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The Prevention of Default

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Default is a major problem in many capital markets. In some cases it is so severe that it even prevents the formation of a market. The purpose of this paper is to consider the effect of this problem on contracts between lenders and borrowers.

It could be argued that default will not be a problem provided there are criminal penalties which are effectively enforced and sufficiently severe. However, a number of objections can be made to this. Firstly, the penalties may be very difficult to enforce. Payments on loans are due on a particular date which is known to both parties in advance. The borrower usually knows some time beforehand that he will default and has the opportunity to leave the country, or to cover his tracks and disappear in some other way. Although this problem of discovering defaulters may be offset to some extent by the imposition of severe penalties on those apprehended, this may be unacceptable to a society for humanitarian reasons. In an uncertain world default may be unintentional. If the expected penalties are high then people may be discouraged from taking loans, so that the formation of a capital market is again hampered or prevented.

For reasons such as these, criminal penalties are likely to be an ineffective way of solving the problem of default. Instead it seems reasonable that lenders will ensure that contracts are specified in such a way that the borrower will have an incentive to make the required payments.

Collateral is perhaps the most obvious way in which this can be achieved. In this case the borrower surrenders to the lender, the deeds of ownership of property worth at least as much as the total value of the loan repayments. If these are not met the lender acquires the property.

This way of preventing default would appear to be extremely widespread. However, it has the major defect that it limits access to loans to those who already have substantial wealth and who are therefore less likely to want to borrow. This raises the question of whether there is any way to prevent the default of people without collateral.

In cases where lenders have no security it seems unlikely that they will lend to people who have already defaulted since this is an indication that they are likely to do so again. If the rule that only those who have not defaulted can borrow without collateral is adopted by lenders then it may be possible to design profitable contracts which prevent default.

If a borrower knows that when he defaults he will in future be unable to gain

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access to the capital market, a contract will be enforceable provided the current payment is less than the value of future access to loans. What sort of contract does this restriction lead to?

To investigate this question the following simple model can be used. The capital $K$ which is lent and borrowed is taken to be perfectly durable and immobile so that it is only possible to default on the rent or interest. It can be thought of as something like land or a factory which has already been built. This is less restrictive than would appear at first sight since the theory can be applied to the more general situation where the capital is initially mobile, as for example in the case where money is lent, provided that the lender ensures the borrower turns it into an immobile form.

Time is divided into discrete production periods with inputs being supplied at the beginning and outputs produced at the end.

Output $Y$ is initially taken to depend only on the amount of capital used

$$Y = Y(K) \quad Y_K > 0; \quad Y_{KK} < 0$$ (1)

There is one generation of infinitely long-lived risk-neutral people.

The rate of interest which is determined by people’s time preference is $d$. The value $V$ in the current period, denoted period 1, of a stream of returns $r$ from time $t$ onwards is therefore

$$V = rD(t)$$ (2)

where

$$D(t) = \sum_{m=t}^{\infty} \frac{1}{(1 + d)^{m-1}}$$ (3)

It is assumed there is an infinitesimal transaction cost to default so that when the gains from defaulting are equal to the losses transactors do not default.

In order for a permanent contract, with a rent of $r$ and capital $K$, to be enforceable the borrower’s current payment must be less than or equal to the value of his future access to the capital market since if he defaults he will not be able to borrow any more. Hence

$$rK \leq [Y - rK]D(2)$$ (4)

Rearranging and using the fact that $D(1) = 1 + D(2)$ it follows that

$$rK \leq \frac{D(2)}{D(1)} Y$$ (5)

Profit maximization on the part of the lender requires that (5) be satisfied with an equality so that

$$rK = \sigma'Y$$ (6)

where

$$\sigma' = \frac{D(2)}{D(1)}$$ (7)
Thus the payment for the capital borrowed is equal to a share $\sigma'$ of the output.

In equilibrium the return to renting to those with collateral and to those without must be the same. If $r_c$ is the rental to those with collateral, the amount of capital $K'$ lent to those without will be given by

$$\frac{\sigma' Y(K')}{K'} = r_c \tag{8}$$

Borrowers without collateral thus have the amount they borrow specified by lenders in contrast to people with wealth who, provided they have enough collateral to cover their rent, are able to specify their own demands for capital.

In the case considered above where everybody is the same the contract that will be offered to those without collateral will involve a rent $r_c$ and $K'$ of capital. If output can be costlessly observed which is taken to be the case throughout this paper, an equivalent contract will involve the payment to the lender of a share $\sigma'$ of the output achieved with $K'$.

If the model is now extended to the case where abilities differ so that with a given amount of capital the outputs of borrowers are no longer the same, this equivalence of share and rent contracts no longer holds. The production function (1) is changed to

$$Y = Y(A, K) \quad Y_K > 0; \quad Y_{KK} < 0 \tag{1^*}$$

where $A$ is an index of the ability of the borrower. This is taken to have a distribution with density function $f(A)$ and upper and lower bounds $A_{\text{max}}$ and $A_{\text{min}}$ respectively. The total population is normalized at 1.

A higher value of $A$ is not necessarily taken to correspond to a higher value of output at all values of $K$. In other words production functions may cross so that information about individuals’ relative outputs with one amount of capital does not imply anything about their relative outputs with any other amount of capital. One consequence of this assumption is that screening will be extremely costly since learning somebody’s production function will involve observing output at many values of $K$.

If abilities are unknown by both borrowers and lenders and can only be observed at great cost so that screening is ruled out, everybody without collateral will be offered the same uniform contract. It can be shown that whereas with one ability group either a share or a rent agreement could be used, with many abilities profit maximization requires that a share contract is used. If a uniform rent is charged then for those groups such that $rK < \sigma' Y(A, K)$ the contract will not be profit maximizing and for those such that $rK > \sigma' Y(A, K)$ borrowers will default and the yield will be zero. Share contracts extract as much as possible from each borrower without precipitating default. This can be seen formally in the following way.

For given $K$, let $\phi(Y)$ be the payment schedule from borrower to lender. This includes renting as a special case when $\phi(Y) = b$ where $b$ is a constant. As in (4), in order to prevent default it is necessary that

$$\phi(Y) \leq [Y - \phi(Y)]D(2) \tag{9}$$
Rearranging and assuming lenders are profit maximizers as before gives

$$\phi(Y) = \sigma' Y$$

(10)

Lenders have the alternative of renting to borrowers with collateral. For low levels of output the total value of payments from a share contract may be less than if the borrower defaults and the capital is lent out in the second and subsequent periods to somebody with collateral at a rent \( r_c \). Thus if \( K \) is the amount of capital lent then for \( Y \) such that

$$\frac{\sigma' Y}{K} D(1) < r_c D(2)$$

(11)

it is better for lenders to discontinue the contract hence causing borrowers to default. Rearranging (11) this condition simplifies to

$$Y < r_c K$$

(12)

The payment schedule should therefore specify that the borrower must produce at least \( r_c K \) in order for the contract to continue. The optimal schedule is then

$$\phi(Y) = \sigma' Y \quad \text{for} \quad Y \geq r_c K$$

(13)

Anybody producing less than \( r_c K \) will default since in future he will be unable to borrow because of his low output. The convention of only lending to people who have not defaulted is then rational and hence endogenous since the only people who default are those who aren't sufficiently productive to produce \( r_c K \) and it is not worth lending to them.

As in (8) the amount of capital lent under the contract is determined by the requirement that borrowers be indifferent between lending to those with collateral and to those without. In the case where lenders are risk neutral this means that the expected returns must be the same. Hence \( K' \) is given by

$$\frac{\sigma' E^*}{K'} D(1) + \alpha r_c D(2) = r_c D(1)$$

(14)

where

$$E^* = \int_{A^*}^{A_{\max}} Y(A, K') f(A) \, dA$$

(15)

$$\alpha = \int_{A_{\min}}^{A^*} f(A) \, dA$$

(16)

and \( A^* \) is defined by

$$Y(A^*, K') = r_c K'$$

(17)

In the derivation of (13) as the optimal contract it was assumed that neither lenders nor borrowers had any information concerning the ability of the latter. Whereas it is perhaps reasonable to suppose that lenders have no direct information it is not so plausible that borrowers have none. If there is an asymmetry of information the analysis is greatly complicated because there then also exists
an adverse selection problem. Nevertheless it can be shown (see Allen [1]) that the main conclusions remain valid provided borrowers are sufficiently uncertain concerning their production possibilities. This is the case because of the conflict of interest between borrowers and lenders concerning default.

It may only be marginal for a borrower whether to default or not since in both cases he receives something: if he makes the current payment he can borrow and earn profits in future periods; if he defaults he keeps the payment but cannot subsequently gain access to the capital market. In the same situation there can be a much greater difference for the lender between defaulting and not defaulting since in one case he receives a full payment but in the other, nothing at all. When considering alternatives the borrower attaches the wrong values as far as the lender is concerned to possibilities of default. Lenders will therefore ignore borrowers’ attempts to convey information concerning their abilities and will specify the contract in the same way as if borrowers had no information.

So far only those with no wealth for collateral have been considered; it is possible to extend the theory above to those who only have a small amount of wealth.

The value of capital as collateral $V_c$ is given by

$$V_c = r_c D(2)$$  \hfill (18)

since if the borrower defaults the lender is only able to start obtaining a return in the period after the current one. Hence given ownership of capital $K_0$ it is possible to borrow $K_B$ with collateral at rent $r_c$ where

$$K_B = K_0 D(2)$$  \hfill (19)

It has been shown above that for those without any collateral at all the lender’s profit maximizing contract is a pure share contract. For those who have a small amount of collateral $K_0$, $K_B$ may also be small so that borrowers would like to borrow more than their collateral allows them. In this case it can be shown that the optimal financial structure involves a mixture of equity and fixed interest debt, with the debt/equity ratio depending on the borrower’s wealth.

Given the likely asymmetry of information postulated above, the borrower has a better idea of what his output will be than the lender. If the borrower expects the return on lending to himself to be greater than $r_c$ he will not only wish to use his own capital but will also want to rent as much as possible at $r_c$ since this will increase his profits. Similarly if he expects the yield to be less than $r_c$ he would be better off renting his own capital to somebody with collateral and not borrowing any at $r_c$. However, it will not be possible to do this since it is an implicit signal that the borrower expects a return less than $r_c$ and he will find it difficult to obtain loans. Given wealth $K_0$, people must lend this to themselves and also use it as collateral to rent $K_B$ at the fixed rate $r_c$. As before, the payment schedule $\phi$ for the capital borrowed without collateral must then satisfy

$$\phi(Y) \leq [Y - r_c K_B - \phi(Y)] D(2)$$  \hfill (20)

Rearranging and taking an equality

$$\phi(Y) = \sigma'(Y - r_c K_B)$$  \hfill (21)
The lender should again specify a lower bound on output at the point where it is more profitable to discontinue the contract, precipitate the borrower's default and then rent out in subsequent periods to people with collateral. This level $Y_L$ is given by

$$\sigma'(Y_L - r_c K_B) \frac{1}{K - K_B} D(1) = r_c D(2)$$

(22)

where $K$ is the total capital used.

This simplifies as before so that the optimal schedule is

$$\phi(Y) = \sigma'(Y - r_c K_B) \quad \text{for} \quad Y \geq r_c K$$

(23)

The total amount of capital lent is again determined by the requirement that lenders be indifferent between contracts

$$\frac{\sigma' E^*}{K' - K_B} D(1) + ar_c D(2) = r_c D(1)$$

(24)

The debt/equity ratio is $K_B/(K' - K_B)$. Assuming $f(A)$ is the same for all wealth levels it can be shown by implicitly differentiating (24) and using the fact that the average product is greater than the marginal product that

$$0 < \frac{K_B}{K'} \frac{\partial K'}{\partial K_B} < 1.$$  

(25)

It follows directly from the first inequality and (19) that as $K_0$ and hence $K_B$ increase, the total amount that can be borrowed goes up. Also since

$$\frac{\partial [K_B/(K' - K_B)]}{\partial K_B} = \frac{K'}{(K' - K_B)^2} \left(1 - \frac{K_B}{K'} \frac{\partial K'}{\partial K_B}\right)$$

(26)

it follows from the second inequality that the debt/equity ratio increases as $K_0$ and $K_B$ increase.

Allen [1] contains a number of other extensions and relaxations of assumptions such as the effect of there being some criminal penalties for default. Although with such changes the value of $\sigma'$ can differ and the optimal contract may involve fixed elements, the analysis remains essentially similar.

In conclusion, it has been argued that default is a serious problem in capital markets, which cannot be solved effectively using criminal penalties. Instead it is likely that where possible collateral will be required for loans in order for lenders to be sure of receiving payment. If the convention is adopted of only lending to those who have not defaulted there may be profitable contracts which allow people to borrow without collateral. The lender's profit maximizing contract in this case is a share contract with a lower bound on output. If the borrower owns a small amount of collateral a mixed debt equity loan is optimal with the debt-equity ratio increasing with the wealth of the borrower.

REFERENCES