

Equilibrium Asset Prices under Imperfect Corporate Control

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Abstract

Shareholders have imperfect control over the decisions of the management of a firm. We integrate a widely accepted version of the separation of ownership and control – Jensen’s (1986) free cash flow theory – into a dynamic equilibrium model and study the effect of imperfect corporate control on asset prices and investment. We assume that firms are run by empire-building managers who prefer to invest all free cash flow rather than distributing it to shareholders. Shareholders are aware of this problem but it is costly for them to intervene to increase earnings payouts. Our corporate finance approach suggests that the aggregate free cash flow of the corporate sector is an important state variable in explaining asset prices and investment. We show that the business cycle variation in free cash flow helps to explain the cyclical behavior of interest rates and the yield curve. The stochastic variation in free cash flow sheds light on risk premia on corporate bonds and out-of-the-money put options. We also show that the financial friction causes cash-flow shocks to affect investment, and causes otherwise i.i.d. shocks to be transmitted from period to period. Unlike the existing macroeconomics literature on financial frictions, the shocks propagate through large firms and during booms.

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1 Introduction

In neoclassical models there usually is no meaningful concept of the “firm” since managers (implicitly) act in shareholders’ interests, maximizing firm value. Managers rank projects by net present value, and only undertake projects with positive net present value. The marginal rate of transformation of production is equal to the cost of capital faced by a firm. For asset pricing purposes, this relation has been exploited to provide a link between economic fundamentals such as investment and productivity, and asset returns (e.g., Cox, Ingersoll, and Ross (1985) and Cochrane (1991)). But the evidence contradicts this conceptualization of the link. One of the strongest empirical findings in Economics is the evidence from research on investment that firms with more cash on hand invest more.¹ This deviation from the Modigliani and Miller paradigm can be explained by corporate control problems, suggesting that a richer concept of the “firm” needs to be incorporated into investment and asset pricing models. This paper investigates how corporate control problems in firms affect equilibrium asset prices and investment.

In the corporate finance literature the cost of capital for a firm is not the marginal rate of transformation of production. Because of the separation between ownership and control (first identified by Berle and Means (1932))² agency problems arise and managers may choose projects that, while in their own interests, do not maximize firm value. Investors try to align the incentives of managers with their own goals through corporate governance mechanisms (e.g., independent directors on the board, charters that facilitate the market for corporate control, bank monitors, etc.) and through financial contracts (the firm’s capital structure, executive compensation incentive contracts, etc.) Nevertheless, corporate control frictions remain a problem. So, for asset pricing and investment theory the separation between ownership and control means that the marginal rate of transformation will not

¹The link between investment and measures of corporate cash or retained earnings is a strong empirical regularity. It was noted as early as Tinbergen (1939, p. 49), who finds “that the fluctuations in investment activity are in the main determined by the fluctuations in profits earned in industry as a whole some months earlier,” or, in Meyer and Kuh (1957, p. 192), who conclude that “the investment decision is subject to a multiplicity of influences ” but that, at business cycle frequencies, there was “a clear tendency for liquidity and financial considerations to dominate the investment decision in the short run.” Fazzari, Hubbard, and Petersen (1988) provide more recent evidence, or see Hubbard (1998) and Stein (2001) for surveys. This link is not only true at the firm level, but also at the industry level and at the aggregate level. See Chirinko (1993) and Caballero (1991).

²Also see La Porta, Lopez-de-Silanes, and Shleifer (1999) who emphasize another control problem, namely, that minority shareholders often have no ability to control dominant shareholders. Stein (2001) surveys the literature.

equal the cost of capital for a firm. The pricing kernel for an economy with imperfect corporate control will be different.

How then are asset prices determined in an economy with imperfect corporate control? To answer this question we integrate one of the most widely accepted versions of the corporate control problem, Jensen's (1986, 1993) free cash flow theory, with a widely used equilibrium asset pricing model, that of Cox, Ingersoll, and Ross (hereafter, CIR) (1985). Free cash flow is the total return from operations net of contractual payments to investors. In Jensen's theory, managers are empire builders: they always want to invest as much as possible, and will not release cash to the shareholders unless forced to do so.³ Shareholders attempt to control managerial decision-making through costly monitoring (Jensen focuses on leverage as a device to force managers to release cash to investors), but they cannot perfectly control their managers. In our model, the aggregate free cash flow in the corporate sector arises as an important state variable. Since managers exercise control over the free cash flow, this state variable measures the severity of the corporate control problem. We show that the business cycle variation in free cash flow can explain a number of empirical asset pricing and investment regularities.

As in CIR, we assume a constant-returns-to-scale production process with a stochastically evolving productivity parameter. Unlike CIR, the production technology in our model is not directly operated by consumers, but instead is run by an empire-building manager. We model the limitations on the ability of investors to control the firm's investment and payout policy. We assume that a costly auditing/monitoring technology is available to investors. Payouts are costly in proportion to the size of the payout. Each period investors hire some number of auditors, at a cost, to monitor the firm. The auditors have a technology to seize output and then transfer the resources back to consumers. However, each auditor can be responsible for only a limited amount of output. Thus, if the realized output is unusually high, the firm will retain some earnings. The manager invests this free cash flow, along with any new infusions from consumers. The next period, consumers again choose some number of auditors to hire, and so on. The critical feature of the model is that

³The idea that managers are "empire-builders" has a fairly long history; e.g., see Baumol (1959), Marris (1964), Williamson (1964), and Donaldson (1984). There have been a number of empirical tests of the free cash flow hypothesis. Lang, Stulz, and Walking (1991) find that when an acquiring firm in a merger has a low value of Tobin's q , and relatively high cash flows, the announcement effect is negative. Blanchard, Lopez-de-Silanes and Shleifer (1994) look at a small sample of firms that receive a windfall amount of cash in a legal settlement. The firms in their sample have low values of q , but still the managers do not return the cash to the shareholders. Other examples include Lang and Litzenberger (1989) and Mann and Sicherman (1991).

investment is chosen not only based on expected productivity, but also in part based on any free cash flow retained from past production.

A basic prediction of models that have formalized Jensen’s ideas is that investors force release of free cash flow by requiring the firm to increase leverage. Debt is a “hard claim” because managers must meet debt payments or face ouster (see, for example, Stulz (1990), Harris and Raviv (1990), Hart and Moore (1995), and Zweibel (1996)). In our model, the firm’s output is only released to investors by employing auditors. Thus our auditing technology works similarly to the hard claim of debt, but more broadly represents a variety of control mechanisms (e.g., debt, hostile takeovers, leverage buyouts, a change of board membership or charter and so on).

Our results contrast with those of a standard business cycle model, such as CIR, where asset prices and investment are driven by changes in the expected marginal rate of transformation (productivity). For example, the CIR account of the procyclical interest rate is that cyclical movements in productivity drive it. This seems very unrealistic for the short rate.^{4,5} In our model, the corporate control problem means that when free cash flow is high, investment may be higher than shareholders would optimally prefer. While shareholders can extract more resources from the firm, this will only occur in the future. The result is that shareholders can end up holding “too many” financial claims on the corporate sector and to induce them to do this, interest rates will have to be high. Thus when free cash flow is high, as in a cyclical peak, investment demand is high, and therefore interest rates are high. Likewise, the model generates a term spread with the well-known leading indicator property.⁶

⁴It is hard to believe that a deep technological parameter such as productivity moves as much as the real interest rate at business cycle frequencies. For example, in the current recession, the one-year nominal interest rate is 1.3 percent (there is no one-year inflation-indexed bond, but with inflation positive, we can conclude that the real interest rate is less than 1.8 percent). The ten-year rate on the U.S. Treasury inflation-indexed bond is close to three percent. In the summer of 1996, both the one-year interest rate from inflation-indexed bonds, as well as the ten-year rate were close to four percent. It is plausible that the drop in the long-term rate is driven by technological factors, but it is hard to believe that the decrease in the short-term rate is completely due to the same factors.

⁵The same issue arises in the real business cycle literature (of which CIR is one example). Large productivity shocks are needed at fairly short intervals to quantitatively account for business cycle fluctuations. Many macroeconomists have criticized this as unreasonable, and researchers have sought amplifications mechanisms whereby small shocks can have large effects. Mechanisms involving aspects of corporate net worth or retained earnings have been studied by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

⁶The procyclical real interest rate, as well as behavior of the yield curve, may also be driven by counter-cyclical monetary policy. However, as is widely appreciated in the literature, monetary policy is endogenous

The variation in the corporate control problem over the business cycle also has implications for risk premia. The distinctive feature of our model is that investors are effectively “debt-holders” on the productive sector: They bear downside risk, but do not fully share in the upside risk, as in these states managers exercise control over free cash flow. This means that securities that match this pattern of payoffs have high risk premia. Corporate bonds have high risk premia. Out of the money put options also have high risk premia, so that implied volatilities on options match the empirically observed volatility “smirk.” Finally, we show that equity securities exhibit mean reversion in returns.

The model’s implications for quantities are also in line with business cycle facts. Aggregate investment and free cash flow are positively correlated. Also, investment is more volatile than consumption. Because of the financial friction, production shocks are amplified and otherwise i.i.d. shocks persist beyond a period. All of these implications are consistent with the macroeconomic literature on financial frictions (e.g. Bernanke and Gertler (1989) and Kiyotaki and Moore (1997)). One important difference between our work and the existing literature is that we study the frictions caused by imperfect corporate control and empire-building managers. The existing literature has focused mostly on costly external finance. Casual empiricism suggests that the costly external finance problem arises more for small firms, while the corporate control problem arises for large firms. So on the one hand, our results reinforce the existing results. On the other hand, our model of imperfect corporate control seems like a better description of developed countries like the U.S.

We investigate asset prices in a framework in which investment and the characteristics of firms play the primary role. However, the model’s implications for asset prices still derive from the marginal rate of substitution of the representative consumer. As will be clear later on, the model’s implications for consumption do not match the smooth aggregate consumption data. We don’t view this as a problem.

It is well understood that the failures of the standard consumption based asset pricing model could be due to the difficulty of identifying the marginal investor in asset markets and measuring his marginal rate of substitution. Aggregate consumption is not a valid proxy when there is limited market participation. One approach to this problem is to measure the consumption of a smaller sample of stockholders (see Mankiw and Zeldes, 1991, or Vissing-Jorgensen, 2002). While some progress has been made via this approach, the difficulty of measurement in consumer data makes the estimates very noisy (see Vissing-Jorgensen, 2002, for a discussion of these points).

and driven by expectations of private sector activity. At an extreme, our model might say that the monetary authority simply chooses the money supply so that the interbank rate aligns with our “real” interest rate.

We view our methodology as complementary to the limited-market participation literature. An alternative way to arrive at the consumption of stockholders is to write an economic model of the production side of the economy, and infer the transfers from the productive sector to these stockholders. The implications of these transfers can be inferred from their effects on asset prices, without explicitly identifying the relevant consumption flows. Granted the inference is gained only through the economic model. However, for empirical work, data from the production side may be better and offer a substantive alternative to the consumption-based models. We think this advantage is sufficient to warrant further investigation.

There is a growing asset pricing literature that explicitly incorporates the production side. Cochrane (1991), Berk, Green, and Naik (1999), Gomes, Yaron, and Zhang (2001), and Holmstrom and Tirole (2001) are examples of papers in this literature. In some of these papers it is the marginal rate of transformation of the firm that is the central focus. But as we have noted, the separation of ownership and control will generally mean that this marginal rate of transformation will not be equal to the cost of capital for a firm. We are closest in spirit to the papers of Holmstrom and Tirole (2001) and Gomes, Yaron, and Zhang (2001) in that we study the asset pricing implications of a corporate financing friction.

2 The model

2.1 Preferences and Technology

We consider an infinite horizon discrete time economy with a single consumption good. There is unit measure of consumers with preferences at time t of,

$$\sum_{s=t}^{\infty} \beta^{s-t} \log C_s \quad (1)$$

There is a constant returns to scale production technology available each period that converts one unit of consumption good at time t into z_{t+1} units of goods at time $t+1$. z_{t+1} is a random variable that is distributed as follows:

$$\log z_{t+1} = \log \rho_{t+1} + \epsilon_{t+1} \quad \text{where} \quad \epsilon_{t+1} \sim \mathcal{N}(0, \sigma^2) \quad \text{and is i.i.d across } t. \quad (2)$$

ρ_{t+1} is a state variable that represents the expected productivity of investment. It is known at time t , i.e. when investment occurs. The productivity state variable evolves as follows,

$$\log \rho_{t+1} = \log \rho_t + \nu_t, \quad (3)$$

where,

$$\nu_t \sim \mathcal{N}(0, \sigma_\rho^2) \quad \text{i.i.d across } t \text{ and independent of } \epsilon_t.$$

We introduce a free cash flow friction to arrive at asset pricing implications of the separation of ownership and control. Managers, not consumers, control corporate assets and, in particular, make investment decisions. The preferences of the manager are those of an “empire-builder,” in the spirit of Jensen (1986). That is, the manager prefers to invest all available corporate resources so as to maximize firm size. Shareholders have only one lever to control this investment. When they invest in the firm at time t , they simultaneously choose a number of “auditors” to hire for the upcoming period. Auditors have a technology to observe goods as they are produced and then to ring fence these produced goods so they do not fall under the control of the empire-building manager. However, each auditor can only be responsible for a maximum of one unit of the good. Moreover, hiring an auditor is costly, as auditors consume a fraction $1 - \gamma$ of the goods they seize.

Hiring more auditors means that, in expectation, investors will receive higher payouts next period. In effect, investors choose the auditors depending on how large a payout they would like next period. For this reason, we refer to the hiring of auditors as the *payout policy*.

At date $t - 1$ the investors hand over some resources to the manager. The manager adds this to retained earnings from past production and, given his empire-building tendencies, invests all of it in production (amounting to I_{t-1}). Simultaneously the investors hire $d_t I_{t-1}$ auditors for the next period.

At t , production returns $z_t I_{t-1}$. If $z_t \geq d_t$, all of the auditors are occupied in observing and seizing output goods. Thus investors are returned γd_t goods per unit. The firm retains earnings of $E_t = (z_t - d_t) I_{t-1}$. If $z_t < d_t$, only $z_t I_{t-1}$ auditors are occupied, and investors are returned γz_t goods, while the firm retains no earnings.

On the one hand if investors chose to set d_t to infinity, E_t will be zero, investors will receive $(1 - \gamma) z_t I_{t-1}$, and thus investors will have full control of corporate investment for period t . On the other hand, the investors will be spending too many resources on auditing. For suppose that investors were planning to infuse some resources into the firm in period t to fund new investment. In this case, investors would be better off by choosing a smaller d_t (in period $t - 1$) so that the firm retained positive retained earnings to fund this new investment. Doing this avoids paying $1 - \gamma$ to the auditors on the funds which in any case would have been returned to the firm to fund new investment. In this modeling, $1 - \gamma$ parameterizes the extent of the corporate control problem.

Our strategy for characterizing asset prices is three-fold: (1) In the next two sub-sections we determine (C_t, d_{t+1}) as solutions to a planning problem; (2) Then we briefly discuss how the planning solution may be decentralized; (3) Finally, the solution to the planning problem is used to derive a pricing kernel that will allow us to price the various assets of interest.

We consider a planner who only maximizes the welfare of the consumer, while being constrained to operate production under the friction of separation of ownership and control. The controls are (C_t, d_{t+1}) given state variables of past investment, I_{t-1} , expected productivity, ρ_t , and production returns, z_t .

We use the following notation in the paper: Capital letters such as I_t or C_t represent aggregate quantities; Lower case letters such as d_t or c_t are per unit of investment quantities.

2.2 Discussion

The CIR benchmark is achieved when we set $\gamma = 1$. In this case it is costless for investors to hire auditors and as a result they have full control of the firm's investment policies. Relative to CIR, we have made a significant simplification by eliminating the correlation between ν_t and ϵ_t . This removes some of the richness in their interest rate dynamics, but it helps focus purely on the cash flow effects.

Macroeconomics models of investment make the realistic assumption that capital depreciates only partially each period. On the other hand, in the CIR model capital depreciates fully each period. An alternative interpretation is that capital does not depreciate, but can be costlessly transformed back into consumption goods. We also make this assumption.

We adopt the perspective that the consumers (outside claimants) choose payout policy. In practice, managers do not just exercise their power by controlling investment. They also influence the firms' capital structure and dividend policy in ways that may not be aligned with shareholders' interests. Furthermore, there are states of the world such as bankruptcy where the debtholders and other stakeholders influence payout policy. We abstract from these other effects in order to capture some of the richness of corporate finance, but in a model that deviates minimally from the neoclassical CIR model.

There is a lag in the model between observing the realized free cash flow and choosing auditors to release free cash flow next period. That is, investors cannot deploy auditors to immediately release any free cash flow from firms. This is an important aspect of our model, as it creates a link between retained earnings based on past production and current investment. i.i.d. shocks are transmitted from period to period. In practice, investors cannot observe the level of free cash flow and then choose leverage; leverage is chosen in advance. More broadly, changing institutions and mechanisms for corporate control takes

time. Our auditing technology works similarly. Lamont (2000) presents evidence that the correlation between investment and cash flow is really a correlation between *unexpected* investment and unexpected cash flows at a one-year horizon. Firms make investment plans one-year ahead, and unexpected cash flow over this year correlates with a deviation between actual investment and the investment plans. Similarly, we think of the time periods in our model in units of one-year.

2.3 The planning problem

The representative consumer chooses consumption and payout policy to solve:

$$\max_{\{C_t, d_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log C_t$$

where,

$$C_t = \max[z_t - d_t, 0]I_{t-1} + W_t - I_t.$$

I_t is period t investment by the firm, and W_t is the period t remunerations to consumers,

$$W_t = \gamma I_{t-1} (d_t 1_{z_t \geq d_t} + z_t 1_{z_t < d_t}).$$

We now move to a recursive representation of this problem. Given C_t , period t investment by the firm is

$$I_t = \max[z_t - d_t, 0]I_{t-1} + W_t - C_t. \quad (4)$$

For future reference we define retained earnings per unit of capital as,

$$e_t = \max[z_t - d_t, 0]. \quad (5)$$

Thus it should be clear that investment at date t is just retained earnings from last period plus new injections of funds from investors. Substituting the expression for W_t into the investment equation (4) gives,

$$I_t = z_t I_{t-1} - (1 - \gamma) \min[z_t, d_t] I_{t-1} - C_t.$$

Investment this period is equal to resources produced from last period's investment minus audit costs minus consumption.

The investment equation can be written more concisely as,

$$I_t = I_{t-1} y_t - C_t, \quad (6)$$

where,

$$y_t = z_t - (1 - \gamma) \min[z_t, d_t] \quad (7)$$

is a “transformed” version of actual production returns, z_t . It is the actual production returns minus a term representing the audit costs associated with the payout policy. Equivalently, it is the return from the point of view of the consumer.

Our assumptions on the auditing technology impose the following key friction: investors can only make positive infusions so as to increase resources available for investment. This is equivalent to saying that,

$$C_t \leq W_t.$$

The only case in which this constraint will bind is when $z_t > d_t$ and there is too much retained earnings in the firm so that the investor would ideally like to have some of this back in terms of consumption. Focusing just on this situation, $W_t = \gamma I_{t-1} d_t$, and we can rewrite the constraint as,

$$C_t \leq \gamma I_{t-1} d_t. \quad (8)$$

During period t investors choose how much to consume and payout policy for period $t + 1$. The consumption decision, along with past investment decisions and decisions over payout policy, leads to a new level of capital at the end of period t (see equation (6)). Investors make these decisions knowing ρ_{t+1} (expected productivity from t to $t + 1$). Let $J(I_t, \rho_{t+1}, d_{t+1})$ be the expected value of I_t units of capital at the start of date $t + 1$, given d_{t+1} and ρ_{t+1} .

The recursive representation of our problem is,

$$J(I_{t-1}, \rho_t, d_t) = E_{t-1} \left[\max_{C_t, d_{t+1}} \{u(C_t) + \beta J(I_t, \rho_{t+1}, d_{t+1})\} | \rho_t \right] \quad (9)$$

given the investment accumulation equation (6) and subject to the constraint (8).

Proposition 1 *The solution to the problem, (9), is as follows. Investment and new capital are:*

$$I_t = I_{t-1} (y_t - \min[(1 - \beta)y_t, \gamma\theta\rho_t]). \quad (10)$$

Consumption is:

$$C_t = I_{t-1} \min[(1 - \beta)y_t, \gamma\theta\rho_t] \quad (11)$$

and total required payouts are:

$$I_{t-1} d_t = I_{t-1} \theta \rho_t \quad (12)$$

where θ is a constant, and y_t is defined in (7).

Proof: See the Appendix.

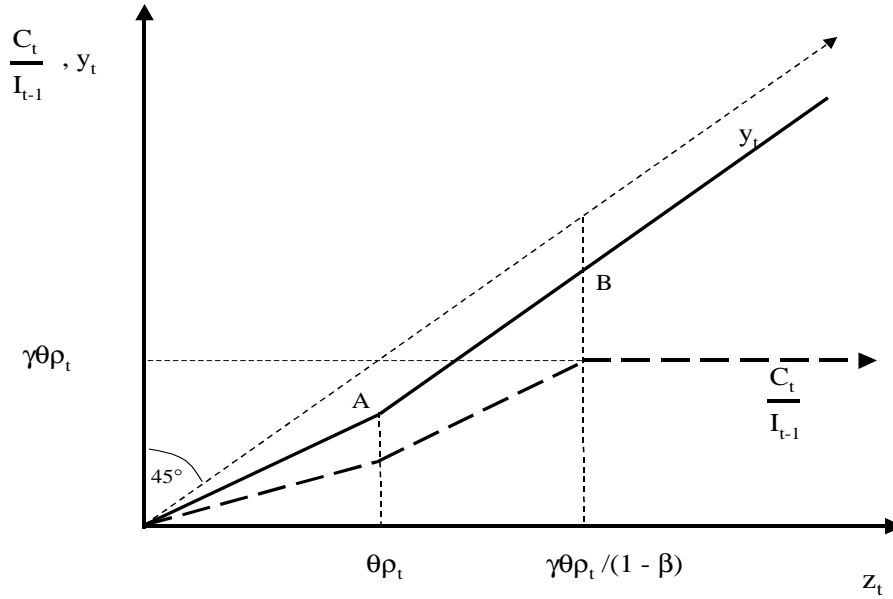


Figure 1

Figure 1 graphs consumption per unit of (past) investment as well as y_t as a function of z_t . Notice that there are three segments in the graph. The first region (to the left of A) is where z_t falls below d_t . In this case investors extract $z_t(1 - \gamma)$ from the firm and consume a fraction $1 - \beta$ of this amount. The second region (between A and B) is where returns exceed required payouts, however as retained earnings are small, investors choose to infuse some resources into the firm. The final region (to the right of B) is where retained earnings are high enough so that investors would ideally like to extract some more resources from the firm, but are unable to given the free cash flow friction. In this case investors simply consume their payouts of $\gamma I_{t-1} d_t$.

Aggregate consumption (per unit of I_{t-1}) resembles a put-option on the date- t output of y_t . It is linear in y_t upto a point and then is flat. Investment is the difference between the date- t output and consumption. It is also linear in y_t but then turns sharply steeper at the point where consumption flattens out.

We refer to the region to the right of point B as the **over-investment region** and the region to the left of point B as the **optimal-investment region**. In the former region, relative to consumers' preferred choices, firms are over-investing because of the manager's empire building preferences. In the latter region, consumers control investment and investment is optimal.

Lemma 1 (CIR Case)

When $\gamma = 1$, the economy collapses to the CIR model. Investment and consumption are as follows:

$$I_t = I_{t-1}z_t\beta, \quad (13)$$

$$C_t = I_{t-1}z_t(1 - \beta). \quad (14)$$

Proof: When $\gamma = 1$, there is no cost of increasing θ , and thus claimholders may as well set $\theta \rightarrow \infty$. Then $y_t = z_t$ and from the equations of Proposition 1, $C_t = I_{t-1}z_t(1 - \beta)$, while $I_t = I_{t-1}z_t\beta$. ■

3 Aggregate quantities and pricing kernel

3.1 Investment and consumption

Investment in our model is a function of current corporate cash-flow as well as last period's investment. The correlation between investment and cash-flow is fairly well documented, and is a natural prediction of models with financial frictions. The cash-flow shock only lasts one-period in our model because, upon the realization of unusually high cash flow at date t , investors simply increase the payout policy for period $t + 1$. It may be interesting to study a model in which the intervention technology had more lags, as this will lead to richer dynamics of shocks.

Macroeconomic models with financial frictions such as Bernanke and Gertler (1989) or Kiyotaki and Moore (1997) also predict these sorts of effects for aggregate investment. One important difference between our model and the existing literature is that our effects arise from free-cash-flow problems as opposed to costly external finance. In the corporate finance literature, both types of friction frictions have received equal attention. Our paper is one of the first to point out the implications of free-cash-flow problems for aggregate quantities (see also Philippon, 2003).

In the costly-external-finance model, frictions get worse during downturns. In the free-cash-flow model, frictions get worse during booms (although, the effects of over-investment during booms is perhaps most apparent during downturns). Our model predicts that investment is more responsive to cash flow during booms, as opposed to downturns. In fact, comparing equations (10) to equation (14) we see that the main kick in our model is on the positive shocks. Investment in our model is a convex function of shocks. Casually, our model seems to fit stories regarding accelerating liquidity and investment during booms.

These effects can be empirically tested and distinguished. However, we think of our model as a complement rather than a substitute to the costly external finance models. They probably work in different types of firms. In the corporate finance literature, free-cash-flow problems have been mostly associated with large firms, while costly external finance problems have been associated with smaller firms. Thus these two types of models have different implications for which types of firms propagate aggregate shocks.

The convexity of investment as a function of shocks also implies that consumption is a concave function of shocks (see Figure 1). It is easy to show that this also implies that consumption is a concave function of measures of wealth/liquidity. Since Keynes, the concavity of the consumption function has been viewed as a desirable property of models. In the consumption literature, models with precautionary savings motives have been shown to generate the concave consumption function (e.g., see Gourinchas and Parker, 2002). Our model does so because of the free-cash-flow problem.

3.2 Decentralization and the pricing kernel

One possible decentralization of the planning solution of the previous section is as follows. At date t , all claimholders of the firm have a meeting in which they decide on resources to inject into the firm for that period and on the payout policy/number of auditors to hire for the following period. Since the claimholders are identical, the policy will be approved unanimously. The manager then invests the firm's retained earnings plus any additional resources infused by the claimholders. Consumers are then free to trade a full set of contingent claims against all possible realizations of output. Between t and $t+1$, auditors observe and seize the designated amount of output goods, and then return them to claimholders. Finally, any contingent claims that are bought or sold between consumers are settled, and consumption takes place.

Since consumers can trade a full set of contingent claims against all output realizations, the pricing kernel can be defined in terms of the marginal rate of substitution of the representative consumer,

$$m_{t+1} = \frac{\beta u'(C_{t+1})}{u'(C_t)} = \beta \frac{I_{t-1}}{I_t} \frac{\min[(1-\beta)y_t, \gamma\theta\rho_t]}{\min[(1-\beta)y_{t+1}, \gamma\theta\rho_{t+1}]}. \quad (15)$$

As we have noted in the introduction, we view the C_t in the pricing kernel of equation (15) as corresponding to the consumption of participants in the asset market, as opposed to measured aggregate consumption. Thus, we don't view the smoothness of the aggregate consumption as a problem for our model (e.g, suppose that non-asset-market participants

simply consumed a constant endowment). We view our methodology as complementary to the limited-market participation literature (e.g. Vissing-Jorgensen, 2002). As opposed to directly measuring the consumption of participants in asset-markets, our approach is to arrive at their consumption through an economic model of the production side of the economy which predicts the transfers from the productive sector to these stockholders as a function of production side variables. The implications of these transfers can be inferred from their effects on asset prices, without explicitly identifying the relevant consumption flows.

4 Term structure

We begin our characterization of asset prices by computing the riskless short term interest rate, and then move on to describe the entire riskless yield curve. Our main interest is in describing the cyclical behavior of interest rates and term spreads and their correlation with aggregate investment and free-cash-flow.

4.1 Short rate

It is helpful to define,

$$\psi_t = \min((1 - \beta)\frac{y_t}{\rho_t}, \gamma\theta) \quad (16)$$

and rewrite the pricing kernel as,

$$m_{t+1} = \beta \frac{I_{t-1}}{I_t} \frac{\rho_t \psi_t}{\rho_{t+1} \psi_{t+1}}. \quad (17)$$

Interest rates are:⁷

$$\frac{1}{1 + r_t} = \beta \frac{I_{t-1}}{I_t} E_t \left[\frac{\rho_t \psi_t}{\rho_{t+1} \psi_{t+1}} \right].$$

Proposition 2 (Short rate)

There are two regimes depending on the relative magnitudes of $y_t(1 - \beta)$ and $\rho_t \gamma \theta$. In the case where $y_t(1 - \beta) < \rho_t \gamma \theta$ – that is the optimal-investment region – the short rate is:

$$\log(1 + r_t) = \log \rho_{t+1} - \log(1 - \beta) - \log E[\psi^{-1}]. \quad (18)$$

In the case where $y_t(1 - \beta) \geq \rho_t \gamma \theta$ – the over-investment region – the short rate is:

$$\log(1 + r_t) = \log \rho_{t+1} + \log \frac{e_t}{\theta \gamma \rho_t} - \log \beta - \log E[\psi^{-1}]. \quad (19)$$

⁷Note that ψ_t is i.i.d. across time. See Lemma 4 in the Appendix.

A little algebra verifies that interest rates coincide at the boundary between these two regions. The short rate has a simple expression then: it is only a function of e_t (realized retained earnings from (5)) and ρ_{t+1} (a proxy for expected production returns). If earnings are unexpectedly high, interest rates are high; if not they are just proportional to ρ_{t+1} , the expected production returns for the next period.

Note that ψ in the interest rate expression in Proposition 2 is a function of γ . If take the limiting case of $\gamma = 1$, then we arrive at the CIR model:

Lemma 2 (CIR Short rate)

The short rate in the case where $\gamma = 1$ coincides with the CIR short rate:

$$\log(1 + r_t) = \log \rho_{t+1} - \frac{\sigma^2}{2} \quad (20)$$

Proof: When $\gamma = 1$, $\theta \rightarrow \infty$. Thus, $y_t = z_t$ and $\psi_t = (1 - \beta) \frac{z_t}{\rho_t}$. From Proposition 2, if θ is large, the short rate expression for the region to the left of point B (see (18)) applies:

$$\begin{aligned} \log(1 + r_t) &= \log \rho_{t+1} - \log(1 - \beta) - \log E[\psi^{-1}] \\ &= \log \rho_{t+1} - \log E\left[\frac{\rho_t}{z_t}\right] \\ &= \log \rho_{t+1} - \log E[e^{\epsilon_t}] \\ &= \log \rho_{t+1} - \frac{\sigma^2}{2}. \end{aligned}$$

■

Comparison of the interest rate expressions in the Proposition 2 and Lemma 2 reveals precisely the difference between our model and CIR. First, in the optimal-investment region, the expressions for interest rates are similar. Expected productivity drives all stochastic variation in the interest rate. The only difference is that the value of $E[\psi^{-1}]$ is higher in the free-cash-flow model, which means the level of interest rates is slightly lower. In the over-investment region, shocks to both productivity as well as retained earnings drive interest rates.

We can illustrate these economic effects using a static loanable funds model. Consider first Figure 2, which represents the CIR model. On the horizontal axis is investment or loans. On the vertical axis is the interest rate. Consumers' supply of loanable funds is upward-sloping in the interest rate. The demand for funds is perfectly elastic at a level determined by the expected productivity of investment (ρ_{t+1}). A shock to ρ_{t+1} shifts up both the demand and supply curves. In the log case, the shift is of the same amount so that investment remains unchanged (see Proposition 1), but the interest rate rises by the amount of the shock.

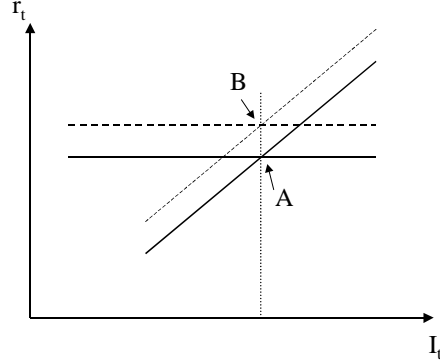


Figure 2: Productivity shocks in CIR

Figure 3 illustrates the effects of shocks to both productivity as well as retained earnings in the free-cash-flow model. The left-hand panel are shocks within the optimal-investment region. Note that the demand for funds is L-shaped. Firms will always invest their retained earnings, regardless of the interest rate. This sets a minimum amount of investment. At low interest rates, we are back in the CIR case: Firms find that their productivity is high relative to the interest rate and as such their investment demand is perfectly elastic. The L-shaped demand curve is the CIR case with a minimum investment level equal to retained earnings.

A shock to expected productivity moves both supply and demand curves up and shifts equilibrium from point A to point B. The net effect on the interest rate is the same as in the CIR case (Figure 2). There is also a shock to retained earnings. But it has no effect on the equilibrium in this region.

The right-hand panel illustrates the effect of these shocks when the retained earnings shock is large enough to take the economy into the over-investment region. At the margin, managers invest all of their retained earnings. Thus interest rates rise not only because of the rise in expected productivity, but also to induce investors to accept the higher required savings level.

Plausibly, at business cycle frequencies, shocks to earnings are much larger than shocks to expected productivity. An asset pricing model such as CIR links all cyclical variation in interest rates just to variation in ρ_{t+1} , as can be seen in the diagram. However, in the over-investment region of the free-cash-flow model, large shocks to retained earnings have large effects on interest rates. Thus a success of the free-cash-flow model is that it links

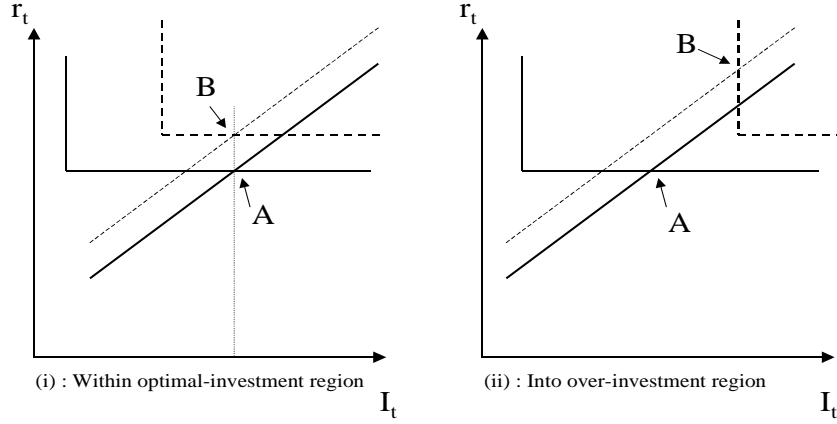


Figure 3: Simultaneous shocks to earnings and productivity

cyclical variation in retained earnings to the short rate: When retained earnings are high, as in a cyclical peak, the short rate is also high; When retained earnings are low, the short rate is low.

The CIR model reconciles the data by positing a correlation structure in the underlying productivity shocks. For example, ρ_t is typically a mean-reverting process in CIR. Of course, this is just by assumption. Our model isolates how otherwise i.i.d. shocks when combined with the free-cash-flow friction result in a mean reverting interest rate process. We have focused on the i.i.d. case in order to make this point most clearly.

Our mechanism of retained earnings and investment demand as being the driving force for the procyclical interest rate is also in contrast to the standard consumption asset pricing model (e.g, the habit formation models of Constantinides, 1990, Sundaresan, 1989, and Campbell and Cochrane, 1999). There, the procyclical interest rate is often rationalized as follows. Fixing equity investment opportunities, if investors receive a negative income shock (i.e. cyclical trough), they will have higher risk aversion, consequently the risk premium will be higher and the interest rate will be lower in the trough.

If one introduces production opportunities into the consumption/endowment economies common in the asset pricing literature, then consumers will have a strong desire to smooth consumption via the production technology. Boldrin, Christiano, and Fisher (2001) make this point in reference to habit formation preferences. If the production technology is modeled as in CIR, then the marginal rate of transformation will determine interest rates (as in

CIR). We see our model as providing one avenue to introduce production into the standard endowment economies and break the strong link between the marginal rate of transformation and asset prices.

4.2 Slope of the yield curve

In order to study the slope of the yield curve, we now derive the expression for the τ -period interest rate.

$$\frac{1}{(1+r_{t,\tau})^\tau} = E_t \left[\frac{\beta^\tau u'(C_{t+\tau})}{u'(C_t)} \right]. \quad (21)$$

Lemma 3 (Long-rate)

The τ -period interest rate is:

$$\begin{aligned} \log(1+r_{t,\tau}) = & -\log\beta + \log E\left[\left(\frac{y}{\rho} - \psi\right)^{-1}\right] - \frac{1}{\tau} \log E_t \left(\prod_{s=t+1}^{s=t+\tau} \rho_s \right)^{-1} \\ & - \frac{1}{\tau} \log \psi_t - \frac{1}{\tau} \log E_t[\psi_{t+\tau}^{-1}]. \end{aligned} \quad (22)$$

Proof: See the Appendix.

For τ large the long term interest rate from (22) is approximately,

$$\begin{aligned} \log(1+r_{t,\tau}) & \approx -\log\beta + \log E \left[\left(\frac{y}{\rho} - \psi \right)^{-1} \right] - \frac{1}{\tau} \log E_t \left(\prod_{s=t+1}^{s=t+\tau} \rho_s \right)^{-1} \\ & \approx \log \rho_{t+1} - \log\beta + \log E \left[\left(\frac{y}{\rho} - \psi \right)^{-1} \right] - \frac{\tau}{3} \sigma_\rho^2. \end{aligned}$$

The last term is a constant that is linear in τ . Thus productivity is the primary determinant of long term interest rates. This is the same as in the CIR model. The term $E \left[\left(\frac{y}{\rho} - \psi \right)^{-1} \right]$ is also a constant, because $\frac{y_t}{\rho_t} - \psi_t$ is an i.i.d random variable (see Lemma 4 in the Appendix).

Proposition 3 (Term spread)

The term spread measured as (long - short) is: If $(1-\beta)y_t \leq \rho_t \gamma \theta$ (optimal-investment region),

$$term = +\log \frac{1-\beta}{\beta} + \log E \left[\left(\frac{y}{\rho} - \psi \right)^{-1} \right] - \log E[\psi^{-1}] - \frac{\tau}{3} \sigma_\rho^2;$$

If $(1-\beta)y_t > \rho_t \gamma \theta$ (over-investment region),

$$term = -\log \left(\frac{e_t}{\theta \gamma \rho_t} - 1 \right) + \log E \left[\left(\frac{y}{\rho} - \psi \right)^{-1} \right] - \log E[\psi^{-1}] - \frac{\tau}{3} \sigma_\rho^2.$$

Since changes in ρ_{t+1} have the same effect on both long and short rates, productivity changes have no effects on the term spread. In our free-cash-flow model, all movements in the term spread are driven by variation in retained earnings.

The retained earnings effect on the short rate lasts only one period. As previously noted, this is because in the period following a big retained earnings innovation, investors will hire sufficient auditors to reduce the free-cash-flow problem. Our intuition is that if the free-cash-flow problem took longer to resolve so that firms were over-investing for several periods, there would be persistence beyond one period.⁸

Another mechanism to arrive at persistence is to suppose that capital did not depreciate fully and was irreversible. There is a large investment literature which makes the case for this friction at the level of an individual firm (see Caballero, 1999, Abel and Eberly, 1996, Dixit and Pindyck, 1994). We can imagine introducing this additional feature into our model. Initially, a high retained earnings shock leads firms to over-invest. If this leads to a persistently larger than optimal capital stock, the interest rate would have to remain high in order to induce investors to hold the larger capital stock. On the other hand, such persistence would also lead to zero net investment over a number of periods, which is clearly counterfactual. For this reason, the investment literature has only been concerned with micro-level irreversibility.

It is also instructive to compare our effect to what would arise in a model with physical irreversibility in capital investment, but without our free-cash-flow friction. This is interesting because our over-investment problem seems similar to a model of physical irreversibility, but with the following difference. In our model, the free-cash-flow problem means that investors are unable to reduce firm investment below the high level chosen by the manager. With physical irreversibility, investors are unable to reduce investment below zero.

Suppose that the CIR model was modified to have irreversible physical capital. In that model, the constraint will bind following a negative realization of output – as opposed to a positive realization of output, as in our free-cash-flow model. Agents would like to consume part of their physical capital, but can not. In order to make them hold the physical capital, interest rates would have to rise. Thus high interest rates would coincide with low output realizations (troughs). This is counterfactual.

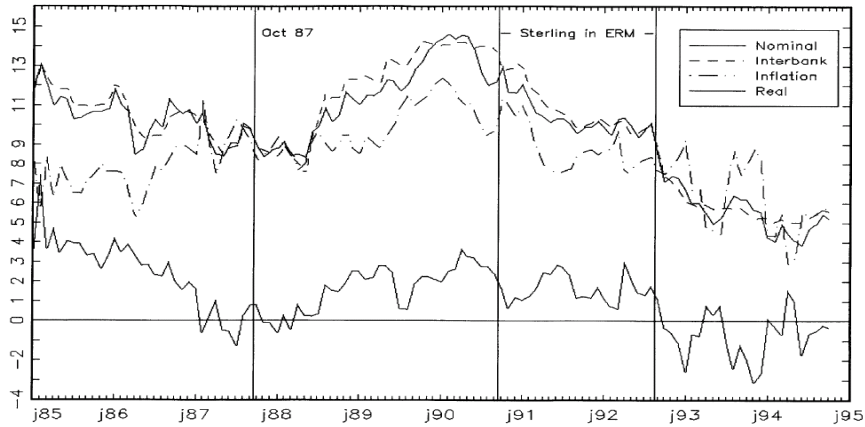


Figure 4: UK Estimated real and nominal 3-month interest rates.
Source: Campbell and Barr (1997, Figure 2)

4.3 Interest rates over the business cycle

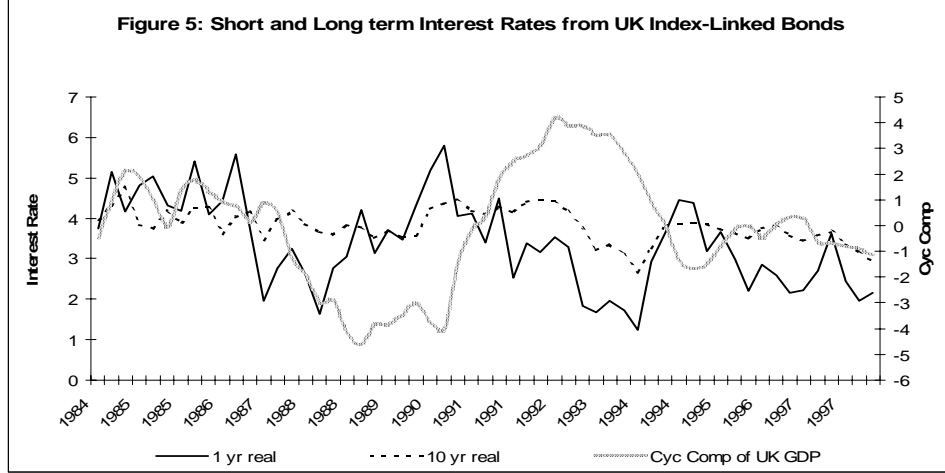
The procyclical behavior of nominal interest rates as well as the leading indicator property of the yield curve are fairly well documented. It is correct to point out that ours is a model of the real term structure. However, the results we have outlined are also consistent with the behavior of real interest rates.

Figures 4 and 5 illustrate the variation in real short and long term interest rates over the business cycle. The UK government introduced inflation-indexed bonds in 1984, whose prices allow for a direct estimate of the market real interest rate. Figure 4 is extracted from Campbell and Barr's (1997, Figure 2) study of the term structure of real interest rates in the UK. The figure graphs the nominal 3-month interest rate as well as the estimated 3-month real interest rate. The figure illustrates that the short term real and nominal interest rates moved together over this period. Figure 5 plots the 1-year and 10-year real rates estimated from the inflation-indexed bonds for a longer sample period.⁹ The interest rate series are plotted versus the cyclical component of UK GDP, extracted using the Hodrick-Prescott filter.

The two stylized facts we note from these figures are: (1) Short term real and nominal interest rates are procyclical; and (2) The short term real rate is far more variable than the

⁸For example, if the cost of auditing function is convex in the number of auditors hired, then investors will reduce the free-cash-flow over many periods, and there will be persistence.

⁹The real interest data was provided by Steve Shaefer. See Brown and Shaefer (1994) for estimation details. See either Brown and Shaefer or Campbell and Barr (1997) for a description of the UK Index-linked Gilt market.



long term real rate.

5 Risk premia

We consider the implications of our kernel for the pricing of risky securities. The distinctive feature of our model is that the free-cash-flow friction leaves consumers as “debt-holders” on the productive sector. This helps us to rationalize high corporate bond spreads, as well as mean reversion in stock returns and the volatility smirk of equity index options.

To begin, we note that the debt and equity securities we price are derivative securities. We price securities whose payoff is some function of y_t (i.e. GDP at date t). This is unusual because the equilibrium asset pricing literature usually identifies equity with the claim on the aggregate consumption stream, or equivalently, as the value of capital.

The reason we opt for pricing y_t -derivatives is that the value of capital in our model is always one. In the optimal-investment region, since investors lend consumption goods to be transformed into capital using a constant-returns-to-scale technology, the price of capital is fixed at one. In the over-investment region, this logic does fail. However, since investor preferences are log, the income and substitution effects exactly cancel and again the price of capital is one. Thus, equity defined as the value of capital will never show capital gains in our economy (although dividends will be state dependent). This is an unappealing feature of the model, but is the price we pay for simplicity. We discuss the issue further in the conclusion.

5.1 Corporate bond spreads

Let us consider pricing a one-period security whose payoff is as follows:

$$P_{t+1} = \min \left[1, \frac{y_{t+1}}{F} \right].$$

This is a corporate bond of one dollar of face on a company whose value next period is perfectly correlated with aggregate output. F is the default point on the bond. Its price today is just,

$$E_t[m_{t+1}P_{t+1}].$$

We can likewise price a riskless bond giving a price of $E_t[m_{t+1}] = \frac{1}{1+r_t}$. Define the spread between the corporate and riskless bonds as,

$$S_t = 1 - \frac{E_t[m_{t+1}P_{t+1}]}{E_t[m_{t+1}]}.$$

Note that this is somewhat unusual in that we look at the ratio of two bond prices as opposed to the difference in the reciprocals of the two prices. While in the data these two measures of the spread will be almost identical, in our analytical model this is the natural definition for analytical tractability.

We can rewrite the spread in a standard single-beta representation as follows (see Cochrane, 2001):

$$\begin{aligned} S_t &= E_t[m_{t+1}(1 - P_{t+1})]/E_t[m_{t+1}] \\ &= E_t[1 - P_{t+1}] + \frac{\text{cov}[m_{t+1}, 1 - P_{t+1}]}{E[m_{t+1}]} \\ &= E_t[1 - P_{t+1}] + \beta_{1-P,m} \left(\frac{\text{var}[m_{t+1}]}{E[m_{t+1}]} \right). \end{aligned}$$

where $\frac{\text{var}[m_{t+1}]}{E[m_{t+1}]}$ is the risk premium associated with holding the pricing kernel.

The spread is compensation for both the chance of default as well a risk premium for the loading of corporate bond defaults on the pricing kernel. Thus, the β is the coefficient on a regression of the corporate bond payoff on the pricing kernel:

$$\beta_{1-P,m} = \frac{\sigma(1 - P_{t+1})}{\sigma(m_{t+1})} \text{corr}[m_{t+1}, 1 - P_{t+1}].$$

If agents were risk neutral so that the pricing kernel was linear, then S_t would be a pure default premium (i.e. $1 - E_t[P_{t+1}]$). Since both $1 - P_{t+1}$ and m_{t+1} are decreasing functions of y_{t+1} , the correlation coefficient is positive. This means that the β is positive and the risk premium component of the spread is also positive. We have chosen to focus on a corporate

bond whose payoff is only composed of aggregate risk, because this maximizes the risk premium component of the spread.

We are going to contrast this spread when bonds are valued under both the free-cash-flow pricing kernel and the CIR pricing kernel. The two pricing kernels are just,

$$m_{t+1}^{CF} = \beta \frac{I_{t-1}}{I_t} \frac{\rho_t \psi_t}{\rho_{t+1} \psi_{t+1}}, \quad (23)$$

and,

$$m_{t+1}^{CIR} = \beta \frac{I_{t-1}}{I_t} \frac{(1-\beta)y_t}{(1-\beta)y_{t+1}}. \quad (24)$$

m_{t+1}^{CIR} can also be obtained by setting θ to infinity in m_{t+1}^{CF} .

At time t all of the variables with the exception of $\rho_{t+1}\psi_{t+1}$ and y_{t+1} are known. Thus, the β -expression can be rewritten as,

$$\beta_{1-P,m} = \frac{\sigma(1-P_{t+1})}{\sigma(m_{t+1})} \text{corr} \left[\frac{1}{\rho_{t+1}\psi_{t+1}}, 1-P_{t+1} \right] \quad (25)$$

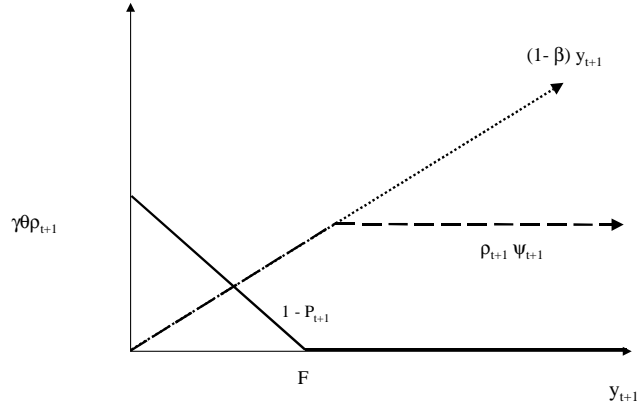


Figure 6: CIR and CF Pricing Kernels

Figure 6 graphs $\rho_{t+1}\psi_{t+1}$ and y_{t+1} along with $1 - P_{t+1}$. Note that $\rho_{t+1}\psi_{t+1}$ is equal to $(1-\beta)y_{t+1}$ in the optimal investment region, and uniformly less in the over-investment region. More importantly, $\rho_{t+1}\psi_{t+1}$ is more correlated with $1 - P_{t+1}$ than is $(1-\beta)y_{t+1}$. This implies that the correlation term is higher under the free-cash-flow pricing kernel than the CIR kernel.

From the figure we can also see that the volatility of the free-cash-flow kernel is lower than the CIR kernel. This implies that the β is higher under the free-cash-flow kernel than the CIR kernel.

Whether this translates into a higher corporate bond spread depends on what happens to the risk premium on the pricing kernel ($\frac{var[m_{t+1}]}{E[m_{t+1}]}$) across these two models. We have seen that $\sigma(m_{t+1})$ and the interest rate are lower under the free-cash-flow kernel. This means that the risk premium is actually lower under the free-cash-flow kernel. Thus, the end result on corporate bond spreads is ambiguous.

We will discuss the risk premium on the kernel in more depth below. However, if we fix the risk premium across both the CIR and the free-cash-flow model, the β effect dominates. In a model where investors have log preferences, the return on the wealth portfolio (i.e. market return) is the reciprocal of the return on the pricing kernel (see, e.g., Cochrane, 2001). Thus the experiment of fixing the risk premium is close to, but not exactly the same as, fixing the equity risk premium in the model.

Proposition 4 (*Corporate spreads*)

Fixing the risk premium of the pricing kernel, the corporate bond spread is higher in the free-cash-flow model than in the CIR model.

Intuitively, investors in the free-cash-flow world face a friction that means they don't fully share the upside of good production returns. The friction leaves them looking like "debt-holders" on the aggregate economy. In this environment, debt securities match their risk more closely and as a result investors demand a higher risk premium on these securities.

The largest effect will occur for bonds whose default point (F) is at the start of the over-investment region. Moreover the effect will have an inverse-U shape. This is a testable implication of the model. It says that a regression of the realized returns on corporate bonds versus the stock market would show the highest β 's for bonds which default at the point where the yield curve changes from upward to downward sloping.

Collin-Dufresne, Goldstein and Martin (2001) find that firm-level variables have surprisingly little explanatory power for changes in corporate bond spreads. In contrast, macro variables, such as the return on the stock market, have much more explanatory power. Our previous result provides one explanation for this finding. Strictly speaking the result implies that only the stock market return should affect corporate bond spreads. However, adding an idiosyncratic component to P_{t+1} will change that. More importantly, if we move away from log preferences, the pricing kernel will include both the stock market return as well as aggregate free-cash-flow. Thus free-cash-flow should also have explanatory power for changes in corporate bond spreads. This remains to be checked.

5.2 Stock returns: Mean reversion, volatility smirk and skewness

We now consider equity derivative securities whose payoff is linearly increasing in y_{t+1} .

We divide the time between t and $t + 1$ into a series of infinitesimally small intervals. At each of these dates, agents can trade a full set of contingent claims against all output realizations at $t + 1$. We take the limit, so that trading occurs continuously between t and $t + 1$. Let $s \in [t, t + 1]$ index time. Between t and $t + 1$, the information flow concerning output realizations is described by a filtration generated by a standard Brownian motion process, which we denote as $\{Z_s\}$.

At each s , we consider pricing the security whose payoff at $s = t + 1$ is $P_s = y_{t+1}$. Notice that consumption does not take place until $t + 1$. Thus we price this risky security using the riskless bond (i.e. the security that pays one at $t + 1$) as the numeraire.

We have seen that the free-cash-flow pricing kernel is given by (23). At time t , the only uncertainty in the pricing kernel of m_{t+1} is in y_{t+1} . For short, let us write this as,

$$m_{(s=t+1)} = A \max \left[\frac{D}{y_{t+1}}, 1 \right],$$

where, A and D are constants,¹⁰ and derive the price of the risky asset at time s , using the bond as numeraire, as

$$P_s = \frac{E_s[m_{t+1}y_{t+1}]}{E_s[m_{t+1}]}.$$

Let $P_s^* = E_s[y_{t+1}]$ (i.e. the actuarially fair price of the risky asset). Then,

$$P_s = P_s^* \left(1 + \frac{\text{cov}_s(m_{t+1}, y_{t+1})}{E_s[m_{t+1}]P_s^*} \right) \equiv P_s^* h(P_s^*, s) \quad (26)$$

It should be clear that the value of $h(\cdot)$ is less than or equal to one, implying that the risky asset has a positive risk premium.

Proposition 5 (*Risk premium*)

$h(P_s^*, s)$ is monotonically increasing in P_s^* . For high values of P_s^* , since the pricing kernel is flat with respect to y_{t+1} , the value of $h(\cdot)$ approaches one.

Proof: Consider the derivative of $h(\cdot)$ with respect to P_s^* ,

$$\frac{\partial h}{\partial P_s^*} = \frac{1}{E_s[m_{t+1}]P_s^*} \frac{\partial \text{cov}_s(m_{t+1}, y_{t+1})}{\partial P_s^*} - \frac{\text{cov}_s(m_{t+1}, y_{t+1})}{E_s[m_{t+1}]P_s^*} \frac{\partial E_s[m_{t+1}]P_s^*}{\partial P_s^*}.$$

Referring back to Figure 6, we see that the covariance of m_{t+1} with y_{t+1} is negative. Moreover, for high values of P_s^* (more precisely, y_{t+1}), the covariance is zero, while the covariance

¹⁰More precisely, $D = \frac{\gamma \theta \rho_{t+1}}{1 - \beta}$ and $A = \frac{\beta I_{t-1} \rho_t \psi_t}{\gamma I_t \theta \rho_{t+1}}$.

is most negative for low values. Thus, $\frac{\partial \text{cov}_s(m_{t+1}, y_{t+1})}{\partial P_s^*}$ is positive, while $\text{cov}_s(m_{t+1}, y_{t+1})$ is negative. It is fairly straightforward to show that $E_s[m_{t+1}]P_s^*$ is increasing in P_s^* .¹¹ ■

The result implies that good news about cash flows (P_s^*) raises P_s more than P_s^* , while bad news about cash flows causes P_s to fall more than P_s^* . It also implies that there is mean reversion in stock returns. News that pushes down the stock price will lead to a higher conditional stock return, so that the negative return will be followed by a positive return.

It is worth pointing out that in the CIR model with log investors, there would be no mean reversion. This is because $h(\cdot)$ will be constant, and not a function of P_s^* . The result is due to the free-cash-flow friction we have assumed.

The result that stock prices are more sensitive to news can also be stated in terms of volatilities. Both P_s and P_s^* can be written as diffusions in Z_s ($\frac{dP_s}{P_s} = \mu_s dt + \sigma_s dZ_s$, and similarly for P_s^*). If we apply Ito's Lemma to equation (26) and retain only the dZ_s terms, we arrive at,

$$\sigma_s dZ_s = \left(1 + \frac{P_s^*}{h(P_s^*, s)} \frac{\partial h}{\partial P_s^*}\right) \sigma_s^* dZ_s. \quad (27)$$

σ_s and σ_s^* are the volatilities of P_s and P_s^* , respectively.

The term $1 + \frac{P_s^*}{h(P_s^*, s)} \frac{\partial h}{\partial P_s^*}$ represents the fraction of the volatility of the asset price that is due to the pricing kernel. That is, if m_{t+1} is constant, then $h(\cdot)$ will also be a constant (equal to one). This will mean that $\sigma_s = \sigma_s^*$, so that the volatility of the asset price is due solely to changes in the conditional expectation of its payoff (y_{t+1}).

Since we have shown that $\frac{\partial h}{\partial P_s^*}$ is positive, it is straightforward that $\sigma_s \geq \sigma_s^*$. More interestingly, since for high values of P_s^* , we have that $h(\cdot)$ is constant, it follows that σ_s equals σ_s^* for high values of P_s^* . This tells us that (subject to a caveat) volatility is higher when the stock price is low than when it is high.¹²

¹¹The algebra is as follows. Since $P_s^* = E[y_{t+1}]$, we can write,

$$y_{t+1} = P_s^* x$$

where, x is a random variable that is positive, has mean of one, and is independent of the information filtration at time s . Denote its distribution function as $G(x)$. Then,

$$\begin{aligned} \frac{\partial E_s[m_{t+1}]P_s^*}{\partial P_s^*} &= E_s[m_{t+1}] + P_s^* \frac{\partial E_s[m_{t+1}]}{\partial P_s^*} \\ &= \int_0^\infty A \max\left[\frac{D}{P_s^* x}, 1\right] dG(x) + P_s^* \frac{\partial}{\partial P_s^*} \int_0^{D/P_s^*} A \max\left[\frac{D}{P_s^* x}, 1\right] dG(x) \\ &= A \frac{D}{P_s^*} \left(1 - G\left(\frac{D}{P_s^*}\right)\right) > 0. \end{aligned}$$

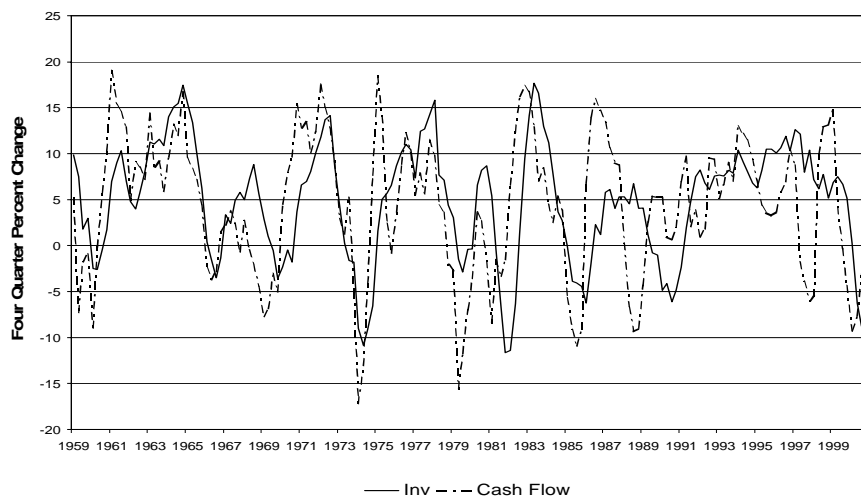
¹²The caveat is as follows. In the CIR model with log preferences, σ_s is equal to σ_s^* . As P_s^* approaches

Out-of-the-money equity index put-options command higher implied volatilities than out-of-the-money call options (the “volatility smirk.”) That is, the volatilities implied by the Black-Scholes-Merton option pricing model vary across different strike prices, for the same underlying asset. This pattern has been widely documented in the literature. For example, see Dupire (1994) and Jackwerth and Rubinstein (1996). Bates (1996) provides a survey.

Our results provide an economic explanation for the smirk. Referring back to Figure 6, we see that the Arrow-Debreu prices are high for low values of y_{t+1} , while they are equal to probabilities for high values of y_{t+1} . Alternatively, given investors debt-like claim on aggregate returns, they bear downside risk without sharing in upside risk. This implies that they have higher demand for put options than call options. Thus it is immediate that out-of-the-money puts will have higher implied volatilities than out-of-the-money calls.

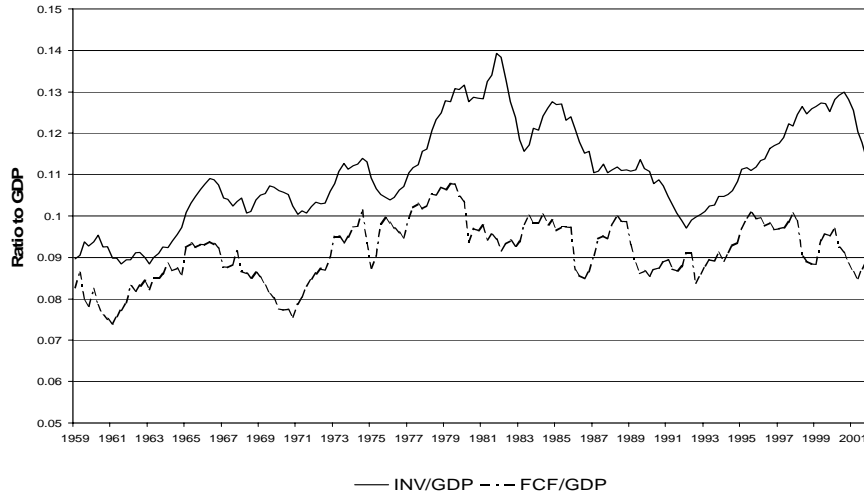
6 Are movements in corporate free-cash-flow sizeable enough?

Figure 7: Free Cash Flow and Investment, 4Qtr Change



We use quarterly measures of aggregate cash flow and investment from the Bureau of Economic Analysis, National Income and Product Accounts. The investment series we use is for non-residential fixed investment. We do not consider broader measures of investment, which would include consumer durables such as residential housing, in order to focus on zero, our free-cash-flow model approaches the CIR model. Thus the volatility relation is non-monotonic. However, this non-monotonicity is due to the log preference assumption. In a range where the free-cash flow friction is the dominant concern, volatility will be decreasing in P_s^* .

Figure 8: Aggregate Free Cash Flow and Investment, Ratio to GDP



the business investment which is subject to corporate control problems. The cash-flow measure is the net cash-flow of the corporate sector as reported in the NIPA Table 1.14, and corresponds to the measure of cash-flow commonly used in the investment literature.

Figure 7 graphs the four-quarter change in the real value of the two series. As documented in the investment literature, the two series are highly correlated (see, for example, Chirinko, 1993, Caballero, 1999, or Fazzari, Hubbard and Petersen, 1988).

Figure 8 graphs the ratios of our free-cash-flow measure to GDP versus non-residential fixed investment to GDP. The investment series varies from 8.3% to 13.9% of GDP. The mean of the ratio is 10.7%. The free-cash-flow series varies from 7.4% to 10.8% of GDP, and the mean of this ratio is 9.1%. As another point of comparison, personal savings rates as a fraction of disposable income have averaged about 8% over the last 50 years (source: BEA). Thus the variation in corporate free-cash-flow is large in comparison to both actual investment and savings.

7 Conclusion

We have studied the effect of imperfect corporate control on asset pricing and investment in a tractable dynamic equilibrium model. In our model, outside investors have limited control over managers and therefore only imperfectly control payout policy. This imperfection allows managers to use their discretion over free cash flow. They always invest as much as they can, consistent with the observation that firms with more cash invest more. Despite the simplicity of the preferences, the results are encouraging. We are able to match some

stylized facts concerning the term structure of interest rates, as well the behavior of risk premia. We are also able to match basic macroeconomic facts regarding investment.

For asset pricing, our results show how the introduction of corporate finance frictions may be fruitful. Although there are alternative existing explanations for all of the asset pricing facts we replicate, it is encouraging that the single free-cash-flow friction has explanatory power for a number of phenomena.

While our goal was to produce a tractable model, the simplicity of the model does lead to some clear failings. As we have mentioned, the price of capital in our model is always one. On the one hand, we know exactly why this occurs in our model. In the CIR model, the price of capital is also fixed at one, and ours is a departure from the CIR model. On the other hand, this means we are unable to address the equity premium puzzle. Moreover, conditional on variables at time t , the Hansen-Jagannathan bound computed from our pricing kernel is tighter in the free-cash-flow model than in the CIR model. That is, $\frac{\sigma(m)}{E(m)}$ is lower under the free-cash-flow kernel. This means that the conditional market risk premium is lower in our model and is certainly a failure of the model. It seems clear that to address the equity premium we will have to depart from log preferences, perhaps to a successful specification such as habit formation.

Our model differs from the endowment economies that are common in the asset pricing literature in that we have explicitly modeled the production side. In a sense, we have tied our hands by modeling both sides of the economy. In a frictionless world, asset prices must also match producers' marginal rate of transformation. Boldrin, Christiano, and Fisher (2001) show that adding a production sector to a model with habit-formation preferences can reduce risk premia dramatically because investors have strong incentives to use the production side to smooth consumption. A contribution of our paper is to show how corporate finance imperfections may prevent this consumption smoothing.

Finally, there is a great deal of evidence on how variables from the production side of the economy help to explain both the cross-section and time-series of asset returns. Variables such as size or book-to-market may have something to do with corporate finance imperfections. However, without an explicit model linking corporate financing imperfections with asset prices, it has not been possible to say how. We hope our model is a step towards addressing these issues.

The macroeconomics literature on financial frictions has for the most part focused on the friction of costly-external-finance (see for example, Bernanke and Gertler (1989) or Kiyotaki and Moore (1997)), as opposed to the free-cash-flow friction we study. In the cor-

porate finance literature, both types of financial frictions have received equal attention. Our paper is one of the first to study the implications of free-cash-flow problems for aggregate quantities. It is interesting that the free-cash-flow model also generates amplification and persistence of macroeconomic shocks. However, the propagation mechanisms differ across these two types of frictions. In our model, shocks are propagated through large firms (as opposed to small firms) and during booms (as opposed to downturns). We plan to study these distinctions in further detail in future research.

A Appendix

A.1 Proof of Proposition 1

Our solution approach is to guess the value function, then use the guess to arrive at expressions for the optimal controls (C_t, d_{t+1}) , and finally to substitute these controls back into the Bellman equation to verify the guess. Since preferences are log, we make the following guess for the value function:

$$J(I, \rho, d) = A \log I + B(\rho, d).$$

Let us also make the guess that $C_t = I_{t-1} \min[\hat{c}_t y_t, \gamma d_t]$, where \hat{c}_t depends only on t . Thus,

$$I_t = I_{t-1}(y_t - \min[\hat{c}_t y_t, \gamma d_t]),$$

and

$$\begin{aligned} u(C_t) + \beta J(I_t, \rho_{t+1}, d_{t+1}) &= \log I_{t-1} + \log \min[\hat{c}_t y_t, \gamma d_t] + \beta A \log I_{t-1} \\ &\quad + \beta A \log(y_t - \min[\hat{c}_t y_t, \gamma d_t]) + \beta B(\rho_{t+1}, d_{t+1}). \end{aligned} \quad (28)$$

Since,

$$A \log I_{t-1} + B(\rho_t, d_t) = J(I_{t-1}, \rho_t, d_t) = E_{t-1}[u(C_t) + \beta J(I_t, \rho_{t+1}, d_{t+1}) | \rho_t], \quad (29)$$

we can collect the terms on $\log I_{t-1}$ and match coefficients to find that $A = \frac{1}{1-\beta}$.

Dropping the terms in (28) that do not contain \hat{c}_t , we can then maximize pointwise for each y_t , to find a solution for consumption. In the range where $\gamma d_t > \hat{c}_t y_t$, we solve,

$$\max_{\hat{c}} \{ \log \hat{c} y_t + \beta A \log(1 - \hat{c}) y_t \}$$

giving $\hat{c}_t = (1 - \beta)$. Obviously for $\gamma d_t < \hat{c}_t y_t$, we don't need to solve for \hat{c}_t . Thus,

$$C_t = I_{t-1} \min[(1 - \beta) y_t, \gamma d_t].$$

To arrive at the expression for the payout policy, return to (29) and substitute in the expression for A . The $\log I_{t-1}$ terms drop out, and we are left with,

$$B(\rho_t, d_t) = E_{t-1} [\log \min[\hat{c}_t y_t, \gamma d_t] + \beta A \log(y_t - \min[\hat{c}_t y_t, \gamma d_t]) + \beta B(\rho_{t+1}, d_{t+1}) | \rho_t].$$

At date t , d_{t+1} is chosen to maximize $B(\rho_{t+1}, d_{t+1})$. Similarly, at date $t - 1$, d_t is chosen to maximize $B(\rho_t, d_t)$. Let us focus on this latter problem, given that we have the expression for $B(\rho_t, d_t)$ above. Suppose we can write the solution for d_t as,

$$d_t = \theta \rho_t.$$

Then, substituting in for \hat{c}_t and dropping the constant $B(\rho_{t+1}, \theta \rho_{t+1})$, gives:

$$d_t = \operatorname{argmax}\{E_{t-1}[\log \min[(1 - \beta)y_t, \gamma d_t] + \beta A \log(y_t - \min[(1 - \beta)y_t, \gamma d_t]) | \rho_{t-1}]\}.$$

θ solves:

$$\theta = \operatorname{argmax}\{E[\log \min[(1 - \beta)\frac{y_t}{\rho_t}, \gamma \theta] + \beta A \log(\frac{y_t}{\rho_t} - \min[(1 - \beta)\frac{y_t}{\rho_t}, \gamma \theta])]\}.$$

Then we note the following useful property:

Lemma 4

$$\frac{y_t}{\rho_t} = \left(\frac{z_t}{\rho_t} - (1 - \gamma) \min\left[\frac{z_t}{\rho_t}, \theta\right] \right)$$

Since $\frac{z_t}{\rho_t}$ is an i.i.d. random variable (see (2)), $\frac{y_t}{\rho_t}$ is also an i.i.d. random variable.

Given the lemma, θ is independent of time t state variables, which verifies our guess that d_t is linear in ρ_t . This completes the proof of Proposition 1. ■

A.2 Proof of Lemma 3

From previous expressions we have that,

$$\begin{aligned} C_t &= I_{t-1} \rho_t \psi_t \\ C_{t+\tau} &= I_{t+\tau-1} \rho_{t+\tau} \psi_{t+\tau}. \end{aligned}$$

Now the investment equation is,

$$I_t = \rho_t I_{t-1} \begin{cases} \beta \frac{y_t}{\rho_t} & \text{if } \gamma \theta > (1 - \beta) \frac{y_t}{\rho_t} \\ \frac{y_t}{\rho_t} - \gamma \theta & \text{if } \gamma \theta \leq (1 - \beta) \frac{y_t}{\rho_t} \end{cases}$$

which can be written more concisely as,

$$I_t = \rho_t I_{t-1} \left(\beta \frac{y_t}{\rho_t} + \max[(1 - \beta) \frac{y_t}{\rho_t} - \gamma \theta, 0] \right) = \rho_t I_{t-1} \left(\frac{y_t}{\rho_t} - \psi_t \right),$$

where as we have noted before (see Lemma 4), $\frac{y_t}{\rho_t} - \psi_t$ is an i.i.d. random variable. Then,

$$I_{t+\tau-1} = I_{t-1} \left(\prod_{s=t}^{s=t+\tau-1} \rho_s \right) \left(\prod_{s=t}^{s=t+\tau-1} \frac{y_s}{\rho_s} - \psi_s \right).$$

In terms of consumption,

$$C_{t+\tau} = I_{t-1} \left(\prod_{s=t}^{s=t+\tau-1} \rho_s \right) \left(\prod_{s=t}^{s=t+\tau-1} \frac{y_s}{\rho_s} - \psi_s \right) \rho_{t+\tau} \psi_{t+\tau}.$$

Substituting all this back into the interest rate expression, (21), gives,

$$\frac{\beta^\tau u'(C_{t+\tau})}{u'(C_t)} = \beta^\tau \left(\prod_{s=t+1}^{s=t+\tau} \rho_s \right)^{-1} \left(\prod_{s=t}^{s=t+\tau-1} \frac{y_s}{\rho_s} - \psi_s \right)^{-1} \left(\frac{\psi_t}{\psi_{t+\tau}} \right).$$

Taking expectations and then logs we arrive at (22). ■

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