Two-Class Voting: A Mechanism for Conflict Resolution

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We discuss two-class voting procedures where voters are divided into classes and a separate majority is required in each class. Examples include Chapter 11 bankruptcy proceedings and some political mechanisms. We investigate how voting mechanisms aggregate information dispersed among voters when voters have conflicts of interests as well as different information regarding a proposal. We find that two-class voting provides a significant improvement over one-class voting in all situations where voters have significant conflicts of interests, and where the voters are relatively evenly divided between interest groups. However, two-class voting is inefficient absent conflicts of interests. (*JEL* G30, G34, D72)

In many situations voting contests are resolved by grouping voters into separate classes, so that the proposal put to a vote is only passed if it receives a majority from each class of voters. A primary example is Chapter 11 of the U.S. Federal Bankruptcy Reform Act of 1978, where claim-holders are divided into classes according to the seniority of their claims to vote on a reorganization proposal. The proposal is accepted only if it receives a majority from each class of claim-holders. There are several related examples. Covenants for preferred stock sometimes require that mergers be approved by common stockholders and preferred stockholders separately.[1] In some countries constitutional amendments need separate approval from different bodies: for instance, in Belgium the Walloon and the Flemish region of the country need to pass certain laws separately.[2] Finally, in bicameral legislatures, two chambers of a parliament have to pass the same legislative proposal.[3]

The purpose of this paper is to study two-class voting as a mechanism to resolve conflicts of interests and to aggregate dispersed information.[4] We show that it is more efficient to group voters into classes whenever the conflict of interests between different groups is sufficiently strong and interest groups are of similar size. In this case one-class voting is not an efficient mechanism for aggregating dispersed information as some voters do not reveal their information through their vote, so their information

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[1] See Frank H. Easterbrook and Daniel R. Fischel (1983, 1991) (especially chapter 3 and references therein) for a general discussion of voting in corporations.

[2] This "strengthened special majority" procedure applies only to some laws that affect the power of the regions. These require that (i) 50 percent of each language group should be present, (ii) 50 percent of each language group votes in favor, and (iii) the total of "yes" votes in both groups exceeds two-thirds of all votes cast. We are grateful to Marc Zenner for providing us with this information.

[3] See George Tsebelis and Jeannette Money (1997) for a survey of the institutions of bicamerialism and analytic approaches to two-chamber parliaments. Their table 2.1 (pp. 48–52) summarizes the institutions of 52 countries with a bicameral parliament as of December 1995.

[4] We use the terms "unicameral" and "one-class," respectively, "bicameral" and "two-class" synonymously.

does not bear on the final outcome. Two-class voting may alleviate this problem of strategic witholding of information and incorporate the information of all voters.

To illustrate this point, we discuss a specific example. Consider the creditors of a firm in Chapter 11 bankruptcy who have to determine whether a reorganization proposal should be accepted, or whether the firm should be liquidated (the default outcome). Suppose there are two states of the world, and in one state all creditors benefit from accepting the reorganization proposal, whereas in the other state liquidation is optimal. The state is unknown and each creditor observes a noisy signal that is informative about which alternative would make her better off. Creditors also have conflicting interests, because the reorganization proposal asks senior creditors to give up a larger part of their claims than junior creditors compared to their expected payoff if the firm is liquidated. Since the potential benefits from reorganization are spread unevenly across creditors, the group that stands to benefit more will require less evidence in favor of reorganization, other things being equal. Therefore, to vote in favor of the proposal, senior creditors require that the proposal increase firm value with a higher likelihood (say, 70 percent) whereas junior creditors are content with a lower likelihood (say, 30 percent).[5]

When comparing the efficiency of voting rules we assume the perspective of a social planner who is interested only in maximizing firm value and ignores the distributional consequences of the proposal for different classes of creditors.[6] The planner would therefore wish to accept the proposal for some intermediate probability of the state where reorganization increases firm value (say, 50 percent).

In order to see the potential advantage of two-class voting, consider a standard one-class voting rule where a majority of all creditors has to vote "yes" to pass the proposal. We discuss why this rule is inefficient if creditors have conflicting interests.[7] Each creditor focuses on those outcomes where the vote is a "close race," so that she has the decisive vote. (She is said to be "marginal" or "pivotal" then.) This is the only outcome where a creditor can change her utility through her vote, so she forms beliefs about what must be true about the world in a state where her vote matters.[8] Assume there is a large number of creditors in each class. Then the information that senior and junior creditors infer from a close race is very similar because their own signals matter only very little. Therefore, all creditors approach the problem with similar beliefs, but their criteria for acceptance are different.

Suppose now that junior creditors have more votes than senior creditors, so with a simple majority rule the proposal can be accepted even if all senior creditors vote "no." Then the outcome is a close race only if the proposal is marginally acceptable to junior creditors. Every senior creditor understands that her vote counts only if the proposal is marginally acceptable to junior creditors, so it has a likelihood of about 30 percent to increase firm value. Then the proposal is not acceptable to senior creditors who require more favorable evidence (70 percent probability). Hence, senior creditors always vote against the reorganization proposal, even when (with hindsight) they would benefit from accepting it. Conversely, suppose senior creditors command the majority of all votes. Then the vote is a close race whenever reorganization is marginally acceptable to senior creditors. Junior creditors conclude that their vote counts only in situations where the proposal increases their payoffs, so a junior creditor always votes in favor of reorganization, even if her own signal alone suggests that liquidation is superior. Hence, no matter whether the votes of

[5] However, we only look at cases where information potentially matters and decisions are not determined entirely by the distributional consequences of a proposal.

[6] This is *ex post* efficient in the event of bankruptcy. It may be socially beneficial from an *ex ante* point of view to design bankruptcy laws so that the value of the firm is not maximized *ex post,* e.g., in order to give managers an incentive to invest in firm-specific human capital (Elazar Berkovitch et al., 1997).

[7] Timothy J. Feddersen and Wolfgang Pesendorfer (1997) analyze the tension between information-based and partisan voting for one-class voting games.

[8] Similarly, in auctions bidders condition on what must be true about the world when they submit the winning bid in order to avoid the "winner's curse." For a detailed discussion of the similarities between bidding in auctions and voting see Feddersen and Pesendorfer (1996a).

senior creditors are required to pass the reorganization proposal or not, the information of one group of creditors has no impact on the final decision if conflicts of interests are sufficiently strong.

It is here that two-class mechanisms like those used in Chapter 11 bankruptcy proceedings can help. In such a setting, each creditor has an impact on the final decision whenever she has the decisive vote in her own class *and* the other class approves. For example, a senior creditor knows that her vote matters only if a majority of junior creditors approves reorganization. However, even in that case, she cannot infer the victory margin among junior creditors. Hence, being a marginal voter in a two-class game means something very different from being a marginal voter in a one-class game. Furthermore, the vote amongst the senior creditors is close if reorganization is marginally acceptable by the (higher) standards of *their* group. Similarly, the vote amongst the junior creditors is close under beliefs that are less favorable about the reorganization proposal compared to the beliefs required by senior creditors. Hence, the class in which a voter is marginal matters. All creditors can now vote in favor if they observe a positive signal and reject it otherwise. Therefore, in a two-class set-up, voting decisions become more informative about the signals creditors observe, and this is critical for the efficiency of two-class voting.

There is a countervailing effect that reduces the efficiency of two-class voting. To see this, suppose that many creditors observe signals in favor of reorganization, but most of them are senior creditors. Most junior creditors observe signals that favor liquidation. Then the reorganization proposal can fail because it does not obtain the required majority of junior creditors. In two-class voting, it is not only the number of favorable signals that matters, but also the distribution of signals across classes, resulting in inefficient outcomes that are not present in a one-class set-up. We refer to this inefficiency in two-class voting as a "coordination problem."

We study a simple, generic model that generalizes this example and analyze the two opposing forces identified above. Firstly, two-class voting helps to aggregate information dispersed among voters with conflicting interests. Secondly, separating voters into classes creates a coordination problem. We show which of these opposing effects dominates in different circumstances and demonstrate that two-class voting is better than one-class voting if and only if conflicts of interests are sufficiently strong. Moreover, two-class voting is strictly better if in addition certain symmetry requirements are met, most importantly, that interest groups are equal in size.

We discuss the literature in Section I. Section II introduces the model and discusses the social planning problem. Section III presents some preliminary analysis and derives the best response functions for two-class voting games. Section IV discusses the properties of equilibria for one-class and two-class voting games. In Section V we compare the efficiency of one-class and two-class voting games and state our main result. Section VI illustrates some additional properties of the model. Section VII concludes. All proofs and technical results are collected in the Appendix.

I. Related Literature

The analysis of information aggregation through voting dates back to the Condorcet Jury Theorem which states that a majority-based rule is less likely to make a "mistake" than any single decision maker. Several proofs have been offered for variations and extensions of this claim.[9] One shortcoming of these papers is that they implicitly assume that each voter votes sincerely, conditioning her vote only on her own information. Recently, a second generation of papers showed that such "naive" voting is often irrational.[10] David Austen-Smith and Jeffrey S. Banks (1996) and Feddersen and Pesendorfer (1998) investigate the features of strategic voting without conflicts of interests. They show that strategic voting can aggregate information perfectly if the correct majority rule is used. Peter J. Coughlin (2000) extends this model and analyzes the implications of modifying the voting mechanism to allow for mistrials and pre-vote communications among voters.

[9] See Alvin K. Klevorick et al. (1984); H. Peyton Young (1988); and Krishna K. Ladha (1992).

[10] See Austin-Smith and Banks (1996), Feddersen and Pesendorfer (1996a, b, 1997, 1998), Andrew McLennan (1998), and Roger B. Myerson (1998a, b, 2000).

Feddersen and Pesendorfer (1996a, 1998) study the asymptotic features of information aggregation in models with conflicting interests. They show that the probability of electing the wrong candidate converges to zero in an arbitrarily large electorate. Our modeling framework is closer to Feddersen and Pesendorfer (1998) and Austen-Smith and Banks (1996). However, our approach differs from theirs in two ways. First, we focus on nonasymptotic features of information aggregation and show that one-class voting wastes large amounts of private information in the presence of conflicts of interests. Secondly, and more importantly, we show that it is possible to enhance information aggregation through a bicameral system.

In a more recent paper, Michael Suk-Young Chwe (1999) also analyzes the problem of conflicts of interests between a majority and a minority with different preferences or beliefs. He derives optimal mechanisms and introduces the possibility that minorities obtain special privileges that increase their inclination to reveal information. (We discuss his results in greater detail below.) Asher Wolinsky (2000) discusses a model where a decision maker extracts information from a number of experts whose preferences differ from those of the decision maker. He shows that separating experts into groups can enhance information aggregation, but the experts in his model do not have conflicting interests. Similarly, Jean-Jacques Laffont and David Martimort (1998, 1999) analyze information aggregation by parliamentary bodies through one or several regulators. They show that regulatory capture can be mitigated by delegating regulation to several independent agencies, which reduces informational rents associated with knowing all relevant information. However, the authors do not extend their results to voting.

The literature on bicameralism has addressed similar institutions but from a different angle than our paper. Daniel Diermeier and Myerson (1999) show in a model of self-interested legislative chambers that supermajority requirements and committees can enhance the bargaining position of a chamber in a bicameral system. Tsebelis and Money (1997) survey institutional and theoretical findings. Their models focus on intercameral bargaining between legislative chambers and on the process of obtaining agreement in case of a stalemate. They find that bicameral legislatures tend to preserve the status quo compared to unicameral legislatures and that bicameral systems tend to concentrate multidimensional conflicts of interests on one privileged dimension. This literature is complementary to our approach as none of these studies are related to information aggregation and the Condorcet Jury Theorem.

II. The Model

Assume there are two types $\tau \in \{j, k\}$ of voters who jointly decide on a proposal. There are M_τ voters of type τ, $M_j + M_k$ in total. The impact of the proposal on each voter's welfare depends on her type and on a state of nature $s \in \{\ell, \mathrm{h}\}$. All voters share a common prior and both states are equally probable.[11] We normalize to zero the utility of both players for the case where the proposal is rejected and denote the utility from accepting the proposal in state s for voters of type τ by U_τ^s, where

$$(1) \qquad U_\tau^{\mathrm{h}} = 1 - q_\tau \qquad \tau = j, k$$

$$U_\tau^{\ell} = -q_\tau \qquad \tau = j, k.$$

We assume that q_τ is strictly inside the unit interval, hence, all voters have a positive payoff in the high state h and a negative payoff in the low state ℓ. We say that voters have conflicting interests if $q_k \neq q_j$. Each voter i observes a binary signal $\sigma_i \in \{\mathrm{g}, \mathrm{b}\}$ where

$$(2) \quad \Pr(\sigma_i = \mathrm{g} | s = \mathrm{h}) = \Pr(\sigma_i = \mathrm{b} | s = \ell)$$

$$= 1 - \varepsilon \qquad 0 < \varepsilon < \tfrac{1}{2}.$$

Therefore, the signal indicates the correct state with probability $1 - \varepsilon$. The efficiency criterion which we use as a benchmark for comparing voting mechanisms is the equally weighted average of the utilities of all voters. We take the perspective of a social planner who can observe all $M_j + M_k$ signals. Since all signals have the same precision $1 - \varepsilon$, the planner's beliefs are a function of the total number of signals $M_j + M_k$ and the number of good signals n, where

[11] The assumption of equal probabilities is inessential.

$n \in \{0, 1, ..., M_j + M_k\}$. The planner would accept the proposal whenever it increases the sum of the utilities of all voters. Define aggregate payoffs from the point of view of the social planner by using

$$(3) \qquad \bar{q} = \frac{M_j}{M_j + M_k} q_j + \frac{M_k}{M_j + M_k} q_k.$$

Given our assumptions on q_τ it is immediate that $\bar{q}$ lies also strictly within the unit interval. The planner accepts the proposal if

$$(4) \quad [(1 - \bar{q})\Pr(s = \text{h}|n) - \bar{q}\ \Pr(s = \ell|n)] \geq 0$$

and rejects it otherwise. It is immediate from (4) that the social planner will accept the proposal if and only if $\Pr(s = \text{h}|n) \geq \bar{q}$. The objective expressed in (4) is clearly plausible in the context of Chapter 11 bankruptcy regulation, where the social objective is to maximize the value of the firm. Since $\Pr(s = \text{h}|n)$ increases in n, the social planner's decision rule can be summarized by a cutoff value n^* such that he will accept the proposal if and only if the number of good signals is at least n^*. He would accept the proposal even if all signals are bad, provided $\bar{q}$ is close to zero and the number of signals $M_j + M_k$ is sufficiently small. (Then $n^* = 0$.) Conversely, the planner would reject the proposal even if all signals are good whenever $\bar{q}$ is close to 1 and $M_j + M_k$ is small. In either case, the information contained in all signals together would not be sufficient to outweigh the prior.

We consider two types of voting rules and always assume that all votes are cast simultaneously and each voter has one vote. If voting takes place in one class, then acceptance of the proposal requires that at least a proportion α of all voters votes in favor of the proposal. Define $a = (M_j + M_k)\alpha$ and assume that a is an integer. We refer to this mechanism as one-class or unicameral voting. Alternatively, voters are grouped in classes according to their type, with type j voters in one class and type k voters in the other. Then acceptance of the proposal requires that at least a proportion α_τ of the voters of each class τ vote in favor of the proposal. (Again, $a_\tau \equiv \alpha_\tau M_\tau$ is assumed to be an integer.) We refer to this mechanism as two-class or bicameral voting. Clearly, this mechanism assumes that voters' types are associated with some observable characteristic.

A pure strategy for each voter is a mapping from the signal space, {g, b}, into the binary action space, {yes, no}. A mixed strategy is a mapping from the signal space {g, b} into the unit interval, denoted by ω^σ, a probability of voting "yes." In this paper we restrict attention to symmetric equilibria.[12] In every symmetric equilibrium voters of the same type who observe the same signal follow the same strategy. Symmetric equilibria are completely characterized by two strategies $\{\omega_\tau^{\text{g}}, \omega_\tau^{\text{b}}\}$, $\tau = j, k$.

In all voting models there are equilibria where each voter votes for the same alternative irrespective of her private information. Then no voter can influence the outcome, so this is always an equilibrium. We will restrict attention to equilibria where at least one type of voters votes "responsively" (i.e., the voter's decision is a function of her private information). Hence, we require that $\omega_\tau^{\text{g}} \neq \omega_\tau^{\text{b}}$ holds for at least one type. We refer to those equilibria as "responsive" in which at least one type votes responsively and where a type that votes nonresponsively does not decide alone on the outcome. This rules out equilibria where one group always votes "no" and effectively vetoes the proposal. Our definition is equivalent to assuming that there is a strictly positive probability of being pivotal for at least one type of voters so that some information always bears on the decision. Denote by π_τ^s the probability that a type τ voter votes "yes" in state s. Then we have

$$(5) \qquad \pi_\tau^{\text{h}} = (1 - \varepsilon)\omega_\tau^{\text{g}} + \varepsilon\omega_\tau^{\text{b}}$$

$$\pi_\tau^{\ell} = \varepsilon\omega_\tau^{\text{g}} + (1 - \varepsilon)\omega_\tau^{\text{b}}$$

and our definition of responsive voting strategies is equivalent to $\pi_\tau^{\text{h}} \neq \pi_\tau^{\ell}$. We follow the convention in the literature and refer to a strat-

[12] It is well known from the voting literature that games like the one considered here have a large number of asymmetric equilibria. Maug (1999) analyzes the asymmetric pure-strategy equilibria for a similar game and shows that their properties are very similar to the symmetric mixed-strategy equilibria characterized by Feddersen and Pesendorfer (1998). Myerson (1998a) objects to asymmetric equilibria as being nonrobust.

egy where $\omega_\tau^g = 1$ and $\omega_\tau^b = 0$ as "sincere" voting.

III. Analysis

We start by analyzing the way each voter conditions on her information (signal σ) and on being pivotal. We denote by

$$F_\tau = \sum_{i=a_\tau}^{M_\tau} \binom{M_\tau}{i} [\pi_\tau^h]^i [1 - \pi_\tau^h]^{M_\tau - i}$$

the probability of class τ approving the proposal in state h, and by

$$G_\tau = \sum_{i=a_\tau}^{M_\tau} \binom{M_\tau}{i} [\pi_\tau^\ell]^i [1 - \pi_\tau^\ell]^{M_\tau - i}$$

the probability of class τ approving the proposal in state ℓ.[13] Denote the probability of a type τ voter to be pivotal in state h by

$$\text{(6)} \quad \Pr(\text{piv}_\tau | \text{h}) = F_{\tau'} \binom{M_\tau}{a_\tau - 1} \times (\pi_\tau^h)^{a_\tau - 1}(1 - \pi_\tau^h)^{M_\tau - a_\tau}.$$

A voter of type τ becomes pivotal if (i) class τ' has passed the proposal (this happens with probability $F_{\tau'}$), and (ii) exactly $a_\tau - 1$ voters of her own class have voted in favor and all others have voted "no," the probability of which is given by the remaining part of (6). A similar expression obtains for the probability of being pivotal in state ℓ, $\Pr(\text{piv}_\tau|\ell)$ (exchange $F_{\tau'}$ with $G_{\tau'}$ and π_τ^h with π_τ^ℓ). Denote beliefs conditional on all three pieces of information (signal σ, pivotality in class τ, acceptance by class τ') by β_τ^σ. Then,

$$\text{(7)} \quad \beta_\tau^g = \frac{\Pr(\text{piv}_\tau|\text{h})(1 - \varepsilon)}{\Pr(\text{piv}_\tau|\text{h})(1 - \varepsilon) + \Pr(\text{piv}_\tau|\ell)\varepsilon}$$

and we obtain the equivalent expression for β_τ^b simply by replacing ε with $1 - \varepsilon$. Voters of type τ vote in favor of the proposal if and only if

$$\text{(8)} \quad \beta_\tau^\sigma(1 - q_\tau) - (1 - \beta_\tau^\sigma)q_\tau \geq 0.$$

Condition (8) is equivalent to $\beta_\tau^\sigma \geq q_\tau$. We can therefore interpret q_τ as a cutoff value for voters' beliefs β_τ^σ for them to vote in favor of the proposal. Hence, if $\beta_\tau^\sigma > q_\tau$, then voters of type τ vote in favor with probability 1 ($\omega_\tau^\sigma = 1$), if $\beta_\tau^\sigma < q_\tau$, they always vote against ($\omega_\tau^\sigma = 0$) and they randomize only if $\beta_\tau^\sigma = q_\tau$. It is evident from these expressions that $\beta_\tau^g > \beta_\tau^b$, since we require the signal to be informative [$\varepsilon < ½$ from (2)]. Hence, in a symmetric equilibrium where type τ voters vote responsively, any optimal strategy conforms to one of the following descriptions:

1. Voters randomize only after the bad signal and vote "yes" after the good signal.
2. Voters randomize only after the good signal and vote "no" after the bad signal.
3. Voters vote sincerely.

It can never be the case that voters randomize after both signals. This result follows immediately from the fact that no voter can be indifferent at both realizations of the signal. Furthermore, voters are more optimistic following a good signal. Therefore, each voter randomizes after observing the bad signal only if she always votes "yes" after observing the good signal. Similarly, she is more pessimistic after observing the bad signal, so she sometimes votes "no" after observing the good signal only if she always votes "no" after observing the bad signal. It is now legitimate to define strategies as:

$$\text{(9)} \quad \omega_\tau = \omega_\tau^b + \omega_\tau^g \in [0, 2]$$

since we can always recover the strategies as $\omega_\tau^g = \min\{\omega_\tau, 1\}$, $\omega_\tau^b = \max\{\omega_\tau - 1, 0\}$. We will frequently refer to voters who have a lower ω_τ as voting more "conservatively."

LEMMA 1 (Beliefs): *Beliefs β_τ^σ are decreasing in ω_τ and in $\omega_{\tau'}$.*

[13] F_τ and G_τ are cumulative distribution functions of order statistics. We discuss the properties of order statistics in greater detail in the mathematical Appendix.

We first explain why β_τ^σ is decreasing in the strategy of the other class, $\omega_{\tau'}$. A voter conditioning on approval by the other class becomes more optimistic about being in state h relative to her prior. Moreover, these beliefs become stronger if the other class votes more conservatively (ω_τ^b and ω_τ^g are lower). If voters of class τ vote in favor of the proposal with a lower probability, then approval by this class conveys stronger evidence in support of being in the high state h because it takes more positive signals to generate the same probability of approval. Conversely, if class τ voters are very likely to vote "yes," then approval by class τ becomes less informative about being in the good state, so β_τ is lower. The same logic carries over to the relationship between β_τ^σ and ω_τ. If all other voters in her class vote more conservatively (vote "yes" with a lower probability), then any particular voter will conclude from being pivotal that more positive signals have been observed by the other voters in her class. Therefore, β_τ^σ is higher if all other voters in the same class τ vote against the proposal more often.

Next, we denote the best-response function for class τ by ω_τ^*.[14]

LEMMA 2 (Best-Response Functions): *The best-response function of class* τ, ω_τ^*, *is a (weakly) decreasing function of the strategy of class* τ', $\omega_{\tau'}$.

Consider the case of a type j voter who observes that each type k voter votes for the proposal with a higher probability. Then the type j voter becomes less optimistic about being in the good state from Lemma 1. This implies immediately that a type j voter now requires more positive information about the proposal from her own class in order to stay indifferent between her choices. This is only possible if all other type j voters now vote more conservatively, so that being pivotal in her own class increases the odds of being in the good state. Then she becomes pivotal only if more of the other type j voters have observed the good signal. As a result, the strategies of type j voters and type k voters become "strategic substitutes," using a characterization familiar from oligopoly theory (Jeremy I. Bulow et al., 1985).

[14] There exists a unique best response for type τ to a given $\omega_{\tau'}$, as long as type τ votes responsively (see Lemma B1).

IV. Existence and Properties of Equilibria

With the help of the lemmas we proved in the previous section we can now prove the existence of equilibrium for two-class voting games.

PROPOSITION 1 (Equilibrium): *There exists a symmetric responsive equilibrium of the two-class voting game whenever the number of both types of voters is sufficiently large.*

Suppose there were only very few type j voters in the electorate. Recall that β_j^σ is falling in ω_j from Lemma 1, so that β_j^σ is maximized for $\omega_j = 0$. Then, for q_j large enough and a sufficiently high error of the signal, ε, we would have that $\beta_j^g < q_j$, even for very low acceptance probabilities ω_τ^g. In this case, there can never be a sufficient number of positive signals collected by type j voters such that they would vote in favor of the proposal, so they would veto it on the basis of their prior. In the same way, if q_τ is close to zero for both groups, then small electorates would accept the proposal on the basis of their prior. Therefore, we need a minimum size of the electorate for a responsive equilibrium to exist.[15]

We prepare for the efficiency comparisons in Section V by showing that two-class voting can never implement the social optimum:

PROPOSITION 2 (Information Aggregation): *There is no pair of majority rules* a_k, a_j *such that two-class voting achieves the first best with respect to the social welfare criterion.*

The intuition for this result is simple and is demonstrated in Figure 1. The horizontal axis of

[15] Note that our definition of responsiveness does admit equilibria where one class always passes the proposal and only the other class votes responsively. In fact, there exists a unique equilibrium for each class with this property. See Lemma B2. The third type of equilibrium involves both classes voting responsively. We will have more to say on the last type in Section V.

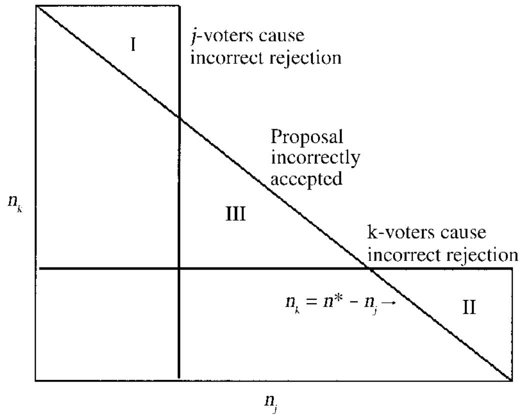

FIGURE 1. ERRORS WITH TWO-CLASS VOTING

Figure 1 shows the number n_j of good signals received by type j voters, the vertical axis represents the number n_k of those received by type k voters. The diagonal line indicates the location of all (n_j, n_k)-tuples so that $n_j + n_k = n^*$; this line represents perfect information aggregation. The proposal should only be accepted if (n_j, n_k) lies to the right and above this diagonal line. Two-class voting can only implement the optimum if all voters vote sincerely, since some information is lost through randomization. Voters will accept the proposal whenever both classes pass it, so $n_j \geq a_j$ (to the right of the vertical line) and $n_k \geq a_k$ (above the horizontal line). Areas I and II in the graph indicate those cases where the proposal is incorrectly rejected ($n_\tau < a_\tau$ for one class), whereas area III indicates distributions of signals where the proposal is incorrectly accepted. Clearly, there is no combination of arranging horizontal and vertical lines that could replicate the diagonal line.

Note that this argument is general and makes no reference to voters' preferences. On the other hand, the next proposition rests crucially on the fact that voters have conflicting interests. This is expressed by the fact that their payoffs from accepting the proposal are different. From (1) and since q_j, q_k are strictly within the unit interval, all voters would accept the proposal if they knew the state with certainty. However, given uncertainty about the state, they apply different criteria for voting "yes" from (8), because they have conflicting interests ($q_k \neq q_j$). In order to facilitate the exposition, we assume without loss of generality that $q_k > q_j$ and measure conflicts of interests by the difference $q_k - q_j$. We can now demonstrate the following result about one-class voting:

PROPOSITION 3 (One-Class Voting with Conflicts of Interests): *If conflicts of interests are sufficiently strong, then at most one type of voters votes responsively in any equilibrium of the one-class voting game. For any majority requirement* $\alpha \in (0, 1)$, *there exists a unique symmetric responsive equilibrium if the electorate is sufficiently large. In this equilibrium, type* j *voters always vote for the proposal if* $(M_j + M_k)\alpha > M_j$. *Otherwise type* k *voters always vote against the proposal.*

Proposition 3 demonstrates that conflicts of interests can be sufficiently significant so that one-class voting develops an important asymmetry. Only one type of voters votes responsively, whereas voters of the other type vote only according to their preferences, independently of the signal they receive. This result is generic and important for our subsequent argument as it reveals the defect of one-class voting that two-class voting can potentially improve on.

A voter of type τ votes in favor of the proposal only if her posterior belief is greater than or equal to her hurdle value q_τ, and votes against otherwise.[16] Being pivotal in a one-class voting game carries similar information for both types. For large enough electorates, this similarity is very high and is not fully outweighed by the signal, especially if the signal is imprecise (ε is high). However, for stronger conflicts of interests, the hurdle values q_τ are very different for different types [see also (8)]. Hence, if beliefs are similar but hurdle rates are different for both types, then it is not possible for both types to vote responsively.

V. Comparing the Efficiency of Voting Rules

In this section we compare and contrast the efficiency properties of two-class and one-class voting games with respect to the social welfare

[16] Posterior beliefs in a one-class voting game have expressions similar to the definition of β_τ^σ [see (7)].

criterion expressed in (4). The comparison of voting rules (one-class voting vs. two-class voting) involves comparisons of sets of outcomes since we can always choose different majority requirements and may sometimes have multiple equilibria. We will therefore say that a voting rule A dominates another voting rule B whenever we can find a set of majority requirements and a corresponding equilibrium with voting rule A that supports a better outcome than *any* outcome that can be supported with rule B. Effectively, the most efficient outcome with rule B becomes the benchmark for rule A (and vice versa). However, we do not require that rule A is an improvement for *any* choice of majority requirements and for *any* equilibrium.[17]

We first establish a comparison for the case without conflicts of interests that follows immediately from the previous section:

PROPOSITION 4: *If there are no conflicts of interests so that $q_j = q_k$, then one-class voting is strictly better than two-class voting with respect to the social welfare criterion.*

Proposition 4 follows from the fact that, absent conflicts of interests, one-class voting implements the social planner's choice whereas two-class voting cannot do that (Proposition 2).[18] Although information aggregation in a two-class voting game is never perfect from a social planner's perspective, this inefficiency vanishes asymptotically:

PROPOSITION 5: *Fix any pair of majority requirements α_j, α_k strictly inside the unit interval. The probability of accepting the proposal in state ℓ and the probability of rejecting the proposal in state* h *both converge to zero as both M_j and M_k become large for any sequence of symmetric responsive equilibria.*

This result establishes a property that parallels related results for one-class voting games.[19] Proposition 5 is important because it demonstrates robustness. Information aggregation may not be optimal because voters choose their voting strategies according to their own preferences and not according to those of the social planner. Also, majority rules α_τ may be suboptimal. Although these distortions reduce efficiency, they do not create a systematic bias and efficiency is attained asymptotically.

From Proposition 4, two-class voting can be a superior mechanism only if there are sufficient conflicts of interests so that unicameral voting is no longer optimal. We have already established that one-class voting with sufficiently strong conflicts of interests will have only one group of voters voting responsively (Proposition 3). We obtain the following proposition.

PROPOSITION 6: *If conflicts of interests are sufficiently strong so that there is no symmetric responsive equilibrium of the one-class voting game where both types of voters vote responsively, then two-class voting is weakly better than one-class voting with respect to the social welfare criterion.*

Suppose conflicts of interests are significant and type τ voters do not vote responsively in any one-class voting game. Then two-class voting can duplicate the outcome and efficiency level of such a one-class voting game by setting $\alpha_\tau = 0$ and choosing $\alpha_{\tau'}$ appropriately. Hence, any result that can be achieved with one-class voting where voters of one group ignore their information can be replicated by a two-class voting game.

Our discussion of welfare so far rests on a specific social welfare criterion that gives equal weights to all voters [see (3) and (4)]. We have not used this restriction so far, but we do require it for our next result.[20] We find a set of suffi-

[17] In fact, for any voting rule we can always find majority requirements that lead to very inefficient outcomes. Also, we focus on those two-class voting equilibria where both classes vote responsively whenever they exist. Those two-class voting equilibria where only one class votes responsively can never improve on the one-class voting outcome.

[18] Optimality of one-class voting follows immediately from Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998).

[19] For example, see Feddersen and Pesendorfer (1998). Like them and other models with discrete signals, we have to exclude unanimity ($\alpha = 1$) as a potential majority rule. See Yılmaz (1998) for a model with continuous signals that can allow $\alpha = 1$.

[20] We can show that this restriction is not necessary as all results we have stated so far would also hold if we gave arbitrary weights to all voters. Propositions 1, 3, and 5 make

cient conditions under which two-class voting is strictly better than one-class voting with respect to maximizing the social welfare criterion. We say that the electorate is symmetric if $M_j = M_k$ and $q_k + q_j = 1$. Also, we say that conflicts of interests are strong if $q_j < \varepsilon$. Now we are ready to state the main result of the paper:

THEOREM 1: *If conflicts of interests are strong, the electorate is symmetric, and M_τ is sufficiently large, then two-class voting is strictly better than one-class voting with respect to the social welfare criterion.*

The superiority of two-class voting depends on a number of conditions. Firstly, conflicts of interests need to be sufficiently strong, otherwise one-class voting could implement the optimum (see Proposition 4). Secondly, if we had only few voters and imprecise signals then a responsive equilibrium cannot exist due to lack of information to outweigh the priors. This is why we need sufficiently many voters (see also Proposition 1).

Now, recall the discussion of one-class voting with conflicts of interests following Proposition 3. Being pivotal in a one-class game conveys similar information to both types of voters, but their criteria for voting in favor are very different if conflicts of interests are strong. For example, a type k voter may infer that the proposal is marginally acceptable (or unacceptable) to type j voters, but this is not relevant for her own decision. It is here where two-class voting can be beneficial because it allows the beliefs of different types of voters to be different when they condition on being pivotal. Now a type k voter becomes pivotal whenever $a_k - 1$ other voters of her type have accepted the proposal, and this fact carries information about the desirability of the proposal to herself. Then it is rational to also consider her own information for making a decision. Therefore, more information is incorporated in the final outcome.[21]

no reference to the social welfare criterion. No part of the proofs of Propositions 2, 4, and 6 uses the weights of the social welfare criterion.

[21] Chwe (1999) discusses other mechanisms to enhance the inclination of minorities to reveal information.

Finally, we need to understand why the result requires symmetric electorates. The problem with having different numbers of voters in each class is that it may lead to equilibria where one class always passes the proposal independently of its information. Suppose class j is much larger than class k. When type k voters condition on acceptance by class j, they form beliefs that are sufficiently positive such that no amount of negative information collected within their own class could outweigh this belief. Then the same outcome can always be replicated by one-class voting and no advantage for two-class voting exists. Hence, the dominance of two-class over one-class voting is only weak for the general case. Clearly, the fact that we obtain strict dominance shows that we can expect the result to hold also if we perturb the game and allow for slightly asymmetric electorates. The next section performs some numerical analysis to explore this issue.

VI. Numerical Illustrations

In Section V we provide sufficient conditions for our main result (Theorem 1). We conjectured that the symmetry conditions could be relaxed. In this section we complement our discussion with some numerical examples and illustrate how far we can deviate from symmetric electorates and still conclude that two-class voting is strictly better than one-class voting. In particular, we relax the condition that there are equal numbers of voters of each type.

We start by formally defining the criterion we wish to use to evaluate voting rules for alternative parameterizations of the model. Let the expected loss associated with a particular voting rule be the weighted average of the probabilities of incorrect rejection in state h (type I error) and of incorrect acceptance in state ℓ (type II error). The weights are the aggregate payoffs from the social planner's criterion [see (3)]. We denote the minimized loss by L^*_{Social}, where

$$(10)\quad L^*_{\text{Social}} = \min_a\{(1-\bar{q})\Pr(n < a|\text{h}) + \bar{q}\,\Pr(n \geq a|\ell)\}$$

$\Pr(n < a|\text{h})$ is the probability of a type I error, and $\Pr(n \geq a|\ell)$ is the probability of a type II error. For one-class and two-class voting we establish the lowest expected loss that can be achieved using these voting rules. Hence, analogously to (10) we define:

$$(11)\quad L^*_{\text{Twoclass}} = \min_{a_j, a_k}\{(1-\bar{q})[1 - F_jF_k] + \bar{q}G_jG_k\}.$$

It follows from our definitions in Section III that F_jF_k and G_jG_k are the probabilities of passing the proposal in state h and state ℓ respectively. Hence, we choose the majority rules a_j, a_k that maximize the social planner's objective which is equivalent to minimizing the weighted sum of type I and type II errors. In the same way, we define the expected loss for one-class voting:

$$(12)\quad L^*_{\text{Oneclass}} = \min_{a,\tau}\{(1-\bar{q})(1-F_\tau) + \bar{q}G_\tau\}$$

where τ is the type of those voters who vote responsively. Comparing voting rules on the basis of the best outcomes they can implement is consistent with our welfare analysis in Section V.

We wish to study how the expected loss defined in (10)–(12) changes for varying compositions of the electorate, holding conflicts of interests constant. We define conflicts of interests as constant whenever $q_k - q_j$ is constant across different electorates. We can compute the two-class voting equilibrium for each vector of parameters $\{q_\tau, M_\tau, a_\tau, \varepsilon\}$ and then calculate (11). We could not prove that the equilibrium where both classes vote responsively is unique, but we have never encountered multiple equilibria.

We specify that $\varepsilon = 0.4$, $q_k - q_j = 0.25$ and $\bar{q} = 0.5$.[22] We use a grid of different sizes for the electorate where M_τ ranges from 10 to 500 for both types. We are interested in the relative improvement of two-class voting over one-class voting and compute the following statistic:

$$(13)\quad \text{Percentage Improvement} = 100\,\frac{L^*_{\text{Oneclass}} - L^*_{\text{Twoclass}}}{L^*_{\text{Oneclass}} - L^*_{\text{Social}}}.$$

This statistic is normalized to lie between zero and 100.[23] If two-class voting achieves the social optimum, then the percentage improvement equals 100. If two-class voting is ineffective and leads to the same errors as one-class voting, then the percentage improvement equals zero. We can interpret this statistic as measuring the extent to which two-class voting helps to close the gap between one-class voting and the social optimum. Figure 2 shows that the relative improvement is larger if the electorate is large and evenly split between the two classes. The percentage improvement approaches 100 as the electorate becomes large. For the parameters chosen here it becomes 98.7 for $M_j = M_k = 500$. For large electorates, the two-class voting mechanism behaves like the social planner: the difference in the incidence of errors between the social planner and two-class voting becomes negligible relative to the incidence of errors with one-class voting. Also, convergence is fast for evenly split electorates, and relatively slow for unevenly split electorates. Our analysis identifies the coordination problem as the source of inefficiencies in two-class voting games. This effect relies crucially on the possibility that the good (respectively, bad) signals are not distributed in proportion to the numbers of voters in each class. As the electorate becomes large, such an uneven distribution of signals becomes less likely, so the coordination problem becomes unimportant for large electorates.

VII. Discussion and Conclusion

This paper has analyzed the comparative advantages of one-class voting and two-class voting in a situation where individuals do not only have differential information, but also different interests with respect to the proposal decided by a vote. The general finding is that it is often useful to segregate voters into homogeneous

[22] We have performed (but do not report) simulations for different values of ε (from 0.38 to 0.495) and for different values of $q_k - q_j$ (from 0.25 to 0.95). The qualitative features remain the same.

[23] This assumes that conflicts of interests are strong, so one-class voting cannot be efficient.

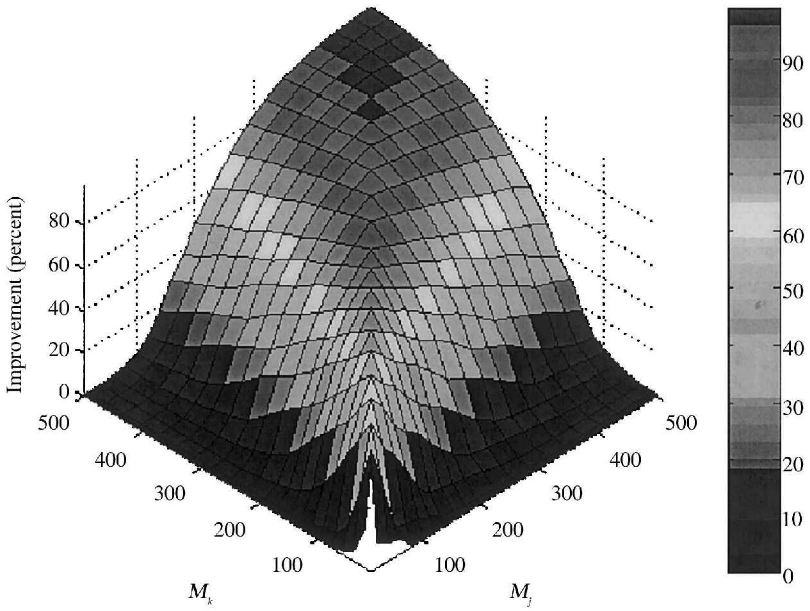

FIGURE 2. IMPROVEMENT OF TWO-CLASS OVER ONE-CLASS VOTING

classes with respect to their preferences. Absent conflicts of interests, two-class voting is inefficient as it results in coordination failure.

Our results are consistent with some institutions found in practice. These include Chapter 11 bankruptcy proceedings and separate votes by common and preferred stock on a merger proposal. Based on our results, bicameral parliamentary systems are useful only if the chambers represent different interests, yet each chamber is relatively homogeneous. For example, a lower house that represents federal interests and an upper house that represents regional interests would be beneficial for voting on a proposal where distributional issues between the federal and the regional level are at stake. However, if the proposal had consequences for the distribution across regions (e.g., northern versus southern regions), then the upper house would not be homogeneous with respect to conflicting interests, and our results would suggest that such an arrangement is inefficient. The same applies to constitutional assemblies. The Belgian constitution discussed in the introduction seems to be a good application of our results: distributional issues between the language regions are resolved by requiring separate majorities from each group to pass a constitutional amendment.

Our analysis is also supported by the fact that unicameral systems with opposing interest groups often develop procedures that emulate two-class voting. Parliamentary systems gave rise to the development of parties that represent interest groups. They take a ballot within the party before they vote in the general assembly. In most countries some laws and constitutional amendments require supermajorities. These pass only with the approval of all major parties. Hence, first interest groups (parties) vote and then voting in the general assembly is along party lines. Similarly, German corporations have worker and employer representatives on their supervisory boards. Typically, each group meets before the actual board meeting to exchange opinions on agenda items. Universities often make decisions at faculty meetings, but opinions (information) are often exchanged at less formalized departmental meetings that seem to represent more homogeneous interest groups. In all cases, information aggregation first takes place at the level of homogeneous interest groups (parties, departments) before the final decision is made according to formal procedures.

At this point we need to emphasize that we have not established that two-class voting is necessarily an optimal mechanism. Most likely, in a superior mechanism voters would agree on the proposal in conjunction with a scheme of offsetting transfers. These would compensate for the distributional consequences of a proposal and mitigate conflicting interests. We do not see examples of transfers of this kind, probably because eliminating conflicting interests is not generally possible.

Another limitation is the sequential nature of many voting processes, since, for example, the different chambers of parliaments often vote sequentially rather than simultaneously. Eddie Dekel and Michele Piccione (2000) show that simultaneous and sequential voting games produce identical symmetric equilibria within a one-class framework. In one-class voting, observing prior votes does not give useful information in addition to being pivotal. However, this is not the case for two-class voting games. Hence, it is an open issue how sequential voting would affect our results.

In our analysis we restrict attention to a basic two-class voting rule. Clearly, several variations of this rule are possible. For example, we could have contingent majorities where the majority requirement in one class depends on the outcome in the other class (e.g., accept if both classes have voted with at least 50 percent in favor, or if one class has voted at least 75 percent in favor).[24] Alternatively, we could require that all voters together must pass the proposal with a certain majority in addition to approval by each class.[25] We conjecture that both modifications of the basic two-class voting rule could help to alleviate the coordination problem we identified above and therefore increase the efficiency of two-class voting. We leave these questions to future research.

[24] Effectively, the "minority caucus" proposed by Chwe (1999) is such a contingent majority requirement: in his mechanism the proposal is accepted if either (1) it receives an overall majority, or (2) it falls short of the overall majority by at most one vote, but obtains a supermajority in the smaller class ("minority").

[25] See the remarks on the Belgian constitution in the introduction for an example.

Appendix A: Technical Results on Order Statistics

This Appendix summarizes the technical results about order statistics required for our argument. We first define

$$F_\tau^i = \binom{M_\tau}{i} [\pi_\tau^{\mathrm{h}}]^i [1 - \pi_\tau^{\mathrm{h}}]^{M_\tau - i}$$

and

$$G_\tau^i = \binom{M_\tau}{i} [\pi_\tau^{\ell}]^i [1 - \pi_\tau^{\ell}]^{M_\tau - i}$$

so that $F_\tau = \sum_{i=a_\tau}^{M_\tau} F_\tau^i$ and $G_\tau = \sum_{i=a_\tau}^{M_\tau} G_\tau^i$ are the cumulative distribution functions (c.d.f.) of the a_τth order statistic from a continuous population with c.d.f. π_τ^{h} and π_τ^{ℓ}, respectively.[26] Let $\beta_\tau = F_\tau/(F_\tau + G_\tau)$ be the probability of being in state h given class τ has accepted the proposal. We denote the corresponding probability density functions of the order statistics by f_τ and g_τ respectively:

$$f_\tau = \frac{\partial F_\tau}{\partial \omega_\tau} \quad g_\tau = \frac{\partial G_\tau}{\partial \omega_\tau}. \tag{A1}$$

LEMMA A1: *For any* $\omega_\tau \in (0, 2)$,

$$\frac{F_\tau^i}{G_\tau^i} < \frac{F_\tau^{i+1}}{G_\tau^{i+1}}.$$

PROOF OF LEMMA A1:

$$\frac{\dfrac{F_\tau^{i+1}}{G_\tau^{i+1}}}{\dfrac{F_\tau^i}{G_\tau^i}} = \frac{\pi_\tau^{\mathrm{h}}(1 - \pi_\tau^{\ell})}{\pi_\tau^{\ell}(1 - \pi_\tau^{\mathrm{h}})} > 1$$

since $\pi_\tau^{\mathrm{h}}/\pi_\tau^{\ell} > 1$ and $(1 - \pi_\tau^{\ell})/(1 - \pi_\tau^{\mathrm{h}}) > 1$ given $\varepsilon < \frac{1}{2}$ from (2).

[26] Note that F_τ is a function of several variables. Therefore, it is more appropriate to write $F_\tau(\omega_\tau, a_\tau)$. For simplicity, we will use F_τ whenever it is not confusing.

LEMMA A2: *For any* a_τ *and* $\omega_\tau \neq 1$,

$$\frac{f_\tau}{g_\tau} \leq \frac{F_\tau^{a_\tau}}{G_\tau^{a_\tau}}.$$

PROOF OF LEMMA A2:

Note that

$$\text{(A2)} \quad \frac{f_\tau}{g_\tau} = \left(\frac{\pi_\tau^{\mathrm{h}}}{\pi_\tau^{\ell}}\right)^{a_\tau - 1} \left(\frac{1-\pi_\tau^{\mathrm{h}}}{1-\pi_\tau^{\ell}}\right)^{M_\tau - a_\tau} \frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau}.$$

For $\omega_\tau < 1$, we have

$$\frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau} = \frac{1-\varepsilon}{\varepsilon} = \frac{\pi_\tau^{\mathrm{h}}}{\pi_\tau^{\ell}}$$

implying

$$\frac{f_\tau}{g_\tau} = \frac{F_\tau^{a_\tau}}{G_\tau^{a_\tau}}.$$

For $\omega_\tau > 1$, we have

$$\frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau} = \frac{\varepsilon}{1-\varepsilon} < 1 < \frac{\pi_\tau^{\mathrm{h}}}{\pi_\tau^{\ell}}$$

implying

$$\frac{f_\tau}{g_\tau} < \frac{F_\tau^{a_\tau}}{G_\tau^{a_\tau}}.$$

LEMMA A3: *For any* a_τ *and* ω_τ, F_τ/G_τ *is strictly decreasing in* ω_τ.

PROOF OF LEMMA A3:

For any $\omega_\tau \neq 1$, the previous two lemmas imply that f_τ/g_τ is strictly less than F_τ/G_τ. Therefore, $f_\tau G_\tau$ is strictly less than $g_\tau F_\tau$. Note that

$$\frac{\partial}{\partial \omega_\tau}\left(\frac{F_\tau}{G_\tau}\right) = \frac{f_\tau G_\tau - g_\tau F_\tau}{(G_\tau)^2}$$

is negative, since $f_\tau G_\tau - g_\tau F_\tau < 0$. For $\omega_\tau = 1$, the fraction F_τ/G_τ is continuous and nondifferentiable.

LEMMA A4: *For any* a_τ *and* $\omega_\tau \neq 1$, *both* F_τ^i/G_τ^i *and* f_τ/g_τ *are strictly decreasing functions of* ω_τ.

The proof of this lemma is omitted as it follows immediately from differentiation of (A2) with respect to ω_τ.

LEMMA A5: *For a given* ω_τ, *increasing the majority requirement from* a_τ *to* a'_τ (*weakly*) *lowers both* F_τ *and* G_τ. *Furthermore,*

$$\frac{F_\tau(\omega_\tau, a_\tau) - F_\tau(\omega_\tau, a'_\tau)}{G_\tau(\omega_\tau, a_\tau) - G_\tau(\omega_\tau, a'_\tau)} < \min\left\{\frac{F_\tau(\omega_\tau, a_\tau)}{G_\tau(\omega_\tau, a_\tau)}, \frac{F_\tau(\omega_\tau, a'_\tau)}{G_\tau(\omega_\tau, a'_\tau)}\right\}.$$

PROOF OF LEMMA A5:

Both F_τ^i and G_τ^i are nonnegative. Thus the first part of the lemma follows. For the second part note that

$$\text{(A3)} \quad \frac{F_\tau(\omega_\tau, a_\tau) - F_\tau(\omega_\tau, a'_\tau)}{G_\tau(\omega_\tau, a_\tau) - G_\tau(\omega_\tau, a'_\tau)} = \frac{\sum_{i=a_\tau}^{a'_\tau - 1} F_\tau^i}{\sum_{i=a_\tau}^{a'_\tau - 1} G_\tau^i}.$$

From Lemma A1, this expression must be strictly less than

$$\min\left\{\frac{F_\tau(\omega_\tau, a_\tau)}{G_\tau(\omega_\tau, a_\tau)}, \frac{F_\tau(\omega_\tau, a'_\tau)}{G_\tau(\omega_\tau, a'_\tau)}\right\}.$$

LEMMA A6:

For a given ω_τ, *increasing the majority requirement from* a_τ *to* a'_τ *increases both* F_τ/G_τ *and* f_τ/g_τ.

PROOF OF LEMMA A6:

The first part follows immediately from Lemma A5. For the second part note that f_τ/g_τ is

$$\text{(A4)} \quad \left(\frac{\pi_\tau^{\mathrm{h}}}{\pi_\tau^{\ell}}\right)^{a_\tau - 1} \left(\frac{1-\pi_\tau^{\mathrm{h}}}{1-\pi_\tau^{\ell}}\right)^{M_\tau - a_\tau} \frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau}.$$

Given that $\pi_\tau^{\mathrm{h}}/\pi_\tau^{\ell} > 1$ and $(1-\pi_\tau^{\mathrm{h}})/(1-\pi_\tau^{\ell}) < 1$, f_τ/g_τ must increase as we increase a_τ.

APPENDIX B: PROOFS

LEMMA B1: *If type* τ *votes responsively, then the best response of type* τ, ω_τ^*, *is unique in a two-class voting game and can be characterized as a solution to the following maximization problem:*

$$\arg\max_{\omega_\tau}[\beta_{\tau'}(1-q_\tau)F_\tau-(1-\beta_{\tau'})G_\tau q_\tau]. \tag{B1}$$

PROOF OF LEMMA B1:

First, note that

$$\left.\frac{f_\tau}{g_\tau}\right|_{\omega_\tau<1}=\left(\frac{\pi_\tau^{h}}{\pi_\tau^{\ell}}\right)^{a_\tau-1}\left(\frac{1-\pi_\tau^{h}}{1-\pi_\tau^{\ell}}\right)^{M_\tau-a_\tau}\frac{1-\varepsilon}{\varepsilon}$$

$$\left.\frac{f_\tau}{g_\tau}\right|_{\omega_\tau>1}=\left(\frac{\pi_\tau^{h}}{\pi_\tau^{\ell}}\right)^{a_\tau-1}\left(\frac{1-\pi_\tau^{h}}{1-\pi_\tau^{\ell}}\right)^{M_\tau-a_\tau}\frac{\varepsilon}{1-\varepsilon}.$$

Therefore,

$$\beta_\tau^{g}=\frac{\beta_{\tau'}}{\beta_{\tau'}+(1-\beta_{\tau'})\left.\dfrac{g_\tau}{f_\tau}\right|_{\omega_\tau<1}} \tag{B2}$$

$$\beta_\tau^{b}=\frac{\beta_{\tau'}}{\beta_{\tau'}+(1-\beta_{\tau'})\left.\dfrac{g_\tau}{f_\tau}\right|_{\omega_\tau>1}}.$$

Then the optimality conditions [stated after (8) above] can be rewritten as

$$\frac{\beta_{\tau'}}{\beta_{\tau'}+(1-\beta_{\tau'})\dfrac{g_\tau}{f_\tau}}=q_\tau$$

in the sense that the left-hand side equals β_τ^{g} when $\omega_\tau<1$ and β_τ^{b} when $\omega_\tau>1$. Simplifying terms, we can rewrite the equilibrium condition as

$$\frac{F_{\tau'}f_\tau}{G_{\tau'}g_\tau}=\frac{q_\tau}{1-q_\tau}. \tag{B3}$$

Note that the first-order condition of the optimization in (B1) is identical to (B3). Finally, for a given $\omega_{\tau'}$, $(G_{\tau'}/F_{\tau'})q_\tau/(1-q_\tau)$ is a constant. Since f_τ/g_τ is decreasing (from Lemma A4) there is a unique solution of (B3).

PROOF OF LEMMA 1:

From Lemma A3 $\beta_{\tau'}$ is decreasing in $\omega_{\tau'}$. Also, β_τ^σ is an increasing function of $\beta_{\tau'}$ [see (B2)]. Thus, β_τ^σ is decreasing in $\omega_{\tau'}$. From Lemma A4, f_τ/g_τ is decreasing in ω_τ; hence, it is immediate from (B2) that so is β_τ^σ.

PROOF OF PROPOSITION 2:

From Lemma A3 $F_{\tau'}/G_{\tau'}$ is decreasing in $\omega_{\tau'}$. Then, to satisfy (B3), f_τ/g_τ must increase. From Lemma A4 this implies that ω_τ^* has to decrease.

LEMMA B2: *Fix any* $\alpha_\tau \in (0, 1)$. *Then, there exist* M_j' *and* M_k' *such that for all* $M_j > M_j'$ *and* $M_k > M_k'$, *there is a unique symmetric responsive equilibrium in which type* τ' *voters always vote for the proposal* (*i.e.,* $\omega_{\tau'} = 2$).

PROOF OF LEMMA B2:

From (B3), a symmetric responsive equilibrium is characterized by the solution of

$$\frac{f_\tau}{g_\tau} = \frac{q_\tau}{1 - q_\tau}$$

given that $F_{\tau'}/G_{\tau'} = 1$. Therefore, there does not exist a responsive equilibrium as long as one of the two inequalities

$$\text{(i)} \quad \frac{f_\tau}{g_\tau} > \frac{q_\tau}{1 - q_\tau}$$

or

$$\text{(ii)} \quad \frac{f_\tau}{g_\tau} < \frac{q_\tau}{1 - q_\tau}$$

holds for all ω_τ. Note that

$$\text{(B4)} \qquad \frac{f_\tau}{g_\tau} = \left(\frac{\pi_\tau^{\mathrm{h}}}{\pi_\tau^{\ell}}\right)^{\alpha_\tau M_\tau - 1} \left(\frac{1 - \pi_\tau^{\mathrm{h}}}{1 - \pi^{\ell}}\right)^{M_\tau(1-\alpha_\tau)} \frac{\partial \pi^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau}$$

and

(B5)

$$\lim_{\omega_\tau \uparrow 2} \left(\frac{1 - \varepsilon + \varepsilon(\omega_\tau - 1)}{\varepsilon + (1 - \varepsilon)(\omega_\tau - 1)}\right)^{\alpha_\tau M_\tau - 1} \left(\frac{\varepsilon \omega_\tau}{(1 - \varepsilon)\omega_\tau}\right)^{M_\tau(1-\alpha_\tau)} \frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau} = \left(\frac{\varepsilon}{1 - \varepsilon}\right)^{M_\tau(1-\alpha)} \frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau}.$$

Given that $(\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau)/(\partial \pi_\tau^{\ell}/\partial \omega_\tau)$ is constant (either $\varepsilon/(1 - \varepsilon)$ or $(1 - \varepsilon)/\varepsilon$) and $\varepsilon/(1 - \varepsilon) < 1$, f_τ/g_τ can get arbitrarily small for large M_τ and large $\omega_\tau(\omega_\tau \uparrow 2)$. Similarly,

$$\text{(B6)} \qquad \lim_{\omega \uparrow 0} \frac{f_\tau}{g_\tau} = \left(\frac{1 - \varepsilon}{\varepsilon}\right)^{\alpha_\tau M_\tau - 1} \frac{\partial \pi_\tau^{\mathrm{h}}/\partial \omega_\tau}{\partial \pi_\tau^{\ell}/\partial \omega_\tau}.$$

Thus, f_τ/g_τ can get arbitrarily large for large M_τ and small $\omega_\tau(\omega_\tau \uparrow 0)$. Therefore, there exists M_τ' large enough that neither

$$\text{(i)} \quad \frac{f_\tau}{g_\tau} > \frac{q_\tau}{1 - q_\tau}$$

nor

$$\text{(ii)} \quad \frac{f_\tau}{g_\tau} < \frac{q_\tau}{1 - q_\tau}$$

holds, implying existence. Uniqueness follow immediately from Lemma A4.

PROOF OF PROPOSITION 1:

We have already established the existence of a certain type of a symmetric responsive equilibrium in two-class voting in Lemma B2.

PROOF OF PROPOSITION 3:

If there is a conflict of interests, then the symmetry condition does not require that both types play the same strategy. Therefore, we have to introduce new notation. Let $\rho_\tau^s = \sum_{i=a-M_{\tau'}-1}^{M_\tau - 1} \rho_\tau^s(i)$ be the probability that a voter of type τ is pivotal in state s where

$$\text{(B7)} \qquad \rho_\tau^s(i) = \binom{M_\tau - 1}{i} [\pi_\tau^s]^i [1 - \pi_\tau^s]^{M_\tau - 1 - i} \binom{M_{\tau'}}{i} [\pi_{\tau'}^s]^{a-i-1} [1 - \pi_{\tau'}^s]^{M_{\tau'} - a + i + 1}.$$

The first term of ρ_τ^s, $\rho_\tau^s(a - M_\tau - 1)$, is the probability of having all type τ' voters and $a - M_\tau - 1$ of type τ voters voting "yes." Then, similar to equation (B3), the first-order condition for a symmetric responsive equilibrium becomes $(\rho_\tau^{\mathrm{h}}/\rho_\tau^{\ell})C_\tau = q_\tau/(1 - q_\tau)$, where C_τ is $\varepsilon/(1 - \varepsilon)$ if $\omega_\tau > 1$ and $(1 - \varepsilon)/\varepsilon$ if $\omega_\tau < 1$.

We proceed in three steps. Step 1 shows that if $\omega_\tau < \omega_{\tau'}$, then

$$\frac{\rho_\tau^{\mathrm{h}}}{\rho_\tau^{\ell}} \leq \frac{\rho_{\tau'}^{\mathrm{h}}}{\rho_{\tau'}^{\ell}} < \left(\frac{1 - \varepsilon}{\varepsilon}\right) \frac{\rho_\tau^{\mathrm{h}}}{\rho_\tau^{\ell}}.$$

In step 2 we show that one type of voters does not vote responsively in equilibrium. Step 3 shows that if $q_\tau > q_{\tau'}$ and $(M_{\tau'} + M_\tau)\alpha > M_{\tau'}$, then type τ' voters always vote for the proposal; otherwise type τ voters always vote against the proposal. Then, uniqueness of equilibrium completes our argument.

Step 1: We start by comparing the ratio of the last terms of ρ_τ^{h} and ρ_τ^{ℓ} with the ratio of the first term of $\rho_{\tau'}^{\mathrm{h}}$ and $\rho_{\tau'}^{\ell}$, and so on; that is, we compare $\rho_\tau^{\mathrm{h}}(M_\tau - m)/\rho_\tau^{\ell}(M_\tau - m)$ with $\rho_{\tau'}^{\mathrm{h}}(a - M_{\tau'} - 2 + m)/\rho_{\tau'}^{\ell}(a - M_{\tau'} - 2 + m)$ for all $m \in \{1, \dots, M_\tau + M_{\tau'} - a + 1\}$:

$$\frac{\rho_\tau^{\mathrm{h}}(M_\tau - m)}{\rho_\tau^{\ell}(M_\tau - m)} = \frac{[\pi_\tau^{\mathrm{h}}]^{M_\tau - m}[1 - \pi_\tau^{\mathrm{h}}]^{m-1}[\pi_{\tau'}^{\mathrm{h}}]^{a - M_{\tau'} + m - 1}[1 - \pi_{\tau'}^{\mathrm{h}}]^{M_\tau + M_{\tau'} - a + 1 - m}}{[\pi_\tau^{\ell}]^{M_\tau - m}[1 - \pi_\tau^{\ell}]^{m-1}[\pi_{\tau'}^{\ell}]^{a - M_{\tau'} + m - 1}[1 - \pi_{\tau'}^{\ell}]^{M_\tau + M_{\tau'} - a + 1 - m}}$$

$$\frac{\rho_{\tau'}^{\mathrm{h}}(a - M_{\tau'} - 2 + m)}{\rho_{\tau'}^{\ell}(a - M_{\tau'} - 2 + m)} = \frac{[\pi_\tau^{\mathrm{h}}]^{M_\tau - m + 1}[1 - \pi_\tau^{\mathrm{h}}]^{m-1}[\pi_{\tau'}^{\mathrm{h}}]^{a - M_{\tau'} + m - 2}[1 - \pi_{\tau'}^{\mathrm{h}}]^{M_\tau + M_{\tau'} - a + 1 - m}}{[\pi_\tau^{\ell}]^{M_\tau - m + 1}[1 - \tau_\tau^{\ell}]^{m-1}[\pi_{\tau'}^{\ell}]^{a - M_{\tau'} + m - 2}[1 - \pi_{\tau'}^{\ell}]^{M_\tau + M_{\tau'} - a + 1 - m}}.$$

Therefore,

$$\text{(B8)} \qquad \frac{\rho_\tau^{\mathrm{h}}(M_\tau - m)/\rho_\tau^{\ell}(M_\tau - m)}{\rho_{\tau'}^{\mathrm{h}}(a - M_{\tau'} - 2 + m)/\rho_{\tau'}^{\ell}(a - M_{\tau'} - 2 + m)} = \frac{\pi_\tau^{\ell}\pi_{\tau'}^{\mathrm{h}}}{\pi_\tau^{\mathrm{h}}\pi_{\tau'}^{\ell}}.$$

If $\omega_\tau, \omega_{\tau'} \leq 1$, then $\pi_\tau^h/\pi_\tau^\ell = \pi_{\tau'}^h/\pi_{\tau'}^\ell$, so that

$$\text{(B9)} \qquad \frac{\rho_\tau^h(M_\tau - m)/\rho_\tau^\ell(M_\tau - m)}{\rho_{\tau'}^h(a - M_{\tau'} - 2 + m)/\rho_{\tau'}^\ell(a - M_{\tau'} - 2 + m)} = 1.$$

If $\omega_{\tau'} > \max\{1, \omega_\tau\}$, then

$$\frac{\pi_\tau^h}{\pi_\tau^\ell} \in \left(\frac{\pi_{\tau'}^h}{\pi_{\tau'}^\ell}, \left(\frac{1-\varepsilon}{\varepsilon}\right) \frac{\pi_{\tau'}^h}{\pi_{\tau'}^\ell}\right)$$

so that

$$\text{(B10)} \qquad \frac{\rho_{\tau'}^h(a - M_{\tau'} - 2 + m)/\rho_{\tau'}^\ell(a - M_{\tau'} - 2 + m)}{\rho_\tau^h(M_\tau - m)/\rho_\tau^\ell(M_\tau - m)} \in \left(1, \frac{1-\varepsilon}{\varepsilon}\right).$$

Thus,

$$\frac{\rho_{\tau'}^h}{\rho_{\tau'}^\ell} \in \left[\frac{\rho_\tau^h}{\rho_\tau^\ell}, \left(\frac{1-\varepsilon}{\varepsilon}\right) \frac{\rho_\tau^h}{\rho_\tau^\ell}\right)$$

if $\omega_\tau < \omega_{\tau'}$.

Step 2: Let $q_\tau > q_{\tau'}$. Then, we argue that we cannot have $\omega_{\tau'} < \omega_\tau$ in equilibrium. If $\omega_{\tau'} < \omega_\tau \leq 1$ or $1 \leq \omega_{\tau'} < \omega_\tau$, then

$$\frac{\rho_\tau^h}{\rho_\tau^\ell} C_\tau \leq \frac{\rho_{\tau'}^h}{\rho_{\tau'}^\ell} C_{\tau'}.$$

Similarly, if $\omega_{\tau'} < 1 < \omega_\tau$, then

$$\frac{\rho_\tau^h}{\rho_\tau^\ell} C_\tau < \frac{\rho_{\tau'}^h}{\rho_{\tau'}^\ell} C_{\tau'}.$$

However, neither can hold in equilibrium since $q_k > q_j$. Therefore, we must have $\omega_{\tau'} \geq \omega_\tau$ in equilibrium. Hence, we have $\rho_\tau^h/\rho_\tau^\ell \leq \rho_{\tau'}^h/\rho_{\tau'}^\ell$ in equilibrium. The ratio between the left-hand side of the two first-order conditions (one for each type) is bounded above:

$$\text{(B11)} \qquad \frac{\dfrac{\rho_\tau^h}{\rho_\tau^\ell} C_\tau}{\dfrac{\rho_{\tau'}^h}{\rho_{\tau'}^\ell} C_{\tau'}} \leq \left(\frac{1-\varepsilon}{\varepsilon}\right)^2.$$

Fix $q_\tau - q_{\tau'} = 1 - 2\varepsilon$. Note that the ratio between the left-hand side of the two first-order conditions, $[q_\tau/(1 - q_\tau)]/[q_{\tau'}/(1 - q_{\tau'})]$, has a minimum of $[(1 - \varepsilon)/\varepsilon]^2$ at $q_\tau = 1 - \varepsilon$. Therefore, for all $q_\tau - q_{\tau'} > 1 - 2\varepsilon$,

$$\frac{q_\tau}{1-q_\tau}\bigg/\frac{q_{\tau'}}{1-q_{\tau'}} > \left(\frac{1-\varepsilon}{\varepsilon}\right)^2.$$

Thus, at most one of the first-order conditions can hold. This implies that voters of one type do not vote responsively.

Step 3: We will start by showing that $\rho_\tau^{\mathrm{h}}/\rho_\tau^{\ell}$ is a decreasing function of ω_τ. Note that $\rho_\tau^{\mathrm{h}}(i)/\rho_\tau^{\ell}(i)$ can be simplified as

$$\frac{[\pi_\tau^{\mathrm{h}}]^i[1-\pi_\tau^{\mathrm{h}}]^{M_\tau-1-i}}{[\pi_\tau^{\ell}]^i[1-\pi_\tau^{\ell}]^{M_\tau-1-i}}\,\Phi(\pi_{\tau'}^{\mathrm{h}}, \pi_{\tau'}^{\ell}). \tag{B12}$$

Recall that

$$\frac{[\pi_\tau^{\mathrm{h}}]^i[1-\pi_\tau^{\mathrm{h}}]^{M_\tau-1-i}}{[\pi_\tau^{\ell}]^i[1-\pi_\tau^{\ell}]^{M_\tau-1-i}} = \frac{f_\tau(a_\tau = i+1)}{g_\tau(a_\tau = i+1)}\,\frac{\partial\pi_\tau^{\ell}/\partial\omega_\tau}{\partial\pi_\tau^{\mathrm{h}}/\partial\omega_\tau}.$$

From Lemma A4, this term is a decreasing function of ω_τ. Therefore, $\rho_\tau^{\mathrm{h}}(i)/\rho_\tau^{\ell}(i)$ must be a decreasing function of ω_τ. Denote by $\eta_\tau^{s}(i)$ the partial derivative of $\rho_\tau^{s}(i)$ with respect to ω_τ. Then we have $\eta_\tau^{\mathrm{h}}(i)\rho_\tau^{\ell}(i) - \eta_\tau^{\ell}(i)\rho_\tau^{\mathrm{h}}(i) < 0$, or, equivalently,

$$\frac{\eta_\tau^{\mathrm{h}}(i)}{\eta_\tau^{\ell}(i)} < \frac{\rho_\tau^{\mathrm{h}}(i)}{\rho_\tau^{\ell}(i)}.$$

Since this holds for all i then we have

$$\frac{\sum_i \eta_\tau^{\mathrm{h}}(i)}{\sum_i \eta_\tau^{\ell}(i)} < \frac{\sum_i \rho_\tau^{\mathrm{h}}(i)}{\sum_i \rho_\tau^{\ell}(i)}.$$

Therefore, $\rho_\tau^{\mathrm{h}}/\rho_\tau^{\ell}$ is a decreasing function of ω_τ.[27] From Step 2, at least one type of voters votes nonresponsively whenever the conflict of interests is large enough. Without loss of generality let $q_k > q_j$. Then, for any $a = (M_j + M_k)\alpha$, either (i) $a > M_j$ or (ii) $a \le M_j$ holds. We will show that in the first case type j, and in the second case type k voters will not vote responsively. We start with case (i) and construct the equilibrium.[28] The first-order condition never holds for type j voters and thus their best response will be $\omega_j = 2$. Note that

$$\lim_{\omega_j \uparrow 2} \frac{\rho_k^{\mathrm{h}}}{\rho_k^{\ell}}\,C_k = \frac{f_k(a_k = a - M_j)}{g_k(a_k = a - M_j)}.$$

Recall that

$$\frac{f_k(a_k = a - M_j)}{g_k(a_k = a - M_j)} = \frac{q_k}{1-q_k}$$

[27] Similarly, $\rho_\tau^{\mathrm{h}}/\rho_\tau^{\ell}$ is a decreasing function of $\omega_{\tau'}$.

[28] The argument for case (ii) is analogous.

is the equilibrium condition for one-class voting with identical preferences when only type k voters vote with majority requirement $a_k = a - M_j$. Therefore, type k voters' best response to $\omega_j = 2$ is $\hat{\omega}_k$, the equilibrium strategy in a one-class voting game with type k voters only and $a_k = a - M_j$. For uniqueness it suffices to use the fact $\rho_j^{\mathrm{h}}/\rho_j^{\ell}$ is a decreasing function of ω_j and to check that we cannot have an equilibrium where type k voters vote nonresponsively ($\omega_k = 0$) and where type j voters vote responsively.

PROOF OF PROPOSITION 5:

The probability of rejecting the proposal in state h is $1 - F_jF_k$, and the probability of accepting the proposal in state ℓ is G_jG_k. Therefore, it suffices to show that both F_j and F_k converge to 1 and either G_j or G_k converges to 0. If only one type votes responsively (say j), then $F_k = G_k = 1$. Furthermore, from Feddersen and Pesendorfer (1998), F_j converges to 1 and G_j converges to 0. Thus, the only case left is both types voting responsively. First, note that both f_τ and g_τ are single-peaked functions and their maximum values are reached at $\pi_\tau^{\mathrm{h}} = \alpha_\tau$ and $\pi_\tau^{\ell} = \alpha_\tau$ for arbitrarily large number of voters. Let $\underline{\omega}_\tau$ denote the solution of $\pi_\tau^{\mathrm{h}} = \alpha_\tau$. Similarly, $\bar{\omega}_\tau$ is the solution of $\pi_\tau^{\ell} = \alpha_\tau$. Thus, we have $\underline{\omega}_\tau < \bar{\omega}_\tau$. From Lemma A4 $f_\tau/g\tau$ is a decreasing function of ω_τ and thus for any $\omega_\tau \leq \underline{\omega}_\tau \lim_{M_\tau \uparrow \infty}(f_\tau/g_\tau) \uparrow \infty$. Since $F_{\tau'}/G_{\tau'} \geq 1$, the solution to (B3) $\omega_\tau^* > \underline{\omega}_\tau$. Thus, $\lim_{M_\tau \uparrow \infty} F_\tau \uparrow 1$. Similarly, $\lim_{M_{\tau'} \uparrow \infty} F_{\tau'} \uparrow 1$. Note that any $\omega_\tau \geq \bar{\omega}_\tau \lim_{M_\tau \uparrow \infty} g_\tau/f_\tau \uparrow \infty$ since g_τ/f_τ is an increasing function. Therefore, in order to satisfy (B3), $\lim_{M_{\tau'} \uparrow \infty} F_{\tau'}/G_{\tau'} \uparrow \infty$. Thus, $\lim_{M_{\tau'} \uparrow \infty} G_{\tau'} \uparrow 0$, completing the argument.

PROOF OF THEOREM 1:

In the symmetric case

$$\frac{q_k}{1 - q_k} \Big/ \frac{q_j}{1 - q_j} > \left(\frac{1 - \varepsilon}{\varepsilon}\right)^2$$

simplifies to $q_k > 1 - \varepsilon$. Therefore, if $q_k > 1 - \varepsilon$, then one type of voters votes nonresponsively from the argument in Proposition 3. Without loss of generality let us assume that type j voters vote nonresponsively in a one-class voting game (i.e., type j voters vote for the proposal independently of their signal). Consequently, using one-class voting with a as the majority requirement is equivalent to having only type k voters vote on the proposal with majority requirement $a_k = a - M$. Note that from Lemma B1 the equilibrium strategy for type k voters $\hat{\omega}_k$ can be characterized as

$$\hat{\omega}_k = \arg\max \tfrac{1}{2}[F_k(1 - q_k) - (1 - G_k)q_k] \tag{B13}$$

so that the first-order condition for (B13) becomes $f_k/g_k = q_k/(1 - q_k)$. Therefore, the expected social loss (per capita) in a one-class voting game is equal to $\frac{1}{4}[(1 - F_k) + G_k]$, where F_k and G_k are calculated for $a_k = a - M$ and $\omega_k = \hat{\omega}_k$. Next, we will show that the same expected social loss can be achieved by letting only type j voters vote with majority rule $a_j = M - a_k + 1$. Using $q_j = 1 - q_k$, the equilibrium strategy of type j voters $\hat{\omega}_j$ is

$$\hat{\omega}_j = \arg\max \tfrac{1}{2}[F_jq_k - (1 - G_j)(1 - q_k)]. \tag{B14}$$

Using $a_j = M - a_k + 1$, G_j becomes

$$\sum_{i = M - a_k + 1}^{M} \binom{M}{i} [\pi_j^{\ell}]^i [1 - \pi_j^{\ell}]^{M - i}.$$

Note that

$$\sum_{i=1}^{M} \binom{M}{i} [\pi_j^{\ell}]^i [1 - \pi_j^{\ell}]^{M-i} = 1.$$

Therefore, $1 - G_j$ is equal to

$$\sum_{i=1}^{M-a_k} \binom{M}{i} [\pi_j^{\ell}]^i [1 - \pi_j^{\ell}]^{M-i}.$$

Next we would like to note that π_j^{ℓ} is equivalent to $1 - \pi_k^{\mathrm{h}}$ if we set $\omega_j = 2 - \omega_k$; that is, $\pi_j^{\ell}(\omega_j = 2 - \omega_k) = 1 - \pi_k^{\mathrm{h}}(\omega_k)$. Therefore, $1 - G_j(\omega_j = 2 - \omega_k, a_j = M_j - a_k + 1) = F_k(\omega_k, a_k)$. Similarly, we have $F_j(\omega_j = 2 - \omega_k, a_j = M_j - a_k + 1) = 1 - G_k(\omega_k, a_k)$. Hence $\omega_j = 2 - \hat{\omega}_k$ must be the solution to equation (B14) since $\hat{\omega}_k$ is the solution to equation (B13). Furthermore, the expected social loss from letting only type j voters vote with majority requirement $a_j = M - a_k + 1$,

$$\text{(B15)} \qquad \tfrac{1}{4}[(1 - F_j(\hat{\omega}_j, a_j = M - a_k + 1)) + G_j(\hat{\omega}_j, a_j = M - a_k + 1)]$$

is equal to that of letting only type k voters vote,

$$\text{(B16)} \qquad \tfrac{1}{4}[(1 - F_k(\hat{\omega}_k, a_k)) + G_k(\hat{\omega}_k, a_k)].$$

Next, as a benchmark, we would like to calculate the expected social loss for two-class voting with $(a_k, a_j = M - a_k + 1)$ being the majority requirements when each class votes as if their class is the only class that votes. Let Λ denote this benchmark loss:

$$\text{(B17)} \qquad \Lambda \equiv \tfrac{1}{4}\{[1 - F_k(\hat{\omega}_k, a_k)F_j(\hat{\omega}_j, a_j = M - a_k + 1)]$$
$$+ G_k(\hat{\omega}_k, a_k)G_j(\hat{\omega}_j, a_j = M - a_k + 1)\}$$
$$= \tfrac{1}{4}\{[1 - F_k(\hat{\omega}_k, a_k)] + G_k(\hat{\omega}_k, a_k)\}.$$

Recall that this expression is equal to (B16), the expected social loss (per capita) in the equilibrium of the one-class voting game with $a = M + a_k$ with type j voters voting for the proposal independently of their signals. Therefore, all that remains to be shown is that there exists a pair of majority requirements (a_k^*, a_j^*) such that two-class voting results in an expected social loss strictly less than Λ.

Note that the first-order condition for type j voters in a two-class voting game is

$$\text{(B18)} \qquad \frac{F_k f_j}{G_k g_j} = \frac{q_j}{1 - q_j}$$

as opposed to $f_j/g_j = q_j/(1 - q_j)$, the first-order condition for one-class voting. Given that f_j/g_j is a decreasing function of ω_j and $F_k/G_k > 1$, the solution to (B18), ω_j^*, must be larger than $\hat{\omega}_j$

if we use the same majority rule $a_j = M - a_k + 1$. From Lemma A6 and Proposition 2, the majority requirement, a_j^*, that will induce $\omega_j^* \leq \hat{\omega}_j$ must be less than $M - a_k + 1$. With identical arguments to those in Lemma B2, there exists an M large enough so that there exists an α_j^* and therefore $a_j^* < a_j$ such that $\omega_j^* \leq \hat{\omega}_j$ satisfies the first-order conditions. Therefore, we have

(B19) $$\frac{G_j(\omega_j^*, a_j^*) - G_j(\hat{\omega}_j, a_j)}{F_j(\omega_j^*, a_j^*) - F_j(\hat{\omega}_j, a_j)} \equiv \frac{\Delta G_j}{\Delta F_j} = \frac{\sum_{i=a_j^*}^{a_j-1} G_j^i(\hat{\omega}_j)}{\sum_{i=a_j^*}^{a_j-1} F_j^i(\hat{\omega}_j)} \equiv \frac{\Delta G_j(\hat{\omega}_j)}{\Delta F_j(\hat{\omega}_j)}.$$

Given that $1 - G_j(\hat{\omega}_j, a_j) = F_k(\hat{\omega}_k, a_k)$ and $F_j(\hat{\omega}_j, a_j) = 1 - G_k(\hat{\omega}_k, a_k)$, we have

(B20) $$\frac{\Delta G_j(\hat{\omega}_j)}{\Delta F_j(\hat{\omega}_j)} = \frac{\sum_{i=a_k+1}^{M-a_j^*+1} F_k^i(\hat{\omega}_k)}{\sum_{i=a_k+1}^{M-a_j^*+1} G_k^i(\hat{\omega}_k)} \equiv \frac{\Delta F_k(\hat{\omega}_k)}{\Delta G_k(\hat{\omega}_k)}.$$

From Lemma A1,

$$\frac{\Delta F_k(\hat{\omega}_k)}{\Delta G_k(\hat{\omega}_k)} < \frac{F_k(\hat{\omega}_k)}{G_k(\hat{\omega}_k)}.$$

Therefore, we have $\Delta G_j/\Delta F_j < F_k(\hat{\omega}_k)/G_k(\hat{\omega}_k)$. Arranging terms yields

(B21) $$G_k(\hat{\omega}_k)\Delta G_j < F_k(\hat{\omega}_k)\Delta F_j.$$

On the other hand, the expected loss for type k voters in a two-class voting game with $(a_k^* = a_k, a_j^*)$ is

(B22) $$\beta_j(\omega_j^*)[1 - F_k(\omega_k^*)]q_j + [1 - \beta_j(\omega_j^*)]G_k(\omega_k^*)q_k.$$

This expression must be less than or equal to

(B23) $$\beta_j(\omega_j^*)[1 - F_k(\hat{\omega}_k)]q_j + [1 - \beta_j(\omega_j^*)]G_k(\hat{\omega}_k)q_k$$

since otherwise each type k voter will deviate to $\hat{\omega}_k$. Therefore, we have

(B24) $$[1 - \beta_j(\omega_j^*)]\Delta G_k q_k \leq \beta_j(\omega_j^*)\Delta F_k q_j$$

where $\Delta F_k = F_k(\omega_k^*) - F_k(\hat{\omega}_k) > 0$ and $\Delta G_k = G_k(\omega_k^*) - G_k(\hat{\omega}_k) > 0$. Arranging and simplifying results in

(B25) $$\Delta G_k \leq \frac{F_j(\omega_j^*)}{G_j(\omega_j^*)} \Delta F_k \frac{q_j}{q_k}.$$

Substituting $F_j(\omega_j^*, a_j^*) = F_j(\hat{\omega}_j, a_j) + \Delta F_j$ and $G_j(\omega_j^*, a_j^*) = G_j(\hat{\omega}_j, a_j) + \Delta G_j$ we have

$$\text{(B26)} \qquad \Delta G_k \leq \frac{F_j(\hat{\omega}_j) + \Delta F_j}{G_j(\hat{\omega}_j) + \Delta G_j} \Delta F_k \frac{q_j}{q_k}.$$

Given that $q_j/q_k < 1$, we have

$$\Delta G_k < \frac{F_j(\hat{\omega}_j) + \Delta F_j}{G_j(\hat{\omega}_j) + \Delta G_j} \Delta F_k.$$

Thus, we obtain

$$\text{(B27)} \qquad \Delta G_k G_j(\hat{\omega}_j) + \Delta G_j \Delta G_k - [F_j(\hat{\omega}_j)\Delta F_k + \Delta F_j \Delta F_k] < 0.$$

Adding this to equation (B21), and adding $1 - F_k(\hat{\omega}_k)F_j(\hat{\omega}_j) + G_k(\hat{\omega}_k)G_j(\hat{\omega}_j)$ to both sides yields

$$\text{(B28)} \quad [1 - F_k(\omega_k^*)F_j(\omega_j^*)] + G_k(\omega_k^*)G_j(\omega_j^*) < [1 - F_k(\hat{\omega}_k)F_j(\hat{\omega}_j)] + G_k(\hat{\omega}_k)G_j(\hat{\omega}_j).$$

Multiplying both sides by $\frac{1}{4}$ concludes the proof, since the right-hand side is equal to Λ and the left-hand side is the expected social loss in two-class voting.

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