Abstract

I study optimal housing and portfolio choice under stochastic inflation and real interest rates. Renters allocate financial wealth to stocks and bonds with different maturities. Homeowners also choose the mortgage type. I show that hedge demands and financial constraints vary over an investor’s lifetime, giving rise to a pronounced life-cycle pattern in the optimal housing, stock, bond, and mortgage choice. Young homeowners take an adjustable-rate mortgage (ARM) and invest financial wealth predominantly in stocks. Later in the life cycle bonds play an important role, mainly as a hedge against changing real interest rates and house prices. Fairly risk-tolerant homeowners still prefer an ARM, while more risk-averse investors rather choose a combination of an ARM and a fixed-rate mortgage.

JEL classification: G11, E43

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1 Introduction

For many investors the house is their largest asset, and the mortgage on the house is their largest liability. Both the house and the associated mortgage are likely to have a major impact on the optimal financial portfolio choice. This paper shows that besides stocks, bonds and mortgages play an important role in a homeowner’s financial portfolio. Together, the bonds and the mortgage determine the duration of the overall portfolio, which is important for hedging real interest rate risk. In addition, the bonds and the mortgage may provide a partial hedge against house price changes. I show that hedge demands and financial constraints vary over an investor’s lifetime, giving rise to a pronounced life-cycle pattern in the optimal housing, stock, bond, and mortgage choice.

This paper combines the main features of two recent strands in the portfolio choice literature. Papers in the first strand investigate life-cycle portfolio choice taking into account the role of housing.\(^1\) Cocco (2005) investigates the joint decision on owner-occupied housing and portfolio choice. Yao and Zhang (2005a,b) also model the housing tenure choice.\(^2\) These papers restrict the financial asset menu to stocks and cash, and do not consider the choice between an adjustable-rate and a fixed-rate mortgage. I extend these papers by adding bonds with different maturities to the asset menu, studying mortgage choice, and modelling the interaction of the housing return with financial asset returns in a more sophisticated manner. The second strand illustrates the importance of bonds for a long-term investor. Examples are Brennan and Xia (2002) and Campbell and Viceira (2001).\(^3\) Both papers use a two-factor model for the nominal interest rate similar to mine. A long-term investor holds bonds not only to exploit the risk premium, but also to hedge changes in the investment opportunity set. My paper extends these papers to a life-cycle setting with risky housing and labor income. In addition, I take into account the housing tenure, house size, and mortgage choice.

I consider the following model. An investor receives stochastic, exogenous labor income until retirement and derives utility from both housing and other goods consumption. In-

\(^{1}\)Brueckner (1997) and Flavin and Yamashita (2002) focus on the housing and financial portfolio choice in a static, one-period, mean-variance setting.

\(^{2}\)Hu (2005) investigates housing and portfolio choice in a five-period model. Cauley, Pavlov, and Schwartz (2005) assume a fixed housing position, and study a model where homeowners can sell a fractional interest in their house.

\(^{3}\)Sangvinatsos and Wachter (2005) and Koijen, Nijman, and Werker (2006) extend these studies by allowing for time variation in risk premia. Koijen, Nijman, and Werker (2006) also take into account labor income, but in their terminal wealth utility specification they abstract from the empirically-observed, hump-shaped age pattern.
vestors dynamically decide on their housing tenure, house size, financial portfolio, and other goods consumption. For homeowners the house not only provides housing services, but also entails a risky investment. An investor can change her tenure and house size only at a transaction cost. This cost is larger when moving to an owner-occupied house than when moving to a rental house. Renters choose how to allocate financial wealth to stocks, bonds with different maturities, and cash. Financial positions can be adjusted without transaction cost. Negative positions are precluded. Homeowners also choose the mortgage type and size. A homeowner may take a mortgage loan up to the market value of the house minus a down payment. I allow for an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a combination of the two (hybrid mortgage). A homeowner can adjust her mortgage type and size at zero cost, as is typically the case for a home line of credit. The ARM is modelled as a negative cash position, and the FRM as a negative position in a long-term bond.

For the asset price dynamics I extend the Brennan and Xia (2002) model with a house price and labor income process. Nominal bonds are priced by a two-factor model for the term structure of interest rates. I use expected inflation and real interest rate as factors. In contrast to a one-factor model, this model provides a rationale for holding nominal bonds with different maturities. Importantly, it also allows me to investigate the implications of different types of mortgages. I also model unexpected inflation, house price risk, labor income risk and stock market risk, leading to a total of six sources of uncertainty. This structure enables me to realistically examine the interaction between different asset prices. The parameter values for these price dynamics are largely based on estimates by Van Hemert, De Jong, and Driessen (2005), who use US data. In accordance with other papers, we find a faster mean-reversion in the real interest rate than in the expected inflation rate. As I show, this implies that a portfolio consisting of a long position in a short-term bond and a short position in a long-term bond can be constructed with the property that it has a negative exposure to real interest rate shocks and a zero exposure to expected inflation rate shocks. An investor who desires to hedge real interest rate shocks can create this hedge portfolio by choosing the appropriate mortgage, even when negative positions in other financial assets are precluded.

The main results of the paper can be summarized as follows. The motivation to hold risky assets varies over an investor’s lifetime, giving rise to a clear life-cycle pattern in her optimal house, stock, bond and mortgage choice. An investor starts adult life with little

\[\text{Source: Brennan and Xia (2002) or Campbell and Viceira (2001).}\]
financial wealth and large human capital, making her severely borrowing constrained. The investor starts out renting the house she lives in. Over time more labor income is earned and the investor starts to save for the down payment on an owner-occupied house. In this period she becomes less borrowing constrained, but is still very short-sale constrained. Taking into account her large human capital, the investor chooses an almost 100% stock allocation in order to exploit the equity premium, which is set at 4% in my analysis.

Per-period, out-of-pocket housing costs for a given house size are smaller when owning than when renting. This makes the investor so eager to buy her first house that the move from a rental to an owner-occupied house often involves moving to a smaller house, for which she is just able to pay the required down payment. The young homeowner optimally chooses an ARM of maximum size, irrespective of risk aversion. This allows a homeowner to exploit the risk premium on stocks and bonds.

As a homeowner builds up more financial wealth, she typically decides to move to a bigger owner-occupied house. With the larger physical (financial plus housing) capital and smaller human capital, the desire to take risk and exploit risk premia decreases, while the desire to hedge against falling real interest rates becomes more important. Initially a homeowner chooses a long-term bond for this hedge. Long-term bonds also have substantial exposure to expected inflation risk and investors capture the associated risk premium. When approaching retirement age the allocation starts to shift towards short-term bonds which have smaller exposure to expected inflation shocks. A fairly aggressive homeowner will still hold a considerable amount of long-term bonds and stocks at retirement. A more risk-averse homeowner, who is mostly concerned with hedging real interest rate risk, will almost completely shift to short-term bonds. Moreover, to further improve the effectiveness of the real interest rate hedge, she desires to short-sell the long-term bond. The optimal mortgage for this more risk-averse homeowner consequently changes from a pure ARM to a hybrid mortgage, modelled as a short position in both cash and a long-term bond.

Towards the end of her lifetime the investor sells her house and starts renting again. This enables her to consume all her wealth, including the down payment on the previously owned house. In anticipation of this sell, the investor adjusts her financial portfolio to hedge against house price falls.

The above analysis enables me to explain two empirical stylized facts in the US. First it rationalizes why young investors take more frequently an ARM. Young homeowners have large human capital and therefore a leveraged desire to exploit the risk premium in their
financial wealth. The ARM provides this leverage. Older homeowners are more concerned with adverse shifts in the real interest rate they earn on their accumulated capital. I show that an FRM, in conjunction with a position in short term bonds, allows them to hedge against falling real interest rates. Second, many investors simultaneously hold both a long position in fixed-income securities, e.g. by holding bonds in their pension account, and a short position in fixed-income securities by having an FRM on their house. Provided that the maturity of the FRM is larger than that of the long position in bonds, this long-short position facilitates the hedging of real interest rate risk.

In addition to the above-mentioned papers, this paper also relates to Campbell and Cocco (2003). In this paper the choice between an FRM and an ARM involves a trade off between what they refer to as wealth and income risk. An FRM has a variable real value, leading to wealth risk. An ARM has an almost fixed real value, but has, in their set up, short-term variability in real payments, leading to income risk. My mortgage analysis differs from Campbell and Cocco (2003) in several important ways. Campbell and Cocco (2003) do not consider stocks and bonds, and assume all other financial wealth is invested in cash. In contrast, I consider mortgage choice as part of the overall financial portfolio choice. While Campbell and Cocco (2003) incorporate persistent shocks to the expected inflation only, I allow for persistent shocks in the real interest rate as well. Together the bonds and mortgage determine the duration of the overall portfolio, which is important for hedging real interest rate risk. Even though there is no income risk of the above kind in my model, these considerations make the choice between an ARM and an FRM interesting in my set up. Moreover, in contrast to Campbell and Cocco (2003), I allow for a housing tenure and house size choice, which enables me to study mortgage choice in a broader context.

Finally this paper relates to Van Hemert, De Jong, and Driessen (2005), who study a homeowner’s optimal portfolio choice assuming (i) utility of terminal wealth, (ii) no labor income, (iii) fixed housing investment. Similar to this paper, we use a two-factor model to describe bond prices and model an ARM (FRM) as a short position in cash (a long-term bond). In contrast to Van Hemert, De Jong and Driessen (2005), I use a life-cycle setting with stochastic labor income and find a pronounced life-cycle pattern in optimal choices. Moreover, I allow for a housing tenure and house size choice.

The structure of this paper is as follows. Section 2 presents the model. Section 3 discusses the estimation of the model parameters. Section 4 contains the main results, and section 5 provides additional analyses. Section 6 concludes.
2 The economic model

I study optimal financial planning for an investor from time 0 to time $T = 60$ years, corresponding to age 20 to 80. The investor dynamically chooses (i) housing tenure, (ii) house size, (iii) financial portfolio, and (iv) consumption. I interpreted the house size as a one-dimensional representation of the overall quality of the house. When the investor decides to move, i.e. change house size or housing tenure, transaction costs are incurred.

2.1 Preferences

The investor derives utility from housing services and other goods consumption, $c$. The real price of consumption goods is chosen to be the numeraire and the real price of a unit of housing is denoted $q$. I denote the house size at time $t$ by $H_t$. Following Cocco (2005), and Yao and Zhang (2005a) preferences over housing and other goods consumption are represented by the Cobb-Douglas function

$$U_t = \int_t^T \beta^{s-t} u(c_s, H_s) ds,$$

(1)

$$u(c, H) = \frac{(c^{1-\psi} H^\psi)^{1-\gamma}}{1-\gamma},$$

(2)

where $U_t$ is lifetime utility evaluated at time $t$, time $T$ is the time of death which is assumed to be known in advance, $u$ is the Bernoulli utility function, $\beta$ is the subjective discount rate, $\gamma$ is the coefficient of relative risk aversion, and $\psi$ is the relative preference for housing consumption.\footnote{In contrast, Lustig and Van Nieuwerburgh (2005), and Piazzesi, Schneider, and Tuzel (2006) use the more general constant elasticity of substitution (CES) utility function in their studies on the role of housing in asset pricing. Piazzesi, Schneider, and Tuzel (2006) estimate a value for the intratemporal elasticity parameter only slightly above one; the value that corresponds to the special case of Cobb-Douglas preferences. To enhance comparison with the more-related papers of Cocco (2005), and Yao and Zhang (2005a), I use Cobb-Douglas preferences, even though either utility specification would be computationally feasible.} In the base case I abstract from a bequest motive. Instead, I will investigate this in section 5.\footnote{The empirical evidence for a strong, intentional bequest motive is mixed. See for example Hurd (1989) and Bernheim (1991) for negative and positive evidence respectively.}

The intratemporal elasticity of substitution between housing and other goods consumption equals one, which is a well-known property of the Cobb-Douglas function. However, the intertemporal elasticity of substitution is governed by $1/\gamma$. In this paper I focus attention to investors with a stronger desire to smooth consumption than the log investor.
(1/\gamma < 1), or, in terms of willingness to take risk, investors more risk averse than the log investor (\gamma > 1). We have 1/\gamma < 1 \Leftrightarrow u_{cH} < 0. Suppose that frictions in housing market causes the investor to live in a small house in period one and to move to a larger house in period two. With an intertemporal elasticity of substitution smaller than one, the investor optimally spends more on other goods consumption while in the small house in period one, at the cost of the spending on other goods in period two.

2.2 Asset price dynamics

I consider an economy with six sources of uncertainty represented by innovations in six Brownian motions. I assume the investor takes price processes as given. Furthermore, I assume that the risk premia on the sources of uncertainty are constant. Financial asset and house prices are determined by the dynamics of the first five sources of uncertainty. For these dynamics I use the setup of Van Hemert, De Jong and Driessen (2005), who in turn extend Brennan and Xia (2002) with an additional source of uncertainty to capture house price risk. The five variables that determine asset prices are: nominal stock return \( S \), instantaneous real interest rate \( r \), instantaneous expected inflation rate \( \pi \), nominal house price \( Q \), and the price level \( \Pi \). The equations driving these variables are given by

\[
dS/S = [R_f + \sigma_S \lambda_S] dt + \sigma_S dz_S, \tag{3}
\]

\[
dr = \kappa (\bar{r} - r) dt + \sigma_r dz_r, \tag{4}
\]

\[
d\pi = \alpha (\bar{\pi} - \pi) dt + \sigma_\pi dz_\pi, \tag{5}
\]

\[
dQ/Q = [R_f + \sigma_Q \lambda_Q - r^{imp}] dt + \sigma_Q dz_Q, \tag{6}
\]

\[
d\Pi/\Pi = \pi dt + \sigma_\Pi dz_\Pi, \tag{7}
\]

where \( R_f \) is the return on the nominal risk free asset (referred to as cash), \( \lambda_S \) and \( \lambda_Q \) are nominal risk premia, \( r^{imp} \) is the imputed rent, \( dz \)'s are changes in standard Brownian motions \( z \) and the \( \sigma \)'s capture the volatility of the processes. The imputed rent term in (6) represents the benefits from the housing services, as measured by the market.\(^7\) We can

\(^7\)The imputed rent is the value the market attaches to the net benefits provided by the house. Equation (6) is simply the first-order condition for the house price being equal to the present value of all future imputed rents. Put differently, the infinitesimal expected total return, as set by the market, equals \( E[dQ/Q + r^{imp} dt] = [R_f + \sigma_Q \lambda_Q] dt \), where the right-hand side captures the familiar compensation for the time value of money, \( R_f \), and risk, \( \sigma_Q \lambda_Q \).
orthogonalize (6) and (7) to

\[
\frac{dQ}{Q} = [R_f + \theta' \lambda - r_{\text{imp}}]\,dt + \theta'\,dz, \tag{8}
\]
\[
\frac{d\Pi}{\Pi} = \pi dt + \xi'\,dz, \tag{9}
\]

with \(\theta = (\theta_S, \theta_r, \theta_\pi, \theta_v, 0)'\), \(\xi = (\xi_S, \xi_r, \xi_\pi, \xi_u)'\), \(\lambda = (\lambda_S, \lambda_r, \lambda_\pi, \lambda_u)'\) and \(z = (z_S, z_r, z_\pi, z_v, z_u)\), such that \(dz_v\) is orthogonal to \(dz_S, dz_r, dz_\pi\), and \(dz_u\) is orthogonal to \(dz_S, dz_r, dz_\pi\) and \(dz_v\). Defining the covariance matrix of \(dz\)

\[
\rho = \begin{pmatrix}
\rho_{S,r,\pi} & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \tag{10}
\]

we have \(\sigma_Q^2 = \theta'\rho\theta\) and \(\sigma_\Pi^2 = \xi'\rho\xi\).

Brennan and Xia (2002) show that the nominal price at time \(t\) of a discount bond with a $1 nominal payoff maturing at time \(T\), denoted as \(P_{tT}\), satisfies

\[
dP_{tT}/P_{tT} = [R_f - B\sigma_r\lambda_r - C\sigma_\pi\lambda_\pi]\,dt - B\sigma_r\,dz_r - C\sigma_\pi\,dz_\pi, \tag{11a}
\]
\[
B(T-t) = \kappa^{-1}\left(1 - e^{-\kappa(T-t)}\right), \tag{11b}
\]
\[
C(T-t) = \alpha^{-1}\left(1 - e^{-\alpha(T-t)}\right), \tag{11c}
\]

where \(B\) and \(C\) are functions of the time to maturity \(T - t\). The return processes for bonds with different maturities differ only in their loadings on \(dz_r\) and \(dz_\pi\). When there are no constraints on position size, any desired combination of loadings on \(dz_r\) and \(dz_\pi\) can be accomplished by positions in any two bonds with different maturities.

The real returns on stocks, bonds, and the house can easily be obtained using the dynamics for the price level, equation (9), and applying Ito’s lemma. I use uppercase letters for nominal variables and the corresponding small case letter for their real counterpart. The nominal interest rate satisfies \(R_f = r + \pi - \xi'\lambda\), and for example the real return on stocks is given by

\[
ds/s = \left[r + \sigma_S (\lambda_S - \xi_S) - \xi' (\lambda - \rho\xi)\right]\,dt + \sigma_S\,dz_S - \xi'\,dz. \tag{12}
\]

In equation (12) \(\lambda_S - \xi_S\) and \(\lambda - \rho\xi\) are the real risk premia associated with the Brownian motions \(z_S\) and \(z\) respectively.
2.3 Investment Opportunity Set

I denote the real market value of the house by $w^H = qH$, where $q$ was the real house price and $H$ the house size. Real financial wealth is denoted by $w^F$. The menu of available financial assets consists of stocks, 3-year bonds, 10-year bonds, and cash. The two bonds are assumed to be zero-coupon bonds. The allocation to these four assets is denoted by $x = (x^{stock}, x^{3ybond}, x^{10ybond}, x^{cash})$. We have

$$w^F = x^{stock} + x^{3ybond} + x^{10ybond} + x^{cash}$$  \hspace{1cm} (13)

The infinitesimal real financial return, $r^F(x)$, for a given asset allocation $x$ is given by

$$r^F(x) = [r + (\sigma_F(x) - \xi^f) (\lambda - \rho \xi)] dt + [\sigma_F(x) - \xi^f] d\xi,$$  \hspace{1cm} (14)

where $\sigma_F(x)$ is the vector of risk exposures for the nominal financial return. Using (3) and (11a), it is given by

$$\sigma_F(x) = \begin{pmatrix}
  x^{stocks} \sigma_S \\
  [-x^{3ybond} B(3) - x^{10ybond} B(10)] \sigma_r \\
  [-x^{3ybond} C(3) - x^{10ybond} C(10)] \sigma_\pi \\
  0 \\
  0
\end{pmatrix}.$$  \hspace{1cm} (15)

Notice that the real financial return is independent of the expected inflation rate, $\pi$. The same holds for the real return on the house, which implies that the real investment opportunity set in my model is independent of the prevailing expected inflation rate.

I assume that renters cannot take short positions in any of the financial assets, i.e. we have

for renters:

$$x^{stock} \geq 0$$  \hspace{1cm} (16a)
$$x^{3ybond} \geq 0$$  \hspace{1cm} (16b)
$$x^{10ybond} \geq 0$$  \hspace{1cm} (16c)
$$x^{cash} \geq 0$$  \hspace{1cm} (16d)

Equations (13) and (16a)-(16d) imply that a renter cannot borrow against human capital,
\[ w^F \geq 0. \]

Homeowners can take a mortgage loan up to a fraction \( 1 - \delta \) of the market value of the house, where \( \delta \) is the minimum down payment fraction. They can use the proceeds to consume or to invest in stocks, bonds, and cash. I include the (negative) market value of the mortgage in my definition of financial wealth, which therefore can become negative. Total (financial plus housing) wealth, however, cannot be less than the minimum down payment of \( \delta \) times the value of the house.

A homeowner can choose between an adjustable-rate mortgage (ARM), a fixed-rate mortgage (FRM), and a hybrid mortgage which is a combination of an ARM and an FRM. I model an ARM (FRM) as a short position in cash (10-year bond), i.e. the (relative) increase in the market value of the loan equals the return on cash (10-year bond). Doing so, I implicitly make two simplifying assumptions. First, I abstract from the prepayment option that is associated with FRMs in some countries, most notably the US. Prepayment behavior by US investors is far from optimal, giving rise to a large literature on mortgage-backed securities pricing. Second, I equate the borrowing and lending rate. On an after-tax basis, the differential between these rates, the mortgage premium, should equal the default premium plus a profit margin for the lender minus government subsidization on mortgage debt. Defaults do not occur in my model, so this should be taken out of the equation. In do not explicitly model the government subsidization, like income tax deductibility of mortgage interest payments, and take the short-cut assumption that the profit margin and government subsidies are exactly offsetting.

Following Cocco (2005) I assume that a homeowner can costlessly adjust the mortgage, as is typically the case for a home line of credit. Since I also allow for hybrid mortgages, the investor basically can take a negative cash and 10-year bond position, each and added up not to exceed \( (1 - \delta) \) times the market value of the house. We have

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8 Practitioners use the term *hybrid* mortgage for a slightly different product: a mortgage with a fixed rate for the first few years and a floating rate thereafter.

9 Allowing for the prepayment option would make the outstanding mortgage balance a state variable. This would make the numerical evaluation of the model intractable.

10 For an early reference, see e.g. Schwartz and Torous (1989).

11 In the US, subsidies are not only provided directly through income tax deductions of mortgage interest rate payments, but also indirectly through the Government-Sponsored Enterprises (GSEs), like the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FMAC), that hold or guarantee around 40\% of all US residential mortgages (Frame and Wall (2002)).

12 Focussing on the subsidy through the deductibility of mortgage interest payment from taxable income, Amromin, Huang and Sialm (2006) argue that a significant number of US households can perform what they call a tax arbitrage by cutting on their mortgage payments and investing the proceeds in tax-deferred accounts.
for homeowners:

\[
\begin{align*}
    x^\text{stock} & \geq 0 \\
    x^\text{3ybond} & \geq 0 \\
    x^\text{10ybond} & \geq -(1 - \delta) w^H \\
    x^\text{cash} & \geq -(1 - \delta) w^H \\
    x^\text{10ybond} + x^\text{cash} & \geq -(1 - \delta) w^H
\end{align*}
\] (17a)-(17e)

Equations (13) and (17a)-(17e) imply that a homeowner can borrow up to market value of the house minus the down payment, i.e. \( w^F \geq -(1 - \delta) w^H \). Comparing constraints (16a)-(16d) for a renter with constraints (17a)-(17e) for a homeowner, notice that owner-occupied housing alleviates the short-sale constraint on the 10-year bond and cash.

2.4 Housing costs

The per-period, out-of-pocket housing expenses are a fraction \( \zeta(I) \) of the market value of the house. It depends on the housing tenure indicator variable \( I \), which is defined to be one for an investor who is currently owning and zero for renters. For both homeowners and renters I assume a constant value over time, and denote it by \( \zeta^{\text{own}} \) and \( \zeta^{\text{rent}} \) respectively, i.e.

\[
\zeta(I) = I \zeta^{\text{own}} + (1 - I) \zeta^{\text{rent}}.
\] (18)

For homeowners the housing expenses represent a maintenance cost, incurred to keep the house at a constant quality. For renters the housing expenses represent the rental cost, which should cover the landlord’s maintenance cost, opportunity cost, and possibly a profit margin. Therefore we typically have \( \zeta^{\text{rent}} > \zeta^{\text{own}} \).

When moving, the investor pays (receives) the increase (decrease) in owner-occupied housing wealth. In addition, a one-time transaction cost is incurred. I consider it a move when the investor decides to change housing tenure, house size, or both. I define the new housing tenure indicator variable, \( I^{\text{new}} \), as being one (zero) when the investor moves to an owner-occupied (a rental) house. The new house size is denoted by \( H^{\text{new}} \) and the total costs are

\[
m = I q^H - I^{\text{new}} q H^{\text{new}} + q H^{\text{new}} \nu(I^{\text{new}})
\] (19)

The first two terms add up to minus the change in owner-occupied housing wealth. The
third term represents the transaction cost. It equals a fraction $\nu^{\text{own}} (\nu^{\text{rent}})$ of the market value of the new house when the investor moves to an owner-occupied (a rental) house, i.e.

$$\nu^n = \nu^{\text{new}} \nu^{\text{own}} + (1 - \nu^{\text{new}}) \nu^{\text{rent}}$$

(20)

Typically we have larger moving costs for the case the investor buys the new house, i.e. $\nu^{\text{own}} > \nu^{\text{rent}}$. When there is no move we have $m = 0$.

### 2.5 Labor income

The sixth source of uncertainty captures labor income risk, which I assume is exogenous. Real labor income, $l$, is assumed to be subject to permanent shocks only. In addition, real labor income has a deterministic component $g(t) dt$ that captures the hump-shaped pattern of labor income. We have

$$\frac{dl}{l} = g(t) dt + \sigma_l dz_l \quad \text{for } t \leq 45$$

(21)

$$l = 0 \quad \text{for } t > 45$$

(22)

where time $t = 45$ corresponds to the retirement age of 65. After retirement labor income is assumed to be zero. That is, I study an investor who saves for her own retirement. Equivalently, I study the joint investment problem for an investor and a pension fund investing on the investor’s behalf, without separating these two parties explicitly. The advantage of this approach is that I do not need have to make simplifying assumptions on the dynamic asset allocation strategy of the pension fund, but rather provide advice to the pension fund how best to invest on behalf of its participants at different stages of life.

Labor income is assumed to be correlated with real house price innovations, but not with stocks, bonds, and the price level, i.e.

$$\frac{dl}{l} = g(t) dt + \rho_{q_l} \sigma_l \left[ \frac{\xi_u}{\sqrt{\xi_u^2 + \xi_v^2}} dz_v - \frac{\xi_v}{\sqrt{\xi_v^2 + \xi_u^2}} dz_u \right] + \sqrt{1 - \rho_{q_l}^2} \sigma_l dz_k \quad \text{for } t \leq 45$$

(23)

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13 Bodie, Merton and Samuelson (1992) show that endogenous labor income may increase the optimal risk taking in the financial portfolio.

14 Viceira (2001), Yao and Zhang (2005a) and Munk and Sørensen (2005) assume stochastic shocks to permanent labor income only as well. Cocco, Gomes and Maenhout (2005), Campbell and Cocco (2003) and Cocco (2005) also allow for transitory, individual labor income shocks.

15 The validity of this assumption depends on the specific investor at hand. A stock broker would be a classical counterexample where one would expect a non-zero correlation between labor income and stock returns.
with \( dz_k \) is the component of labor income shocks orthogonal to \( dz \), and with \( \rho_{ql} \) the correlation of labor income with real house price innovations. Notice that the term in square brackets is chosen such that the correlation with realised inflation, as given by equation (9), is zero.

2.6 Optimization problem

The investor maximizes lifetime utility (1) over real consumption, \( c \), portfolio choice, \( x \), and the new housing tenure and size, \( I^{\text{new}} \) and \( H^{\text{new}} \) (when she decides to move). This is subject to the following real financial wealth dynamics

\[
dw^F = w^F r^F (x) + w^H \zeta(I) \, dt + m + ldt - cdt
\]

The five right-hand-side terms in (24) represent the financial portfolio return (as defined in (14) and (15)), out-of-pocket, per-period housing expenses (as defined in (18)), moving costs (as defined in (19), and (20)), labor income (as defined in (21), (22), and (23)) and consumption, respectively. Real housing wealth is given by \( w^H = qH \), where the dynamics of the real house price, \( q \), are determined by the nominal house price dynamics, as given in (8), deflated by price level changes, as given in (9).

At any time the investor’s asset allocation is subject to restrictions (16a)-(16d) when currently renting and (17a)-(17e) when currently owning. Finally, through identity (13), the restrictions on the asset positions put a lower bound on real financial wealth, which puts an upper bound on real consumption, \( c \), in (24). By moving to a smaller house, investors can always attain strict positive consumption, and hence the problem is well defined.

2.7 Solution method

The state variables for the investor’s investment problem are given by the current housing tenure, \( I \), financial wealth, \( w^F \), housing wealth, \( w^H \), labor income, \( l \), real house price, \( q \), real interest rate, \( r \), and time \( t \). From the financial wealth dynamics provided above, it is immediate that a strategy for other goods and housing consumption \( \{c_t, H_t\}_t^T \) is sustainable starting in state \( (I, w^F, w^H, l, q, r, t) \) if and only if the consumption strategy \( \{vc_t, vH_t\}_t^T \) is sustainable starting in state \( (I, vw^F, vw^H, vl, q, r, t) \) for any \( v > 0 \). Similarly, a consumption strategy \( \{c_t, H_t\}_t^T \) is sustainable starting in state \( (I, w^F, w^H, l, q, r, t) \) if and only if the consumption strategy \( \{ct, vH_t\}_t^T \) is sustainable starting in state \( (I, w^F, w^H, l, q/v, r, t) \) for
any $v > 0$. Exploiting that lifetime utility, as given in equation (1), is homogeneous of degree $1 - \gamma$ in $\{c_t, H_t\}_t^T$ and homogeneous of degree $(1 - \gamma) \psi$ in $\{H_t\}_t^T$, we can write the indirect utility function as,

$$
\max_{\{c_t, x_t, (H^{\text{new}}_t), (I^{\text{new}}_t)\}} U_t = \left( \frac{w}{q^\psi} \right)^{1-\gamma} J(I, y, h, r, t),
$$

(25)

$$
w \equiv w^F + w^H,
$$

(26)

$$
h \equiv w^H/w,
$$

(27)

$$
y \equiv w/l,
$$

(28)

where $w$ is total real wealth, $h$ is the housing-to-wealth ratio, $y$ is the wealth-to-income ratio, and $J$ is the part of the indirect utility function that cannot be determined in closed form. So, I am able to separate out two state variables, which makes the model tractable.

Terminal utility $J(I_T, y_T, h_T, r_T, T)$ is known. It is zero in the base case where the investor derives no utility from leaving a bequest. To determine indirect utility, $J$, for $t < T$, I choose a grid over $y, h, r,$ and $t$, and use backward induction. I denote the time step size by $\Delta t$. For every point on the grid I solve

$$
\left( \frac{w_t}{q^\psi_t} \right)^{1-\gamma} J(I_t, y_t, h_t, r_t, t) = \max_{c_t, x_t, (H^{\text{new}}_t), (I^{\text{new}}_t)} \left( \frac{c_t^{1-\psi} (H^{\text{new}}_t)^\psi}{1-\gamma} \right)^{1-\gamma} \Delta t
$$

(29)

$$
+ E_t \left[ \beta^\Delta t \left( \frac{w_{t+\Delta t}}{q^\psi_{t+\Delta t}} \right)^{1-\gamma} J(I_{t+\Delta t}, y_{t+\Delta t}, h_{t+\Delta t}, r_{t+\Delta t}, t + dt) \right],
$$

subject to the above-mentioned constraints on the choice variables and dynamics for the state variables. Without loss of generality I can normalize $w_t = 1$ and $q_t = 1$. Further details on the solution method are provided in appendix A.

### 3 Calibration

The calibrated parameter values for the asset price dynamics and labor income process are presented in table 1. The values for the real interest, expected inflation, and unexpected inflation rate are taken from Van Hemert, De Jong and Driessen (2005). In this paper we use quarterly US data on nominal interest rates and inflation from 1973Q1 to 2003Q4 and back
out the real interest and expected inflation rate with a Kalman filter technique. Notice that the mean reversion in the real interest rate, as governed by $\kappa$, is around 1.1 years. This is much faster than the mean reversion in the expected inflation rate, as governed by $\alpha$, which is around 12.6 years. This is in accordance with e.g. Brennan and Xia (2002) and Campbell and Viceira (2001). This has important implications for the nominal bonds, which are priced by a two-factor model with the real interest and expected inflation rate as factors (see equations (11a)-(11c)). In particular, the speed of mean reversion in a factor determines the exposure of bond prices to shocks in the factor. Table 2 provides numerical estimates for this. Notice from this table that a portfolio consisting of a $1 position in a 3-year bond and a $-2.8/7.7 position in a 10-year bond has the property that it has a negative exposure to real interest and a zero exposure to expected inflation rate shocks. It is therefore the portfolio that hedges against negative shocks to the real interest rate without incurring expected inflation risk. In addition to the parameters provided by Van Hemert, De Jong and Driessen (2005), I set the nominal unexpected inflation premium, $\lambda_u$, equal to zero.

I calibrate the parameter governing stock and house price dynamics to quarterly US data from 1980Q2 to 2003Q4. For the stock data I use an index comprising all NYSE, AMEX and NASDAQ firms. For house price data I use a repeated-sales index for houses in Atlanta, Boston, Chicago and San Francisco. I have no data on market imputed rent, but for the financial asset allocation $\theta_v \lambda_v - \tilde{r}^{imp}$ and not $\lambda_v$ and $\tilde{r}^{imp}$ separately is relevant. I estimate $\theta_v \lambda_v - \tilde{r}^{imp}$ from the data and without loss of generality set $\tilde{r}^{imp}$ equal to the mean real interest rate $\bar{r}$. The nominal risk premium on the housing investment is slightly negative, $\sigma_Q \lambda_Q - \tilde{r}^{imp} = -0.53\%$, implying the average house price appreciation is slightly below the risk free rate. The low excess return on housing is at odds with the current popular perception that the housing investment is a sure bet with a high return. Himmelberg, Mayer, and Sinai (2005) discuss the up- and down-swings in US house prices and argue that the about one-third of the recent run-up in house prices merely reflects a return to the house price level before the down-swing in the early nineties. Notice that the average total excess return on the house, $\sigma_Q \lambda_Q$, is still positive as long as the imputed rent is larger than 0.53%.

Based on the house price index the annual house price change has a standard deviation

---

16 For more details I refer to Van Hemert, De Jong and Driessen (2005)
17 I would like to thank Kenneth R. French for making this data available at his website.
18 I would like to thank the Case-Shiller-Weiss company for providing us with this data.
19 Brunnermeier and Julliard (2006) argue that houses are subject to mispricing caused by investors suffering from money illusion.
of 2.67%. Case and Shiller (1989) argue that the standard deviation of individual house price changes are close to 15.00%, like individual stocks. Because price changes of different houses are far from perfectly correlated, aggregation leads to a considerable reduction of the variability. Since we are interested in the dynamics of an individual house, we correct this series by simply scaling house price shocks with a factor $15.00\% / 2.67\% = 5.6$ around its mean. I calculate correlations with house price innovation on a yearly instead of a quarterly basis to account for the effect that house prices may adjust slower to news than financial assets. Extending the calibration horizon beyond one year makes little difference. Nominal house price changes are found to be negatively correlated with real interest rate shocks and positively correlated with expected inflation shocks. The scaling of house prices might lead to coefficients of correlation with financial asset prices that are biased upwards in size. The financial portfolio best offsetting changes in the nominal house price under this assumption is short stocks, short 3-year bonds, long 10-year bonds and short cash.\footnote{This can be easily obtained by solving $\sigma_F(x) = -\theta$ for $x_{\text{stocks}}$, $x_{\text{3ybond}}$, $x_{\text{10ybond}}$, where $\sigma_F(x)$ is given in equation (15).} For an investor expecting to downsize her housing position this is the appropriate hedge portfolio. For an investor who is expecting to buy a (bigger) house in the near future the opposite position is needed. However, one should bear in mind that even with these upward biased correlations, it is just a partial hedge and that most of the housing risk is in fact idiosyncratic and unhedgeable. As an alternative parameterisation I consider zero correlations between housing and all other assets, as presented in parentheses in table 1.

I consider a horizon of $T = 60$ years, corresponding to age 20 to 80. The investor is assumed to retire at time $t = 45$. I follow Munk and Sørensen (2005) by adapting the estimated labor income profile of Cocco, Gomes and Maenhout (2005) to a continuous-time setting. The deterministic part of the change in labor income is given by

$$g(t) = b + 2c(t + 20) + 3d(t + 20)^2.$$ \hspace{1cm} (30)

where $t + 20$ is the age of the investor. Cocco, Gomes and Maenhout (2005) estimate $b$, $c$ and $d$ for three groups characterized by the highest level of education achieved: no high school, high school, and college. I focus on the high school group. I follow Munk and Sørensen (2005) and set the income rate volatility at $\sigma_I = 0.10$. Recall that post-retirement income is assumed to be zero. Cocco (2005) and Ortalo-Magné and Rady (2006) document a positive correlation between house price and labor income shocks. In his model, Cocco (2005) uses a perfect correlation between housing and aggregate labor income shocks (for tractability.
reasons) and imperfect correlation between housing and temporary labor income shocks. Yao and Zhang (2005a) use a correlation of 0.2, which I follow.\footnote{Spiegel (2001) illustrates in a general equilibrium model how house prices and local economic growth can be linked.}

Table 3 provides the other parameter values. For the risk aversion parameter I examine two values: $\gamma = 3$ for an aggressive and $\gamma = 9$ for a more risk-averse investor. For the parameter governing housing preferences I choose $\psi = 0.2$, which is the same as in Yao and Zhang (2005a). Cocco (2005) chooses $\psi = 0.1$. The subjective discount rate is set at $\beta = 0.96$. Following Yao and Zhang (2005a), the rent rate is $\zeta^{\text{rent}} = 6\%$, maintenance costs are $\zeta^{\text{own}} = 1.5\%$, transaction costs when moving to an owner-occupied house are $\nu^{\text{own}} = 6\%$ and the down payment on the house is $\delta = 20\%$, all as a percentage of the market value of the house. Cocco (2005) chooses 1\%, 8\% and 15\% for $m$, $\nu^{\text{own}}$ and $\delta$ respectively. Yao and Zhang (2005a) assume a zero transaction cost for moving to a rental house. Taking into account the cost of for example moving furniture and in-house painting, I consider a modest $\nu^{\text{rent}} = 1\%$ more reasonable.

4 Results

In this section I present the solution to the base case model presented in section 2 using the calibrated parameter values presented in section 3. The solution comprises the optimal housing tenure, house size, financial portfolio, and consumption choice, all conditional on the state of the world. The non-separable state variables for the problem are, the current housing tenure indicator variable, $I$, the wealth-to-income ratio, $y$, the housing-to-wealth ratio, $h$, the real interest rate, $r$, and time. The number of non-separable state variables exceeds the dimension of the world we live in, which makes it impossible to show the full solution in one graph. Instead I will illustrate the model implications in several graphs and tables. For the graphs I simulate paths for the non-separable state variables using derived optimal choices. Along with values for the non-separable state variables on a particular path, I obtain the values for the choice variables and separable state-variables. I will show results from age 20 to 80 for the mean investor, determined by averaging the state and choice variables over 10,000 (simulated) investors. The starting values are presented in table 4. The investor starts with $7,500 wealth. She rents a house worth $45,000 (wealth, $w$, times housing-to-wealth ratio, $h$). Her wage is $15,000 per year (wealth, $w$, divided by the wealth-to-income ratio, $y$). The real interest rate is at its long-term mean value of
\[ \bar{r} = 2.26\%. \] Time and the real house price are normalised at zero and one respectively.

Figure 1 shows consumption and house size for the mean investor with risk aversion parameter \( \gamma = 3 \). These are the two variables that directly enter the investor’s lifetime utility function, as presented in equation (1). Figure 2 shows the annualised move rate for the mean investor and the fraction of investors owning the house they live in. Young investors have large human capital and little wealth, which makes them borrowing constraint. Over time the investor’s wage increases and we see in figure 1 both housing and other goods consumption rise between age 20 and 25. In figure 2 we see that no investor owns in this phase of life. Investors have too little wealth saved to pay the down payment on a reasonable size house. Some investors move to a bigger rental house in this period though. Recall that moves are generated for endogenous reasons only in my model. Between ages 25 and 35 investors buy their first home. Most of the time this is a smaller home than the one they were renting just before. We can see this by the decline in house size in figure 1 or the many moves down around this age in figure 2. Owning involves lower out-of-pocket, per-period expenses than renting does. This makes investors eager to buy, even if they have not enough wealth to pay for the down payment on a house as big as the one they are renting. Around age 40 most investors own the house they live in. The mean house size rises until age 60 because only later in life investors have enough wealth to pay for the down payment on a house of the desired size. In this age category other goods consumption decreases slightly reflecting a decreasing desire to substitute for the initially small house. See equation (1) and the discussion below that for a further discussion on this. House size is then fairly constant until age 70, and starts decreasing after that. Since in the base case we assume no bequest motive, investors want to consume all their wealth before they die. Because of the compulsory down payment on the house, the investor optimally decreases house size (and therefore down payment) and eventually starts renting towards the end of her life. In figure 2 the move from an owner-occupied to a rental house is visible by the large moving rate around age 78. Because lower per-period, out-of-pocket housing expenses when owning, housing wealth is released fairly late in life. This causes consumption to be large in the last period of life. An additional reason for other goods consumption being large late in life is again the substitution motive, i.e. compensating for a smaller house in that phase of life.

Next I discuss the portfolio choice and wealth accumulation for the mean investor with risk aversion parameter \( \gamma = 3 \) and \( \gamma = 9 \), presented in figures 3 and 4 respectively. Portfolio shares add up to one. The (negative) mortgage position exactly cancels against the part of housing wealth that exceeds the dashed, horizontal line for the total portfolio share.
equals one. Consequently, net housing wealth is exactly the part of housing wealth that is underneath this dashed, horizontal line. I also plot total wealth accumulated.

In the first years investors have very little wealth compared to the value of their human capital. This creates a desire to leverage risk taking in the financial portfolio in order to reap the risk premium. Stocks have the highest risk premium in the presented model. Both the aggressive $\gamma = 3$ and the more risk-averse $\gamma = 9$ investor hold predominantly stocks when they are young. Investor cannot hedge against rent price (which is tied to house price) increases because this involves a negative 10-year bond position. See also the discussion in section 3 on the house price hedge portfolio implied by the calibrated parameter values. Labor income shocks cannot be hedged either with financial assets.

Between age 25 and 35 a house is purchased. Both the $\gamma = 3$ and the $\gamma = 9$ investor choose an adjustable-rate mortgage in this phase of life, reflecting the desire to leverage the risk exposure. The financial portfolio still consists mainly of stocks, but there is also a small holding of 10-year bonds. This 10-year bond position hedges against real interest changes. The investor prefers the 10-year bond to the 3-year bond for its larger risk premium associated with the larger exposure to expected inflation shocks. See table 2 for the exact exposures of these two bonds to real interest and expected inflation risk. Notice that the hedging demand is bigger for the more risk averse $\gamma = 9$ investor. As wealth is accumulated between age 40 and 65 and human capital is capitalised, the hedge demand against falling real interest rates increases and the desire for a leveraged stock exposure decreases. For the $\gamma = 3$ investor this results in increasing the 10-year bond holdings. In contrast, the $\gamma = 9$ investor gradually switches to 3-year bonds between age 55 and 65. The reason for this difference is twofold. First the more aggressive $\gamma = 3$ investor is more willing to bear the expected inflation risk of the 10-year bond and reap the associated risk premium. Second, the more aggressive $\gamma = 3$ investor has larger stock holdings, leaving her with less financial wealth to construct the hedge portfolio against falling interest rates, which in turn induces her to invest in the bond with the largest exposure to real interest rate, which is the 10-year bond (see table 2). Notice that the 10-year bond position for the $\gamma = 9$ investor becomes negative around the retirement age of 65. That is, no longer is a pure adjustable-rate mortgage optimal, but the investor rather holds a hybrid mortgage. The optimal mortgage choice at retirement is consistent with results presented in Van Hemert De Jong, and Driessen (2005), who abstract from labor income. In that paper we show that there is a large welfare loss when no hybrid mortgage is available and the investor has to choose either an ARM or an FRM. The composition of the bond portfolio before the sale
of the house around age 78 clearly differs from the composition of the bond portfolio after
the sale. This shows that part of the bond holdings towards the end of life is motivated by
hedging house price falls in anticipation of selling the house. Since I abstract from longevity
risk, the investor is able to exhaust her savings fully, as can be seen by the zero wealth at
age 80.

The above analysis provides interesting insights into two empirical stylized facts in
the US that are sometimes considered puzzling. First it rationalizes why on average more
young investors take ARMs.22 Young homeowners have large human capital and therefore
a leveraged desire to exploit the risk premium in their financial wealth. The ARM provides
this leverage. Older homeowners are more concerned with adverse shifts in the real interest
rate they earn on their accumulated capital. An FRM, in conjunction with a position in
short term bonds, allows them to hedge against falling real interest rates. Second, many
investors simultaneously hold both a long position in fixed-income securities, e.g. by holding
bonds in their pension account, and a short position in fixed-income securities by having
an FRM on their house. Above analysis, and the discussion of table 2, shows that such a
long-short position helps in hedging real interest rate risk without incurring much inflation
rate risk. That is, provided that the maturity of the FRM is larger than that of the long
position in bonds.

Next I briefly discuss how the stock allocation in above analysis compares to the empirical
evidence. The stock allocation in figures 3 and 4 is decreasing with age. The empirical
evidence for this is being debated. Agnew, Balduzzi, and Sunden (2003) argue that the age
pattern in the share allocated to stocks is hump shaped, peaking around age 50. Ameriks
and Zeldes (2004) argue that this pattern could be due to a cohort effect, and stress the
impossibility to disentangle age, time, and cohort effects, dubbed the identification problem.
Moreover, Ameriks and Zeldes argue that the pattern is flat conditional on stock market
participation. Most at odds with the empirical evidence on stocks allocation is the near
100% stock allocation at the start of life in figures 3 and 4. Gomes and Michaelides (2005)
show that a fixed stock market participation, a feature absent in my model, can bring their
model predictions on stock holdings close to observed values.

I now turn the attention to the impact of house size on optimal portfolio choice. Table
5 shows the optimal portfolio choice for a homeowner at the retirement age of 65 years for
different housing-to-wealth ratios. Again I consider both an aggressive $\gamma = 3$ (panel A) and
a more risk-averse $\gamma = 9$ (panel B) investor. The real interest rate and wealth-to-income

22See e.g. the opinion survey commissioned by the Consumer Federation of America (2004).
ratio are fixed and set at the mean value of the previous simulation exercise. That is, the real interest rate is set equal to its long-term mean, \( r = \bar{r} \), and the wealth-to-income ratio is set equal to \( y = 18 \) and 20 for the \( \gamma = 3 \) and \( \gamma = 9 \) investor respectively. I also report the utility loss \( UL \), measured as the certainty equivalent loss in (housing plus other goods) consumption. In this case the utility loss compares the utility in the optimal housing-to-wealth ratio, \( h^{\text{optimal}} \), to the utility in a particular housing-to-wealth ratio, \( h \), taking the values of the other state variables constant, i.e.

\[
UL = \left( \frac{J(I, h, y, r, t)}{J(I, h^{\text{optimal}}, y, r, t)} \right)^{\frac{1}{1-\gamma}} - 1.
\]  

(31)

At any time an investor has the possibility to move to the house size that is optimal given the values of the other state variables. However, because moving involves transaction costs, there is range for the housing-to-wealth ratio where the investor optimally chooses not to move. When the housing-to-wealth ratio is outside this range, she optimally chooses to move to a house that brings her inside the range again. Notice that the range is more narrow for the more risk-averse \( \gamma = 9 \) investor compared to the aggressive \( \gamma = 3 \) investor. The optimal housing-to-wealth ratio is lower for the \( \gamma = 9 \) investor. At retirement the investor is typically overexposed to house price risk, in the sense that the value of the house exceeds the present value of future housing consumption. This makes a more risk-averse investor less willing to own a large house.

For larger housing-to-wealth ratios the investor can take a larger mortgage. However, because of the required down payment, the investor has less wealth available to take long positions in financial assets. Recall that the size of the mortgage may not exceed \( 1 - \delta \) times the housing wealth at any time, not only at moments the investor adjusts her mortgage size. With less financial wealth available the investor tends to shift her bond portfolio to long-term bonds which have a larger absolute loading on real interest rate risk. Doing so she can maintain the appropriate hedge against changing interest rates. This results in the general tendency to increase the maturity of bond portfolio with the housing-to-wealth ratio. Stock allocation tends to be crowded out more for larger housing to wealth ratios, consistent with the empirical evidence.\(^{23}\) However, there is an additional, superposed, effect that blurs the picture somewhat. Investors act less risk averse close to the border of the no-move region than close to the optimal housing to wealth ratio.\(^{24}\) In panel A this is best visible by

\(^{23}\)See e.g. Heaton and Lucas (2000) and Kullmann and Siegel (2003).

\(^{24}\)See e.g. Grossman and Laroque (1990) for a study on optimal behavior in the presence of an illiquid asset like a house.
noticing that the 10-year bond allocation at the left border of the no-move region \((h = 0.2)\) is larger than at \(h = 0.3\) and \(0.4\). In panel B it is more clearly visible by noticing the slightly rising stock allocation near the right border of the no-move region \((h = 0.4\) vs. \(h = 0.5)\).

5 Sensitivity analyses and extensions

The modelling framework presented above allows for many possible sensitivity analyses and extensions. In this section I choose the following three: idiosyncratic house price risk, bequest motive, and suboptimal mortgage choice.

5.1 Idiosyncratic house price risk

In this subsection I investigate the portfolio choice when house price risk is idiosyncratic. There are three motivations to do so. First, there is likely to be considerably heterogeneity among investors with regard to the co-movement of their house price with financial assets and labor income. For example, Davidoff (2006) determines the coefficient of correlation between house prices and labor income in the US and finds a wide dispersion in this coefficient across industries and region. The analysis in this subsection helps us to identify to what extent and in what phase of life this heterogeneity in house price co-movement with other assets leads to heterogeneity in the optimal portfolio choice. Second, as discussed in section 3, the calibrated values for the correlation between house price and financial asset price changes reflect an upperbound because of the scaling technique used. The analysis here allows us to check the consequences of working with this upperbound. Third, comparing the portfolio choice for the \(\gamma = 3\) investor in the previous section (figure 3) with the portfolio choice with fully idiosyncratic house price risk here, allows us better to identify which part of the portfolio choice in figure 3 was driven by a house price hedge demand.

Figure 5 shows the portfolio choice and wealth accumulation for the mean \(\gamma = 3\) investor with idiosyncratic house price risk. Comparing this figure with figure 3, which shows the mean \(\gamma = 3\) investor with partially hedgeable house price risk, we see that investors takes into account they will downsize their housing wealth and eventually rent again during retirement. This creates a motive to hedge against falling house prices. As discussed in section 3 this calls for a long 10-year bond and a short 3-year bond position. Indeed the 10-year bond allocation is lower and the 3-year bond allocation larger in figure 5 where there is no additional hedge demand because of the idiosyncratic risk of the housing
investment. This hedge demand is also detectable in figures 3 and 4, where the 10-year bond position decreases and the 3-year bond position increases once the house is sold. Further comparing Figures 3 and 5 we see little differences between the two graphs earlier in life, indicating that hedging house price risk with financial assets does not play a very important role in the accumulation phase of life.\(^{25}\)

### 5.2 Bequest motive

In this subsection I study optimal portfolio choice for an investor who derives utility from leaving terminal wealth \(w_T\) as bequest. Denoting the utility derived from leaving a bequest by \(V\), lifetime utility is then given by

\[
U_t = \int_t^T \beta^{s-t} \left( \frac{1-\psi}{1-\gamma} \right)^{1-\gamma} ds + \beta^{T-t} V(w_T, q_T, r_T). \tag{32}
\]

I follow Yao and Zhang (2005a) and assume that the beneficiaries purchase a real annuity, which in turn is used to pay for 15 years of other goods consumption and housing rental costs. The payout rate for a $1 wealth annuitized wealth will depend on the real interest rate at time \(T\), and is denoted by \(K(r_T)\). An analytical expression for the payout rate is provided in appendix B.

Assuming the same Cobb-Douglas preferences over housing consumption to other goods for the bequest function, a fraction \(\psi\) is spent on rental costs and a fraction \(1-\psi\) on other goods consumption. As before denoting the rental costs as a fraction of the house price by \(\zeta_{rent}\), the utility from bequest is given by

\[
V(w_T, q_T, r_T) = \int_T^{T+15} \beta^{s-T} \left( \frac{[(1-\psi) K(r_T) w_T]^{1-\psi} \left[ \psi K w_T / (\zeta_{rent} q_T) \right]^\psi \right)^{1-\gamma} ds
\]

\[
= \left( \frac{w_T}{q_T^\psi} \right)^{1-\gamma} \left( \frac{K(r_T)}{(\zeta_{rent})^\psi} \right)^{1-\gamma} \frac{\beta^{15-1} - 1}{\ln \beta} \left( 1 - (1-\psi)^{1-\psi} \psi \right)^{1-\gamma} \tag{33}
\]

Notice that the same homogeneity properties apply as before, and therefore that the indirect utility is again of the functional form given in (25).

---

\(^{25}\)As Sinai and Souleles (2005) notice, owning itself may provide a hedge against future housing costs risk, which in turn might influence the tenure decision. Pelizzon and Weber (2006) estimate the difference between the value of the house and the present value of future housing consumption costs using Italian household portfolio data.
Figure 6 shows the portfolio choice and wealth accumulation for the mean $\gamma = 3$ investor. Comparing it with figure 3, which shows the mean $\gamma = 3$ investor without bequest motive, we notice three main differences. First, with a bequest motive the investor accumulates considerably more wealth. Moreover, it doesn’t run down its accumulated wealth after the retirement age of 65. Second, the investor doesn’t switch to renting again at the end of life. The main motivation to switch to renting without a bequest motive was that this enabled the investor to consume all of the housing equity. This motivation is clearly less relevant with bequest motive. Third, with a bequest motive the portfolio choice after retirement remains fairly constant, in contrast to situation without a bequest motive. Notice that before retirement the portfolio choice with and without bequest motive are very similar.

5.3 Suboptimal mortgage choice

The share of newly originated mortgages that is of the FRM type in the US has varied between 30% and 90% over the last two decades, see e.g. Campbell (2006). In some other countries, e.g. the UK, ARMs are far more common. In the above analysis investors chose more often the ARM, see figures 3 and 4, mainly because financing your house with an FRM is more expensive because you pay the risk premium on long term bonds. In a speech to the Credit Union National Association, Alan Greenspan (2004), former chairman of the Federal Reserve, criticises the preference of US households for FRMs: "... the traditional fixed-rate mortgage might be an expensive method of financing a home.", and "Fixed-rate mortgages seem unduly expensive to households in other countries.". The prepayment option, abstracted from in the analysis above, is not likely to make the FRM cheaper than the ARM. First, an investor pays in advance a premium to compensate the lender for writing the prepayment option. Second, households are suboptimal in exercising this prepayment option, see e.g. Schwartz and Torous (1989).

In this subsection I study an investor who, suboptimally, always chooses an FRM to finance her house. Figure 7 shows the portfolio choice and wealth for the mean $\gamma = 3$ investor. Comparing this figure to the case of optimal mortgage choice, presented in figure 3, we notice substantial differences. Investors start owner-occupying their house only later in life, they pay down the mortgage quicker, and accumulate less wealth during their working life. The utility loss, evaluated at age 20, expressed as certainty equivalent loss in (housing plus other goods) consumption is as large as $-2.84\%$. I consider this result supporting the critical comments expressed by Alan Greenspan.
6 Conclusion

I investigated housing and portfolio choice under stochastic inflation and real interest rates. Both housing and financial portfolio choice show a clear life-cycle pattern. When just entering the labor force an investor is very borrowing constrained and prefers to rent. After having saved for the down payment she buys a house and enjoys lower per-period, out-of-pocket housing expenses. At the very end of life she starts renting again, which enables her to consume the down payment on the previously owned house.

Young homeowners choose an ARM and invest financial wealth mainly in stocks. At retirement bonds play an important role, mainly as hedge against changes in the real interest rate. The mean-reversion in the real interest rate is faster than in the expected inflation rate. This implies that the sensitivity to real interest rate shocks relative to the sensitivity to expected inflation rate shocks will be higher for short-term bonds than for long-term bonds. The absolute sensitivity to both shocks is higher for a long-term bond though. An aggressive investor, who is still very financially constrained at retirement, mainly chooses 10-year bonds for the hedge against real interest rates and continues to finance her house with an ARM. A more risk-averse investor prefers short-term bonds to hedge real interest changes and switches to a hybrid mortgage, being a combination of an ARM and an FRM.

The choice on mortgage type is first and foremost a choice between different interest rate products and, as I showed, should therefore be analysed in conjunction with the other financial decisions. However, there might be additional effects from which I abstracted in the current analysis. First, the payments on an FRM are higher than on an ARM for a normal, upward-sloping, nominal interest rate curve. In countries where mortgage payments are tax deductible this might result in larger tax benefits for homeowners financing their house with an FRM. Second, holders of an FRM might have a prepayment option, which in turn will give rise to a premium on the mortgage payments. Third, in reality some homeowners default on their mortgage. Incorporating these effects is a challenging avenue for future research.

Appendix A: solution method

As a timing convention, I assume that state variables in period $t$ are defined after the labor income of period $t$ (equalling $I_t \Delta t$) is received, but before period-$t$ consumption, per-period out-of-pocket housing, and moving costs are incurred. This puts an attainable lower bound
on the wealth-to-income ratio of \( y_t \geq l_t \Delta t / l_t = \Delta t \). I choose a 60-point grid on \([\Delta t, 50]\) for \( y \). By choosing the lower limit of the grid for \( y \) equal to the attainable lower bound \( \Delta t \), I don’t have to extrapolate on the lower end of the grid, which greatly enhances the precision of the numerical procedure. The upper limit of the grid for \( y \) is set large enough so that it is never reached when simulating paths for the state variables. The grid for \( h \) is chosen differently for homeowners and renters, i.e. depends on \( I \). For homeowner I choose \([0, 1/\delta]\) with step size 0.1. The upper limit \( 1/\delta \) is the largest attainable value considering the down payment requirement. I will choose \( \delta = 0.2 \), hence \( 1/\delta = 5 \). For homeowners I choose \([0, 25]\) with step size 0.5. For the real interest rate \( r \) I choose a 5-point grid around the unconditional mean, \( \bar{r} \). I choose an interval length of one month, i.e. \( \Delta t = 1/12 \).

For the optimization I combine two methods, (i) the Direction Set (Powell’s) Method in multidimensions, which uses a numerically-determined derivative of the objective function to determine the search direction, and (ii) the Downhill Simplex Method in multidimensions (Nelder and Mead (1965)), which doesn’t use any derivative information. See Press et al. (1992) for an excellent introduction to these methods. I find that these methods are complimentary in their ability to find the global maximum in different states of the world, and combining them works very well. I also consider many different starting values to initiate the optimization algorithm, including the optimal values for the choice variables one period ahead.

The conditional expectation in (29) is approximated using a 3-point Gaussian quadrature for each of the six sources of uncertainty represented by the six Brownian motions (Tauchen and Hussey (1991)). To determine \( J(I_{t+\Delta t}, y_{t+\Delta t}, h_{t+\Delta t}, r_{t+\Delta t}, t+\Delta t) \) for values of \( y_{t+\Delta t}, h_{t+\Delta t} \) and \( r_{t+\Delta t} \) not on the grid, I use linear interpolation on \( \log(J) \). The possibility to move causes the indirect utility to be kinked on the boundaries of the no-move region, making cubic spline interpolation techniques less suitable. At a given point in time \( t \), the solution to (29) for different values for \( I, y, h, \) and \( r \) can be determined simultaneously, making parallel computation techniques feasible. The calculations are performed on 60 parallel-connected computers with two 3.4 \( GHz \) processors each. I use the \( C \) programming language. One run takes approximately 10 hours.
Appendix B: payout rate real annuity

Denoting the real price at time $T$ for an inflation-indexed bond with a $\$1$ (real) payoff at time $T + \tau$ by $i_{T,T+\tau}$, the total payout rate of a $\$1$ annuitized is given by

$$K(r_T) = \left( \int_T^{T+R} i_{T,T+\tau} d\tau \right)^{-1}$$  \hspace{1cm} (34)

The real pricing kernel that prices financial securities, $M_t$, has dynamics

$$\frac{dM}{M} = -rdt + (\xi - \rho^{-1}\lambda)'dz$$  \hspace{1cm} (35)

Now

$$i_{T,T+\tau} = \mathbb{E}_T \left[ \frac{M_{T+\tau}}{M_T} \right]$$

$$= \exp \left\{ \mathbb{E}_T \left[ \ln \left( \frac{M_{T+\tau}}{M_T} \right) \right] + \frac{1}{2} \text{Var}_T \left[ \ln \left( \frac{M_{T+\tau}}{M_T} \right) \right] \right\}, \hspace{1cm} (36)$$

where the second step follows from the lognormality of $M_{T+\tau}/M_T$. Expressions for both the expectation and variance of the log pricing kernel are straightforward to derive (see e.g. Brennan and Xia (2002)). We have that

$$i_{T,T+\tau} = \exp \left\{ A(\tau) - B(\tau)(r_T - \bar{r}) \right\}, \hspace{1cm} (37)$$

where $B(\tau) = \kappa^{-1} (1 - e^{-\kappa \tau})$, as before, and

$$A(\tau) = -\bar{r} \tau - \frac{\sigma_r^2}{4\kappa^3} \left[ 2\kappa (B(\tau) - \tau) + \kappa^2 B^2(\tau) \right] - \frac{\sigma_x}{\kappa} (\xi - \rho^{-1}\lambda)' \rho e_2 (\tau - B(\tau)), \hspace{1cm} (38)$$

with $e_2$ the unit vector with a 1 as its second element and zeros elsewhere.

References


Kullman, Cornelia, and Stephan Siegel (2003), Real estate and its role in household portfolio choice, Working paper.

Munk, Claus and Carsten Sørensen (2005), Dynamic asset allocation with stochastic income and interest rates, Working Paper.


Sinai, Todd, and Nicholas S. Souleles (2005), Onwer-occupied housing as a hedge against rent risk, Quarterly Journal of Economics, 120(2), 763-789.


This table presents the calibrated parameter values for the asset price dynamics. Parameter values for the real interest, expected inflation, and unexpected inflation rate are taken from Van Hemert, De Jong, and Driessen (2005), who use quarterly US data on nominal interest rates and inflation from 1973Q1 to 2003Q4. The house price dynamics are calibrated to Case-Shiller-Weiss repeated-sales data for Atlanta, Boston, Chicago, and San Francisco, from 1980Q2 to 2003Q4. For stocks an index comprising all NYSE, AMEX, and NASDAQ firms over the same sample period as the house price data is used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>(Alternative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock return process: $dS/S = (R_f + \sigma_S \lambda_S) dt + \sigma_S dz_S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.1748</td>
<td></td>
</tr>
<tr>
<td>$\lambda_S$</td>
<td>0.2288</td>
<td></td>
</tr>
<tr>
<td>Real riskless interest rate process: $dr = \kappa (\bar{r} - r) dt + \sigma_r dz_r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>0.0226</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.6501</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.0183</td>
<td></td>
</tr>
<tr>
<td>$\lambda_r$</td>
<td>-0.3035</td>
<td></td>
</tr>
<tr>
<td>Expected inflation process: $d\pi = \alpha (\bar{\pi} - \pi) dt + \sigma_\pi dz_\pi$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>0.0351</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0548</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>0.0191</td>
<td></td>
</tr>
<tr>
<td>$\lambda_\pi$</td>
<td>-0.1674</td>
<td></td>
</tr>
<tr>
<td>House price process: $dQ/Q = (R_f + \sigma_Q \lambda_Q - r^{imp}) dt + \sigma_Q dz_Q = (R_f + \theta^{Q} \lambda - r^{imp}) dt + \theta^{Q} dz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_S$</td>
<td>0.0079</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>-0.0129</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\theta_\pi$</td>
<td>0.0427</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\theta_v$</td>
<td>0.1418</td>
<td>(0.1500)</td>
</tr>
<tr>
<td>$\lambda_v$</td>
<td>0.1315</td>
<td>(0.1150)</td>
</tr>
<tr>
<td>$r^{imp}$</td>
<td>0.0226</td>
<td></td>
</tr>
<tr>
<td>$\sigma_Q$</td>
<td>0.1500</td>
<td></td>
</tr>
<tr>
<td>$\lambda_Q$</td>
<td>0.1150</td>
<td></td>
</tr>
<tr>
<td>Realized inflation process: $d\Pi/\Pi = \pi dt + \sigma_\Pi dz_\Pi = \pi dt + \xi^{Q} dz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_S$</td>
<td>-0.0033</td>
<td></td>
</tr>
<tr>
<td>$\xi_r$</td>
<td>0.0067</td>
<td></td>
</tr>
<tr>
<td>$\xi_\pi$</td>
<td>0.0012</td>
<td></td>
</tr>
<tr>
<td>$\xi_v$</td>
<td>-0.0236</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\xi_u$</td>
<td>0.0474</td>
<td>(0.0530)</td>
</tr>
<tr>
<td>$\lambda_u$</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\Pi$</td>
<td>0.0535</td>
<td></td>
</tr>
<tr>
<td>Real labor income process: $dl/l = g(t) dt + \sigma_l dz_l$, where $g(t) = b + c(t + 20) + 3d(t + 20)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.1682</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>-0.00323</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>0.000020</td>
<td></td>
</tr>
<tr>
<td>Correlations:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{Sr}$</td>
<td>-0.1643</td>
<td></td>
</tr>
<tr>
<td>$\rho_{S\pi}$</td>
<td>0.0544</td>
<td></td>
</tr>
<tr>
<td>$\rho_{r\pi}$</td>
<td>-0.2323</td>
<td></td>
</tr>
<tr>
<td>$\rho_{ql}$</td>
<td>0.2000</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>
Table 2: Bond price exposure to real interest and expected inflation rate shocks.
This table shows the exposure of the 3- and 10-year nominal bond price to shocks in the real interest and expected inflation rate. The exposure to these shocks for a bond with maturity $\tau$ is given by $B(\tau)$ and $C(\tau)$ respectively (see equations (11a)-(11c)). The value for the mean reversion is taken from table 1. Notice that a portfolio consisting of a $1 position in a 3-year bond and a $-2.8/7.7 position in a 10-year bond has the property that it has a negative exposure to real interest rate shocks and a zero exposure to expected inflation rate shocks.

<table>
<thead>
<tr>
<th>Bond maturity, $\tau$</th>
<th>Price exposure to shocks in</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real interest rate, $B(\tau)$</td>
<td>Expected inflation, $C(\tau)$</td>
</tr>
<tr>
<td>3 years</td>
<td>$-1.3$</td>
<td>$-2.8$</td>
</tr>
<tr>
<td>10 years</td>
<td>$-1.5$</td>
<td>$-7.7$</td>
</tr>
</tbody>
</table>

Table 3: Choice of other parameters.
This table reports the parameter values that need to be set in addition to the calibrated parameters governing the asset price dynamics.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>3 or 9</td>
</tr>
<tr>
<td>Housing preferences</td>
<td>$\psi$</td>
<td>0.20</td>
</tr>
<tr>
<td>Subjective discount rate</td>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>Rental rate</td>
<td>$\zeta_{rent}$</td>
<td>6.0%</td>
</tr>
<tr>
<td>Maintenance rate</td>
<td>$\zeta_{own}$</td>
<td>1.5%</td>
</tr>
<tr>
<td>Move to rent cost</td>
<td>$\nu_{rent}$</td>
<td>1.0%</td>
</tr>
<tr>
<td>Move to own cost</td>
<td>$\nu_{own}$</td>
<td>6.0%</td>
</tr>
<tr>
<td>Minimum down payment</td>
<td>$\delta$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 4: Starting values for simulation exercise.
This table presents the starting values at age 20 used for the simulation exercise. The separable state variables do not affect the solution up to a multiplicative factor. Total wealth is included in order to interpret the variables in dollar terms. The zero value for housing tenure indicator indicates the investor is renting at the starting age of 20.

<table>
<thead>
<tr>
<th>State variable</th>
<th>Symbol</th>
<th>Value</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing tenure indicator</td>
<td>$I$</td>
<td>0</td>
<td>Non-separable, binary</td>
</tr>
<tr>
<td>Wealth-to-labor income ratio</td>
<td>$y$</td>
<td>0.5</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Housing-to-wealth ratio</td>
<td>$h$</td>
<td>6.0</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>$r$</td>
<td>$\bar{r} = 2.26%$</td>
<td>Non-separable</td>
</tr>
<tr>
<td>Time</td>
<td>$t$</td>
<td>0</td>
<td>Non-separable, normalisation</td>
</tr>
<tr>
<td>Real total wealth</td>
<td>$w$</td>
<td>$$7,500$</td>
<td>Separable</td>
</tr>
<tr>
<td>Real house price</td>
<td>$q$</td>
<td>1.0</td>
<td>Separable, normalisation</td>
</tr>
</tbody>
</table>
Table 5: Portfolio choice for different housing-to-wealth ratios.

This table reports the optimal portfolio choice for a homeowner of age 65 at different housing-to-wealth ratios. The labor-to-income ratio is set equal to the mean of the simulation analysis, i.e. \( y = 16 \) and 18 for the \( \gamma = 3 \) and \( \gamma = 9 \) investor respectively. The real interest rate is set equal to the long run mean \( \bar{r} \). In addition it presents the utility loss \( UL \), measured as the certainty equivalent loss of having a house size different from the optimal house size (recognizable by a zero value for \( UL \)). Move indicates an investor optimally chooses to change house size in a particular state of the world.

**Panel A: the investor has risk aversion \( \gamma = 3 \)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{stock} )</td>
<td>move</td>
<td>0.38</td>
<td>0.35</td>
<td>0.33</td>
<td>0.33</td>
<td>0.32</td>
<td>0.30</td>
<td>0.27</td>
<td>move</td>
</tr>
<tr>
<td>( x_{3ybond} )</td>
<td>move</td>
<td>0.13</td>
<td>0.16</td>
<td>0.15</td>
<td>0.10</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>move</td>
</tr>
<tr>
<td>( x_{10ybond} )</td>
<td>move</td>
<td>0.44</td>
<td>0.42</td>
<td>0.43</td>
<td>0.46</td>
<td>0.53</td>
<td>0.55</td>
<td>0.56</td>
<td>move</td>
</tr>
<tr>
<td>( x_{cash} )</td>
<td>move</td>
<td>-0.16</td>
<td>-0.24</td>
<td>-0.32</td>
<td>-0.40</td>
<td>-0.48</td>
<td>-0.56</td>
<td>-0.64</td>
<td>move</td>
</tr>
<tr>
<td>( UL )</td>
<td>move</td>
<td>-1.72</td>
<td>-0.53%</td>
<td>0.00%</td>
<td>-0.19%</td>
<td>-0.85%</td>
<td>-1.62%</td>
<td>-2.20%</td>
<td>move</td>
</tr>
</tbody>
</table>

**Panel B: the investor has risk aversion \( \gamma = 9 \)**

<table>
<thead>
<tr>
<th>Variable</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{stock} )</td>
<td>move</td>
<td>0.11</td>
<td>0.11</td>
<td>0.10</td>
<td>0.11</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>( x_{3ybond} )</td>
<td>move</td>
<td>0.84</td>
<td>0.82</td>
<td>0.80</td>
<td>0.78</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>( x_{10ybond} )</td>
<td>move</td>
<td>-0.12</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.01</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>( x_{cash} )</td>
<td>move</td>
<td>-0.04</td>
<td>-0.15</td>
<td>-0.25</td>
<td>-0.39</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
<tr>
<td>( UL )</td>
<td>move</td>
<td>-0.49%</td>
<td>0.00%</td>
<td>-0.53%</td>
<td>-1.43%</td>
<td>move</td>
<td>move</td>
<td>move</td>
<td>move</td>
</tr>
</tbody>
</table>
Figure 1: Housing and other goods consumption for the mean $\gamma = 3$ investor. This figure shows housing and other goods consumption for an investor with a coefficient of relative risk aversion equal to three. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 2: Move rate and fraction owning for the mean $\gamma = 3$ investor.

This figure shows the move rate and fraction owning for an investor with a coefficient of relative risk aversion equal to three. The move rate is annualised and split into house size increases (move up) and decreases (move down). Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 3: Portfolio choice and wealth for the mean $\gamma = 3$ investor.

This figure shows the portfolio choice and wealth accumulation for an investor with a coefficient of relative risk aversion equal to three. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 4: Portfolio choice and wealth for the mean $\gamma = 9$ investor.
This figure shows the portfolio choice and wealth accumulation for an investor with a coefficient of relative risk aversion equal to nine. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 5: Portfolio choice and wealth for the mean $\gamma = 3$ investor with idiosyncratic house price risk.

This figure shows the portfolio choice and wealth accumulation for the case of fully idiosyncratic house price risk (alternative calibrated parameters in table 1). The investor has a coefficient of relative risk aversion equal to three. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 6: Portfolio choice and wealth for the mean $\gamma = 3$ investor with bequest motive.

This figure shows the portfolio choice and wealth accumulation for an investor with a bequest motive. The investor has a coefficient of relative risk aversion equal to three. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.
Figure 7: Portfolio choice and wealth for the mean $\gamma = 3$ investor who suboptimally always chooses an FRM.
This figure shows the portfolio choice and wealth accumulation for an investor who suboptimally always chooses an FRM. The investor has a coefficient of relative risk aversion equal to three. Results are calculated by taking the average value over 10,000 investors, using the derived optimal policy functions and for each investor simulating a different path for the exogenous stochastic processes. Starting values at age 20 are given in table 4.