across shareholders) is not important for the qualitative nature of the results.[5] Indeed, similar results

[5]In a related paper, Marquez and Yılmaz (2006) analyzed a similar setting in which the raider also has private information and showed that there exists a pooling equilibrium in which the tender offer does not reveal the raider's information. Moreover, this pooling equilibrium is the only "robust" equilibrium.

obtain even if all shareholders share the same information and if this information is perfect. The noisy signals case we study allows us to derive implications concerning the distribution of rents between the raider and shareholders as a function of shareholders' private information.

The rest of the paper is organized as follows. Section 2 lays out the basic framework. Section 3 presents a benchmark with symmetric information. Section 4 contains the main analysis of the role of private shareholder information. The case where the raider can destroy value upon acquiring the firm is studied in Section 5. Section 6 concludes. All omitted proofs are in the Appendix.

2. THE MODEL

2.1. *Preferences*

There is a firm with n shareholders, each of whom owns a single share of the firm. In addition, there is a raider who wishes to purchase and run the firm, and an incumbent who currently manages the firm. Let $\omega \in \Omega = \{0, 1\}$ be the true state of the world. Given the true state of the world, each shareholder prefers the manager who is capable of producing higher cash flows. For simplicity, we normalize the per-share value to 0 under the incumbent management. The raider, if successful in taking over the firm, is expected to generate a firm value of V_ω, with per-share value of $v_\omega = V_\omega/n$. We assume that $V_1 > V_0 = 0$. The true state of the world is unknown to either the raider or the shareholders. Let $\lambda \in (0, 1)$ stand for the probability that $\omega = 1$ and assume that λ is common knowledge. While the raider may add value by managing the firm, we also assume that he may like to take over partly due to a private benefit, $B \geq 0$, which he receives if he acquires control, for which he needs to acquire at least half of the outstanding shares.[6] We assume that everyone is risk-neutral.

2.2. *Information*

Conditional on the true state of the world, each shareholder i receives a signal $s_i \in [0, 1]$ independently drawn from an identical distribution. Let $f(s)$ be the density function for the probability of receiving signal s if $\omega = 1$ is the true state of the world. Therefore, $F(\cdot)$ stands for the cumulative probability function of s if $\omega = 1$. Similarly, we use $g(s)$ as the density function for the probability of receiving signal s if $\omega = 0$ is the true state of the world, with $G(\cdot)$ being the cumulative probability function in this state of the world. We

[6]Allowing for supermajority rules does not qualitatively change our results, as long as the rule does not require unanimity (see Bond and Eraslan (2007), for a discussion of the role of unanimity rules in a bargaining setting). The only effect of introducing a supermajority rule is that greater supermajority rules require higher offer prices to maintain the same probability of a successful takeover. Such a higher offer price, however, also decreases the likelihood of a value-increasing takeover, since it increases the minimum size of the private benefit that is needed.

assume that $f(\cdot)$ and $g(\cdot)$ are continuous, and that $\frac{f(\cdot)}{g(\cdot)}$ is strictly increasing in the interval [0, 1] (monotone likelihood ratio (MLR) condition). Furthermore, $\frac{f(0)}{g(0)} = 0$ and $\frac{f(1)}{g(1)} = \infty$, so that extreme signals are highly informative about the underlying state of the world. Each shareholder updates his belief given his signal s and we let $\beta(\omega|s)$ stand for this posterior belief.

2.3. *Tender Offers*

The raider makes a tender offer at price per share $p \in [0, \infty)$, agreeing to buy any and all shares that are tendered (an unconditional offer).[7] Note that offering a price above v_1 is suboptimal, so that without loss of generality we can restrict attention to $p \in [0, v_1]$. Following a tender offer, each shareholder receives his private signal and then all shareholders decide simultaneously whether to tender or to reject the offer. Let $\sigma_i : [0, 1] \to \{\text{tender}, \text{keep}\}$ denote a (pure) strategy for shareholder i in a tendering subgame. We will focus on the symmetric Nash equilibria of this game.

3. THE SYMMETRIC INFORMATION BENCHMARK

We begin our analysis by focusing on the benchmark case where there is symmetric information, so that all information is public and each participant—the raider and all shareholders—observes *every* signal that is available. Formally, this means that the raider's as well as shareholder i's information set comprises $S_n = \{s_j\}_{j=1}^n$. Beliefs are formed by Bayesian updating and, with a slight abuse of notation, we let $\beta(\omega|S_n)$ stand for this posterior belief, which must be the same for everyone since there is no private information. One immediate implication is that, for $\omega = 1$, $p\lim \beta(1|S_n) = 1$, and similarly, for $\omega = 0$, $p\lim \beta(1|S_n) = 0$, since as n increases the probability that observing all the signals does not perfectly reveal the underlying state becomes vanishingly small.

We can now analyze shareholders' tendering decisions as well as the raider's bidding behavior. This setting is analogous to the perfect information case studied by Bagnoli and Lipman (1988), where the true state ω is common knowledge, so that the extent of the raider's value improvement is known by all. Bagnoli and Lipman focused primarily on symmetric equilibria and showed that the raider's expected profit converges to zero when he has no private benefits ($B = 0$) but can increase firm value.[8] Moreover, implicit in their analysis

[7]Conditional offers in which the raider offers to purchase any tendered shares only if they are enough for him to gain control are strictly dominated by unconditional offers in this setting. See Marquez and Yılmaz (2007a) for an analysis of conditional versus unconditional offers.

[8]They also show that asymmetric equilibria may exist where exactly the number of shares necessary to effect a takeover are tendered, and the takeover occurs with probability 1. In these equilibria, each shareholder is pivotal, allowing the raider to earn positive profits. However, the return to each shareholder is very different depending on whether he tenders or not in these equilibria despite these shareholders being ex ante symmetric.

is the finding that the takeover bid of a value-increasing raider succeeds with a probability that is bounded away from zero as the number of shareholders increases. Therefore, under symmetric information, efficient takeovers occur with a strictly positive probability even as the raider's profit becomes vanishingly small. The results of Bagnoli and Lipman carry over to our setting if we simply replace the known value of the firm under the raider with the expected post-takeover value given observation of the signals, $E[V|S_n]$.

To see this more formally, we follow Bagnoli and Lipman (1988) by noting that if tendering is to occur in any symmetric equilibrium, it must be that for any $n > 2$ and $p \in (0, E[v|S_n])$ each shareholder plays a mixed strategy in the unique symmetric equilibrium of the tendering subgame. Let ϕ stand for the probability of a given shareholder tendering. Then the expected cost of buying all of the tendered shares is $n\phi p$ given price p. Thus, the expected profit for the raider from the shares bought is

$$E[v|S_n]\sum_{i=n/2}^{n}\binom{n}{i}\phi^i(1-\phi)^{n-i}i - n\phi p.$$

Note that the first term is just the expected value of the shares purchased, conditional on the takeover being successful. ϕ is determined by the indifference condition for shareholders between tendering and not tendering which, for any price p, is given by

$$p = E[v|S_n]\sum_{i=n/2}^{n-1}\binom{n-1}{i}\phi^i(1-\phi)^{n-1-i}. \tag{1}$$

The sum on the right-hand side corresponds to the probability that at least $\frac{n}{2}$ of the remaining $n-1$ shareholders tender, which corresponds to the probability that a shareholder attaches to the success of the tender offer conditional on knowing that he has not tendered. Substituting for p, the expected profit due to the shares bought is

$$E[v|S_n]\left(\sum_{i=n/2}^{n}\binom{n}{i}\phi^i(1-\phi)^{n-i}i - n\phi\sum_{i=n/2}^{n-1}\binom{n-1}{i}\phi^i(1-\phi)^{n-1-i}\right). \tag{2}$$

We show in the Appendix that this profit expression converges to zero for any price $p \in (0, E[v|S_n])$ (see the proof of Proposition 1).[9] There are two effects at work here that drive this result. When the takeover is successful, which occurs with positive probability, the raider offers a price lower than the full post-takeover value and captures rents. However, these rents are offset by the losses

[9]If n is an odd number, then the summations start with $i = \frac{n+1}{2}$ instead of $i = \frac{n}{2}$.

the raider incurs when an insufficient number of shares are tendered, given that the price offered must be higher than the value of the firm under current management. As n increases, these two effects exactly offset each other and the raider becomes approximately indifferent between all possible prices.

While Bagnoli and Lipman (1988) did not consider the case where the raider has a private benefit, it is straightforward to see how their analysis changes when such a benefit is added. Since, in the absence of a private benefit, expected profits for the raider are zero in the limit anyway, adding a positive private benefit ($B > 0$) leads the raider to maximize the probability that his offer will be successful. This is achieved by always bidding the entire post-takeover value of the firm so that shareholders capture all the surplus. In other words, adding even an arbitrarily small private benefit to the symmetric information case leads to efficient takeovers: offers are always successful and at prices equal to the post-takeover value.

We can now summarize the results for the symmetric information case. Since the per-share value $v_\omega = V_\omega / n$ always converges to zero as n increases, we focus instead on $P = np$, which denotes the total dollar value (i.e., the price) the raider offers for the entire equity stake of the firm.

PROPOSITION 1: (i) *For* $B = 0$, *the raider's profit converges to* 0 *as* $n \to \infty$. *The probability of a successful takeover, however, remains strictly between* 0 *and* 1. (ii) *For any* $B > 0$, $\lim_{n\to\infty} P = \lim_{n\to\infty} E[V|S_n]$. *Furthermore, the probability of a successful takeover attempt converges to* 1 *for all states*.

It is worth noting that, since shareholders extract the entire value of the firm for $B > 0$, the expected value gain that shareholders receive converges to V_1 when the state is $\omega = 1$, which occurs with ex ante probability λ. When the state is $\omega = 0$, the price converges to $V_0 = 0$, so that shareholders obtain no increase in their wealth with probability $1 - \lambda$. Letting W represent the ex ante wealth (or value) gain for shareholders as a result of a tender offer, we can now conclude that $W = \lambda V_1 > 0$, and note that W just corresponds to the ex ante value of the firm $\bar{V} = E[V]$ under the raider.

4. ASYMMETRIC INFORMATION AND TENDER OFFERS

We next consider the case where each shareholder's signal is private, so that there is asymmetric information across shareholders. We first characterize shareholders' tendering strategies given a tender offer price p as a function of their private information. We then calculate the raider's equilibrium profit given his optimal choice of price.

4.1. *Shareholders' Tendering Strategies*

The first result establishes a threshold structure in shareholders' tendering strategies.

PROPOSITION 2: *For any offer price p, in the unique equilibrium of the tendering subgame there exists a cutoff signal s^* such that $\sigma_i^*(s) = tender$ for all $s < s^*$ and $\sigma_i^*(s) = keep$ for all $s > s^*$.*

PROOF: Let $q(\omega, \sigma^*, m)$ be the probability of having at least m shares tendered in the conjectured equilibrium σ^* given state ω. The expected value of a nontendered share is

$$\sum_{\omega \in \Omega} \beta(\omega|s) q\left(\omega, \sigma^*, \frac{n}{2}\right) v_\omega.$$

On the other hand, the expected value of tendering is just the price offered p. The difference, $D(s, \sigma^*)$, between not tendering and tendering then becomes

$$D(s, \sigma^*) = \sum_{\omega \in \Omega} \beta(\omega|s) q\left(\omega, \sigma^*, \frac{n}{2}\right) v_\omega - p = \beta(1|s) q\left(1, \sigma^*, \frac{n}{2}\right) v_1 - p, \tag{3}$$

where $\beta(1|s) = \frac{\lambda f(s)}{\lambda f(s) + (1-\lambda) g(s)}$.

Note that $\beta(1|s)$ and therefore $D(s, \sigma^*)$ are strictly increasing in s for all $s \in (0, 1)$ due to MLR. Furthermore, for $s = 0$, $D(0, \sigma^*) = -p < 0$, while for $s = 1$, $D(1, \sigma^*) = q(1, \sigma^*, \frac{n}{2}) v_1 - p$. Note that if there does not exist an $s^* < 1$ such that $D(1, \sigma^*) \geq 0$, then it must be that $D(s, \sigma^*) < 0$ for all s. This implies that $q(1, \sigma^*, \frac{n}{2})$ must be equal to 1, implying that $D(1, \sigma^*) = v_1 - p > 0$ for all $p \in (0, v_1)$, thus contradicting the assumption that $D(1, \sigma^*) < 0$. Therefore, for any $p \in (0, v_1)$ there exists an s^* such that $D(s, \sigma^*) = 0$, verifying that σ^* is an equilibrium. Uniqueness of the symmetric Nash equilibrium follows immediately given that $\beta(1|s)$ and $q(\omega, \sigma^*, \frac{n}{2})$ are increasing in s and s^*, respectively. *Q.E.D.*

This proposition states that, for any price offered by the raider, only shareholders whose signals suggest a relatively low post-takeover value will be willing to tender. Shareholders with signals suggesting a high post-takeover value will prefer to keep their shares and benefit from the price appreciation following an acquisition. In essence, for given strategies of the other shareholders, any individual shareholder is more tempted to hold out when his signal is high. This gives rise to the threshold structure identified in the proposition.

Proposition 2 identifies a "lemons problem" in takeover bidding. Importantly, this issue arises directly as a result of the free-rider problem associated with dispersed ownership. To see why, note that the price mechanism implied by a tender offer is essentially a "take it or leave it" mechanism. With only one shareholder, the raider could entirely avoid the lemons problem by making an offer that is infinitesimally greater than 0. Since the single shareholder's decision is clearly pivotal to whether the takeover succeeds or not, he would find it optimal to tender independently of his information. However, as ownership becomes more dispersed, the likelihood that any single shareholder is pivotal goes down, and shareholders have an opportunity to free-ride on their peers.[10] Each shareholder will therefore only tender when his information indicates that the post-takeover value will be low.

COROLLARY 1: *The cutoff signal s^* characterized by Proposition 2 is an increasing function of p, with $\lim_{p\to v_1} s^* = 1$.*

This corollary establishes that increasing the offer price increases the likelihood that any given shareholder will tender his share. The intuition for this result is that by raising the price, the raider increases the cost of holding out to a shareholder, thus making him more willing to tender. At the extreme, bidding the highest possible post-takeover value (i.e., $p = v_1$) leads all shareholders to tender, since there is no further gain from holding out.

Given Proposition 2 and Corollary 1, we can now invert expression (3), defining the cutoff signal s^*, $D^U(s^*, \sigma^*) = 0$, to express p as a function of s^*:

$$p(s^*) = \beta(1|s^*)v_1 \sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s^*)(1 - F(s^*))^{n-1-i}. \tag{4}$$

Equation (4) gives us an indifference condition for shareholders in that a shareholder with signal s^* is indifferent between tendering at the price p or keeping his share. It is therefore the analog of (1) for the asymmetric information case, being virtually identical for the case where $\lambda \to 0$ or 1. The sum on the right-hand side corresponds to the probability that at least half of the remaining $n-1$ shareholders tender, given the threshold strategy s^*. Equivalently, this represents the probability that an individual shareholder attaches to the success of the tender offer, conditional on knowing that he has not tendered. The first term, $\beta(1|s^*)$, is simply the posterior belief that the state is high for a shareholder with signal $s = s^*$. The product is therefore the expected value of a nontendered share to a shareholder with a signal s^*.

4.2. *Equilibrium Price and Raider Profit*

Since from now on we will be using primarily the cutoff rule in most of our expressions, in what follows we abuse notation slightly by just using s in place

[10]There are other mechanisms for making shareholders pivotal and thus reducing or possibly even eliminating the lemons problem induced by free-riding. One of the simplest is to use a 100% conditional offer, so that no shares are purchased unless every shareholder agrees to tender his share. Such an offer, however, suffers from obvious practical difficulties in that it is highly likely that there will always be some subset of shareholders who either remain uninformed about the tender offer or do not behave optimally.

of s^* wherever there is no risk of confusion. We can now calculate the expected profit to the raider, Π:

$$
\begin{aligned}
(5) \qquad \Pi = {} & \lambda v_1 \sum_{i=n/2}^{n} \binom{n}{i} F^i(s)(1-F(s))^{n-i} i \\
& - p\Biggl(\lambda \sum_{i=0}^{n} \binom{n}{i} F^i(s)(1-F(s))^{n-i} i \\
& \qquad + (1-\lambda) \sum_{i=0}^{n} \binom{n}{i} G^i(s)(1-G(s))^{n-i} i\Biggr) \\
& + B\Biggl(\lambda \sum_{i=n/2}^{n} \binom{n}{i} F^i(s)(1-F(s))^{n-i} \\
& \qquad + (1-\lambda) \sum_{i=n/2}^{n} \binom{n}{i} G^i(s)(1-G(s))^{n-i}\Biggr).
\end{aligned}
$$

This equation represents the analog of (2) for the case where each shareholder's information is private and the raider possibly has a private benefit from acquiring the target firm. The first term reflects the expected value of the shares that are purchased, which is just the expected number of shares that are tendered times the value of each share conditional on the takeover being successful in the good state, when the takeover adds value. The second term is the price paid times the expected number of shares bought on aggregate, which are paid for whether the takeover is successful or not. The third term is the expected value of any private benefits of control, which accrue in either state ($\omega = 0$ or 1) but only if at least half the shares are tendered so that the takeover is successful.

It is useful to divide the analysis at this point into two cases: $B = 0$ and $B > 0$. We start with the former, for which we have the following result.

PROPOSITION 3: *Let* $B = 0$. *Then for any* p, $\lim_{n\to\infty} \Pi(s^*(p)) \leq 0$.

The proposition establishes that, for widely held firms, no tender offer can make positive profits. The intuition stems from the lemons problem discussed earlier. For any price that is offered, a larger fraction of the shares are tendered in the bad state than in the good state, since on average a larger number of shareholders will have sufficiently low signals (i.e., negative information) in the bad state. Given this, the share of the surplus going to the raider in the good state can never be enough to compensate him for the cost of the shares in the bad state.

Put differently, a raider who learns that his offer is successful at a given price should lower his expectation on the post-takeover value, since the offer's success means that a majority of shareholders have observed sufficiently low signals. Consequently, he should consider lowering his bid. However, even at a lower price the raider would still face bad news if he learns that his offer is accepted by a majority of shareholders, since those shareholders must have even lower signals. In this sense, this problem can also be viewed as a "winner's curse" in that the raider must recognize that his offer succeeding in fact represents bad news, since at least half of the shareholders must have observed relatively low signals. Consequently, he must revise downward his expected value of the firm. Moreover, this happens for any price offered, since the lower is the price, the worse is the news that the offer has gone through.

Assume therefore from now on that $B > 0$. Using equation (9) in the Appendix, the expected value of the shares that are purchased by the raider can be written as

$$\lambda V_1 F(s) \sum_{i=n/2-1}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}.$$

Defining $\tilde{n} = n(\lambda F(s) + (1-\lambda)G(s))$ as the expected number of shares that are purchased, this translates into an expected value for each share purchased of

$$\tilde{v} = v_1 \frac{\lambda F(s) \sum_{i=n/2-1}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}}{\lambda F(s) + (1-\lambda)G(s)}.$$

We denote by p^* the price that maximizes the raider's profit for any given $B > 0$. In what follows, we focus as before on $P^* = np^*$, which represents the total dollar value the raider offers for the entire equity stake of the firm. For comparability, we normalize the expected value of each share that is tendered by the total number of shares, $\tilde{V} = n\tilde{v}$.

Define $\bar{s}$ such that $\int_0^{\bar{s}} f(s)\,ds = \frac{1}{2}$. We can now establish our main result.

PROPOSITION 4: *For any $B > 0$, there exists $p \in (0, v_1)$ and $\overline{n}$ such that, for all $n > \overline{n}$, $\Pi(s^*(p)) > 0$. Furthermore, let $\bar{B} = (V_1/\lambda)[\beta(1|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})) - \lambda F(\bar{s})]$. Then we have the following cases*:

(i) *For $0 < B < \bar{B}$, $\lim_{n\to\infty} P^* = \lim_{n\to\infty} \tilde{V} = 0$. The probability of a successful takeover attempt converges to* 1 *when $\omega = 0$ and converges to* 0 *when $\omega = 1$.*

(ii) *For $B > \bar{B}$,*

$$\lim_{n\to\infty} P^* = \beta(1|\bar{s})V_1 > \frac{\lambda F(\bar{s})}{\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})} V_1 = \lim_{n\to\infty} \tilde{V}.$$

The probability of a successful takeover attempt converges to 1 *for all states*.

In contrast to the result in Proposition 3, this last result shows that for any private benefit $B > 0$, positive expected profits in the takeover of a widely held firm are possible. When the raider derives a private benefit from the acquisition, there is always a price sufficiently low that in the limit guarantees the offer will be accepted at least in the bad state and that allows the raider to enjoy his private benefit with probability $1 - \lambda$. Therefore, a necessary and sufficient condition for a takeover bid of a widely held firm to be profitable is that $B > 0$.

However, Proposition 4 also presents a stark dichotomy in the raider's optimal pricing strategy as a function of the private benefit B in that his behavior changes in a discrete way around the cutoff value $\bar{B}$. For values of B below this cutoff, the raider offers the minimum amount possible, with a price approaching 0 as n increases. For these low values of the private benefit, we find then that the limiting price as the number of shareholders increases converges to the expected value of the firm conditional on the success of the takeover. However, the outcome does not allocate the firm in an efficient manner: in the limit, the tender offer is successful if and only if the takeover adds no value. Acquisitions that increase the most value, those where $V = V_1 > 0$, do not take place.

For B above the cutoff $\bar{B}$, the raider offers a strictly positive price that reflects much of the value increase in the good state. Essentially, when the private benefit B is relatively large, the raider finds it optimal to bid a higher price and induce tendering more often than when B is low. We find now that the limiting price converges to the lowest price necessary to guarantee that the offer always succeeds. We therefore have allocative efficiency, since the tender offer is always successful and it is ex ante optimal for the firm to be taken over. In other words, all value-increasing acquisitions take place.[11] We also observe that the equilibrium price converges to a value greater than the expected value of the tendered shares, so that the acquirer overpays for the firm. However, this price is strictly less than V_1, implying that shareholders fail to extract the full surplus from a value-increasing raider (we discuss this issue further in Section 4.4).

The reason for the abrupt transition in the raider's bidding strategy stems from the behavior of the order statistics that determine the probability of success of the tender offer in the good state. From (4) we see that for any P such that $s^* < \bar{s}$, as n increases, the right-hand side converges to zero, since the probability of a successful takeover in the good state (i.e., when at least 50% of the shares are tendered) becomes vanishingly small. Conversely, for P sufficiently large that $s^* \geq \bar{s}$, as n increases, the right-hand side converges to $\beta(1|s^*)v_1$, since the probability of a successful takeover in the good state approaches 1. From the raider's perspective, therefore, as n increases, the only question is

[11] However, if we allow $v_0 < 0$, then both value-increasing and -decreasing takeover attempts may be successful. Therefore, even efficiency may not hold in more general settings. We discuss this case further in Section 5.

whether to bid high enough to induce at least half the shareholders to tender in the good state. If so, which occurs when B is high enough, the raider wishes to do so at the least cost, which implies bidding $P^* = \beta(1|\bar{s})V_1$. If not, which occurs when B is low, the raider may as well bid the lowest possible amount, which is achieved by offering a price P that gets closer and closer to zero as n increases.

Given this limiting behavior of the probability of a successful takeover, we can restate the intuition for these two cases as follows. With a relatively low price the offer will succeed only if the value of the shares after the takeover is expected to be low, since many individuals have observed a low signal. When B is small, it is optimal for the raider to offer a price only slightly higher than the value of the shares. The raider loses money on each share, but this loss can be made arbitrarily small by reducing the price, taking it in the limit to 0, the post-takeover value when $\omega = 0$. As a consequence, even a small benefit can be enough to compensate the raider for the infinitesimally small loss. While the price can never be exactly right because of the free-riding problem, it can be made "almost" right by reducing it, thus making sure that the takeover is successful only in the bad state.

When B is high, the fact that a low bid leads the takeover to be unsuccessful with probability λ (the ex ante probability that the post-takeover value of the firm is high) implies too much of a loss for the raider. Therefore, the raider will want to make sure that the takeover always succeeds, even if this implies offering a price which is too high. In other words, the potential gain of the private benefit B is sufficiently large that it is no longer so crucial to minimize the loss on a per-share basis.

The results above characterize the behavior of the total value offered for the firm in equilibrium, P^*, and show that the raider offers a higher price as his private benefit increases. In the limit, of course, this effect is pushed to the extreme, as characterized in Proposition 4. For completeness, we show that a similar result also holds for the case of finite n, in that comparative statics results show that the equilibrium share price depends on the level of private benefits in an intuitive way.

PROPOSITION 5: *For any finite n, the equilibrium price p^* is increasing in B.*

4.3. *Comparison to Symmetric Information Benchmark*

From the analysis in Section 3 we know that, for the symmetric information case, allowing the raider to have even an arbitrarily small private benefit leads to takeover attempts that are always successful at prices that equal the post-takeover value. By contrast, Proposition 4 shows that this need not be the case when shareholders have private information. With private information, prices deviate from post-takeover firm value as the raider's private benefit increases, since shareholders use their information in deciding whether to tender or not.[12] We also show that the probability of realizing a value-increasing takeover converges to zero when the raider's private benefit is strictly positive but low, implying that the efficient allocation of the firm is never achieved, quite unlike the symmetric information case.

For completeness, we note that a similar result to that of the symmetric information case can be derived within the private information framework discussed above for the case where information problems become vanishingly small as a corollary to Proposition 4 by letting $\lambda \to 0$ or 1. In this case, it is straightforward to see that, as $\lambda \to 1$, $\bar{B} \to 0$ and we are always in case (ii) of the proposition, even for an arbitrarily small private benefit: the probability of a value-increasing raider taking over converges to 1 as n goes to infinity. Moreover, the price converges to the post-takeover value, which in this case is equal to $V_1 > 0$, so that the raider makes no profit beyond the value of his private benefit (this is similar to the symmetric information case studied in Section 3). At the other extreme, as $\lambda \to 0$, we have that $P^* \to 0$ as well for either case of Proposition 4 so that again the price converges to the post-takeover value.

4.4. *Private Information and Value Extraction*

From Proposition 4 we obtain that for $B > \bar{B}$, the equilibrium price converges to a value greater than that of the tendered shares, so that the raider overpays for the shares purchased. However, this price is strictly less than V_1, implying that shareholders fail to extract the full surplus from those raiders who increase the most value. Nevertheless, shareholders gain as a result of their private information when the raider's private benefit is sufficiently high, as described below.

As in Section 3, define W to represent the ex ante wealth gain to shareholders from the presence of the raider. First, consider the case where $\omega = 0$. For the symmetric information case we have determined that, conditional on $\omega = 0$, $W \to 0$ as $n \to \infty$. By contrast, with asymmetric information, for $\omega = 0$ we have that $W > 0$ for $B > \bar{B}$, since the raider will bid $P^* > 0$ and a majority of shareholders will tender. Conversely, consider the case where $\omega = 1$. For the symmetric information case we know that $W \to V_1$ as $n \to \infty$, while under asymmetric information $W \to \frac{1}{2}P^* + \frac{1}{2}V_1$, since half of the shareholders tender at the offered price, with the other half holding out and receiving the post-takeover value. Whether shareholders on net receive more or less than $\bar{V}$, the ex ante value of the firm, depends of course on the value of P^*. Note, however, that since the firm is always purchased when $B > \bar{B}$, total ex ante surplus must be $B + \bar{V}$: the private benefit plus the ex ante value of the firm conditional on

[12]This does not necessarily imply that shareholders are made worse off as a result of their private information. We analyze this issue in the next subsection.

success of the takeover. We can now establish the following result concerning the distribution of surplus.

COROLLARY 2: *If $B > \bar{B}$, then with asymmetric information, $W = \lambda\bar{B} + \bar{V} > \bar{V}$: The value gain to shareholders is higher than the ex ante firm value under the raider. The payoff to the raider is $B - \lambda\bar{B} > 0$.*

PROOF: Recall that we defined $\tilde{n} = n(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))$, which is the expected number of shares purchased by the raider in equilibrium. The total dollar payment to shareholders is therefore

$$\tilde{n}\frac{P^*}{n} = (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))\beta(1|\bar{s})V_1.$$

Furthermore, conditional on $\omega = 1$, a fraction $(1 - F(\bar{s}))$ of shareholders hold out at the offer price P^* and instead obtain the post-takeover value V_1. The ex ante expected value gain to these shareholders is $\lambda(1-F(\bar{s}))V_1$. Therefore, the total gain to shareholders is

$$\begin{aligned}
&\tilde{n}\frac{P^*}{n} + \lambda(1 - F(\bar{s}))V_1 \\
&\quad = (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))\beta(1|\bar{s})V_1 + \lambda(1 - F(\bar{s}))V_1 \\
&\quad = \lambda\left(\frac{V_1}{\lambda}\big[\beta(1|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})) - \lambda F(\bar{s})\big]\right) + \lambda V_1 \\
&\quad = \lambda\bar{B} + \bar{V},
\end{aligned}$$

as desired.

Finally, note that since the takeover attempt is always successful, the total surplus must be $B + \bar{V}$, which implies that the raider receives $B - \lambda\bar{B} > 0$.

Q.E.D.

The corollary demonstrates that shareholders benefit from their private information by allowing them to extract greater value from the raider. As in the symmetric information benchmark, shareholders extract the full ex ante value of the firm, $\bar{V}$. However, their private information also allows them to extract a fraction of the raider's private benefit B, since the raider needs to overpay for the shares he purchases to ensure a successful takeover attempt.

From the corollary, it is also straightforward to see that an increase in the information value of shareholders' signals allows them to extract greater surplus when B is large. One way to observe this is by considering an improvement in the information content of the signal that increases the threshold signal $\bar{s}$, representing the point where $F(\bar{s}) = \frac{1}{2}$. As $\bar{s} \to 1$ (or as $\bar{s}$ increases), the price offered, P^*, converges to the value of the firm in the good state conditional on the success of the takeover offer, V_1. This increase is reflected in an increase in $\bar{B}$ and thus an increase in the surplus obtained by shareholders.

These rents, of course, must reduce the efficiency of the takeover market, since they reduce the return to the raider and thus the likelihood that a value-increasing takeover will occur. This is seen very clearly by focusing on the case where $B < \bar{B}$, for which we have the following result.

COROLLARY 3: *If* $0 < B < \bar{B}$, *then with asymmetric information the value gain to shareholders is zero*: $W = 0$. *The payoff to the raider is* $B(1 - \lambda) > 0$.

PROOF: For this case, from Proposition 4 we have that the total ex ante surplus is $B(1-\lambda)$, since the takeover occurs only for $\omega = 0$. Moreover, for $B < \bar{B}$, the price offered converges to 0 as $n \to \infty$, so that shareholders obtain virtually no surplus. *Q.E.D.*

This result points to an inefficiency created by the existence of private information in that value-increasing takeovers fail to occur when private benefits are low. In this case, the raider only receives his private benefit B in the bad state, $\omega = 0$. Unlike the symmetric information case, the total surplus being created now is just $B(1 - \lambda)$, which is strictly lower than the ex ante surplus that is available, $B + \bar{V}$.

5. ROBUSTNESS: VALUE-DESTROYING RAIDERS

We now consider the case where $V_0 < 0$, so that the raider may destroy value if the takeover is successful. As a starting point, note that there may be multiple equilibria even when information is symmetric. Consider a raider who is expected to destroy value. If he offers a negative price at least as large as the value under his management, there is an equilibrium in the tender offer subgame such that nobody tenders: Not tendering is a best response if a shareholder knows that the raider will not be able to take over. However, there is also an equilibrium in the tender offer subgame such that everyone tenders: Tendering is a best response if a shareholder knows that the value-destroying raider is going to take over. The same multiplicity problem exists in our setting as well.

In what follows, we restrict attention to the equilibrium in which no tendering occurs following a negative offer price (although similar conclusions hold also for the case where the offer price is allowed to be negative). In this case, the raider's expected profit following a negative offer price is zero, so without loss of generality we can restrict attention to offers with nonnegative prices only. We can now characterize shareholders' tendering strategies. For $V_0 \neq 0$, a shareholder's incentive to hold out versus tendering, for any price p, is described by

$$D(s, \sigma^*) = \beta(1|s)q\left(1, \sigma^*, \frac{n}{2}\right)v_1 + \beta(0|s)q\left(0, \sigma^*, \frac{n}{2}\right)v_0 - p, \quad (6)$$

which is similar to expression (3), but includes the additional term $\beta(0|s)q(0, \sigma^*, \frac{n}{2})v_0$ representing the payoff to a shareholder when he holds out and the takeover is successful in state $\omega = 0$. Much as in the previous section, it is straightforward to show that for any positive offer price p, setting (6) equal to zero defines a unique threshold signal s^* such that, in equilibrium, shareholders with signal $s < s^*$ tender, while those with signal $s > s^*$ do not.

We can now calculate the expected profit of the raider, Π^-, following a positive offer price. Much as in the previous section, we have that

$$\Pi^- = \Pi + (1-\lambda)v_0 \sum_{i=n/2}^{n} \binom{n}{i} G^i(s)(1-G(s))^{n-i} i, \tag{7}$$

where Π is as defined above in equation (5) for the case where $V_0 = 0$. Equation (7) highlights the fact that the raider's profit when $V_0 < 0$ is similar to the case where $V_0 = 0$ with the addition of the second term to reflect the value destruction when $\omega = 0$. The last term therefore represents the expected value of the shares purchased when $\omega = 0$: the per-share value v_0 times the expected number of shares, all conditional on the takeover occurring. Since $v_0 < 0$, this additional term must be negative.

As in the previous section, we can invert expression (6), defining the cutoff signal s^*, $D(s^*, \sigma^*) = 0$, to express p as a function of s^*:

$$\begin{aligned} p(s^*) &= \beta(1|s^*)v_1 \sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s^*)(1-F(s^*))^{n-1-i} \\ &\quad + \beta(0|s^*)v_0 \sum_{i=n/2}^{n-1} \binom{n-1}{i} G^i(s^*)(1-G(s^*))^{n-1-i}. \end{aligned} \tag{8}$$

This expression is similar to (4) with the addition of the second term to reflect the expected value of the shares in state $\omega = 0$. The final summation therefore corresponds to the probability that at least half of the remaining $n-1$ shareholders tender, given the threshold strategy s^* and conditional on the bad state $\omega = 0$. The term $\beta(0|s)$ is simply the posterior belief that the state is low for a shareholder with signal $s = s^*$. We can now state the following result.

PROPOSITION 6: *There exists* $\underline{B} > 0$ *such that for any* $B \le \underline{B}$, $\lim_{n\to\infty} \Pi^- \le 0$.

This results differs from the earlier case where $V_0 = 0$ in which, for any positive private benefit $B > 0$, there is always a price that leads to positive expected profits. The intuition for this result once again stems from the lemons problem discussed earlier, in that the success of the tender offer signals to the raider that a majority of shareholders observed relatively low signals, suggesting he may have overpaid for the shares. However, unlike the previous case, when the raider destroys value in state $\omega = 0$, he loses value on all the shares he purchases, and this remains true even if the price he pays is arbitrarily close to zero for those shares. The raider therefore needs a sufficiently large private benefit to compensate him for the expected losses.

Much like in the previous section, it can be shown that for intermediate values of the private benefit, the raider can in fact make nonnegative expected profits. However, unlike the case where $V_0 = 0$, an equilibrium in which the raider successfully takes over only when $\omega = 0$ and at a price arbitrarily close to zero is no longer feasible. To see why, suppose to the contrary that the probability of taking over converges to 0 for $\omega = 1$. In that case, the benefit of holding out for a shareholder with a sufficiently high signal becomes arbitrarily small as n goes to infinity. By contrast, the cost of not tendering remains strictly negative since $V_0 < 0$. But this contradicts the fact that for any $p > 0$, there must be some cutoff signal s^* such that $D(s^*, \sigma^*) = 0$. Therefore, even for arbitrarily small prices, there must be a positive probability of a successful takeover when $\omega = 1$, which arises because the possibility that value may be destroyed upon the success of the offer encourages shareholders to tender.

In equilibrium, the minimum private benefit that just breaks even for the raider must reflect that when the tender offer is successful in state $\omega = 1$, some value will be created and some profits will be earned by the raider. Conditional on $\omega = 0$, however, the raider makes losses. Since the price at which the raider purchases the shares is arbitrarily small, these losses also represent overall value destruction, that is, $V_0 + B < 0$. The following proposition formalizes this argument.

PROPOSITION 7: *Let* $|V_0| < \frac{\beta(1|\bar{s})}{\beta(0|\bar{s})} V_1$. *There exists a* $\Delta > 0$ *such that for* $B \in (\underline{B}, \underline{B} + \Delta)$, *the following cases exist*:

(i) *The probability of a successful takeover converges to* 1 *when* $\omega = 0$ *and is strictly positive*, *but less than 1*, *when* $\omega = 1$.

(ii) *The probability of a successful takeover converges to* 0 *when* $\omega = 1$ *as* $V_0 \to 0$.

(iii) *Conditional on* $\omega = 0$, *a takeover destroys value*, *that is*, $V_0 + \underline{B} < 0$.

This finding confirms that our main result in Proposition 4 is robust to allowing for a raider who sometimes destroy firm value: for low values of B, either no takeover occurs or the likelihood of the takeover occurring is significantly higher when $\omega = 0$ than when $\omega = 1$. Even though value destruction by the raider leads to greater shareholder tendering and thus a positive probability of success when $\omega = 1$, it is still true that value-increasing takeovers are less likely to be successful than value-destroying ones. Moreover, the proposition establishes that not only do shareholders lose value, but that, conditional on $\omega = 0$, the change in total welfare, $V_0 + \underline{B}$, is negative. The restriction on $|V_0|$ is simply to ensure that, conditional on the cutoff signal $\bar{s}$, the takeover does not destroy firm value ex ante.

For the case where B is large, it is straightforward to see that for B large enough, it will be optimal to take over in both states of the world, since only then will the raider be assured of receiving his private benefit. The argument is similar to that in the second part of Proposition 4. Note, however, that firm value is destroyed when $\omega = 0$, since $V_0 < 0$, so that efficiency ex post no longer holds.

As a final point, we note that our main inefficiency result from this section and the last—that value-increasing takeovers are less likely to be successful—also arises in other contexts. In particular, a similar inefficiency result holds for the case where V_0 is strictly positive, thus establishing that such inefficiencies arise regardless of the value being generated by the raider. Likewise, it can be shown that the result is not restricted to the case where only shareholders have information. Rather, it is robust to the possibility that the raider has additional private information about some component of the value he may create upon acquiring the firm. For details on both of these cases, see Marquez and Yılmaz (2007b).

6. CONCLUSION

This paper has analyzed takeover bidding when shareholders have private information. The main result is that a tender offer is more likely to be successful when the raider adds no value to the firm. In particular, when private benefits are small, the raider can take over only when he is not expected to create any value. Only when private benefits are large will value-increasing takeovers be profitable.

The main driving force in these results stems from a lemons problem that arises as a result of the free-rider problem in takeovers, in that shareholders who expect a higher value for their shares post-takeover will be unwilling to tender at the offered price. This lemons problem implies that a raider finds it difficult to acquire the firm when doing so would increase the firm's value. In contrast to the symmetric information case, though shareholders may be better off, they are unable to extract full surplus from a value-increasing raider.

APPENDIX

Mathematical Digression

We would like to show that

$$\sum_{i=k}^{n} \binom{n}{i} \alpha^i (1-\alpha)^{n-i} i = \alpha n \sum_{i=k-1}^{n-1} \binom{n-1}{i} \alpha^i (1-\alpha)^{n-1-i}, \tag{9}$$

where $\alpha \in [0, 1]$. Start by noting that

$$\sum_{i=k}^{n} \binom{n}{i} \alpha^i (1-\alpha)^{n-i} i = \alpha n \sum_{i=k}^{n} \frac{(n-1)!}{i!(n-i)!} \alpha^{i-1} (1-\alpha)^{n-i} i.$$

Canceling out the i terms in the numerator and denominator, this expression can be further written as $\alpha n\sum_{i=k}^{n}\frac{(n-1)!}{(i-1)!(n-i)!}\alpha^{i-1}(1-\alpha)^{n-i}$. By changing the limits of the summation, this expression can also be written as $\alpha n\sum_{i=k-1}^{n-1}\frac{(n-1)!}{i!(n-1-i)!}\alpha^{i}(1-\alpha)^{n-1-i}=\alpha n\sum_{i=k-1}^{n-1}\binom{n-1}{i}\alpha^{i}(1-\alpha)^{n-1-i}$, as desired.

A direct implication of this fact is that

$$\sum_{i=1}^{n}\binom{n}{i}\alpha^{i}(1-\alpha)^{n-i}i=\alpha n.$$

Omitted Proofs

PROOF OF PROPOSITION 1: The proof is similar to the analysis in Bagnoli and Lipman (1988), so we only sketch the main parts of the argument. As argued above, for $p=E[v|S_n]\sum_{i=n/2}^{n-1}\binom{n-1}{i}\phi^{i}(1-\phi)^{n-1-i}$ and $B=0$, we can write the raider's profits as

$$E[v|S_n]\left(\sum_{i=n/2}^{n}\binom{n}{i}\phi^{i}(1-\phi)^{n-i}i-n\phi\sum_{i=n/2}^{n-1}\binom{n-1}{i}\phi^{i}(1-\phi)^{n-1-i}\right).$$

Using expression (9), the above equation simplifies to $E[v|S_n]\binom{n}{n/2}\phi^{n/2}(1-\phi)^{n/2}\frac{n}{2}$. Note that this term is nonnegative for all $\phi\in[0,1]$ and furthermore it is maximized at $\phi=\frac{1}{2}$. Therefore, following an optimal offer price, the expected profit due to shares bought is $E[V|S_n]\binom{n}{n/2}(\frac{1}{2})^{n+1}$. From Sterling's approximation we have that $\lim_{n\to\infty}E[V|S_n]\binom{n}{n/2}(\frac{1}{2})^{n+1}=0$. Therefore, the expected profit due to the shares bought must converge to zero for any price $P\in(0,E[V|S_n])$ as $n\to\infty$. Furthermore, note that the probability of a successful takeover is bounded away from 1. This establishes the first part of the proposition.

Consider now the case where $B>0$. Since in the absence of a private benefit profits converge to zero for any possible price, for any $B>0$ there must exist $\bar{n}$ such that for any $n>\bar{n}$, it is optimal for the raider to offer $p=E[v|S_n]$. Doing so guarantees the success of the offer and allows the raider to obtain his private benefit B. *Q.E.D.*

PROOF OF COROLLARY 1: To show that s^* is an increasing function of p, note simply that if $D(s,\sigma;p')=0$ for some $p=p'$, then $D(s,\sigma;p)<0$ for all $p>p'$. Since β is increasing in s, the result will be true as long as q is increasing in the cutoff signal s^*. Since q is an order statistic specifying the probability that at least a certain number of shares are tendered in equilibrium, this probability must be increasing in the value of the cutoff signal s^* below which shareholders tender, thus establishing the first part. For the second part, note that as p goes to v_1, we need $\beta(1|s)q(1,\sigma^*,\frac{n}{2})$ to go to 1 as well so that $D(s,\sigma^*)\geq 0$. But note that as s and s^* go to 1, both $\beta(1|s)$ and $q(1, \sigma^*, \frac{n}{2})$ converge to 1, thus establishing the second part of the result. *Q.E.D.*

PROOF OF PROPOSITION 3: Using expression (9), equation (5) simplifies to

$$\begin{aligned}\Pi &= \lambda F(s) n v_1 \sum_{i=n/2-1}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i} \\ &\quad - pn(\lambda F(s) + (1-\lambda)G(s)) \\ &\quad + B\Bigg(\lambda \sum_{i=n/2}^{n} \binom{n}{i} F^i(s)(1-F(s))^{n-i} \\ &\qquad + (1-\lambda) \sum_{i=n/2}^{n} \binom{n}{i} G^i(s)(1-G(s))^{n-i}\Bigg).\end{aligned}$$

Substituting for $p(s^*)$ from (4) and using the fact that $V_1 = nv_1$, we can now write Π as

$$\begin{aligned}\Pi(s) &= V_1\big[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))\big] \\ &\quad \times \sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i} \\ &\quad + V_1 \lambda F(s) \binom{n-1}{n/2-1} F^{n/2-1}(s)(1-F(s))^{n/2} \\ &\quad + B\Bigg(\lambda \sum_{i=n/2}^{n} \binom{n}{i} F^i(s)(1-F(s))^{n-i} \\ &\qquad + (1-\lambda) \sum_{i=n/2}^{n} \binom{n}{i} G^i(s)(1-G(s))^{n-i}\Bigg).\end{aligned} \tag{10}$$

Differentiating the first term in square brackets with respect to s, we obtain

$$\begin{aligned}&\big[\lambda F - \beta(1|s)(\lambda F + (1-\lambda)G)\big]' \\ &\quad = -\frac{\lambda(1-\lambda)[f'g - fg']}{[\lambda f + (1-\lambda)g]^2}(\lambda F + (1-\lambda)G).\end{aligned} \tag{11}$$

The term $f'g - fg'$ is strictly positive for all n given MLR. Consequently, the derivative in equation (11) is negative for all n. We also know that $[\lambda F(0) - \beta(1|0)(\lambda F(0) + (1-\lambda)G(0))] = 0$. Therefore, $[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))]$ must be not only a decreasing function of s, but also negative for all

$s > 0$. Since the term $\sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}$ is strictly positive for any given n for all $s > 0$, this implies that the entire first term of $\Pi(s)$ in (10) must be negative for all $s > 0$ and for all n.

We can now calculate the limiting value for $\Pi(s)$. From Sterling's approximation it is easy to see that $\lim_{n\to\infty} \binom{n-1}{n/2-1} F^{n/2-1}(s)(1-F(s))^{n/2} = 0$. Therefore, $\lim_{n\to\infty} \Pi(s)$ becomes

$$
\begin{aligned}
(12)\qquad & V_1\big[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))\big] \\
& \times \lim_{n\to\infty}\left(\sum_{i=n/2}^{n-1}\binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}\right) \\
& + B\lim_{n\to\infty}\left(\lambda\sum_{i=n/2}^{n}\binom{n}{i} F^i(s)(1-F(s))^{n-i}\right. \\
& \qquad\qquad \left. + (1-\lambda)\sum_{i=n/2}^{n}\binom{n}{i} G^i(s)(1-G(s))^{n-i}\right).
\end{aligned}
$$

From above, we know that for all n the first term is negative, so that its limit as $n \to \infty$ must be nonpositive. Therefore, for $B = 0$ we can now conclude that $\lim_{n\to\infty} \Pi(s) \le 0$. *Q.E.D.*

PROOF OF PROPOSITION 4: Assume that $B > 0$. Note that, for any s such that $F(s) < \frac{1}{2} < G(s)$, by the law of large numbers (LLN) the first term in expression (12) converges to 0 as n gets arbitrarily large, whereas the second term converges to $(1-\lambda)B$. In other words, for some $s < \bar{s}$, expression (12) converges by the LLN to $(1-\lambda)B$ as $n \to \infty$. Therefore, for $p(s)$ we must have $\lim_{n\to\infty} \Pi(s^*(p)) > 0$, which establishes that there is always some price at which equilibrium profits are positive when $B > 0$.

For the rest, note that for $s > \bar{s}$, (12) converges to $V_1[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))] + B$ as $n \to \infty$. Therefore, one necessary condition for the raider to find it optimal to induce a majority of shareholders to tender in both states is that there exists an $s > \bar{s}$ such that

$$V_1\big[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))\big] + B > (1-\lambda)B.$$

From equation (11), we know that the left-hand side is a decreasing function of s. Therefore, the cutoff private benefit of control is characterized by the equality

$$V_1\big[\lambda F(\bar{s}) - \beta(1|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))\big] + \bar{B} = (1-\lambda)\bar{B},$$

yielding $\bar{B} = (V_1/\lambda)[\beta(1|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})) - \lambda F(\bar{s})]$ so that for $B > \bar{B}$ the raider prefers to induce $s^* > \bar{s}$ rather than $s^* < \bar{s}$.

To obtain $\lim_{n\to\infty} \tilde{V}$, recall that $\tilde{V} = n\tilde{v}$, so that

$$\lim_{n\to\infty} \tilde{V} = V_1 \lim_{n\to\infty}\Bigg(\frac{\lambda F(s)}{\lambda F(s) + (1-\lambda)G(s)} \times \sum_{i=n/2-1}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}\Bigg).$$

For the first part, note that for p such that $F(s^*(p)) < \frac{1}{2}$, it is immediate by the LLN that

$$\lim_{n\to\infty} \tilde{V} = V_1 \lim_{n\to\infty}\Bigg(\frac{\lambda F(s)}{\lambda F(s) + (1-\lambda)G(s)} \times \sum_{i=n/2-1}^{n-1} \binom{n-1}{i} F^i(s)(1-F(s))^{n-1-i}\Bigg) = 0.$$

The equilibrium price is $P^* = \beta(1|s^*)V_1 \sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s^*)(1-F(s^*))^{n-1-i}$, so that for p such that $F(s^*(p)) < \frac{1}{2}$, $\lim_{n\to\infty} P^* = V_1 \lim_{n\to\infty}(\beta(1|s^*)\sum_{i=n/2}^{n-1}\binom{n-1}{i} \times F^i(s^*)(1-F(s^*))^{n-1-i}) = 0$. Therefore, $\lim_{n\to\infty} P^* = \lim_{n\to\infty} \tilde{V} = 0$, as desired.

For the second part, note that p must be such that $F(s^*(p)) > \frac{1}{2}$. Then by the LLN the price converges to $\beta(1|s^*)V_1$ for all $s^* > \bar{s}$ since from equation (4) we have that $p(s^*) = \beta(1|s^*)v_1 \sum_{i=n/2}^{n-1}\binom{n-1}{i} F^i(s^*)(1-F(s^*))^{n-1-i}$. Taking the lowest possible price such that $F(s^*(p)) > \frac{1}{2}$, the optimal price np^* converges to $\beta(1|\bar{s})V_1$. We then have

$$\lim_{n\to\infty} \tilde{V} = V_1 \frac{\lambda F(\bar{s})}{\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})},$$

as claimed. Finally, note that

$$\beta(1|\bar{s}) = \frac{\lambda f(\bar{s})}{\lambda f(\bar{s}) + (1-\lambda)g(\bar{s})} > \frac{\lambda F(\bar{s})}{\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})} \quad \Leftrightarrow \quad \frac{f(\bar{s})}{g(\bar{s})} > \frac{F(\bar{s})}{G(\bar{s})},$$

which is satisfied by MLR. Therefore, $\beta(1|\bar{s})V_1 > \frac{\lambda F(\bar{s})V_1}{\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})}$, as desired.

Finally, we show that offer prices $P \in (0, \beta(1|\bar{s})V_1)$ are suboptimal as n gets arbitrarily large. For any $P \in (0, \beta(1|\bar{s})V_1)$, we have

$$\frac{P}{\beta(1|s^*)V_1} = \sum_{i=n/2}^{n-1} \binom{n-1}{i} F^i(s^*)(1-F(s^*))^{n-1-i} = q\Big(1, \sigma^*, \frac{n}{2}\Big).$$

Moreover, as $n \to \infty$, $s^* < \bar{s}$ for all $P \in (0, \beta(1|\bar{s})V_1)$. Expression (12) then converges by the LLN to

$$V_1\big[\lambda F(s^*) - \beta(1|s^*)(\lambda F(s^*) + (1-\lambda)G(s^*))\big]\frac{P}{\beta(1|s^*)V_1} + B(1-\lambda) \quad \text{as} \quad n \to \infty.$$

However, we know from above that the term $[\lambda F(s^*) - \beta(1|s^*)(\lambda F(s^*) + (1-\lambda)G(s^*))]$ is negative for all $s > 0$. Therefore, the raider's profit is maximized by choosing P to be arbitrarily close to zero as n increases. An analogous argument can be used to show that for all $B > \bar{B}$, offer prices $P < \beta(1|\bar{s})V_1$ are less profitable than $\beta(1|\bar{s})V_1$ as n gets arbitrarily large. *Q.E.D.*

PROOF OF PROPOSITION 5: Since the cutoff signal s^* is monotonically increasing in p, we can focus on choosing an optimal s^*, which is characterized by the first order condition

$$\frac{\partial \Pi}{\partial s} = 0,$$

with second order condition $\partial^2\Pi/\partial s^2 < 0$. Define $H = \frac{\partial \Pi}{\partial s}$ and note that, at the optimal cutoff signal, $H \equiv 0$. We can therefore use the implicit function theorem to find

$$\frac{\partial s^*}{\partial B} = -\frac{\partial H/\partial B}{\partial H/\partial s}.$$

Since $\frac{\partial H}{\partial s} < 0$ by the second order condition, we know that the sign of $\frac{\partial s^*}{\partial B}$ will be the same as the sign of $\frac{\partial H}{\partial B}$. From equation (5),

$$\begin{aligned}\frac{\partial H}{\partial B} &= \frac{\partial}{\partial s}\left(\lambda \sum_{i=n/2}^{n}\binom{n}{i}F^i(s)(1-F(s))^{n-i}\right.\\ &\qquad\left. + (1-\lambda)\sum_{i=n/2}^{n}\binom{n}{i}G^i(s)(1-G(s))^{n-i}\right)\\ &= \frac{n!}{(\frac{n}{2}-1)!(\frac{n}{2})!}\big(\lambda f(s)F^{n/2-1}(s)(1-F(s))^{n/2}\\ &\qquad + (1-\lambda)g(s)G^{n/2-1}(s)(1-G(s))^{n/2}\big)\\ &> 0.\end{aligned}$$

Therefore, we have $\frac{\partial H}{\partial B} > 0$ for all $s^* \in (0,1)$, implying that $\frac{\partial p^*}{\partial B} > 0$. *Q.E.D.*

PROOF OF PROPOSITION 6: Substituting for $p(s^*)$ from (8) and, as above, using expression (9), we can now write $\lim_{n\to\infty} \Pi^-$ as

$$
\begin{aligned}
(13)\qquad & V_1\big[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))\big] \\
&\quad \times \lim_{n\to\infty}\left(\sum_{i=n/2}^{n-1}\binom{n-1}{i}F^i(s)(1-F(s))^{n-1-i}\right) \\
&\quad + V_0\big[(1-\lambda)G(s) - \beta(0|s)(\lambda F(s) + (1-\lambda)G(s))\big] \\
&\quad \times \lim_{n\to\infty}\left(\sum_{i=n/2}^{n-1}\binom{n-1}{i}G^i(s)(1-G(s))^{n-1-i}\right) \\
&\quad + B\lim_{n\to\infty}\left(\lambda\sum_{i=n/2}^{n}\binom{n}{i}F^i(s)(1-F(s))^{n-i}\right. \\
&\quad\qquad \left. + (1-\lambda)\sum_{i=n/2}^{n}\binom{n}{i}G^i(s)(1-G(s))^{n-i}\right).
\end{aligned}
$$

Recall, from the proof of Proposition 3, that for all n the term $[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))]$ is zero at $s = 0$ and strictly decreasing for all $s > 0$. Similarly, the term $[(1-\lambda)G(s) - \beta(0|s)(\lambda F(s) + (1-\lambda)G(s))]$ is zero at $s = 0$, and from MLR we know that this term is strictly increasing for all $s > 0$. Moreover, it is of equal magnitude but of opposite sign as $[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))]$. Therefore, $V_1[\lambda F(s) - \beta(1|s)(\lambda F(s) + (1-\lambda)G(s))]$ and $V_0[(1-\lambda)G(s) - \beta(0|s)(\lambda F(s) + (1-\lambda)G(s))]$ are both negative for all $s > 0$ and for all n.

We can now conclude that for any s such that $\frac{1}{2} < G(s)$, by the LLN as n gets arbitrarily large the second term in the above expression converges to a strictly negative number and the third term converges to a value which is proportional to B and no smaller than $(1-\lambda)B$. The first term is always nonpositive. Therefore, there exists a minimum level of B such that the raider's expected profit is positive. *Q.E.D.*

The proof of Proposition 7 makes use of the following result.

LEMMA 1: *There does not exist a price* $p > 0$ *such that* $\lim_{n\to\infty} \frac{q(1,\sigma^*,n/2)}{q(0,\sigma^*,n/2)} = 0$.

PROOF: Suppose to the contrary. Recall that $D(s, \sigma^*) = \beta(1|s)q(1, \sigma^*, \frac{n}{2}) \times v_1 + \beta(0|s)q(0, \sigma^*, \frac{n}{2})v_0 - p$. If $\frac{q(1,\sigma^*,n/2)}{q(0,\sigma^*,n/2)}$ converges to zero, then it must be that $D(s, \sigma^*) < 0$ for all s. This contradicts the existence of a cutoff signal such that $D(s^*, \sigma^*) = 0$. *Q.E.D.*

PROOF OF PROPOSITION 7: From Lemma 1, we know that if $V_0 < 0$, then for any $p > 0$ we must have $q(1, \sigma^*, \frac{n}{2}) > 0$. Therefore, s^* must converge to $\bar{s}$. Consequently, we must have $D(\bar{s}, \sigma^*) = 0$. Furthermore, $q(0, \sigma^*, \frac{n}{2})$ converges to 1 for any $p > 0$, so that $q(1, \sigma^*, \frac{n}{2})$ must converge to $-(\beta(0|\bar{s})V_0)/(\beta(1|\bar{s})V_1)$. This completes parts (i) and (ii) of the proposition.

From equation (13), for zero expected profits we have

$$\underline{B} = \frac{(V_1 - V_0)[(1-\lambda)G(\bar{s}) - \beta(0|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))]}{\lambda q(1, \sigma^*, \frac{n}{2}) + (1-\lambda)}.$$

Substituting $V_1 = -(\beta(0|\bar{s})V_0)/(\beta(1|\bar{s})q(1, \sigma^*, \frac{n}{2}))$ and letting $q(1, \sigma^*, \frac{n}{2}) = 1$ we have

$$V_0 + \underline{B} = V_0\left(1 - \frac{(1-\lambda)G(\bar{s}) - \beta(0|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))}{\beta(1|\bar{s})}\right).$$

Therefore, $V_0 + \underline{B} < 0$ as long as $\beta(1|\bar{s}) > (1-\lambda)G(\bar{s}) - \beta(0|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))$. Note that $\beta(1|\bar{s})[1 - (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))] > -\lambda F(\bar{s})$ since the left-hand side is positive and the right-hand side is negative. Adding and subtracting $(1-\lambda)G(\bar{s})$ to the right-hand side, we have

$$\begin{aligned}&\beta(1|\bar{s})\big[1 - (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s}))\big] \\ &\quad > (1-\lambda)G(\bar{s}) - (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})).\end{aligned}$$

Rearranging terms we have

$$\begin{aligned}\beta(1|\bar{s}) > {}&(1-\lambda)G(\bar{s}) + \beta(1|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})) \\ &- (\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})).\end{aligned}$$

Substituting $\beta(1|\bar{s}) = 1 - \beta(0|\bar{s})$ in the right-hand side gives us

$$\beta(1|\bar{s}) > (1-\lambda)G(\bar{s}) - \beta(0|\bar{s})(\lambda F(\bar{s}) + (1-\lambda)G(\bar{s})).$$

Therefore, we must have $V_0 + \underline{B} < 0$. From continuity, the inequality holds for some $q(1, \sigma^*, \frac{n}{2}) < 1$, thus establishing part (iii). *Q.E.D.*

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