New or Used? Investment with Credit Constraints*

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Abstract

This paper studies the choice between investment in new and used capital. We argue that used capital inherently relaxes credit constraints and thus firms which are credit constrained invest in used capital. Used capital is cheap relative to new capital in terms of its purchase price but requires substantial maintenance payments later on. The timing of these investment cash outflows makes used capital attractive for credit constrained firms. While used capital is expensive when evaluated using the discount factor of an agent with a high level of internal funds, it is relatively cheap when evaluated from the vantage point of a credit constrained agent with few internal funds. We provide an overlapping generations model and determine the price of used capital in equilibrium. Agents with less internal funds are more credit constrained, invest in used capital, and start smaller firms. Empirically, we find that the fraction of investment in used capital is substantially higher for small firms.

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1 Introduction

Why invest in used capital rather than new capital? We argue that firms which are credit constrained purchase used capital because the timing of the necessary cash outflows is such that used capital relaxes credit constraints. The purchase price of used capital is lower than that of new capital. However unlike new capital, used capital requires substantial maintenance payments down the road. The timing inherent in these cash flows makes used capital more attractive to firms which are credit constrained at the time of investment. By investing in used capital, constrained firms can operate at a larger scale.

We develop an overlapping generations model in order to study the decision to invest in new or used capital along with the equilibrium in the market for used capital. New and used capital are perfect substitutes in production, but used capital requires a maintenance payment subsequent to purchase. Agents can borrow only against a fraction of the resale value of capital and differ in their initial endowments of internal funds. We find that agents which have few internal funds are credit constrained, invest in used capital, and start smaller firms.

In equilibrium, used capital is expensive when valued using the discount factor of an unconstrained firm, but cheap when valued at the discount factor of a constrained firm. The opposite is true for new capital. The variation in the multiplier on the credit constraint introduces a wedge between the valuations of constrained and unconstrained investors. We show that any agent who invests in used capital in fact must be credit constrained. Thus, one can observe how credit constrained a firm is through revealed preference in the choice between new and used capital. The convenience yield used capital provides by relaxing constrained agents' credit constraints pushes up the equilibrium price of used capital so that it becomes expensive for unconstrained agents. Moreover, since capital can be sold as used capital after use in production, the higher equilibrium price of used capital in the presence of constrained agents makes investment in new capital even more attractive to unconstrained agents.

We provide a condition for the value of used capital to deviate from that of new capital when both are evaluated by an unconstrained agent, that is, we provide a condition for the equilibrium price of used capital to exceed its value to unconstrained agents. We call this a *credit constrained pricing equilibrium* since the fact that there are credit constrained investors in the economy is reflected in the pricing of used capital. We call an equilibrium where unconstrained agents value new and used capital equally an *unconstrained pricing equilibrium*. We also characterize in closed form the levels of internal funds for which agents invest in used or new capital, or both, in each type of equilibrium. In both cases, the fraction of capital investment comprised by used capital is decreasing in firm size, and increasing in how credit constrained the firm is.

We find empirically that the fraction of investment comprised by used capital is indeed decreasing with firm size. We use data from the Annual Capital Expenditures Survey (ACES), which samples nonfarm businesses, and data from the Vehicle Inventory and Use Survey (VIUS), both by the Bureau of the Census. The fraction of capital expenditures on used capital is considerably (about five times) larger for businesses with no employees than for businesses with employees. This is true for structures, equipment, and total capital expenditures. Moreover, on average about thirty percent of aggregate used capital expenditures are done by small businesses, while these businesses on average contribute only about eight percent of aggregate capital expenditures. Similarly, owners of smaller vehicle fleets purchase a much larger fraction of their fleet used than those with large fleets. We argue that, as in our model, these small businesses are more likely to be credit constrained, and therefore purchase more used capital which in turn relaxes these constraints.

Differences in factor prices may also give rise to variation in firms' decision to invest in new vs. used capital. Bond (1983, 1985) develops a model with exogenous heterogeneity in factor prices where firms or sectors with low capacity utilization and low labor costs but high capital costs choose used capital since low labor costs and capacity utilization rates are complementary with the associated production downtime. Variation in factor prices is not correlated with how quickly machines can be fixed, or with productivity in maintaining them. In contrast to our paper, the focus in Bond's model, which is static, is on the relative magnitude of maintenance costs, not on the timing of such payments. Factor price variation has been used to understand trade in used capital across countries, where variation in such prices may be considerable, by Sen (1962) and Smith (1976).

Our work is also related to studies of vintage capital, durable goods, and the effects of credit constraints on investment. In standard vintage capital models the choice between new and used capital is determined by preferences for different vintages. Similarly, the choice between high and low quality durables is often modeled using exogenous preferences for quality. The results in this paper shed light on how these preferences over new and used capital, or goods of high and low quality, might be determined by underlying constraints on the investment decision, and hence complements these studies.[1] Finally, our paper is also related to studies of the effects of credit constraints on investment decisions.[2] The results of our model imply that the degree of a firm's credit constraints can be identified through revealed preference in the choice between new and used capital. This is important because although shifts in investment opportunities drive total capital expenditures, it is not clear that they should affect the new vs. used composition of such expenditures.

The remainder of the paper is organized as follows: In Section 2 we describe our model of new and used capital investment decisions, along with the associated equilibrium in the used capital market. We characterize constrained and unconstrained pricing equilibria. We present a numerical example of an economy in Section 3. In Section 4 we discuss the empirical evidence and conclude in Section 5.

2 Credit Constraints and the Purchase of Used Capital

In this section we consider an economy in which agents can choose between investing in new and used capital in order to study the effect of credit constraints on this choice.

[1]For vintage capital models, see Benhabib and Rustichini (1991), Cooley, Greenwood, and Yorukoglu (1997), Campbell (1998), and Jovanovic (1998). For models of durable goods markets with exogenous preferences over quality, see, for example, Hendel and Lizzeri (1999) for a dynamic adverse selection model with sorting of new and used goods to heterogeneous consumers, Stolyarov (2002) for a model where goods deteriorate with age and volume varies with vintage, and Berkovec (1985), Porter and Sattler (1999), and Adda and Cooper (2000) for simulated discrete choice models and associated empirical tests in the automobile market.

[2]For a survey, see Hubbard (1998). For empirical tests of constrained and unconstrained firms' investment Euler equations, see Whited (1992) and Bond and Meghir (1994). For models describing the effects of variation in net worth on credit constraints, see Townsend (1979), Gale and Hellwig (1985), and Bernanke and Gertler (1989). For evidence that leverage affects overall investment in the trucking industry, see Zingales (1998). For models and evidence of distortions in durable goods consumption from credit constraints, see, for example, Chah, Ramey, and Starr (1995) and Campbell and Hercowitz (2003). Finally, for a general model of investment with a wedge between the purchase price and sale price of capital in the absence of credit constraints see Abel and Eberly (1994).

2.1 The Environment

Consider an economy with overlapping generations. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. At each point in time t, a generation with a continuum of agents with measure one is born. Generations live for one period, that is, for two dates. Agents have identical preferences and access to the same productive technologies, but differ in the idiosyncratic endowment of consumption goods that they are born with, i.e., in the amount of internal funds that they have. The preferences of an agent born in generation t are

$$u(c_t) + \beta u(c_{t+1})$$

where u is strictly increasing and concave and satisfies $\lim_{c \to 0} u'(c) = +\infty$.[3] At time t, each agent observes his idiosyncratic endowment $e \in \mathcal{E} \subset \mathbb{R}_+$ which is distributed independently and identically with density $\pi(e)$ on $\mathcal{E}$.

At time t, each young agent chooses how much to invest in new and used capital for use in production at time $t+1$. The price of new capital is normalized to 1. Used capital, on the other hand, can be bought at a price p_u, which will be determined in equilibrium. Used capital will turn out to be cheaper than new capital in terms of its purchase price, i.e., $p_u < 1$, in equilibrium, but it requires maintenance one period after it is bought. That is, investment in a unit of used capital at time t, requires payments of p_u at time t and $m_u > 0$ at time $t+1$. New and used capital are assumed to be perfect substitutes in production. Thus, an agent who invest $i_{u,t}$ in used capital and $i_{n,t}$ in new capital will have a total amount of capital $k_t = i_{u,t} + i_{n,t}$. Capital generates output of $f(k_t) = k_t^\alpha$, where $\alpha \in (0, 1)$, at time $t + 1$. Capital depreciates at a rate of δ, such that the agent will have $k_{t+1} = (1 - \delta)k_t$ units of capital at time $t + 1$. The agent can sell the depreciated capital at time $t + 1$ as used capital to agents from the next generation. Notice that both new and used capital are sold as used capital after use in production. The idea is that except for the original owner who buys capital new, capital requires maintenance one period after the capital is purchased. Once capital has been previously owned, it is used and it does not matter how many previous owners there were.

Furthermore, an agent can borrow or save at a rate of return $R = \beta^{-1}$, which we fix exogenously to focus on the equilibrium in the used capital market. An agent can

[3] Strict concavity of the utility function is not necessary for our results as long as the production function is concave. Indeed, we could assume linear preferences, i.e., risk neutrality. We discuss below how this can be seen using the marginal rates of transformation as discount rates, rather than marginal rates of substitution.

however only borrow against a fraction $0 \leq \theta < 1$ of the resale value of capital and can not borrow against future output. Thus, the agent needs to provide collateral for loans he takes out and the extent of collateralization is limited. This constraint can be motivated by assuming that lenders can only seize a fraction θ of the capital in case of default, which limits how much the agent can credibly promise to repay to that amount.[4] This defines the credit constraint considered here. Agents' investment in used or new capital is constrained by the amount of their initial endowment of internal funds and their limited ability to borrow. Note that $\theta = 0$ is a special case of our model where agents can not borrow at all, neither against output nor against capital. All the results that we obtain in this paper apply to this special case as well.

We consider a stationary equilibrium where the price of used capital p_u is determined such that all the used capital sold by agents in generation t is bought by agents in generation $t+1$. The equilibrium will be stationary in the sense that all quantities, such as investment, the capital stock, and the volume of trade in used capital are constant across periods. We provide a formal definition of a stationary equilibrium in Section 2.3.

2.2 The Agent's Problem

Consider the problem of an agent in generation t, $t \in \{0, 1, 2, \dots\}$. Since we are studying a stationary model and all generations are identical, we will consider the problem of generation 0 to simplify notation. Taking the price of used capital p_u as given, the agent's problem is one of maximizing utility by choice of consumptions $\{c_0, c_1\}$, investment in used capital i_u and new capital i_n, and borrowing b, given their initial endowment of internal funds e. Specifically, the agent's problem is

$$\max_{c_0, c_1, i_u, i_n, b} u(c_0) + \beta u(c_1)$$

subject to

$$c_0 + p_u i_u + i_n \leq e + b \tag{1}$$

$$c_1 + m_u i_u + Rb \leq k^\alpha + p_u k(1-\delta) \tag{2}$$

where $k \equiv i_u + i_n$ and

$$c_0, c_1 \geq 0 \tag{3}$$

$$i_u, i_n \geq 0, \tag{4}$$

[4]See Hart and Moore (1994) and Kiyotaki and Moore (1997) for models in a similar spirit.

and

$$Rb \leq \theta p_u k(1-\delta). \tag{5}$$

Equations (1-2) are the budget constraints for time 0 and 1, with associated multipliers μ_0 and μ_1, respectively. The constraints (3-4) require non-negativity of consumption and investment. Constraint (5) is the borrowing constraint which restricts borrowing to a fraction θ of the resale value of capital (in present value terms). The non-negativity constraints on consumption (see equation (3)) do not bind since we have assumed that the utility function satisfies an Inada condition. The multipliers associated with the non-negativity constraints for used and new capital investment are λ_u, λ_n, respectively. The multiplier on the credit constraint is λ_b. When this constraint binds ($\lambda_b > 0$), we will say that the agent is credit constrained, and, that the agent is more credit constrained the larger this multiplier is. Note that at most one of the non-negativity constraints on investment in new and used capital (equation (4)) will be binding, since $\lim_{k\to 0} f'(k) = +\infty$.

2.3 Stationary Equilibrium

An economy can be described by the agent's utility function and discount rate along with the technology parameters for the production function, depreciation, and the used capital maintenance cost, the collateralization rate, and the support and distribution over initial endowments of internal funds. Thus, an economy $\mathbb{E}$ is defined by $\mathbb{E} = \{u(\cdot), \beta, \alpha, \delta, m_u, \theta, \mathcal{E}, \pi(e)\}$.

Definition 1 *A stationary equilibrium for an economy $\mathbb{E}$ is a used capital price p_u and an allocation $\{c_0^*, c_1^*, i_n^*, i_u^*, b^*\}$ of consumptions $\{c_0^*(e), c_1^*(e)\}$, investments in new and used capital $\{i_n^*(e), i_u^*(e)\}$, and borrowing $\{b^*(e)\}$ for all $e \in \mathcal{E}$ such that the following conditions are satisfied:*

(i) *The allocation $\{c_0^*, c_1^*, i_n^*, i_u^*, b^*\}$ solves the problem of each agent in generation t, $\forall e \in \mathcal{E}, t$.*

(ii) *The price of used capital p_u is such that the market for used capital clears given p_u, i.e., the amount of used capital sold by generation t equals the amount of used capital bought by generation $t+1$, $\forall t$:*

$$\sum_{e\in\mathcal{E}} \pi(e) i_n^*(e)(1-\delta) + \sum_{e\in\mathcal{E}} \pi(e) i_u^*(e)(1-\delta) = \sum_{e\in\mathcal{E}} \pi(e) i_u^*(e).$$

The right hand side of the market clearing condition in the above definition is the aggregate amount of used capital bought by each generation. The left hand side is the aggregate amount of capital sold by each generation, which is the sum of the aggregate amount of investment in new capital net of depreciation and the aggregate amount of investment in used capital net of depreciation.

If used capital were not cheap in terms of its purchase price, i.e., if $p_u \geq 1$, then all agents would buy new capital only, since new capital does not involve maintenance costs. However, if used capital were too cheap, specifically if the purchase price of used capital plus the maintenance costs discounted at β were strictly less than the cost of buying new capital, i.e., if $p_u + \beta m_u < 1$, then all agents would buy used capital and there would be no investment in new capital. Thus, in equilibrium, $1 - \beta m_u \leq p_u < 1$. Stated formally:

Proposition 1 *The price of used capital satisfies* $1 - \beta m_u \leq p_u < 1$ *in equilibrium.*

The proofs of this proposition and all other formal statements are in the appendix unless noted otherwise. Notice that the discount factor for time 1 payoffs of an agent who is not credit constrained is β and the price of used capital in an economy without credit constraints would hence be $p_u + \beta m_u = 1$. The case of interest here is the other case, in which $p_u + \beta m_u > 1$, which we will refer to as a *credit constrained pricing equilibrium* since in this case used capital is not priced as if there were no credit constraints. We study the properties of credit constrained pricing equilibria below. We also discuss the properties of *unconstrained pricing equilibria*, i.e., equilibria where $p_u + \beta m_u = 1$, and conditions under which an equilibrium with credit constrained pricing obtains, i.e., conditions such that $p_u + \beta m_u > 1$.

2.4 Characterization

In this section we first characterize the used and new capital investment decision and equilibrium in a credit constrained pricing equilibrium. Next, we provide an analogous characterization of an unconstrained pricing equilibrium, and finally we provide conditions for a credit constrained pricing equilibrium to obtain.

Characterization of a Credit Constrained Pricing Equilibrium

We first characterize the solution to the agent's problem in a credit constrained pricing equilibrium, i.e., under the assumption that $p_u + \beta m_u > 1$. We show that in such an equilibrium, it is agents with few internal funds who invest in used capital. Indeed, agents with internal funds below a certain threshold $\bar{e}_u$ invest only in used capital. Agents with internal funds in an intermediate range, i.e., between $\bar{e}_u$ and $\bar{e}_n > \bar{e}_u$, invest in both new and used capital. Agents with internal funds above $\bar{e}_n$ invest in new capital only. Furthermore, the size of an agent's firm measured in units or value of capital is increasing in e. The size of an agent's firm is strictly increasing below $\bar{e}_u$, is constant between $\bar{e}_u$ and $\bar{e}_n$, and then strictly increasing again above $\bar{e}_n$ until internal funds reach $\bar{\bar{e}} > \bar{e}_n$. Agents with internal funds above $\bar{\bar{e}}$ are unconstrained and their level of investment, and hence the size of their firm, is constant and equal to the unconstrained optimal firm size. Thus, agents with few internal funds are credit constrained, start smaller firms, and invest in used capital.

First, note that in a credit constrained pricing equilibrium, any agent who invests a positive amount in used capital must be credit constrained, that is, the multiplier on that agent's credit constraint must be strictly positive. This is stated formally in the following proposition.

Proposition 2 *Suppose $p_u + \beta m_u > 1$. If $i_u(e) > 0$, then $\lambda_b(e) > 0$.*

Thus, one can observe how credit constrained an agent in this economy is through revealed preference in their choice between new and used capital.[5] This seems an interesting implication, since identifying credit constrained firms has remained a challenge in the corporate finance literature.[6]

Next, we characterize the solution to the agent's problem as a function of e, the endowment or internal funds of the agent. The characterization is summarized in the next proposition:

Proposition 3 *Suppose $p_u + \beta m_u > 1$. There exist three cutoff levels of internal funds $\bar{e}_u < \bar{e}_n < \bar{\bar{e}}$ such that the solution to the agent's problem satisfies:*

(i) *For $e \leq \bar{e}_u$, $i_u > 0$, $i_n = 0$, and $b = \beta\theta p_u i_u(1-\delta)$. Moreover, $\frac{di_u}{de} > 0$.*

(ii) *For $\bar{e}_u < e < \bar{e}_n$, $i_u > 0$, $i_n > 0$, and $b = \beta\theta p_u(i_u + i_n)(1-\delta)$. Moreover, $i_u + i_n = \bar{k}$, where $\bar{k} = \left(\alpha^{-1}\left(\frac{m_u}{1-p_u}(1-\beta\theta p_u(1-\delta)) - p_u(1-\delta)(1-\theta)\right)\right)^{\frac{1}{\alpha-1}}$, $\frac{di_u}{de} < 0$, and $\frac{di_n}{de} > 0$.*

[5] See Attanasio, Goldberg, and Kyriazidou (2000) for a related study of the decision to borrow in the auto loan market as a function of loan price and maturity for constrained and unconstrained consumers.

[6] See for example, Fazzari, Hubbard, and Petersen (1988), Kaplan and Zingales (1997), Lamont, Polk, and Saá-Réquejo (2001), and Whited and Wu (2003).

(iii) *For* $\bar{e}_n \leq e \leq \bar{\bar{e}}$, $i_n > 0$, $i_u = 0$, *and* $b = \beta\theta p_u i_n(1-\delta)$. *Moreover,* $\frac{di_n}{de} > 0$.

(iv) *For* $e > \bar{\bar{e}}$, $i_n > 0$, $i_u = 0$, *and* $b < \beta\theta p_u i_n(1-\delta)$. *Moreover,* $i_n = \bar{\bar{k}} = (\alpha^{-1}(\beta^{-1} - p_u(1-\delta)))^{\frac{1}{\alpha-1}}$.

Finally, $\bar{\bar{e}} = \bar{\bar{k}}(1-\beta\theta p_u(1-\delta)) + \bar{\bar{k}}^{\alpha} + p_u\bar{\bar{k}}(1-\delta)(1-\theta)$, *and, if* $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, *then* $\bar{e}_u = p_u\bar{k}(1-\beta\theta(1-\delta)) + (\bar{k}^{\alpha} + p_u\bar{k}(1-\delta)(1-\theta) - m_u\bar{k})\left(\frac{\beta m_u}{1-p_u}\right)^{-\frac{1}{\gamma}}$ *and* $\bar{e}_n = \bar{k}(1-\beta\theta p_u(1-\delta)) + (\bar{k}^{\alpha} + p_u\bar{k}(1-\delta)(1-\theta))\left(\frac{\beta m_u}{1-p_u}\right)^{-\frac{1}{\gamma}}$.

The last part of Proposition 3 is useful for studying the numerical example provided in Section 3 below where we specify $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

The decision to invest in used and new capital depends on the agent's initial endowment of internal funds. Because of the credit constraint, the values of used vs. new capital are agent specific and depend on the level of internal funds. That is, the multipliers on the borrowing constraints drive wedges between the valuations of agents with differing levels of internal funds. This is related to the result in the literature studying investment with financing constraints which shows that constrained firms make investment decisions which reflect the higher discount rate induced by binding credit constraints.[7] This can be seen by considering the first order conditions with respect to i_u and i_n which can be written as:

$$\mu_0 p_u + \mu_1 m_u - \lambda_u = \mu_1(f'(k) + p_u(1-\delta)) + \lambda_b\theta p_u(1-\delta), \tag{6}$$

$$\mu_0 - \lambda_n = \mu_1(f'(k) + p_u(1-\delta)) + \lambda_b\theta p_u(1-\delta). \tag{7}$$

The terms on the right hand side of equations (6) and (7) reflect the return on investing in used and new capital, respectively, in terms of output, resale value of capital, and shadow value of collateral. Since used and new capital are assumed to be perfect substitutes in production the returns are the same. The terms on the left hand side reflect the cost of investing in used capital $\mu_0 p_u + \mu_1 m_u$ and new capital μ_0, respectively. New and used capital can be valued for agents in each of the three endowment regions using the appropriate marginal rates of substitution. Since agents with internal funds between $\bar{e}_u$ and $\bar{e}_n$ invest in both new and used capital, the multipliers λ_u and λ_n are both zero and hence the shadow price of used capital for these agents satisfies:

$$p_u + \left[\beta\frac{u'(c_1(e))}{u'(c_0(e))}\right] m_u = 1, \tag{8}$$

[7] See, for example, Whited (1992) and Bond and Meghir (1994).

where we have used the fact that $\mu_0(e) = u'(c_0(e))$ and $\mu_1(e) = \beta u'(c_1(e))$. This means that from the vantage point of an agent in this intermediate range, the shadow price of used capital equals the shadow price of new capital. Notice that the shadow price of used capital is agent specific and depends on the agent's endowment.

Agents with internal funds less than $\bar{e}_u$ however invest in used capital only and in this range $\lambda_n > 0$. Thus, from their vantage point used capital is relatively cheap, i.e., the shadow price of used capital satisfies:

$$p_u + \left[\beta \frac{u'(c_1(e))}{u'(c_0(e))}\right] m_u < 1. \tag{9}$$

Finally, for agents who invest in new capital only, i.e., agents with internal funds exceeding $\bar{e}_n$, $\lambda_u > 0$ and thus these agents consider new capital relatively cheaper than used capital:

$$p_u + \left[\beta \frac{u'(c_1(e))}{u'(c_0(e))}\right] m_u > 1. \tag{10}$$

New and used capital can also be valued for agents in each of the three endowment regions using the appropriate marginal rates of transformation.[8] For agents with endowments between $\bar{e}_u$ and $\bar{e}_n$ we have:

$$p_u + \left[\frac{p_u(1-\beta\theta(1-\delta))}{f'(k(e)) + p_u(1-\delta)(1-\theta) - m_u}\right] m_u = p_u + \left[\frac{1-\beta\theta p_u(1-\delta)}{f'(k(e)) + p_u(1-\delta)(1-\theta)}\right] m_u = 1.$$

These agents invest in both new and used capital, hence the value of a unit of used capital must equal the value of a unit of new capital evaluated at the marginal rate of transformation implied by the level of capital chosen by agents with endowments in this region. Notice that these are the marginal rates of transformation for internal funds. For example, an extra unit of used capital costs p_u, but allows extra borrowing of $\beta\theta p_u(1-\delta)$, and thus costs $p_u(1-\beta\theta(1-\delta))$ in internal funds. The return on used capital is $f'(k)$ in terms of output, $p_u(1-\delta)(1-\theta)$ in terms of resale value net of loan repayment, and requires a maintenance payment of m_u, thus $f'(k) + p_u(1-\delta)(1-\theta) - m_u$ overall. The marginal rate of transformation for new capital has a similar interpretation. Recall that the optimal choice for capital (the sum of new and used capital investment) is increasing in initial endowment. For agents with internal funds less than $\bar{e}_u$ we have:

$$p_u + \left[\frac{p_u(1-\beta\theta(1-\delta))}{f'(k(e)) + p_u(1-\delta)(1-\theta) - m_u}\right] m_u < 1.$$

[8]Marginal rates of transformation have also been used to value assets in the production based asset pricing literature following Cochrane (1991, 1996), including Restoy and Rockinger (1994) and Gomes, Yaron, and Zhang (2003).

Thus, used capital is cheaper than new capital valued at the marginal rate of transformation for the most constrained agents. By investing in used capital, constrained agents can operate larger firms. Notice that with concave production technologies, even risk neutral agents would pay a premium for used capital if they were credit constrained. Finally, for agents with internal funds exceeding $\bar{e}_n$ we have:

$$p_u + \left[\frac{1 - \beta\theta p_u(1-\delta)}{f'(k(e)) + p_u(1-\delta)(1-\theta)}\right] m_u > 1,$$

which means that used capital is more expensive than new capital when valued by these unconstrained agents.

Two implications of the equilibrium pricing of used capital in a credit constrained pricing equilibrium are notable here. First, used capital is made expensive to unconstrained investors by the fact that it provides a convenience yield to constrained investors by relaxing their credit constraints and this makes these agents willing to pay more for used capital. Second, the premium at which used capital trades means that unconstrained agents invest more in equilibrium because they can sell capital at a premium in the used capital market. Of course, this is also true for used capital investment, however in this case the premium affects both the purchase and selling price.

Characterization of an Unconstrained Pricing Equilibrium

We now consider the properties of an equilibrium with unconstrained pricing. The characterization is quite similar to the one in a credit constrained pricing equilibrium. In particular, agents with few internal funds are credit constrained, buy only used capital, and start smaller firms. There are two main differences, however. The first difference is that investment in new capital is not uniquely determined for all agents. The minimum amount an agent invests in used capital, however, is uniquely determined and has the same properties as before. The minimum investment in used capital is 100% of investment for agents with internal funds below some threshold $\bar{e}_u$, then decreases over an interval of intermediate values of internal funds between $\bar{e}_u$ and $\bar{e}_n$, and is zero above $\bar{e}_n$. The second difference is that in an unconstrained pricing equilibrium there is no region besides the region below $\bar{e}_u$ where an agent's total investment is increasing. The third region of Proposition 3 thus collapses, i.e., $\bar{e}_n = \bar{\bar{e}}$ using the notation of that proposition. The characterization of the agent's problem is summarized in Proposition 4 below.

To see that investment in new capital is not uniquely determined consider the following argument: Any agent who is willing to invest a positive amount in new capital would be indifferent between doing so and raising the investment in used capital by a small amount while reducing his investment in new capital by the same small amount and reducing borrowing (or increasing savings) by the difference. Specifically, increasing i_u by Δ and reducing i_n by the same amount frees up $(1-p_u)\Delta$ units of consumption at date 0. Reducing borrowing (or increasing savings) b by that amount leaves consumption at date 0 unchanged, and pays off $R(1-p_u)\Delta$ at date 1. Maintenance costs at date 1 increase by $m_u\Delta$, but the reduction in borrowing (or increase in savings) exactly covers that. Thus consumption at date 1 is not affected either.

We denote the minimum investment in used capital, which is determined uniquely given the agent's internal funds e, by i_u^{min} and the corresponding maximum investment in new capital by i_n^{max}. Similarly, we denote the implied maximum borrowing by b^{max}. The solution can then be characterized as follows:

Proposition 4 *Suppose $p_u+\beta m_u = 1$. There exist two cutoff levels of internal funds $\bar{e}_u < \bar{e}_n$ such that the solution to the agent's problem satisfies:*

(i) *For $e \leq \bar{e}_u$, $i_u > 0$, $i_n = 0$, and $b = \beta\theta p_u i_u(1-\delta)$. Moreover, $\frac{di_u}{de} > 0$.*

(ii) *For $\bar{e}_u < e < \bar{e}_n$, $i_u^{min} > 0$, $i_n^{max} > 0$, and $b^{max} = \beta\theta p_u(i_u+i_n)(1-\delta)$. Moreover, $i_u + i_n = \bar{k}$, where $\bar{k} = (\alpha^{-1}(\beta^{-1} - p_u(1-\delta)))^{\frac{1}{\alpha-1}}$, $\frac{di_u^{min}}{de} < 0$, and $\frac{di_n^{max}}{de} > 0$.*

(iii) *For $e \geq \bar{e}_n$, $i_n^{max} > 0$, $i_u^{min} = 0$, and, for $e > \bar{e}_n$, $b^{max} < \beta\theta p_u(i_u+i_n)(1-\delta)$. Moreover, $i_u + i_n = \bar{k}$ and $\frac{di_n^{max}}{de} = 0$.*

Finally, $\bar{e}_u = p_u\bar{k}(1-\beta\theta(1-\delta)) + \bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta) - m_u\bar{k}$ and $\bar{e}_n = \bar{k}(1-\beta\theta p_u(1-\delta)) + \bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta)$.

Conditions for a Credit Constrained Pricing Equilibrium

We can now determine the conditions for the equilibrium to have unconstrained pricing versus credit constrained pricing. The maximum aggregate amount of new capital sold after one period given unconstrained pricing, i.e., $p_u + \beta m_u = 1$, is

$$\sum_{e\in\mathcal{E}} \pi(e) i_n^{max}(e)(1-\delta)$$

while the minimum aggregate net amount of used capital investment given unconstrained pricing is

$$\sum_{e\in\mathcal{E}} \pi(e) i_u^{min}(e)\delta.$$

Notice that both these expressions involve only parameters since $p_u = 1 - \beta m_u$ in an unconstrained pricing equilibrium. From Definition 1, market clearing requires that

$$\sum_{e\in\mathcal{E}} \pi(e) i_n^*(e)(1-\delta) = \sum_{e\in\mathcal{E}} \pi(e) i_u^*(e)\delta.$$

Thus, if

$$\sum_{e\in\mathcal{E}} \pi(e) i_n^{max}(e)(1-\delta) \geq \sum_{e\in\mathcal{E}} \pi(e) i_u^{min}(e)\delta,$$

then unconstrained agents are willing to invest more than enough in new capital to satisfy the net demand for used capital. This means that the marginal used capital investor is an unconstrained agent and hence we have an equilibrium with unconstrained pricing. However, if the converse is true, then some constrained agents need to invest in new capital and hence the marginal agent pricing used capital is constrained. Hence, a credit constrained pricing equilibrium obtains under the following condition:

Condition 1 $\quad \sum_{e\in\mathcal{E}} \pi(e) i_n^{max}(e)(1-\delta) < \sum_{e\in\mathcal{E}} \pi(e) i_u^{min}(e)\delta.$

3 Numerical Example

To illustrate and compare credit constrained and unconstrained pricing equilibria, we present two example economies. We will refer to the credit constrained pricing equilibrium as "CCPE," and the unconstrained pricing equilibrium as "UPE." In particular, we study the premium used capital trades at in the CCPE economy and the decision to invest in used vs. new capital as a function of agents' initial internal funds. The only difference in primitives between the two economies is the distribution over endowments of internal funds. To construct a CCPE, we know that Condition 1 must be satisfied. In the CCPE economy, the distribution over endowments is exponential on the state space, so that there are more agents with low endowments than high endowments, whereas endowments are distributed uniformly in the UPE economy. Thus, the internal funds distribution determines whether used capital trades at a premium and the magnitude of this premium. This has a couple of interesting implications. First, in the cross section, it implies that used capital should trade at a higher premium in industries with more firms with low levels of internal funds. Second, over the business cycle, it implies that the relative shadow price of used capital is, ceteris paribus, higher in recessions, which means that used capital is an even worse deal for unconstrained firms in bad times.

For preferences, we specify that $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ so that the last part of Proposition 3 applies, and set $\gamma = 2$. We think of one period as a year and set $\beta = 0.96$. Used and new capital are perfect substitutes in the production function, $f(k) = k^{\alpha}$, with $\alpha = 0.33$. All capital depreciates at a rate $\delta = 0.12$. The differences between used and new capital are that new capital is purchased at a price of one and requires no maintenance, while the price of used capital is determined in equilibrium and incorporates the maintenance cost m_u which we set to 0.50. We set the collateralization rate θ equal to 0.20, such that agents can promise to pay up to 20% of the resale value of their capital. We solve the model on a discretized state and choice space. The choice space for investment in new and used capital (i_u and i_n) and borrowing (b) is chosen to impose the borrowing constraint and such that the boundaries do not impose additional constraints at the solution. We endow agents with initial internal funds between 0.05 and 1.75. For the CCPE economy the distribution over endowments is exponential on the state space, with 50% of agents endowed with internal funds of 0.20 or less. The UPE economy has uniformly distributed endowments, so that 50% of agents are endowed with internal funds of 0.90 or less. Table 1 presents the parameters which define the two example economies.

We first discuss the results for the CCPE economy. Used capital trades at a premium of 2.85% in the CCPE. Under unconstrained pricing, the price of used capital equals the price of new capital (one) minus βm_u, which equals 0.52 under our assumptions, while in the CCPE $p_u = 0.5485$ and the shadow price of used capital is $p_u + \beta m_u = 1.0285$. The fact that agents with low endowments, who are credit constrained, can start larger firms by investing in used capital (that is, used capital relaxes credit constraints for these agents) means that used capital earns a convenience yield which raises the equilibrium price of used capital. The price of used capital in the CCPE and UPE economies is presented in Table 2 along with the other equilibrium implications for the two economies.

Next, we describe the investment decisions by agents in the CCPE. As stated in Proposition 3, there are three increasing cutoff levels for internal funds, defining four investment regions. Figure 1 presents the investment and savings decisions, along with the shadow price of used capital, and the multiplier on the borrowing constraint, in the CCPE economy as a function of agents' initial endowments of internal funds. The top left panel plots new and used capital investment. For low endowment levels, below $\bar{e}_u = 0.8469$, agents invest only in used capital and investment is increasing in this range. At higher levels of internal funds, between $\bar{e}_u$ and $\bar{e}_n = 1.2134$, agents invest an increasing amount in new capital, and a decreasing amount in used capital. Total investment is constant at $\bar{k}$ which is 0.3914. Used capital investment reaches zero at $\bar{e}_n$ and agents with endowments greater than $\bar{e}_n$ invest only in new capital. Between $\bar{e}_n$ and $\bar{\bar{e}}$ investment in new capital increases until total investment reaches $\bar{\bar{k}} = 0.4554$ at $\bar{\bar{e}}$ and capital investment is constant thereafter. Agents in this region are unconstrained and begin to borrow less than their borrowing capacity. The top right panel of Figure 1 plots borrowing. Agents with internal funds below $\bar{\bar{e}}$ borrow up to their borrowing capacity and borrowing is increasing in this range. Above $\bar{\bar{e}}$, borrowing is decreasing and agents with lots of internal funds save positive amounts. The middle left panel plots the fraction of capital expenditures comprised by used capital, which decreases monotonically with internal funds. The bottom left panel describes why this is the case along with what motivates the investment decision described above, by plotting the shadow prices for used and new capital as a function of internal funds. The shadow price of new capital is one, and the shadow price of used capital, described in equations (8)-(10), is increasing with the level of internal funds. The shadow price of used capital is less than one for agents who invest only in used capital, equals one in the intermediate region where agents are indifferent between used and new capital, and exceeds one for agents who invest only in new capital and for agents who are unconstrained. Finally, the middle right panel of Figure 1 plots the multiplier on agents' borrowing constraints. This multiplier monotonically decreases with internal funds, and reaches zero for unconstrained agents, who borrow less than their borrowing capacity.

We turn now to the results of the UPE economy for comparison. A UPE is defined by the fact that used capital sells for $1 - \beta m_u = 0.52$. The other results for the UPE are similar to those for the CCPE, and are presented in the bottom panel of Table 2. However, as described in Proposition 4, investment in new capital and savings are only uniquely defined for agents with internal funds less than $\bar{e}_u = 0.8820$, who do not purchase any new capital. Agents with higher endowments are indifferent between appropriately combining reduced borrowing with used capital and investing in new capital. Also, total investment is increasing in initial internal funds only for agents who invest only in used capital. Still, as in the CCPE economy, agents with low endowments invest exclusively in used capital, are constrained, and start smaller firms. Moreover, above $\bar{e}_u$ the minimum investment in used capital is monotonically decreasing in agents' endowment, and reaches zero at $\bar{e}_n = 1.3001$, where maximum borrowing no longer exhausts agents' borrowing capacity. Analogous to Figure 1 for the CCPE economy, Figure 2 presents the investment and borrowing decisions, along with the shadow price of used capital, and the multiplier on the borrowing constraint, in the UPE economy as a function of agents' initial endowment of internal funds. Notice that $\bar{e}_u$ is higher in the UPE than in the CCPE so agents with higher endowment levels invest exclusively in used capital. This is because the price of used capital is not inflated by the effect of credit constraints. For the same reason, in the UPE unconstrained agents are willing invest in both new and used capital, in contrast to the CCPE. In fact, only unconstrained agents are willing to invest in both used and new capital. To see this, compare the top left and middle right panels of Figure 2, which plot investment and the multiplier on the borrowing constraint, respectively.

4 Evidence on Investment in New and Used Capital

Our model predicts that firms with less internal funds are credit constrained, invest more in used capital, and start smaller firms. We will focus on the prediction that the fraction of investment comprised by used capital and firm size are negatively related, i.e. that used to total capital expenditures are decreasing with firm size. First, it is important to note that small firms are indeed important in used capital markets. On average about thirty percent of aggregate used capital expenditures is done by small businesses, while these businesses on average contribute only about eight percent of aggregate total capital expenditures.[9]

Table 3 presents the composition of capital expenditures on new and used structures and equipment for companies with and without employees. Clearly, small companies (those without employees, or with less than five employees) invest a larger

[9]See the Annual Capital Expenditures Survey reports at http://www.census.gov/csd/ace/ace-pdf.html. The sample frame for companies with employees was slightly more than 5.6 million and for companies without employees about 20.3 million, in 2002, and the survey covers a broad range of industries.

fraction of their capital expenditures in used capital. The fraction of capital expenditures comprised by used capital for small firms is about five times the fraction for large firms. This pattern is robust across years and is similar for structures and equipment. By comparing the statistics reported for two different size cutoffs in 1996 we can see that this fact is also robust to the cutoff for large vs. small firms. Table 4 presents similar statistics for trucks dedicated to business use.[10] Overall, about 50% of trucks were purchased used. This fraction decreases monotonically with fleet size. For example, businesses with fleets of 10 to 24 purchased about 38% of their trucks used, whereas businesses with fleets of 100-499 vehicles purchased only about 24% of their trucks used. Figure 3 displays this information graphically, and for different body types. Businesses with the smallest fleets buy at least 40% of trucks of all body types used, while businesses with the largest fleets buy no more than 37% of trucks of any body type used. Thus, we conclude that small firms invest more in used capital than large firms.

The idea that small firms are more likely to be credit constrained is supported by the studies of the effects of credit constraints on investment. For example, Whited and Wu (2003) report that for both their index of credit constraints, as well as the Kaplan and Zingales (1997) index used by Lamont, Polk, and Saá-Réquejo (2001), average firm assets decrease monotonically with the degree of financial constraints. Moreover, classic models of borrowing constraints link internal funds to the degree of financial constraints.[11] Likewise, in our model small firms are constrained and have a larger multiplier on their borrowing constraint. The effect of credit constraints is manifested in the composition of investment in terms of new and used capital and in firm size.

5 Conclusions

This paper develops a model of the decision to invest in new vs. used capital when used capital has a lower purchase price, but requires maintenance payments later on. We find that used capital is valuable to credit constrained agents because it relaxes borrowing constraints. Used capital allows constrained agents to operate larger scale firms by deferring some of the capital costs. This is interesting because it implies

[10] Data are from the public use microdata file of the 1997 Vehicle Inventory and Use Survey by the Bureau of the Census.

[11] See Townsend (1979), Gale and Hellwig (1985), and Bernanke and Gertler (1989).

that firms' credit constraints can be measured by the composition of their capital expenditures. We find that agents with low levels of internal funds invest more in used capital, are credit constrained, and operate smaller scale firms. Credit constraints imply that discount factors are firm specific and used capital can thus seem cheap from the vantage point of a constrained firm while unconstrained firms consider it expensive. We present evidence that used capital indeed comprises a much larger fraction of capital expenditures for small firms who are likely to face binding credit constraints. The distribution of internal funds determines whether the equilibrium of our model displays credit constrained or unconstrained pricing. Thus, the distribution of internal funds determines the premium at which used capital trades. This implies that in the cross section used capital should be relatively more expensive in industries with a large fraction of small, credit constrained firms. Similarly, over the business cycle, we expect used capital to trade at a higher premium in recessions when credit constraints are more binding. Finally, our results shed light on the choice between capital vintages and consumer durables of different quality, which are typically motivated by exogenous variation in preferences for quality.

Appendix

Proof of Proposition 1. Notice that the objective of the agent's problem is concave and the constraint set convex and hence the first order conditions are necessary and sufficient. The first order conditions with respect to i_u and i_n are

$$\mu_0 p_u = \mu_1(\alpha k^{\alpha-1} + p_u(1-\delta) - m_u) + \lambda_u + \lambda_b \theta p_u(1-\delta) \tag{11}$$
$$\mu_0 = \mu_1(\alpha k^{\alpha-1} + p_u(1-\delta)) + \lambda_n + \lambda_b \theta p_u(1-\delta) \tag{12}$$

and with respect to b is $\mu_0 = \mu_1 \beta^{-1} + \lambda_b \beta^{-1}$, where μ_t is the multiplier on date t consumption, λ_u and λ_n are the multipliers on the non-negativity constraints for i_u and i_n, respectively, and λ_b is the multiplier on the borrowing constraint. Subtracting (11) from (12) gives

$$\mu_0(1-p_u) = \mu_1 m_u + \lambda_n - \lambda_u. \tag{13}$$

Thus, if $p_u \geq 1$, then $\lambda_u > 0$ (the strict inequality follows from the fact that $\mu_1 > 0$) and hence $i_u = 0$ for all $e \in \mathcal{E}$, which is impossible in equilibrium. If $p_u < 1 - \beta m_u$, then using this inequality and (13), we have

$$\mu_0 \beta m_u < \mu_0(1-p_u) = \mu_1 m_u + \lambda_n - \lambda_u$$

and substituting for μ_0 using the first order condition with respect to b we get $\lambda_b m_u < \lambda_n - \lambda_u$. Thus, $\lambda_n > 0$ and hence $i_n = 0$ for all $e \in \mathcal{E}$ which is again impossible in equilibrium. $\square$

Proof of Proposition 2. Since $i_u > 0$, $\lambda_u = 0$, where we have suppressed the dependence on e to simplify notation. Equation (13) together with $p_u + \beta m_u > 1$ and $\mu_0 = \mu_1 \beta^{-1} + \lambda_b \beta^{-1}$ then imply that

$$\mu_1 m_u + \lambda_n = \mu_0(1-p_u) < \mu_0 \beta m_u = \mu_1 m_u + \lambda_b m_u$$

or $\lambda_n < \lambda_b m_u$ which implies $\lambda_b > 0$. $\square$

Proof of Proposition 3. Note that the objective is continuous and strictly concave and that the constraint set is convex and continuous. Thus, by the theorem of the maximum (see, e.g., Stokey, Lucas, and Prescott (1989)), the maximizing choices are continuous in e.

First, suppose $i_u > 0$ where we suppress the dependence on e for simplicity. Then, by Proposition 2, $\lambda_b > 0$, i.e., $b = \beta\theta p_u(i_u + i_n)(1-\delta)$. Consider the case where

$i_n = 0$. Then the first order condition with respect to i_u, equation (11), can be written as

$$\begin{aligned} u'(e - p_u i_u(1-\beta\theta(1-\delta)))p_u(1-\beta\theta(1-\delta)) &= \beta u'(i_u^\alpha + p_u i_u(1-\delta)(1-\theta) - m_u i_u) \times \\ &\quad (\alpha i_u^{\alpha-1} + p_u(1-\delta)(1-\theta) - m_u) \quad (14) \end{aligned}$$

where we substituted for λ_b using $\mu_0 = \mu_1\beta^{-1} + \lambda_b\beta^{-1}$. By totally differentiating we get

$$\begin{aligned} \frac{di_u}{de} &= \frac{u''(c_0)p_u(1-\beta\theta(1-\delta))}{u''(c_0)(p_u - \beta\theta p_u(1-\delta))^2 + \beta u''(c_1)(f'(k) + p_u(1-\delta)(1-\theta) - m_u)^2 + \beta u'(c_1)f''(k)} \\ &> 0. \qquad (15) \end{aligned}$$

Next, consider the case where both $i_u > 0$ and $i_n > 0$, such that the first order conditions with respect to i_u and i_n are, again substituting for λ_b,

$$\begin{aligned} \mu_0 p_u(1-\beta\theta(1-\delta)) &= \mu_1(\alpha k^{\alpha-1} + p_u(1-\delta)(1-\theta) - m_u) \quad (16) \\ \mu_0(1-\beta\theta p_u(1-\delta)) &= \mu_1(\alpha k^{\alpha-1} + p_u(1-\delta)(1-\theta)) \quad (17) \end{aligned}$$

Dividing equation (16) by equation(17) implies

$$\frac{p_u - \beta\theta p_u(1-\delta)}{1-\beta\theta p_u(1-\delta)} = 1 - \frac{m_u}{\alpha k^{\alpha-1} + p_u(1-\delta)(1-\theta)} \quad (18)$$

and thus k is uniquely determined and constant in this range and equals $\bar{k} = \left(\alpha^{-1}(\frac{m_u}{1-p_u}(1-\beta\theta p_u(1-\delta)) - p_u(1-\delta)(1-\theta))\right)^{\frac{1}{\alpha-1}}$. Totally differentiating (17) implies that

$$\frac{d\mu_0}{de}(1-\beta\theta p_u(1-\delta)) = \frac{d\mu_1}{de}(\alpha\bar{k}^{\alpha-1} + p_u(1-\delta)(1-\theta)),$$

and thus $\frac{d\mu_0}{de}$ and $\frac{d\mu_1}{de}$ have the same sign. Since it is not possible that the agent's consumption at both dates decreases in e, the sign must be negative, which in turn implies that $\frac{dc_1}{de} > 0$. Totally differentiating the time 1 budget constraint gives $\frac{dc_1}{de} = -m_u\frac{di_u}{de}$ and thus $\frac{di_u}{de} < 0$ and $\frac{di_n}{de} > 0$.

At the upper boundary of this region, the capital $\bar{k}$ is entirely made up by new capital. Equation (17) can then be written as

$$\begin{aligned} u'(e - \bar{k}(1-\beta\theta p_u(1-\delta)))(1-\beta\theta p_u(1-\delta)) &= \beta u'(\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta)) \times \\ &\quad (\alpha\bar{k}^{\alpha-1} + p_u(1-\delta)(1-\theta)) \quad (19) \end{aligned}$$

which implicitly defines the upper bound on internal funds $\bar{e}_n$. If $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and using equation (18), we can solve for $\bar{e}_n$: $\bar{e}_n = \bar{k}(1 - \beta\theta p_u(1-\delta)) + (\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta))\left(\frac{\beta m_u}{1-p_u}\right)^{-\frac{1}{\gamma}}$. At the lower boundary of the region, the agent only invest in used capital and hence

$$\begin{aligned} u'(e - p_u\bar{k}(1-\beta\theta(1-\delta))) &= \beta u'(\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta) - m_u\bar{k}) \times \\ &\quad (\alpha\bar{k}^{\alpha-1} + p_u(1-\delta)(1-\theta)) \end{aligned} \tag{20}$$

which implicitly defines the lower bound on internal funds $\bar{e}_u$. Proceeding as above, we obtain $\bar{e}_u = p_u\bar{k}(1-\beta\theta(1-\delta)) + (\bar{k}^\alpha + p_u\bar{k}(1-\delta)(1-\theta) - m_u\bar{k})\left(\frac{\beta m_u}{1-p_u}\right)^{-\frac{1}{\gamma}}$. Comparing (19) and (20) implies $u'(\bar{e}_u - p_u\bar{k}(1-\beta\theta(1-\delta))) > u'(\bar{e}_n - \bar{k}(1-\beta\theta p_u(1-\delta)))$ or $\bar{e}_u - p_u\bar{k}(1-\beta\theta(1-\delta)) < \bar{e}_n - \bar{k}(1-\beta\theta p_u(1-\delta))$ and thus $\bar{e}_u < \bar{e}_n$.

Finally, suppose $i_n > 0$ and $i_u = 0$. Consider the case where $\lambda_b > 0$ and hence $b = \beta\theta p_u i_n(1-\delta)$. Then equation (12), again substituting for λ_b, can be written as

$$\begin{aligned} u'(e - i_n(1-\beta\theta p_u(1-\delta)))(1-\beta\theta p_u(1-\delta)) &= \beta u'(i_n^\alpha + p_u i_n(1-\delta)(1-\theta)) \times \\ &\quad (\alpha i_n^{\alpha-1} + p_u(1-\delta)(1-\theta)). \end{aligned}$$

By totally differentiating we get

$$\begin{aligned} \frac{di_n}{de} &= \frac{u''(c_0)(1-\beta\theta p_u(1-\delta))}{u''(c_0)(1-\beta\theta p_u(1-\delta))^2 + \beta u''(c_1)(f'(k) + p_u(1-\delta)(1-\theta))^2 + \beta u'(c_1)f''(k)} \\ &> 0. \end{aligned}$$

In the case where $\lambda_b = 0$ we have $\mu_0 = \beta^{-1}\mu_1$ and hence $c_0 = c_1$ and the agent is unconstrained. The first order condition with respect to i_n then implies that

$$1 = \beta(\alpha k^{\alpha-1} + p_u(1-\delta)),$$

which means that investment is constant and $\bar{\bar{k}} = (\alpha^{-1}(\beta^{-1} - p_u(1-\delta)))^{\frac{1}{\alpha-1}}$. Notice also that in a credit constrained pricing equilibrium $\bar{k} < \bar{\bar{k}}$. At the lower boundary of this region, savings b equal $\beta\theta p_u\bar{k}(1-\delta)$ and since $u'(c_0) = u'(c_1)$ we have

$$u'(e - \bar{\bar{k}}(1-\beta\theta p_u(1-\delta))) = u'(\bar{\bar{k}}^\alpha + p_u\bar{\bar{k}}(1-\delta)(1-\theta))$$

which implicitly defines $\bar{\bar{e}} = \bar{\bar{k}}(1-\beta\theta p_u(1-\delta)) + \bar{\bar{k}}^\alpha + p_u\bar{\bar{k}}(1-\delta)(1-\theta)$. In a credit constrained pricing equilibrium $\bar{e}_n < \bar{\bar{e}}$.

Since the maximizing choices are continuous functions, we conclude that the agent invests in used capital only below $\bar{e}_u$, invests in new and used capital between $\bar{e}_u$ and

$\bar{e}_n$, and in new capital only above $\bar{e}_n$. Moreover, the agent is credit constrained below $\bar{\bar{e}}$ and unconstrained above that value. $\square$

Proof of Proposition 4. Recall from equation (13) that

$$\mu_1 m_u + \lambda_n - \lambda_u = \mu_0(1 - p_u) = \mu_1 m_u + \lambda_b m_u$$

where the second equality uses the fact that $\mu_0 = \mu_1\beta^{-1} + \lambda_b\beta^{-1}$ and $p_u + \beta m_u = 1$. Hence, $\lambda_b m_u = \lambda_n - \lambda_u$. But this implies that $\lambda_u = 0$, $\forall e \in \mathcal{E}$, since λ_n and λ_u can not both be strictly positive. Moreover, $\lambda_n = 0$ if and only if $\lambda_b = 0$.

Suppose $\lambda_b > 0$ and hence $b = \beta\theta p_u i_u(1-\delta)$ and $i_n = 0$. Then, i_u solves equation (14) and, totally differentiating, we have $\frac{di_u}{de} > 0$ (see equation (15)).

Suppose $\lambda_b = 0$, and hence $\lambda_n = 0$ and $\mu_0 = \mu_1\beta^{-1}$. Equation (12) then implies that $1 = \beta(\alpha k^{\alpha-1} + p_u(1-\delta))$ which is solved by $\bar{k}$ as defined in the proposition. Now agents in this range are indifferent between investing in new and used capital at the margin. However, we can determine the minimum used capital investment that is required for given e. At the margin, investing in new capital instead of used capital is equivalent to investing in used capital and reducing borrowing by the difference $1 - p_u$. Thus, the way to obtain a capital stock of $\bar{k}$ while saving the minimum amount is by investing in used capital only. At the lower boundary of the region, the agent invest in used capital only and borrowing is $b^{max} = \beta\theta p_u\bar{k}(1 - \delta)$. Moreover, since $\lambda_b = 0$, we have

$$u'(e - p_u\bar{k}(1 - \beta\theta(1 - \delta))) = u'(\bar{k}^\alpha + p_u\bar{k}(1 - \delta)(1 - \theta) - m_u\bar{k})$$

which defines $\bar{e}_u = p_u\bar{k}(1 - \beta\theta(1 - \delta)) + \bar{k}^\alpha + p_u\bar{k}(1 - \delta)(1 - \theta) - m_u\bar{k}$. Thus, the minimum used capital investment at $\bar{e}_u$ is $i_u^{min} = \bar{k}$. Above $\bar{e}_u$, the minimum used capital investment, which implies $b^{max} = \beta\theta p_u\bar{k}(1 - \delta)$, decreases since c_1 must be increasing in e and $\frac{dc_1}{de} = -m_u\frac{di_u^{min}}{de}$. At the upper boundary of this region, the agent invests in new capital only and $b^{max} = \beta\theta p_u\bar{k}(1 - \delta)$, and

$$u'(e - \bar{k}(1 - \beta\theta p_u(1 - \delta))) = u'(\bar{k}^\alpha + p_u\bar{k}(1 - \delta)(1 - \theta))$$

which is solved by $\bar{e}_n = \bar{k}(1 - \beta\theta p_u(1 - \delta)) + \bar{k}^\alpha + p_u\bar{k}(1 - \delta)(1 - \theta) > \bar{e}_u$. Above $\bar{e}_n$, $i_u^{min} = 0$, $i_n^{max} = \bar{k}$, and $b^{max} < \beta\theta p_u\bar{k}(1 - \delta)$. $\square$

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Table 1: Parameter Values for Example Economies

Preferences	β	σ	
	0.96	2.00	
Technology	α	δ	m_u
	0.33	0.12	0.50
Collateralization Rate	θ		
	0.20		
Discretized State Space	i_u, i_n	b	
	$[0 : 0.002 : 0.5]$	$[-0.2 : 0.002 : 0,\ 0.001 : 0.001 : 0.05]$	
	Distribution of Internal Funds		
Credit Constrained Pricing Equilibrium	e	$\pi(e)$	
	$[0.05 : 0.05 : 1.75]$	$\propto [\exp(-0.05), \ldots, \exp(-1.75)]$	
Unconstrained Pricing Equilibrium	e	$\pi(e)$	
	$[0.05 : 0.05 : 1.75]$	$[1/35, \ldots, 1/35]$	

Table 2: Equilibrium Implications

Panel A: Credit Constrained Pricing Equilibrium

Price of Used Capital	p_u		
	0.5485		
Cutoff Levels of Internal Funds	$\bar{e}_u$	$\bar{e}_n$	$\bar{\bar{e}}$
	0.8469	1.2134	1.3604
Levels of Capital	$\bar{k}$	$\bar{\bar{k}}$	
	0.3914	0.4554	

Panel B: Unconstrained Pricing Equilibrium

Price of Used Capital	p_u	
	0.5200	
Cutoff Levels of Internal Funds	$\bar{e}_u$	$\bar{e}_n$
	0.8820	1.3001
Level of Capital	$\bar{k}$	
	0.4265	

Table 3: Capital Expenditures for New and Used Capital: Companies with and without Employees

This table describes the composition of capital expenditures on new and used structures and equipment for companies with and without employees. Total expenditures is the sum of structures and equipment. Category numbers represent percent of total expenditures on structures and equipment comprised by each of the four capital categories (used structures, new structures, used equipment and new equipment) for all companies, and companies with and without employees. Data is from the Annual Capital Expenditure Survey 1995-2002 published by the Bureau of the Census. Numbers marked with an asterisk appear in 1995 and 1996 for the statistics for which the cutoff between small and large firms was less or greater than five employees. In 1996, statistics were reported for cutoffs of both zero and five employees.

		Year								
		1995*	1996*	1996	1997	1998	1999	2000	2001	2002
	Total									
Used	All Companies	9.6%	7.7%	7.1%	6.3%	8.3%	5.8%	7.0%	6.0%	8.1%
	Without Employees	27.1%*	22.4%*	24.0%	24.2%	19.0%	25.9%	26.4%	30.8%	30.7%
	With Employees	5.8%*	5.0%*	4.7%	4.0%	7.4%	4.3%	5.7%	4.7%	6.1%
New	All Companies	90.4%	92.3%	92.9%	93.7%	91.7%	94.2%	93.0%	94.0%	91.9%
	Without Employees	72.9%*	77.6%*	76.0%	75.8%	81.0%	74.1%	73.6%	69.2%	69.3%
	With Employees	94.1%*	95.0%*	95.3%	96.0%	92.6%	95.7%	94.3%	95.3%	93.9%
	Structures									
Used	All Companies	3.9%	2.7%	2.5%	2.2%	4.6%	2.3%	3.0%	2.5%	4.1%
	Without Employees	11.6%*	7.2%*	7.4%	7.8%	5.8%	8.1%	8.9%	10.3%	15.0%
	With Employees	2.3%*	1.9%*	1.8%	1.4%	4.5%	1.8%	2.6%	2.1%	3.1%
New	All Companies	28.1%	29.0%	27.7%	29.2%	29.3%	28.3%	28.4%	30.3%	30.5%
	Without Employees	27.9%*	31.0%*	31.7%	29.5%	32.9%	28.2%	28.1%	20.6%	26.4%
	With Employees	28.2%*	28.6%*	27.1%	29.1%	29.0%	28.3%	28.4%	30.8%	30.9%
	Equipment									
Used	All Companies	5.7%	5.0%	4.7%	4.2%	3.7%	3.6%	4.0%	3.5%	4.0%
	Without Employees	15.5%*	15.2%*	16.7%	16.3%	13.2%	17.7%	17.5%	20.5%	15.7%
	With Employees	3.5%*	3.0%*	3.0%	2.6%	2.9%	2.5%	3.1%	2.6%	3.0%
New	All Companies	62.2%	63.3%	65.2%	64.5%	62.4%	65.9%	64.7%	63.7%	61.4%
	Without Employees	45.0%*	46.5%*	44.2%	46.3%	48.1%	45.9%	45.5%	48.6%	42.9%
	With Employees	65.9%*	66.4%*	68.1%	66.8%	63.6%	67.3%	65.9%	64.5%	63.0%

Table 4: Fraction of Trucks Purchased Used

The table shows the fraction of trucks purchased used as percentage of all trucks purchased new or used for all trucks and depending on the fleet size, which is the number of trucks and trailers operated by a truck owner for his/her entire operation. The table also shows the 25th percentile, median, and 75th percentile of the fraction of trucks purchased used across 32 different body types of trucks (e.g., pickup, panel or van, ...). Data is from the public use microdata file of the Vehicle Inventory and Use Survey of the 1997 Economic Census published by the Bureau of the Census. We report results for trucks reported as operated for business use only.

		All Trucks	By Body Type		
			25th Percentile	Median	75th Percentile
Overall		50.49%	41.98%	52.07%	67.07%
By Fleet Size	1	56.97%	64.26%	75.19%	85.57%
	2-5	56.61%	54.78%	69.31%	76.52%
	6-9	55.57%	51.92%	57.54%	71.95%
	10-24	37.96%	42.00%	49.04%	63.39%
	25-99	33.62%	32.20%	46.75%	51.45%
	100-499	24.08%	23.20%	29.89%	41.75%
	500-999	17.79%	14.60%	30.03%	41.59%
	1,000-4,999	12.77%	7.98%	13.67%	24.23%
	5,000-9,999	2.70%	2.51%	17.43%	28.42%
	10,000 or more	4.08%	4.43%	7.00%	15.67%

Figure 1: Investment in New and Used Capital in a Credit Constrained Pricing Equilibrium

Top Left Panel: Investment in new capital (dash dotted), used capital (solid), and total investment (dotted) as a function of the amount of internal funds. Middle Left Panel: Investment in used capital as percentage of total investment. Bottom Left Panel: Agent specific shadow price of new capital (dotted) and used capital (solid). Top Right Panel: Borrowing. Middle Right Panel: Multiplier on the borrowing constraint $\lambda_b(e)$ (normalized by the marginal utility of consumption at time 0) as a function of the amount of internal funds.

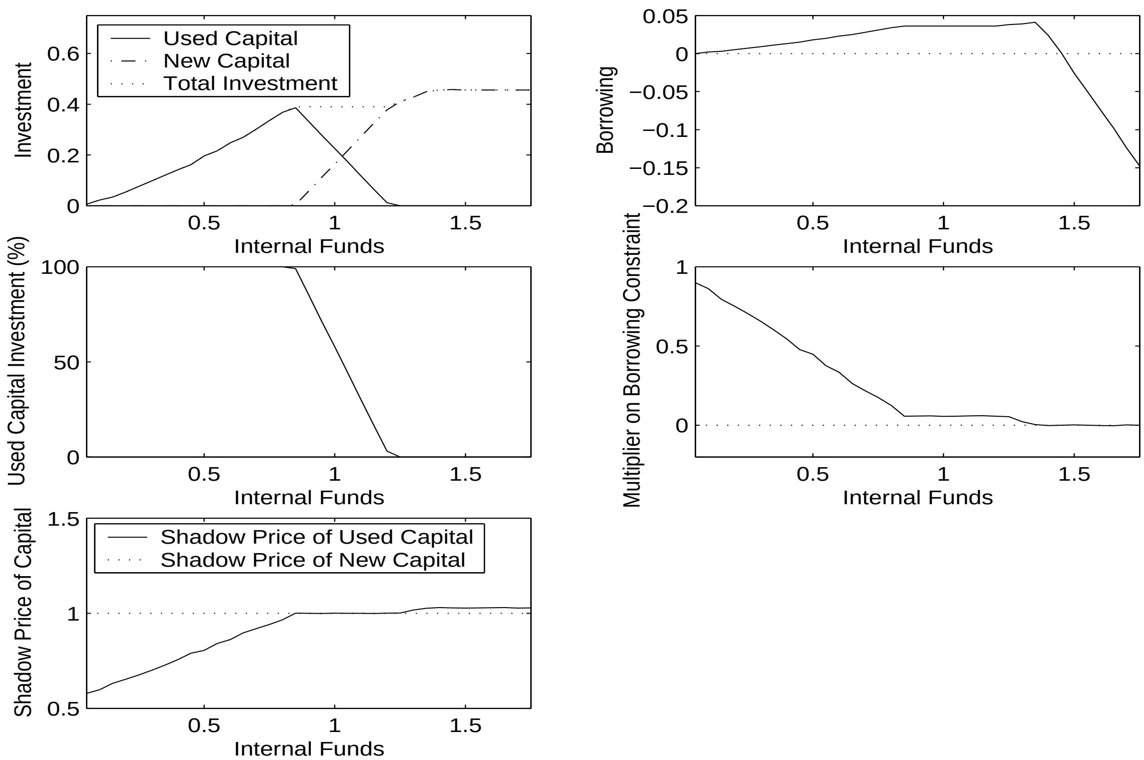

Figure 2: Investment in New and Used Capital in an Unconstrained Pricing Equilibrium

Top Left Panel: Maximum investment in new capital (dash dotted), minimum investment in used capital (solid), and total investment (dotted) as a function of the amount of internal funds. Middle Left Panel: Minimum investment in used capital as percentage of total investment. Bottom Left Panel: Agent specific shadow price of new capital (dotted) and used capital (solid). Top Right Panel: Maximum borrowing. Middle Right: Multiplier on the borrowing constraint $\lambda_b(e)$ (normalized by the marginal utility of consumption at time 0) as a function of the amount of internal funds.

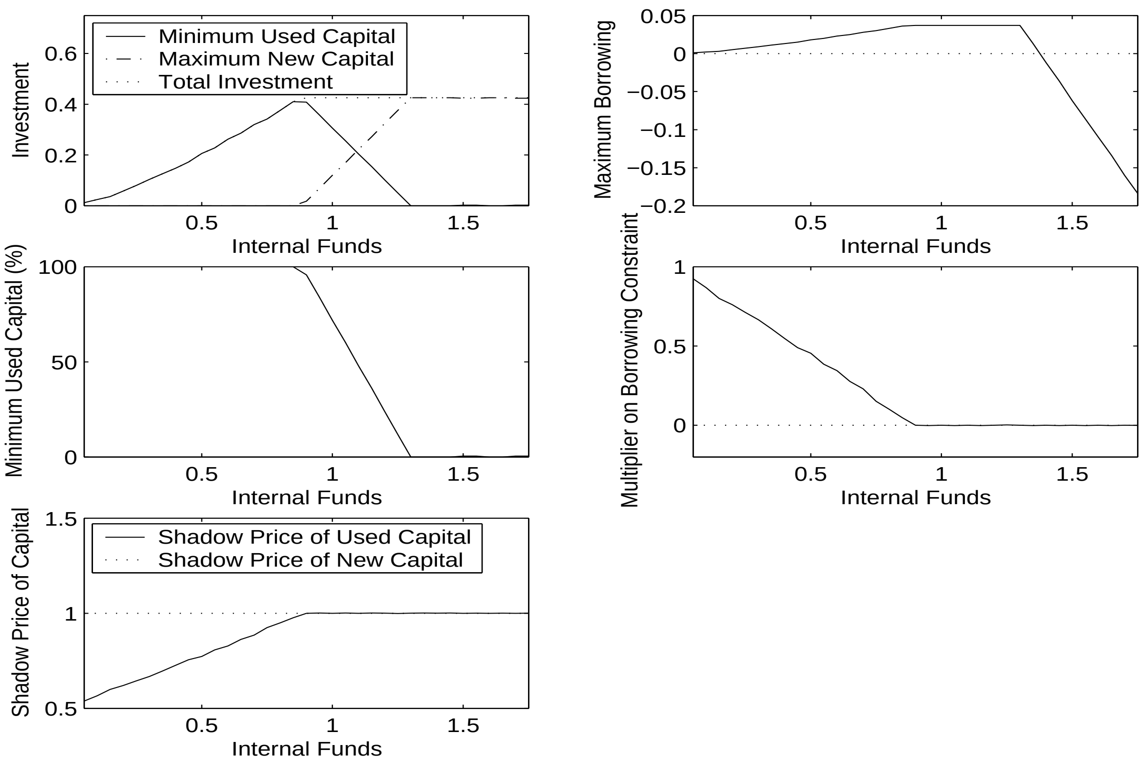

Figure 3: Fraction of Trucks Purchased Used versus Fleet Size

Plotted series are the fraction of trucks purchased used as percentage of all trucks purchased new or used graphed against the natural logarithm of the fleet size, which is the number of trucks and trailers operated by a truck owner for his/her entire operation. We use the midpoint of the reporting category as the fleet size. Solid bold line is the fraction of all trucks purchased used and dotted lines are fraction of trucks purchased used for 32 different body types of trucks (e.g., pickup, panel or van, ...). Data is from the public use microdata file of the Vehicle Inventory and Use Survey of the 1997 Economic Census published by the Bureau of the Census. We report results for trucks reported as operated for business use only.