Monetary Policy Risks in the Bond Markets and Macroeconomy

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Abstract

What is the role of monetary policy fluctuations for the macroeconomy and bond markets? To answer this question we estimate a novel asset-pricing framework which incorporates a time-varying parameter Taylor rule, macroeconomic factors, and risk pricing restrictions of the recursive preferences. We find that in the data, monetary policy fluctuations significantly impact inflation uncertainty and bond price exposures to economic risks, but do not have a sizeable effect on the first moments of macroeconomic variables. Monetary policy fluctuations contribute about 20% to the persistent variations in bond risk premia. A significant component of the monetary policy impact on risk premia comes from its effect on inflation volatility.

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**Introduction**

There is a significant evidence that monetary policy fluctuates over time. In certain periods the monetary authority reacts more strongly to fundamental concerns about real economic growth and inflation, thus affecting the dynamics and the risk exposures of the bond markets. Yet, there is no conclusive evidence on how these policy fluctuations impact the economy and asset prices. To assess the role of a time-varying monetary policy, we develop an economically motivated asset-pricing model which incorporates the link between the monetary policy fluctuations, aggregate macroeconomic variables, and nominal bond yields. We estimate the model using macroeconomic, forecast, and term structure data, and quantify the conditional implications of the monetary policy fluctuations above and beyond standard macro-finance risk channels. We find that while monetary policy fluctuations are not significantly related to the first moments of the macroeconomic variables, the inflation uncertainty and bond price exposures to economic risks significantly increase in aggressive relative to passive regimes. Consequently, through their direct impact on bond risk exposures and indirect effect on the quantity of inflation risk, fluctuations in monetary policy have a sizeable contribution to the time-variation in the levels of yields and bond risk premia.

Our asset-pricing framework features a novel recursive-utility based representation of the stochastic discount factor (SDF), the exogenous dynamics of the macroeconomic factors, and the time-varying Taylor rule for the interest rates. Specifically, the stochastic discount factor incorporates pricing conditions of the recursive-utility investor, but does not force an inter-temporal restriction between the short rate and the fundamental macroeconomic processes. This representation is an alternative to the decompositions in Bansal, Kiku, Shaliastovich, and Yaron (2013) and Campbell, Giglio, Polk, and Turley (2012), and identifies long-run cash-flow, long-run interest rate news, and the uncertainty news as the key sources of risk for the investor. Our representation of the SDF is particularly convenient for our analysis. Similar to the reduced-form, no-arbitrage models of the term structure, our representation allows us to exogenously and flexibly model the dynamics of the short rates and the macroeconomic state variables. On the other hand, our stochastic discount factor is economically motivated, and the sources and the market prices of risks are disciplined to be consistent with recent economic term-structure models.

To model the short rate, we assume a forward looking, time-varying Taylor rule in

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1 See Singleton (2006) for the review of the no-arbitrage term structure models.
which the sensitivities of the short rate to expected real growth and expected inflation can vary across the monetary policy regimes. We further consider an exogenous specification of the dynamics of the real growth and inflation, which features persistent fluctuations in the conditional means and volatilities of the economic states. As in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005), we allow for inflation non-neutrality, so that expected inflation can have a negative impact on future real growth. This economic channel plays an important role to explain a positive bond risk premium and a positive slope of the nominal term structure. Following Bansal and Shaliastovich (2013), we also incorporate exogenous fluctuations in inflation volatility which drive the quantity of macroeconomic risks and bond risk premia. Novel in our paper, we allow the conditional expectations of future consumption and inflation as well as the inflation volatility to directly depend on the monetary policy regime. In this sense, we introduce a link between the time-varying monetary policy, the exposures of bond prices to economic risks, and the movements in the levels and volatilities of the underlying economic factors.

To estimate the model and assess the role of the monetary policy fluctuations, we utilize quarterly data on realized consumption and inflation, the survey expectations on real growth and inflation, and the data on bond yields for short to long maturities. The model-implied observation equations are nonlinear in the states, and all economic factors are latent. Similar to Schorfheide, Song, and Yaron (2013) and Song (2014), we rely on Bayesian MCMC methods to draw model parameters, and use particle filter to filter out latent states and evaluate the likelihood function.

We find that the estimation produces plausible parameter values and delivers a good fit to the observed macroeconomic and yield data. The expected consumption, expected inflation, and inflation volatility are very persistent, and expected inflation has a strong and negative feedback to future expected consumption. We further find substantial fluctuations in the monetary policy across the regimes. Interestingly, monetary policy fluctuations do not have a sizeable effect on the first moments of the macroeconomic variables: the expectations of future real growth and inflation are not significantly different across the regimes. On the other hand, inflation uncertainty significantly increases by about a quarter, and interest rates respond stronger to expected real growth and inflation risks in aggressive relative to passive regimes. Indeed, the median short rate loadings on expected real growth and expected inflation are equal to 0.7 and 1.7, respectively, in aggressive regimes, which are significantly larger than the estimated loadings of 0.5 and 0.9 in passive regimes.

These differences in inflation volatility and bond sensitivities have important implica-
tions for the dynamics of bond yields. We document that the loadings of bond yields and bond risk premia are magnified in aggressive relative to passive regimes. This results in elevated means and volatilities of bond yields and bond risk premia in aggressive regimes. Introducing time-variation in monetary policy increases variability of bond risk premia by about 20%. The time-varying bond exposures to expected inflation risk and the time-varying quantity of inflation risk due to monetary policy fluctuations both contribute about equally to a rise in bond risk premia. Interestingly, the variations in inflation and real growth sensitivities of interest rates have opposite effects on the bond risk premia. We further show that the bond risk premia can go negative when the inflation premium is relatively small. In the data, the model-implied bond risk premia turn negative post 2005 when the conditional inflation volatility is below the average, and the economy is in a passive regime.

Related Literature. There are several contributions of our approach to the existing literature. First, we rely on a novel representation of the stochastic discount factor which allows us to incorporate a flexible dynamics of a time-varying Taylor rule and macroeconomic factors, and yet impose economic pricing restrictions from the recursive preferences. Second, we consider the interaction between monetary policy fluctuations and movements in stochastic volatility, expected growth, and expected inflation. Finally, we estimate the model using the macroeconomic and asset price data, and perform a quantitative assessment of the importance of monetary policy fluctuations for the levels and volatilities of bond yields and bond risk premia.

Our paper is related to a large and growing macro-finance literature which studies the role of monetary policy for macroeconomic fundamentals and asset prices. In the context of general equilibrium models, Song (2014) considers a long-run risks type model to investigate a role of monetary policy and macroeconomic regimes for the dynamics of bond and equity prices, and specifically for the comovement between bond and equity returns. This work does not consider the link between monetary policy and macroeconomic uncertainties, which we find to be important for the movements in the bond risk premia. Campbell, Pflueger and Viceira (2014) use a New Keynesian habit formation model to study the variation in stock and bond correlation and movements in the bond risk premia across monetary policy regimes. The model is calibrated to target interest rate rules across data subsamples. In our work, persistent changes in monetary policy regimes which affect the dynamics of the macroeconomy and bond yields are taken into account when evaluating
the Euler equation, and represent a priced source of risks for the investor. Within a DSGE framework, time-variation of the monetary policy is also considered in Andreasen (2012), Chib, Kang, and Ramamurthy (2010), and Palomino (2012), while constant coefficient Taylor rule are featured in general equilibrium models such as Rudebusch and Swanson (2012) and Gallmeyer et al. (2006). Kung (2015) embeds a constant coefficient Taylor rule in a production-based asset pricing model, and considers the monetary policy impact on the term structure of interest rates. Relative to this literature, our paper entertains an alternative and more flexible representation of the stochastic discount factor and macroeconomic factors, and further incorporates a link between inflation uncertainty and the monetary policy fluctuations.

In terms of the reduced-form term structure literature, Ang, Boivin, Dong, and Loo-Kung (2011) highlight the importance of a time-varying monetary policy for the shape of the nominal term structure and the levels of the bond risk premia. They document substantial fluctuations in Fed’s response to inflation, while the variations in policy stance to output gap shocks are much smaller. They do not entertain movements in macroeconomic volatilities. In our framework, we find that monetary policy coefficients to persistent growth and inflation risks are both quite volatile, and contribute to the fluctuations in the risk premia, alongside with movements in fundamental uncertainties. Bikbov and Chernov (2013) incorporate time-variation in monetary policy and stochastic volatilities of the fundamental shocks, and argue that interest rate data play an important role to identify movements in the underlying regimes. We follow their insight and use bond price data jointly with the macro and survey observations to identify the model parameters and states. Different from their paper, we take an economically-motivated, long-run risks pricing approach to the term structure, and allow for the link between monetary policy and economic uncertainty. Bekaert and Moreno (2010) and Ang, Dong, and Piazzesi (2005) study the role of the monetary policy for the term structure using constant-coefficient Taylor rules and reduced-form specifications of the pricing kernel.

Our paper focuses on both the time-varying macroeconomic volatilities and regime shifts due to monetary policy as the main drivers of the bond risk premia. Hasseltoft (2012) and Bansal and Shaliastovich (2013) document the importance of the fluctuations in macroeconomic uncertainty for the time-variation in bond risk premia. In an alternative approach, Bekaert, Engstrom, and Xing (2009), Bekaert and Grenadier (2001) and Wachter (2006) use time-varying habits-formation models to study fluctuations in the bond risk premia. There is a large literature which incorporates discrete regimes changes into the

In terms of the earlier literature, Hamilton (1989) was the first to perform a Markov-switching, regime shift model using purely macroeconomic data in a traditional VAR setting, finding that state parameters correspond to peaks and troughs in the business cycle. The seminal work of Sims and Zha (2006) extended the Markov-switching to a Bayesian framework with a structural VAR setup. The large conclusion of this work was that monetary policy shifts have been brief if at all existent. In fact the model that fits the best is one where there is stochastic volatility in the disturbances of fundamental variables. Related to that, Primiceri (2005) shows that while monetary policy significantly varies over time, it appears to have little effect on the real economy. Consistent with these works, we find little effect of monetary policy for the levels of real growth and inflation. However, we find a significant effect on the macroeconomic uncertainty, and the conditional dynamics of bond prices and bond risk premia.

Our paper is organized as follows. The next section discusses the economic model. In the following two sections, we provide an overview of our estimation method and discuss our results. The last section concludes.

1 Economic Model

1.1 Stochastic Discount Factor

We derive a convenient representation of the stochastic discount factor, similar to Bansal et al. (2013), which incorporates pricing conditions of the recursive‑utility investor, but which does not force an inter‑temporal restriction between the short rate and the fundamental macroeconomic processes. This approach allows us to specify a flexible link between short rates and macroeconomic factors, which can be identified directly in the data. At the same time, we maintain economic restrictions on the fundamental risk sources and their market prices of risks, useful for pricing long‑term bonds.
As shown in Epstein and Zin (1989), under the recursive utility the log real stochastic discount factor (SDF) is given by,

\[ m_{t+1}^r = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \]  

(1)

where where \( \Delta c_{t+1} \) is the real consumption growth, and \( r_{c,t+1} \) is the return to the aggregate wealth portfolio. Parameter \( \gamma \) is a measure of a local risk aversion of the agent, \( \psi \) is the intertemporal elasticity of substitution, and \( \delta \in (0,1) \) is the subjective discount factor. For notational simplicity, parameter \( \theta \) is defined as \( \theta = \frac{1-\gamma}{1+\psi} \).

To obtain nominal SDF, we subtract inflation rate \( \pi_{t+1} \) from the real SDF:

\[ m_{t+1} = m_{t+1}^r - \pi_{t+1} \]

\[ = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1} - \pi_{t+1}. \]  

(2)

The nominal SDF is driven the consumption growth, inflation rate, and the unobserved return on the wealth portfolio.

Next we incorporate the budget constraint and the Euler equation for the short rates to characterize the expectations of the SDF, \( E_t m_{t+1} \), and the innovation into the SDF, \( N_{m,t+1} = m_{t+1} - E_t m_{t+1} \), in terms of the observed dynamics of the macroeconomic factors and the short rate.

The standard first-order condition implies that for any nominal return \( r_{t+1} \), the Euler equation should hold:

\[ E_t [\exp(m_{t+1} + r_{t+1})] = 1. \]

(3)

Using this condition for the one-period short-term nominal interest rate, \( r_{t+1} = i_t \), we obtain that:

\[ E_t m_{t+1} = -i_t - V_t. \]

(4)

The last component \( V_t \) is the entropy of the SDF:

\[ V_t = \log E_t(\exp(N_{m,t+1})). \]  

(5)

Up to the third-order terms, \( V_t \) is driven by the variance of the SDF; indeed, in a conditionally Gaussian model, \( V_t \) is exactly equal to half of the conditional variance of the
stochastic discount factor. This is why we refer to this component as capturing uncertainty or volatility risks.

Now let us relate the SDF innovation $N_{m,t+1}$ to the fundamental economic shocks, consistent with the recursive utility specification in (2). Consider the news into the current and future expected stochastic discount factor, $(E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j m_{t+j+1}$. Based on the recursive utility formulation in (2), it is equal to,

$$(E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j m_{t+j+1} = \frac{\theta}{\psi} (E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j \Delta c_{t+j+1}$$

$$+ (\theta - 1)(E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j r_{c,t+j+1} - (E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j \pi_{t+j+1}.$$ (6)

Note that the return to the consumption claim $r_{c,t+1}$ satisfies the budget constraint:

$$r_{c,t+1} = \log \frac{W_{t+1}}{W_t} \approx \kappa_0 + wc_{t+1} - \frac{1}{\kappa_1} wc_t + \Delta c_{t+1},$$ (7)

where $wc$ is the log wealth-consumption ratio and the parameter $\kappa_1 \in (0, 1)$ corresponds to the log-linearization coefficient in the investor's budget constraint. Iterating this equation forward, we obtain that the cash-flow news, defined as the current and future expected shocks to consumption, should be equal to the current and future expected shocks to consumption return:

$$N_{CF,t+1} = (E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j \Delta c_{t+j+1} = (E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j r_{c,t+j+1}.$$ (8)

With that, the right-hand side of (6) simplifies to,

$$(E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j m_{t+j+1} = -\frac{\theta}{\psi} N_{CF,t+1} + (\theta - 1) N_{CF,t+1} - N_{\pi,t+1}$$

$$= -\gamma N_{CF,t+1} - N_{\pi,t+1},$$ (9)

where the long-run inflation news are defined as,

$$N_{\pi,t+1} = (E_{t+1} - E_t) \sum_{j=0}^\infty \kappa_1^j \pi_{t+j+1}.$$ (10)
The news into the current and future expected stochastic discount factor, which is on the left-hand side of equation (9), incorporates current SDF shock $N_{m,t+1}$ and shocks to future expectations of the SDF, which we can characterize using the representation in (4):

$$
(E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_j^1 m_{t+j+1} = N_{m,t+1} - (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_j^1 (i_{t+j} + V_{t+j})
$$

$$
= N_{m,t+1} - N_{i,t+1} - N_{V,t+1},
$$

where the interest rate and volatility news are defined as follows:

$$
N_{i,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_j^1 i_{t+j}, \quad N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_j^1 V_{t+j}.
$$

Hence, we can represent the SDF shock as,

$$
N_{m,t+1} = -\gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1},
$$

and the total SDF is given by:

$$
m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + (N_{i,t+1} - N_{\pi,t+1}) + N_{V,t+1}.
$$

That is, under the recursive utility framework, the agent effectively is concerned about the long-run real growth news $N_{CF,t+1}$, long-run risk free rate news (inflation-adjusted short rate news $N_{i,t+1} - N_{\pi,t+1}$), and long-run uncertainty news $N_{V,t+1}$. The market price of the cash-flow risks is equal to the risk-aversion coefficient $\gamma$, while the market prices of both the interest rate and volatility shocks are negative 1: the marginal utility increases one-to-one with a rise in uncertainty or interest rates.

It is important to emphasize that the SDF representation above is common to all the recursive-utility based models. Indeed, to derive it we only used the Euler condition and the budget constraint, and did not make any assumptions about the dynamics of the underlying economy. In general equilibrium environments, the macroeconomic model assumptions are going to determine the decomposition of these underlying cash-flow, interest rate, and volatility risks into primitive economic shocks.

In our empirical approach, we rely on the SDF representation (14), instead of a more primitive specification in (2). This allows us to model short-term interest rates exogenously together with consumption and inflation processes, and yet maintain the pricing implica-
tions of the recursive utility SDF. Notably, the uncertainty term is still endogenous in our framework, as the volatility term $V_t$ and the innovations $N_{V,t+1}$ should be consistent with the entropy of the SDF in (5).

1.2 Economic Dynamics

In this section we specify the exogenous dynamics of consumption, inflation, and the short rates. This, together with the specification of the SDF and the Euler condition, allows us to solve for the prices of long-term nominal bonds.

We first specify a Markov chain to represent the time-variation in monetary policy regimes $s_t$. We assume $N$ states with a transition matrix, $T$, given by:

$$ T = \begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{pmatrix} = \begin{pmatrix} : & : & : & : \\ T_1 & T_2 & \cdots & T_N \\ : & : & : & : \end{pmatrix} $$

where each $\pi_{ji}$ indicates the probability of moving from state $i$ to state $j$. Each column, $T_i$, is the vector of probabilities of moving from state $i$ to all other states next period.

We next specify the exogenous dynamics of the macroeconomic state variables. Our asset-pricing framework underscores the importance of long-run, persistent movements in consumption, inflation, and fundamental volatility. To capture these risks, we directly specify exogenous processes for consumption and inflation, which feature persistent fluctuations in their expectations and volatilities. In this sense, our approach is different from “New Keynesian” based models which feature alternative empirical specifications for output gap and a Phillips curve, such as those in Bikbov and Chernov (2013), Campbell, Pflueger, and Viceira (2013), and Bekaert and Moreno (2010).

Specifically, similar to Bansal and Shaliastovich (2013), we specify an exogenous dynamics for consumption and inflation, which incorporates persistent movements in the conditional expectations and volatilities. The realized consumption and inflation are given by,

$$ \Delta c_{t+1} = \mu_c(s_t) + x_{c,t} + \sigma^*_c \epsilon_{c,t+1} $$
$$ \pi_{t+1} = \mu_\pi(s_t) + x_{\pi,t} + \sigma^*_\pi \epsilon_{\pi,t+1}. \quad (15) $$
Notably, we allow the components of the conditional means to depend on the monetary regime \( s_t \), which allows us to identify the variation in expectations of future consumption and inflation related to monetary policy. Fundamentals are also driven by "non-policy" shocks such as movements in persistent expected components, \( x_{i,t} \), as well as \( \epsilon_{i,t+1} \) which represent i.i.d. Gaussian short-run shocks.

The dynamics of the expected consumption and expected inflation states \( X_t = [x_{ct}, x_{\pi t}]' \) follows a VAR(1) process:

\[
\begin{bmatrix}
  x_{c,t+1} \\
  x_{\pi,t+1}
\end{bmatrix} =
\begin{bmatrix}
  \Pi_{cc} & \Pi_{c\pi} \\
  \Pi_{\pi c} & \Pi_{\pi \pi}
\end{bmatrix}
\begin{bmatrix}
  x_{c,t} \\
  x_{\pi,t}
\end{bmatrix} + \Sigma_t \epsilon_{t+1}.
\]

(16)

This representation allows us to capture persistence of expected consumption and inflation risks, as measured by the values of \( \Pi_{cc} \) and \( \Pi_{\pi \pi} \). Further, this specification can also incorporate “inflation non-neutrality,” that is, a negative response of future expected consumption to high expected inflation \( (\Pi_{c\pi} < 0) \). As shown in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005), inflation non-neutrality is important to account for the bond market data. For simplicity, the persistence matrix \( \Pi \) is constant. The model can be extended to accommodate regime-dependent persistence coefficients.

In general, the volatilities of expected real growth and inflation states are given by:

\[
\Sigma_t =
\begin{pmatrix}
  \sigma_{c,0} & 0 \\
  0 & \sigma_{\pi,t}\n\end{pmatrix} =
\begin{pmatrix}
  \sqrt{\delta^c(s_t) + \tilde{\sigma}^2_{c,t}} & 0 \\
  0 & \sqrt{\delta^\pi(s_t) + \tilde{\sigma}^2_{\pi,t}}\n\end{pmatrix}
\]

Each conditional variance depends on the monetary policy regime and is also driven by an orthogonal component \( \tilde{\sigma}_{i,t} \). This component captures movements in macroeconomic volatilities which are independent from the monetary policy. Their dynamics is specified as follows:

\[
\begin{align*}
  \tilde{\sigma}^2_{ct} &= \tilde{\sigma}^2_{c,0} + \varphi_c \tilde{\sigma}^2_{c,t-1} + \omega_c \eta_{\sigma c t}, \\
  \tilde{\sigma}^2_{\pi t} &= \tilde{\sigma}^2_{\pi,0} + \varphi_\pi \tilde{\sigma}^2_{\pi,t-1} + \omega_\pi \eta_{\sigma \pi t}.
\end{align*}
\]

(17)

For simplicity, the exogenous volatilities are driven by Gaussian shocks. The specification can be extended to square root processes, as in Tauchen (2005), or positive Gamma shocks.

Notably, in our specification macroeconomic volatilities can be systematically differ-

\[\text{In empirical implementation, we focus on the time-variation in inflation volatility, and set consumption volatility to be constant.}\]
ent across the monetary policy regimes. This captures the link between the fluctuations in monetary policy and aggregate economic uncertainty. Many of the existing specifications which incorporate fluctuations in monetary policy and volatilities do not entertain comovements between the two (see e.g., Song (2014) and Bikbov and Chernov (2013)).

Finally, we specify the dynamics of the short rate. It follows a modified Taylor rule, in which the monetary authority reacts to expected growth and expected inflation, and the stance of the monetary policy can vary across the regimes. Specifically,

\[ i_t = i_0 + \alpha_c(s_t) (x_{ct} + \mu_c(s_t)) + \alpha_\pi(s_t) (x_{\pi,t} + \mu_\pi(s_t)) \]

(18)

\[ \equiv \alpha_0(s_t) \]

The loadings \( \{\alpha_c(s_t), \alpha_\pi(s_t)\} \) are the key regime-dependent parameters of monetary policy. The interpretation of this Taylor rule is that the short rate loads stochastically on both expected growth and inflation. The justification for a “forward-looking” Taylor rule has been empirically founded and shown in Clarida, Gali and Gertler (2003). For parsimony, we abstract from other sources of variation in the interest rate rule, such as monetary policy shocks, dependence on lag rates, etc. While they can be easily introduced in our framework, we opt for a simpler specification to focus on the time-variation in the growth and inflation loadings of the Taylor rule and their relation to the macroeconomy and bond markets.

### 1.3 Model Solution

In Appendix we show that the long-run cash-flow, inflation, interest rate, and volatility news can be expressed in terms of the underlying macroeconomic, interest rate, and regime-shift shocks. Specifically, the cash flow, inflation news and interest rates news are given by,

\[ N_{CF,t+1} = F_{CF,0}(s_{t+1}, s_t) + F_{CF,\varepsilon}(s_{t+1}, s_t)\varepsilon_t + \sigma_c^* \varepsilon_{c,t+1}, \]  

(19)

\[ N_{\pi,t+1} = F_{\pi,0}(s_{t+1}, s_t) + F_{\pi,\varepsilon}(s_{t+1}, s_t)\varepsilon_t + \sigma_\pi^* \varepsilon_{\pi,t+1}, \]  

(20)

\[ N_{I,t+1} = F_{I,0}(s_{t+1}, s_t) + F_{I,X}(s_{t+1}, s_t)X_t + F_{I,\varepsilon}(\ldots)\varepsilon_t + \sigma_{\varepsilon}^* \varepsilon_{\varepsilon,t+1}, \]  

(21)

where the functions \( F \) depend on the policy regimes and model parameters. Notably, the sensitivities of the long-term news to primitive economic shocks in general depend on current and future monetary policy regimes.
We can also show that the uncertainty term \( V_t \) is in general given by,

\[
V_t(s_t, X_t, \sigma_{c,t}^2, \sigma_{\pi,t}^2) = V_0(s_t) + V_1(s_t)X_t + V_2c(s_t)\sigma_{c,t}^2 + V_2\pi(s_t)\sigma_{\pi,t}^2, \tag{22}
\]

so that the volatility news are given by,

\[
N_{V,t+1} = F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)X_t + F_{v,c}(s_{t+1}, s_t)\sigma_{c,t}^2 + F_{v,\pi}(s_{t+1}, s_t)\sigma_{\pi,t}^2 \\
+ F_{v,\epsilon}(\ldots)\Sigma_t\epsilon_{t+1} + F_{v,\eta_c}(\ldots)\omega_c\eta_{c,t+1} + F_{v,\eta_\pi}(\ldots)\omega_\pi\eta_{\pi,t+1}. \tag{23}
\]

The coefficients are determined as part of the model solution, and are given in the Appendix.

Combining all the components together, we can represent the SDF in terms of the underlying macroeconomic, interest rate, and regime shift shocks:

\[
m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1} \\
= S_0 + S'_1X_t + S_{1,c}\sigma_{c,t}^2 + S_{1,\pi}\sigma_{\pi,t}^2 + S'_{1,\epsilon}\Sigma_t\epsilon_{t+1} + S_{2,\eta_c}\omega_c\eta_{c,t+1} + S_{2,\eta_\pi}\omega_\pi\eta_{\pi,t+1} \tag{24} \\
- \gamma \sigma^*_c\epsilon_{c,t+1} - \sigma^*_\pi\epsilon_{\pi,t+1}.
\]

The SDF loadings and the market prices of risks depend on the primitive parameters of the model. In this sense, the pricing restrictions of the recursive utility provide economic discipline on the dynamics of our pricing kernel. Notably, because short rate loadings are time-varying, the SDF coefficients generally depend on monetary policy regimes. In a model with constant Taylor rule coefficients, the volatility of the SDF and the asset risk premia fluctuate only because the volatilities of expected growth and expected inflation are time-varying. In the model with time-varying monetary policy, the SDF volatility varies also due to movements in the Taylor rule coefficients.

### 1.4 Nominal Term Structure

In our model, log bond prices, \( p^n_t \), and the bond yields \( y^n_t = -\frac{1}{n}p^n_t \) are (approximately) linear in the underlying expected growth, expected inflation, and volatility states, and the loadings vary across the regimes:

\[
y^n_t = -\frac{1}{n}p^n_t = A^n(s_t) + B^n_X(s_t)X_t + B^n_{c}(s_t)\sigma_{c,t}^2 + B^n_{\pi}(s_t)\sigma_{\pi,t}^2. \tag{25}
\]
For $n = 1$ we uncover the underlying Taylor rule parameters:

\[
\begin{align*}
A^1(s_t) & = \alpha_0(s_t), \\
B^1_X(s_t)' & = \alpha(s_t)', \\
B^1_{\sigma_C}(i) & = 0, \\
B^1_{\sigma_\pi}(i) & = 0.
\end{align*}
\]

We can further define one-period excess returns on $n$–maturity bond,

\[
rx_{t\to t+1, n} = ny_{t,n} - (n - 1)y_{t+1,n-1} - yt, 1.
\]

The expected excess return on bonds is approximately equal to,

\[
E_t(rx_{t\to t+1, n}) + \frac{1}{2} Var_t(rx_{t+1, n}) \approx -Cov_t(m_{t+1}, rx_{t+1, n})
\]
\[
= Cons(s_t) + r_{\sigma_C}(s_t)\tilde{\sigma}_c^2 + r_{\sigma_\pi}(s_t)\tilde{\sigma}_\pi^2.
\]

The risk premia in our economy are time varying because there are exogenous fluctuations in stochastic volatilities, and because bond exposures fluctuate across monetary policy regimes. The second, monetary policy channel is absent in standard macroeconomic models of the term structure which entertain constant bond exposures and rely on time-variation in macroeconomic volatilities to generate movements in the risk premia (see e.g., Bansal and Shaliastovich (2013), Hasseltoft (2012)). In the next section we assess the importance of the monetary policy risks to explain the term structure dynamics, above and beyond traditional economic channels.

2 Model Estimation

2.1 Data Description

We use macroeconomic data on consumption and inflation, survey data on expected real growth and expected inflation, and asset-price data on bond yields to estimate the model. For our consumption measure we use log real growth rates of expenditures on non-durable goods and services from the Bureau of Economic Analysis (BEA). The inflation measure corresponds to the log growth in the GDP deflator. The empirical measures of the ex-
pectations are constructed from the cross-section of individual forecasts from the Survey of Professional Forecasts at the Philadelphia Fed. Specifically, the expected real growth corresponds to the cross-sectional average, after removing outliers, of four-quarters-ahead individual expectations of real GDP. Similarly, the expected inflation is given by the average of four-quarters-ahead expectations of inflation. The real growth and inflation expectation measures are adjusted to be mean zero, and are rescaled to predict next-quarter consumption and inflation, respectively, with a loading of one. The construction of these measures follows Bansal and Shaliastovich (2009). Finally, we use nominal zero-coupon bond yields of maturities one through five years, taken from the CRSP Fama-Bliss data files. We also utilize the nominal three-month rate from the Federal Reserve to proxy for the short rate. Based on the length of the survey data, our sample is quarterly, from 1969 through 2014.

Table 1 shows the summary statistics for our variables. In our sample, the average short rate is 5.2%. The term structure is upward sloping, so that the five-year rate reaches 6.4%. Bond volatilities decrease with maturity from 3.3% at short horizons to about 3% at five years. The yields are very persistent. As shown in the bottom panel of the Table, real growth and inflation expectations are very persistent as well. The AR(1) coefficients for the real growth and inflation forecasts are 0.87 and 0.98, respectively, and are much larger than the those for the realized consumption growth and inflation. Figure 1 shows the time series of the realized and expected consumption growth and inflation rate. As shown on the Figure, the expected states from the surveys capture low frequency movements in the realized macroeconomic variables.

2.2 Estimation Method

In our empirical analysis of the model, we focus on the stochastic volatility channel of the expected inflation, and set the volatility of the expected real growth to be constant.

To identify the volatility level parameters, we set the monetary policy component of the inflation volatility in state one to be zero; to identify the regimes, we impose that the short rate sensitivity to expected inflation is highest in regime 2. Finally, we set the log-linearization parameter $\kappa_1$ to a typical value of .99 in the literature.

To estimate the model and write down the likelihood of the data, we represent the evolution of the observable macroeconomic, survey, and bond yield variables in a convenient

\footnote{Identification of real volatility is challenging in bond market data alone. In a related framework, Song (2014) incorporate equity market data, which are informative about movements in real uncertainty, to help estimate real volatility.}
state-space form:

(Measurement)

\[ y_{t+1}^{1:Ny} = A_{x}^{1:Ny}(s_{t+1}) + B_{x}^{1:Ny}(s_{t+1})X_{t+1} + B_{\sigma_{\pi}}^{1:Ny}(s_{t+1})\tilde{\sigma}_{\pi,t+1}^{2} + \Sigma_{u,y}u_{t+1,y}, \]
\[ \Delta c_{t+1} = \mu_{c}(s_{t}) + \epsilon_{1}^{c}X_{t} + \sigma_{c}^{*}\epsilon_{c,t+1}, \]
\[ \pi_{t+1} = \mu_{\pi}(s_{t}) + \epsilon_{2}^{\pi}X_{t} + \sigma_{\pi}^{*}\epsilon_{\pi,t+1}, \]
\[ x_{\text{SPF cons},t+1} = \mu_{c}(s_{t+1}) - E[\mu_{c}(s_{t+1})] + x_{c,t+1} + \sigma_{u,xc}u_{t+1,xc}, \]
\[ x_{\text{SPF infl},t+1} = \mu_{\pi}(s_{t+1}) - E[\mu_{\pi}(s_{t+1})] + x_{\pi,t+1} + \sigma_{u,x\pi}u_{t+1,x\pi}, \]

(Transition)

\[ X_{t+1} = \Pi X_{t} + \Sigma_{t}(\tilde{\sigma}_{\pi t,s_{t}})\epsilon_{t+1}, \]
\[ \tilde{\sigma}_{\pi t}^{2} = \tilde{\sigma}_{\pi 0}^{2} + \varphi_{\pi}\tilde{\sigma}_{\pi,t-1}^{2} + \omega_{\pi}\eta_{t}, \]
\[ s_{t} \sim \text{Markov Chain} (\mathbb{P}_{s}), \]

where \( Ny \) is the number of bond yields in the data. In the estimation we allow for Gaussian measurement errors on the observed yields and survey expectations, captured by \( u_{t+1,y} \) and \( u_{t+1,xc,x\pi} \). For parsimony and to stabilize the chains, we fix the volatilities of the measurement errors to be equal to \( 20\% \) of the unconditional volatilities of the factors. As we describe in the subsequent section, the ex-post measurement errors in the sample are much smaller than that.

The set of parameters, to be jointly estimated with the states, is denoted by \( \Theta \), is given by:

\[ \Theta = \{ \Pi, \delta^{\alpha_{\pi}}, \tilde{\sigma}_{\pi 0}^{2}, \varphi_{\pi}, \sigma_{c}^{*}, \sigma_{\pi}^{*}, i_{0}, \gamma, \mu_{c}^{1:N}, \mu_{\pi}^{1:N}, \omega_{c}^{1:N}, \omega_{\pi}^{1:N}, \mathbb{P}_{s} \}. \]

The estimation problem is quite challenging due to the fact that the observation equations are nonlinear in the state variables, and the underlying expectation, volatility, and regime state variables are latent. Because of these considerations, we cannot use the typical Carter and Kohn (1994) methodology which utilizes smoothed Kalman filter moments to draw states. Instead, to estimate parameters and latent state variables we rely on a Bayesian MCMC procedure using particle filter methodology to evaluate the likelihood function. As in Andrieu et al. (2010) and Fernandez-Villaverde and Rubio-Ramirez (2007), we embed the particle filter based likelihood into a Random Walk Metropolis Hastings algorithm and sample parameters in this way. Schorfheide et al. (2013) and Song (2014) entertain similar approaches to estimate versions of the long-run risks model.
3 Estimation Results

3.1 Parameter and State Estimates

Table 2 shows the moments of the prior and posterior distributions of the parameters. We chose fairly loose priors which cover a wide range of economically plausible parameters to maximize learning from the data. For example, a two-standard deviation band for the persistence of the expected inflation and expected consumption ranges from 0.5 to 1.0. The prior means for the scale parameters are set to typical values in the literature, and the prior standard deviations are quite large as well. Importantly, we are careful not to hardwire the fluctuations in monetary policy and their impact on macroeconomy and bond prices through the prior selection. That is, in our prior we assume that the monetary policy coefficients are the same across the regimes, and are equal to 1 for expected inflation and 0.5 for expected growth. Likewise, our prior distribution for the role of monetary policy on inflation volatility is symmetric and is centered at zero, and there is no difference in the conditional means of consumption and inflation across the regimes. Hence, we do not force any impact through the prior, and let the data determine the size and the direction of the monetary policy effects.

The table further shows the posterior parameter estimates in the data. The posterior median for the risk aversion coefficient is 13.6, which is smaller than the values entertained in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005). The expected consumption, expected inflation, and inflation volatility are very persistent: the median AR(1) coefficients are above 0.95. The expected inflation has a negative and non-neutral effect on future real growth: $\Pi_{t+1}$ is negative, consistent with findings in Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2005).

We further find that there are substantial fluctuations in the monetary policy in the data. The monetary policy regimes are quite persistent, with the probability of remaining in a passive regime of 0.955, and in the aggressive regime of 0.958. There is a significant difference in monetary policy across the regimes. Indeed, the median short rate loadings are equal to .75 and 1.68 on the expected growth and expected inflation, respectively, in aggressive regimes, which are larger than .54 and .94 in passive regimes. These differences are very significant statistically. Overall, our estimates for these regime coefficients corroborate the prior evidence for Taylor rule coefficients on inflation being above one; see e.g. Cochrane (2011), Gallmeyer et al. (2006), and Backus, Chernov and Zin (2013).

In terms of the impact of monetary policy on the macroeconomy, we find that the
expected consumption and expected inflation are somewhat lower in aggressive regimes. This is consistent with the evidence in Bikbov and Chernov (2013) who show that future output and inflation tend to decrease following a monetary policy shock. However, in our estimation the difference in expectations is not statistically significant across the regimes, mirroring the findings in Primiceri (2005) that monetary policy appears to have little effect on the levels of economic dynamics. On the other hand, we find that inflation volatility is quite different across the regimes. The value of \( \delta^\pi \) is positive and significant statistically and economically: total inflation volatility rises by about a quarter in aggressive regimes.

Our filtered series for the latent expected growth, expected inflation, inflation volatility, and monetary policy regimes are provided in Figures 2-4. The estimated expectations are quite close to the data counterparts, and are generally in the 90% confidence set. Some of the noticeable deviations between the model and the data include post-2007 period, when model inflation expectations are systematically below the data. Notably, this is a period of a zero lower bound and unconventional monetary policy, which are outside a simple Taylor rule specification considered in this model.

The exogenous component of inflation volatility is plotted in Figure 3. It is apparent that non-policy related volatility spikes up in the early to mid 1980’s and gradually decreases over time. The inflation volatility is quite low in the recent period, which reflects low variability in inflation expectations in the data.

Finally, we provide model-implied estimates of the monetary regime in Figure 4. The figure suggests that a shift to an aggressive regime occurred in the late-70’s / early-80’s period, in accordance with the Volcker period. In mid 90’s, there was a shift to a passive regime, consistent with the anecdotal evidence regarding the Greenspan loosening. These findings are consistent with the empirical evidence for the monetary policy regimes in Bikbov and Chernov (2013). In the crisis period our estimates suggest a passive regime, consistent with the observed evidence of lower levels and volatilities of the bond yields and risk premia in this period.

### 3.2 Model Implications for Bond Prices

Figure 5 shows the time series of model-implied yields. The model matches the yields quite well in the sample: the average pricing errors range from 0.08% for 1-year yields to about 0.03% for 3-year yields, and a good fit is apparent from the Figure. As shown in Figure 4, Branger, Schlag, Shaliastovich, and Song (2015) discuss the impact of a zero lower bound on the inference of economic states and model-implied yields in a related framework.
the model generates an unconditional upward sloping term structure and a downward sloping volatility term structure. These patterns are consistent with the data.

We next consider the conditional dynamics of bond prices implied by the model. In Figure 7 we report standardized bond loadings on the expected growth, expected inflation and inflation volatility. The Figure shows that bond yields increase at times of high expected real growth. This captures a standard inter-temporal trade-off effect: at times of high expected real growth agents do not want to save, so bond prices fall and yields increase. Because we are looking at the nominal bonds which pay nominal dollars, their prices fall at times of higher anticipated inflation, so bond yields also increase with expected inflation. Finally, while short rates do not respond to inflation volatility, long term yields increase at times of high volatility of inflation. This reflects a positive risk premium component which is embedded in long term yields, and which increases at time of high inflation volatility.

Interestingly, all the bond loadings are uniformly larger in aggressive relative to passive regimes. Hence, a higher sensitivity of short-term bonds to expected consumption and expected inflation risks in aggressive regime, embedded in the Taylor rule, persists across all the bond maturities. As bonds are riskier in aggressive regimes, the average levels and volatilities of bond yields are higher in aggressive relative to passive regimes, as shown in Figure 6.

3.3 Model Implications for Bond Premia

In the benchmark model, the market price of the expected growth risk is positive, while the market prices of risks are negative for expected inflation and volatility risks. Indeed, high marginal utility states are those associated with low expected real growth, high expected inflation, or high inflation volatility. As bond yield loadings are all positive to these risks, it implies that the bond exposure to expected real growth contributes negatively to bond risk premia, while bond exposures to inflation risks contribute positively to the bond risk premia. Table 3 shows the average bond risk premium in the model, and its decomposition into the underlying economic sources of risk. Quantitatively, the expected inflation risk premium is quite large, so the average bond risk premia are positive.

One of the key model parameters which determines the magnitude of the inflation premium, and thus the level of the risk premia and slope of the nominal term structure, is the inflation non-neutrality coefficient $\Pi_{ct}$. When this parameter is negative, as in the benchmark model, high expected inflation is bad news for future real growth. The inflation
non-neutrality implies that investors are significantly concerned about expected inflation news. Long-term bonds which are quite sensitive to expected inflation are thus quite risky, and require a positive inflation premium. In the middle panel of Table 3, we show the risk premia implications when the inflation non-neutrality parameter is set to zero. In this case, expected inflation risk premium is virtually zero, the bond risk premia are negative, and the entire term structure is downward sloping.

We plot the in-sample risk premia in Figure 8. Consistent with the above discussion, the bond risk premia are positive on average, and the term-structure of bond risk premia is upward-sloping, so that long-term bonds are riskier and have higher expected excess returns than short-term bonds. The risk premia fluctuate over the sample, and can even go negative, as in in the post 2000 sample when the volatility of expected inflation is quite below its average, and the economy is in a passive regime.

We next quantify the contribution of the monetary policy risks to the levels of the bond risk premia. Specifically, we set all the regime-dependent coefficients to be equal to their unconditional means, based on the median set of parameters. This includes the regime-shifting Taylor rule coefficients, the policy component of expected inflation volatility, as well as drift components in the fundamental consumption and inflation processes. As displayed in the last panel, we find that risk premia decrease, with larger absolute differences at the long end of the curve. However, the impact of the time-variation in monetary policy to the levels of the risk premia is quite modest, about 10-15 basis points.

On the other hand, we find that monetary policy fluctuations contribute significantly to the time-variation in bond risk compensation. Similar to the regime dependent structures in Bansal and Zhou (2002) and Dai et al. (2007), the time-variation in monetary policy coefficients creates nonlinearities in yields via regime dependent bond loadings that affect the fluctuations in the risk premia. We can think of this as a time-varying risk exposure channel, which is different from a time-varying quantity of risk generated through the conditional volatility present in the inflation expectations. Both of these channels help generate risk premia variability.

To examine the quantitative impact of monetary policy on risk premia fluctuations, in Table 4, we present the volatilities of risk premia under different model specifications, in sample and population. First, we consider a case where all the regime-shifting parameters are set to the constant unconditional averages, and only the non-policy portion of inflation volatility is present. Next we consider the case where we add the time-varying short rate sensitivity related to inflation, $\alpha_\pi$. Subsequently, in the third line, we allow for
time-variation in the policy portion of inflation volatility, $\delta^\pi$. The fourth line represents the case with variation in the growth-related Taylor rule coefficient, while the baseline configuration additionally allows for regime-dependent movements in expected growth and inflation. We find that only allowing for exogenous inflation volatility generates about 80% of the risk premia variation in population, and about 70% to 75% of the variation in sample. Incorporating movements in the inflation-related Taylor rule coefficient increases the variance of the risk premia by about 15% of the total risk premia variance. Interestingly, movements in the policy portion of inflation volatility then contribute an additional 15%. Allowing for the movements in the short rate sensitivity to growth brings down the risk premia variability back to value in the benchmark model, and incorporating movements in expected consumption and inflation does not materially alter the volatility of the risk premia. In total this suggests that the effects of monetary policy on risk premia variability is substantial – about 20% of baseline in total.

The impact of monetary policy on bond risk premia is quite substantial in the sample as well, as shown in Figure 9. Here the solid line is the case with only exogenous inflation volatility, while the dashed and circled lines represent cases with movements in inflation-related policy variables and the baseline parameters. The central takeaway is that adding monetary policy fluctuations increases the volatility of the risk premia. Relative to the model with constant monetary policy, the benchmark bond risk premia rise in the 1980s, and fall below zero, at a greater degree in the recent period.

In our model, the risk premia volatility is higher when the short rate sensitivity to growth is constant. To help interpret this result, consider the bond risk premia decomposition:

$$rp^c_t = Cons(s_t) + r_{\sigma_c}(s_t)\tilde{\sigma}_c^2 + r_{\sigma_\pi}(s_t)\tilde{\sigma}_\pi^2.$$ 

The first component captures the risk premia due to the inflation volatility and regime shifts risks. The second component captures the risk premia due to the expected consumption risks. This is constant within the regime because the amount of expected consumption risks, the real growth volatility, is assumed to be constant. Finally, the last component contains the compensation for the expected inflation risks. It is driven by the monetary policy fluctuations, and the movements in exogenous inflation volatility. As we showed in Table 3, the risk premium portion coming from the expected real growth risks is negative while that from expected inflation risk is positive: $r_{\sigma_c} < 0, r_{\sigma_\pi} > 0$. Further, both coefficients
become larger, in absolute value, in aggressive regimes, as bond riskiness increases. Hence, in the benchmark model movements in expected growth risk premia offset the movements in expected inflation risk premia. When short-rate sensitivity to real growth is constant, the expected real growth component of the bond premia is also constant, and thus the volatility of the risk premia increases.

The above decomposition also shows that bond risk premia can turn negative at times when expected inflation compensation is relatively low. This is more likely to happen in passive regimes, at times of low inflation volatility, and for long-maturity bonds, as shown in Figure 10. This is why the in-sample bond risk premia turn negative post 2000 when inflation volatility is low, and the economy is in the passive regime.

4 Conclusion

We estimate a novel, structurally motivated asset-pricing framework to assess the role of monetary policy fluctuations for the macroeconomy and bond markets. We find substantial fluctuations in the monetary policy in the data. Interestingly, monetary policy fluctuations do not seem to have a sizeable effect for the first moments of the macroeconomic variables: the expectations of real growth and inflation are not significantly different across the regimes. On the other hand, inflation uncertainty significantly increases, and interest rates respond stronger to economic risks in aggressive relative to passive regimes. The monetary policy fluctuations help increase persistent variations in the bond risk premia, and the policy fluctuations in regard to real growth and inflation concerns have offsetting effects on the level and volatility of the bond premia.

Our empirical findings for the impact of monetary policy on the expectations and volatilities of the macroeconomic variables have important implications for the conduct of monetary policy and the understanding of the transmission of the monetary policy risks. We leave the study of the economic mechanisms which can explain our evidence for future research. Further, in our paper we focus on a “conventional” monetary policy represented by changing coefficients in the Taylor rule. Other policy issues, such as a zero lower bound and unconventional policy channels, can play an important role to explain the recent evidence, and are also left for future research.
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Appendix

A Analytical Model Solution

In this section we present the details of the model solution.

A.1 Long-Run Cash Flow and Inflation News

Recall that the cash flow news is solved through

\[ N_{CF,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_i \Delta e_{t+j+1} \]

\[ = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_i \mu_c(s_{t+j}) + (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_i e'_t \chi_{t+j} + \sigma^*_c \epsilon_{t+1}. \]  

(28)

We start by calculating the first portion of the news related to \( \mu_c(\ldots) \). Note that,

\[ (E_{t+1} - E_t) \mu_c(s_{t+1}) = \mu_c(k) - \sum_{j=1}^{N} \pi_{ji} \mu_c(j) = \mu_c(k) - T'_i \mu_c = (e'_k - T'_i) \mu_c, \]

\[ (E_{t+1} - E_t) \mu_c(s_{t+2}) = T'_k \mu_c - \sum_{j} \pi_{ji} \sum_{j} \pi_{jj} \mu_c(j) = T'_k \mu_c - T'_i T'_j \mu_c, \]  

(29)

\[ \ldots \]

\[ (E_{t+1} - E_t) \mu_c(s_{t+j}) = [T'_k (T'_j)^{j-2} - T'_i (T'_j)^{j-1}] \mu_c \text{ for } j > 1. \]

Summing over \( j \), we obtain that,

\[ (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^j_i \mu_c(s_{t+j}) = \left\{ \kappa_1 (e'_k - T'_i) + \kappa_2 (T'_k - T'_i T'_j) (I - \kappa_1 T')^{-1} \right\} \mu_c. \]  

(30)

On the other hand, the revisions in the expectations of the continuous factors are given by,

\[ (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_i e'_t X_{t+j} = \kappa_1 e'_1 (I - \kappa_1 \Pi)^{-1} \Sigma_t \epsilon_{t+1}. \]  

(31)

Hence, the long-run cash flow news is given by,

\[ N_{CF,t+1} = \left\{ \kappa_1 (e'_k - T'_i) + \kappa_2 (T'_k - T'_i T'_j) (I - \kappa_1 T')^{-1} \right\} \mu_c \]

\[ + \kappa_1 e'_1 (I - \kappa_1 \Pi)^{-1} \Sigma_t \epsilon_{t+1} + \sigma^*_c \epsilon_{c,t+1} \]

\[ = F_{CF,0}(s_{t+1}, s_t) + F_{CF,1}(\ldots)' \Sigma_t \epsilon_{t+1} + \sigma^*_c \epsilon_{c,t+1}. \]  

(32)
In an analogous way we can show that the long-run inflation news is:

\[
N_{\pi,t+1} = \left\{ \kappa_1 (\epsilon_k - T_t') + \kappa_1^2 (T_k' - T_t' T_t') (I - \kappa_1 T_t')^{-1} \right\} \mu \pi \\
+ \kappa_1 \epsilon_2 (I - \kappa_1 \Pi)^{-1} \Sigma_t \epsilon_{t+1} + \sigma_\pi^* \epsilon_{\pi,t+1}
\]

(33)

A.2 Long-Run Nominal Interest Rate News

The interest rate news is specified by:

\[
N_{I,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_j^j \left[ \alpha_0(s_{t+j}) + \alpha(s_{t+j})' x_{t+j} \right].
\]

(34)

Similar to the cash flow news, the first part is equal to,

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_j^j \alpha_0(s_{t+j}) = \left\{ \kappa_1 (\epsilon_k - T_t') + \kappa_1^2 (T_k' - T_t' T_t') (I - \kappa_1 T_t')^{-1} \right\} \alpha_0.
\]

(35)

To compute the second component, note that:

\[
(E_{t+1} - E_t) \alpha(s_{t+1})' x_{t+1} = \alpha_k' x_{t+1} - \sum_{j=1}^{N} \pi_{ji} \alpha(j)' (\Pi X_t) \\
= \alpha_k' (\Pi X_t + \Sigma_t \epsilon_{t+1}) - T_t' \alpha (\Pi X_t) = (\alpha_k' - T_t' \alpha) \Pi X_t + \alpha_k' \Sigma_t \epsilon_{t+1},
\]

where \( \alpha \) indicates the stacked matrix of \( \alpha(j)' \) of all states. For the \( t + 2 \) state,

\[
(E_{t+1} - E_t) \alpha(s_{t+2})' x_{t+2} = T_k' \alpha \Pi X_{t+1} - \sum_{j=1}^{N} \sum_{\tilde{j}} \pi_{ji} \alpha(\tilde{j})' (\Pi^2 X_t) \\
= T_k' \alpha \Pi X_{t+1} - T_t' T_t' \alpha \Pi^2 X_t \\
= (T_k' \alpha \Pi - T_t' T_t' \alpha \Pi) \Pi X_t + T_k' \alpha \Pi \Sigma_t \epsilon_{t+1}.
\]

(37)

More generally for \( j \geq 2 \),

\[
(E_{t+1} - E_t) \alpha(s_{t+j})' x_{t+j} = [T_k' - T_t' T_t'] \left[ (T')^{j-1} \alpha \Pi^j \right] X_t + T_k' (T')^{j-2} \alpha \Pi^{j-1} \Sigma_t \epsilon_{t+1}.
\]

(38)

Denote the infinite sums, \( \{ S_{0,t}^1, S_{0,t}^2 \} \) which can be solved through the Ricatti equations,

\[
S_{0,t}^1 = \sum_{j=2}^{\infty} (T')^{j-1} \alpha \Pi^j \kappa_1' = \kappa_2^2 I' \alpha \Pi^2 + \kappa_1 T_t' S_{0,t}^1 \Pi,
\]

(39)

\[
S_{0,t}^2 = (T')^{-1} S_{0,t}^1 (\Pi)^{-1}.
\]

(40)
After summing across \( j \) we obtain that,

\[
N_{I,t+1} = \left\{ \kappa_1 (e_k' - T'_i) + \kappa^2_1 (T'_k - T'_i T') (I - \kappa_1 T')^{-1} \right\} \alpha_0 \\
+ \left\{ \kappa_1 [a_k' \Pi - T'_i \alpha \Pi] + [T'_k - T'_i T'] \Sigma^1_{0,I} \right\} X_t \\
+ \left\{ \kappa_1 \alpha'_k + T'_k \Sigma^2_{0,I} \right\} \Sigma_t \epsilon_{t+1}
\] (41)

\[
N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_1 V_{t+j}.
\] (43)

A.3 Long-Run Volatility News

We guess and verify that the uncertainty term \( V_t \) satisfies,

\[
V_t(s_t, X_t, \tilde{\sigma}^2_{e,t}, \tilde{\sigma}^2_{\pi,t}) = V_0(s_t) + V_1(s_t)' X_t + V_2c(s_t) \tilde{\sigma}^2_{e,t} + V_2\pi(s_t) \tilde{\sigma}^2_{\pi,t},
\] (42)

where \( \{V_0, V_1, V_2c, V_2\pi\} \) are the volatility loadings that are determined endogenously.

The long-run volatility news is given by,

\[
N_{V,t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa^j_1 V_{t+j}.
\] (43)

Similar to cash-flow and interest rate news, we can solve for the components of the volatility news as follows. The first term is given by,

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^j_1 V_0(s_{t+j}) = \left\{ \kappa_1 (e_k' - T'_i) + \kappa^2_1 (T'_k - T'_i T') (I - \kappa_1 T')^{-1} \right\} V_0,
\] (44)

where \( V_0 \) denotes the stacked matrix of \( V_0(j)' \). The second portion of the volatility news is given by:

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa^j_1 V_1(s_{t+j})' X_{t+j} = \left\{ \kappa_1 [V'_1, k \Pi - T'_i V'_1 \Pi] + [T'_k - T'_i T'] \Sigma^1_{0,V} \right\} X_t \\
+ \left\{ \kappa_1 V'_1, k + T'_k \Sigma^2_{0,V} \right\} \Sigma_t \epsilon_{t+1},
\] (45)

where the volatility loadings \( \{S^1_{0,V}, S^2_{0,V}\} \) are given by,

\[
S^1_{0,V} = \kappa^2_1 T'V_1 \Pi^2 + \kappa_1 T'S^1_{0,V} \Pi, \quad \text{and}
\] (46)

\[
S^2_{0,V} = (T')^{-1} S^1_{0,V} (\Pi)^{-1}.
\] (47)

The final two portions of the volatility news can be derived in an analogous way. They are equal
\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2c}(s_{t+j}) \tilde{\sigma}_{c,t+j}^2 = \left\{ \kappa_1 \left[ V_{2c}(k) \varphi_c - T'_1 V_{2c} \varphi_c \right] + \left[ T'_k - T'_i T' \right] S_{0,v2c} \right\} \tilde{\sigma}_{c,t}^2 \\
+ \left\{ \kappa_1 V_{2c}(k) + T'_k S_{0,v2c}^2 \right\} \omega_c \eta_{c.t+1},
\]

(48)

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j V_{2\pi}(s_{t+j}) \tilde{\sigma}_{\pi,t+j}^2 = \left\{ \kappa_1 \left[ V_{2\pi}(k) \varphi_\pi - T'_1 V_{2\pi} \varphi_\pi \right] + \left[ T'_k - T'_i T' \right] S_{0,v2\pi}^1 \right\} \tilde{\sigma}_{\pi,t}^2 \\
+ \left\{ \kappa_1 V_{2\pi}(k) + T'_k S_{0,v2\pi}^2 \right\} \omega_\pi \eta_{\pi,t+1}.
\]

For \( i = c, \pi \) the vector terms satisfy:

\[
S_{0,v2i}^1 = \kappa_1^i T' V_i \varphi_i^2 + \kappa_1^i T' S_{0,v2i}^1 \varphi_i, \\
S_{0,v2i}^2 = (T')^{-1} S_{0,v2i}^1 (\varphi_i)^{-1}.
\]

(49)

(50)

Hence, the volatility news is given by,

\[
N_{V,t+1} = F_{v,0}(s_{t+1}, s_t) + F_{v,X}(s_{t+1}, s_t)' X_t + F_{v,c}(s_{t+1}, s_t)' \tilde{\sigma}_c^2 + F_{v,\pi}(s_{t+1}, s_t)' \tilde{\sigma}_\pi^2 \\
+ F_{v,\epsilon}(\ldots) \Sigma_{i=1}^T \epsilon_{t+1} + F_{v,\eta_c}(\ldots) \omega_c \eta_{c,t+1} + F_{v,\eta_\pi}(\ldots) \omega_\pi \eta_{\pi,t+1},
\]

(51)

(52)

where the loadings satisfy:

\[
F_{v,0} = \left\{ \kappa_1 \left( c_t' - T'_i \right) + \kappa_1^2 \left( T'_k - T'_i T' \right) (I - \kappa_1 T')^{-1} \right\} V_0,
\]

\[
F_{v,X} = \left\{ \kappa_1 \left[ V_{1,k} \Pi - T'_1 V_1 \Pi \right] + \left[ T'_k - T'_i T' \right] S_{0,v}^1 \right\},
\]

\[
F_{v,c} = \left\{ \kappa_1 \left[ V_{2c}(k) \varphi_c - T'_i V_{2c} \varphi_c \right] + \left[ T'_k - T'_i T' \right] S_{0,v2c}^1 \right\},
\]

\[
F_{v,\pi} = \left\{ \kappa_1 \left[ V_{2\pi}(k) \varphi_\pi - T'_1 V_{2\pi} \varphi_\pi \right] + \left[ T'_k - T'_i T' \right] S_{0,v2\pi}^1 \right\},
\]

\[
F'_{v,0} = \left\{ \kappa_1 V_{1,k}' + T'_k S_{0,v}^2 \right\},
\]

\[
F'_{v,c} = \left\{ \kappa_1 V_{2c}(k) + T'_k S_{0,v2c}^2 \right\},
\]

\[
F'_{v,\pi} = \left\{ \kappa_1 V_{2\pi}(k) + T'_k S_{0,v2\pi}^2 \right\}.
\]

Now we solve for coefficients on the volatility factor. From the definition of the uncertainty component \( V_t \) it follow that,

\[
V_t = \log E_t \exp (N_{m,t+1}).
\]

(54)

The SDF shock be related to the primitive macroeconomic and regimes shocks,

\[
N_{m,t+1} = -\gamma N_{CF,t+1} + N_{I,t+1} - N_{\pi,t+1} + N_{V_t,t+1} \\
= M_0 + M_{1,X}' X_t + M_{1,\sigma_c}' \tilde{\sigma}_c^2 + M_{1,\sigma_\pi}' \tilde{\sigma}_\pi^2 + M_{2,\epsilon}' \Sigma_{i=1}^T \epsilon_{t+1} + M_{2,\eta_c}' \omega_c \eta_{c,t+1} + M_{2,\eta_\pi}' \omega_\pi \eta_{\pi,t+1} \\
- \gamma \sigma_c^2 \epsilon_{c,t+1} - \sigma_\pi^2 \epsilon_{\pi,t+1},
\]

(55)
where each loading in general depends on current and future monetary policy regime:

\[ M_0(s_t, s_{t+1}) = -\gamma F_{CF,0} + F_{I,0} - F_{\pi,0} + F_{v,0}, \]
\[ M_{1,X}(s_t, s_{t+1})' = F'_{I,X} + F'_{v,X}, \]
\[ M_{1,\sigma c}(s_t, s_{t+1}) = F_{v,\sigma c}, \]
\[ M_{1,\sigma \pi}(s_t, s_{t+1}) = F_{v,\sigma \pi}, \]
\[ M_{2,\epsilon}(s_t, s_{t+1})' = -\gamma F'_{CF,\epsilon} + F'_{I,\epsilon} - F'_{\pi,\epsilon} + F'_{v,\epsilon}, \]
\[ M_{2,\eta c}(s_t, s_{t+1}) = F_{v,\eta c}, \]
\[ M_{2,\eta \pi}(s_t, s_{t+1}) = F_{v,\eta \pi}. \]

(56)

Conditioning on next-period regime, one can show that,

\[ V_t = \log E_t \exp (N_{m,t+1}) \]
\[ = \log E_t \exp \left( \tilde{M}_0 + \tilde{M}_{1,X} X_t + \tilde{M}_{1,\sigma c} \tilde{\sigma}^2_{ct} + \tilde{M}_{1,\sigma \pi} \tilde{\sigma}^2_{\pi t} \right), \]

where

\[ \tilde{M}_0 = M_0 + \frac{1}{2} \left[ (M^1_{2,c})^2 \delta^{ac} \alpha_c(s_t) + (M^2_{2,c})^2 \delta^{ac} \alpha_c(s_t) + M^2_{2,\eta c} \omega^2_{\pi} + M^2_{2,\eta \pi} \omega^2_{\pi} \right] + \frac{1}{2} \gamma^2 (\sigma_*)^2 + \frac{1}{2} (\sigma^*)^2, \]
\[ \tilde{M}'_1 = M'_1, \]
\[ \tilde{M}_{1,\sigma c} = M_{1,\sigma c} + \frac{1}{2} \delta^{ac} (M^1_{2,c})^2, \]
\[ \tilde{M}_{1,\sigma \pi} = M_{1,\sigma \pi} + \frac{1}{2} \delta^{ac} (M^2_{2,c})^2. \]

(57)

To integrate out next-period regimes, similar to Bansal and Zhou (2002) and Song (2014), we use the approximation, \( \exp(y) \approx 1 + y \), which holds for small enough \( y \). It follows that,

\[ V_t = T'_t \tilde{M}_0(i) + \left( T'_t \tilde{M}_{1,X}(i) \right) X_t + T'_t \tilde{M}_{1,\sigma c}(i) \tilde{\sigma}^2_{ct} + T'_t \tilde{M}_{1,\sigma \pi}(i) \tilde{\sigma}^2_{\pi t}, \]

(59)

where \( \tilde{M}_0(i) \) is the stacked vector of \( \tilde{M}_0(i, \cdot) \), and \( \tilde{M}_{1,X}(i) \) is the stacked matrix of \( \tilde{M}_{1,X}(i, k)' \).

To guarantee an internally-consistent solution to the model, we equate the volatility specification in (42) to the equation above. This implies that:

\[ V_0(i) = T'_t \tilde{M}_0(i), \]
\[ V_{1X}(i) = T'_t \tilde{M}_{1,X}(i), \]
\[ V_{2c}(i) = T'_t \tilde{M}_{1,\sigma c}(i), \]
\[ V_{2\pi}(i) = T'_t \tilde{M}_{1,\sigma \pi}(i). \]

(60)

for all \( i \). This system is exactly identified, and allows us to endogenously relate volatility news to the primitive shocks and model parameters.
A.4 Nominal Bond Prices

The solution to the stochastic discount factor satisfies,

\[ m_{t+1} = -i_t - V_t - \gamma N_{CF,t+1} + N_{R,t+1} + N_{V,t+1} \]

\[ = S_0 + S'_{1,X} X_t + S_{1,\sigma c} \delta^{2}_{\sigma c,t} + S_{1,\sigma \pi} \delta^{2}_{\sigma \pi,t} + S'_{2,\epsilon} \Sigma_t \epsilon_{t+1} + S_{2,\eta c} \omega_c \eta_{c,t+1} + S_{2,\eta \pi} \omega_\pi \eta_{\pi,t+1} \]

where each of the coefficients are given by,

\[ S_0(s_t, s_{t+1}) = M_0 - \alpha_0 - V_0, \]
\[ S_{1,X}(s_t, s_{t+1})' = M'_{1,X} - \alpha' - V'_1, \]
\[ S_{1,\sigma c}(s_t, s_{t+1}) = M_{1,\sigma c} - V_{2c}, \]
\[ S_{1,\sigma \pi}(s_t, s_{t+1}) = M_{1,\sigma \pi} - V_{2\pi}, \]
\[ S_{2,\epsilon}(s_t, s_{t+1})' = M'_{2,\epsilon}, \]
\[ S_{2,\eta c}(s_t, s_{t+1}) = M_{2,\eta c}, \]
\[ S_{2,\eta \pi}(s_t, s_{t+1}) = M_{2,\eta \pi}. \]  

In the model, log bond prices, \( p^n_t \), are linear in states, and the bond loadings vary with the monetary policy regime:

\[ p^n_t = \tilde{A}^n(s_t) + \tilde{B}'_X(s_t) X_t + \tilde{B}'_{\sigma c}(s_t) \delta^{2}_{\sigma c} + \tilde{B}'_{\sigma \pi}(s_t) \delta^{2}_{\sigma \pi}, \]

where for the short term bond \( n = 1 \) the loadings satisfy

\[ \tilde{A}^n(s_t) = -\alpha_0(s_t), \]
\[ \tilde{B}'_X(s_t) = -\alpha(s_t)', \]
\[ \tilde{B}'_{\sigma c}(s_t) = 0, \]
\[ \tilde{B}'_{\sigma \pi}(s_t) = 0. \]  

Using similar approach as before to solve for the bond prices, we obtain that the bond loadings for longer maturities satisfy,

\[ \tilde{A}^n(i) = \frac{1}{2} \left( \gamma^2 (\sigma^*_c)^2 + (\sigma^*_\pi)^2 \right), \]

\[ + \sum_k \pi_{ki} \left\{ \tilde{S}_0 + \frac{1}{2} \left[ \left( \tilde{S}^{i}_{1,\epsilon} \right)^2 \delta^{\alpha c} \alpha_c(s_t) + \left( \tilde{S}^{i}_{2,\epsilon} \right)^2 \delta^{\alpha \pi} \alpha_\pi(s_t) + \tilde{S}^2_{2,\eta c} \omega^2_c + \tilde{S}^2_{2,\eta \pi} \omega^2_\pi \right] \right\}, \]

\[ \tilde{B}'_{X}(i) = \sum_k \pi_{ki} \tilde{S}'_{1,X}, \]

\[ \tilde{B}'_{\sigma c}(i) = \sum_k \pi_{ki} \left\{ \tilde{S}_{1,\sigma c} + \frac{1}{2} \delta^{\sigma c} \left( \tilde{S}^{i}_{1,\epsilon} \right)^2 \right\}, \]

\[ \tilde{B}'_{\sigma \pi}(i) = \sum_k \pi_{ki} \left\{ \tilde{S}_{1,\sigma \pi} + \frac{1}{2} \delta^{\sigma c} \left( \tilde{S}^{i}_{1,\epsilon} \right)^2 \right\}. \]
Tables and Figures

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>3M</th>
<th>1Y</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
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<td><strong>Bond Yields:</strong></td>
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<td>Mean (%)</td>
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<td>5.90</td>
<td>6.09</td>
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<td>6.39</td>
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<td>Std. Dev. (%)</td>
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<td>3.33</td>
<td>3.28</td>
<td>3.18</td>
<td>3.09</td>
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<td>AR(1)</td>
<td>.942</td>
<td>.954</td>
<td>.961</td>
<td>.965</td>
<td>.967</td>
<td>.969</td>
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<table>
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<tr>
<th></th>
<th>△c</th>
<th>π</th>
<th>x&lt;sub&gt;c&lt;/sub&gt;</th>
<th>x&lt;sub&gt;π&lt;/sub&gt;</th>
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<td><strong>Macro and Survey Data:</strong></td>
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<td>Mean (%)</td>
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<td>Std. Dev. (%)</td>
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Table 2: Prior and Posterior Distributions

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<th>Prior Distr.</th>
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<th>50%</th>
<th>95%</th>
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<td>(\Pi_{cc})</td>
<td>(N^T)</td>
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<td>.2</td>
<td>.956</td>
<td>.976</td>
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<tr>
<td>(\Pi_{\pi c})</td>
<td>(N^T)</td>
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<td>0</td>
</tr>
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<td>(\Pi_{c\pi})</td>
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<td>-.015</td>
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<td><strong>Non-Policy Volatility Parameters:</strong></td>
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<tr>
<td>(\tilde{\sigma}_c^2\times 10^5)</td>
<td>(IG)</td>
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<td>.02</td>
<td>.011</td>
<td>.018</td>
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<tr>
<td>(\tilde{\sigma}_\pi^2\times 10^5)</td>
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<td>.01</td>
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<td><strong>Regime-Shifting Coefficients:</strong></td>
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<td>(\mu_c(s_1))</td>
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<td>.001</td>
<td>.0024</td>
<td>.0042</td>
</tr>
<tr>
<td>(\mu_c(s_2))</td>
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<td>.001</td>
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<tr>
<td>(\delta^c(s_2)\times 10^5)</td>
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<td>.006</td>
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<td>(\alpha_c(s_1))</td>
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<td>(\alpha_\pi(s_2))</td>
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<td>(\pi_{11})</td>
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<td><strong>Other Parameters:</strong></td>
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<td>(\gamma)</td>
<td>(G)</td>
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<td>12.92</td>
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<td>(i_0)</td>
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<td>.01</td>
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<td>-.0008</td>
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</table>

The table summarizes the prior and posterior distributions for the model parameters. \(G\) refers to Gamma distribution, \(N\) to Normal distribution, \(N^T\) is truncated (at zero and/or one) Normal distribution, and \(IG\) is Inverse-Gamma. Dashed line indicates that the parameter value is fixed.
Table 3: Risk Premia Levels

<table>
<thead>
<tr>
<th>Baseline Parameters:</th>
<th>n = 6M</th>
<th>1Y</th>
<th>3Y</th>
<th>5Y</th>
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</thead>
<tbody>
<tr>
<td>Exp Growth Shocks (%)</td>
<td>-.205</td>
<td>-.601</td>
<td>-2.02</td>
<td>-3.19</td>
</tr>
<tr>
<td>Exp Inflation Shocks (%)</td>
<td>.324</td>
<td>.920</td>
<td>2.74</td>
<td>3.91</td>
</tr>
<tr>
<td>Other Shocks (%)</td>
<td>0.00</td>
<td>0.03</td>
<td>.430</td>
<td>1.09</td>
</tr>
<tr>
<td>Total (%)</td>
<td>.119</td>
<td>.348</td>
<td>1.16</td>
<td>1.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inflation Neutrality:</th>
<th>n = 6M</th>
<th>1Y</th>
<th>3Y</th>
<th>5Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp Growth Shocks (%)</td>
<td>-.205</td>
<td>-.601</td>
<td>-2.02</td>
<td>-3.19</td>
</tr>
<tr>
<td>Exp Inflation Shocks (%)</td>
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<td>.022</td>
<td>.026</td>
<td>-.009</td>
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<tr>
<td>Other Shocks (%)</td>
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<tr>
<td>Total (%)</td>
<td>-.196</td>
<td>-.579</td>
<td>-1.99</td>
<td>-3.20</td>
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</table>

<table>
<thead>
<tr>
<th>Constant Monetary Policy:</th>
<th>n = 6M</th>
<th>1Y</th>
<th>3Y</th>
<th>5Y</th>
</tr>
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<tbody>
<tr>
<td>Exp Growth Shocks (%)</td>
<td>-.205</td>
<td>-.601</td>
<td>-2.02</td>
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<tr>
<td>Exp Inflation Shocks (%)</td>
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<td>Other Shocks (%)</td>
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<tr>
<td>Total (%)</td>
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<td>.311</td>
<td>1.07</td>
<td>1.71</td>
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</table>

The table shows the decomposition of the average bond risk premia into the risk contributions due to expected growth shocks, expected inflation shocks, and the remaining shocks (inflation volatility and the regime shifts). The Baseline case refers to the benchmark estimation of the model. For the Constant Monetary Policy case the regime-dependent coefficients are fixed at their unconditional averages. For the Inflation Neutrality case the feedback between expected consumption and expected inflation is set to zero. All statistics are in annual terms and in percentages, and are computed at the median parameter draw.
Table 4: Risk Premia Volatilities

<table>
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<tr>
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<th>n = 6M</th>
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<th>3Y</th>
<th>5Y</th>
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<tr>
<td><strong>In-Sample:</strong></td>
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<tr>
<td>% of Baseline</td>
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<tr>
<td>Movements in $\tilde{\sigma}_{\pi t}$</td>
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<td>69.7</td>
<td>72.7</td>
<td>74.5</td>
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<tr>
<td>Movements in ${\tilde{\sigma}<em>{\pi t}, \alpha</em>{\pi t}}$</td>
<td>95.1</td>
<td>94.8</td>
<td>93.7</td>
<td>92.9</td>
</tr>
<tr>
<td>Movements in ${\tilde{\sigma}<em>{\pi t}, \delta</em>{\pi t}, \alpha_{\pi t}}$</td>
<td>109.5</td>
<td>109.2</td>
<td>108.4</td>
<td>107.7</td>
</tr>
<tr>
<td>Movements in ${\tilde{\sigma}<em>{\pi t}, \delta</em>{\pi t}, \alpha_{\pi t}, \alpha_{ct}}$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Baseline (%)</td>
<td>.231</td>
<td>.649</td>
<td>1.87</td>
<td>2.62</td>
</tr>
</tbody>
</table>

| **Population:**          |        |     |     |     |
| % of Baseline            |        |     |     |     |
| Movements in $\tilde{\sigma}_{\pi t}$ | 79.9   | 80.8| 83.4| 85.0|
| Movements in $\{\tilde{\sigma}_{\pi t}, \alpha_{\pi t}\}$ | 94.9   | 94.6| 93.8| 93.3|
| Movements in $\{\tilde{\sigma}_{\pi t}, \delta_{\pi t}, \alpha_{\pi t}\}$ | 110.6  | 110.2| 108.8| 107.8|
| Movements in $\{\tilde{\sigma}_{\pi t}, \delta_{\pi t}, \alpha_{\pi t}, \alpha_{ct}\}$ | 100.0  | 100.0| 100.0| 100.0|
| Baseline (%)             | .178   | .503| 1.47| 2.06|

This table shows the in-sample and population risk premia volatilities in the restricted models which incorporate time-varying exogenous inflation volatility ($\tilde{\sigma}_{\pi}$), monetary portion of inflation volatility ($\delta_{\pi}$), Taylor rule coefficients on inflation and consumption ($\alpha_{\pi}, \alpha_{c}$). The baseline model additionally includes movements in the consumption and inflation drifts ($\mu_{c}, \mu_{\pi}$). The bottom line represents the annualized volatility, in percentage terms. The top lines represent volatilities as a percentage of the baseline value.
The top panel of the figure shows the realized (dashed line) and expected (solid line) consumption growth. The bottom panel shows the realized and expected inflation. Real growth and inflation expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4. The variables are demeaned, and are reported at the annual basis in percentage terms.
Figure 2: Estimated Macroeconomic States

Real Growth:

Inflation:

The top panel of the figure shows the expected real growth in the survey data (dashed line), and the estimated posterior median from the model (solid line). The bottom panel shows the expected inflation in the survey data and in the model. Grey region represents posterior (5%, 95%) credible sets. Data expectations are constructed from the Survey of Professional Forecasters. Quarterly observations from 1968Q3 to 2013Q4. The variables are reported at the annual basis in percentage terms.
Figure 3: Estimated Inflation Volatility

The figure shows the estimated posterior median of the exogenous component of inflation volatility. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4. The variables are reported at the annual basis in percentage terms.
The figure shows the estimated posterior median of the monetary policy regime. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.
The figure shows the nominal bond yields in the data (red line), and the estimated posterior median from the model. Grey region represents posterior (5%, 95%) credible sets. Quarterly observations from 1968Q3 to 2013Q4.
Figure 6: Unconditional Levels and Volatilities of Yields

(a) Levels

(b) Volatilities

The figure shows model-implied unconditional levels and volatilities of bond yields across monetary policy regimes.
The figure shows the model-implied bond loadings on the expected real growth, expected inflation and inflation volatility, and the unconditional bond yields in aggressive and passive regimes. Bond loadings are standardized to capture a one standard deviation movement in each factor, and are computed at the median parameter draw.
Figure 8: Model-Implied Risk Premia

The figure shows the estimated bond risk premia in the sample. We display the one-quarter risk premia for one-, three-, and five-year to maturity bonds. All model-implied values are computed at median parameter values and states.
This figure displays the in-sample time series of the five year bond risk premia in the restricted model which incorporates only time-varying exogenous inflation volatility ($\tilde{\sigma}_\pi$); and the model which also adds monetary portion of inflation volatility ($\delta \pi$) and Taylor rule coefficients on inflation $\delta\pi$. The baseline model additionally includes movements in the Taylor rule coefficients to real growth, and monetary-policy components of the consumption and inflation drifts ($\mu_c, \mu_\pi$). The economic states correspond to the benchmark estimate of the model.
Figure 10: Conditional Risk Premia in the Model

The figure shows the model-implied risk premia for one- and five-year bonds with respect to standardized movements of inflation volatility. The solid lines are the risk premia that result in passive regimes while the dashed ones result in aggressive regimes.