Research articles

Measurement distortion and missing contingencies in optimal contracts*

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Summary. Theory suggests that optimal contracts should include many contingencies to achieve optimal risk sharing. However, in practice, few contracts are as complex as theory suggests. This paper develops a model which is consistent with this observation. The lack of risk sharing results from the interplay of two factors. First, contingencies must be based on information produced by measurement systems, which may be manipulable. Second, when two parties to a contract meet, they often have incomplete information. The type of contract offered may reveal information about the party who proposes it. Different types of agents have different preferences over contingent contracts, because they have different abilities to manipulate the measurement system. These differences in preferences allow the parties to signal their types through the contracts they offer. Non-contingent contracts may be chosen in equilibrium because they are the only contracts which do not give any type an incentive to distort the measurement system and, hence, do not reveal information about the party proposing the contract.

I Introduction

Contract theory suggests that optimal contracts are often extremely complicated (Hart and Holmstrom 1987). In particular, Holmstrom (1979) and Shavell (1979) have shown that, in the presence of moral hazard, optimal contracts will be contingent on all relevant information. Even if we ignore agency problems, simple risk-sharing arguments suggest that optimal contracts should include many contingencies. But whatever the implications of contract theory, it is a commonplace that in practice contracts are much simpler. In particular, they leave out

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contingencies that have the potential to improve risk-sharing. Examples include the use of firm fixed price contracts in defense procurement and the use of the standard debt contract in corporate finance.  

In this paper, we develop a model in which the absence of contingencies that would allow better risk-sharing is an equilibrium phenomenon. A central role in the theory is played by measurement systems. We start with the idea that it is impossible to write contracts that are contingent on “states of nature” since states of nature are not verifiable. The contingencies incorporated in actual contracts depend on signals produced by measurement systems, which reveal the state of nature only indirectly, if at all. For instance, the cost reimbursement contracts used in the procurement process require accounting systems to measure costs. Similarly, if bond repayments are contingent on accounting profits, there has to be an accounting system to produce the profit figures.

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1 Risk-sharing is a major concern of contractors developing and producing weapons for the U.S. Defence Department. In his classic study of the weapons acquisition process, Scherer (1964) describes the very limited number of types of contract used by the Defence Department. Most are variants on the polar extremes of the Cost Plus Fixed Fee (CPFF) contract and the Firm Fixed Price (FFP) contract. The contractors' desire to avoid the risk of cost overruns explains the use of the CPFF contract, despite the fact that the CPFF appears to offer no incentive to the contractor to minimize costs. The FFP contract, on the other hand, offers strong incentives but no risk-sharing. Scherer's study describes a number of other types of contracts that combine the features of these two polar cases; but the interesting fact is that the CPFF and FFP contracts are used at all. It might be thought that the FFP, which admits no contingencies, will be used only when the uncertainty about cost is minimal. But the fact that realized profit rates are substantially higher for these contracts suggests that, on the contrary, there is a significant risk premium built in, presumably because the contractor is bearing substantial risk. In any case, the contracts are simpler than one would expect on the basis of optimal contract theory.

2 Under a standard debt contract, the borrower is committed to a fixed schedule of repayments of principal and interest, independently of the state of nature. Optimal risk-sharing requires that borrower and lender have equal marginal rates of substitution between states. Under a debt contract, this condition will clearly be violated, except in very special cases. In addition, the borrower may declare bankruptcy in some states because he cannot meet the non-contingent payment. In that case, both parties may incur substantial deadweight costs.

Some writers have argued that the standard debt contract is an optimal incentive-compatible contract when there is incomplete information (see Townsend 1979; Diamond 1984; Gale–Hellwig 1985). But this argument does not explain why the debt contract does not make repayments contingent on readily available information, such as company profits. It is the absence of this sort of contingency that requires explanation.

3 A practical example of how such contingencies might be used to improve the risk-sharing is provided by the income bond (Dewing 1911; Dewing 1955; McConnel and Schlarbaum 1987; De and Kale 1990). The income bond combines the advantages of debt and preferred stock. Income bonds are a contractual obligation of the issuer, so the interest payments are tax deductible. On the other hand, interest is paid only if earned, which makes the borrower's repayment contingent on accounting profits. Income bonds have been issued in certain periods, but their use has been extremely limited. Before the First World War, income bonds were used with some success by railway companies during reorganizations (McConnell and Schlarbaum 1987). Income bonds were used in the nineteen-thirties, by companies restructuring their debt, and in the nineteen-forties, mainly by railways undergoing reorganization. A few strong companies have issued income bonds in the ensuing decades, but despite the availability of the income bond “technology” and the theoretical advantages it offers, it has fallen into disuse.
A measurement system does not necessarily reveal the true state of nature. In the first place, the system may be "noisy", so that it produces at best imperfect information about the variables that really matter to the contracting parties. Second, and more importantly, the system may be manipulable, that is, it can be distorted by one of the parties to the contract.

The fact that measurement systems are manipulable is an important part of our explanation of why some contingencies are not more widely used. But it is not the whole story. Distorting the information produced by the measurement system is not without cost and these costs will limit the amount of distortion. As long as there is some information in the distorted signal, it should be used in the contract, according to the Holmstrom–Shavell theorem. Thus, in order to explain the complete absence of a contingency based on manipulable data, something more is needed. In this paper, we appeal to adverse selection.

When two parties to a contract first meet, they have incomplete information. The proposal to include a certain kind of contingency may be interpreted by one party as a bad signal about the other. Rather than send a bad signal, which could lead to the rejection of the entire contract, the party proposing the contract may choose to exclude the contingency. The interplay between measurement distortion and incomplete information is crucial in obtaining these results. As we pointed out above, the mere possibility of distorting the signal on which a contingency is based will not eliminate that contingency from the equilibrium contract. Similarly, it is well known that adverse selection may reduce the amount of insurance contained in equilibrium contracts, but it will not lead to missing contingencies in our sense. What we show is that different types of agents pool in equilibrium and that they pool at the non-contingent contract, i.e., they all choose a contract offering no insurance. At a contingent contract, different types of agents have positive incentives to distort and furthermore, their marginal incentives are different. At the margin, "bad" types have greater incentives to increase the contingency of the contract than "good" types. This allows "good" types to separate themselves by offering less contingent contracts and so prevents pooling at contingent contracts. What makes the non-contingent contract special in our theory is that it is the only contract at which no type has an incentive to distort. So it is the interaction of adverse selection with measurement distortion that produces a novel result.

Note that this result can obtain even if the cost of distortion is very high. In fact, one can make the costs of distortion in the model arbitrarily large. The amount of distortion that will occur in equilibrium will then be very small, but

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4 This point is discussed more fully in Sect. II.
5 The clearest example of this kind of signaling effect seems to arise in the case of income bonds. The evident resistance of the financial community to the issue of income bonds is conventionally attributed to their association with the reorganization of bankrupt or struggling companies. They are said to have "the smell of death about them". (See McConnell and Schularbaum 1987 and the quotations therein). In other words, the attempt to issue these securities is interpreted as a bad signal about the issuer.
6 See Sect. IV for a fuller discussion.
the results described above will still hold. What matters is the difference in distortion costs between types.

There are undoubtedly many reasons why contracts have missing contingencies. In this paper we offer one kind of explanation. A related approach is found in Holmstrom and Milgrom (1990), in the context of principal-agent problems. They argue that if the agent engages in several tasks, some of which can be monitored more accurately than others, the use of high-powered incentives may distract the agent from productive, poorly monitored tasks to spend more time on less productive, better monitored tasks. Dewatripont and Maskin (1989) consider the case where the parties to a contract can decide ex ante which variables will be observed. They argue that it may be optimal not to observe certain variables, because this information gives rise to opportunities for renegotiation. There is also a number of papers in which pure adverse selection arguments are used to show why contracts have missing contingencies. These theories are discussed in greater detail in Sect. IV.

In this paper, we describe a simple contracting problem that is intended to serve as a paradigm of the general phenomenon. The model is introduced in Sect. II. It describes the choice of a contract for the delivery of a good. The supplier has private information about the quality of his good and his ability to distort the measurement system. The contract specifies the delivery price of the good, contingent on data about the supplier's costs produced by a (manipulable) measurement system. We define a non-contingent contract to be one in which the delivery price is independent of information that is available to everyone (i.e., the data produced by the measurement system).

Analytical results are presented in Sect. III. We show that there exists a pooling equilibrium in which a non-contingent contract is chosen by all supplier types. This equilibrium satisfies the (quite strong) Universal Divinity criterion of Banks–Sobel (1988). Furthermore, in any equilibrium satisfying the (relatively weak) Intuitive Criterion of Cho–Kreps (1987), the best types must always choose a non-contingent contract. Under certain conditions, equilibria satisfying the Intuitive Criterion can be ranked according to the payoffs of the different supplier types. A pooling equilibrium, in which all types choose the same, non-contingent contract, yields higher payoffs to each type of supplier. It could be argued, for this reason, the pooling equilibrium is more likely than the (partially) separating equilibria.

Section IV concludes the paper with a discussion of related work on adverse selection. Proofs are gathered in the Appendix.

II A contracting problem with measurement distortion

The contracting framework

Two agents want to write a contract for the supply of an indivisible good. The agents are referred to in the sequel as the purchaser and the supplier, respectively. The purchaser wants to obtain one unit of the good from the supplier. The supplier
can produce one unit of the good at a random cost, measured in terms of "money". If the good is not produced, there is no cost.

The supplier is risk averse and the purchaser is risk neutral, so there is an opportunity for mutually beneficial risk sharing. However, although the supplier can observe the cost shock directly, the purchaser cannot. As a result, the contract cannot be made directly contingent on the cost of production. In order to incorporate risk sharing into the contract, it is necessary to make use of a measurement system. A crucial element in the story is the assumption that the supplier can distort the output of the measurement system. If the supplier could distort the measurement system as much as he liked, without cost, then there would be no possibility of writing a useful contingent contract. But distortion is assumed to take effort and effort is costly. Thus, the informational content of the system may be reduced but it will not be eliminated.

The sort of measurement systems we have in mind are exemplified by accounting systems. Various devices have been created to maintain the integrity of accounting systems. Companies are required to have their books audited by qualified accountants and the legal system imposes punishments on executives who falsify the books. So distorting accounting data is not simply a matter of changing some numbers. To manipulate accounting data without getting caught, managers have to operate the firm in a suboptimal manner. There are two kinds of costs involved. First, an accounting system is both a record of the firm's activity for the benefit of claimants and a tool for management. Degrading the accounting system by keeping poor records may result in less effective management. Second, in order to manipulate cost or profit data, the firm may have to make a decision that is economically inefficient. For example, firms may effectively cross-subsidize projects when there are joint costs to be allocated. But in order to have joint costs, it will be necessary to use some joint inputs for both projects, even if this is not productively efficient. For both reasons, there are real costs of manipulating the accounting data.

The second crucial element of the story is that the ability to distort the data produced by the measurement system is correlated with the supplier's type. More precisely, different types of supplier produce different qualities of goods and the quality of the good is negatively correlated with the ability to distort. The sort of situation we have in mind is the following. Company A is a high quality company that has invested in information systems and a highly professional management over a number of years. As a result, it has good cost and quality control, which allow it to develop a high quality product. At the same time, the institutional structure of the firm (the existence of high quality data and the professionalism of the staff) makes it costly to distort the true costs of production. Company B, on the other hand, is badly managed. It has poor accounting and information systems and management is not very competent either. These characteristics mean that their product is likely to be inferior and, for the same reason, it is relatively easy for them to manipulate accounting data.

Indirect evidence that supports our view comes from standard accounting practice. In conducting an audit for financial statements, external auditors focus on the effectiveness of the internal control provided by the firm's accounting
systems. If these are adequate, the auditors do not waste much time doing many other checks, since the likelihood is that the financial statements will be accurate. The following is a representative statement:

One of the most widely accepted concepts in the theory and practice of auditing is the importance of the client's accounting system and related internal controls to generate reliable financial information. If the auditor is convinced the client has an excellent system, one which includes controls for providing reliable data and for safeguarding assets and records, the amount of evidence to be accumulated can be significantly less than for a system that is not adequate. In some instances the controls may be so inadequate as to preclude conducting an effective audit.


In other words, an accounting system that provides effective internal control is harder to manipulate. Since internal cost and quality control lead to better quality products, broadly defined, this suggests that measurement distortion and product quality will be negatively correlated.

Of course, one could easily tell a story where measurement distortion and quality were positively correlated. For example, a skilled staff might be better at fooling the market with accounting tricks. The results then would be quite different. We do not want to claim that negative correlation is the only case worth considering, only that it is not implausible and produces interesting results.

The formal model

There is a finite number of different types of the supplier indexed by \( k = 1, \ldots, \ell \). We let \( K \) stand for the set of types. For any finite set \( S \), \( \mathcal{A}(S) \) denotes the set of probability distributions on \( S \). The prior probability distribution of the supplier's type is denoted by \( \nu \in \mathcal{A}(K) \).

The cost function is denoted by \( C(\theta) \), where \( \theta \) is a real random variable, continuously distributed with support \([\underline{\theta}, \bar{\theta}]\). For simplicity, we assume that \( C(\cdot) \) takes on a finite number of values indexed by \( c_1, \ldots, c_n, \ldots, c_{\ell} \). Let \( E_i = C^{-1}(c_i) = \{ \theta | C(\theta) = c_i \} \) denote the set of states \( \theta \) in which the cost is equal to \( c_i \). Again for simplicity, we assume that \( E_i \) is an interval for each \( i = 1, \ldots, n \).

The output of the measurement system is represented by a signal \( S \). The signal is publicly observed and is correlated with the supplier's cost. However, the signal can be distorted by the supplier. If \( \theta \) is the true value of the cost shock and \( d \) is

\[ \text{The crucial assumption is that the signal produced by the measurement system takes on a finite number of values. From this it follows that a contract can be described by a finite number of parameters so the supplier's strategy set and the domain of the purchaser's strategies are both contained in a finite dimensional space. Obviously, the extension to an infinite dimensional contract space would pose formidable technical difficulties. Even under the assumption of risk neutrality, the analysis of infinite dimensional contracts is not easy. Cf. Nachman-Neue (1990). Finiteness does not seem a restrictive assumption, because a continuum can always be approximated by a sufficiently large but finite set. Also, the measurement systems we are interested in, such as those that produce accounting data, do seem to produce discrete signals. What is possibly restrictive is the assumption that distortion is continuous while signals are not. This allows for the possibility that in some states, very small distortions can lead to large changes in signals. What is needed for our results, however, is the property that some distortion will occur under any contingent contract. For this, discreteness is not necessary. Instead, it is enough to assume that the marginal cost of distortion is zero when } d = 0. \]
the amount of distortion caused by the supplier then the signal produced by the measurement system is a function of the distorted shock \( \theta + d \). In the absence of distortion, the measurement system is perfectly informative, that is, \( S(\theta) = C(\theta) \). We assume that the supplier's effort is proportional to the amount of distortion and that his utility depends on this effort.

The supplier's preferences are described by an additively separable von Neumann–Morgenstern utility function: \( U_k(c,d) = U(c) - T_k(d) \), where \( c \) is the supplier's final wealth. Without loss of generality, we can normalize utility so that \( U(0) = 0 \). Note that preferences over wealth are independent of type.\(^8\)

The purchaser places a value \( v_k \) on the good produced by a supplier of type \( k \). If he purchases the good from supplier \( k \) at a price of \( p \), then his utility is \( v_k - p \). Otherwise his utility is zero.

The following regularity assumptions are maintained throughout:

(A.1) \( v_k < v_{k+1} \) for \( k = 1, \ldots, \ell - 1 \).

(A.2) (i) \( T_k(0) = 0 \) and \( T_k(d) > 0 \) for any \( d \neq 0 \) and \( k = 1, \ldots, \ell \);

(ii) For any \( d, d' \) such that \( |d'| < |d| \),

\[
T_k(d) - T_k(d') < T_{k+1}(d) - T_{k+1}(d').
\]

(A.3) \( U(\cdot) \) and \( T_k(\cdot) \) are \( C^1 \); \( U(\cdot) \) is strictly concave and increasing; \( T_k(\cdot) \) is strictly convex.

The meaning of (A.1) is obvious. (A.2i) says that distortion is costly and, together with (A.3), implies that the costs of distortion are strictly increasing in the absolute value of the distortion. (A.2ii) says, roughly, that the marginal costs of distortion are strictly increasing in type. Higher types have higher marginal costs of distortion. But note that we only compare absolute values. This imposes a kind of symmetry on costs of distortion. Again, (A.3) is self-explanatory. The combination of (A.1) and (A.2ii) implies that product quality and the capacity to distort the measurement system are negatively correlated.

We also assume that mutually advantageous trade is possible between the purchaser and every type of supplier. The most that the purchaser is willing to pay for a unit of the good from a supplier of type \( k \) is \( v_k \). Then trade is always possible if

\[
\mathbb{E}\{U(v_k - C(\theta))\} > 0 \quad \forall k \in K,
\]

where \( \mathbb{E}\{\cdot\} \) denotes the expectation operator with respect to \( \theta \). This assumption implies that every type of supplier will always obtain a positive payoff in equilibrium, since he can always demand a fixed price \( p = v_1 \) and the purchaser will accept.

A contract specifies the payment for the good, contingent on the observed value of the signal, assuming that trade takes place. If the good is not produced, there

\[\text{8}\text{ In an earlier version of this paper (Allen and Gale 1990), we indicated how our results could be extended to deal with type dependent preferences. The analysis is quite complicated and the results much less "clean", so we have not included them in the present version.}\]
is no payment. The contract can be identified with a vector \( P = (P_1, \ldots, P_n) \), where \( P_i \) is the payment required when \( S = c_i \). For some purposes, it is convenient to treat the contract as a function of the distorted shock \( \theta + d \). In that case, we write the payment as \( P(\theta + d) \), where of course \( P(\theta + d) = P_i \) for all \( \theta + d \in E_i \).

The trading process is represented by a simple game. The supplier proposes a contract which the purchaser can accept or reject. If the contract is rejected, the game ends. Otherwise, the good is produced, the supplier observes the cost shock and decides how much effort to exert to distort the signal. Finally, the signal is observed, the good is exchanged and payment is made.

The entire sequence of events in the contracting and production process is summarized by the time-line in Fig. 1.

We analyze this game using the concept of Perfect Bayesian Equilibrium. As applied to the complete extensive form game, Perfect Bayesian Equilibrium requires that four conditions be satisfied. First, given that a contract \( P \) has been accepted and a cost shock \( \theta \) has been observed, the supplier must choose a distortion \( d \) optimally, as a function of \( P \) and \( \theta \). Second, once the supplier has proposed a contract \( P \), the purchaser’s decision to accept or reject the contract must be made optimally, given the purchaser’s beliefs about the supplier’s type and his anticipation of the supplier’s distortion of the signal. Third, the supplier must propose an optimal contract, given the optimal response of the purchaser and the anticipated optimal distortion of the supplier himself. Finally, the purchaser’s beliefs must be consistent with the equilibrium strategies of the supplier.

The reduced form game

The analysis of this game is greatly simplified by the fact that it can be reduced to a game that is formally equivalent to a standard signaling game. Once we note that the supplier’s optimal distortion is uniquely determined as a function of \( \theta \) and \( P \), we can eliminate explicit reference to the measurement distortion decision.

| Supplier | Supplier offers contract to purchaser. | Purchaser accepts or rejects contract. | Supplier learns state \( \theta \) and chooses distortion \( d \). | Good is produced. | Signal \( \theta + d \) is observed. | Payment is made. |

Fig. 1

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9 This is not the only game form that could be used. One could assume the distortion decision is made before the cost shock is known. The results would be much the same, with one qualification which should be noted. Because the distortion decision is made ex post, after the cost shock has been observed, there will always be some values of \( \theta \) for which distortion is optimal, even if the costs of distortion are large. If the distortion decision were made ex ante, before the observation of the cost shock, it might be optimal to choose no distortion because of the costs. To avoid this degenerate case, it would be necessary to assume the marginal cost of distortion is zero when \( d = 0 \). This would ensure that some distortion is always optimal in contingent contracts. In general, information arrives and decisions are made at several points. One would need a much more complex dynamic game to explore this structure.
What remains is the contracting process, which is effectively a signaling game. This allows us to apply familiar techniques that have been developed to test the stability of equilibria in signaling games. This does not mean that measurement distortion is irrelevant to the explanation of missing contingencies. On the contrary, it is crucial to explaining the special structure of the signaling game and hence to the results obtained below.

To see informally why the supplier's optimal distortion is uniquely determined as a function of $\theta$ and $P$, consider the case where the cost of production is either high or low:

$$C(\theta) = \begin{cases} C_H & \text{if } \theta \geq 0; \\ C_L & \text{if } \theta < 0, \end{cases}$$

where $0 < C_L < C_H$ and the signal $S = C$ as usual.

In this case, a contract can be represented by an ordered pair $P = (P_H, P_L)$, where $P_i$ is the payment when the signal takes the value $c_i$. Suppose that a contract $P = (P_H, P_L)$ with $P_H > P_L$ has been accepted, the good has been produced and the supplier observes that the cost shock is $\theta$. He has to decide whether and by how much to distort the signal. Obviously, if the supplier chooses to distort the signal, he will use the smallest effort needed to send the high signal. Thus, the distortion will be $d = -\theta$ for some range $\bar{\theta} \leq \theta < 0$ and $d = 0$ for $\theta < \bar{\theta}$, where $\bar{\theta}$ is the value of $\theta$ at which the cost of distortion is equal to the benefit of obtaining $P_H$ rather than $P_L$. He will never distort the signal when $\theta \geq 0$ since this involves positive effort and gains nothing. A similar argument applies in the case $P_H < P_L$.

There is a unique choice of distortion effort $d$ and a unique payoff, for both the supplier and the purchaser, for almost every value of $\theta$ and $P$.

When dealing with the general case, it is useful to represent the supplier's choice variable as the value of the signal $S$, rather than the amount of distortion $d$. Consider the decision of the supplier once the contract $P$ has been accepted and the cost shock $\theta$ has been observed. The supplier can choose among $n$ different payments by choosing an appropriate distortion $d$. The minimum cost of obtaining $P_i$ is given by

$$T_{ki}(\theta) = \inf_d \{ T_k(d) | \theta + d \in E_t \}.$$

Then the supplier will choose the value of $i$ that maximizes

$$U(P_i - C(\theta)) - T_{ki}(\theta).$$

In defining the minimum cost, the discontinuity of the signal $S(\theta + d)$ apparently prevents the existence of a minimum. There are various ways this could be dealt with; we shall simply ignore it and treat the minimum as if it were well defined.\(^{10}\) This is a purely technical problem which is not important for the analysis.

\(^{10}\) One way of dealing with the discontinuity is to define an equilibrium distortion to be optimal for a supplier if it is the limit of distortions that yield maximum utility asymptotically. Alternatively, one could treat the signal $S$ as a correspondence that is bi-valued at 0. These all amount to the same thing and add nothing to the theory.
To reduce the extensive form game to a two-stage game, we must show that the supplier's optimal distortion decision is essentially unique.

**Lemma 1.** Suppose that a contract $P$ has been accepted. Then, for all but a finite number of values of $\theta$, there is a unique value of $i$ that maximizes $U(P_i - C(\theta)) - T_{ki}(\theta)$.

**Proof.** See the Appendix.

Let $H$ denote the set of functions from $\mathbb{R} \times \mathbb{R}^n$ to $\{e_1, \ldots, e_n\}$ where $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ is an $n$-vector with a one in the $i$-th place and zeros elsewhere. If $h \in H$ then we interpret $h(\theta, P) = (h_1(\theta, P), \ldots, h_n(\theta, P))$ as the choice of signal to send, conditional on the contract $P$ and the cost shock $\theta$. In other words, $h_i(\theta, P)$ is equal to 1 if the supplier chooses to send the signal $c_i$ and is zero otherwise.

Since we have shown the selection function $h$ is essentially unique for any contract $P$, we can express both players’ equilibrium payoffs as functions of the equilibrium contract $P$. Let $U^*(P, k)$ denote the expected utility of a supplier of type $k$ if a contract $P$ is accepted by the purchaser. Let $V^*(P, k)$ denote the expected utility of the purchaser who has accepted a contract $P$ from a supplier of type $k$. Then we can define the (reduced form) payoff functions $U^*$ and $V^*$ by putting

$$U^*(P, k) = \mathbb{E} \left[ \max_i \{ U(P_i - C(\theta)) - T_{ki}(\theta) \} \right]$$

and

$$V^*(P, k) = \mathbb{E} \left[ v_k - \sum_{i=1}^{n} h_i(\theta, P)P_i \right],$$

where, for each $k \in K$, $h$ is chosen to be the unique best response for a supplier of type $k$.

From now on, we use these reduced-form payoff functions to describe the reduced-form game. In this game, the supplier proposes a contract $P$, which the purchaser accepts or rejects. If the contract is accepted, the payoffs are $U^*(P, k)$ and $V^*(P, k)$. Otherwise, both agents receive zero.

**Equilibrium**

A pure strategy for the supplier, in the reduced-form game, is a choice of contract $P \in \mathbb{R}$. A pure strategy for the purchaser, in the reduced form game, is a decision rule $\alpha: \mathbb{R} \to \{0, 1\}$, with the interpretation that $\alpha(P) = 0$ means the contract $P$ is rejected and $\alpha(P) = 1$ means it is accepted. In what follows, we only consider pure strategy equilibria.

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The maximum payoff the supplier can obtain from a contract $P$ is obviously well defined, whether or not his optimal distortion is uniquely determined by $(\theta, P)$. On the other hand, the payoff received by the purchaser may not be. If the supplier is indifferent between two different actions which have different payoffs for the purchaser, we cannot represent the payoff to the purchaser as a function of the contract. Even if we were to make an arbitrary selection of the supplier's optimal action, the payoff would not be a continuous function of the contract. To reduce the game to a well behaved signaling game, it is crucial that the supplier's optimal distortion be unique.
Let $\Delta(K)$ denote the set of probability distributions on the set $K$. The purchaser’s beliefs about the supplier’s type are represented by a function $\mu: \mathbb{R}^n \to \Delta(K)$. For any contract $P$, $\mu(P) = (\mu(P, 1), \ldots, \mu(P, t)) \in \Delta(K)$, with the interpretation that $\mu(P, k)$ is the subjective probability that the supplier’s type is $k$, given that he has proposed $P$. Extend the purchaser’s payoff function to $\mathbb{R}^n \times \Delta(K)$ by putting
\[
V^*(P, m) = \sum_{k \in K} V^*(P, k)m(k),
\]
where $V^*(P, m)$ denotes expected utility when beliefs are given by $m \in \Delta(K)$. A Perfect Bayesian Equilibrium is defined to be a strategy profile $(\{P^k\}, \pi)$ together with an assignment of beliefs $\mu$ such that the following conditions are satisfied:

(E.1) \hspace{1cm} P^k \in \text{Arg Max}_P \alpha(P)U^*(P, k), \quad \forall k \in K; \\
(E.2) \hspace{1cm} \alpha(P) \in \text{Arg Max}_{\alpha \in \{0,1\}} aV^*(P, \mu(P)), \quad \forall P \in \mathbb{R}^n; \\
(E.3) \hspace{1cm} \forall k \in K, \quad \forall P \in \mathbb{R}^n, \quad \text{if } P^k = P \text{ then } \mu(P, k) = \frac{v_k}{\sum_{j : \pi_j = P} v_j}.

The first condition simply requires the supplier to choose his contract optimally. The second condition requires the purchaser’s acceptance decision to be optimal, given his beliefs, for every contract, not just those that are proposed in equilibrium. The third condition requires beliefs to be consistent with equilibrium strategies. (Recall that we only consider pure strategy equilibria, so Bayes’ rule reduces to the formula in (E.3.).)

III Analysis of the reduced form game

In this section, we shall argue that the most plausible equilibria involve all supplier types pooling and offering the same, non-contingent contract. This argument depends on a number of results. First, it is shown that there exists a pooling equilibrium where all types use the same non-contingent contract and that this equilibrium is stable in the quite strong sense of Universal Divinity. Second, there cannot be pooling at a contingent contract in any equilibrium that is stable in the weak sense of the Intuitive Criterion. Third, even in these (weakly stable) equilibria with contingent contracts, the best type or range of types chooses a non-contingent contract. Finally, it is shown that all supplier types are better off in a Universally Divine pooling equilibria, where only a non-contingent contract is used, than in any other equilibrium satisfying the Intuitive Criterion.

Refinements
We start by considering the meaning and formal definition of Universal Divinity and the Intuitive Criterion. The notion of Universal Divinity is weaker than stability in the sense of Kohlberg–Mertens (1986) but stronger than the Intuitive Criterion. Universal Divinity provides a criterion for deciding which type is most
likely to make a particular deviation from equilibrium, and concentrates the purchaser’s beliefs on that type. Type \( k \) is assumed to be “unlikely” to choose a non-equilibrium contract \( P \) if, whenever type \( k \) weakly (strictly) prefers that contract for some best response from the purchaser, there is some type \( k' \neq k \) that strictly (strictly) prefers it. This is a very strong refinement of equilibrium. It eliminates a strategy for a type as long as there is some combination of types that in aggregate seem more likely to use the strategy.

The following account is based on Cho–Sobel (1990). Choose some fixed but arbitrary equilibrium \( (\{P^k\}, \pi, \mu) \). Let \( u^*_k \) denote the equilibrium payoff for a supplier of type \( k \). Let \( \text{BR}(P, m) \) denote the set of best responses for the purchaser, to the contract \( P \), when beliefs are given by \( m \). That is,

\[
\text{BR}(P, m) = \text{Arg Max}_{a \in [0, 1]} aV^*(P, m),
\]

Let \( \Delta(K') \) denote the set of probability distributions in \( \Delta(K) \) concentrated on the set \( K' \subseteq K \), and let

\[
\text{BR}(P, K') = \bigcup \{a \in \text{BR}(P, m) | m \in \Delta(K') \}
\]
denote the set of best responses for the purchaser for some beliefs in \( \Delta(K') \). Note that the definition of \( \text{BR}(P, K') \) allows the purchaser to use a mixed strategy as his best response, even though mixed strategies are not used in equilibrium. For any \( k = 1, \ldots, \ell \) and any \( P \in \mathbb{R}^n \), let

\[
R(k|P) = \{a \in \text{BR}(P, K) | U^*(P, k) > u^*_k \}
\]

and

\[
R^0(k|P) = \{a \in \text{BR}(P, K) | U^*(P, k) = u^*_k \}.
\]

Universal Divinity eliminates a strategy \( P \) for the supplier of type \( k \) if

\[
R(k|P) \cup R^0(k|P) = \bigcup_{k' \neq k} R(k'|P).
\]

An equilibrium is said to be Universally Divine if it remains an equilibrium after the removal of all strategies satisfying this condition, for each type of supplier. If a strategy \( P \) is eliminated for type \( k \), but not for all types, then equilibrium beliefs must satisfy \( \mu(P, k) = 0 \). If the strategy is eliminated for all types, the purchaser’s beliefs conditional on that strategy become irrelevant.

Next we define the Intuitive Criterion of Cho–Kreps (1987). The Intuitive Criterion is weaker than Universal Divinity, so any equilibrium that satisfies the latter must satisfy the former. The rationale for the Intuitive Criterion comes from the idea that some supplier types may be able to signal their types credibly (and profitably) by deviating from their equilibrium strategies. Suppose that there exists an out-of-equilibrium contract \( P \) and a set of supplier types \( K' \), such that the following is true. Some type \( k \) in \( K' \) would get more than his equilibrium payoff if the purchaser believed his type belonged to \( K' \) and responded optimally to the offer. On the other hand, any type \( k \notin K' \), would be worse off whatever the purchaser’s beliefs and optimal response. Then type \( k \) would reason “If I choose \( P \), the purchaser should believe that I belong to \( K' \) and then his optimal response
must make me better off. So I should deviate to $P$." This equilibrium will fail the Intuitive Criterion.

An equilibrium $\{(P^k), x, \mu\}$ fails to satisfy the Intuitive Criterion if and only if there exist a proper subset $K'$ of $K$ and a contract $P$ such that:

(i) $\forall k \in K \setminus K', \ aU^*(P, k) < u_k^*$, for any $a \in BR(P, K)$;

(ii) $\exists k \in K', \ aU^*(P, k) > u_k^*$ for any $a \in BR(P, K)$.

We are assuming here that the contract is accepted with probability one in equilibrium. This will be true for the equilibria considered in the sequel.

**Pooling equilibrium**

An equilibrium $\{(P^k), x, \mu\}$ is called a pooling equilibrium if all supplier types choose the same contract, that is, $P^k = P$ for every $k$ in $K$. Note that in a pooling equilibrium, the purchaser may receive some positive surplus, that is, we may have $V^*(P, v) > 0$.

A contract $P$ is said to be non-contingent if it does not make any use of publicly available information, that is, $P_i = P_j$, for all $i$ and $j$. Otherwise it is said to be contingent. Note that the definition says nothing about whether the payments actually made in equilibrium are contingent. For example, even if the contract is contingent, the supplier can ensure he gets the same payment for every value of $\theta$ simply by distorting the signal. However, this cannot happen in “stable” equilibria: the equilibrium payments will be contingent if and only if the contract is contingent.\(^\text{12}\)

It is easy to show that there exist many equilibria in which only non-contingent contracts are used. Only two additional conditions are required. To define these, suppose that every type $k$ proposes the same non-contingent contract $P^*$, where $P^*_i = p$ for all $i = 1, \ldots, n$. Also, let $\bar{u}_k$ denote the maximum expected utility a supplier of type $k$ can obtain by offering an acceptable contract to the purchaser when the purchaser knows his type. That is,

$$\bar{u}_k = \sup_{P} \{ U^*(P, k) | V^*(P, k) \geq 0 \}.$$

The two conditions needed for the existence of (multiple) pooling equilibria are as follows. First, the purchaser must be willing to accept $P^*$ conditional on his belief that every type is offering this contract:

$$\sum_{k \in K} v_k u_k \geq p.$$
Second, it must be the case that the worst type cannot do better by deviating from $P^*$ and revealing his type:

$$\mathbb{E}U(p - C(\theta)) \geq \bar{u}_1.$$ 

These two conditions imply that the worst type of supplier is “sufficiently bad” relative to the average that he cannot gain by deviating from the pooling contract. In other words, the loss of insurance from choosing the non-contingent contract is offset by the “adverse selection” gain from hiding his type.

In what follows, we shall assume that whenever all types pool at a non-contingent contract $P$, the constant payment $p$ satisfies (A.5).

(A.5) There exists some value of $p$ such that $\sum_{k \in K} v_k v_k \geq p$ and $\mathbb{E}U(p - C(\theta)) \geq \bar{u}_1$.

**Proposition 1.** There exists a pooling equilibrium $(\{P^*\}, \alpha, \mu)$ in which all types choose a non-contingent contract $P^*$. Suppose that $P^*$ satisfies (A.5) and that for any contract $P \neq P^*$ the purchaser’s beliefs are concentrated on the worst type $k = 1$. Then the equilibrium is Universally Divine.

**Proof.** See Appendix.

The proposition can be illustrated by constructing a pooling equilibrium for the two-dimensional example discussed in Sect. II. (We shall use this special case to illustrate each of the results in turn). To keep things simple, suppose there are two types of supplier, good (G) and bad (B). The two types are equally likely. (A.1) and (A.2) imply that the bad type produces an inferior good and has lower costs of distortion than the good type. Then $0 < v_B < v_G$. Let $P^* = (p, p)$ denote the non-contingent contract, where $p \leq (v_B + v_G)/2$.

The equilibrium strategies are defined as follows. The strategy for both types of supplier is to propose the contract $P^*$. The purchaser’s strategy is to accept any contract $P$ if $V^*(P, B) \geq 0$ or $P = P^*$ and reject the rest. The purchaser is assumed to believe that any contract $P \neq P^*$ is offered by the worst type.

The pooling equilibrium is illustrated in Fig. 2. Note that the indifference curves of the two types are tangent at the equilibrium contract $P^*$. At any point other than $P^*$, the bad type’s indifference curve lies below the good type’s indifference curve. These facts follow directly from the form of the utility functions. Both types get the same payoff from a non-contingent contract, since there is no incentive to distort and they have the same utility of wealth. On the other hand, at any contingent contract, the bad type gets a higher payoff than the good type, simply because his cost of distortion is lower. Thus, if the good type prefers a contingent contract to $P^*$, the bad type must strictly prefer it.

It is clear that $P^*$ is an equilibrium contract. First, by construction, $P^*$ gives the purchaser a non-negative payoff, so he cannot do better than to accept it. Also, given his beliefs, he must reject any contract that gives him a negative payoff when it is offered by the bad type. Second, by construction, $P^*$ is preferred by either type of supplier to any other contract the purchaser will accept. We assumed in (A.5) that the bad type prefers $P^*$ to any contract in his acceptance region $\{P : V^*(P, B) \geq 0\}$. If the good type prefers any contract $P$ in this acceptance region
Missing contingencies in optimal contracts

![Graph showing indifference curves and acceptance regions for two suppliers: good and bad.]

Fig. 2. A stable equilibrium with pooling at a non-contingent contract. A pooling equilibrium with two types of supplier, good (G) and bad (B) is illustrated. Both types of supplier get the contract \( P^* = (p, p) \). The purchaser accepts this contract because it gives him non-negative profits. The only other contracts the purchaser accepts are in the bad type's acceptance region, i.e., they give him non-negative profits when he believes the supplier's type is B. Clearly, neither type of supplier will prefer one of these contracts, so they offer \( P^* \) in equilibrium.

to \( P^* \), so must the bad type, contradicting our assumption. Thus, for any acceptable contract \( P \), suppliers of both types will prefer \( P^* \) to \( P \). Finally, by (A.4), \( P^* \) gives them all a non-negative payoff, so it must be optimal for them to propose it.

Since there are many equilibria, each of them supported by more or less arbitrary off-the-equilibrium-path beliefs, it is not surprising that non-contingent contracts are used in some equilibria. However, as the proposition shows, the pooling equilibrium is in fact robust to precisely the sort of arguments that are often used to eliminate “unreasonable” off-the-equilibrium-path beliefs. This can perhaps be grasped intuitively from Fig. 2. The pooling equilibrium is supported by the purchaser's belief that any contract \( P \neq P^* \) is offered by the bad type. Is this reasonable? If \( P \) is preferred to \( P^* \) by the good type, it is strictly preferred by the bad type. Thus, it could be argued that the bad type is more likely to offer \( P \) than the good type and that the purchaser's beliefs should be concentrated on the bad type. In fact, this is what Universal Divinity requires. In this sense, the purchaser's beliefs are reasonable. If \( P \) is preferred to \( P^* \) by the bad type and not by the good type, the purchaser's beliefs would seem, a fortiori, to be reasonable. If neither type prefers \( P \) to \( P^* \), the purchaser's beliefs are irrelevant.

The crucial element, of course, is the ability of the supplier to distort the signal on which contingencies are based. The bad type has a stronger preference for the inclusion of contingencies in contracts simply because he has a greater ability to exploit any contingency for his own advantage. Since this effect is symmetric with respect to the inclusion of positive or negative insurance, it explains the stability of an interior pooling equilibrium.

The next step in the analysis is to show that other pooling equilibria do not satisfy reasonable stability properties.
Proposition 2. Suppose there is an equilibrium in which different types of supplier pool at a contingent contract. Then the equilibrium fails to satisfy the Intuitive Criterion.

Proof. See Appendix.

It might be thought, from the work of Cho–Kreps (1987) and Cho–Sobel (1990), that there will be other, stable, pooling equilibria, since the supplier’s preferences do not satisfy the single-crossing property. In fact, as Proposition 2 shows, there are not. The reason is simply that the tangency of indifference curves illustrated in Fig. 2 occurs only on the diagonal, i.e., only on the set of non-contingent contracts. Elsewhere, a version of the single-crossing property will be satisfied. More precisely, above the diagonal the bad type’s indifference curve will be flatter than the good type’s and below the diagonal it will be steeper. Suppose that both types of supplier pool at a contingent contract like P in Fig. 3. The good type would be strictly better off at a contract like $P'$ in the diagram, if it were accepted, whereas the bad type would be strictly worse off whatever the purchaser’s response. Furthermore, if $P'$ is sufficiently close to $P$, the purchaser will prefer $P'$ offered by the good type to $P$ offered by both types. The Intuitive Criterion requires the purchaser to reason as follows “If the good type offers contract $P'$ and I believe that he is offering the contract, then I will accept it and he will be better off than at $P$. On the other hand, if the bad type were to offer this contract, he would be worse off no matter what my response. Therefore, I should believe that only the good type will offer $P'$.” If the purchaser’s beliefs satisfy this property, he must accept $P'$ in equilibrium so pooling at $P$ cannot be an equilibrium.

![Diagram](image)

**Fig. 3.** An unstable equilibrium with pooling at a non-contingent contract. Suppose that two types of supplier (good and bad) pool at the contingent contract $P$. Then there exists a contract $P'$ that is preferred to $P$ by the good type, is acceptable to the purchaser if he believes it is offered by the good type and is strictly worse than $P$ for the bad type. The Intuitive Criterion says that the purchaser should believe $P'$ will be offered only by the good type and accept $P'$ if it is offered. But $P$ cannot be an equilibrium outcome with these beliefs.
Separating equilibria
Although there are no stable equilibria that involve pooling at contingent contracts, there will exist some stable equilibria in which contingent contracts are chosen. In these equilibria, the choice of a contingent contract reveals the supplier's type. The amount of separation in these equilibria varies. What we can show is that, even in a separating equilibrium, at least one type chooses a non-contingent contract and possibly several types pool at a non-contingent contract.

Proposition 3. Suppose that an arbitrary equilibrium satisfies the Intuitive Criterion. Then at least one type of supplier (the "best" type) must choose a non-contingent contract. Furthermore, the set of types pooling at this contract is an interval of the form \( \{k^{NC}, \ldots, \ell^C\} \).

Proof. See Appendix.

In the special case with two supplier types, there is a unique separating equilibrium that satisfies the appropriate stability criterion. This equilibrium is illustrated in Fig. 4. The good type chooses a non-contingent contract \( P^{NC} \) and the bad type chooses a contingent contract \( P^C \). \( P^C \) is the best contract the bad type can get accepted, given that the purchaser knows his type. \( P^{NC} \) is the best non-contingent contract for the good type that does not attract the bad type. The purchaser is willing to accept \( P^{NC} \) since it gives him a strictly positive payoff. By

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Fig. 4. A stable equilibrium with separation between contingent and non-contingent contracts. A stable equilibrium with two supplier types, good and bad is illustrated. The good supplier gets a non-contingent contract \( P^{NC} \) and the bad supplier gets a contingent contract \( P^C \). \( P^C \) is the best contract (from the supplier's point of view) that the purchaser will accept, given that the supplier's type is bad. \( P^{NC} \) is the best non-contingent contract that will not attract the bad type of supplier. It is acceptable to the purchaser because it is more attractive than \( P^* \) and it is offered by the good type. The only other contracts the purchaser will accept are those that yield non-negative profit when the supplier's type is known to be bad, i.e., contracts that lie in the bad type's acceptance region. It is clear that both types (weakly) prefer their equilibrium choices to one of these contracts. Finally, it can be seen that both types prefer the pooling equilibrium to the separating equilibrium.
assumption, the bad type prefers $P^\ast$ to $P^C$. Since the bad type is indifferent between $P^C$ and $P^{NC}$, $P^\ast$ must be preferred to $P^C$ by both types.

Proposition 3 tells us that pooling will occur at a single non-contingent contract, if it occurs at all. The next result gives a sufficient condition for type $k$ to be part of the pooling set.

**Proposition 4.** Suppose that an equilibrium satisfies the Intuitive Criterion. Suppose that $I^k$ is the non-contingent contract that makes type $k$ indifferent with his equilibrium contract. Then if $I^k$ lies strictly within the acceptance region for type $k + 1$, i.e., $V^\ast(I^k, k + 1) > 0$, the type $k + 1$ supplier chooses the non-contingent contract $I^k$ in equilibrium, i.e., $k + 1 \geq k^{NC}$.

**Proof.** See Appendix.

As a corollary of this result, if $V^\ast(I^1, k) \geq 0$ for some $k > 1$, then type $k$ chooses the non-contingent contract in equilibrium. Thus, if type 1 is “bad” enough relative to the rest, all types $k > 1$ will pool at the non-contingent contract. Similarly, if type 2 is “bad” enough relative to better types, all types $k > 2$ will pool at a non-contingent contract. The proposition gives us a sufficient condition for any type to belong to the set pooling at the non-contingent contract.

The next result shows that, from the perspective of the different types of supplier, the pooling equilibrium dominates all the stable equilibria that involve some separation. This result is interesting in its own right and, as we have mentioned, may be an additional argument in favor of the pooling equilibrium.

**Proposition 5.** Suppose that some types separate in an equilibrium satisfying the Intuitive Criterion. In the given equilibrium, every type of supplier is worse off than he would be in the pooling equilibria described in Proposition 1.

**Proof.** See Appendix.

In terms of Fig. 4, the bad type prefers $P^\ast$ to $P^C$ by (A.5). Since the bad type is indifferent between $P^C$ and $P^{NC}$, he must prefer $P^\ast$ to $P^{NC}$. Then the good type prefers $P^\ast$ to $P^{NC}$, so both types are better off in the pooling equilibrium.

To sum up, even if there is separation, there will be some use of non-contingent contracts. In addition, the fact that both types of supplier are worse off in the separating equilibrium may suggest that the separating equilibrium is less plausible than the pooling equilibrium. The argument is as follows. In order to support the separating equilibrium, the purchaser must reject non-contingent contracts that are preferred by both types. This is rational if his beliefs are sufficiently negative, i.e., put sufficiently high weight on the bad type. But when faced with a non-contingent contract that both types prefer, why should he assume that this is a deviation by the bad type? It might be more reasonable to make the following “forward induction” argument: “Every type prefers the pooling equilibrium to the separating equilibrium. Therefore, if I observe the pooling contract, it is probably because all types are coordinating on the pooling equilibrium. If so, my beliefs ought to be determined by my prior probabilities and I should accept the contract.” If the purchaser accepts this sort of argument, the pooling equilibrium will be observed.
IV Discussion

The idea that adverse selection may lead to missing contingencies has been explored by a number of writers. The difference between the various models lies mainly in the economic structure that is used to support missing contingencies as an equilibrium phenomenon. To clarify the contribution of the present paper, we briefly review the literature known to us.

Spier (1990) has studied a contracting problem closely related to the one described in Sect. II. Translating her assumptions into the language of this paper, we obtain the following structure. A supplier and a purchaser want to trade one unit of an indivisible good. The cost of production is uncertain and takes on two values, high and low. There are two types of supplier, a bad type and a good type. The supplier knows his type; the purchaser does not. The probability of having high costs is $\pi_B$ for the bad type and $\pi_G$ for the good type, with $0 < \pi_G < \pi_B < 1$. The purchaser gets utility $v > 0$ from one unit of the good. To explain the use of non-contingent contracts in equilibrium, Spier assumes that writing a contingent contract involves a fixed cost. Adverse selection reduces the value of contingent contracts (the good type will take less insurance to signal his type). This increases the set of parameter values for which a non-contingent contract is used in equilibrium. The crucial assumption is that the purchaser is indifferent to the supplier’s type when a non-contingent contract is used. If different types of suppliers produce different goods, her results can be reversed. In other words, adverse selection can lead to the use of more contingencies, not less.\(^{13}\)

Spier’s work focuses on separating equilibria.\(^{14}\) Another line of research has focused on pooling equilibria. In these papers we find that equilibrium contracts are on the boundary of the contract space.\(^{15}\) Most arguments against pooling rely

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\(^{13}\) In Spier’s model, the non-contingent contract $(v, v)$ represents a lower bound on the equilibrium outcome that both types of supplier can receive. Since the two types differ only in the probability distribution of costs, the purchaser is indifferent about the supplier’s type when he is offered the non-contingent contract. Thus, he will always accept an offer of the contract $(v, v)$. Adverse selection can force the low-cost supplier’s contract toward the non-contingent contract, but not beyond it. This property disappears if the two types differ in some other dimension. Suppose that the low-cost supplier produces the high-quality good. Let the values be $v_G$ and $v_B$ with $v_G > v_B$. The low-cost supplier can no longer be assured of getting a non-contingent contract in which he is paid the value of his good. If he is confused with the bad type, he may be forced to offer a much worse contract, even if it does not contain any contingencies. In order to separate himself from the bad type, the good type may be willing to accept negative insurance, i.e., contingencies that increase his risk. By contrast, for the same parameter values, he might choose a non-contingent contract under complete information.

\(^{14}\) These are the only stable equilibria in her model.

\(^{15}\) In Spier’s model, the non-contingent contract is a kind of boundary case. The non-contingent contract, under which the supplier is paid a fixed price $v$, independently of costs, is always acceptable to the purchaser. This contract provides a lower bound to the low-cost supplier’s equilibrium payoff. Adverse selection can force the low-cost supplier toward this lower bound, but no further. If the model is changed, so that the non-contingent contract is no longer on the boundary of the set of contracts that can be observed in equilibrium, the non-contingent contract ceases to have a distinguished place in the theory.
on the single-crossing property. But the single-crossing property loses its force on the boundary of the contract space. Pooling can occur at boundary contracts, even when it could not occur in the interior.

Nachman and Noe (1990) describe a model in which a single firm and a single investor negotiate over the form of contract to use for financing investment in the firm. Both agents are risk neutral. There are two types of firms, high-risk and low-risk. In a stable equilibrium, both types of firm pool at the standard debt contract. In a certain sense, this is the least contingent contract allowed: the firm makes a constant payment to the investor when this is feasible and makes the greatest feasible payment when it is not. This contract is also on the boundary of the set of admissible contracts. Contracts are assumed to make the firm’s repayment monotonically non-decreasing in the firm’s revenue. The low-risk firm would like to signal its type by offering a contract that paid more in the low states and less in the high states, but this is incompatible with feasibility and monotonicity.

Without the monotonicity condition, the stable equilibrium would be separating and neither type of firm would choose a debt contract. This illustrates the importance of the boundary constraint and suggests a certain fragility of this kind of result.

De and Kale (1990) exploit a similar idea in their analysis of debt packaging. They assume that the firm seeking finance has a choice of two contracts. It can either choose Fixed-Periodic-Obligation Debt (FPOD) contracts or No-Periodic-Obligation Debt (NPOD) contracts or some combination of the two. Once the firm has made its choice, the market determines the coupons the firm must pay in each period. Under certain conditions, it can be shown that the unique equilibrium that is stable in the sense of Universal Divinity (Banks–Sobel, 1988) is one in which FPOD contracts, i.e., non-contingent contracts, are used. The explanation of these results, like those of Nachman–Noe, turns on the fact that the equilibrium contract is on the boundary of the contract space, in this case because of the restriction on the forms of the contracts.

Stiglitz–Weiss (1981) and DeMeza–Webb (1987) have used a similar idea to obtain pooling equilibria with credit rationing. In both cases, the power of the single-crossing property is vitiated by adopting assumptions that ensure the equilibrium occurs on the boundary of the contract space. Like the Nachman–Noe and De-Kale constructions, these equilibria seem a little fragile, since small changes in the model could ensure that the equilibrium contract was located in the interior of the contract space. In the case of Stiglitz–Weiss, this possibility has been studied by Bester (1985).

To sum up, these papers have made important contributions to our understanding of the ways in which incomplete information limits the use of contingencies in optimal contracts. However, they may be fragile solutions to the extent that they rely, in different ways, on the use of “corner solutions”. One of the contributions of the present model is to provide an explanation that does not rely on a boundary property. In other words, we have described a stable equilibrium in which the non-contingent contract is in the interior of the contract set.
Appendix

Proof of Lemma 1
The proof requires us to examine a number of cases. Let $i, j$ and $h$ ($i \neq j$) be arbitrarily given and suppose that $\theta$ belongs to $E_h$. Let $\bar{E}$ denote the closure of $E$. Let $x_i$ and $x_j$ be the points in $\bar{E}$ and $\bar{E}_j$, respectively, that minimize the distance to $\theta$. Then the supplier will be indifferent between $P_i$ and $P_j$ if and only if

$$U(P_i - C_h) - T_k(\theta) = U(P_j - C_h) - T_k(\theta).$$

Case (i). Suppose first of all that $i = h$. (The case $j = h$ is exactly similar). Then

$$U(P_h - C_h) = U(P_j - C_h) - T_k(\theta).$$

Since $T_k(d)$ increases when the absolute value of $d$ increases, it is clear that there can be at most one value of $\theta$ for which this condition is satisfied.

Case (ii). In what follows, we can suppose without loss of generality that $i \neq h \neq j$. Suppose that $\theta$ lies between $x_i$ and $x_j$, say $x_i < \theta < x_j$. Then

$$U(P_i - C_h) - U(P_j - C_h) = T_k(x_i - \theta) - T_k(x_j - \theta).$$

It is clear from the fact that the distortion cost is increasing in absolute distortion that an increase in $\theta$ will raise $T_k(x_i - \theta)$ and reduce $T_k(x_j - \theta)$ and that a decrease in $\theta$ will have the opposite effects. There is at most a single value of $\theta$ for which this equation can be satisfied.

Case (iii). Now suppose that both $x_i$ and $x_j$ lie to the right of $\theta$. (The case where both lie to the left is exactly similar). Then

$$U(P_i - C_h) - U(P_j - C_h) = T_k(x_i - \theta) - T_k(x_j - \theta).$$

The strict convexity of the distortion cost ensures that there is at most one value of $\theta$ in $E_h$ that can satisfy the equation. If $x_i < x_j$, say, then an increase in $\theta$ will decrease both $T_k(x_i - \theta)$ and $T_k(x_j - \theta)$, but the decrease in $T_k(x_j - \theta)$ will be greater than the decrease in $T_k(x_i - \theta)$. Exactly the same argument works if $\theta$ decreases or if $x_j < x_i$.

We have shown that for any choice of $i, j$ and $h$ there is at most one value of $\theta$ in $E_h$ at which the supplier is indifferent between $P_i$ and $P_j$. There is a finite number of choices of $i, j$ and $h$. Thus, under the maintained assumptions, there is a unique optimal distortion for the supplier for all but a finite number of values of $\theta$. □

Proof of Proposition 1
Let $P^*$ be defined by $P^*_i = p$ for all $i = 1, \ldots, n$, where $p$ satisfies the conditions of (A.5). The equilibrium strategies are defined as follows.

$$P^* = P^*, \quad \forall k = 1, \ldots, \ell;$$

$$x(P) = \begin{cases} 1 & \text{if } P = P^* \text{ or } V^*(P, 1) \geq 0, \\ 0 & \text{otherwise}; \end{cases}$$
\[
\mu(P, k) = \begin{cases} 
v(k) & \text{if } P = P^*; \\
1 & \text{if } P \neq P^* \text{ and } k = 1; \\
0 & \text{if } P \neq P^* \text{ and } k \neq 1.
\end{cases}
\]

In other words, the strategy for all types of supplier is to propose the contract \(P^*\). The purchaser’s strategy is to accept any contract \(P\) if \(V^*(P, 1) \geq 0\) or \(P = P^*\) and reject the rest. Any type of supplier who deviates from the equilibrium strategy \(P^*\) is believed to be the worst type \(k = 1\).

To see that this is a Perfect Bayesian Equilibrium, consider first the purchaser’s strategy. By construction, \(P^*\) gives the purchaser a non-negative payoff, so he cannot do better than to accept it. Since he believes any other contract to be offered by the worst type, it is optimal for him to reject unless \(V^*(P, 1) \geq 0\). Thus, the purchaser’s strategy is a best response given his beliefs.

Next consider the suppliers. At the pooling contract, there is no distortion and all types of suppliers get the same expected utility. Any preferred deviation must be to a contingent contract. By (A.5), the worst type \((k = 1)\) prefers \(P^*\) to any contract that the purchaser will accept. Since any other type has higher costs of distortion, his expected utility at a non-contingent contract \(P\) will be lower than type 1’s. Thus, it cannot be optimal for any type of supplier to deviate from \(P^*\). Finally, (A.5) implies that \(p > v_1\) so by (A.4) all supplier types receive a positive payoff and it must be optimal for them to propose \(P^*\).

Finally, note that beliefs are consistent with the equilibrium strategies, that is, \(\mu(P^*) = v\). This completes the proof that \((\{P^k\}, \alpha, \mu)\) is a PBE.

Now suppose that \((\{P^k\}, \alpha, \mu)\) is any pooling equilibrium in which \(P^*\) satisfies (A.5) and any deviations from \(P^*\) are assumed to be by the worst type \(k = 1\). To show this equilibrium is Universally Divine, we need the fact that for any contract \(P \neq P^*\), \(U^*(P, k) \geq U^*(P^*, k)\) for \(k > 1\) implies that \(U^*(P, 1) > U^*(P^*, 1)\). This condition is trivially satisfied for non-contingent contracts, so consider a contingent contract \(P\). If \(P\) is accepted, there must be positive distortion, since the distortion costs are arbitrarily small for some values of \(\theta\). Now, if type \(k\) weakly prefers \(P\) to \(P^*\) then any type \(k' < k\) must strictly prefer it, since \(k'\) has lower costs of distortion than \(k\). Thus,

\[
U^*(P, 1) > U^*(P, k) \quad \text{for any } k > 1.
\]

There are now two cases to consider. \(R(1|P) \cup R^0(1|P) = \phi\) implies that \(R(k|P) = \phi\) for any \(k = 2, \ldots, \ell\) and the required condition is obviously satisfied. On the other hand, if \(R(1|P) \cup R^0(1|P) \neq \phi\), then \(V^*(P, 1) < 0\) and \(U^*(P, 1) \geq u_1^*\) for some \(a \in BR(P, K)\). Then we cannot have

\[
R(1|P) \cup R^0(1|P) \subset \bigcup_{k' \neq 1} R(k'|P).
\]

So the pooling equilibrium satisfies the definition of Universal Divinity. \(\square\)
Proof of Proposition 2
Recall that

\[ U^*(P, k) \equiv \max_{h \in H} \mathbb{E} \left\{ \sum_{i=1}^{n} h_i(\theta, P)[U(P_i - C(\theta)) - T_k(\theta)] \right\} \]

\[ \equiv \mathbb{E} \left\{ \max_i [U(P_i - C(\theta)) - T_k(\theta)] \right\}. \]

In proving Proposition 2, we need a formula for the derivative of \( U^* \) with respect to \( P \). The derivative is denoted by \( \partial U^*(P, k) \) and defined to be the linear function from \( \mathbb{R}^n \) to \( \mathbb{R}^n \) with the property that

\[ U^*(P + z, k) - U^*(P, k) = \partial U^*(P, k)(z) + o(z), \quad \forall z \in \mathbb{R}^n. \]

Lemma 2. \( U^*(\cdot, k) \) is a continuously differentiable function for any value of \( k \). The derivative is given by the formula:

\[ \partial U^*(P, k)(z) = \mathbb{E} \left\{ \sum_{i=1}^{n} h_i(\theta, P)U'(P_i - C(\theta))z_i \right\}, \quad \forall z \in \mathbb{R}^n, \]

where \( h \) is the optimal selection function for type \( k \).

The proof of the Lemma is omitted. The argument is standard and requires the Lebesgue dominated convergence theorem.

Turning to the proof of the Proposition, suppose, contrary to what is to be proved, that two or more types of supplier offer the same, contingent contract \( P^C \). Let \( \kappa \) denote the highest type for whom the choice of \( P^C \) is optimal. The strategy of the proof is to show that there exists a contract \( P' \) very close to \( P^C \) which is preferred by the supplier of type \( \kappa \) and by the purchaser and is worse for any types \( k < \kappa \). Any contract sufficiently close to \( P^C \) which is offered only by type \( \kappa \) must be strictly preferred by the purchaser, since the higher type distorts less than the lower types and produces a good of higher value. Then the Intuitive Criterion is violated. The rest of the proof is devoted to constructing the contract \( P' \) with the desired properties. Without loss of generality, we can assume that the supplier of type \( \kappa \) actually offers \( P^C \) in equilibrium. The other case, where he is indifferent between \( P^C \) and \( P^* \), is identical.

Let \( P^C_m \) and \( P^C_M \) denote the minimum and maximum values of \( P^C_i \), respectively; let \( m \) and \( M \) denote the values of \( i \) for which the minimum and maximum, respectively, are attained; let \( E_i = \bigcup_{i=1}^{s} E_{i} \) for \( s = m, M \) and let \( A_{k_s} \) denote the values of \( \theta \) for which type \( k \) chooses a signal that results in the payment \( P^C_{i_s} \), for \( s = m, M \) and any \( k \).

Claim 1. For any \( k < \kappa \), \( A_{k_s} \subseteq A_{k_m} \); the inclusion is strict\(^{16}\) if \( A_{k_m} \) has positive measure. Clearly, no one will send the signal \( c_i \) for \( i \in m \) unless \( \theta \in E_i \). Thus, if type \( \kappa \) distorts his signal, it must be optimal for type \( k < \kappa \) to do so and this proves that \( A_{k_m} \subseteq A_{k_m} \). Now suppose that \( A_{k_m} \) has positive measure. By choosing \( \theta \in E_{m} \)...

\(^{16}\) That is, the measure of \( A_{k_m} \) is strictly less than the measure of \( A_{k_m} \).
very close to the boundary, we can make the cost of distortion so small that type $\kappa$ will choose to distort. Then, by continuity, there exists $\hat{\theta} \in E_m$ at which type $\kappa$ is indifferent between distorting and not distorting. For any $\theta \in E_m$ sufficiently close to $\hat{\theta}$, types $k < \kappa$ will strictly prefer to distort, as claimed.

**Claim 2.** For any $k < \kappa$, $A_{k,M} \supseteq A_{\kappa,M}$ and the inclusion is strict if the complement of $A_{\kappa,M}$ has positive measure. Clearly, $E_M \subseteq A_{k,M}$ for any type $k$. If type $\kappa$ chooses to send the signal $c_i$ for $i \in M$ and $\theta \notin E_M$ then any type $k < \kappa$ must also choose to do so. To see this, let $d$ denote the distortion chosen by type $\kappa$ and $d'$ the distortion chosen by type $k < \kappa$. It cannot be optimal to have $|d'| > |d|$ because this involves a larger distortion for a smaller payoff. Conversely, for any $|d'| < |d|$, (A.3) implies that

$$T_k(d) - T_k(d') < T_k(d) - T_k(d'),$$

so the optimality condition

$$U(P_M - C(\theta)) - T_k(d) \geq U(P(\theta + d') - C(\theta)) - T_k(d')$$

implies that

$$U(P_M - C(\theta)) - T_k(d) > U(P(\theta + d') - C(\theta)) - T_k(d').$$

This shows that $A_{k,M} \supseteq A_{\kappa,M}$ for $k < \kappa$. If the complement of $A_{\kappa,M}$ is non-null, there must exist some $\hat{\theta}$ at which type $\kappa$ is indifferent between sending the signal $c_i$ for $i \in M$ and sending some other signal. Then, for any value of $\theta$ sufficiently close to $\hat{\theta}$, types $k < \kappa$ will strictly prefer to send the signal $c_i$ for some $i \in M$.

Now define the alternative contract $P'$ by putting

$$P'_i = \begin{cases} P^C_i & \text{for } i \notin M, M; \\ P^C_m - \varepsilon_M & \text{for } i \in M; \\ P^C_m + \varepsilon_m & \text{for } i \in m. \end{cases}$$

Suppose that the signal $c_m$ is not used by any type offering the contract $P^C$ in equilibrium, that is, $A_{k,m}$ is the null set. Then put $\varepsilon_M = 0$ and raise $\varepsilon_m$ until type $\kappa$ is just indifferent between distorting the signal and not distorting the signal for some $\theta$ in $E_m$. Raising $\varepsilon_m$ a little more will make $\kappa$ better off but will not affect the utility of any type $k < \kappa$, since they strictly prefer distortion for any $\theta \in E_m$. Then we have found a contract $P'$ with the desired properties. There is no loss of generality, therefore, in assuming in the sequel that $A_{k,m}$, and hence the complement of $A_{k,M}$, has positive measure.

For any choice of $\varepsilon_m$ and $\varepsilon_M$ and any type $k$,

$$U^*(P', k) - U^*(P^C, k)$$

$$= \mathbb{E}\{h_m(\theta, P')U'(P^C_m - C(\theta))\varepsilon_m - h_m(\theta, P)U'(P^C_m - C(\theta))\varepsilon_M\} + o(\varepsilon_m, \varepsilon_M)$$

by Lemma 2, where $h_{k_s}$ is the optimal selection function of the set $s = m, M$ for type $k$. From Claims 1 and 2 we can see that $\varepsilon_m$ and $\varepsilon_M$ can be chosen so that $U^*(P', \kappa) > U^*(P^C, \kappa)$ and $U^*(P', k) < U^*(P^C, k)$ for $k < \kappa$, as required. □
Proof of Proposition 3
Once again, the proof is by contradiction. Suppose that the highest type \( k = \ell \) has chosen a contingent contract \( P^C \) in equilibrium. Let \( P^{NC} \) denote the best non-contingent contract that is not strictly preferred by type \( \ell \) to his equilibrium contract. Then every type \( k < \ell \) must strictly prefer his equilibrium contract to \( P^{NC} \). This follows since each type \( k < \ell \) weakly prefers his equilibrium contract to \( P^C \) and if \( \ell \) weakly prefers \( P^C \) to \( P^{NC} \), then every type \( k < \ell \) must strictly prefer \( P^C \) to \( P^{NC} \).

Since there is no pooling at contingent contracts, the worst type 1 reveals his type by proposing the contract \( P^1 \) in equilibrium. Assumption (A.5) tells us that type 1 is worse off at \( P^1 \) than at the non-contingent contract \( P^* \) chosen in the pooling equilibrium. Then, \( P^{NC} \) must be strictly worse than \( P^* \) for the supplier and therefore strictly better than nothing for the purchaser. Hence, the contract \( P^{NC} \) must be accepted by the purchaser if he believes it is offered by type \( \ell \); it will never be offered by types \( k < \ell \), whatever the response of the purchaser; and it is at least as good as the equilibrium contract \( P^C \) for the supplier of type \( \ell \), if it is accepted. We have not shown that the the supplier will be strictly better off at \( P^{NC} \), but a small increase in the contract price will be make type \( \ell \) strictly better off without changing any of the other conditions. Thus, the Intuitive Criterion is violated, a contradiction.

The last claim follows from the fact that if type \( k \) weakly prefers a non-contingent contract \( P^{NC} \) to a contingent contract \( P^C \), then any type \( k' > k \) strictly prefers \( P^{NC} \) to \( P^C \). ∎

Proof of Proposition 4
Suppose that \( V^*(l^k, k + 1) > 0 \). If, contrary to what we want to prove, type \( k + 1 \) chooses a contingent contract \( P^{k+1} \), then by a previous argument \( U^*(P^{k+1}, k + 1) \geq U^*(l^k, k + 1) \) implies that \( U^*(P^{k+1}, k') > U^*(l^k, k') \) for every type \( k' < k + 1 \), contradicting the definition of \( l^k \). So \( P^{k+1} \) must be a non-contingent contract. Since we know from Proposition 3 that the set of types choosing a non-contingent contract is an interval, we can assume without loss of generality that each type \( k' < k + 1 \) chooses a contingent contract. If \( P^{k+1} \) is worse than \( l^k \), then all types \( k' < k + 1 \) must strictly prefer their equilibrium contracts to \( P^{k+1} \), even if we improve it a little. Hence, there exists a contract \( P \) that is strictly preferred to \( P^{k+1} \) by \( k + 1 \), strictly inferior to the equilibrium contract for every \( k' < k + 1 \) and satisfies \( V^*(P', k') > 0 \) for every \( k' > k \). Then the Intuitive Criterion is violated. ∎

Proof of Proposition 5
From Propositions 2 and 3 we know that, in the given separating equilibrium, at least one type of supplier chooses a non-contingent contract \( P^{NC} \) and there is no pooling at contingent contracts. In particular, the worst type, \( k = 1 \), is revealed, and must get the best contract that is acceptable to the purchaser, conditional on his type. That is, the supplier of type 1 gets a payoff of \( \bar{u}_1 \).

In the pooling equilibrium described in Proposition 1, every type of supplier receives the same payoff, call it \( u^* \). By (A.5), \( u^* > \bar{u}_1 \). Therefore at least one type is better off in the pooling equilibrium. To see that the other types are also better
off in the pooling equilibrium, note first that the non-contingent contract \( P_{NC} \) must be strictly worse than \( P^* \); otherwise, the worst type, \( k = 1 \), would choose it. So any type that chooses \( P_{NC} \) is strictly worse off than in the pooling equilibrium. Second, suppose there is some type \( k > 1 \) who chooses a contingent contract \( P^k \) in the given equilibrium and that, contrary to what is to be proved, type \( k \) weakly prefers \( P^k \) to \( P^* \). Then by a familiar argument (used in the proof of Proposition 3), the worst type must strictly prefer \( P^k \) to \( P^* \). But since the equilibrium conditions require that \( P^1 \) is weakly preferred to \( P^k \) by type 1, \( P^1 \) must be strictly preferred to \( P^* \) by type 1, contradicting our hypothesis. This completes the proof.

References


De, S., Kale, J.: Security design and equilibrium refinements: implications for debt packaging. GSB, University of Wisconsin–Madison (mimeo), 1990


Dewatripont, M., Maskin, F.: Multidimensional screening, observability and contract renegotiation. Harvard University (mimeo), 1989

Dewing, A.: The position of income bonds, as illustrated by those of the Central of Georgia Railway. Q. J. Econ. 15, 396–405 (1911)


Holmstrom, B., Milgrom, P.: Multi-task principal agent problems: incentive contracts, asset ownership and job design. SOM, Yale University (mimeo), 1990


Nachman, D., Noe, T.: Design of securities under asymmetric information. Georgia Institute of Technology (mimeo), 1990


Scherer, F.: The weapons acquisition process: economic incentives. GSBA, Cambridge, MA: Harvard University 1964

