Incentives, Organization, and Public Economics

Papers in Honour of Sir James Mirrlees

Edited by
PETER J. HAMMOND
GARETH D. MYLES

OXFORD UNIVERSITY PRESS
15 Capital Structure and Imperfect Competition in Product Markets

FRANKLIN ALLEN

1. Introduction

It is often acknowledged that the question of how firms choose their capital structure has not been satisfactorily answered yet; current theories seem to be unable to explain the financing decisions firms actually make. The debate has mainly centred around two apparent empirical regularities. First, many firms paying corporate taxes use only a small amount of debt. Debt ratios are typically of the order of 20 to 30 per cent (see e.g. Wald 1999). Second, similar firms in the same industry often have significantly different capital structures (see e.g. Myers 1984).

The most widely taught theory of capital structure is still Modigliani and Miller (1958) extended to include corporate taxes and bankruptcy costs. Firms trade off the corporate tax advantage of debt arising from interest being tax deductible, against the costs of bankruptcy. Given the first regularity, these costs must be large for the theory to be plausible. In a number of early papers, such as Scott (1976) and Kim (1978), liquidation costs, which are potentially large, are the main costs the authors seem to have in mind. A possible reason is that bankruptcy is often followed by liquidation. However, as Haugen and Senbet (1978) stress liquidation is a capital budgeting decision whereas bankruptcy is a transfer of assets. Standard models do not explain why bankruptcy should cause liquidation. Given this, the trade-off theory is difficult to reconcile with the first observation since the other costs appear to be small. As Myers (1984) argues, it also seems inconsistent with the second since it suggests similar firms should have similar capital structures. In fact there are significant differences between similar firms.

These and other difficulties have led to alternative theories being developed. One approach is to include personal as well as corporate taxes (see e.g. Stiglitz 1973; King 1974; Miller 1977; and De Angelo and Masulis 1980). By retaining earnings and in various other ways, firms can help their shareholders avoid the personal tax on equity income. To the extent that firms do this, the corporate tax advantage to debt is mitigated by a personal tax advantage to equity. Provided most personal taxes on equity income are avoided, these theories do not require bankruptcy costs to be large to be consistent with the first observation. The problem is that the empirical evidence on dividends suggests that only a limited amount of personal tax on equity income is avoided.

I would like to thank R. Heinkel, P. Knez, A. Posilewaite, C. Spatt, and J. Williams, participants at a number of seminars and particularly M. Roe for many helpful comments and suggestions. Financial support from the NSF is gratefully acknowledged.
(see e.g. Peterson, Peterson, and Ang 1985). Moreover, the predictions of the theory are again difficult to reconcile with the second observation.

Other approaches are concerned with asymmetric information and with contracting difficulties (see e.g. Jensen and Meckling 1976; Leland and Pyle 1977; Myers and Majluf 1984; Titman 1984; and Leland 1998). These theories provide plausible explanations of phenomena at a detailed level. However, it can be argued that the severity of these problems is not sufficient for them to be the primary explanation for the observations mentioned above. Many of these theories abstract from taxes. If these were included, the apparent magnitude of tax shields suggests that direct solutions such as more acquisition of information might be better than having a low debt ratio.

A theory of capital structure is presented below where bankruptcy causes partial or complete liquidation. It is based on imperfections in firms’ product markets. Most corporations, particularly large ones, operate in product markets with only a few firms. For simplicity, a linear duopoly model with a Cournot market structure is analysed. In addition, it is assumed that because of the way the bankruptcy laws operate, which is described below, the effect of bankruptcy is to delay investment decisions. This delay is not costly in itself. However, given the imperfectly competitive product market, it means the bankrupt firm is at a strategic disadvantage and is forced to contract if fixed costs are small or liquidate if they are large. Thus when choosing the amount of debt to use, firms trade off the corporate tax advantage against these liquidation costs. The nature of equilibrium also depends on the fixed costs of capacity. If these are small, the equilibrium is symmetric so both firms adopt the same strategy. If they are large the equilibrium is asymmetric. One firm goes for the tax advantage of debt and the other goes for the strategic advantage of equity. Thus, even though the firms are similar, they can have different capital structures.

A number of other papers also consider models of investment and financing in a duopolistic industry. Brander and Lewis (1986) and Maksimovic (1988) show how debt acts as a pre-commitment device and this leads to more aggressive product market behaviour. These papers explain why firms find the use of debt advantageous but the puzzle has been why it is not used more. In the current paper, greater debt increases the probability of bankruptcy and liquidation which is costly. Thus more debt will be associated with less aggressive product market behaviour subsequently. The theory is consistent with the observation that firms use little debt despite its apparent tax advantages. Fudenberg and Tirole (1986) and Bolton and Scharfstein (1990) develop theories where an agency problem between the firm and providers of finance leads to financial constraints and these may encourage product market competitors to engage in predatory pricing. Higher debt leads a firm to engage in less aggressive product market behaviour because access to capital is decreased. Rotemberg and Scharfstein (1990) consider a model where investors are not as well informed as firms about future profitability. The firms choose their product market actions to manipulate investors’ beliefs and maximize the firm’s stock market value. This can lead to either more or less aggressive product market strategies depending on the nature of inferences drawn by investors from the performance of different firms in the industry. In contrast, the model presented below does not rely on asymmetric information. More importantly,
the trade-off considered is between the tax benefit of debt and costs of liquidation rather
than between the manager-shareholder agency problem and the predatory behaviour
of competitors. Glazer (1994) focuses on the role of long-term debt and shows that
the accumulated profit is important in determining product market behaviour in this
case. Here the focus is on short-term debt and accumulated profits do not play any
role.

The paper proceeds as follows. Section 2 outlines the model. In Section 3 equilib-
rium is analysed. Section 4 discusses the relationship between the results and empirical
evidence. Finally, Section 5 contains concluding remarks.

2. The Model

There are two identical firms denoted j and k. The market structure is Cournot. Each
period the firms produce quantities \( q_j \) and \( q_k \), place them on the market and together
these determine the market clearing price \( p \).

Demand for the industry is stochastic. The demand curve has the form

\[
p = \alpha - \beta(q_j + q_k),
\]

(1)

where the intercept \( \alpha \) is a random variable and the slope \( \beta \) is a constant. The density
function of \( \alpha \) is linear:

\[
f(\alpha) = \alpha - \alpha_i \text{ for } \alpha_i \leq \alpha \leq \alpha_i + 1
\]

(2)

\[
= \alpha_i + 2 - \alpha \text{ for } \alpha_i \leq \alpha \leq \alpha_i + 2,
\]

(3)

where \( \alpha_i = \alpha_i + 1 \).

This density function is illustrated in Figure 15.1. It is assumed that \( \alpha_i \) is
sufficiently large that in equilibrium \( p > 0 \) for all possible realizations of \( \alpha \). The real-
izations of \( \alpha \) in each period are independently and identically distributed.

A firm's output is equal to its capacity which is determined by its initial investment
\( I_j \). The cost of capacity is a linear function

\[
I_j = 0 \text{ for } q_j = 0
\]

\[
= \phi + \mu q_j \text{ for } q_j > 0,
\]

where \( \phi \) is the fixed cost and \( \mu \) is the constant marginal cost.
There are two periods with period 1 starting at date 0 and finishing at date 1 and period 2 starting at date 1 and finishing at date 2. The sequence of events in each period is the following:

1. Initially firms make their investment decisions and determine their capacities. At the same time they make their financing decisions. The capital markets meet, the firms issue the securities corresponding to their decisions, and use the proceeds to pay the costs of their investments. All this is done before \( \alpha \) is realized.

2. Output is produced; \( \alpha \) is realized.

3. The two firms' quantities \( q_i \) and \( q_s \), together with \( \alpha \), determine the price. Revenues \( p q_i \) and \( p q_s \) are received. These are used to pay the corporate income tax and make payments on the securities issued at stage 1.

Capital markets are perfectly competitive and frictionless. Investors are risk-neutral and securities sell for their expected value. The interest rate is zero.

Firms have two financing instruments, debt and equity. Debt is a promise to repay \( D_i \) at the end of the period or the firm goes bankrupt and the bondholders acquire it. Equity receives the profits (if any) remaining at the end of the period after taxes and payments to debtholders.

At the start of each period, before firms make their investment and financing decisions, they are entirely owned by equity holders. When firms finance their investment they issue debt with par value \( D_i \). If the debt is risky, its market value \( D_i^m \) will be less than \( D_i \). The remainder required to finance the investment, \( I_i - D_i^m \), is raised by issuing equity. Since the capital markets are perfect, the original shareholders' total expected pay-off is the expected value of the firm. Hence their objective when they make the investment and financing decisions is to maximize this.

The corporate tax rate is \( \tau \). Investment costs are tax deductible. Nominal debt \( D_i \) is also deductible so that there is a tax bias in favour of debt finance. The taxes paid by a firm are

\[
T_i = \tau [p q_i - (\phi + \mu q_i) - D_i].
\]  

Taxes are paid when the tax base is positive and subsidies are received when it is negative. To prevent firms from issuing an infinite amount of debt \( D_i \), and claiming the infinite subsidy this would give rise to, there is an upper bound on the amount of debt that is tax deductible

\[
D_i \leq D_s.
\]  

This corresponds to the limited loss carry-over provisions in the actual code.

For simplicity, there are no personal taxes on either debt or equity income or equivalently the marginal personal rate on equity income is the same as the marginal personal rate on debt income.

Bankruptcy occurs when the total receipts at the end of the period from selling the goods less any tax payments (or plus any subsidies) are insufficient to repay the debt

\[
p q_i - T_i \leq D_i.
\]
This condition assumes that equity holders do not have the opportunity to raise funds against future profits and use them to repay the debt and possibly prevent bankruptcy. This is the simplest case to consider. If future profits were included in the bankruptcy condition, there would still be a determinate bankruptcy point and a similar analysis to that below would be possible.

The condition also assumes that the equity holders cannot repurchase the debt at its current market value just before bankruptcy as suggested by Haugen and Senbet (1978, 1988). The justification for this is that it will be difficult for a firm to repurchase its debt securities when it is near bankruptcy if they are held by more than one person. This is because, similarly to Grossman and Hart (1980), there is a free-rider problem. Given some people accept the firm’s repurchase offer, anybody who holds out and refuses it can expect the value of their securities to increase. If the firm were to go bankrupt, its assets would be divided between fewer bonds and so the pay-off on each would be increased. If bankruptcy is avoided, each bond owner has an incentive to hold out for the entire savings in bankruptcy costs. Roe (1987) terms this the ‘buoying up effect’. The end-result of it is that standard debt cannot be repurchased when it is widely held.

It is not possible to use non-standard forms of debt contract which eliminate the free-rider problem and permit bankruptcy to be avoided. The reason is the restrictions placed on bond contracts by the Trust Indenture Act of 1939. Roe (1987) points out the purpose of these restrictions is to precipitate bankruptcy. The proponents of the law suggested that to prevent minority bondholders being cheated, it is necessary to ensure that any change in a bond’s terms occur in bankruptcy under the supervision of the court rather than before bankruptcy. Without such protection it was argued that equity holders could, for example, obtain a majority of a firm’s bonds and vote to expropriate the minority bondholders. In fact for this and other reasons (see Roe (1987) for a complete account) it was standard practice prior to 1939 to issue bonds which did not allow bankruptcy to be avoided. Thus not only is debt of this type not observed currently, it was rare historically.

The crucial assumption of the model is that the effect of bankruptcy is to cause a delay in the bankrupt firm’s investment decision. The basis for this is the operation of the bankruptcy laws. Under Chapter 11 of the current code a plan of reorganization must be agreed upon after bankruptcy occurs. This outlines a new financial structure for the firm and any necessary investments or sales of assets. The plan must be approved by two-thirds of each class of claimant. If it is rejected, an amended plan is produced and the process repeated until one is approved. Once the plan is agreed upon the judge must also give her consent to it. The firm can then emerge from bankruptcy and continue operations normally. Although it is possible for those in control of the firm to sell assets or raise money outside of the plan, the approval of the judge must be obtained and this is only given in special cases. Moreover, even if this is obtained it can be challenged by the security holders of the firm. If any claimants’ bargaining power is reduced by the change they will have an incentive to do this. (See Baird and Jackson (1985) for a full description of the operation of Chapter 11.)
The bankruptcy process is protracted and usually takes between two and three years to complete (see e.g. Eberhart, Moore, and Roenfeldt 1990 and Franks and Torous 1989, 1994). It is this lengthy procedure and the restrictions placed on the firm while it is going through it that is the basis for the assumption that the investment decisions of the bankrupt and solvent firm are not simultaneous.

When a firm’s debt is not widely held, then attempts to recapitalize and avoid bankruptcy may be successful. However, the negotiations to achieve such work-outs can be complex and drawn out because they involve bargaining over the division of the firm between security holders. To the extent that these negotiations delay investment the theory below remains valid when financial distress occurs even if actual bankruptcy is avoided.

Thus in the model, the effect of bankruptcy (or financial distress) at the end of a period is to delay the investment decision at the start of the next period. If both firms are in the same position at the end of period 1 so they are either both solvent or both bankrupt, they make their decisions simultaneously at the start of period 2, and so play a Nash game. However, if one firm is solvent and the other is bankrupt so its investment decision is delayed, they play a Stackelberg game with the solvent firm as leader and the bankrupt one as follower. Hence if one firm is bankrupt, it is at a strategic disadvantage.

3. Equilibrium

A perfect Nash equilibrium concept is used. The solution procedure involves solving backwards by first looking at the possibilities in period 2 and then analysing the decisions for period 1 given that the firms play equilibrium strategies in the period 2 sub-games. The equilibrium of the model depends crucially on the level of the fixed cost of capacity $\phi$. This determines the outcome of the Stackelberg subgame the firms play in period 2 if one of them goes bankrupt at the end of period 1. If there is no fixed cost, it is always profitable for the follower to remain in business at a reduced size. However, if the fixed cost is sufficiently large, it is optimal for the leader to expand and make the profits of the follower fall below the level of the fixed cost. This causes the follower to liquidate and the leader becomes a monopolist. The magnitude of the fixed cost therefore determines the marginal cost of bankruptcy and hence the form of equilibrium.

In Section 3.1 the case with no fixed costs is analysed. The opposite extreme where the fixed costs are large is considered in Section 3.2. Section 3.3 deals with the intermediate case. Finally, Section 3.4 contains a brief summary.

3.1. No fixed cost of capacity

In this subsection it is assumed

$$\phi = 0$$  \hspace{1cm} (7)
so there is only a marginal cost of capacity.

First, consider decisions for period 2, which are made at \( t = 1 \). Since the only effect of bankruptcy is to delay investment decisions in the subsequent period, it follows that bankruptcy in period 2 has no effect so that firms will always use as much debt as possible. To save having to repeatedly write a lump-sum subsidy component, \( \tau D_c \) in profit expressions, it is simplest to assume that in period 2, \( D = D_c = 0 \). At \( t = 1 \) the only decision firms are then concerned with is the level of investment \( I \) or equivalently their period 2 capacity \( q_2 \).

There are two possibilities at \( t = 1 \). Either both firms are solvent or both are bankrupt in which case they play a Nash game. Alternatively, one firm is solvent and the other is bankrupt in which case they play a Stackelberg game. Consider the Nash game first. Each firm chooses its second-period capacity to maximize expected profits given the other firm’s capacity:

\[
\max_{q_2} (1 - \tau)(\bar{\alpha} - \beta(q_2 + q_{2b}) - \mu_2 q_2,)
\]

taking \( q_{2b} \) as given.

The first-order condition implies

\[
q_{2b} = \frac{\bar{\alpha} - u}{2\beta} - \frac{q_{1b}}{2}.
\]

Similarly for \( q_{2s} \). Solving these simultaneously gives the standard results

\[
q_{1b} = \frac{\bar{\alpha} - \mu}{3\beta},
\]

\[
E \pi j = (1 - \tau) \frac{Z}{q^*},
\]

where \( E \pi j \) is the expected profit of \( j \) in period 2 and

\[
Z = \frac{(\bar{\alpha} - \mu)^2}{\beta}.
\]

The other possibility is that one firm, say \( k \), is solvent and the other, \( j \), is bankrupt. Firm \( k \) acts as leader and makes its investment decision while \( j \) is tied up in bankruptcy court. When \( j \) finally makes its investment decision it takes \( k \)'s as given. Firm \( k \) takes this into account when it makes its decision initially in the usual Stackelberg way. Firm \( j \)'s decision as follower is

\[
\max_{q_j} (1 - \tau)(\bar{\alpha} - \beta(q_j + q_{2b}) - \mu_2 q_j,)
\]

taking \( q_{2b} \) as given. Hence

\[
q_{1j} = \frac{\bar{\alpha} - u}{2\beta} - \frac{q_{1b}}{2}.
\]

Then firm \( k \)'s decision as leader is
\[
\max_{\epsilon_i} \left[ \alpha - \beta \left( \frac{\alpha - \mu}{2\beta} - \frac{q_{2i}}{2} \right) - \mu \right] q_{2i}.
\]

It follows
\[
q_{2i} = \frac{\alpha - \mu}{2\beta}
\]
\[
E \pi_{2i} = (1 - \tau) \frac{Z}{8}
\]
\[
q_{2} = \frac{\alpha - \mu}{4\beta}
\]
\[
E \pi_{2i} = (1 - \tau) \frac{Z}{16}.
\]

The effect of a single firm going bankrupt can be seen by comparing (8) and (9) with (12)–(15). The leader expands its capacity above that in the Nash case. The follower is at a strategic disadvantage, and as a result has a lower capacity than before. The bankrupt firm is worse off since its profits are reduced from \((1 - \tau)Z/9\) to \((1 - \tau)Z/16\). The solvent firm is made better off since its profits are increased from \((1 - \tau)Z/9\) to \((1 - \tau)Z/8\). Hence the delay in a firm's investment decision, although not costly in itself, means bankruptcy is undesirable.

Next consider the decisions of the firms at \(t = 0\). In period 1 it is assumed that \(D_j > 0\). The analysis will be concerned with interior solutions such that (5) does not bind.

In period 1 firm \(j\)'s expected profits are
\[
E \pi_j = (1 - \tau)(\alpha - \beta(q_j + q_k) - \mu)q_j + \tau D_j,
\]
where \(q_j\) and \(q_k\) are the firms' period 1 capacities and \(D_j\) is \(j\)'s period 1 debt.

The realization of the demand parameter \(\alpha\) at the end of period 1 determines whether or not the firms go bankrupt and hence the type of game played and the expected profits in period 2. In order to derive an expression for \(j\)'s expected profits in period 2 evaluated at \(t = 0\), it is necessary to define the level of demand, \(\alpha_j^*\), such that firm \(j\) goes bankrupt at the end of period 1. Using (1), (4), (6) with an equality, and (7),
\[
\alpha_j^* = \frac{D_j}{q_j} + \beta(q_j + q_k) - \frac{\tau}{1 - \tau} \mu.
\]

For values of \(\alpha\) above \(\alpha_j^*,\) firm \(j\) is solvent; for values below it is bankrupt. Similarly for \(k\). If \(\alpha_j^* \geq \alpha_k^*\) then
\[
E \pi_{2j} = (1 - \tau) \left[ \int_{\alpha_j}^{\alpha_j^*} \frac{Z}{9} f(\alpha) d\alpha + \int_{\alpha_j^*}^{\alpha_{j+2}} \frac{Z}{16} f(\alpha) d\alpha + \int_{\alpha_{j+2}}^{\alpha_{j+2}} \frac{Z}{9} f(\alpha) d\alpha \right].
\]
As illustrated in Figure 15.2, for $\alpha$ between $\alpha_i$ and $\alpha^*_i$ both firms go bankrupt and in period 2 they play a Nash game so $j$'s expected profits are $(1 - \tau)Z/9$. From $\alpha^*_i$ to $\alpha^*_j$ firm $j$ is bankrupt but $k$ is not. They play a Stackelberg game with $j$ as a follower so its expected profits are $(1 - \tau)Z/16$. For $\alpha$ above $\alpha^*_j$ they again play a Nash game.

For $\alpha^*_j \leq \alpha^*_i$,

$$E\pi_{2j} = (1 - \tau) \left[ \frac{\alpha^*}{9} f(\alpha) d\alpha + \frac{\alpha^+_j}{8} \frac{Z}{f(\alpha) d\alpha} + \frac{\alpha^+_j + 1}{9} \frac{Z}{f(\alpha) d\alpha} \right].$$ (19)

This is the same as when $\alpha^*_i \geq \alpha^*_i$ except that between $\alpha^*_j$ and $\alpha^*_i$, $j$ is the leader so its expected profits are $(1 - \tau)Z/8$.

Since the objective of the initial shareholders is to maximize the total value of the firm as explained in Section 2, firm $j$’s decision problem at $t = 0$ for choosing its period 1 capacity and debt is

$$\max \ E\pi_j = E\pi_{1j} + E\pi_{2j}$$

taking $q_j$ and $D_j$ as given.

The equilibria of interest are those where firms are at interior optima and so attention is restricted to these. Two types are possible. The first is symmetric with $\alpha^*_i = \alpha^*_j$ so both have the same capacity and debt. In the other $\alpha^*_i > \alpha^*_j$ (say) so that the equilibrium is asymmetric. One firm goes for the tax advantage of debt and the other goes for the equity advantage of being leader if demand is low.

In a symmetric equilibrium, each firm has the same level of debt and capacity. For an equilibrium to exist each firm must perceive that if it increases or reduces its debt or capacity, taking those of the other firm as given, then its profits will fall.

First consider firm $j$’s debt decision. Differentiating (17) gives:

$$\frac{\partial \alpha^*_j}{\partial D_j} = \frac{1}{q_j}$$

$$\frac{\partial \alpha^*_i}{\partial D_j} = 0.$$

Hence if $j$ increases its debt a small amount then $\alpha^*_j > \alpha^*_i$ and so (18) is the relevant expression for period 2’s profits. Thus
\[ \frac{\partial E \pi_j^+}{\partial D_j} = \tau - m_f f(\alpha_j^+ - \alpha_i) \frac{\partial \alpha_j^+}{\partial D_j} = \tau - m_f \frac{(\alpha_j^+ - \alpha_i)}{q_i}, \]  

where \( m_f = (1 - \tau) \frac{Z}{9} - (1 - \tau) \frac{Z}{16} = (1 - \tau) \frac{7}{144} Z. \)  

The first term, \( \tau \), in (20) is the tax benefit in period 1 from increasing debt. The second term is the loss in expected profits in period 2 arising from the fact that firm \( j \) will go bankrupt in more states. Instead of having expected profits of \((1 - \tau)Z/9\) when playing a Nash game, \( j \) only expects \((1 - \tau)Z/16\) as a Stackelberg follower and the difference between them is \((1 - \tau)(7/144)Z\).

In contrast if \( j \) reduces its debt then \( \alpha_j^+ < \alpha_j^* \) and (19) is the relevant expression for period 2 profits. Thus

\[ \frac{\partial E \pi_j^-}{\partial D_j} = \tau - m_L \frac{(\alpha_j^+ - \alpha_i)}{q_i}, \]  

where \( m_L = (1 - \tau) \frac{2}{144} Z. \)  

In this case the change in expected profits resulting from bankruptcy, \( m_L \), is the difference between those from being a Stackelberg leader, \((1 - \tau)Z/8\), and those in the Nash game, \((1 - \tau)Z/9\).

It can be seen

\[ m_f > m_L \]  

and

\[ \frac{\partial E \pi_j^+}{\partial D_j} - \frac{\partial E \pi_j^-}{\partial D_j} = (m_L - m_f) \frac{(\alpha_j^+ - \alpha_i)}{q_i} < 0 \]  

so that \( E \pi_j \) is kinked at \( \alpha_j^+ = \alpha_j^* \). Moreover, the kink is concave so that it can correspond to a maximum.

Changes in capacity can be similarly analysed. Since these affect the levels of demand the two firms go bankrupt at, there is again a kink. An increase results in a smaller absolute effect on profits than a reduction. This means the kink is concave so that it can correspond to a maximum. These changes of capacity and the other features of the symmetric equilibria of the model are considered in detail in the Appendix.

The second type of equilibrium that could exist is asymmetric with \( \alpha_j^+ \neq \alpha_k^+ \). This possibility is considered in detail in the Appendix. It is shown formally that there is an asymmetric equilibrium cannot exist when \( m_f > m_L \). To see why this is, suppose for the sake of discussion \( \alpha_j^+ > \alpha_k^+ \). In such an equilibrium firm \( j \) goes bankrupt in states that \( k \) doesn't. Consider firm \( j \)'s debt decision. If the firm increases its debt, \( \alpha_j^+ \) rises and its pay-off in the marginal states switches from the Nash level of \((1 - \tau)Z/9\) to the Stackelberg follower level of \((1 - \tau)Z/16\). A small reduction in debt leads to the opposite, namely a switch from the Nash level to the Stackelberg follower level. Thus
\[
\frac{\partial E \pi^*_j}{\partial D_j} = \frac{\partial E \pi^*_k}{\partial D_k} = \frac{\partial E \pi^*_i}{\partial D_i} = \tau - m_r \frac{(\alpha^*_r - \alpha_i)}{q_i} = 0. \tag{26}
\]

In contrast to the symmetric case, the right- and left-hand derivatives are equal and there is no kink. Similarly for firm \(k\). As it alters its debt level, the change is from the Nash to the Stackelberg leader outcome.

\[
\frac{\partial E \pi^*_i}{\partial D_i} = \frac{\partial E \pi^*_k}{\partial D_k} = \frac{\partial E \pi^*_r}{\partial D_r} = \tau - m_l \frac{(\alpha^*_l - \alpha_i)}{q_i} = 0. \tag{27}
\]

For an interior equilibrium to exist, the marginal costs of bankruptcy for the two firms must be equal

\[
m_r \frac{(\alpha^*_r - \alpha_i)}{q_i} = m_l \frac{(\alpha^*_l - \alpha_i)}{q_i}. \tag{28}
\]

Now \(m_r > m_l\) and \(\alpha^*_r > \alpha^*_l\). The only possibility if there is to be an equilibrium is that \(q_i\) is sufficiently larger than \(q_i\). It is shown in the Appendix that this is not possible.

The basic problem is that firm \(j\) not only has a high cost of bankruptcy \(m_r\), it also has a high marginal probability of going bankrupt, \((\alpha^*_r - \alpha_i)\). In contrast, \(k\) has both a low cost of bankruptcy, \(m_l\), and a low marginal probability of bankruptcy, \((\alpha^*_l - \alpha_i)\). Firm \(j\)'s marginal bankruptcy cost of debt is therefore higher than \(k\)'s. However, in equilibrium they must be equal since both firms receive the same marginal benefit from the tax shield provided by debt. Thus no asymmetric equilibrium can exist.

The results of this section are summarized by the following proposition.

**Proposition 1.** When \(\phi = 0\) the only type of equilibrium that exists is symmetric with \(\alpha^*_r = \alpha^*_l\). At each firm optimum profits are a non-differentiable function of capacity and debt.

### 3.2. High fixed costs of capacity

In this subsection the fixed cost of capacity is such that

\[
\frac{Z}{16} \leq \phi \leq \frac{Z}{9}. \tag{29}
\]

The right-hand inequality is necessary to ensure that both firms are viable in a Nash equilibrium. The significance of the left-hand inequality will be seen below.

As before, in order to find the equilibrium it is necessary to first consider investment decisions at \(t = 1\). If both are solvent or both are bankrupt, the firms play a Nash game similarly to (10) except with the inclusion of \(\phi\). Each firm’s optimal capacity is again as in (8) and their expected profits are

\[
E \pi_{ij} = (1 - \tau) \left( \frac{Z}{9} - \phi \right).
\]
When one firm, say $k$, is solvent and the other, $j$, is bankrupt they play a Stackelberg game with $k$ as leader and $j$ as follower. In this case if $k$ sets $q_{2k} = (\bar{\alpha} - \mu)/2\beta$ as in (12) then it follows that the highest profits $j$ can obtain with a positive output are

$$E\pi_{ij} = (1 - \tau) \left( \frac{Z}{16} - \phi \right).$$

The left hand inequality of (29) implies $E\pi_{ij} \leq 0$ and $j$'s optimal strategy in this case is to liquidate so that

$$q_{2j} = 0; E\pi_{ij} = 0.$$  

The $q_{2k}$ in (12) was derived on the assumption that $q_{2k} = (\bar{\alpha} - \mu)/4\beta$. However, it turns out that even if $q_{2j} = 0$ firm $k$ can do no better than $q_{2k} = (\bar{\alpha} - \mu)/2\beta$. This can easily be seen from the first-order condition for $k$'s problem as a monopolist. Thus $k$'s profits when it is the Stackelberg leader are

$$E\pi_{2k} = (1 - \tau) \left( \frac{Z}{4} - \phi \right).$$

The analysis of decisions at $t = 0$ is then similar to that in Section 3.1. Expected profits in the first period are as in (16) except for the inclusion of the fixed cost $\phi$. If $\alpha_j^* \equiv \alpha_j^*$, expected profits in the second period are

$$E\pi_{2j} = (1 - \tau) \left( \frac{Z}{9} - \phi \right) \left[ \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha + \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha \right].$$

Profits are of this form since for $\alpha$ between $\alpha_j^*$ and $\alpha_j^*$, $j$ is forced to liquidate and receives nothing in the second period. If $\alpha_j^* \equiv \alpha_j^*$, $j$ is the leader between $\alpha_j^*$ and $\alpha_j^*$ and receives $(1 - \tau)(Z/4 - \phi)$ so

$$E\pi_{2j} = (1 - \tau) \left[ \frac{Z}{9} - \phi \right] \left[ \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha + \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha \right]$$

$$+ \left( \frac{Z}{4} - \phi \right) \int_{\alpha_1}^{\alpha_2} f(\alpha) d\alpha.$$ 

For symmetric equilibria the partial derivatives of $E\pi_j$ with respect to $D_j$ are the same as in (25) and (27) except now

$$m_r = (1 - \tau) \left( \frac{4}{36} Z - \phi \right)$$

$$m_i = (1 - \tau) \left( \frac{5}{36} Z \right).$$

In contrast to (24),
\[ m_r < m_z. \]

There cannot be any symmetric equilibria since from (25) the kinks at firms’ optima are now convex and correspond to minima rather than maxima.

Next consider the possibility of an asymmetric equilibrium. The analysis is similar to that in Section 3.1 except now \( m_r \) and \( m_z \) are given by (30) and (31), respectively. Since it is possible for (28) to be satisfied an asymmetric equilibrium can exist. This is shown formally in the Appendix. The basic reason is that firm \( j \) which has a high marginal probability of bankruptcy now has a low marginal cost of bankruptcy; firm \( k \), which has a low marginal probability, has a high cost. It is therefore possible for both firms to have a marginal bankruptcy cost of debt equal to the marginal tax benefit.

The results of this subsection are summarized by the following proposition.

**Proposition 2.** If \( Z/16 \leq \phi \leq Z/9 \) the only type of equilibrium that exists is asymmetric. If \( \alpha^*_k > \alpha^*_j \) (say), then for realizations of \( \alpha \) in period 1 such that \( \alpha^*_k < \alpha < \alpha^*_j \), the bankrupt firm \( j \) liquidates at \( t = 1 \) and \( k \) becomes a monopolist in period 2.

### 3.3. Intermediate fixed costs of capacity

This subsection considers the case where

\[ 0 < \phi < \frac{Z}{16}. \tag{32} \]

With no fixed costs of capacity, a bankrupt firm acting as follower chooses a smaller capacity in the equilibrium at \( t = 1 \) than it would if it were not bankrupt. With a high fixed cost, a bankrupt firm acting as follower liquidates. With intermediate fixed costs, it is shown both of these are possible. Below a certain level \( \phi^*_j \) the follower stays in the market; above it, the firm leaves. Similarly, the equilibrium is symmetric for \( \phi \) below a certain level \( \phi^*_k > \phi^*_j \) and asymmetric above.

Consider the decision of a bankrupt firm, say \( j \), at \( t = 1 \) given that the solvent firm \( k \) has already chosen capacity \( q_{k \mu} \). It can either set \( q_{j \mu} = 0 \), or it can set it so that (11) is satisfied and profits are maximized given a positive capacity. Its profits in the latter case are

\[ E \pi_{2, j} = (1 - \tau) \left[ \beta \left( \frac{\bar{\alpha} - \mu}{2 \beta} - \frac{q_{2 \mu}}{2} \right) - \phi \right]. \]

Hence if

\[ \left( \frac{\bar{\alpha} - \mu}{\beta} - \frac{q_{2 \mu}}{2} \right) \preceq \frac{4 \phi}{\beta} \tag{33} \]

the firm liquidates; otherwise it stays in the market.

In Section 3.2 it was shown that if \( \phi > Z/16 \) and \( k \) chooses the monopoly capacity of \((\bar{\alpha} - \mu)/2\beta\), the follower liquidates. If \( \phi = 0 \), \( k \)’s optimal strategy is also to
choose a capacity of \((\bar{\alpha} - \mu)/2\beta\) but now \(j\) remains in the market. For intermediate \(\phi\), \(k\) has two possible courses of action. First, it could set \(q_k\) so that \((33)\) is satisfied. This pushes \(j\) out of the market and \(k\) becomes a monopolist. Alternatively, it could allow \(j\) to remain in the market. In order to determine which of these it should do, it is necessary to find the profitability of each.

If \(k\) is a monopolist its profits are
\[ E\pi_{jk} = (1 - \tau)[(\bar{\alpha} - \mu - \beta q_{jk})(q_{jk} - \phi)]. \]

For \(k\) to be able to make a positive profit \(q_{jk} < (\bar{\alpha} - \mu)/\beta\). Given this, \((33)\) is satisfied if
\[ q_{jk} \geq \frac{\bar{\alpha} - \mu}{\beta} - 2\left(\frac{\phi}{\beta}\right)^{1/2}. \quad (34) \]

When \(\phi < Z/16\) the right-hand side of this inequality is greater than \((\bar{\alpha} - \mu)/2\beta\). For such \(q_{jk}\), \(k\)'s profits as a monopolist are a decreasing function of capacity. Hence if \(k\) wants to force \(j\) to liquidate it should choose \(q_{jk}\) so that \((34)\) is satisfied with an equality. Its expected profits are then
\[ E\pi_{jk}^* = (1 - \tau)\left[2\left(\frac{\phi}{\beta}\right)^{1/2}(\bar{\alpha} - \mu) - 5\phi\right]. \]

If \(k\) allows \(j\) to remain in the market, it can be shown similarly to the case where \(\phi = 0\) that its optimal capacity is again \((\bar{\alpha} - \mu)/2\beta\) and its expected profits are
\[ E\pi_{jk}^* = (1 - \tau)\left[\frac{Z}{8} - \phi\right]. \]

It follows from these that for \(\phi\) satisfying \((32)\)
\[ E\pi_{jk}^* \equiv E\pi_{jk}^+ \text{ as } \phi = \phi_1^*, \]
where \(\phi_1^* = \frac{1}{4}\left(\frac{1}{2} - \frac{1}{8\phi^*}\right)^2 Z = 0.0054Z.\)

All this implies the following proposition.

**Proposition 3.** In the Stackelberg equilibrium at \(t = 1\), if \(0 < \phi < \phi_1^*\) the solvent firm chooses a capacity of \((\bar{\alpha} - \mu)/2\beta\) and the bankrupt firm stays in the market at a reduced size. For \(\phi_1^* < \phi < Z/16\) it is optimal for the solvent firm to expand capacity above \((\bar{\alpha} - \mu)/2\beta\) and force the bankrupt firm to liquidate.

The other question of interest concerns whether equilibrium at \(t = 0\) in this intermediate case is symmetric or asymmetric. It can be seen from the analyses of the previous subsections that this is determined by whether \(m_P\) is above or below \(m_1\). If \(m_P > m_1\), the equilibrium is symmetric; if \(m_P < m_1\), it is asymmetric.

For \(\phi < \phi_1^*\), \(m_P\) and \(m_1\) are as in \((21)\) and \((23)\) and the equilibria are symmetric. For \(\phi > \phi_1^*\),
\( m_r = (1 - \tau) \left( \frac{Z}{9} - \phi \right) \)

\( m_L = (1 - \tau) \left[ 2 \left( \frac{\phi}{\beta} \right)^{V_2} (\alpha - \mu) - \frac{Z}{9} - 4\phi \right] \).

Hence

\( m_r \leq m_L \) as \( \phi = \phi^* \),

where \( \phi^* \) is the value of \( \phi \) such that \( m_r = m_L \) or

\[ \frac{\phi^*}{\beta} \left( \alpha - \mu \right) - 2 \frac{Z}{9} - 3\phi^* = 0 \]

and

\[ \phi^* = 0.0198Z. \]

This gives the following.

**Proposition 4.** For \( 0 \leq \phi \leq \phi^* \) the equilibrium at \( t = 0 \) is symmetric and for \( \phi^* < \phi < Z/16 \) it is asymmetric.

### 3.4. Summary

The results of this section are summarized diagrammatically in Figure 15.3. The crucial determinant of the form of equilibrium is the fixed cost of capacity. For \( 0 \leq \phi \leq \phi^* \) a solvent firm acting as a Stackelberg leader at \( t = 1 \) chooses a capacity of \( (\alpha - \mu)/2\beta \). The bankrupt firm acting as follower is forced to reduce its size

**Fig. 15.3.** The relationship between fixed costs and equilibrium.
relative to the equilibrium if it were not bankrupt, but still remains in the market. For \( \phi^* < \phi < Z/16 \) the solvent firm expands its capacity above \((\alpha - \mu)/2\beta\) in order to force the bankrupt firm to liquidate. For \( Z/16 \leq \phi \leq Z/9 \) the solvent firm produces the monopoly output which is again \((\alpha - \mu)/2\beta\) and the bankrupt firm's best response is to liquidate.

As far as the equilibrium at \( t = 0 \) is concerned, it is symmetric so that firms have the same capacity, use the same amount of debt, and so on if \( 0 \leq \phi \leq \phi^* \). It is asymmetric if \( \phi^* < \phi < Z/9 \). One firm goes for the tax advantage of debt and the other goes for the equity advantage of being able to force the levered firm out of the market if demand turns out to be low.

4. Relationship to Empirical Evidence

The model developed above involves a number of simplifying assumptions such as there being two periods, linear demand, and density functions and so on. These are designed to allow theoretical results to be derived. Their disadvantage is that, as it stands, the model is an unsuitable basis for an empirical investigation. However, it is possible to demonstrate that the theory is broadly consistent with the two regularities mentioned initially. It is also consistent with a number of recent studies on product and capital market interactions.

The first observation was that most tax-paying firms use predominantly equity finance, debt ratios are typically around 20 to 30 per cent. In the model, firms making their capital structure decision in the first period, take into account that an increase in their debt level will cause them to go bankrupt in more states and as a result they will be forced to either partially or completely liquidate. Since liquidation costs can be significant, this implies the marginal bankruptcy costs of debt can be large. Even at low debt ratios the model therefore provides a rationale for why marginal bankruptcy costs might be large enough to offset the tax advantage of debt.

The second observation was that similar firms in the same industry often have different capital structures. In asymmetric equilibria of the model the two firms can have capital structures that differ significantly, which is consistent with this.

There have been a number of empirical studies considering the relationship between product markets and financial markets. Opler and Titman (1994) find that highly levered firms lose significant market share to their less levered rivals during economic downturns. This is consistent with the model above when firms are in bankruptcy and also if they delay investments when they are in financial distress. Chevalier (1995) and Phillips (1995) find that firms that have dramatically increased debt invest less than competitors with lower debt. Kovenock and Phillips (1997) find that after debt recapitalizations firms in highly concentrated industries are more likely to close plants and reduce investment while rival firms with less debt are less likely to do these things. Khanna and Tice (1998) discover that higher leverage makes firms less aggressive. These studies are thus broadly consistent with the results of the model presented here which suggests that more levered firms are less aggressive.
5. Concluding Remarks

This paper models bankruptcy as causing a delay in investment which in itself is costless. If the product market is imperfectly competitive, this delay puts a bankrupt firm at a strategic disadvantage. In particular, the bankrupt firm is either partially or completely pushed out of the market and forced to liquidate by the solvent one. In such cases the total costs of bankruptcy include these costs of liquidation. In equilibrium firms determine their optimal capital structure by weighing the tax advantages of debt against its bankruptcy and liquidation costs.

For many years the most widely taught theories of capital structure have involved a trade-off of one sort or another. In essence the model presented above is also a trade-off theory. It is more complex than traditional theories in that it suggests firms take account of their strategic position within an industry. The effect of this is that bankruptcy leads to liquidation. As a result the theory provides a rationale for why liquidation costs should be included in bankruptcy costs and hence why these may be large. It is also consistent with the fact that similar firms in the same industry often have such different capital structures.

APPENDIX

A1. Symmetric equilibria

In this subsection symmetric equilibria of the model where firms are at interior optima are analysed. It is shown that such equilibria can only exist when \( m_r > m_l \).

The effect of changes in debt on profits are given in (20) and (22). Next consider changes in capacity. Now,

\[
\frac{\partial \alpha^*_j}{\partial q_j} = \frac{D_j}{q_j} + \beta < \frac{\partial \alpha_j^*}{\partial q_j} = \beta.
\]

For increases in capacity (19) is therefore the relevant expression for \( E\pi_j^* \).

Using the fact that in symmetric equilibria \( \alpha^*_j = \alpha^*_j \) it can be shown

\[
\frac{\partial E\pi_j^*}{\partial q_j} = (1 - \tau)[\bar{\alpha} - \mu - \beta(2q_j + q_j)] + m_f(\alpha^*_j - \alpha) \frac{D_j}{q_j}.
\]

For reductions in capacity (18) is relevant, so

\[
\frac{\partial E\pi_j^*}{\partial q_j} = (1 - \tau)[\bar{\alpha} - \mu - \beta(2q_j + q_j)] + m_f(\alpha^*_j - \alpha) \frac{D_j}{q_j}.
\]

Hence

\[
\frac{\partial E\pi_j^*}{\partial q_j} - \frac{\partial E\pi_j^*}{\partial q_j} = (m_l - m_f)(\alpha^*_j - \alpha) \frac{D_j}{q_j}.
\]
Similarly for firm $k$. There is again a kink that can correspond to a maximum if it is concave.

It follows from (25) and (36) that an equilibrium can only exist if $m_r > m_L$. Otherwise the left-hand derivatives are less than the right-hand ones and the point could not be a maximum.

When firm $j$ increases its debt it goes bankrupt in more states than $k$ and becomes the follower in these. When it decreases its capacity firm $j$ also goes bankrupt in more states than $k$. Hence one equilibrium is where $\partial E\pi_j/\partial D_j$ and $\partial E\pi_j/\partial q_j$ are both zero. Solving (20) and (35) simultaneously and using the definition of $\alpha_j^*$ in (17) it can be shown

$$\alpha_j^* - \alpha_j = \frac{\tau(1 - \tau)(\alpha - \mu) + \tau\alpha_j + \tau^2\mu/(1 - \tau)}{(3 - \tau)\beta m_r - \tau^2}$$

$$q_j = \frac{m_r(\alpha_j^* - \alpha_j)}{\tau}$$

$$D_j = [\alpha_j^* - 2\beta q_j + \tau\mu/(1 - \tau)]q_j.$$ 

The other possibility involves $\partial E\pi_j/\partial D_j$ and $\partial E\pi_j/\partial q_j$ both equal to zero. The equilibrium is as in (37)–(39) except $m_r$ is replaced by $m_L$.

It can be seen that there always exist solutions to (37)–(39). However, these only correspond to an interior equilibrium if

$$0 \leq \alpha_j^* - \alpha_j \leq 1.$$ 

Otherwise equilibrium involves corner solutions to the firms' debt or capacity choice problems.

### A2. Asymmetric equilibria

In this subsection it is shown that asymmetric equilibria where firms are at interior optima can only exist when $m_r < m_L$. As in the text the case considered is $\alpha_j^* > \alpha_k^*$.

Consider first the case in Section 3.1 where $m_r > m_L$. The first-order conditions corresponding to debt choices are given by (26) and (27). For capacity choices they are

$$\frac{\partial E\pi_j}{\partial q_j} = (1 - \tau)(\alpha - \mu - \beta(2q_j + q_d)) + \beta m_r(\alpha_j^* - \alpha_j^*) + \left(\frac{D_j}{q_j}\right)m_r(\alpha_j^* - \alpha_j) = 0$$

$$\frac{\partial E\pi_k}{\partial q_k} = (1 - \tau)(\alpha - \mu - \beta(q_j + 2q_d)) + \beta m_r(\alpha_k^* - \alpha_k^*) + \left(\frac{D_j}{q_k}\right)m_r(\alpha_k^* - \alpha_k) = 0.$$ 

Solving all of these simultaneously and using the definitions of $\alpha_j^*$ and $\alpha_k^*$ it can be shown
\[ \alpha_i^* - \alpha_i = C \left( \beta m_F + \frac{\beta m_L}{\tau} - \tau \right) \]  
(41)

\[ \alpha_i^* - \alpha_i = C \left( \beta m_L + \frac{\beta m_F}{\tau} - \tau \right) \]  
(42)

\[ q_j = m_F \frac{(\alpha_j^* - \alpha_i)}{\tau} \]

\[ q_i = m_F \frac{(\alpha_i^* - \alpha_i)}{\tau} \]

\[ D_j = [\alpha_j^* - \beta(q_j + q_i) + \tau(\mu)\mu/(1 - \tau)]q_i \]

\[ D_i = [\alpha_i^* - \beta(q_j + q_i) + \tau(\mu)\mu/(1 - \tau)]q_i \]

where

\[ C = \frac{(1 - \tau)\alpha - [1 - \tau - \tau^2/(1 - \tau)]\mu + \tau\alpha_i}{(\tau - 2\beta m_F/\tau)(\tau - 2\beta m_L/\tau) - \beta^2(m_F - m_L/\tau)(m_L - m_F/\tau)}. \]

If \( C < 0 \) then it can be shown only corner solutions are possible. This follows from the fact that given (2) and \( \alpha_j^* > \alpha_i^* \) it is necessary for an interior solution that

\[ 0 < \alpha_i^* - \alpha_i < \alpha_j^* - \alpha_i \leq 1. \]  
(43)

This along with (41) and (42) implies that if \( C < 0 \), then

\[ \tau^2 > \beta m_F + \tau \beta m_L \]

so

\[ \tau > (\beta m_F)^{1/2}. \]  
(44)

Similarly

\[ \tau > (\beta m_L)^{1/2}. \]  
(45)

Also (26), (27), and (43) imply

\[ q_i \leq \frac{m_F}{\tau} \]  
(46)

\[ q_i \leq \frac{m_L}{\tau} \]  
(47)

Using (44)–(47) and the definitions of \( m_F, m_L, \) and \( Z \) it follows

\[ \alpha - \mu - \beta(q_j + 2q_i) > 0. \]

This and \( \alpha_j^* > \alpha_i^* \) mean that (40) cannot be satisfied since all the terms on the left-hand side are positive. Hence for interior asymmetric solutions it is necessary that
\[ C > 0 \] (48)

From (41) and (42)

\[ \alpha^*_p - \alpha^*_s = -C \left( \frac{1}{\tau} - 1 \right) (m_p - m_s). \] (49)

Using (48) \( 0 < \tau \leq 1 \) it follows that if \( m_p > m_s \), this cannot be satisfied and no asymmetric equilibrium can exist.

Next consider the case in Section 3.2 where \( m_p < m_s \). Here (49) means that an interior asymmetric equilibrium can exist provided \( \tau \) is sufficiently small so that \( C > 0 \) and \( \tau \leq \beta(m_s + m_p/\tau) \). For large \( \tau \) either one or both firms set \( D = D_s \) in equilibrium and are at a corner solution to their optimization problems.

References


