Valuation and Capital Structure

- In the real world with market imperfections, the value of a project using debt can be written as

\[ V_L = V_U + PV(\text{ITS}) - PV(\text{E[Costs Financial Distress]}) - PV(\text{Agency costs of debt}) + PV(\text{Agency benefits of debt}) \]

- In this lecture we will examine several discounted cash flow methods to estimate the value of levered projects or firms
- These methods are
  - weighted average cost of capital method (WACC)
  - adjusted present value approach (APV)
  - flow to equity (FTE)
- These supplement the comparables methods of valuation
  - use of P/E, P/S, or P/BV ratios of comparable firms
    - Value estimates are forecasts of E, S, or BV times average multiple
- To start we will keep things simple
  - Corporate taxes will be the only market imperfection
    - This means that we only worry about PV(ITS) right now
Valuation with Corporate Taxes

- In the following examples, the goal is to capture the value of the leverage decision in the project’s value
  - leverage adds value through interest tax shields, ITS
    - it also impacts value through expected costs of distress and other transaction costs, as well as agency costs and impacts on value through asymmetric information
  - to start we are only going to consider ways to capture the expected tax advantage to debt
    - the other effects (costs) will be ignored or are assumed to be captured in expected FCF forecasts
  - each of the methods we will look at will account for the additional value of the PV(ITS) to the project in a different way
    - WACC will do it by creating an artificially lower discount rate than the true cost of capital to discount the unlevered FCF
    - APV will do it by calculating the PV(ITS) separately and adding it to unlevered firm value estimated from unlevered FCF and \( r_U \)
    - Flows to Equity will calculated the levered FCF to Equity and discount it at \( r_E \) and then add the PV of Debt to get enterprise value

Project: Heart Watch

- Bodnar Co. has developed a new wrist watch that constantly monitors your blood pressure
  - basic assumptions about planned project
    - technology behind this design will be good for 3 years of production and sales
    - annual sales expected to be $110M with COGS of $60M and operating expenses (SGA) of $10M
    - development costs for production/sales will be $4.3M today
    - production will require $60M in new equipment that can be fully depreciated over 3 years to zero residual value
      - zero salvage value so depreciation is $20M per year
    - NWC will be $5M and is constant over project life
    - tax rate for company is 30%
    - market risk of this project expected to be similar to Bodnar Co’s other lines of business so its \( r_D \) and \( r_E \) can be used
    - Bodnar Co will not change its D/E ratio as result of project
Project Forecasts

- Project’s forecasts of expected unlevered FCFs

<table>
<thead>
<tr>
<th>Year</th>
<th>Incremental Earnings ($MM)</th>
<th>Free Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1) Sales</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>2) COGS</td>
<td>(60)</td>
<td></td>
</tr>
<tr>
<td>3) Gross Profit</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>4) Operating Expenses</td>
<td>(4.3)</td>
<td>20</td>
</tr>
<tr>
<td>5) Depreciation</td>
<td>(20)</td>
<td>20</td>
</tr>
<tr>
<td>6) EBIT</td>
<td>(4.3)</td>
<td>20</td>
</tr>
<tr>
<td>7) Income Tax (30%)</td>
<td>1.3</td>
<td>14</td>
</tr>
<tr>
<td>8) Unlevered Net Income</td>
<td>(3)</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>9) Plus depreciation</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>10) Less Chg NWC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11) Less Capital Expend</td>
<td>(60)</td>
<td>5</td>
</tr>
<tr>
<td>12) FCF</td>
<td>(68)</td>
<td>34</td>
</tr>
</tbody>
</table>

Bodnar Co’s MV B/S and Costs of Capital

- Bodnar’s B/S prior to undertaking new project (MV)

<table>
<thead>
<tr>
<th>Assets ($MM)</th>
<th>Liabilities ($MM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus cash</td>
<td>40</td>
</tr>
<tr>
<td>Other Assets</td>
<td>400</td>
</tr>
<tr>
<td>Total Assets</td>
<td>440</td>
</tr>
<tr>
<td>Debt</td>
<td>200</td>
</tr>
<tr>
<td>Equity</td>
<td>240</td>
</tr>
<tr>
<td>Total Liabs+Eq</td>
<td>440</td>
</tr>
</tbody>
</table>

- separate surplus cash from other assets (i.e., operating assets)
  - Other Assets = NWC (cur assets – cur liabs) + PPE + Intangibles
- surplus cash is really negative debt so really D = 200 - 40 = 160
  - D = 160 and E = 240 => Enterprise Value = D + E = 400

- return information
  - assume \( r_f = 6\% \) and \( (E[r_m] - r_f) = 5\% \)
  - from regression of past returns on market returns, Bodnar Co’s levered equity has \( \beta_E = 1.2 \) => \( r_E = 12\% \)
  - equity has had average D/E ~ 0.66
- Bodnar Co’s cost of debt, \( r_D \), is currently 7% so \( \beta_D = 0.2 \)
  - beta of debt comes from solving CAPM for \( \beta_D \) using expected \( r_D \)
    \[ \beta_D = (r_D - r_f) / (E[r_m] - r_f) = (7\% - 6\%) / 5\% = 0.2 \]
**WACC**

- The WACC method for valuation uses the (after-tax) cost of capital, $r_{WACC}$ to discount the unlevered FCF.
- Bodnar Co’s after-tax weighted average cost of capital incorporating the tax shield into average cost of capital gives us
  \[ r_{WACC} = \frac{E}{D+E} r_E + \frac{D}{D+E} r_D (1 - \tau_C) \]
  \[ r_{WACC} = \frac{240}{400} \times 12\% + \frac{160}{400} \times 7\% (1 - 30\%) = 9.16\% \]
  - we can use existing firm’s data because of earlier assumptions
- note this is less than the true weighted average cost of equity and debt
  \[ WACC_{NOTAX} = \frac{E}{D+E} r_E + \frac{D}{D+E} r_D = r_U = r_A \]
  \[ WACC_{NOTAX} = r_U = r_A = \frac{240}{400} \times 12\% + \frac{160}{400} \times 7\% = 10\% \]
  - in fact we can see the $r_{WACC}$ reduces the asset’s true cost of capital, $r_U$, by exactly the impact of the tax shield term
  \[ r_{WACC} = r_U - \frac{D}{D+E} \cdot r_D \cdot \tau_C \]
  \[ r_{WACC} = 10\% - \frac{160}{400} \times (7\%) \times (30\%) = 9.16\% \]

**Value of Heart Watch Project using WACC**

- To value project using WACC method we take PV of project unlevered future E(FCF) using $r_{WACC}$
  \[ V_0 = \frac{E_0(FCF_1)}{1+r_{WACC}} + \frac{E_0(FCF_2)}{(1+r_{WACC})^2} + \frac{E_0(FCF_3)}{(1+r_{WACC})^3} = 34/(1.0916) + 34/(1.0916)^2 + 39/(1.0916)^3 = \$89.7M \]
  - with $C_0 = -$68M, NPV of project to the firm is $= \$21.7M
  - NPV = $C_0 + PV(Future FCF) = -$68 + $89.7 = $21.7M
- using WACC approach is suitable when
  - the new project is of similar risk as the firm existing assets
  - firm maintains a constant D/E ratio

- **Maintaining constant leverage** ($D/E$ ratio = 2/3 or $D/V = 0.40$)
  - with project of value $89.7M, the firm’s assets rise by this amount
    - this increases the left side of the MV balance sheet
  - to keep the leverage of the firm at desired level, $D/V = 0.40$, the firm will have to add adjust right hand side of balance sheet
    - adjust mix of D and E to keep $D/E = 2/3$ or $D/V = 0.40$ along with constraint that MV Assets = $D + E$
Maintaining D/E

- Taking the project, $89.7M in assets is added to B/S
  - MV of (non cash) assets rise from $400 to $489.7
  - to maintain D/V ratio of 0.40, firm must add $89.7 x 0.40 = $35.9 in debt to its B/S today
    - borrow an additional $35.9M
      - D rises from 200 to 235.9
      - E rises by initial equity of $32.1 ($68 – $35.9) plus $21.7 in NPV so \( \Delta E = 53.8 \), => E rises from $240 to $293.8
  - or firm could use $35.9M of its $40M surplus cash to fund project to maintain desired leverage
    - again the firm would need to contribute $32.1 in new equity
      - the MV of equity thus rises by 32.1 + the NPV of the project, 21.1
      - \( \Delta E = 32.1 + 21.7 = 53.8 \) so E => $293.8M

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus cash</td>
<td>40</td>
</tr>
<tr>
<td>Other Assets</td>
<td>400</td>
</tr>
<tr>
<td>Heart Watch</td>
<td>89.7</td>
</tr>
<tr>
<td>Total Assets</td>
<td>529.7</td>
</tr>
<tr>
<td>Surplus cash</td>
<td>4.1</td>
</tr>
<tr>
<td>Other Assets</td>
<td>400</td>
</tr>
<tr>
<td>Heart Watch</td>
<td>89.7</td>
</tr>
<tr>
<td>Total Assets</td>
<td>493.8</td>
</tr>
<tr>
<td>Surplus cash</td>
<td>40</td>
</tr>
<tr>
<td>New Debt</td>
<td>35.9</td>
</tr>
<tr>
<td>Equity</td>
<td>293.8</td>
</tr>
<tr>
<td>Total D+E</td>
<td>529.7</td>
</tr>
<tr>
<td>Surplus cash</td>
<td>4.1</td>
</tr>
<tr>
<td>New Debt</td>
<td>25.6</td>
</tr>
<tr>
<td>Equity</td>
<td>293.8</td>
</tr>
<tr>
<td>Total D+E</td>
<td>493.8</td>
</tr>
</tbody>
</table>

Debt Capacity

- The use of WACC method requires that the firm maintain constant leverage, D/V
  - this implies that the project has a given debt capacity
    - an amount of debt that it allows the firm to take each period
      - debt capacity, \( D_t = d \times V_t^L \)
        - where \( d \) = firm’s desired D/V ratio
        - \( V_t^L = \text{expected value at time } t = \text{PV of FCF in periods } t+1 \text{ onward} \)
        - note \( V_t^L = \frac{\text{FCF}_{t+1} + V_{t+1}^L}{(1+r_{\text{WACC}})} \) where \( V_{t+1}^L = \text{value of FCF in years } t+2 \text{ onward} \)
      - for this project the debt capacity each period (with \( d = 40\% \)) is

<table>
<thead>
<tr>
<th>Project Debt Capacity</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free cash flow</td>
<td>-68</td>
<td>34</td>
<td>34</td>
<td>39</td>
</tr>
<tr>
<td>Levered value, ( V^L )</td>
<td>89.7</td>
<td>63.9</td>
<td>35.7</td>
<td>-</td>
</tr>
<tr>
<td>Debt capacity (@ ( d = 40% ))</td>
<td>35.9</td>
<td>25.6</td>
<td>14.3</td>
<td>-</td>
</tr>
</tbody>
</table>
Adjusted Present Value

- APV is an alternative valuation method in which we first determine unlevered value, $V_U$, and then adjust this value for PV(ITS) and other influences
  
  $V_L = V_U + PV(ITS) - PV(\text{distress, agency and transaction costs})$

  - a.k.a “divide and conquer” approach

- how do we determine $V_U$?
  - $V_U$ is just the PV(future FCF) discounted at $r_U (= r_A)$
  
  2 ways to estimate $r_U$
  
  1. WACC(tax) = $[E/(D+E)] r_E + [D/(D+E)] r_D = r_U = r_A$
     
     $r_U = \frac{240/400}{12\%} + \frac{160/400}{7\%} = 10\%$
  
  2. $\beta_U = \frac{E}{D+E} \beta_E + \frac{D}{D+E} \beta_D$
     
     $\beta_U = \frac{240/400}{1.2} + \frac{160/400}{0.2} = 0.8 = \beta_A$
     
     $r_U = r_F + \beta_U (E[R_M] - r_F) = 6\% + 0.8 (5\%) = 10\% = r_A$

  $V_{u0} = \frac{E_0(FCF_1)}{1+r_U} + \frac{E_0(FCF_2)}{(1+r_U)^2} + \frac{E_0(FCF_3)}{(1+r_U)^3} = 88.3M$

Determining PV(ITS)

- To determine PV(ITS), we need to specify the interest tax shield the firm get each year
  
  - this will be a function of the interest rate, the tax rate and the debt outstanding at the end of the previous year
  
  - interest tax shield in year $t = D_{t-1} \times r_D \times \tau_C$
  
  - the debt levels are determined from the debt capacity calculation

<table>
<thead>
<tr>
<th>Interest Tax Shields</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt capacity, $D_t$</td>
<td>35.9</td>
<td>25.6</td>
<td>14.3</td>
<td>-</td>
</tr>
<tr>
<td>Interest paid, at $r_D = 7%$</td>
<td>2.51</td>
<td>1.79</td>
<td>1.00</td>
<td>-</td>
</tr>
<tr>
<td>Interest tax shield @ $\tau_C = 30%$</td>
<td>0.75</td>
<td>0.54</td>
<td>0.30</td>
<td>-</td>
</tr>
</tbody>
</table>

- when the firm maintains a target leverage ratio ($D/V$), its future interest tax shields are as risky as its asset value, so the ITS should be discounted at the assets’ cost of capital, $r_U$

  $PV(ITS) = 0.75/1.10 + 0.54/1.10^2 + 0.30/1.10^3 = 1.4M$

- APV valuation:

  $V_L = V_U + PV(ITS) = 88.3 + 1.4 = 89.7M$

  » same as the WACC method estimate
Flow to Equity Method

- A final DCF method for determining enterprise value is the calculate equity value and add debt
  - whereas the previous to methods both used unlevered cash flows, the flows to equity (FTE) method measures the free cash flows to the equity holders (FCFE)
    - these flows take account of all the payments to and from the debtholders
      - this is how the interest tax shields are taken into account
    - these equity cash flows are then discounted at the required return for levered equity (given D/E assumption)
    - the resulting estimate for equity value is then added to the amount of debt suggests by the D/E ratio to determine \[ V_L = PV(\text{Free cash flows to equity @ } r_E) \]
    \[ V_L = E + D \]

Free Cash Flow to Equity (FCFE)

- FCFE is constructed in similar way to FCF, just with the inclusion of interest expense and net borrowings

<table>
<thead>
<tr>
<th>Incremental Earnings</th>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Sales</td>
<td></td>
<td>110</td>
<td>110</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>2) COGS</td>
<td></td>
<td>-60</td>
<td>-60</td>
<td>-60</td>
<td></td>
</tr>
<tr>
<td>3) Gross Profit</td>
<td></td>
<td>50</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>4) Operating Expenses</td>
<td></td>
<td>-4.3</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>5) Depreciation</td>
<td></td>
<td>-20</td>
<td>-20</td>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>6) EBIT</td>
<td></td>
<td>-4.3</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>7) Interest Expense, (D_{t-1} \times r_D)</td>
<td></td>
<td>-2.5</td>
<td>-1.8</td>
<td>-1.0</td>
<td></td>
</tr>
<tr>
<td>8) Pre tax income</td>
<td></td>
<td>-4.3</td>
<td>17.5</td>
<td>18.2</td>
<td>19.0</td>
</tr>
<tr>
<td>9) Income Tax (τC = 30%)</td>
<td></td>
<td>1.3</td>
<td>-5.2</td>
<td>-5.5</td>
<td>-5.7</td>
</tr>
<tr>
<td>10) Unlevered Net Income</td>
<td></td>
<td>-3</td>
<td>12</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Free Cash Flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11) Plus depreciation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12) Less Chg NWC</td>
<td></td>
<td>-5</td>
<td>0</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>13) Less Capital Exp</td>
<td></td>
<td>-60</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14) Plus Net Borrowing, (D_t - D_{t+1})</td>
<td></td>
<td>35.9</td>
<td>-10.3</td>
<td>-11.3</td>
<td>-14.3</td>
</tr>
<tr>
<td>15) Free cash flow to equity</td>
<td></td>
<td>-32.1</td>
<td>21.9</td>
<td>21.5</td>
<td>24.0</td>
</tr>
</tbody>
</table>

\( r_D = 7\% \)

- FCFE also equal: FCF - after tax interest expense + net borrowings
Valuation of Equity

We discount the FCFE at \( r_E \)
- for this project, we can use the parent firm \( r_E \) as the asset risk and leverage of project are same as parent
  - Bodnar Co.'s \( r_E = 12\% \)
- value of equity in project with $35.9M in debt financing with \( r_D \) at 7% and tax rate \( \tau_C = 30\% \)
  \[
  E = E_0(\text{FCFE}_1)/(1+r_E) + E_0(\text{FCFE}_2)/(1+r_E)^2 + E_0(\text{FCFE}_3)/(1+r_E)^3 \\
  = 21.9/(1.12) + 21.5/(1.12)^2 + 24/(1.12)^3 = \$53.8M
  \]
  - same as the equity value of the project from the other methods
- to determine enterprise value, \( V_L = D + E \), we add the initial MV of the debt to this estimate of \( E \)
  - with \( D_0 = $35.9M \)
  \[
  V_L^0 = D + E = $35.9M + $53.8M = $89.7M
  \]
  - same enterprise value as other methods

Adjustments to the Simple Case

The previous examples have been standard cases
- project is scale expanding project of existing firm so can use firm’s \( r_E \) and \( r_D \)
- some firms do not maintain a constant D/V ratio
- these will not be so in all cases
  - if project is not scale expanding, we must determine a project specific cost of capital, \( r_U \)
  - we do this by looking at other single activity firms in the same line of business as the project
    - we can take the weighted average of their return on equity and return on debt to determine \( r_U \)
      - here we will use the firms average D/V and E/V ratios
    - we can estimate the \( \beta_E \) of these firms or their portfolio, and delever this beta to get \( \beta_U \) and use this to estimate \( r_U \)
      - again we use the firms D/E ratios to delever the equity beta
Project Specific Cost of Capital

- Suppose Bodnar Co. was to invest in publishing personal investment books and guides
  - this is a new line of business and not like their existing assets
  - to determine the $r_U$ for these assets they must look at returns to firms in the self-help publishing area
    - consider the return info on firms exclusively in this business

<table>
<thead>
<tr>
<th>Company</th>
<th>Historic $R_E$</th>
<th>Current $R_D$</th>
<th>Hist Avg D/V</th>
<th>$r_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 1</td>
<td>11.55%</td>
<td>6.00%</td>
<td>0.11</td>
<td>10.9%</td>
</tr>
<tr>
<td>Company 2</td>
<td>14.10%</td>
<td>6.85%</td>
<td>0.43</td>
<td>11.0%</td>
</tr>
<tr>
<td>Company 3</td>
<td>12.65%</td>
<td>6.45%</td>
<td>0.24</td>
<td>11.2%</td>
</tr>
<tr>
<td>Average</td>
<td>12.77%</td>
<td>6.43%</td>
<td>0.26</td>
<td>11.1%</td>
</tr>
</tbody>
</table>

- to find $r_U$ we can take an average of each firm’s weighted average of $R_E$ and $R_D$ or the average of $\text{Avg } R_E$ and $\text{Avg } R_D$

$$r_U = (1 - \frac{D}{V}) \times \text{Avg } R_E + \frac{D}{V} \times \text{Avg } R_D$$  [note: $(1 - \frac{D}{V}) = \frac{E}{V}$]

- $r_U$ for this project in the self-help publishing business area would appear to be ~ 11.1%
  - this is fine ONLY IF $r_f$ and $(E[R_M] - r_f)$ are similar as in past

Project Specific Cost of Capital

- The historical average estimate for $R_E$ will be a poor predictor of $r_E$ if market conditions have changed
  - in this case we are better off using the historical data to estimate the $\beta_E$ and then estimating $r_E$ with current info
    - can use Excel to calculate stats or to run a simple regression
  - again we can do this for each firm or for a portfolio of the three firms (arbitrarily weighted – usually equal or MV weights)
    - we also convert $r_D$ into $\beta_D$ using CAPM (approximation)

<table>
<thead>
<tr>
<th>Company</th>
<th>$\beta_E$</th>
<th>$\beta_D$</th>
<th>D/V</th>
<th>$\beta_U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company 1</td>
<td>1.05</td>
<td>0</td>
<td>0.11</td>
<td>0.93</td>
</tr>
<tr>
<td>Company 2</td>
<td>1.54</td>
<td>0.17</td>
<td>0.42</td>
<td>0.96</td>
</tr>
<tr>
<td>Company 3</td>
<td>1.23</td>
<td>0.09</td>
<td>0.25</td>
<td>0.95</td>
</tr>
<tr>
<td>Average</td>
<td>1.27</td>
<td>0.08</td>
<td>0.26</td>
<td>0.96</td>
</tr>
</tbody>
</table>

- the average (or EW portfolio) beta on the equity $\beta_E$ is 1.27

- we recall the unlevered beta of equity, $\beta_{U}$, is also just the weighted average of $\beta_E$ and $\beta_D$

$$\beta_U = (1 - \frac{D}{V}) \beta_E + \frac{D}{V} \beta_D = (0.74) \cdot 1.27 + (.26) \cdot 0.08 = 0.96$$
Project’s $r_U$ and $r_E$

- With an $\beta_U = 0.96$, the project will have an $r_U$ of
  
  \[
  r_U = r_f + \beta_U \times (E[R_M] - r_f) = 6\% + 0.96 \times 5\% = 10.8\%
  \]

  - this is marginally lower than approach using historic $r_E$ because either $r_f$ or $(E[R_M] - r_f)$ is slightly lower now than historically
  - generally determining $r_U$ from betas is safer than from average historic returns

Project’s $r_E$

- having estimated the project’s $r_U$, we can determine the $r_E$ with an estimates of the firm’s post-project D/V and $r_D$
  - if the firm will have a post project D/V = 0.35 (D/E = 0.54) and this is expected to entail a $r_D = 6.6\%$ ($\beta_D = .12$), then
    
    \[
    r_E = r_U + \frac{D}{E} \times (r_U - r_D)
    \]

    \[
    r_E = 10.8\% + (0.54)(10.8 - 6.6) = 13.1\%
    \]

  - or $\beta_E = \beta_U + (D/E)(\beta_U - \beta_D) = 0.96 + (0.54)(0.96 - 0.12) = 1.41$
    
    \[
    r_E = r_f + \beta_E \times (E[R_M] - r_f) = 6\% + 1.41 \times 5\% = 13.1\%
    \]

Project WACC

- Once we have a project’s $r_E$, we can calculate its WACC
  - we just use the project’s $r_E$, and the firm’s post project $r_D$ and D/V and the firm’s tax rate $\tau_C$
    
    \[
    r_{WACC} = \frac{E}{V} \times r_E + \frac{D}{V} \times r_D \times (1 - \tau_C)
    \]

    \[
    r_{WACC} = 0.65 \times 13.1\% + 0.35 \times 6.6\% \times (1 - .30) = 10.1\%
    \]

  - alternatively
    
    \[
    r_{WACC} = r_U - d \times \tau_C \times r_D
    \]

    - where $d$ is the post project target debt-to-value (D/V) ratio = 0.35
      
      - plugging in
        
        \[
        r_{WACC} = 10.8\% - 0.35 \times 0.30 \times 6.6\% = 10.1\%
        \]

  - keep in mind that when redoing WACC for a new D/V, both the $r_E$ and $r_D$ will change with the new D/V
    
    - $r_{WACC}$ will fall with increasing D, but keep in mind that distress costs and agency costs will be rising
Case of Non Constant D/V

- So far we have assumed that D/V will be constant over the life of the project
  - firms can follow other leverage policies
    - constant interest coverage
      - firm sets D so that its interest is always a set proportion of FCF
      - firm sets debt so that given rD, interest payments are 25% of FCF
      - we use FCF rather than EBIT to make closed form solution possible, it will be similar if FCF is a roughly a constant proportion of EBIT
    - pre-determined debt level
      - the firm borrows a specific amount as a result of the project
      - project that borrows to finance itself and then pay off the debt
  - whenever the firm does not follow a constant D/V ratio
    - WACC and FTE will be difficult to use as they entail recalculation of rWACC or rE each period
    - these cases are best suited for use of APV method

Constant Interest Coverage

- If a firm sets its leverage ratio to maintain a constant interest coverage, k, then it want to give the fraction k of its FCF to debt holders each period
  - interest paid in year t = k x FCF
  - note the interest tax shields will be proportional to FCF
    - therefore they should be discounted at same rate as FCF, so they use rU as their discount rate
    - PV(ITS) = PV(τC x k x FCF) = τC x k x PV(FCF)
      = τC x k x VU
  - APV valuation says
    - VL = VU + PV(ITS) - PV(other issues)
    - ignoring the other issues for the moment as we have been doing
    - VL = VU + τC x k x VU = (1 + τC x k) VU
  - for Heart Watch project, if k = 10%, then
    - VL = (1 + τC x k) VU = (1 + (.30)(.10)) x $88.3M = $90.9M
Predetermined Debt Levels

- What if the firm simply sets the debt schedule at the beginning of project?
  - this the case in project finance deals
    - debt related to project is determined at beginning of project and interest payments are set in advance by contract
      - firm pays debt according to predetermined schedule and never alters the debt based upon the performance of the project
    - D could be constant or if debt principal is repaid then D could decline over time
  - thus D levels are known in advance but not the D/V ratio
- assume upon taking the Heart Watch project, Bodnar Co. decides to borrow $36M and pay it off over three years
  - principal repayment is $12M per year for three years

<table>
<thead>
<tr>
<th>Fixed Debt Schedule</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt Outstanding , D</td>
<td>36.0</td>
<td>24.0</td>
<td>12.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Principal repayment</td>
<td>12.0</td>
<td>12.0</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>Interest paid, at rD = 7%</td>
<td>2.52</td>
<td>1.68</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>Interest tax shields @ τc = 30%</td>
<td>0.76</td>
<td>0.50</td>
<td>0.25</td>
<td></td>
</tr>
</tbody>
</table>

Project Value with Predetermined Debt

- We take the PV of the ITS
  - rather than use rU, since the interest payments are set as a function of the known debt schedule, the ITS are only as risky as the debt
    - the risk of the debt is captured by rD, the cost of the debt
      - so we use rD to take the PV of the ITS
    - for Bodnar Co. post - Heart Watch project, its rD = 7%
      - Heart Watch project with this level of borrowing has no impact on its corporate borrowing rate

\[
PV(ITS) = \frac{0.76}{1.07} + \frac{0.50}{1.07^2} + \frac{0.25}{1.07^3} = \$1.4M
\]

- APV value with fixed debt for Heart Watch project
  \[
  V_L = V_U + PV(ITS) - PV(other items)
  \]
  \[
  V_L = 88.3M + 1.4M = \$89.7M
  \]
  - this is same valuation as with D/V = 0.40 though leverage is slightly less in years 2 and 3
Choosing Amongst Methods

- As we have seen, all three methods of valuation produce the same result for similar assumptions
  - when it is appropriate to assume the firm is following a constant D/V ratio, then it will be most convenient to use WACC
  - if the firm is following some other leverage policy or is slowly change leverage policies, the APV is the most straightforward to apply
  - FTE is used mostly in complex situations with securities other than standard debt involved where it is difficult to determine capital structure and interest tax shields

Other Effects of Financing Choices

- Until now we have only considered the ITS benefit of debt
  - use of debt or adding debt to the firm has costs that we need to consider
  - transaction costs of issuing debt and or new equity (no internally generated CF) are incremental to the project and must be deducted from FCFs

<table>
<thead>
<tr>
<th>Financing Type</th>
<th>Underwriting Fees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank loans</td>
<td>&lt; 2%</td>
</tr>
<tr>
<td>Corporate bonds</td>
<td></td>
</tr>
<tr>
<td>Investment grade</td>
<td>1–2%</td>
</tr>
<tr>
<td>Non-investment grade</td>
<td>2–3%</td>
</tr>
<tr>
<td>Equity issues</td>
<td></td>
</tr>
<tr>
<td>Initial public offering</td>
<td>8–9%</td>
</tr>
<tr>
<td>Seasoned equity offering</td>
<td>5–6%</td>
</tr>
</tbody>
</table>

Costs of Financial Distress and Agency Costs

- The use of debt will at some point start to incur non trivial distress costs and or agency costs
  - these are not measurable by any formula, and they differ based upon the specific type of project
    - as a result we have no general formula to take them into account
      - but they must be taken into account once debt is large enough that they are tangible
    - when the debt level gets high enough, and the probability of distress becomes meaningful to investors, expected free cash flows will be reduced by expected costs of financial distress \( \text{E[CFD]} \)
      - thus expected FCF forecasts will be lower for higher leverage levels than for zero leverage
        - costs likely increase slowly at first, but the rise at an increasing rate as D level rises

Leverage and \( r_U \) with Costly Distress

- Financial distress can be random but it is also likely at least partially related to market conditions
  - when a firm gets highly levered, its prospects are likely more correlated with market conditions
    - this means its beta and therefore required return may be higher than other firms with standard leverage even when unlevered
    - thus \( r_U \) may actually increase slightly due to the costs of financial distress as the firm increases leverage (arbitrary adjustments)
    - one needs to consider both impacts on FCF and the cost of capital, \( r_{WACC} \) or \( r_U \) when determining value with high leverage

Example

- firm with perpetual FCFs has \( V_U = $250M \) has \( E(FCF) \) and \( r_U \) as function of level of \( D \) as given

<table>
<thead>
<tr>
<th>Level of Debt ($MM)</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>140</th>
<th>180</th>
<th>220</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCF ($MM)</td>
<td>20</td>
<td>20</td>
<td>19.8</td>
<td>19.4</td>
<td>18.6</td>
<td>17</td>
</tr>
<tr>
<td>( r_U )</td>
<td>8.0%</td>
<td>8.0%</td>
<td>8.2%</td>
<td>8.5%</td>
<td>8.8%</td>
<td>9.5%</td>
</tr>
</tbody>
</table>
Choosing Optimal D/V for Firm

- Since FCF contain costs of increasing D, we can estimate optimal leverage by finding maximum of PV(FCF) + PV(ITS)
  - since project is a perpetuity, the debt will be permanent and constant so PV(ITS) = \( \tau_C \times D \) (\( \tau_C = 30\% \))
  - PV(ITS) = perpetuity of \( r_D \times \tau_C \times D \Rightarrow PV = \frac{r_D \times \tau_C \times D}{r_D} = \tau_C \times D \)

  \[
  \begin{array}{|c|c|c|c|c|c|}
  \hline
  \text{Level of Debt ($MM)} & 0 & 50 & 100 & 140 & 180 & 220 \\
  \hline
  V_U &= E(FCF)/r_U & 250.0 & 250.0 & 241.5 & 228.2 & 211.4 & 178.9 \\
  \hline
  PV(ITS) &= \tau_C \times D & $0.0 & $15.0 & $30.0 & $42.0 & $54.0 & $66.0 \\
  \hline
  V_L &= V_U + PV(ITS) & $250.0 & $265.0 & $271.5 & $270.2 & $265.4 & $244.9 \\
  \hline
  \end{array}
  \]

- given these assumptions, (perhaps from simulations) the optimal level of Debt for the firm would be something around $100M to $140M
- these levels provide maximum value to SHs net of distress and agency costs
  - optimal leverage increases firm value by almost 10%

Summary

- 3 basic advanced DCF valuation methods
  - WACC
    - uses unlevered FCF and discounts at \( r_{WACC} < r_U \) to determine \( V_L \)
  - APV
    - calculates \( V_U \) and then separately estimates PV(ITS) to determine \( V_L \)
    - discount rate for ITS depends on leverage assumption
  - Flows to Equity
    - uses FCFE and discounts at \( r_E \) to estimate MV of equity
      - \( V_L = E + D \)
  - other influences
    - be aware that finance is very good at determine tax shield benefit and issuance costs of D or E but has no formulas for E(CFD) or agency costs

- Be sure to read Chpt 19, a great example of these methods in a realistic situation