1. (a) The yield on the bond (assuming annual compounding) is:

\[ r = (1000/800)^{1/5} - 1 = .04564 \]

(b) With a yield of 4.564%, the present value (that is, the price) of a three year zero-coupon bond with face value 1000 is:

\[ P = 1000/(1.04564)^3 = 874.69. \]

2. (a) The annual return in this case equals the yield to maturity. The yield to maturity is the rate \( r \) that solves:

\[ \$850 = \frac{\$1000}{(1 + r)^3}. \]

Inverting the equation as usual, we find:

\[ r = \left( \frac{\$1000}{\$850} \right)^{1/3} - 1 = 0.056 \]

(b) Recall that the yield to maturity is defined as the interest rate that makes the present value of the payments equal to the price. Hence the yield to maturity (annual compounding) solves

\[ \$975 = \frac{\$70}{(1 + r)} + \frac{\$70}{(1 + r)^2} + \frac{\$1070}{(1 + r)^3}. \]

When we put in \( r = 5.6\% \), we find:

\[ \frac{\$70}{1.056} + \frac{\$70}{1.056^2} + \frac{\$1070}{1.056^3} = \$1037.70 \]

Thus to make the present value equal to the price of \$975, the yield to maturity must be greater than 5.6%, the yield to maturity for security A.
3. (a) With $F$ equal to face value, $P$ equal to purchase price and $t$ equal to years to maturity, the yield to maturity for a zero coupon bond is given by:

$$\text{YTM} = \left(\frac{F}{P}\right)^{\frac{1}{t}} - 1$$

$$= \left(\frac{$1000}{$800}\right)^{\frac{1}{5}} - 1 = 0.0456$$

so

$$\text{YTM} = 4.56\%.$$ 

(b) Holding period return is given by

$$\text{HPR} = \left(\frac{P_t}{P_0}\right)^{\frac{1}{t}} - 1$$

where $P_0$ is the purchase price of the bond, $P_t$ is the selling price, and $t$ is the number of years the bond is held. This bond must also yield 7% to those you sell it to after one year. Using the formula for the price of a zero and recalling that the 5-year bond becomes a 4-year bond after one year, we have:

$$P_t = \frac{$1000}{(1 + 0.07)^4} = $762.90$$

Therefore, the holding period return when you sell this bond after one year assuming yields have increased from 7% is:

$$\text{HPR} = \frac{$762.90}{$800} - 1 = -0.0464$$

Your return is less than the YTM because yields rose and you sold the bond at a lower price. Moreover, you actually lost money.

(c) The selling price after 2 years is:

$$P_t = \frac{$1000}{(1 + 0.07)^3} = $816.30$$

and your holding period return over the two-year period is:

$$\text{HPR} = \left(\frac{$816.30}{$800}\right)^{\frac{1}{2}} - 1 = 0.0101$$

Note that even though rates are still 7%, your holding period return over the two years is positive because the selling price of the bond is now higher than after one year.
(d) Since the yield to maturity on comparable zeros is now 3% and there are two years left to maturity, your selling price is:

\[ P_t = \frac{1000}{(1 + .03)^2} = 942.60 \]

and your holding period return over three years is:

\[ \text{HPR} = \left( \frac{942.60}{800} \right)^{\frac{1}{3}} - 1 = 0.056 \]

(e) After four years your selling price is:

\[ P_t = \frac{1000}{1 + .03} = 970.87 \]

and your holding period return over four years is:

\[ \text{HPR} = \left( \frac{970.87}{800} \right)^{\frac{1}{4}} - 1 = 0.0496 \]

(f) After five years the bond sells for its face value because it can be redeemed for $1000. Therefore, no matter what yields are after five years \( P_t = 1000 \) and your holding period return is:

\[ \text{HPR} = \left( \frac{1000}{800} \right)^{\frac{1}{5}} - 1 = 0.0456 \]

(g) If you sell a bond prior to maturity, the holding period return earned need not equal the yield to maturity implied by the price you paid when you purchased the bond. The calculations in (b) and (c) show that if yields rise, your holding period return is lower while (d) and (e) show that if yields decline your holding period return is higher. Nevertheless, because bond prices are “pulled to par,” if you hold a zero coupon bond to its final maturity your holding period return will equal the yield to maturity calculation on the day you purchased the bond.
4. (a) The $1000 investment in the zero coupon bond will return $1538.62 at the end of the five year period: $1000(1.09)^5 = 1538.62$.

(b) The five-year 9% coupon bond pays a coupon of $90 every year (for five years) in addition to returning the $1,000 face value at maturity. To calculate how much money you will end up with at maturity, sum the values of the individual coupons reinvested from the time received until the maturity of the initial bond. In other words, the $90 coupon received at the end of year one is reinvested for 4 years, which at a nine percent return, will give you $127.04 at year 5. It is helpful to use a diagram to keep track of the cash flows:

<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon</th>
<th>Reinvestment</th>
<th>Value at Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$90</td>
<td>$90(1.09)^4</td>
<td>$127.04</td>
</tr>
<tr>
<td>2</td>
<td>$90</td>
<td>$90(1.09)^3</td>
<td>$116.55</td>
</tr>
<tr>
<td>3</td>
<td>$90</td>
<td>$90(1.09)^2</td>
<td>$106.93</td>
</tr>
<tr>
<td>4</td>
<td>$90</td>
<td>$90(1.09)</td>
<td>$98.10</td>
</tr>
<tr>
<td>5</td>
<td>$1090</td>
<td>$1090</td>
<td>$1090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>$1538.62</td>
</tr>
</tbody>
</table>

The sum of the values at the end of year 5 is $1,538.62.

(c) If the coupons are reinvested at 12%, the value of the payments received in year five are:

<table>
<thead>
<tr>
<th>Year</th>
<th>Coupon</th>
<th>Reinvestment</th>
<th>Value at Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$90</td>
<td>$90(1.12)^4</td>
<td>$141.62</td>
</tr>
<tr>
<td>2</td>
<td>$90</td>
<td>$90(1.12)^3</td>
<td>$126.44</td>
</tr>
<tr>
<td>3</td>
<td>$90</td>
<td>$90(1.12)^2</td>
<td>$112.90</td>
</tr>
<tr>
<td>4</td>
<td>$90</td>
<td>$90(1.12)</td>
<td>$100.08</td>
</tr>
<tr>
<td>5</td>
<td>$1090</td>
<td>$1090</td>
<td>$1090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>$1571.76</td>
</tr>
</tbody>
</table>

The sum of the values at the end of year 5 in this case is $1,571.76.

(d) If the coupons can be reinvested only at 6%, then the total value of the payments
received in year five are:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$90</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$90(1.06)^4 = $113.62</td>
</tr>
<tr>
<td>$90</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$90(1.06)^3 = $107.19</td>
</tr>
<tr>
<td>$90</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$90(1.06)^2 = $101.12</td>
</tr>
<tr>
<td>$90</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$90(1.06) = $95.4</td>
</tr>
<tr>
<td>$1090</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$1090 = $1090</td>
</tr>
<tr>
<td>Total</td>
<td>→</td>
<td>→</td>
<td>→</td>
<td>$1507.34</td>
</tr>
</tbody>
</table>

The sum of the values at the end of year 5 in this case is $1,507.34.

(e) The definition of holding period return is

\[ HPR = \left( \frac{V_t}{V_0} \right)^{\frac{1}{t}} - 1 \]

where \( V_t \) is the value accumulated at the end and \( V_0 \) is the value at the beginning.

Thus, we have for (a) -(d):

a. \( \left( \frac{1538.62}{1000} \right)^{\frac{1}{5}} - 1 = .09 \)

b. \( \left( \frac{1538.62}{1000} \right)^{\frac{1}{5}} - 1 = .09 \)

c. \( \left( \frac{1571.76}{1000} \right)^{\frac{1}{5}} - 1 = .0947 \)

d. \( \left( \frac{1507.34}{1000} \right)^{\frac{1}{5}} - 1 = .0855 \)

We see that the holding period return for a coupon bond equals its yield to maturity only if you can reinvest the coupons at the yield to maturity. Reinvestment at a higher rate produces a holding period return (realized coupon yield) above YTM, while reinvestment at a lower rate produces a holding period return below YTM. For a zero coupon bond the yield to maturity is always equal to the holding period return if the bond is held to maturity.