Imperfect Information, Consumers' Expectations and Business Cycles*

Guido Lorenzoni†

September 2004

Abstract

This paper presents a model of business cycles driven by shocks to consumers' expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The shock to the public signal has the features of a "demand shock": it increases output, employment and inflation in the short run and has no effects in the long run. On the other hand, the aggregate productivity shock has the features of a "supply shock": after a productivity shock output adjusts gradually to its higher long run level, and there is a temporary negative effect on employment and inflation. The fraction of short run fluctuations explained by the public signal shock is non-monotone in the precision of the public signal. For high levels of idiosyncratic uncertainty the model can replicate the variance decomposition obtained in identified VARs.

1 Introduction

This paper analyzes business cycle fluctuations in an economy where consumers have imperfect information regarding the level of aggregate productivity. The model formalizes the old idea that business cycles are driven by changes in consumers' expectations. In particular, it formalizes the idea that changes in consumers' expectations can generate fluctuations in expenditure that drive output temporarily away from a "natural" equilibrium path entirely determined by tastes and technology. In this view cyclical fluctuations in employment and inflation are associated to these temporary deviations of output from the natural path.

*Preliminary version.

†Department of Economics and Woodrow Wilson of Public and International Affairs, Princeton University. Email guido@princeton.edu. The author thanks Marios Angeletos, Olivier Blanchard, Michele Cavallo, Mark Gertler, Christian Hellwig, Nancy Stokey, Ivan Werning and seminar participants at MIT, U. of Virginia, Ohio State University, UCLA, Yale, NYU, "Ente Einaudi"-Rome, and U. of Minnesota for useful comments.

In dynamic stochastic general equilibrium models cyclical fluctuations can be driven by a variety of shocks: technology shocks, preference shocks, shocks to public expenditure, and so on. In these models consumers' expectations move together with aggregate variables but do not provide an independent source of fluctuations. In recent work, Danthine and Donaldson (1998) and Beaudry and Portier (2000, 2003) have studied equilibrium models where changes in expectations regarding *future* productivity have real effects on output. However, equilibrium models of this type tend to generate a negative correlation between consumption and investment and between consumption and labor supply following an "expectation shock". In this type of models news regarding future productivity induce workers to postpone labor effort and to dissave to finance a current increase in consumption[1]. Models based on increasing returns and sunspots have strongly emphasized the role of expectations. However, in a crucial dimension these models are observationally analogous to models with exogenous productivity shocks: a boom is associated to a *contemporaneous* increase in total factor productivity[2].

Recent work by Hall (1997) has emphasized that a large fraction of business cycle fluctuations seems to be accounted for by changes in the marginal rate of substitution between consumption and leisure. Chari, Kehoe and McGrattan (2004) call this variable the "labor wedge" and discuss its role in accounting for business cycles. Smets and Wouters (2003) study a dynamic general equilibrium model with nominal rigidities and allow for various types of shocks, including productivity shocks, preference shocks and mark-up shocks. Also their paper tends to attribute a sizeable fraction of output volatility to preference shocks that induce changes on the consumption-leisure margin.

On the other hand, work based on identified VARs, going back to Shapiro and Watson (1988) and Blanchard and Quah (1989), has emphasized the role of "demand shocks" in business cycle fluctuations. Demand shocks are characterized empirically as shocks that have no effects on output and employment in the long run. Identified demand and supply shocks induce different responses of output, prices and employment. Both output and prices and output and employment are positively correlated following a demand shock and negatively correlated after a supply shock.

This paper shows that introducing imperfect information about *current* productivity in a dynamic equilibrium model it is possible to (1) study consumers' expectations as an independent source of fluctuations, (2) allow for cyclical variation in the "labor wedge" without introducing preference shocks, (3) generate responses of output, prices, and hours in line with existing VAR evidence.

The observation that imperfect information can cause movements in output and hours that are not due to shifts in tastes or technology goes back to Phelps (1969) and Lucas (1972). That idea was developed to study money non-neutrality, that is, the effect of

[1] In models with embodied technical change a future increase in productivity can generate an increase in investment and labor supply, but in this case consumption tends to drop following an expectation shock. See also the discussion in Section 6 of Beaudry and Portier (2003).

[2] See Murphy et al. (1989).

unanticipated monetary policy shocks[3]. In this paper a similar mechanism is applied to study the effects of shifts in consumers expectations. This requires a setup where money velocity is not constant, so that changes in consumers expectations can affect nominal spending. For this reason we use a buffer-stock model of real balances where total spending can adjust to changes in consumers expectations.

The paper considers an economy with sectoral productivity shocks and imperfect information. Each agent observes the productivity shock in his own sector and a noisy public signal of the aggregate productivity shock. The noise in the public signal moves agents' expectations independently of movements in real productivity. This shock generates an increase in output and hours worked with no changes in current productivity. Following a public signal shock average productivity remains constant. However, the average producer believes that productivity has increased in the other sectors, moving up the demand for his own good. He responds by increasing his spending, increasing his level of employment and trying to increase the relative price of his good. This generates an increase in output, in hours worked and in nominal prices.

The presence of imperfect information also changes the response of the economy to an aggregate productivity shock. In particular a permanent productivity shock has a positive effect on output, but has a temporary negative effect on employment and inflation. Following a productivity shock agents' expectations regarding aggregate productivity increases less than one-for-one with productivity itself. The average producer believes that productivity has increased less in the other sectors than in his own. He responds by increasing his spending less than his productivity, decreasing his level of employment and trying to decrease the relative price of his good. This reduces aggregate employment and nominal prices. The negative effect of permanent productivity shocks on employment is consistent with the evidence in Gali (1999) and in Francis and Ramey (2002).[4]

In the model presented there are two types of strategic complementarity. A strategic complementarity in price setting analogous to the one present in sticky price models and a strategic complementarity in spending. A higher degree of complementarity in price setting reduces the effect of a public signal shock on prices and increases its effect on quantities. A higher degree of complementarity in spending, on the other hand, amplifies the effect of a public signal shock on both prices and quantities. The role of imperfect information and higher order expectations in games with strategic complementarity has been recently analyzed in a number of theoretical and applied papers. In particular Morris and Shin (2002) have shown that the presence of strategic complementarities amplifies the effect of public information and slows down the adjustment of aggregate variables towards their "fundamental" value[5]. In this paper the presence of strategic complementarity has an additional effect because it affects the speed of learning in the economy. This is because in our model quantities are an *endogenous signal*. Therefore, when there is a higher degree of

[3] See also Woodford (2002) and Hellwig (2003).

[4] See Chrisitano, Eichenbaum and Vigfusson (2003) for a critical view of this evidence.

[5] We will call the "fundamental" value of a variable the value that would arise under full information.

strategic complementarity observed quantities reflect more the public signal and less the underlying fundamental. As a consequence current quantities are a less precise signal of the fundamental, agents learn more slowly about the fundamental and output stays away longer from its fundamental value. Therefore, a high degree of strategic complementarity generates stronger *and* more persistent effects of the public signal.[6]

This paper is also related to the literature on monetary policy with imperfect information. In recent work Aoki (2000), Svensson and Woodford (2000, 2001) and Reis (2003) have considered sticky price models where a monetary authority has imperfect information regarding macroeconomic fundamentals. In these models monetary policy mistakes arise despite the best intentions of the monetary authority and realized output (or unemployment) can deviate from their natural rate in the short run. In these papers the monetary authority uses its knowledge of the structure of the economy to infer the equilibrium level of output (or employment) from past observations of output and inflation. In our paper the same inference problem is faced by each individual agent. As agents observe past output and inflation they update their beliefs regarding the productivity shock and they converge towards the full information equilibrium.

2 The Model

2.1 Setup

The model is a version of a Dixit-Stiglitz model of monopolistic competition with local productivity shocks. Consider an economy populated by a continuum of infinitely lived households uniformly distributed on the unit square $[0, 1] \times [0, 1]$. The first index denotes island i where the households lives, while the second index denotes variety k produced on island i. Since all households on island i are subject to identical shocks and will have identical behavior it is convenient to save on notation and use only the location index i wherever possible. We will call "household i" the representative household on island i, and "good i" any of the goods produced on island i.[7]

Each household on island i is composed of a consumer and a producer. The producer works N_{it} hours to produce a given variety of good i. The shopper travels each period to m islands, randomly drawn, and consumes all the varieties produced in those islands. Let H_{it} denote the set of islands visited by the shoppers from island i. The assignment of consumers to islands is randomly determined by an independent draw each period. The assignment is determined in such a way that the marginal probability of traveling to any island j is uniform for each consumer i and the marginal probability of selling to a consumer coming

[6] In the asset pricing model by Allen, Morris and Shin (2003) prices are also an endogenous signal, so probably a similar mechanism is at work in their framework.

[7] The assumption of multiple varieties on island i is only made to simplify the monopolistic pricing problem. Specifically, it makes the informational content of observed sales independent of the individual posted prices, thus eliminating any motive for price experimentation on the monopolist side.

from any island j is uniform for each producer i. Moreover, the assignment is determined so that each island is visited by consumers coming from exactly m islands. Figure 1 represents the islands on a circle and illustrates the structure of the exchanges in the case $m = 2$.

Household preferences are represented by the utility function:

$$E\left[\sum_{t=1}^{\infty} \delta^t u(C_{it}, N_{it})\right]$$

where

$$u(C_{it}, N_{it}) = \ln C_{it} - \frac{1}{1+\eta} N_{it}^{1+\eta} \tag{1}$$

and C_{it} is a composite good defined below. This utility function is compatible with balanced growth, this is an important feature of the model, given that we will introduce a non-stationary process for aggregate productivity. This utility function, together with the absence of capital, implies that when agents have full information employment is constant in equilibrium. This simplification allows us to study in insulation the effect of imperfect information on the cyclical behavior of employment.

The composite consumption good is a standard CES aggregate including all the varieties produced in the m islands visited in period t

$$C_{it} = \left(\frac{1}{m} \sum_{j \in H_{it}} \int C_{ijkt}^{\frac{\sigma-1}{\sigma}} dk\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

with $\sigma > 1$, where C_{ijkt} is consumption of variety k on island j by household i.

The production function on island i is

$$Y_{it} = A_{it} N_{it}.$$

All producers on island i receive the same productivity shock A_{it}, but different islands receive different productivity shocks. The productivity parameters A_{it} are the crucial source of uncertainty in this economy.

At date 0 all agents start with the same level of productivity A_0. Then, at the beginning of each period t agents observe their local productivity A_{it}. Let a_{it} denotes the log of A_{it}. Local productivity a_{it} has an aggregate component a_t and an idiosyncratic (island-specific) component ϵ_{it}:

$$a_{it} = a_t + \epsilon_{it}$$

The aggregate component a_t follows the random walk

$$a_t = \gamma + a_{t-1} + u_t$$

where u_t is the aggregate productivity shock. The parameter γ determines the average growth rate. The cross sectional distribution of the ϵ_{it}'s satisfies:

$$\int \epsilon_{it} di = 0.$$

At the beginning of each period agents observe a *public signal* regarding the current productivity shock, the signal s_t is given by

$$s_t = a_t + e_t.$$

The noise in the public signal e_t will be the source of autonomous shifts in consumers' expectations. It is also convenient to assume that after T periods the aggregate productivity shock is observed with no noise. This assumption is only made to simplify the treatment of monetary policy and in all simulations we will take T to be large.

Let $\mathbf{P}_{it}$ denote the vector of the prices of the goods purchased by agent i at date t,

$$\mathbf{P}_{it} = \{P_{jt}\}_{j \in H_{it}}.$$

Household i can observe the following variables: the local productivity a_{it}, the public signal s_t, the price vector $\mathbf{P}_{it}$ and the quantity of the good sold Y_{it}. We will be more specific about the timing of the information flows in the next section.

The aggregate shocks in this economy are represented by the permanent productivity shock u_t and the public signal shock e_t. These shocks are independent and serially uncorrelated random variables with zero mean and variances (σ_u^2, σ_e^2). The idiosyncratic shocks ϵ_{it} are independent across i's for each finite sample of islands, they are also serially uncorrelated and independent of the aggregate shocks. The signal shock e_t is gaussian. Given the financial structure of the economy it is necessary to assume that the technological shocks ϵ_{it} and u_t are bounded, however in the numerical part we will approximate them with Gaussian random variables, in order to apply Kalman filter techniques.

2.2 Trading, Financial Markets, and Monetary Policy

The only asset in the economy is government issued fiat money, or cash. Trading on each island is anonymous and takes the form of spot exchanges of cash for goods. Since producers need no cash all the cash is given to the consumer before the trading phase begins. We assume that there is no record-keeping device and no other asset in the economy, therefore money is the only means of exchange in this economy.

The government is located in a central island. At any time during period t shoppers can travel to the central island and borrow money at a zero interest rate, this money must be repaid before the end of period t[8].

At the beginning of period t the government levies a lump-sum tax T_t, in cash, on all households. The receipts from this tax and the increase in money supply are used to pay the gross interest rate R_t on money balances inherited from last period. Let M_{it} denote the money balances available immediately before the trading phase in period t. The money balances of household i evolve according to:

$$M_{it+1} = R_{t+1}\left(M_{it} + P_{it}Y_{it} - \sum_{j \in H_{it}} P_{jt}C_{ijt}\right) - T_{t+1} \tag{3}$$

and the process for total money supply is given by

$$M_{t+1} = R_{t+1}M_t - T_{t+1}.$$

The assumption of zero-interest borrowing within the period is made so we do not need to introduce a cash-in-advance constraint of the type $\sum_{j \in H_{it}} P_{jt}C_{ijt} \leq M_{it}$. The only constraint on cash balances is the non-negativity constraint on cash balances

$$M_{it} \geq 0.$$

Let us describe in detail the trading sequence and the information flows in this economy:

- At the beginning of period t the government raises taxes and pays interest on nominal balances from the previous period.
- Then, all agents in household i observe the productivity A_{it} and the public signal s_t.
- The producer at each location sets the price P_{it} and stands ready to deliver any quantity of his good at that price[9]. This is called the "pricing stage."
- The shopper takes the money stock M_{it} and travels to the m islands H_{it}. He observes the price vector $\{P_{jt}\}_{j \in H_{it}}$ and exchanges money for cash to purchase the consumption goods $\{C_{ijt}\}_{j \in H_{it}}$. If he needs to, he travels to the central island to borrow cash. This is called the "trading stage." We assume that shoppers do not communicate with producers during the trading stage, so shoppers do not know the quantity produced of their home good when they are making their spending decisions.

[8] This means that the government in the central island has access to some record-keeping device and some punishment mechanism. We are implicitly assuming that these instruments are only used to sustain within-period cash loans.

[9] To be more precise we let producers set a maximum supply $\overline{Y}_{it}$, satisfy any demand in $\left[0, \overline{Y}_{it}\right]$, and adopt some rationing rule if demand exceeds $\overline{Y}_{it}$. As usual in models of price setting we assume that the size of the shocks is small enough that the demand for good i is always smaller than $\overline{Y}_{it}$ in equilibrium.

- The shopper returns home and observes the quantity produced in island Y_{it}. If he has borrowed any money from the central bank he travels to the central island to return it.

This trading mechanism embeds three crucial features: (1) a form of price setting by sellers, (2) money that serves both as means of exchange and as a store of value (3) incomplete financial markets.

The first feature implies that price formation is explicitly modeled. However, price setting and monopolistic competition are not strictly necessary for the analysis. It is possible to construct a competitive model with local productivity shocks with similar features. The crucial assumption is that there is some separation between the shopper and the producer, as in the competitive model of Lucas (1972), so that the producer faces some uncertainty regarding the real terms of trade he is facing. The confusion between nominal and relative prices is crucial in determining a temporary response of output to the noise shock. From a modeling point of view the advantage of a price setting framework is that households acquire information in a sequential fashion: first they make their pricing decisions, then they observe a set of prices and make their quantity decisions. In the competitive model instead the information set of the shopper and of the producers are not nested, which makes the analysis more cumbersome.

The fact that money is the only asset is a useful simplification. The crucial thing is that money velocity is not fixed, so that total nominal spending is allowed to vary even though money supply stays fixed. This is a crucial distinction between the model presented here and the model in Hellwig (2004). When the cash-in-advance constraint is binding the only uncertainty regarding total nominal expenditure comes from exogenous shocks to the money supply. On the other hand, in the present model velocity is allowed to adjust and nominal expenditure needs to be determined endogenously. In particular nominal expenditure will depend on agents' expectations regarding the current and future level of activity. The determination of nominal expenditure in the case of imperfect information is the central focus of this paper.

The assumption of incomplete financial markets is made for simplicity and in order to limit the amount of information revelation through asset prices. By introducing a single nominal asset (money) we do not have to track the heterogeneous portfolios decisions of different consumers, which would complicate the analysis considerably. In this simple setup consumers' behavior is captured by a permanent-income equation. From an informational point of view the introduction of a richer asset structure would have different effects. On the one hand it would help individuals pool their information, on the other hand it may add a source of noisy public signals, given that asset prices are related to expectations of future returns but not necessarily to current productivity. Recent work by Beaudry and Portier (2003) shows that stock prices have features that resemble those of the public signal s_t in our model: they carry information regarding long-run productivity and at the same time have business-cycle short-run effects. Incorporating news driven by asset price movements in a model with imperfect information seems a promising avenue for future research.

Monetary policy in this environment has two roles: it can affect the level of nominal spending through changes in M_t and R_t and it can reveal information about the economy by making monetary interventions a function of private information received by the central bank. In this paper we want to mute both effects and let all the adjustment in nominal spending be associated to changes in velocity. A way of doing that is to fix the nominal interest rate

$$R_t = R$$

and let monetary policy follow a very simple backward looking monetarist rule

$$M_t = A_{t-T}.$$

This rule makes the money stock proportional to productivity in the long run and allows for an equilibrium with stationary velocity and a stationary price level. The crucial aspect of this rule is that it is consistent with an equilibrium where the price level has a tendency to return a long run average $\bar{P}$. This type of price level target has strong stabilizing property, since temporary inflation is automatically associated with an increase in the long run real interest rate. Therefore, it is interesting to notice that still under this rule considerable variation in real expenditure can arise due to changes in expectations. The role of this rule and the effect of alternative policy rules is discussed in the conclusions.

We have assumed that the nominal interest rate is fixed and taxation T_t is set in nominal levels so as to pay a constant interest rate on cash balances, this means that the model is open to indeterminacy. In the following we will assume that the agents coordinate on a given long run price level $\bar{P}$ and abstract from issues of determinacy.

Before defining an equilibrium it is useful to describe more formally the information available to market participants. Producer i makes his pricing decision based on the information in $\mathcal{I}_{it}^{*}=\{A_i^t, \mathbf{P}_i^{t-1}, Y_i^{t-1}, s^t\}$[10]. Shopper i observes the m prices of the goods in H_{it} and makes his quantity decision based on $\mathcal{I}_{it}=\{A_i^t, \mathbf{P}_i^t, Y_i^{t-1}, s^t\}$. We will use the following notation for agents expectations at the pricing stage and at the trading stage:

$$\begin{aligned} E_{it}^{*}[.] &= E[.|\mathcal{I}_{it}^{*}] \\ E_{it}[.] &= E[.|\mathcal{I}_{it}] \end{aligned}$$

Also, we will use the following notation for the cross sectional averages of the first order expectations of any variable x_{t+s}

$$\begin{aligned} x_{t+s|t^*} &= \int E_{it}^{*}[x_{t+s}]\, di \\ x_{t+s|t} &= \int E_{it}[x_{t+s}]\, di. \end{aligned}$$

[10] We use the notation $X^t = \{X_t, X_{t-1}, ...\}$.

2.3 Rational Expectations Equilibrium

Prior to giving a formal definition of equilibrium it is convenient to derive the demand curve faced by monopolist i. Consider consumer j purchasing variety k on island i . Given his level of consumption C_{jt} his demand will be

$$C_{jikt} = \left(\frac{P_{ikt}}{\overline{P}_{jt}}\right)^{-\sigma} C_{jt}$$

where the price index for consumer j, $\overline{P}_{jt}$, is defined as

$$\overline{P}_{jt} = \left(\frac{1}{m}\sum_{l\in H_{jt}}\int P_{lkt}^{1-\sigma}dk\right)^{\frac{1}{1-\sigma}}. \tag{4}$$

Therefore, the demand faced by the producer of good (k,i) at date t is given by

$$Y_{it} = \sum_{j\in\tilde{H}_{it}}\left(\frac{P_{ikt}}{\overline{P}_{jt}}\right)^{-\sigma} C_{jt}. \tag{5}$$

Where $\tilde{H}_{it} = \{j : i \in H_{jt}\}$ is the set of agents that buy good i. In this way we can derive the individual demand for each good conditional on the prices and consumption levels P_{ikt} and C_{it}. From now on, then, we let $P_{ikt} = P_{it}$ and we give an equilibrium definition in terms of the two processes P_{it} and C_{it}.

A *symmetric rational expectations equilibrium* is defined by two stochastic processes P_{it} and C_{it} for each island i that satisfy

$$P_{it} = P_t\left(\mathcal{I}_{it}^*\right) \tag{6}$$

$$C_{it} = C_t\left(\mathcal{I}_{it}\right) \tag{7}$$

such that P_{it} and C_{it} solve the following maximization problem

$$\max_{\{\tilde{P}_{it},\tilde{C}_{it}\}} E\left[\sum_{t=0}^{\infty}\delta^t u(\tilde{C}_{it}, N_{it})\right]$$

$$s.t. \quad Y_{it} = \sum_{j\in\tilde{H}_{it}}\left(\frac{\tilde{P}_{it}}{\overline{P}_{jt}}\right)^{-\sigma} C_{jt}$$

$$M_{it+1} = R\left(M_{it} + \tilde{P}_{it}Y_{it} - \overline{P}_{it}\tilde{C}_{it} - T\right)$$

$$M_{it} \geq 0; \quad M_{i0} = M$$

$$P_{jt} = P(\mathcal{I}_{jt}^*); C_{jt} = C\left(\mathcal{I}_{jt}\right) \text{ for all } j$$

under the measurability constraints implicit in (6)and (7) and where the price index $\overline{P}_{it}$ is defined as

$$\overline{P}_{it} = \left(\frac{1}{m} \sum_{j \in H_{it}} P_{jt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.$$

The aggregate price index and aggregate output are defined as

$$\begin{aligned} P_t &= \left(\int P_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \\ Y_t &= \frac{\int P_{it} Y_{it} di}{P_t}. \end{aligned}$$

Notice that in a symmetric equilibrium the state $\left(a^t, s^t\right) = \{a_t, s_t, a_{t-1}, s_{t-1}, ...a_0, s_0\}$ is sufficient to characterize the aggregate behavior of the economy. In particular for a given state $\left(a^t, s^t\right)$ it is possible to derive the conditional distribution of past prices P_{jt-s} and quantities C_{jt-s} for $s \geq 0$. Using that distribution and the observed prices and quantities a Bayesian agent can derive its posterior regarding the state $\left(a^t, s^t\right)$. Using again the equilibrium functions $P_t(.)$ and $C_t(.)$ he can then derive the future distributions of prices and quantities at all future nodes and solve his optimization problem. Therefore, the decision problem of agent i at date t depends only on his money balances M_{it} his current productivity a_{it} and his beliefs regarding the aggregate state $\left(a^t, s^t\right)$.

In section 3 we modify our basic setting, and allow agents to pool the cash-flow risk across islands. In that case the distribution of M_{it} is degenerate and the steady state can be studied analytically.

In the case of no insurance, however, the distribution of cash balances is non-degenerate and evolves over time. Let Φ_{it} be the CDF that describes the beliefs of agent i at date t regarding the aggregate state $\left(a^t, s^t\right)$. Let Δ_t describe the joint cross sectional distribution of $\frac{M_{it}}{A_t P_t}$ and Φ_{it} at a given point in time. Then a symmetric equilibrium describes a transition map $\mathcal{T}$ for the distribution Δ_t

$$\Delta_{t+1} = \mathcal{T}\left(\Delta_t, \left(a^t, s^t\right), u_{t+1}, e_{t+1}\right)$$

Our working hypothesis, for the case of no insurance, is that there exists an equilibrium where Δ_t has a stationary stochastic steady state. This requires, among other things, that (1) the effect of (a_{t-s}, s_{t-s}) on Δ_t vanishes as s goes to infinity, (2) the dynamics of real balances are non-explosive and (3) the dynamics of beliefs are non-explosive. In the numerical examples this conjectures appear to be validated, see Appendix C.

3 Equilibrium

In order to study the properties of the model it is useful to start with simple cases where the equilibrium can be derived analytically. First, we consider the case of no heterogeneity and full information. This gives us a benchmark where fluctuations are only driven by contemporaneous changes in productivity and where news shocks are irrelevant. Second, we consider an economy with full insurance across islands. This case illustrates well the different effects of productivity shocks and news shocks on output, employment and prices. The main limitation of the full insurance model is that with full insurance agents fully learn aggregate productivity at the end of each trading period. Therefore, the model with full insurance leaves no room to analyze the transmission and propagation of shocks. The model with no insurance, on the other hand, is analytically much richer, since it requires us, in principle, to keep track of the joint distribution of money balances and beliefs. For this reason, we turn to a log-linear approximation that allows us to describe the economy in terms of the evolution of the first moments of agents' beliefs. This log-linear approximation forms the basis for the numerical analysis in next section.

3.1 No Idiosyncratic Shocks

We will consider first the equilibrium of an economy with no local shocks ($\sigma_\epsilon^2 = 0$). With no local shocks all agents are identical and have full information about aggregate productivity. Set the variance σ_ϵ^2 of the local shocks to zero. In a symmetric equilibrium all prices and consumption levels are identical and equal to P_t and C_t.

In general form, the optimality condition for the monopolist pricing problem is

$$E_{it}^* \left[\frac{1}{\overline{P}_{it} C_{it}} (1-\sigma) Y_{it} - \sigma \frac{1}{A_{it}} \left(\frac{Y_{it}}{A_{it}} \right)^\eta \frac{Y_{it}}{P_{it}} \right] = 0. \tag{8}$$

In the case of no local shocks this expression boils down to

$$\frac{\sigma - 1}{\sigma} = \left(\frac{C_t}{A_t} \right)^{1+\eta}$$

which gives constant labor supply

$$N = \left(\frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1+\eta}}.$$

All agents have the same consumption level equal to

$$C_t = Y_t = A_t N$$

and all prices are the same.

Given that there is full information we can assume that monetary policy adjusts immediately to the observed level of productivity and obtain a simple equilibrium with a constant price level. If money supply is given by

$$M_t = A_t$$

then we have a continuum of equilibria where the price level is constant P_t and the velocity of money is constant. Namely, for any level of $\bar{P}$, we have an equilibrium where the ratio of real balances to output is given by

$$\mu = \frac{M_t}{\bar{P} A_t N}$$

In this economy real balances do not serve any precautionary purpose, and agents only hold them to pay future nominal taxes. Therefore, we cannot hope to determine their value in equilibrium.

The nominal interest rate consistent with this equilibria has to satisfy the Euler equation

$$\frac{1}{C_t} = \delta R E_t \left[\frac{1}{C_{t+1}} \right]$$

This condition is satisfied only if $R = R^* = \delta^{-1} e^{\gamma + \frac{1}{2}\sigma_u^2}$. Notice that, in the case of no idiosyncratic risk the interest rate has to be set at $R = R^*$ and the price level and the value of the constant μ are indeterminate in equilibrium. We will have a similar result in the case of full insurance. In the case of idiosyncratic risk and no full insurance, instead, the monetary authority is free to set R in a certain range, and the value of the price level and of real balances will be determined in equilibrium.

3.2 Full Insurance

A second case where the equilibrium can be derived analytically is the case of full insurance. Suppose that the islands on $[0, 1]$ can be divided in groups that replicate exactly the cross sectional distribution of the productivity shocks a_{it}. Suppose that all the consumers in a group can meet at the end of the period and pool their cash balances together. This happens *before* the signal shock and the productivity shocks are realized, but after all trading in the previous period has concluded. An optimal insurance arrangement within the group implies that each household receives the same amount of cash at the end of the period.[11] Since households have to verify each others' shocks to implement the insurance agreement, this setup implies that at the end of each period the productivity levels are commonly observed. Therefore, uncertainty will regard only the current aggregate shock u_t.

Now we can consider a stationary equilibrium where the end-of-period cash balances in each group are constant and equal to money supply. Since aggregate productivity A_{t-1} is revealed to all consumers at the beginning of period t, we allow the monetary authority to

[11] Lucas (1990) was the first to use this device to allow risk pooling across households in a model with trading frictions. The equilibrium will be analogous if we allowed for trade of state contingent claims at the beginning of date t, before the realization of the signal and of the productivity shocks.

observe it and to set the money supply M_t accordingly. In particular, we assume that the monetary rule is[12]

$$M_t = A_{t-1}.$$

For simplicity, we consider the case where all consumers from island i travel and make purchases in a single island, j, that is we let $m = 1$. With a slight abuse of notation let H_{it} denote the island where consumers from island i travel and $\tilde{H}_{it}$ the island of origin of consumers traveling to island i. In this case the first order condition (8) characterizing optimal pricing can be rewritten as

$$P_{it} = -\frac{\sigma}{\sigma - 1}\left(E_{it}^{*}\left[\frac{C_{\tilde{H}_{it},t}}{P_{H_{it},t}C_{it}}\right]\right)^{-1} E_{it}^{*}\left[\left(\frac{C_{\tilde{H}_{it},t}}{A_{it}}\right)^{1+\eta}\right].$$

Thanks to full insurance we can treat each group of households as a single decision maker and the Euler equation for optimal real balances takes the following form

$$\frac{1}{P_{H_{it},t}C_{it}} = \beta R E_{it}\left[\left(\int \frac{1}{P_{H_{jt+1},t+1}C_{jt+1}}dj\right)\right].$$

These two conditions suggest us to find an equilibrium where prices and consumption levels are log normal. Let p_{it} and c_{it} denote the logs of P_{it} and C_{it}. Then we can establish the following.

Proposition 1 *In the case of full insurance there is a value of R such that the price level is stationary and prices and quantities are given by:*

$$\begin{aligned} p_{it} &= \phi_0 + \phi_1 (u_t + \epsilon_{it}) + \phi_2 (u_t + e_t), \\ c_{it} &= a_{t-1} + \psi_0 + \psi_1 (u_t + \epsilon_{it}) + \psi_2 (u_t + e_t) + \psi_3 (p_{jt} - \phi_0), \end{aligned}$$

where $j = H_{it}$.

Let q_{it} denote the intercept of the demand curve for good i (in logs)

$$q_{it} = c_{it} - \psi_3 p_{jt}$$

and let q_t denote the economy wide average level of q_{it}. Then, apart from constant terms, we can write the pricing equation and the Euler equation as

$$\begin{aligned} p_{it} &= \frac{1+\eta}{1-\psi_3\eta}\left[E_{it}[q_t] - a_{it}\right] + \frac{1+\psi_3}{1-\psi_3\eta}E_{it}[p_t] \\ c_{it} &= -p_{jt} + E_{it}[a_t] \end{aligned}$$

(TO BE COMPLETED)

[12] Given that agents are allowed to borrow money inter-periods at zero interest and given that there is no cash-flow uncertainty (due to full insurance) many other monetary rules are consistent with the equilibrium we describe below. This rule has the advantage of keeping the ratio of inside to outside money stable over time.

Proposition 2 *If there is full insurance and the signal is perfectly informative then the equilibrium has two properties: (1) output depends only on current productivity, (2) employment is unaffected by the productivity shock.*

A similar result will hold, in approximation, in the case of no insurance.

The second property depends on the absence of capital and on the form of the preferences (1). As we already observed in this model we are muting the mechanisms that drive employment fluctuations in real business cycle models in order to focus on the consequences of imperfect information and decentralized trading.

3.3 Self Insurance

In the case where no insurance arrangements are allowed consumers will hold precautionary balances to protect themselves against idiosyncratic productivity shocks. Given that the idiosyncratic component of the productivity shock is temporary and that we can set a positive rate of return on real balances we think it is appropriate to use a permanent income approximation for the optimal consumption rule of consumers.

Then equilibrium output and prices can be derived in terms of a log-linear approximation around the equilibrium described in 3.1. The choice of a log-linear approximation is dictated by the presence of imperfect information. When agents use linear decision rules individual decision rules only depend on the first moments of agents' posterior distributions. Linearity is useful both to simplify the inference problem faced by each individual and to simplify the aggregation of the individual decision rules. In particular, when agents use linear decision rules we can show that there is a linear equilibrium where aggregate output and prices, y_t and p_t, are linear functions of the aggregate state $\left(a^t, s^t\right)$.

With idiosyncratic shocks ($\sigma_\epsilon^2 > 0$) the economy has the features of the economies with idiosyncratic shocks and a single bond studied by Bewley (1986), Aiyagari (1994) and Huggett (1993). The difference between our economy and the economies analyzed in those papers is (1) the presence of aggregate shocks, (2) the fact that terms of trade are endogenous, (3) the presence of imperfect information. Notice also, that in those papers the supply of any outside asset (money or government bonds) is fixed and the real interest rate is endogenously determined by market clearing on the asset market, in our environment instead we set the interest rate R and the value of the outside asset, M/P_t, will adjusts in equilibrium thanks to adjustments in the nominal price level.

Here, we can be more precise about the use of the economy with no local shocks as an appropriate benchmark for a linear approximation. Consider an economy with $\sigma_\epsilon^2 > 0$ and $R < R^*$. In this economy, in principle, we could derive the mean of the stationary distribution of the ratio $M/\left(P_t Y_t\right)$. Let that mean be denoted by $\hat{\mu}$. Since real balances are indeterminate in the economy with no local shocks and $R = R^*$ we can consider an equilibrium where $\mu = \hat{\mu}$ and use it as the appropriate benchmark for our log-linearization.[13]

[13] This approach is valid provided that there is path between $\left(R, \sigma_\epsilon^2\right)$ and $(R^*, 0)$ such that the relation

Pricing In log-linear terms the demand for good i (5) takes the form[14]

$$y_{it} = -\sigma \left(p_{it} - \frac{1}{m} \sum_{j \in \tilde{H}_{it}} \overline{p}_{jt} \right) + \frac{1}{m} \sum_{j \in \tilde{H}_{it}} c_{jt} \tag{9}$$

Aggregate output is defined as

$$y_t \equiv \int y_{it} di = \int c_{it} di$$

where the second equality follows from (9). The aggregate price index is defined as

$$p_t = \int p_{it} di.$$

At the pricing (trading) stage agent i estimates that the income of the consumers purchasing product i will be $E_{it} y_t$ ($E^*_{it} y_t$). Since the assignment of consumers to goods is uniform the prices of the goods competing with good i will be estimated at $E_{it} p_t$ ($E^*_{it} p_t$). Therefore we can take expectations at the pricing stage and at the trading stage and obtain:

$$E^*_{it} y_{it} = -\hat{\sigma} \left(p_{it} - E^*_{it} p_t \right) + E^*_{it} y_t \tag{10a}$$

$$E_{it} y_{it} = -\hat{\sigma} \left(p_{it} - E_{it} p_t \right) + E_{it} y_t \tag{10b}$$

where

$$\hat{\sigma} = \sigma (1 - \frac{1}{m}).$$

In log-linear form the pricing equation (**??**) takes the form

$$p_{it} - E^*_{it} \left[p_t \right] = \kappa E^*_{it} \left[c_{it} - a_{it} \right] + \kappa \eta E^*_{it} \left[y_t - a_{it} \right] \tag{11}$$

where

$$\kappa = \frac{1}{1 + \eta \hat{\sigma}} \tag{12}$$

The mechanism behind this equation is analogous to the one in Lucas (1972) and Woodford (2002), in that agents confound relative and absolute price changes. When expected total output y_t is large relative to the individual productivity a_{it} agents attempt to increase the relative price of their good. Since they do not observe the prices set by other agents this will have an effect on the aggregate price level. The spread between consumption and productivity $c_{it} - a_{it}$ has a similar effect because it affects the consumption-leisure margin at the individual level. When an agent plans to consume more than his current productivity

between the parameters and the equilibrium allocation is continuous.

[14] All lowercase variables denote the logarithm of the corresponding uppercase variable in terms of deviation from the steady state with no local shocks.

his marginal cost (in terms of consumption) increases and he attempts to increase his relative price. As we will see c_{it} also depends on the agent's estimate of aggregate demand y_t. Therefore, the general feature of equation (11) is that when an agent perceives that total output y_t is larger than his own productivity the agent will attempt to increase his relative price, and this will generate temporary inflation.

Here, we can briefly return to the discussion of the equilibrium with full information. Suppose there are idiosyncratic shocks but the signal s_t has no noise, so agents can observe the aggregate shock a_t. Then we can show that there is an equilibrium where p_t and y_t only depend on current and past values of the shock a_t. In this case equation (11) boils down to

$$p_{it} - p_t = \kappa \left(c_{it} - a_{it} \right) + \kappa\eta \left(y_t - a_{it} \right)$$

that can be aggregated to

$$0 = \kappa \left(c_t - a_t \right) + \kappa\eta \left(y_t - a_t \right).$$

Using the market clearing condition $c_t = y_t$ we have

$$y_t = a_t.$$

Therefore, as discussed above, under full information output at date t depends only on current productivity and employment is constant and independent of productivity. We now return to the case of imperfect information.

Consumption The characterization of optimal consumption in presence of local productivity shocks and imperfect information is complicated by the fact that agents have to solve an asset accumulation problem taking into account the non-negativity constraint on money holdings. This, in principle, would require to derive the dynamics for the joint distribution of money balances and expectations regarding the aggregate state $\left(a^t, s^t\right)$ across agents. At this stage, we introduce a stark simplification and we work under the assumption that, when σ_ϵ^2 is small and R is close to δ^{-1}, a permanent-income equation in log-linear terms gives a good approximation to the behavior of individual consumption.[15] To derive our permanent-income equation we log-linearize the consumers' Euler equation and the intertemporal budget constraint and abstract from the non-negativity constraint on money balances. By looking at a log-linear approximation to the agents decision rule we are able to characterize the aggregate behavior of prices and output without having to determine the distribution of real balances in equilibrium. In Appendix C we test the accuracy of our permanent-income approximation.

[15] Wang (2003) provides some support to this approach, showing that a permanent-income equation gives a good approximation to a buffer-stock savings problem in the case of small idiosyncratic shocks and when R is close to δ^{-1}.

Log-linearizing the consumers' Euler equation and the intertemporal budget constraint we obtain

$$\begin{aligned}
& c_{it} = E_{it}\left[c_{it+1} + \overline{p}_{it+1} - \overline{p}_{it}\right] \\
& \sum_{k=0}^{\infty} R^{-k}\left[p_{it+k} + y_{it+k} - \overline{p}_{it+k} - c_{it+k}\right] + b_{it} = 0
\end{aligned} \tag{13}$$

where b_{it} denotes the consumer real balances at date t.

Using the expressions above we can express the production and price of good i in all future periods in terms of aggregate output, future productivity and the consumption of agent i. Then we can substitute these expressions in the intertemporal budget constraint and use the Euler equation to solve for expected future consumption[16]. Finally, we obtain the following version of a permanent-income equation

$$\begin{aligned}
c_{it} &= (1-\phi)\left(b_{it} + E_{it}y_t + (1-\hat{\sigma})\left(p_{it} - \overline{p}_{it}\right)\right) + \\
&\quad +\phi\left(1-\beta\right)\sum_{k=1}^{\infty}\beta^{k-1}E_{it}\left[\alpha a_{it+k} + (1-\alpha)\,y_{t+k}\right] + \\
&\quad -\phi\sum_{k=0}^{\infty}\beta^{k}E_{it}\left[\overline{p}_{it+k} - \overline{p}_{it+k+1}\right]
\end{aligned} \tag{14}$$

where the coefficients α, β and ϕ are given by

$$\alpha = \left(1 - \frac{1}{\hat{\sigma}}\right) \in (0,1) \tag{15}$$

$$\beta = R^{-1}$$

$$\phi = \frac{\beta\hat{\sigma}\left(1+\eta\right)}{\beta\hat{\sigma}\left(1+\eta\right) + \left(1-\beta\right)\left(1+\eta\hat{\sigma}\right)} \tag{16}$$

The permanent-income equation (14) can be easily interpreted. Consumer i sets his consumption based on his expectation of current output and a linear combination of his own future productivity and future aggregate output, with coefficients α and $(1-\alpha)$ reflecting the demand elasticity of good i. When the demand for good i is very inelastic the future income of household i will mostly depend on home productivity a_{it}, on the other hand when the demand is very elastic household income will be largely determined by the level of aggregate output. In this model the demand elasticity affects the degree of strategic complementarity between the spending decisions of different households. A very elastic demand determines a high degree of strategic complementarity, and, as we will see a stronger amplification of expectational shocks. The last line of (14) shows that the spending decisions of household i also depend on their expectations regarding future real interest rates.

[16] See the detailed derivations in Appendix A.

Equilibrium Aggregating the permanent-income equation across agents and using the market clearing condition $\int b_{it}di = 0$ we obtain the following equation for aggregate output:

$$\begin{aligned} y_t &= (1-\phi)\, y_{t|t} + \phi \left[\alpha a_{t|t} + (1-\alpha)(1-\beta) \sum_{k=1}^{\infty} \beta^{k-1} y_{t+k|t} \right] + \\ &\quad -\phi \sum_{k=1}^{\infty} \beta^k \left(p_{t+k|t} - p_{t+k+1|t} \right) - \phi \left(p_t - p_{t+1|t} \right) \end{aligned} \tag{17}$$

Substituting in the pricing equation (11)and aggregating across agents we obtain the following expression for the price level

$$\begin{aligned} p_t &= p_{t|t^*} + \kappa \left(\alpha\beta a_{t|t^*} + \alpha (1-\beta)\, a_t + (1-\alpha)(1-\beta) \sum_{k=0}^{\infty} \beta^k y_{t+k|t^*} - a_t \right) + \\ &\quad +\kappa\eta \left(y_{t|t^*} - a_t \right) - \kappa\beta \sum_{k=0}^{\infty} \beta^k \left(p_{t+k|t^*} - p_{t+k+1|t^*} \right) \end{aligned} \tag{18}$$

Once we have a method for computing the average expectations in (17) and (18) the two equations (17), (18) fully characterize the equilibrium dynamics of this economy.

Agents use past information on prices, quantities, private productivity and the public signal in order to compute the expectations in (17) and (18). At the beginning of each period t each agent i observes the vector

$$S_{it} = \begin{pmatrix} s_t & a_{it} - \rho a_{it-1} & q_{it-1} \end{pmatrix}'$$

updates his beliefs and sets the price p_{it}. The variable q_{it} is the observed intercept of the demand for good i at time t. This intercept can be recovered exactly by the monopolist because the slope of the demand curve is known. The variable q_{it} is a linear combination of the price p_{it} and of the quantity sold y_{it} and it reflects information regarding (1) the consumption levels of the consumers shopping for good i and (2) the prices of the goods competing with good i. Since prices are set by agent i the quantity q_{it} is a sufficient statistic for p_{it} and y_{it} and we do not need to keep track of both separately in the history of individual i. In Appendix B we give the explicit expression for q_{it} and write it as

$$q_{it} = q_t + \eta_{it}.$$

The variance of the error term η_{it} is due to two factors. First, the demand is random due to the dispersion of the characteristics of the consumers in $\tilde{H}_{it}$: their expectations, their productivity shocks a_{jt}, their real balances b_{jt}. Second, the demand is random due to the dispersion in the prices of the goods competing with good i[17].

[17] Formally, these are the prices P_{kt} with k in the set

$$\left\{ k : k \neq i, k \in H_{jt} \text{ for some } j \in \tilde{H}_{it} \right\}.$$

During the trading phase shoppers use the additional information contained in the m prices observed. For the purpose of forecasting aggregate variables this information can be summarized in the price index:

$$\overline{p}_{it} = p_t + \zeta_{it}$$

The variance of ζ_{it} is also due to the dispersion of expectations, to the dispersion in a_{jt}, and to the dispersion in b_{jt} across price-setting agents.

In the computation of the equilibrium we will derive the steady state volatility in consumption and prices due to the cross sectional dispersion of expectations and of the idiosyncratic shock and solve for the corresponding volatilities of η_{it} and ζ_{it}. However, the use of a log-linear approximation does not let us derive the steady state distribution of real balances and its correlation with expectations. Therefore, in the computation we omit the volatility in consumption and prices due to the volatility of real balances b_{it}.

4 Equilibrium Dynamics

In this section we use numerical simulations of the basic model in order to study its qualitative and quantitative implications. On the qualitative side, we look at the response of the economy to a signal shock, e_t, and a productivity shock, u_t. We characterize the dynamics of prices, employment and consumers' expectations following the two shocks. Prices and employment increase following a positive noise shock and decrease following a positive productivity shock. Consumers' expectations tend to overreact after a noise shock and to under-react after a productivity shock.

Next, we turn to the quantitative implications and we ask what fraction of output volatility can be explained by the noise shock. We study this question along two dimensions. First, we look at the model implication in terms of the ratio between short run and long run output volatility. Second, we calibrate the model in order to replicate the variance decompositions at various horizons obtained in existing VAR studies.

4.1 Computation and Parameters' Choice

To compute the equilibrium we adapt a method of undetermined coefficients to the case of imperfect information. The computation involves three steps: the definition of an appropriate state space, the solution of a filtering problem for each individual agent, and the solution of the expectational equations (17), (18). We will approximate the equilibrium state space considering a truncated state vector that includes all aggregate variables up to period $t - T$. Let

$$z_t = \begin{pmatrix} p_t & y_t & s_t & a_t \end{pmatrix}'$$

then the vector of state variables is given by:

$$Z_t = \begin{pmatrix} z_t \\ z_{t-1} \\ ... \\ z_{t-T} \end{pmatrix}$$

The law of motion for the state vector can be written as

$$Z_t = AZ_{t-1} + BW_t \tag{19}$$

where

$$W_t = \begin{pmatrix} u_t & e_t \end{pmatrix}'$$

is the vector of aggregate shocks. For given coefficients A and B we can solve the filtering problem for each agent and derive expressions for the average first order expectations $Z_{t|t}$ and $Z_{t|t^*}$ as linear functions of the state vector Z_t. Substituting these expressions in the equilibrium equations (17), (18) we can derive new coefficients for the matrices A and B. We update these coefficients and iterate until convergence is achieved. Our computation method is related to the methods employed by Woodford (2003) and Bacchetta and Van Wincoop (2003). The details of its implementation are contained in Appendix B.

The model is very stylized in at least three respects: there is no capital, labor is immobile across sectors and there are no financial assets aside from the risk free nominal bond. However, we will try to choose realistic values for the model parameters. The parameter β is set at 0.99 so one can interpret the time period as a quarter. The parameter η is set at 0.33 corresponding to a Frisch labor elasticity of 3. The parameter $\hat{\sigma}$ is set equal to 7 which corresponds to a mark-up of 16.6%. These values for η and $\hat{\sigma}$ are in the range of values commonly used in business cycle models with price rigidities. In our model these two parameters play a crucial role because through (12) and (15) they determine the degree of strategic complementarity in pricing and in spending, summarized by the coefficients κ and α. In particular, the parameters chosen imply that $\alpha = 0.86$ and $\kappa = 0.30$ which correspond to moderate levels of strategic complementarity in pricing and spending. The effect of changes in these two parameters on equilibrium dynamics will be analyzed in detail in the next section. For the monetary policy rule we will concentrate for the moment on the case of a passive monetary policy, i.e. $\theta_r = \theta_p = 0$[18].

It remains to choose values for the variances of the shocks. Given the linearity of the model we can set $\sigma_u = 0.1$ as a normalization. For the remaining parameters $\sigma_\epsilon, \rho, \sigma_e$ and m we will experiment with different values. In particular we will try to replicate observed volatility in sectoral price changes and we will attempt to replicate the variance decompositions obtained in Shapiro and Watson (1988) and Gali (1992).

[18] This passive monetary policy opens the door to indeterminacy in the price level. However, we assume that agents coordinate their expecations on a target price level $\overline{P}$ and focus on the corresponding equilibrium leaving aside the problem of how the monetary authority can implement this equilibrium.

4.2 Dynamic Responses

4.2.1 Prices and Employment

Figure 2 shows the responses of output, employment and the price level to a productivity shock u_t (solid line) and to a public signal shock e_t (dashed line).[19] The last panel of Figure 2 illustrates the dynamics of average expectation regarding aggregate productivity, $a_{t|t}$.

First, let us discuss the response of the economy to a noise shock e_t. When the noise shock hits, the average agent believes that aggregate productivity has increased, while his own productivity is unchanged. This has two effects: first, it increases household expenditure given that households expect higher sales in the current and future periods. Second, the average agent believes that his own productivity is smaller than average productivity. He thinks the production of all other goods will increase and his own good will become relatively scarcer. His optimal response is to increase labor supply and increase the relative price of his good. This has the effect of increasing the average level of employment and the average price level.

On the other hand, after a productivity shock u_t output increases but the adjustment to the new level of equilibrium output is gradual. Along the transition path output grows less than actual productivity. The average agent realizes that aggregate productivity has increased, but believes that his individual productivity has increased more. His optimal response is to reduce labor supply and reduce the price of his good. This generates a reduction in aggregate employment and in the price level.

From a qualitative standpoint the conditional covariances of inflation and output are consistent with the evidence from identified VAR exercises if we identify e_t with the demand shock and u_t with a supply shock (see e.g. Table III in Gali (1992)). Following a public signal shock output and inflation have a positive correlation, while following a productivity shock they have a negative correlation. A similar result holds for the correlation of output and employment. These conditional correlations of employment and output after a permanent technology shocks are consistent with the evidence presented in Gali (1999) and Francis and Ramey (2003).

4.2.2 Consumers' Expectations, Overreaction and Underreaction

Let us turn to the model predictions regarding the relative reaction of output and output expectations following noise shocks and productivity shocks. Figure 4 shows the response of output y_t (solid line) and its first order expectation $y_{t|t}$ (dashed line) after a signal shock.

[19] For this simulation we use the parameters:

$$(\sigma_\epsilon, \sigma_e, \rho, m) = (....)$$

In all the figures the impulse-responses represent the effect of a 1 standard-error change to each shock.

After a positive noise shock output expectations tend to increase more than actual output. This is due to the fact that output expectations depend only on expectations regarding the productivity a_t, while output itself depends also on the actual productivity a_t which is unaffected by a signal shock (see equation (17)). Moreover, expected output tends to give more weight to higher order expectations of a_t which tend to move more with the signal e_t.

On the other hand after a positive productivity shock output tends to increase less than expected productivity, as we can see from Figure 5. This is due to the fact that an increase in productivity has a less than one-to-one effect on expected productivity, and a smaller and smaller effect on higher order expectations. Since expected output gives more weight to higher order expectations it will move less than actual output.

Therefore the model predicts that after a noise shock consumers' expectations will overreact while after a productivity shock consumers' expectations will under-react. This predictions are amenable to empirical testing using available measures of consumers' expectations, something we plan to pursue in future work.

4.2.3 Noise Shocks and Persistence

By construction, the signal shock can only affect output in the short run, while all long-run output volatility is due to the productivity shock. An important quantitative question is: What fraction of short-run output volatility can be explained by shocks to the public signal noise? The structure of the model imposes a bound on the fraction of output volatility that can be explained by the signal noise. If the public signal is very noisy agents would disregard it altogether, while if the signal is very precise the economy will converge very fast to the full information equilibrium. In both cases the noise will explain a small fraction of output volatility. Therefore, the question is whether intermediate levels of signal precision can generate realistic values for the fraction of output volatility explained by the noise shock. To address this question we enrich the model by including public statistics in the information set of agents. Namely, we assume that at the end of each period agents observe noisy statistics of output and prices

$$
\begin{aligned}
\tilde{p}_t &= p_t + w_{1t} \\
\tilde{y}_t &= y_t + w_{2t}
\end{aligned}
$$

Figure 3 illustrates the effects of changing the precision of the signal (i.e. changing σ_e) on the dynamic responses to the two shocks[20]. For each value of σ_e the figure plots the output response to a productivity shock (solid line) and to a signal shock (dashed line). In

[20] For this figure we use the set of parameters

$$
(\sigma_\epsilon, \sigma_{w_1}, \sigma_{w_2}) = (7, 2, 2)
$$

We discuss this parameters in the next paragraphs.

the first panel of Figure 3 the public signal is very precise, after a productivity shock the economy converges very fast to the long-run equilibrium and a signal shock has a very small and temporary effect on output. As we move to the second and third panel we see that the effect of a signal shock increases and becomes more persistent. However, in the fourth panel we see that as the noise is very large agents stop relying on the public signal and the impact effect becomes smaller. On the other hand as the quality of the signal deteriorates agents take a longer time to learn the long-run equilibrium, so the demand shock becomes very persistent. At this point agents information is so imprecise that output takes a long time to adjust after a real productivity shock, and after ten quarters output is still 30% below its long-run level.

Table 1 summarizes the result of experimenting with different values for the variance of the various shocks. In the table we report the fraction of forecast variance in output and prices accounted for by the all the "demand" shocks (i.e. e_t, w_{1t} and w_{2t}). For comparison we report the corresponding values reported in Gali (1992) (Table IV). In the first column we report the variance decomposition for the parameters

$$\begin{aligned} \sigma_\epsilon &= 1.8; \sigma_{w_1} = 0.027; \sigma_{w_2} = 0.05 \\ \sigma_e &= 0.2; m = 3 \end{aligned}$$

The parameters $\sigma_{w_1}$and σ_{w_2} were chosen so as to generate realistic values for the noisiness of public statistics regarding output and prices[21], while the parameter σ_ϵ was chosen to generate realistic values for the cross sectional volatility of price changes[22]. The fraction of output volatility explained by the "demand" shocks in the first quarter is equal to 0.26 which is in the range of values found in Shapiro and Watson (1988) and Gali (1992). On the other hand the demand shocks are very short lived. From a quantitative standpoint the main weakness of the model seems to be its inability to match the persistence of the demand shocks observed in the data. In particular with the model parameters chosen above the fraction of output volatility due to demand shocks goes to 0.01 in the second quarter, while, in the variance decomposition in Gali (1992) the fraction of output volatility due to the "IS shock" is still 0.19 after five quarters. In short, in our model agents learn too quickly to generate a realistic persistence for the demand shocks. This is due to several reasons.

[21] In particular we look at the ratio of the noise as a fraction of the total forecast error in the underlying variable $\sigma^2_{w_1}/Var_{t-1}[\tilde{p}_t - \tilde{p}_{t-1}]$ and $\sigma^2_{w_2}/Var_{t-1}[\tilde{y}_t - \tilde{y}_{t-1}]$.

To get realistic orders of magnitude for $\sigma^2_{w_1}$ and $\sigma^2_{w_2}$ one can think that w_{1t} and w_{2t} correspond to the noise associated to the first release of the macroeconomic data by the BEA, and that that noise is eliminated at the time of the last revision. With this interpretation we can rely on the analysis in Runkle (1998) and choose ratios around 40%-45%.

[22] In particular we look at the implications of σ_ϵ for the ratio of $E_{t-1}\int((p_{it} - p_{it-1}) - (p_t - p_{t-1}))^2 di$ to inflation volatility. The data by Klenow and Kryvstov (2003) gives us some idea of the order of magnitude for this statistic. Suppose we take Table 1 in Klenow and Kryvstov (2003), and use the data for "all prices" in the three metropolitan areas and use the absolute mean deviation as a lower approximation to the standard error. Considering that 0.28 is the fraction of goods with price changes, a measure of volatility of price changes for all the goods $(0.28)^{1/2}10.5 = 5.6$ which is 15 times the volatility of inflation.

First of all, the structure of the equilibrium is very simple, the only fundamental shock that agents are trying to learn is the permanent productivity shock. Moreover, we have assumed that all the noise shocks are i.i.d. and all the price and quantity observations made by agents are independent. This allows every agent to collect a large sample of price and quantity observations in a short amount of time. Allowing for a more realistic autocorrelation structure for the shocks w_{1t}, w_{2t} and e_t would both slow down learning and introduce an additional source of persistence. Finally, the model has no propagation mechanism aside from information diffusion, in particular there are no temporary technology shocks and no capital.

The second column of Table 1 illustrates the fact that if either one of the signals (public or private) is sufficiently informative, then agents rely primarily on that signal, learning is very fast and the effect of the other signals is small. In particular, in the second column the public statistics are extremely noisy and agents disregard them. We get similar results when the private signal is very noisy but at least one of the public statistics is sufficiently precise. Therefore, we need larger values for all the noise variances in order to obtain a speed of learning that matches the empirical data. This is illustrated in the third column where we set the parameters

$$\sigma_\epsilon = 7; \sigma_{w_1} = 2; \sigma_{w_2} = 2; \sigma_e = 1.5; m = 3$$

With this set of parameters we obtain a more realistic persistence for the noise shocks and we are able to get a variance decomposition closer to that obtained in VAR exercises.

5 Strategic Complementarity and Speed of Learning

The model displays two types of strategic complementarity. First, there is strategic complementarity in price setting. This type of strategic complementarity has been studied extensively in the literature on sticky prices[23]. Woodford (2002) shows that in presence of imperfect information strategic complementarity leads to slower adjustment of prices and a larger effect on quantities after a monetary shock. A similar mechanism is at work here. If we rewrite the right hand side of equation (18) explicitly in terms of $p_{t|t^*}$ we see that the coefficient for $p_{t|t^*}$ is equal to

$$1 - \kappa \left(\beta - \frac{1 - \beta}{\hat{\sigma}} \right) - \kappa \eta \phi$$

When κ is smaller than one this expression is positive, and prices are strategic complements. In particular if agents expect other agents' prices to adjust slowly after a noise shock, they will also tend to adjust their prices less when they receive a positive signal about productivity. This will dampen the adjustment of the aggregate price level, and increase

[23] See the discussion in Woodford (2003).

the response of quantities to a noise shock. Figure 6 illustrates this mechanism at work[24]. The right hand side panels illustrate the responses of prices and output to a productivity shock (solid line) and to a signal shock (dashed line) using the baseline parameters for η. This corresponds to a level of κ equal to 0.30. The right hand side panels, instead, correspond to a much lower value of η that implies κ equal to 0.74. A larger value for κ implies a lower degree of strategy complementarity across price setters, a greater response of prices and a smaller response of output to a noise shock.

A different type of strategic complementarity arises in the spending decisions of individual agents. From equation (17) we see that an agent optimal consumption depends on his expectations regarding current and future output. In particular, a lower level of α implies a higher degree of strategic complementarity among quantity choices. When α is smaller agents attach a greater weight to higher order expectations regarding productivity. Since higher order expectations are more reactive to the public signal and less to the private signal a lower level of α implies that the quantities react more to the public signal s_t and less to the private signal a_{it}.[25] When we aggregate across agents the average private signal corresponds to actual productivity. Therefore a smaller α implies a smaller effect of the noise shock e_t and a larger effect of the productivity shock u_t. This effect is illustrated in Figure 7 where we compare the response of output (in blue) and of expected productivity $a_{t|t}$ (in green) to the noise shock for various levels of α.[26] It is also useful for the following discussion to plot higher order expectations of productivity, so we report $a_{t|t}^{(10)}$. Notice that we do not report the response of a_t, because this response is always zero after a noise shock.

The mechanism described by Morris and Shin can be described as follows. Abstracting from the effects of the price level, the output y_t is determined as a weighted average of $a_t, a_{t+1} a_{t+2}, ...$ and of all their higher order expectations. The smaller the α the more the weights are shifted towards higher order expectations and the bigger the output response. However, an interesting thing to notice about Figure 7 is that the degree of strategic complementarity not only affects the position of y_t between a_t and $a_{t|t}$, but it also affects the dynamics of $a_{t|t}$. In particular for smaller values of α expected productivity takes longer to adjust to actual productivity.

This points to a second mechanism by which α affects the effect of the noise shock in this economy. This channel is associated to the fact that quantities are an endogenous signal in this economy and therefore quantity decisions affect the speed of learning in the economy. As agents decisions rely more on the public signal and less on the private signal the quantity sold by agent i, y_{it} carries less information regarding the aggregate shock.

[24] The parameters for this figure are $(\sigma_\epsilon, \sigma_{w_1}, \sigma_{w_2}, \sigma_e) = (7, 2, 2, .25)$.

[25] On the role of strategic complementarity in determining agents responses to private and public signals see Morris and Shin (2002) and Allen, Morris and Shin (2003).

[26] We want to study the effect of changing the degree of strategic complementarity on the quantity side, keeping the strategic complementarity in price setting constant. Therefore, we have adjusted the parameter η so that the level of κ is maintained constant at 0.30 in all panels.

As the informativeness of the signal y_{it} deteriorates the economy learns more slowly the equilibrium level of output. Therefore, it takes more time for $a_{t|t}$ to adjust to actual productivity a_t. As we can see in Figure 7 a smaller level of α corresponds to a slower adjustment of $a_{t|t}$ to zero. As a consequence a small level of α is associated to more persistent deviations of output from productivity, both because y_t depends more on $a_{t|t}$ and less on a_t and because the difference between $a_{t|t}$ and a_t is larger.

6 Shocks to Future Productivity

In recent work Beaudry and Portier (2004) have presented empirical evidence that expectations regarding future productivity may be an important source of business cycle fluctuations. In their empirical analysis they identify a shock that affects TFP in the long-run but has no effect on TFP in the short run and show that this shock can explain a large fraction of business cycle variation in output and hours. In our model so far there is no shock with this features: the productivity shock u_t has an immediate impact on productivity which is identical to its long-run impact, while the signal shock e_t has no effect at all on productivity. In this section we modify the process for productivity to allow for news regarding future productivity to enter the model. In particular let x_t represent long-run productivity and follow the random walk

$$x_t = x_{t-1} + u_t$$

aggregate productivity follows the process

$$a_t = (1 - \lambda) x_{t-1} + \lambda a_{t-1} + w_t$$

which includes a temporary shock w_t. Notice that with this specification the shock u_t has no effect on productivity on impact. The public signal now takes the form

$$s_t = x_t + e_t$$

so that agents only receive a signal about long-run productivity but do not observe how fast productivity will adjust. The uncertainty about the speed of adjustment is captured by the temporary shock w_t. If w_t has the same sign as u_t the adjustment to the new productivity level is faster, while if w_t has the opposite sign the adjustment to the new level of productivity will be slower.

Figure 6 shows the responses of productivity, output, prices and employment to the three shocks u_t (solid), w_t (dashed), and e_t (in red)[27]. As we can see the shock u_t has

[27] The parameters for this example are

σ_u	0.100
σ_w	0.015
σ_e	0.150
σ_ϵ	2.000

the features of a demand shock in the short run, namely it increases output, inflation and employment in the same direction. An interesting feature is that this shock generates a cyclical response of employment. In the first periods after the shock spending reacts more than proportionally to the increase in productivity and this pushes employment up, in the following periods, though, realized productivity catches up faster than perceived productivity and therefore employment is temporarily below its long-run equilibrium.

7 Conclusions

[...]

Appendix A

Derivation of the permanent-income equation (14).

First, we derive the intertemporal budget constraint. Using the government budget balance condition $(R-1)(M-T)=T$ we can rewrite the law of motion for real money holdings as

$$\frac{M_{it+1}-M}{\overline{P}_{it+1}} = R\frac{\overline{P}_{it}}{\overline{P}_{it+1}}\left(\frac{M_{it}-M}{\overline{P}_{it}}+\frac{P_{it}}{\overline{P}_{it}}Y_{it}-C_{it}\right)$$

this gives us

$$\sum_{s=t}^{\infty} Q_{t,s}^{i}\left(\frac{P_{is}}{\overline{P}_{is}}Y_{is}-C_{is}\right)=\frac{M_{it}-M}{\overline{P}_{it}}$$

where

$$Q_{t,s}^{i}=\Pi_{l=t}^{s}\left(R\frac{\overline{P}_{it}}{\overline{P}_{it+1}}\right)^{-1}$$

We log-linearize all variables except the real balances around the steady state with no idiosyncratic shocks where

$$\begin{aligned} P_{it} &= M/(\mu A_t) \\ Y_{it} &= C_{it}=A_t. \end{aligned}$$

For the real balances $\frac{M_{it}-M}{\overline{P}_{it}}$ we use instead $b_{it}=\ln\left(1+\frac{M_{it}-M}{A_t\overline{P}_{it}}\right)$. We obtain expression (13) in the text.

Now we can derive the permanent-income equation. For all $k\geq 1$ we can use the law of iterated expectations for agent i and the pricing equation (11) to obtain:

$$E_{it}\left(p_{it+k}-\overline{p}_{it+k}\right)=\frac{1}{1+\eta\hat{\sigma}}E_{it}\left[c_{it+k}-a_{it+k}\right]+\frac{\eta}{1+\eta\hat{\sigma}}E_{it}\left[y_{t+k}-a_{it+k}\right]$$

Then we get the following expression

$$\begin{aligned}
&E_{it}\left[(1-\hat{\sigma})(p_{it+k}-p_{t+k})+y_{t+k}-c_{it+k}\right]= \\
&E_{it}[\frac{(1+\eta)(\hat{\sigma}-1)}{1+\eta\hat{\sigma}}a_{it+k}+\frac{1+\eta}{1+\eta\hat{\sigma}}y_{t+k}-\frac{(1+\eta)\hat{\sigma}}{1+\eta\hat{\sigma}}c_{it+k}]= \\
&\frac{(1+\eta)\hat{\sigma}}{1+\eta\hat{\sigma}}E_{it}[\alpha a_{it+k}+(1-\alpha)y_{t+k}-c_{it+k}] \qquad (20)
\end{aligned}$$

where α is

$$\alpha=\left(1-\frac{1}{\hat{\sigma}}\right).$$

Take expectations E_{it} on the intertemporal budget constraint

$$b_{it}+E_{it}y_t+(1-\hat{\sigma})(p_{it}-\overline{p}_{it})-c_{it}+\sum_{k=1}^{\infty}\beta^k E_{it}\left[(1-\hat{\sigma})(p_{it+k}-p_{t+k})+y_{t+k}-c_{it+k}\right]=0.$$

The expression $\sum_{k=1}^{\infty}\beta^k E_{it}\left[c_{it+k}\right]$ can be solved in terms of current consumption and expected real rates as follows:

$$\begin{aligned}
E_{it}\left[c_{it+1}\right] &= c_{it}+E_{it}\left[\overline{p}_{it+1}-\overline{p}_{it}\right] \\
E_{it}\left[c_{it+k}\right] &= c_{it}+\sum_{s=0}^{k-1}E_{it}\left[\overline{p}_{it+s+1}-\overline{p}_{it+s}\right] \\
\sum_{k=1}^{\infty}\beta^k E_{it}\left[c_{it+k}\right] &= \frac{\beta}{1-\beta}c_{it}+\frac{\beta}{1-\beta}\sum_{k=0}^{\infty}\beta^k E_{it}\left[\overline{p}_{it+k+1}-\overline{p}_{it+k}\right] \qquad (21)
\end{aligned}$$

Therefore, using (20) and (21) one obtains:

$$\begin{aligned}
&b_{it}+E_{it}y_t+(1-\hat{\sigma})(p_{it}-\overline{p}_{it})-c_{it}+\frac{\hat{\sigma}(1+\eta)}{1+\eta\hat{\sigma}}\sum_{k=1}^{\infty}\beta^k E_{it}\left[\alpha a_{it+k}+(1-\alpha)y_{t+k}\right]+ \\
-&\frac{(1+\eta)\hat{\sigma}}{1+\eta\hat{\sigma}}\frac{\beta}{1-\beta}c_{it}-\frac{(1+\eta)\hat{\sigma}}{1+\eta\hat{\sigma}}\frac{\beta}{1-\beta}\sum_{k=0}^{\infty}\beta^k E_{it}\left[\overline{p}_{it+k+1}-\overline{p}_{it+k}\right]=0
\end{aligned}$$

Rearranging, one obtains the permanent-income equation

$$\begin{aligned}
c_{it} &= (1-\phi)\left(b_{it}+E_{it}y_t+(1-\hat{\sigma})(p_{it}-\overline{p}_{it})\right)+\phi(1-\beta)\sum_{k=1}^{\infty}\beta^{k-1}E_{it}\left[\alpha a_{it+k}+(1-\alpha)y_{t+k}\right]+ \\
&-\phi\sum_{k=0}^{\infty}\beta^k E_{it}\left[-\overline{p}_{it+k+1}+\overline{p}_{it+k}\right]
\end{aligned}$$

where

$$\phi = \frac{\beta\hat{\sigma}(1+\eta)}{\beta\hat{\sigma}(1+\eta)+(1-\beta)(1+\eta\hat{\sigma})}.$$

Derivations of the pricing equation.

In order to derive an expression for prices we need to derive an expression for $E_{it}^*[c_{it}]$ in (11).

For all $k \geq 0$ we can use the law of iterated expectations for agent i and the pricing equation (11) to obtain:

$$E_{it}^*\left(p_{it+k}-\overline{p}_{it+k}\right) = \frac{1}{1+\eta\hat{\sigma}}E_{it}^*\left[c_{it+k}-a_{it+k}\right]+\frac{\eta}{1+\eta\hat{\sigma}}E_{it}^*\left[y_{t+k}-a_{it+k}\right]$$

Using the analogous to (20) and (21) under the expectation operator E_{it}^* and using the intertemporal budget constraint one obtains

$$\begin{aligned}
& b_{it}+\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[(1-\hat{\sigma})(p_{it+k}-p_{t+k})+y_{t+k}-c_{it+k}\right]=0 \\
& b_{it}+\frac{\hat{\sigma}(1+\eta)}{1+\eta\hat{\sigma}}\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[\alpha a_{it+k}+(1-\alpha)y_{t+k}\right]+ \\
- \quad & \frac{\hat{\sigma}(1+\eta)}{1+\eta\hat{\sigma}}\frac{1}{1-\beta}E_{it}^*\left[c_{it}\right]-\frac{\hat{\sigma}(1+\eta)}{1+\eta\hat{\sigma}}\frac{\beta}{1-\beta}\sum_{k=0}^{\infty}\beta^k E_{it}\left[-\overline{p}_{it+k+1}+\overline{p}_{it+k}\right]=0
\end{aligned}$$

which gives an expression for $E_{it}^*[c_{it}]$

$$\begin{aligned}
E_{it}^*\left[c_{it}\right] &= \frac{1+\eta\hat{\sigma}}{\hat{\sigma}(1+\eta)}(1-\beta)b_{it}+(1-\beta)\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[\alpha a_{it+k}+(1-\alpha)y_{t+k}\right]+ \\
&\quad -\beta\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[-\overline{p}_{it+k+1}+\overline{p}_{it+k}\right].
\end{aligned} \tag{22}$$

Substituting (22) in (11) gives

$$\begin{aligned}
p_{it} &= E_{it}^* p_t+\frac{1-\beta}{\hat{\sigma}(1+\eta)}b_{it}+\frac{1-\beta}{1+\eta\hat{\sigma}}\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[\alpha a_{it+k}+(1-\alpha)y_{t+k}\right]+ \\
&\quad -\frac{\beta}{1+\eta\hat{\sigma}}\sum_{k=0}^{\infty}\beta^k E_{it}^*\left[-p_{it+k+1}+p_{it+k}\right]+ \\
&\quad +\frac{\eta}{1+\eta\hat{\sigma}}E_{it}^*\left[y_t\right]-\frac{1+\eta}{1+\eta\hat{\sigma}}a_{it}
\end{aligned}$$

and aggregating across consumers gives (18).

Appendix B

The matrices A and B in (19) can be written as:

$$A = \begin{bmatrix} & \tilde{A} & \\ I & & 0 \end{bmatrix}; B = \begin{bmatrix} \tilde{B} \\ 0 \end{bmatrix}.$$

The law of motion (19) must satisfy the restrictions associated to the equations:

$$\begin{aligned} s_t &= a_{t-1} + u_t + e_t \\ a_t &= a_{t-1} + u_t \end{aligned}$$

This means that the last two rows of $\tilde{A}$ are given by

$$\begin{matrix} 0 & 0 & 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 1 & 0 & \ldots & 0 \end{matrix}$$

and the last two rows of $\tilde{B}$ are

$$\begin{matrix} 1 & 1 \\ 1 & 0 \end{matrix}$$

The first two rows of $\tilde{A}$ and $\tilde{B}$ contain the law of motion for p_t and y_t and need to be determined.

Kalman filter

For given A and B we can derive the expressions for the Kalman filter. Agent i observes first the vector of signals S_{it}, where

$$S_{it} = \begin{pmatrix} s_t \\ a_{it} \\ q_{it-1} \end{pmatrix}$$

and then observes $\overline{p}_{it}$.

The demand curve for the monopolist is

$$y_{it} = -\sigma \left(p_{it} - \frac{1}{m^2} \sum_{j \in \tilde{H}_{it}} \sum_{k \in H_{jt}} p_{kt} \right) + \frac{1}{m} \sum_{j \in \tilde{H}_{it}} \left(\hat{c}_{jt} - \phi \frac{1}{m} \sum_{k \in H_{jt}} p_{kt} \right)$$

where the quantities $\hat{c}_{jt} = c_{jt} + \phi \frac{1}{m} \sum_{k \in H_{jt}} p_{kt}$ do not depend directly on the prices p_{kt}. The intercept of the demand curve of monopolist i, q_{it}, is defined as

$$
\begin{aligned}
q_{it} &= y_{it} + \left(\sigma - \frac{\sigma-\phi}{m}\right) p_{it} = \\
&= (\sigma-\phi)\frac{1}{m^2}\sum_{j\in\tilde{H}_{it}}\sum_{k\in H_{jt},k\neq i} p_{kt} + \frac{1}{m}\sum_{j\in\tilde{H}_{it}} \hat{c}_{jt} \\
&= y_t + \left(\sigma - \frac{\sigma-\phi}{m}\right) p_t + \eta_{it}
\end{aligned}
$$

where

$$
\eta_{it} = (\sigma-\phi)\left(1-\frac{1}{m}\right)\frac{1}{m(m-1)}\sum_{j\in\tilde{H}_{it}}\sum_{k\in H_{jt},k\neq i}(p_{kt}-p_t) + \frac{1}{m}\sum_{j\in\tilde{H}_{it}}(\hat{c}_{jt}-\hat{c}_t)
$$

In the pricing stage and in the trading stage he forms the expectations

$$
\begin{aligned}
E^*_{it}Z_t &= E_{it-1}Z_t + C\left(S_{it} - E_{it-1}S_{it}\right) \\
E_{it}Z_t &= E^*_{it}Z_t + D\left(\overline{p}_{it} - E^*_{it}\overline{p}_{it}\right)
\end{aligned}
$$

To derive the Kalman gains C and D we can use the orthogonality conditions

$$
\begin{aligned}
E_{it-1}\left[\left(Z_t - E_{it-1}Z_t - C\left(S_{it}-E_{it-1}S_{it}\right)\right)\left(S_{it}-E_{it-1}S_{it}\right)'\right] &= 0 \\
E^*_{it}\left[\left(Z_t - E^*_{it}Z_t - D\left(p_t - E^*_{it}p_t\right)\right)\left(\overline{p}_{it}-E^*_{it}\overline{p}_{it}\right)'\right] &= 0
\end{aligned}
$$

the law of motion (19) and the relations

$$
\begin{aligned}
S_{it} &= GZ_t + FV_{it} \\
\overline{p}_{it} &= QZ_t + \zeta_{it}
\end{aligned}
$$

where V_t is the vector of idiosyncratic shocks

$$
V_{it} = \begin{bmatrix} \epsilon_{it} \\ \eta_{it} \end{bmatrix}
$$

and G, F and Q are known matrices given by:

$$
\begin{aligned}
G &= \left[\begin{array}{cccccccccccccc} 0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&\hat{\sigma}&1&0&0&0&0&0 \end{array} ; 0_{(5,7*(T-2))}\right], \\
F &= \begin{bmatrix} 0&0 \\ 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & ... & 0 \end{bmatrix}.
\end{aligned}
$$

Let

$$\begin{aligned}
\Omega &= Var_{it-1}\left[Z_t\right] \\
V &= Var_{it}^{*}\left[Z_t\right] \\
\Sigma_1 &= \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_\eta^2 \end{bmatrix}
\end{aligned}$$

then the orthogonality conditions give us the Kalman gains

$$\begin{aligned}
C' &= \left(G\Omega G' + F\Sigma_1 F'\right)^{-1} G\Omega \\
D' &= \left(QVQ' + \sigma_\zeta^2\right)^{-1} QV
\end{aligned}$$

The expressions for the residual variance are

$$\begin{aligned}
V &= \Omega - \Omega G'\left(G\Omega G' + F\Sigma_1 F'\right)^{-1} G\Omega \\
&= \Omega - C(G\Omega G' + F\Sigma_1 F')C' \\
Var_t\left[Z_t\right] &= V - VQ'\left(QVQ' + \sigma_\zeta^2\right)^{-1} QV \\
&= V - D\left(QVQ' + \sigma_\zeta^2\right) D'
\end{aligned}$$

Using the law of motion of Z_t and imposing a steady state condition we obtain

$$Var_t\left[Z_{t+1}\right] = AVA' - AVQ'\left(QVQ'\right)^{-1} QVA' + B\Sigma B' = \Omega.$$

Prices and output

Now we want to express the first order expectations in terms of the current state as

$$\begin{aligned}
Z_{t|t} &= \Xi Z_t \\
Z_{t|t^*} &= \Xi^* Z_t.
\end{aligned}$$

Using the updating equations and aggregating across consumers

$$\begin{aligned}
E_{it}^{*} Z_t &= E_{it-1} Z_t + C\left(S_{it} - E_{it-1} S_{it}\right) \\
E_{it} Z_t &= E_{it}^{*} Z_t + D\left(\overline{p}_{it} - E_{it}^{*} p_t\right)
\end{aligned}$$

we obtain

$$\begin{aligned}
Z_{t|t} &= \left(I - DQ\right)\left(I - CG\right) A Z_{t-1|t-1} + \left(\left(I - DQ\right) CG + DQ\right) Z_t \\
&= \left(I - DQ\right)\left(I - CG\right) A \Xi Z_{t-1} + \left(\left(I - DQ\right) CG + DQ\right) Z_t
\end{aligned}$$

Now since our state variable is truncated we need to use the approximation

$$A\Xi Z_{t-1} \simeq A\Xi U Z_t \tag{23}$$

where $U=\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$. This approximation is accurate if the last rows of A are close to zero. Then we can impose the condition

$$\Xi = (I-DQ)(I-CG)A\Xi U + ((I-DQ)CG + DQ)$$

and obtain values for Ξ.

Furthermore, we can derive

$$\begin{aligned} Z_{t|t^*} &= AZ_{t-1|t-1} + C\left(GZ_t - GAZ_{t-1|t-1}\right) = \\ &= (I-CG)AZ_{t-1|t-1} + CGZ_t \end{aligned}$$

and use the approximation (23) to obtain

$$\Xi^* = (I-CG)A\Xi U + CG.$$

Having expressions for $Z_{t|t}$ and $Z_{t|t^*}$ in terms of the current state variable we can use the equilibrium relations (17), (18) to obtain:

$$\begin{aligned} p_t &= e_p Z_{t|t^*} + \kappa\left((1-\beta)\alpha - 1\right)e_a Z_t + \\ &\quad + \kappa(1-\beta)\alpha\sum_{k=1}^{\infty}\beta^k e_a A^k Z_{t|t^*} + \kappa(1-\beta)(1-\alpha)\sum_{k=0}^{\infty}\beta^k e_y A^k Z_{t|t^*} + \\ &\quad - \kappa\beta\sum_{k=0}^{\infty}\beta^k\left(e_r A^k Z_{t|t^*} - e_p A^{k+1} Z_{t|t^*} + e_p A^k Z_{t|t^*}\right) \\ &= \left[e_p + \kappa\left[(1-\beta)\left(\alpha\beta e_a A + (1-\alpha)e_y\right) - \beta(e_r - e_p(A-I))\right](I-\beta A)^{-1}\right]\Xi^* Z_t + \\ &\quad + \kappa\left((1-\beta)\alpha - 1\right)e_a Z_t \\ &= \Phi_p Z_t \end{aligned}$$

and

$$\begin{aligned} y_t &= (1-\phi)e_y Z_{t|t} + \phi(1-\beta)\sum_{k=1}^{\infty}\beta^{k-1}\left(\alpha e_a A^k Z_{t|t} + (1-\alpha)e_y A^k Z_{t|t}\right) + \\ &\quad - \phi\sum_{k=1}^{\infty}\beta^k\left(e_r A^k Z_{t|t} - e_p A^{k+1} Z_{t|t} + e_p A^k Z_{t|t}\right) - \phi\left(e_r Z_t - e_p Z_{t|t}\right) = \\ &= \left[(1-\phi)e_y + \phi\left((1-\beta)\left(\alpha e_a + (1-\alpha)e_y\right)A - \beta\left(e_r - e_p(A-I)\right)A\right)(I-\beta A)^{-1}\right]\Xi Z_t + \\ &\quad + \phi e_p A\Xi Z_t - \phi e_r Z_t - \phi e_p Z_t \\ &= \Phi_y Z_t \end{aligned}$$

Substituting $Z_t = AZ_{t-1} + BW_t$ on the right hand sides of these equations we obtain new values for the coefficients of the first two rows of A and B. We iterate until we achieve convergence according to the criterion

$$(e_p - \Phi_p)'\,\Omega\,(e_p - \Phi_p) + (e_y - \Phi_y)'\,\Omega\,(e_y - \Phi_y).$$

7.1 Cross-sectional dispersion

The computation above takes σ_ζ^2 and σ_η^2 as given. Here, we derive the cross sectional dispersion of prices and quantities. Given the cross sectional dispersion of prices and quantities one obtains an expression for σ_ζ^2 and σ_η^2. Therefore, to compute the equilibrium of a given economy we need to solve a fixed point problem in terms σ_ζ^2 and σ_η^2. The first step in deriving the volatility of η_{it} and ζ_{it} is to derive the volatility of the individual expectations $E_{it}^* Z_t$ and $E_{it} Z_t$. Let

$$\begin{aligned} E_{it}^* Z_t &= Z_{t|t}^* + J_{it}^* \\ E_{it} Z_t &= Z_{t|t} + J_{it} \end{aligned}$$

We can use

$$\begin{aligned} E_{it}^* Z_t &= E_{it-1} Z_t + C\left(S_{it} - E_{it-1} S_{it}\right) \\ &= (I - CG)\, A E_{it-1} Z_{t-1} + C\left(G Z_t + F V_{it}\right) \\ &= (I - CG)\, A \left(Z_{t-1|t-1} + J_{it-1}\right) + C\left(G Z_t + F V_{it}\right) \end{aligned}$$

and

$$\begin{aligned} E_{it} Z_t &= E_{it}^* Z_t + D\left(\overline{p}_{it} - E_{it}^* p_t\right) \\ &= (I - DQ)\, E_{it}^* Z_t + D\left(p_t + \zeta_{it}\right) \\ &= (I - DQ)\left(Z_{t|t}^* + J_{it}^*\right) + D\left(p_t + \zeta_{it}\right) \end{aligned}$$

and obtain the following recursive expression for the individual forecast errors

$$\begin{aligned} J_{it}^* &= (I - CG)\, A J_{it-1} + C F V_{it} \\ J_{it} &= (I - DQ)\, J_{it}^* + D \zeta_{it} \end{aligned}$$

Let K and K^* be the cross sectional variance covariance matrix of the expectations $E_{it} Z_t$ and $E_{it}^* Z_t$. These two matrices satisfy the equations:

$$\begin{aligned} K^* &= (I - CG)\, A K A' (I - CG) + C F \Sigma_1 F' C' \\ K &= (I - DQ)\, K^* (I - DQ)' + \sigma_\zeta^2 D' D \end{aligned}$$

Using the cross sectional dispersions of the agents' expectations we can derive the cross sectional dispersions of prices and quantities. If we omit the terms including the aggregate shocks and the nominal balances b_{it} we can write prices as:

$$\begin{aligned} & p_{it} = ... + \left(e_p + \kappa \eta e_y + \kappa\left[(1-\beta)\left(\alpha \beta e_a A + (1-\alpha)\, e_y\right) - \beta(e_r - e_p\,(A - I))\right](I - \beta A)^{-1}\right) \times \\ & \times \left((I - CG)\, A J_{it-1} + C F V_{it}\right) - \kappa\left(1 + \eta - \alpha\left(1 - \beta\right)\right) e_\epsilon V_{it} \\ = \;& ... + \Psi_{p1} J_{it-1} + \Psi_{p2} V_{it} \end{aligned}$$

$$p_{it}-p_{it-1}=p_t-p_{t-1}+\Psi_{p1}\left(\left(I-DQ\right)\left(I-CG\right)A-I\right)J_{it-2}+\left(\Psi_{p1}\left(I-DQ\right)CF-\Psi_{p2}\right)V_{it-1}+D\zeta_{it-1}+\Psi_{p2}V_{it}$$

and we can write individual consumption as:

$$\begin{aligned}
c_{it} &= ... - \left(\phi + \left(1-\phi\right)\left(1-\hat{\sigma}\right)\right)\zeta_{it} + \left(1-\phi\right)\left(1-\hat{\sigma}\right)\left(\Psi_{p1}J_{it-1}+\Psi_{p2}V_{it}\right) + \\
&\quad + \left[\left(1-\phi\right)e_y + \phi\left(\left(1-\beta\right)\left(\alpha e_a + \left(1-\alpha\right)e_y\right)A - \beta\left(e_r - e_p\left(A-I\right)\right)A\right)\left(I-\beta A\right)^{-1} + \phi e_p A\right] \times \\
&\quad \times \left(\left(I-DQ\right)\left(I-CG\right)AJ_{it-1} + \left(I-DQ\right)CFV_{it} + D\zeta_{it}\right) \\
&= ... + \Psi_{y1}J_{it-1} + \Psi_{y2}V_{it} + \Psi_{y3}\zeta_{it}
\end{aligned} \tag{24}$$

Consistency of the shocks requires that:

$$\begin{aligned}
\eta_{it} &= \frac{1}{m}\sum_{j\in\tilde{H}_{it}}\left(\Psi_{y1}J_{jt-1}+\Psi_{y2}V_{jt}+\Psi_{y3}\zeta_{jt}\right) + \\
&\quad +\hat{\sigma}\frac{1}{m\left(m-1\right)}\sum_{j\in\tilde{H}_{it}}\sum_{k\in H_{jt}k\neq i}\left(\Psi_{p1}J_{kt-1}+\Psi_{p2}V_{kt}\right)
\end{aligned}$$

and

$$\zeta_{it} = \frac{1}{m}\sum_{j\in H_{it}}\left(\Psi_{p1}J_{jt-1}+\Psi_{p2}V_{jt}\right).$$

From these we derive

$$\begin{aligned}
\sigma_\eta^2 &= \frac{1}{m}\left(\Psi_{y1}K\Psi_{y1}'+\Psi_{y2}\Sigma_1\Psi_{y2}'+\Psi_{y3}^2\sigma_\zeta^2\right)+\hat{\sigma}^2\frac{1}{m\left(m-1\right)}\left(\Psi_{p1}K\Psi_{p1}'+\Psi_{p2}\Sigma_1\Psi_{p2}\right) \\
\sigma_\zeta^2 &= \frac{1}{m}\left(\Psi_{p1}K\Psi_{p1}'+\Psi_{p2}\Sigma_1\Psi_{p2}\right).
\end{aligned}$$

References

[1] S. Rao Aiyagari, 1994, "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659-684.

[2] Jeffery Amato, Stephen Morris and Hyun Song Shin, 2002, "Communication and Monetary Policy." *Oxford Review of Economic Policy* 18, pp. 495-503.

[3] Kosuke Aoki, 2001, “Optimal Monetary Policy Response to Relative-Price Shocks”, *Journal of Monetary Economics*, 1, pp 55-80.

[4] Pierpaolo Benigno, 2003, "Optimal Monetary Policy in a Currency Area," *Journal of International Economics*.

[5] Olivier J. Blanchard, 1989, "A Traditional Interpretation of Macroeconomic Fluctuations" *American Economic Review*, Vol. 79, No. 5, pp. 1146-1164.

[6] Olivier J. Blanchard and Danny Quah, 1989, "The Dynamic Effects of Aggregate Demand and Aggregate Supply Shocks." *American Economic Review*, 79-4, September, pp. 655-673.

[7] Olivier J. Blanchard and Mark W. Watson, 1986, "Are Business Cycles All Alike?"

[8] V. V. Chari, Patrick J. Kehoe and Ellen R. McGrattan, 2004, "Business Cycle Accounting" Federal Reserve Bank of Minneapolis Research Department Staff Report 328.

[9] Lawrence Christiano, Martin Eichenbaum, and Robert Vigfusson, 2003, "What happens after a Technology Shock?," mimeo.

[10] Jean-Pierre Danthine and John Donaldson, 1998, "Productivity Growth, Consumer Confidence, and the Business Cycle," *European Economic Review*, 42, 1113-1140.

[11] Jordi Gali, 1992, "How Well Does the IS-LM Model Fit Postwar U.S. Data?" *The Quarterly Journal of Economics*, Vol. 107, No. 2., pp. 709-738.

[12] Jordi Gali, 1999, "Technology, Employment, and the Business Cycle: Do Technology Schocks Explain Aggregate Fluctuations?" *American Economic Review*, March, pp. 249-271.

[13] Robert E. Hall, 1997, "Macroeconomic Fluctuations and the Allocation of Time" *Journal of Labor Economics*, Vol. 15, No. 1, pp. S223-S250.

[14] Christian Hellwig, 2003 "Public Announcements, Adjustments Delays and the Business Cycle"

[15] Pete Klenow and Oleksiy Kryvtsov, 2003, "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?"

[16] Robert E. Lucas Jr.,1972, "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4, no.2: 103-24.

[17] Robert E. Lucas, Jr., 1975, "An Equilibrium Model of the Business Cycle" *The Journal of Political Economy*, Vol. 83, No. 6., pp. 1113-1144.

[18] Stephen Morris and Hyun Song Shin, 2002, "The Social Value of Public Information", *American Economic Review* 92, pp. 1521-1534.

[19] Kevin M. Murphy, Andrei Shleifer, Robert W. Vishny, 1989, "Building Blocks of Market Clearing Business Cycle Models." NBER Macroeconomic Annual 1989. Cambridge, MA: MIT Press.

[20] Neville Francis and Valerie A.Ramey, 2002, "Is the Technology-Driven Real Business Cycle Hypothesis Dead?"

[21] Edmund S. Phelps et al., 1969, *Microeconomic Foundations of Employment and Inflation Theory*, New York: Norton.

[22] David E. Runkle, 1998, "Revisionist History: How Data Revisions Distort Economic Policy Research", Federal Reserve Bank of Minneapolis Quarterly Review Vol. 22, No. 4, Fall 1998, pp. 3–12.

[23] Matthew D. Shapiro, Mark W. Watson, 1988, "Sources of Business Cycle Fluctuations," NBER Macroeconomics Annual 1988, Vol. 3, edited by Stanley Fischer, pp.111-148. Cambridge, MA: The MIT Press.

[24] Neg Wang, 2003, "Caballero Meets Bewley: The Permanent-Income Hypothesis in General Equilibrium", *American Economic Review.*

[25] Mike Woodford, 2003

[26] Mike Woodford, 2003, *Interest and Prices*, Princeton University Press.

Tables and Figures

	Baseline	No public signals	Large noise	Gali(1992)
	$\sigma_\varepsilon = 1.8$	$\sigma_\varepsilon = 1.8$	$\sigma_\varepsilon = 7$	
	$\sigma_{w1} = 0.03$	$\sigma_{w1} = 100$	$\sigma_{w1} = 2$	
	$\sigma_{w2} = 0.05$	$\sigma_{w2} = 100$	$\sigma_{w2} = 2$	
	$\sigma_e = 0.2$	$\sigma_e = 0.2$	$\sigma_e = 1.5$	
Output				
1 quarter	0.26	0.31	0.50	0.31
2 quarters	0.01	0.09	0.39	---
5 quarters	0.00	0.00	0.19	0.19
Prices				
1 quarter	0.37	0.62	0.44	0.37
2 quarters	0.36	0.61	0.42	---
5 quarters	0.39	0.61	0.40	0.51

Table 1. Variance decomposition for different noise levels.

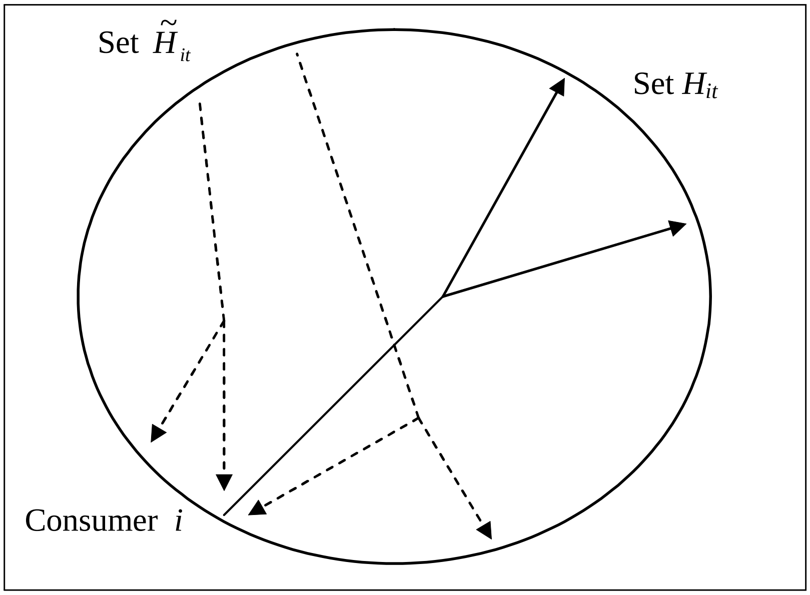

Figure 1. Spatial structure of the model.

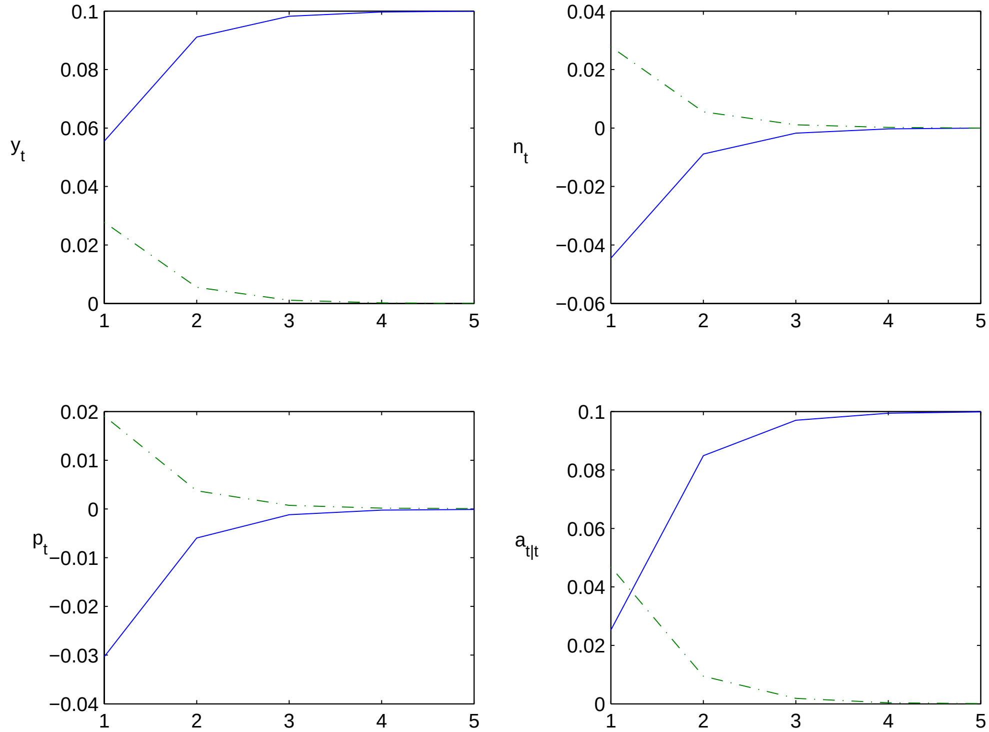

Figure 2. Dynamic responses to a noise shock and to a productivity shock.

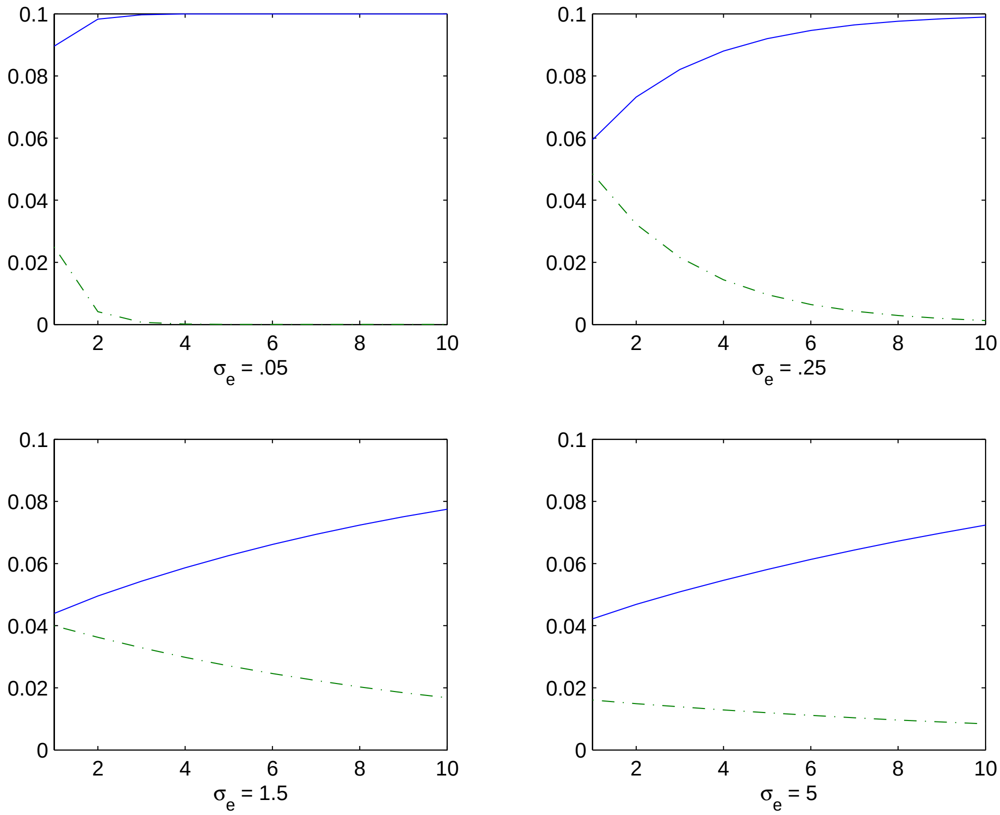

Figure 3. Effects of changing the signal volatility σ_e^2.

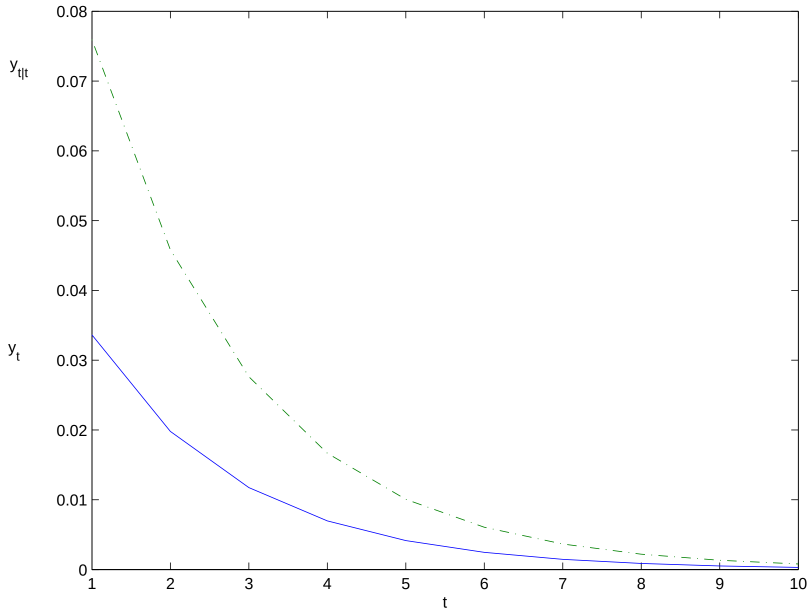

Figure 4. Overreaction of consumers' expectations following a noise shock.

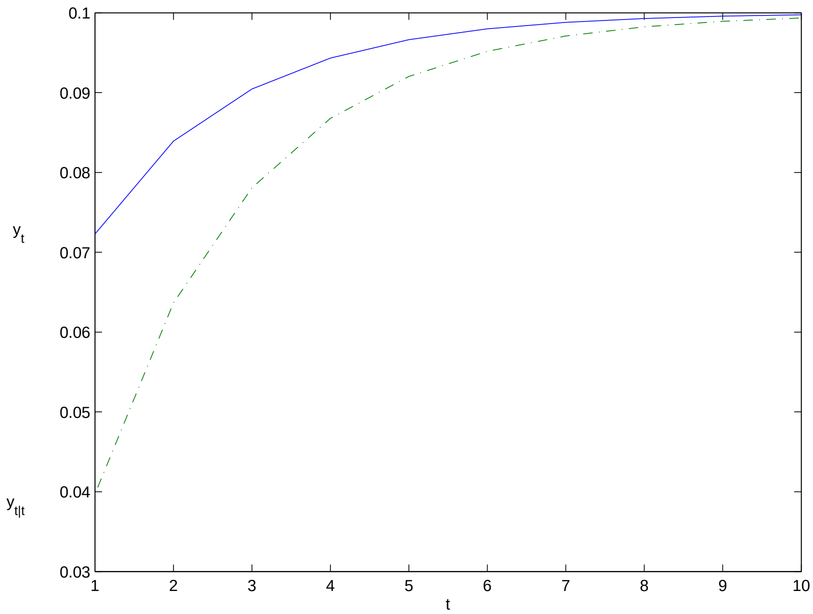

Figure 5. Underreaction of consumers' expectations following a productivity shock.

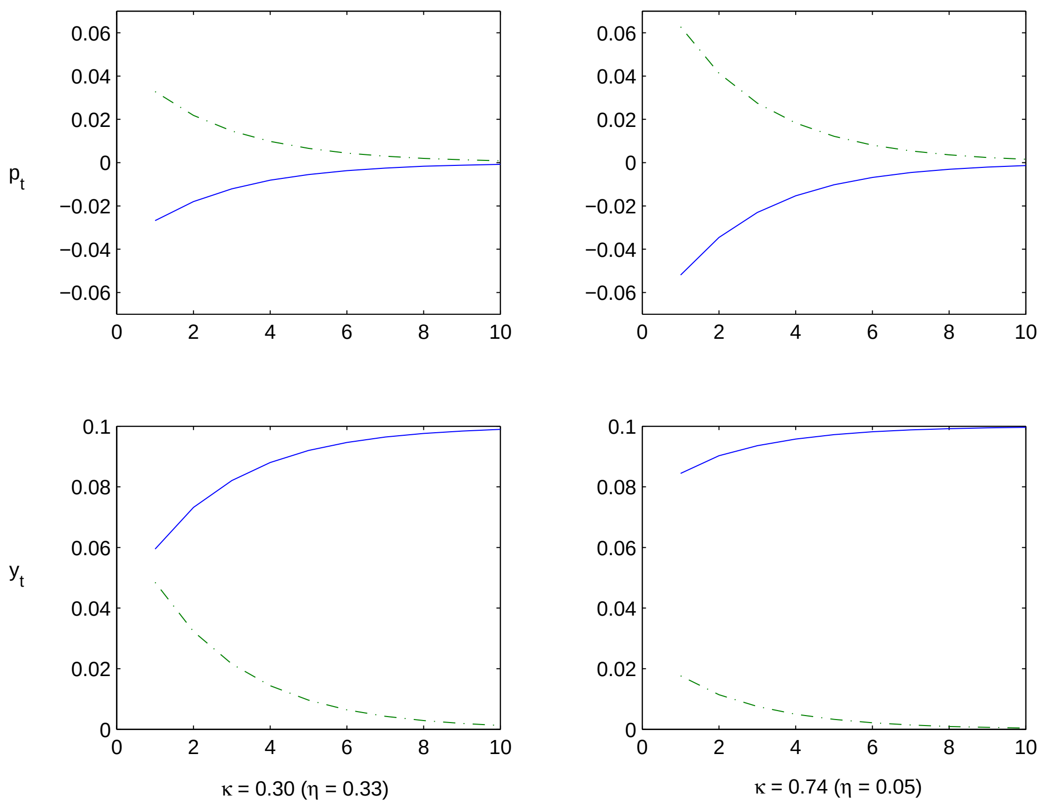

Figure 6. Strategic complementarity in pricing.

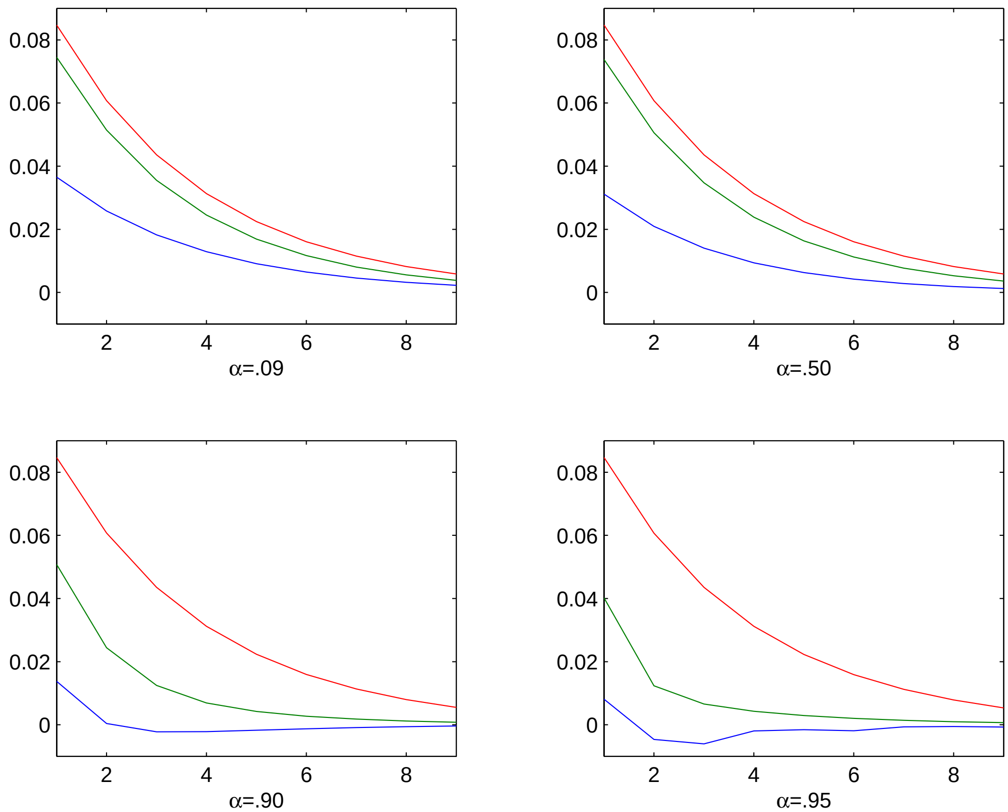

Figure 7. Strategic complementarity in spending.

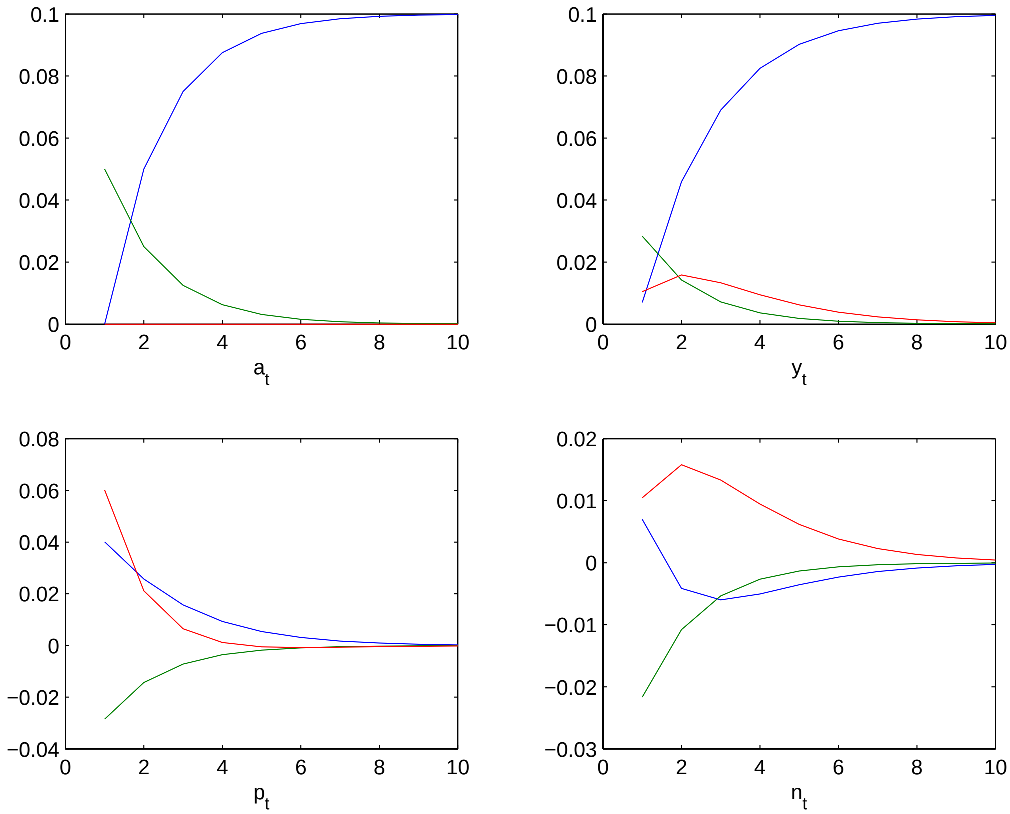

Figure 8. Shocks to future productivity.