Inattentive Consumers

Ricardo Reis*
Harvard University

December 2003

JOB MARKET PAPER

Abstract

This paper studies the consumption decisions of agents who face costs of acquiring, absorbing and processing information. These consumers rationally choose to only sporadically update their information and re-compute their optimal consumption plans. In between updating dates, they remain inattentive. This behavior implies that news disperses slowly throughout the population, so events have a gradual and delayed effect on aggregate consumption. The model predicts that aggregate consumption adjusts slowly to shocks and is excessively sensitive and excessively smooth relative to income. In addition, individual consumption is sensitive to small and unexpected past news, but it is not sensitive to large and predictable events. The model further predicts that some people rationally choose to not plan, live hand-to-mouth, and save less, while other people make plans. The longer are these plans, the more they save. Evidence using U.S. data supports these predictions. Finally, this paper justifies the existence of information and planning costs using a model from computer science, and contrasts the inattentiveness framework with other models of consumption and bounded rationality.

*I am extremely grateful to N. Gregory Mankiw for mentoring my graduate studies, and to Alberto Alesina, Robert Barro, and David Laibson for their constant intellectual stimulation and priceless advice. Susanto Basu, John Campbell, Mariana Colacelli, Benjamin Friedman, Jens Hilscher, Yves Nosbusch, David Romer, Monica Singhal, Adam Szeidl, and Bryce Ward offered useful comments on this paper. The Fundacao Ciencia e Tecnologia, Praxis XXI and the Eliot Memorial fellowship provided financial support. Contact: reis@fas.harvard.edu.

"Attention as the Scarce Resource. [...] Many of the central issues of our time are questions of how we use limited information and limited computational ability to deal with enormous problems whose shape we barely understand."

Herbert A. Simon (1978, page 13)

"Perhaps it is not surprising that many people do not report an expectation given the costs of it."

Sherwin Rosen (1990, page 284)

1 Introduction

Most economists would agree that a rational consumer sets the marginal utility of consuming in the present equal to the discounted marginal utility of consuming in the future times the price of present relative to future consumption. If the future is uncertain, it is expected marginal utility that is relevant, and a crucial component of a model of consumption specifies how agents form expectations. In a pioneering contribution, Hall (1978) assumes that agents form expectations rationally in the Muth-Lucas sense: they have full information on the structure of the economy and use this as their probability model to form expectations in a statistically optimal way. Rational expectations leads to the prediction that consumption should be a martingale: consumption growth should not be predictable over time. Hall's finding that post-war U.S. aggregate consumption is approximately consistent with this prediction was an early success of rational expectations modelling.

Over the past 25 years though, many papers have found problems with the Hall model. Deviations of aggregate consumption from a martingale in the data have been convincingly established, taking the form of either excess sensitivity of consumption to past known information, or excess smoothness in response to permanent income shocks.[1] Campbell and Mankiw (1989, 1990) illustrate these failures by showing that if the world is partially populated by rational expectations agents, then there must be as many irrational consumers who consume their current income every period, in order to match the data on aggregate consumption.

This paper revisits the modelling of expectation formation by consumers. With rational expectations, agents have an unbounded ability to absorb and process information on all the relevant characteristics of the economy, and an unbounded ability to think through this information and calculate optimal forecasts and actions. I assume instead that it is costly for agents to acquire, absorb, and process information in forming expectations and making decisions. In a dynamic setting, while agents with rational expectations undertake these costly activities at every instant in time, in this paper agents rationally choose to update their information and plans infrequently: Expectations are rational, but are only sporadically updated. Following a new event, many agents will be unaware

[1] Consumption is excessively sensitive (Flavin, 1981) if future consumption growth depends on lagged information. It is excessively smooth (Deaton, 1987) if it does not respond one-to-one to shocks to permanent income, and thus is smoother than permanent income.

of the news for a while, and will continue following their outdated plans, only eventually updating their expectations. Agents are inattentive and the information in the economy is sticky, gradually dissipating over time to the entire population. Consumption in turn is excessively sensitive, since when agents adjust plans and consumption, they react to all the information (present and past) since their last adjustment date. Consumption is also excessively smooth, since only a fraction of agents are attentive when there is a shock to permanent income and react to it instantly.

Beyond generating predictions that match the data on individual and aggregate consumption, a further contribution of this paper is that it provides a micro-foundation for time-contingent adjustment rules. Because inattentiveness emphasizes the costs of observing and processing the state of the economy, it naturally justifies agents adjusting to news at certain dates regardless of the state of the economy at those dates. However, when she plans, the inattentive agent optimally decides when to plan again, taking into account the state of the world at the *current* planning date. In traditional models with time-contingent adjustment rules, the adjustment dates are set regardless of the state of the world at *any* date. The inattentiveness model therefore implies recursively time-contingent adjustments, independent of the state of the economy at that date, but dependent on the state of the economy at the last adjustment date. In some special cases, this reduces to the standard time-contingent adjustment model, but even when it does not, the model retains the tractability that has made time-contingent adjustment so popular in the literature.

A few papers have recently explored the potential of modelling inattentiveness. Gabaix and Laibson (2001) assume that investors update their portfolio decisions infrequently, and show that this can explain the puzzling premium of equity over bond returns. Mankiw and Reis (2002, 2003) study inattentiveness on the part of price-setting firms and show that the resulting model of the Phillips curve matches well the dynamics of inflation and output that we observe in the data. Relative to these papers, this paper differs by focusing on consumption decisions and deriving predictions for individual and aggregate consumption, which are empirically tested. Moreover, I do not assume that agents infrequently adjust their plans, but rather I derive this behavior as the optimal response to explicitly modelled costs of planning.

Recent empirical work using microeconomic data has also emphasized that most people are inattentive and that this affects their behavior. Lusardi (1999, 2002) and Ameriks, Caplin, and Leahy (2003a) find that a significant fraction of survey respondents make financial plans infrequently (if at all) and that their planning behavior has a statistically significant and sizeable effect on the amount of wealth they have accumulated. This paper contributes to this literature a theoretical model of costly and infrequent planning. Inattentiveness rationalizes these authors' findings and suggests further implications to test using observations of individual behavior.

This paper is composed of three parts. The first part presents the inattentiveness theory. It starts in Section 2 by presenting the main ideas in a simple model, in order to highlight the intuition behind the results. Section 3 rigorously sets up the problem of an agent facing costs of planning, and derives the optimality conditions describing consumption and planning behavior. It aggregates individual consumption decisions over many such agents to obtain the predictions of the model for the time-series of aggregate consumption, which will later be tested in the data. Section 4 solves the inattentive agent's problem analytically for a particular specification of preferences and uncertainty. This provides further implications and intuition on the effects of costly planning on savings and optimal inattentiveness. Next, I show that the inattentiveness model predicts that if the agent's costs of planning are above a certain threshold, she rationally chooses to never plan, and to live hand-to-mouth, consuming her income every period less a pre-determined amount. The theory section concludes by examining the response of an inattentive agent to "extraordinary events," which occasionally induce large changes in the environment she faces.

In the second part of the paper, I test the implications of the model. In Section 5, I use U.S. aggregate consumption data. I examine whether these data exhibit the slow adjustment and the stickiness of information that the model predicts, and I show that the model can generate the extent of sensitivity and smoothness with respect to income that we observe. Furthermore, I show that the data favors the inattentiveness model over the model of Campbell and Mankiw (1989, 1990). Section 6 discusses studies which used microeconomic data, and shows that the inattentiveness model matches the existing evidence on the sensitivity of individual consumption to past information, the expectations of individuals, and their planning and savings behavior.

The third part of the paper contrasts the inattentiveness model with alternative existing models. Section 7 starts by showing that one specific model of computation widely used in computer science can justify the assumption made on the costs of planning. This leads to a discussion of another model, proposed by Sims (2003), that also limits agents' attention but uses a different framework from electrical engineering. Section 8 compares this paper with other models of consumption and bounded rationality.

Section 9 concludes by collecting the many theoretical results and empirical estimates in the paper into a coherent description of individual and aggregate consumption in the United States. It also discusses directions for future research on inattentiveness.

2 A simple model of inattentiveness

Consider the problem of an agent living forever in discrete time who consumes c_t each period, obtaining utility given by the function $u(c_t)$, which is of the constant absolute risk aversion (CARA) form. This agent discounts future utility by the factor β, and each period she receives stochastic income y_t, which is normally distributed with mean $\bar{y}$ and variance σ^2. She earns returns on her assets, a_t, at the gross interest rate R which equals $1/\beta$, so the subjective discount rate equals the net real interest rate.

Despite being fully rational and making optimal choices, the consumer must pay a monetary cost K whenever she acquires information and makes optimal decisions. This can be thought of as the cost in money and time of obtaining information, processing and interpreting it, and deciding how to optimally act. Facing this cost, the agent must then choose when to plan. Her decision dates are denoted by $D(i)$, and for any t between $D(i-1)$ and $D(i)$ the consumer follows the plan set at time $D(i-1)$. The problem of the agent therefore is:

$$\begin{aligned} V(a_0) &= \max_{\{c_t,s_t\}_{t=0}^{\infty},\{D(i)\}_{i=1}^{\infty}} E\left[\sum_{t=0}^{\infty}\beta^t\left(-\frac{e^{-\alpha c_t}}{\alpha}\right)\right] \\ a_{t+1} &= Ra_t - c_t + y_t - K\iota(t), \end{aligned}$$

where $\iota(t)$ is an indicator function that equals 1 if $t = D(i)$ and is zero otherwise. Note that the agent can choose either consumption or savings $s_t = y_t - c_t$. For now, I focus on picking optimal consumption, but the savings alternative will become relevant later in this Section.

The first-order condition with respect to consumption between two periods t and $t+s$ which are in between planning dates $(D(i-1) < t < t+s < D(i))$ implies:

$$c_t = c_{t+s}. \tag{1}$$

With inattentiveness, consumption stays constant, or more generally, *consumption follows a predetermined path in between planning dates.*[2] Since the consumer does not update her information between t and $t+s$, consumption evolves deterministically between these two periods. The first-order condition with respect to consumption at two planning periods is:

$$e^{-\alpha c_{D(i)}} = E_{D(i)}\left[e^{-\alpha c_{D(i+1)}}\right]. \tag{2}$$

Consumption at planning dates is determined by a stochastic Euler equation. This is of the same form as in the problem without planning costs. Yet now it holds only between two planning dates rather than always, since only at these dates is new information observed by the consumer.

To proceed further, I guess that optimal consumption at adjustment dates is linear in the level of wealth $c_{D(i)} = A + Bw_{D(i)}$. Wealth includes not only her financial assets but also her human capital which includes current and expected future labor income: $w_t = a_t + (y_t - \bar{y})/R + \bar{y}/(R-1)$. The coefficients A and B are to be determined. Iterating on the budget constraint between time 0 and the first decision date D:

$$w_D = R^D w_0 - c_0\frac{1-R^D}{1-R} + \sum_{j=0}^{D-1} R^{j-1}\left(y_{D-j} - \bar{y}\right) - K. \tag{3}$$

Since income is normally distributed, then so will be wealth, and given the guess that consumption is linear in wealth, consumption is also normally distributed. Then, $\exp(-\alpha c_D)$ is log-normally distributed so the log of its expectation equals $-\alpha E_0[c_D] + (\alpha^2/2)Var_0[c_D]$. Calculating these

[2] In the psychology literature, Bargh and Chartrand (1999) describe this as "the unbearable automaticity of being."

moments of consumption using (3), and simplifying the first-order condition (2) to solve for the unknowns in the consumption function, gives:

$$c_t = (R-1)w_D - \frac{(R-1)K}{R^D - 1} - \frac{\alpha\sigma^2}{2R^2}\frac{(R-1)\left(R^D+1\right)}{R+1}$$

If she faces higher planning costs, *the agent plans less often and saves more.* The longer the agent remains inattentive, the larger is her exposure to risk, since she is not reacting to shocks as they occur. This larger risk leads in turn to higher precautionary savings as the agent saves to safeguard herself against a sequence of bad income shocks.

Given the solution for consumption, the value function of the agent is then given by:

$$V(a_0) = \max_D \left\{ -\frac{R}{\alpha(R-1)} e^{-\alpha A - \alpha(R-1)w_0} \right\},$$

Performing the maximization with respect to D gives:

$$\hat{D} = \frac{\ln\left(1 + \sqrt{\frac{2R^2(R+1)K}{\alpha\sigma^2}}\right)}{\ln(R)}. \tag{4}$$

The optimal discrete time inattentiveness interval, D^*, is the integer just before or after $\hat{D}$ that yields the higher value. Equation (4) shows that if the costs of planning are close to zero, $\hat{D}$ is of order $\sqrt{K}$, so *second-order costs of planning lead to first-order periods of inattentiveness.* An agent with even very small costs of planning can be inattentive for a long time since her behavior is close to the full information behavior, so the utility losses from inattentiveness are small. Equation (4) also shows that she will be *inattentive for longer the lower is risk aversion and the lower is income volatility.* The lower these are, the smaller is the effective cost of being inattentive driven by her exposure to risk, and thus the less frequently she adjusts her plans. Moreover *the larger is the interest rate* R, *the shorter the inattentiveness.* While the agent is inattentive, she is not adjusting her savings optimally. The larger the interest rate, the larger the impact of these inefficient savings on her future asset position. Facing a large interest rate, the agent will choose to update more often to avoid her asset position becoming severely sub-optimal.

So far, I have been solving the problem of an agent who chooses consumption plans. Yet, she could instead set plans for her savings. If the agent has full information or if there is no income uncertainty, then the two are indistinguishable. But if the agent is not monitoring her income every instant, she must choose to either set a plan for consumption and let savings adjust to the shocks, or to set a plan for savings and let consumption adjust.

An inattentive saver has $c_t = y_t - s_t$, so since s_t follows a pre-determined path, *the inattentive saver behaves like a hand-to-mouth consumer*, every period consuming her income up to a pre-determined amount. As long as K is not too small, *the inattentive saver rationally chooses to never plan*, every period just consuming her income less a constant amount. To see this, consider an agent who at some period decides not to update her plans. Since assets evolve deterministically and income is white noise, note that in the next period the agent is facing exactly the same problem and thus she must again choose not to update her plans. Iterating on this logic to infinity shows that the saver will either always be attentive, or never update her plans. If K is not too small, she will choose to never plan.

The inattentive consumer is worse off the larger is K, whereas the inattentive saver never plans and so is unaffected by this cost. Therefore, the agent will only choose savings plans if the costs of planning are large enough. This gives the following characterization of behavior in an inattentive economy: some agents have high costs of planning and optimally choose to live hand-to-mouth consuming their income every period and never making plans. The other agents, who have lower planning costs, opt instead for having consumption follow pre-determined plans. Within this group of planners, the lower are the planning costs, the more frequently they update plans and the less they save.

In an economy populated by many inattentive consumers, their consumption decisions can be aggregated to obtain implications for aggregate consumption. This requires a description of how agents differ in the economy, and for now I make the simplest assumption: I assume that all agents are identical so all choose the same D^*, but they are uniformly staggered with respect to their planning dates.[3] Between two successive periods, a fraction $(D^* - 1)/D^*$ of agents will not change their consumption, while a fraction $1/D^*$ updates consumption responding to the information that arrived over the last D^* periods. If C_{t+1} denotes aggregate consumption:

$$C_{t+1} - C_t = \frac{\mu}{D^*} e_{t+1} + \frac{\mu}{D^*} e_t + ... + \frac{\mu}{D^*} e_{t-D^*+1}, \tag{5}$$

where e_t denotes the information that arrived at period t, and μ is the marginal propensity to consume out of that information. Since income is a relevant piece of information, equation (5) shows that *the change in aggregate consumption is sensitive to income up to D^* periods earlier.* If the inattentiveness model describes the data, then conventional tests of excess sensitivity will find that consumption is excessively sensitive to past income, but only up to D^* periods ago. Moreover, since only a fraction of the agents react contemporaneously to changes in income, *consumption will be smoother than income.* From the perspective of the Hall model, consumption will be excessively smooth.

Equation (5) also shows that the change in consumption should be a moving average process of order D^*. *Inattentive aggregate consumption adjusts slowly to shocks*, with a reaction that builds up over time. Given the arrival of a piece of news, only a few agents will be attentive and react instantly to it. The remaining consumers gradually update their plans and adjust consumption so *information disseminates slowly*, and affects aggregate consumption gradually over time.

Finally, a consumer responds to present and past shocks only if she could not predict them

[3] Section 3 allows for a general distribution of individual characteristics and optimal planning dates.

when she last planned. *Past predictable events do not affect present individual consumption changes.* Moreover, if some variables only move infrequently, so the cost of monitoring them is very small, and if movement in these variables leads to large changes in the agent's income, she will pay attention to these variables and react to them instantly. For instance, if the agent suddenly becomes unemployed or wins the lottery, she responds immediately to these easily observed and significant shocks. *Past infrequent and large events do not affect present individual consumption changes.*

Summarized and simplified, these are the main results from the theory of inattentive consumption. The next two Sections set up and solves the problem of the inattentive consumer more rigorously deriving a set of predictions that Sections 5 and 6 test with the data.

3 The formal inattentiveness model

3.1 The set-up of the problem

I model the problem of the inattentive consumer in continuous time, so that the planning dates are chosen from a continuous set. Time is indexed by t on the positive real line while the decision periods are denoted by $D(i)$ where $i \in \mathbb{N}_0$ orders the decision times so that $D(i+1) \geq D(i)$ for all i with $D(0) \equiv 0$. If $d(i)$ denotes the time until the next adjustment, defined recursively as $d(i) = D(i) - D(i-1)$, it is clearly equivalent for the agent to choose the calendar dates of planning, $D(i)$, or the inattentiveness intervals, $d(i)$.

The economy is populated by many infinitely-lived consumers, which can be interpreted as the result of altruistic links between generations. Each instant, the agent consumes an amount of goods c_t, which yields an amount of utility given by the function $u(c_t)$. This function is assumed to be continuous, everywhere twice differentiable, increasing and concave. Future utility is discounted at the rate ρ, either due to impatience or because the current generation discounts the well-being of future generations relative to its own.

Each instant, the agent receives an income flow $y(x)$, and her assets a_t earn returns at the interest rate r. The flow budget constraint is $da_t = (ra_t - c_t + y(x_t))dt$, stating that at each instant, assets increase by the interest earned plus new savings. Borrowing is constrained by the condition that all debts must be repaid, so the agent cannot run Ponzi schemes rolling over debt forever: $\lim_{t\to\infty} e^{-rt} a_t \geq 0$. Income is a function of a state vector x_t which is generated by a continuous time stochastic process, defined on a standard filtered probability space $\{X, F, P\}$ where X is the set of possible states, F is the filtration $F = \{F_t, t \geq 0\}$ where F_t is the σ-algebra through which information on x_t is revealed, and P is the probability measure on F. I will write $y(x_t)$ more compactly as y_t. The notation $E_k[.]$ will be used to denote the expectation conditional on information up until time k: $E_k[y_t] = \int y_t dP(F_k)$. I further assume that the state vector has the Markov property, and, without loss of generality, that it is arranged in such a way that it is first-order Markov. Therefore, a sufficient statistic for the probability of any state $y_t \in Y$ from the perspective of time $k < t$ is the state vector at time k: $P(y_t \mid F_k) = P(y_t \mid x_k)$.

The consumer's choice of planning dates defines a new filtration $\Im = \{\Im_t, t \geq 0\}$ such that $\Im_t = F_{D(i)}$ for $t \in [D(i), D(i+1))$. When the consumer writes a plan at time $D(i-1)$, she decides on a consumption sequence until the next adjustment, $c^i = [c_{D(i-1)}, c_{D(i)})$, and on when to plan again, $D(i)$. The restriction embodied in the existence of a plan is that both of these must be contingent on the information available at time $D(i-1)$: If $\{c, D\} \equiv \{c^i, D(i)\}_{i=1}^{\infty}$, then both c and D must be $\Im$-adapted processes.

Whenever she plans, the consumer incurs a fixed monetary cost given by $K_t \equiv K(x_t)$. This cost will be further discussed in Section 7. It is a function of the state vector x_t, so it can be stochastic and time-varying. If the consumer enters period $D(i)$ with assets given by $a^-_{D(i)}$, her wealth then changes discontinuously to $a^+_{D(i)} = a^-_{D(i)} - K_{D(i)}$.[4] Formally, $a^-_{D(i)}$ is the left-hand side time limit of assets, while $a^+_{D(i)}$ is the right-hand side limit, and they differ by the fixed cost $K_{D(i)}$.

The problem of the consumer can then be compactly written as:

$$\max_{\{c,D\}} E_0 \left[\sum_{i=0}^{\infty} \int_{D(i)}^{D(i+1)} e^{-\rho t} u(c_t) dt \right] \tag{6}$$

$$\text{s.t.} \quad : \quad \{c, D\} \text{ are } \Im\text{-adapted}, \tag{7}$$

$$da_t = (ra_t - c_t + y_t)dt, \tag{8}$$

$$a^+_{D(i)} = a^-_{D(i)} - K_{D(i)}, \text{for all } i \in \mathbb{N}_0, \tag{9}$$

$$\lim_{t \to \infty} e^{-rt} a_t \geq 0, \tag{10}$$

with initial conditions a_0, x_0. It is difficult to solve this problem both because it is hard to impose the measurability restriction (7) and because of the discontinuity in the level of assets at the planning dates (9). To make progress, the problem must be re-stated in a more convenient form.

Start by integrating the law of motion for assets (8) between $D(i)$ and $D(i+1)$, and replace $a^-_{D(i)}$ by $a^+_{D(i)} + K_{D(i)}$, using (9). This gives:

$$a^+_{D(i+1)} = e^{rd(i)} \left(a^+_{D(i)} - \int_0^{d(i)} e^{-rt} c_{D(i)+t} dt + \int_0^{d(i)} e^{-rt} y_{D(i)+t} dt \right) - K_{D(i+1)},$$

thus eliminating the a^-_t variables, so that only a^+_t's are left. Moreover, realize that there is a recursive structure between planning dates so the cumbersome time indices can be dropped by denoting $a^+_{D(i)}$ by a and $a^+_{D(i+1)}$ by a', and similarly $x_{D(i)}$ by x and $x_{D(i+1)}$ by x'. Next, let $V(a, x)$ be the value function associated with this problem. The state vector for this problem is (a, x) since the law of motion for assets and the Markov assumption for the state vector imply that (a, x) is a sufficient statistic for the uncertainty facing the agent until the next planning date.

[4] Implicit in this setup is the assumption that while it is costly to re-write new plans, this can be done in an instant of time. I could assume instead that it takes a fixed interval of time to figure out a plan. While this would require some modifications to the analysis that follows, it would not affect the main conclusions.

With these changes, the problem in (6)-(10) becomes:

$$V(a,x) = \max_{c,d} \int_0^d e^{-\rho t} u(c_t) dt + e^{-\rho d} E\left[V(a', x')\right] \tag{11}$$

$$\text{subject to } a' = e^{rd}\left(a - \int_0^d e^{-rt} c_t dt + \int_0^d e^{-rt} y_t dt\right) - K'. \tag{12}$$

The measurability constraints are imposed by having passed the expectations operator through $\{c, d\}$, so that these choices are made conditional only on the information in (a, x). The only unknown at this planning date is what assets and accumulated income will be by the next planning date. As for the initial conditions, note that since there is planning at time 0, a cost K is incurred at this date so the initial post-planning asset level is $a_0 - K_0$.

The solution to the problem in (11)-(12) will be a pair of functions, $c_t(a, x)$ and $d(a, x)$, determining optimal consumption from time 0 to time d and when the next planning will take place. Consumption at any date between 0 and d is inattentive since it is chosen regardless of the state of the world at that date. In turn, the date of the next adjustment does not depend on the state at that date – adjustment is not state-contingent. However, adjustment is also not purely time-contingent, since the date of the next adjustment depends on the state of the world at the last adjustment. For lack of better words, I describe adjustment with inattentiveness as *recursively time-contingent*: it occurs at a pre-set date which depends recursively on the state at the past planning date. In some cases, $d(a, x)$ is independent of (a, x) in which case the inattentiveness model leads to purely time-contingent adjustment.

3.2 Characterizing the solution

Because (11)-(12) is a familiar dynamic programming problem, familiar optimality conditions characterize the solution. Taking the derivative of (11) with respect to d and setting it equal to zero gives:

$$u(c_d) = \rho E\left[V(a', x')\right] - \frac{\partial}{\partial d} E\left[V(a', x')\right].$$

This first-order condition states that the agent plans to adjust when the marginal cost of adjusting equals the marginal benefit of doing so. On the left-hand side is the marginal cost of adjusting, which is the utility the agent would get if she kept to her outdated consumption plan. On the right-hand side is the marginal benefit of adjusting at time d. The first term is the present flow value of having re-planned and obtained new information, while the second term is the benefit from acquiring this information at d rather than in the next instant when this value has fallen. The cost K enters the first-order condition on the right-hand side by lowering the benefits of planning through the fall in assets by K to a' at the planning date.

The first-order conditions with respect to c_t are:

$$u'(c_t) = e^{(r-\rho)(d-t)} E\left[V_a(a', x')\right], \text{ for } t \in [0, d), \tag{13}$$

where $u'(.)$ is the first derivative of the utility function and $V_a(.)$ is the derivative of the value function with respect to its first argument. Using the fact that $e^{(r-\rho)d}E\left[V_a(a', x')\right]$ is independent of time, take logs and derivatives with respect to time of equation (13) to find that for $t \in [0, d)$ optimality requires that:

$$\frac{du'(c_t)/dt}{u'(c_t)} = -(r - \rho).$$

This is the famous Ramsey (1928) rule. The rate of change of the marginal utility of consumption equals the gap between the agent's impatience and the riskless rate of return.

A third optimality condition comes from the envelope theorem:

$$V_a(a, x) = e^{(r-\rho)d}E\left[V_a(a', x')\right]. \tag{14}$$

Combining (13) and (14) to substitute out the value function for the utility functions gives:

$$u'(c_0) = e^{(r-\rho)d}E\left[u'(c_d)\right].$$

This is also a familiar expression. It is the stochastic Euler equation that arises in the study of consumption under uncertainty. At time 0, a dollar can be used to consume goods yielding $u'(c_0)$ units of utility, or instead it can be saved returning e^{rd} dollars in d periods, which can be used to consume goods at d giving $e^{-\rho d}u'(c_d)$ units of utility in time 0 utility units. Optimal behavior requires that these two uses of funds give the same benefit.

The dynamics of inattentive consumption over time are therefore simple to describe. During the intervals of inattentiveness, consumption evolves just like in the standard consumer problem with certainty. At adjustment dates, consumption evolves just like in the standard consumer problem with uncertainty. Intuitively, between adjustments the agent is not receiving new information so it is as if there is no uncertainty; at adjustments, information is revealed and her optimal choices incorporate it.

Proposition 1 *If the consumer is inattentive between times t and $s > t$, consumption between these periods obeys the deterministic Euler equation:*

$$u'(c_t) = e^{(r-\rho)(s-t)}u'(c_s). \tag{15}$$

If $D(t)$ and $D(s)$ are two planning dates, consumption between these periods obeys the stochastic Euler equation:

$$u'(c_{D(t)}) = e^{(r-\rho)(D(s)-D(t))}E_{D(t)}\left[u'(c_{D(s)})\right]. \tag{16}$$

A final optimality condition is the transversality condition, which requires that present value of a unit of assets at infinity must be zero, for otherwise the agent could have used it to increase consumption and utility:

$$\lim_{t\to\infty}\left[e^{-\rho t}V_a(a_t,x_t)a_t\right]=0. \tag{17}$$

3.3 Aggregate consumption

The economy is populated by many inattentive agents, whose individual behavior is described in Proposition 1. The different consumers have the same preferences but differ for instance in their realization of income shocks and in the costs of planning they face. They therefore differ in how long they stay inattentive and in how much they consume. Obtaining predictions for aggregate consumption is complicated by the non-linearities of the marginal utility function. Following the literature, I work instead with linearized versions of (15) and (16).[5] A first-order Taylor approximation of (15) around the point where $c_t = c_s$ and $r = \rho$ gives:

$$c_s = c_t + \frac{1}{\alpha}(r-\rho)t, \tag{18}$$

where $\alpha = -u''(c_t)/u'(c_t)$ is the coefficient of absolute risk aversion. A similar approximation of (16) leads to:

$$c_{D(s)} = c_{D(t)} + \frac{1}{\alpha}(r-\rho)t + e_{D(s),D(t)}, \tag{19}$$

where $e_{D(s),D(t)} \equiv c_{D(s)} - E_{D(t)}\left[c_{D(s)}\right]$, the innovation to consumption between $D(t)$ and $D(s)$.

Some form of indexing must be defined to keep track of the different agents. The following indexing turns out to be convenient: let j denote how long, starting from $t+1$, one must go back to find the last date when the agent has adjusted, with j lying in the interval $[0, J]$. Similarly, let $i \in [0, I]$ denote how long, starting from t, one must go back to the last adjustment date for that same agent. The change in consumption of an individual agent is denoted by $c_{t+1}(i,j) - c_t(i,j)$. For instance, $c_{t+1}(4, 0.75) - c_t(4, 0.75)$ is the change in the consumption of an agent whose last two adjustments were at $t + 0.25$ and at $t - 4$.

The population of consumers between two time periods always divides itself between two groups. On the one hand, there are the consumers with $j \geq 1$ (and so for whom $i + 1 = j$), which account for a fraction $\bar{\Psi}$ of aggregate consumption. These agents have not adjusted their consumption plans between t and $t + 1$, so using equation (18):

$$c_{t+1}(i,j) - c_t(i,j) = \frac{1}{\alpha}(r-\rho).$$

On the other hand, there is a $(1 - \bar{\Psi})$ fraction of agents who have adjusted at some time between t and $t + 1$ and so for whom $j < 1$. Equation (19) describes the consumption choices of these consumers between $t - i$ and $t + 1 - j$. Since they have not adjusted between $t + 1 - j$ and $t + 1$, equation (18) describes the change in consumption between these two periods. Likewise the

[5] This is not to say that these non-linearities are not important. Attanasio and Weber (1995) argue that they can significantly affect tests of the Hall model.

definition of i implies that equation (18) holds between $t-i$ and t. Combining these three equations gives the relation between consumption at t and $t+1$ for these agents:

$$c_{t+1}(i,j) - c_t(i,j) = \frac{1}{\alpha}(r-\rho) + e_{t+1-j,t-i}.$$

Summing over the two groups of consumers gives aggregate consumption:

$$C_{t+1} - C_t = \text{constant} + (1-\bar{\Psi}) \int_{j=0}^{1} \int_{i=0}^{I} e_{t+1-j,t-i} d\Psi(i,j), \tag{20}$$

where $C_{t+1} - C_t = \bar{\Psi}(r-\rho)/\alpha + (1-\bar{\Psi}) \int_{j=0}^{1} \int_{i=0}^{I} \left[c_{t+1}(i,j) - c_t(i,j)\right] d\Psi(i,j)$, and $\Psi(i,j)$ is the cumulative density function over the consumers for whom $j \in [0,1)$ and $i \in [0,I]$. I assume that this distribution takes only finitely many values, which matches the fact that there are a finite number of people in the world.

If $u_{t+1} \equiv (1-\bar{\Psi}) \int \int e_{t+1-j,t-i} d\Psi(i,j)$ is treated as the error term in a linear regression for consumption growth, the model predicts that $E_{t-I}\left[u_{t+1}\right] = 0$: Consumption growth is unpredictable from the perspective of $t-I$ information. In the full information case ($J = I = 0$), Hall (1978) first derived the implication that any variable dated t or before should not predict consumption growth between t and $t+1$. With inattentive agents, events between $t-I$ and t predict consumption growth, since some consumers who had been inattentive will update their information and plans between times t and $t+1$ and will only then react to the past events.

Proposition 2 *With inattentiveness, aggregate consumption growth between t and $t+1$ should be unpredictable from the perspective of $t-I$ information, where I is the largest amount of time during which agents remain inattentive.*

Breaking the $e_{t+1-j,t-i}$ news into independent increments and assuming that these are homoskedastic, Appendix A shows that:

Proposition 3 *With inattentiveness, aggregate consumption growth can be written as:*

$$C_{t+1} - C_t = \Phi(0)e_{t+1} + \Phi(1)e_t + ... + \Phi(I)e_{t-I+1}, \tag{21}$$

with $\Phi(s) \geq \Phi(s+1) \geq 0$ for $s = 1, 2, ..., I$, while $E_{t-s}\left[e_{t+1-s}\right] = 0$ defines the innovations.

It is appropriate to call the e_t's "news" since they are mutually uncorrelated and are unpredictable one period ahead. The $\Phi(s)$'s correspond approximately to the share of agents in the population that update their information between t and $t+1$ and had last done so at or before $t-s$. Thus, they are non-increasing in s.

Equation (21) reveals another implication of the model for aggregate consumption. When news arrives, consumption rises immediately by $\Phi(0)$. The following period, consumption rises further but now by the smaller amount $\Phi(1)$, and the following period it rises further by the even smaller amount $\Phi(2)$, and so on until I periods after. Aggregate consumption is thus characterized by slow adjustment. It reacts slowly and gradually to shocks, so the impulse response of aggregate consumption to a shock should be increasing for a few periods. Moreover, the impulse response should be concave since the change in consumption gets smaller over time. In contrast, with full information, consumption responds immediately to the news ($\Phi(0) = 1$ and $\Phi(s) = 0$ for $s \geq 1$), since all agents are attentive and so react immediately. A related implication of equation (21) is that consumption growth depends on past news with more recent news receiving a larger weight than older news does. Information disseminates slowly in the inattentive economy, as news gradually spreads and has an impact on consumption choices. Combining these two results:

Proposition 4 *The inattentiveness model predicts that aggregate consumption exhibits:*
a) Slow adjustment - the impulse response of consumption to shocks is increasing and concave.
b) Slow dissemination of information - consumption growth depends on current and past news and the estimates from regressing consumption growth on current and past news are non-increasing in how far in the past the news had arrived.

While the Hall model predicts that aggregate consumption should follow a random walk, equation (21) implies that the change in aggregate consumption should follow an $MA(I)$ process with positive coefficients. The difference between the two is well illustrated by looking at their different predictions for the shape of the normalized power spectrum of aggregate consumption changes.[6] Appendix B derives a formula for the normalized power spectrum corresponding to equation (21), denoted by $f_{\Delta C}(\omega)$. In the full information case, consumption changes are white noise so the power spectrum is horizontal. Gali (1991) uses the power spectrum to test theories of consumption, focusing in particular on the spectrum at frequency zero. He follows Deaton (1987), who examines the excess smoothness ratio $\psi \equiv 1/\sqrt{2\pi f_{\Delta C}(0)}$, interpreted as the square root of the ratio between the variance of changes in consumption and the variance of changes in permanent income.[7] In the Hall model, this ratio equals one, since consumption reacts immediately one-for-one to changes in permanent income. Findings of $\psi < 1$ have therefore been interpreted as suggesting that consumption is excessively smooth relative to income, whereas if $\psi > 1$ consumption is excessively volatile.

[6] The power spectrum of a time series process x_t is defined as

$$h_x(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\omega j},$$

where $\gamma_j = E(x_t - E(x_t))(x_{t-j} - E(x_t))$, the j^{th} autocovariance of x_t. The normalized power spectrum is: $f_x(\omega) = h_x(\omega)/Var(x_t)$. See Priestley (1981) for a discussion of the spectrum and its applications.

[7] The link between ψ and $f_{\Delta C}(0)$ was rigorously established by Gali (1991). Heuristically, the argument goes as follows. The variance ratio of Deaton is: $\psi = \sqrt{Var(\Delta C)/Var(\Delta Y^P)}$, where Y^P denotes permanent income. Gali notes that since the agent faces a budget constraint, changes in permanent income must lead to changes in permanent consumption, so $Var(\Delta Y^P) = Var(\Delta C^P)$. But 2π times the normalized spectrum at frequency zero of consumption changes measures exactly the fraction of the variability of consumption changes driven by permanent movements: $2\pi f_{\Delta C}(0) = Var(\Delta C^P)/Var(\Delta C)$. That $\psi = 1/\sqrt{2\pi f_{\Delta C}(0)}$ then follows immediately.

With inattentive consumers, Appendix B shows that:

$$\psi = \sqrt{\frac{\sum_{i=0}^{I} \Phi(i)^2}{\left[\sum_{i=0}^{I} \Phi(i)\right]^2}}, \tag{22}$$

which is clearly smaller than one, so that with inattentiveness, consumption is excessively smooth. Note that if there is excess smoothness, then it must be that $\Phi(i) \neq 0$ for some $i > 0$, so there is excess sensitivity. Yet, excess sensitivity per se does not necessarily imply excess smoothness. However, if the inattentiveness model is correct so all the $\Phi(i)$ are non-negative, excess sensitivity does imply excess smoothness. In the inattentiveness model, the two concepts are intimately linked, and any particular pattern of coefficients of excess sensitivity implies an exact value for the excess smoothness ratio.[8]

Proposition 5 *In the inattentiveness model:*
a) Changes in aggregate consumption have a normalized power spectrum given by:

$$f_{\Delta C}(\omega) = \frac{1}{2\pi}\left\{1 + 2\frac{\sum_{j=1}^{I}\sum_{k=0}^{I-j} \Phi(k)\Phi(k+j)\cos(\omega j)}{\sum_{k=0}^{I} \Phi(k)^2}\right\}. \tag{23}$$

b) Consumption is excessively smooth, as $\psi < 1$.

Propositions 2 to 5 give a set of predictions that can be tested using aggregate data. Yet the available measurements of consumption do not give consumption at an instant in time, but rather as the sum over a time period. In other words, while the Propositions assert implications for C_{t+1}, the available observations are of $\bar{C}_{t+1} = \int_0^1 C_{t+1-s} ds$. Nevertheless, as Appendix A shows, this only affects equation (21) insofar as it turns the $MA(I)$ process into an $MA(I+1)$ with a new set of coefficients $\tilde{\Phi}(s)$ which are still non-increasing for $s = 1, ..., I+1$. Proposition 2 is modified to assert that measured consumption growth is unpredictable from the perspective of $t - I - 1$ information. The other Propositions are unchanged.[9]

4 Functional forms and further predictions

Further implications of the model require assumptions on the utility function, the stochastic process governing income, and the costs of planning. A particular combination allows for a closed-form solution, while being roughly consistent with the data. I assume that the utility function is of the constant absolute risk aversion (CARA) form:

$$u(c) = -\frac{e^{-\alpha c}}{\alpha},$$

[8] Campbell and Deaton (1989) link excess sensitivity and excess smoothness in the rational expectations model.
[9] Christiano, Eichenbaum, and Marshall (1991) study the effect of time aggregation in the Hall model.

where $\alpha > 0$ is the coefficient of absolute risk aversion. It is well-known that this is one of the few utility functions for which even the full information problem has an analytical solution. Also for tractability, I assume that the costs of planning are fixed at a constant K.

Following Friedman (1957), I assume that income is the sum of two independent components. The first component is permanent income, denoted by y_t^P, which is assumed to follow a driftless Brownian motion with variance σ_P^2 and Wiener increments dz_t^P. This corresponds for instance to changes in employment status or in experience, training or education. The second component is transitory income, y_t^T, which is assumed to follow an Orstein-Uhlenbeck process (a continuous time AR(1)), with mean reversion speed ϕ and independent Wiener impulses $\sigma_T dz_t^T$. Shocks to transitory income affect income only temporarily, and the larger is ϕ the more short-lived their effects are. For instance, these could stand for overtime payment or for occurrences such as illness or winning a lottery prize. If these transitory components are idiosyncratic to the agent, they will aggregate to zero, in which case y_t^P is aggregate income in the economy, but this does not need to be the case. Aggregate but short-lived events, such as weather shocks to productivity, movements in the price level, or business cycles, affect disposable income through y_t^T.

If permanent income is observed at discrete points in time, it generates observations matching a discrete-time random-walk, while transitory income observed in discrete time is an AR(1). Income changes therefore follow an ARMA(1,1) process. MaCurdy's (1982) seminal study of annual earnings in the United States finds that this specification describes the data well.[10] If ϕ is large, income changes will be close to the MA(1) process originally proposed by Muth (1960).

4.1 Optimal inattentiveness and consumption

Defining the consumer's wealth, w_t, as the sum of her assets, a_t, and the present value of her expected income, $y_t^P/r + y_t^T/(r+\phi)$, the law of motion for wealth is:

$$dw_t = (rw_t - c_t)dt + \frac{\sigma_P}{r}dz_t^P + \frac{\sigma_T}{r+\phi}dz_t^T. \tag{24}$$

Whereas generally the agent must keep track of a_t and y_t separately in order to assess how her constraints will evolve, (24) shows that in this case w_t is a sufficient statistic. I can then write the value function as $V(w_t)$, reducing the dimension of the state space. The agent solves the problem:

$$V(w) = \max_{c,d} \int_0^d e^{-\rho t}\left(-\frac{e^{-\alpha c_t}}{\alpha}\right)dt + e^{-\rho d}E\left[V(w')\right], \tag{25}$$

$$\text{subject to } w' = e^{rd}\left[w - \int_0^d e^{-rt}c_t dt + \int_0^d e^{-rt}\left(\frac{\sigma_P}{r}dz_t^P + \frac{\sigma_T}{r+\phi}dz_t^T\right)\right] - K. \tag{26}$$

[10] MaCurdy (1982) also finds that an MA(2) fits the annual PSID earnings observations as well as an ARMA(1,1). His findings are confirmed by Hall and Mishkin (1982), Abowd and Card (1989), and Meghir and Pistaferri (2003). Pischke (1995) obtains similar results using the quarterly income observations in the 1984 SIPP.

Denoting the variance of wealth shocks by $\sigma^2 \equiv \sigma_P^2/r^2 + \sigma_T^2/(r+\phi)^2$, Appendix C proves:

Proposition 6 *In the CARA-utility, ARMA-income, inattentive consumer problem, the optimal inattentiveness intervals are given by:*

$$d^* = \frac{1}{r} \ln \left(1 + \sqrt{\frac{4K}{\alpha \sigma^2}} \right). \tag{27}$$

Optimal consumption between adjustments is:

$$\begin{aligned} c_t^* &= r w_{D(i)} + \frac{(r-\rho)(t-D(i))}{\alpha} - \frac{(r-\rho)}{\alpha r} - \frac{rK}{e^{rd^*}-1} - \frac{\alpha r \sigma^2}{4}(e^{rd^*}+1) && (28) \\ &= r w_{D(i)} + \frac{(r-\rho)(t-D(i))}{\alpha} - \frac{(r-\rho)}{\alpha r} - \frac{r\alpha\sigma^2}{2} - r\sqrt{\alpha\sigma^2 K}, && (29) \end{aligned}$$

for $D(i) < t < D(i+1)$. *The resulting value function is:*

$$V(w_{D(i)}) = -\frac{\exp(-\alpha c_{D(i)}^*)}{\alpha r}. \tag{30}$$

The rational expectations consumer would choose:

$$c_t^* = r w_t - \frac{(r-\rho)}{\alpha r} - \frac{r\alpha\sigma^2}{2}, \tag{31}$$

which follows by setting $K = 0$ and $t = D(i)$ in (29). The first term is the annuity on permanent income, as in Friedman's (1957) permanent income theory of consumption. The last term shows that a larger variance of income leads to lower consumption. This is the precautionary motive for savings – facing an uncertain future, the agent saves to insure herself against the possibility of bad income shocks.

Consumption in (31) differs from that in (28) since the latter responds to current shocks (through w_t), whereas inattentive consumption follows a pre-determined path between adjustments, independent of the arrival of news. Moreover:

Corollary 1 *At time 0, in the CARA-utility, ARMA-income problem, inattentive agents consume less than attentive ones. The larger are the costs of planning, the longer they are inattentive for, and the more they save.*

The lower consumption is due to two reasons, which are captured by the two extra terms in (28) relative to (31). The first reason is that costly planning lowers the agent's wealth, since she must pay an amount K every d^* periods, and lower permanent income reduces optimal consumption. The present value of this periodic expense is given by the second term from the right in the right-hand side of (28). The second reason for lower consumption is that the inattentive agent is more vulnerable to risk, since she only periodically adjusts her behavior to take account of the income shocks that are arriving every instant. The precautionary motive for savings is therefore larger by a factor of $(1 + e^{rd^*})/2$, which is increasing in the length of inattention. Larger costs of planning lead to longer periods of inattentiveness thus strengthening the precautionary motive and raising savings.[11]

Inspecting the optimal inattentiveness in (27) establishes:

Corollary 2 *In the CARA-utility, ARMA-income case, inattentiveness (d^*):*
1. Falls with the volatility of the income shocks (σ^2);
2. Falls with the coefficient of absolute risk aversion (α);
3. Falls with the real interest rate (r);
4. Increases with the costs of planning (K);
5. Is first-order long with only second-order costs of planning.

The intuition behind these results is as follows. The more volatile are income shocks, the more often the agent wants to re-plan so that she is able to adjust her behavior to the arrival of news. In a world that is quickly changing, it is very costly to not pay attention to news so the agent will avoid being inattentive for long. Similarly, if the agent is very risk averse, she will want to lower the risk she faces by updating information more often and responding to shocks faster. This does not imply that higher volatility is beneficial by inducing greater attentiveness. Quite on the contrary, a higher σ^2 unambiguously lowers welfare, since it increases uncertainty which the risk-averse agent dislikes, and moreover it forces her to spend more resources updating plans more frequently. Government policy with inattentive consumers should aim at stabilizing the economy. People can then be inattentive for long and direct their resources towards productive uses, rather than towards planning consumption.

Between planning dates the inattentive consumer (dis)saves all the unexpected changes in income, whereas the full-information consumer (dis)saves only a fraction of the new income. The larger is the interest rate, the larger is the repercussion that this inefficient (dis)saving will have on her future wealth. Facing a high interest rate, the agent will want to adjust more often to avoid past mistakes and to keep her assets under control.

The fourth property of inattentiveness is very intuitive: it states that the more costly it is to plan the less often the agent plans. More interesting is the last property, which shows that even very small costs of planning can lead to considerable inattentiveness. The intuition for this result is similar to that in Mankiw (1985), Akerlof and Yellen (1985) and Cochrane (1989). Inattentiveness leads to consumption differing from its full information optimum. However, since the choices of the inattentive consumer are close to this optimum, this deviation only has a second-order effect on utility. Therefore, even a second-order cost of planning will induce the agent to tolerate the

[11] The inattentiveness model suggests a curious explanation for the decline in the U.S. personal savings rate in the last two decades. If advances in information technology have lowered the costs of obtaining and processing information, then agents should optimally respond by saving less.

second-order costs of being inattentive for a first-order period of time.[12]

To illustrate how large d^* can be, consider the parameter estimates by Pischke (1995). He identified y_t^P with aggregate income and y_t^T with idiosyncratic income, and measured them using aggregate and family income from the National Income and Product Accounts (NIPA) and the Survey of Income and Program Participation (SIPP). He estimates that $\sigma_P = \$45$ and his estimates of the autocorrelation of income changes imply that $\phi = 0.487$ (which implies an AR coefficient in the ARMA for income changes of 0.615).[13] His estimate that income changes have an average standard deviation of $\$2,812$ then implies that $\sigma_T = \$1,962$. I set the quarterly interest rate at 1.5%, approximately its historical value in the United States, and $\alpha = 2/6926$, where $\$6,926$ is mean income in the Pischke sample, so the coefficient of relative risk aversion is about 2. Equation (27) implies that if the costs of updating plans are just $30, the agent stays inattentive for over 2 years. Very small costs of planing can lead to considerable inattentiveness.[14]

4.2 What to plan

With rational expectations or if income is certain, seeing the agent as a consumer or as a saver are two equivalent ways of looking at the same problem. With inattentiveness though, they are no longer equivalent. The agent must choose whether to set a plan for consumption or a plan for savings, letting the other absorb the shocks to income.

An inattentive saver sets plans for savings s_t, subject to the constraint that this choice is conditional on the information at the last planning date. Appendix D solves for the optimal choices of this agent, proving the following:

Proposition 7 *The CARA-utility, ARMA-income, inattentive saver sets consumption:*

$$\hat{c}_t = y_t - \hat{s}_t$$

where optimal savings $\hat{s}_t$ are set conditional on information at the last adjustment date. The optimal inattentiveness $\hat{d} = +\infty$ if:

$$K \geq \frac{\alpha\phi\sigma_T^2}{4(r+2\phi)(r+\phi)^2}.$$

[12] Further deviations from rationality would magnify this inertia. For instance, if agents have hyperbolic discount functions over the future, as in Laibson (1997), small costs of planning will lead to procrastination. Akerlof (1991) and O'Donoghue and Rabin (1999) explore this behavior.

[13] By comparison, using annual earnings from the PSID, MaCurdy (1982) estimates an AR coefficient of 0.216 in the ARMA(1,1), which is close to 0.615^4.

[14] It may be more appropriate to identify the interest rate in the model with a risk-free asset, rather than with the average real interest rate in the U.S. economy. Then, a quarterly interest rate of 0.5% is more adequate. With this lower interest rate, even smaller costs of planning lead to considerable inattentiveness ($12 induces $d^* = 8$ quarters).

Otherwise, $\hat{d}$ is finite and is the unique solution of the equation:

$$re^{2\phi\hat{d}}\left(1-\frac{4(r+2\phi)(r+\phi)^2K}{\alpha\phi\sigma_T^2}\right)=r+2\phi(1-e^{-r\hat{d}}).$$

Since consumption is just $c_t = y_t - s_t$ and s_t is pre-determined, the inattentive saver consumes every period her total income less a pre-determined amount. She lives hand-to-mouth, with a marginal propensity to consume out of income equal to one.

The optimal inattentiveness interval is plotted in Figure 1. After the costs of planning rise above a certain level, inattentiveness by the saver quickly rises to infinity. The intuition for this result comes from realizing that while consumption reacts optimally (one-to-one) to permanent income shocks, it also responds one-to-one to transitory income shocks when the optimal reaction would be to consume only a fraction $r/(r+\phi)$ of these shocks. As the costs of planning and optimal inattentiveness rise, less remains of a transitory shock by the time the agent responds to it. The incentive to update her plans therefore falls as inattentiveness rises, and a small increase in the costs of planning leads to a large increase in inattentiveness. After a certain level, optimal inattentiveness becomes convex in the costs of planning, and shoots to infinity.

If the agent chooses $\hat{d} = +\infty$, she can be described as a *rational non-planner.* She writes a plan once at time 0 and then follows this plan forever. For the parameter estimates in Pischke (1995), she chooses to do so once the costs of planning exceed \$543. Moreover, as Appendix E shows:

Corollary 3 *At time 0, in the CARA-utility, ARMA-income problem, rational non-planners save less than the consumption planners.*

If the agent is given the option of being either an inattentive consumer or an inattentive saver, which will she choose? Appendix E proves the following answer:

Proposition 8 *If $(\phi - r)/(\phi + r) > \sigma_P^2/\sigma_T^2$, the CARA-utility, ARMA-income, inattentive agent prefers consumption plans if the costs of planning are below a threshold $\hat{K}$, and prefers savings plan if the costs of planning are above this threshold. When the agent shifts from consumption to savings plans, her inattentiveness rises discontinuously, and possibly to infinity.*

Since almost all studies of individual income find that transitory shocks are the dominant source of income variation, the parameter restriction in this Proposition reduces to assuming that $\phi > r$. With an annual interest rate of 6%, this requires that transitory income shocks have a half life of no more than 11.5 years. From the other perspective, if $\phi = 0.487$ as estimated by Pischke (1995), the annual interest rate must be lower than 601%. The constraint in Proposition 8 very likely holds given plausible values of the interest rate and the persistence of transitory shocks.

Proposition 8 then states that, according to the model, there are two distinct groups in the population. On the one hand, are those who make financial plans for consumption, updating them sporadically. On the other hand, are those who are inattentive for longer, live hand-to-mouth and save less. This second group may even be composed only of people who rationally choose to never plan:

Corollary 4 *As long as:*

$$\frac{\phi^3 - r^2 (r + 2\phi)}{(r + 2\phi)(r + \phi)^2} > \sigma_P^2 / \sigma_T^2,$$

then agents who choose to be inattentive savers also choose to be rational non-planners.

For the parameter estimates of σ_P^2/σ_T^2 and ϕ found by Pischke (1995), the condition in the corollary holds as long as the annual interest rate is below 232%. It is reasonable to expect that all inattentive savers are rational non-planners.

A convenient way to assess how likely it is to find rational non-planners in the economy is to use the following result, proven in Appendix E:

Proposition 9 *If the conditions in Proposition 8 and Corollary 4 apply, then consumption plans are strictly preferred to savings plans if:*

$$\frac{\sigma_P^2}{\sigma_T^2}(e^{rd^*} - 1) + \left(\frac{r}{r + \phi}\right)^2 e^{rd^*} - \frac{r}{r + 2\phi} < 0, \tag{32}$$

where d^* is the optimal inattentiveness of the consumption-planner.

While this condition involves an endogenous variable (d^*), it only requires knowledge of σ_P^2/σ_T^2 and ϕ from the earnings data, and no information on the degree of risk aversion. Using the benchmark estimates in Pischke (1995) for σ_P^2/σ_T^2 and ϕ, then if the agent would choose to be inattentive for 8 quarters under a consumption plan, she prefers this plan to being a rational non-planner as long as the quarterly real interest rate is below 12.5%. From a different perspective, if the quarterly interest rate is 1.5%, then only if the consumption-planning agent stays inattentive for more than 41 years would she prefer to become a rational non-planner. Some agents may face such high costs of planning and interest rates that they live hand-to-mouth, but these calculations suggest that the majority of the population follows consumption plans.

Aside from strict consumption or savings plans, another plausible form of planning sets a fixed percentage of income to be automatically consumed or saved. The agent still keeps herself from observing and calculating an optimal response to income shocks every instant, but now has savings and consumption absorbing the shock in fixed percentages. In the United States, many workers enroll in fixed percentage contribution IRA plans that resemble these hybrid consumption-savings plans. Consumption is then $c_t = \lambda y_t + \tilde{c}_t$ and at a planning date, the agent now sets a plan for consumption ($\tilde{c}_t$), for the next planning date ($\tilde{d}$), as well as for the fraction of income shocks so be absorbed by consumption (λ). Appendix F solves the problem of this hybrid consumption-savings planner, with CARA utility and ARMA income. The optimal $\tilde{d}$ and λ are state-independent, and

Table 1 displays them for different plausible values of the interest rate and the costs of planning. The ability to choose λ implies that relative to consumption-planning the agent is now inattentive for even longer. The optimal λ in turn are quite small, ranging from 0.02 to 0.19.

Considering savings or hybrid planning then qualifies the prediction in Proposition 2 as follows:

Corollary 5 *If I is the largest amount of time during which planning agents remain inattentive, regressing aggregate consumption growth between t and $t+1$ on expected income as of $t-I$ periods ago, identifies either the fraction of aggregate consumption by hand-to-mouth inattentive savers, or the optimal percentage of income shocks consumed by hybrid planners. Either way, the estimated coefficient should be small.*

4.3 Extraordinary events

For most of the time in her ordinary life, a person is subject to random but small income shocks so it is not too costly to be inattentive. Occasionally though, big things happen in your life. You may lose your job, or win the lottery; your close family may be struck by a serious and expensive disease, or you may receive a sudden inheritance from a distant relative; an unexpected hyperinflation may eat up your purchasing power, or the shares in your small company may be worth a fortune after you come across a great invention. These things make you stop and think: the circumstances around you have changed so radically that old plans must be thrown out of the window and new plans made for the future.

A simple way to model these extraordinary events is by adding to the agent's income an independent Poisson stochastic term with arrival rate δ and jumps u or $-u$ with equal probability. Most of the time (with probability $1-\delta$) no event takes place, but every so often (with probability δ) an extraordinary event occurs which dramatically changes the agent's disposable income and to which she responds instantly. Because most of the time no event occurs, the computational cost of observing this variable is small so this is consistent with the underlying assumption that there are costs of absorbing and processing information. As long as the event is extraordinary (i.e., $\mid u \mid >> 0$), the agent will respond by collecting information and setting a new plan.[15] Further adding the convenient but inessential assumption that the interest rate equals the discount rate, Appendix G proves:

Proposition 10 *With CARA preferences and income following an ARMA(1,1) process plus Poisson extraordinary events, optimal inattentiveness is the minimizer of the function $A(0)$, which*

[15] I could alternatively assume that when an extraordinary event occurs, the agent adjusts consumption to it, but does not collect information on the remaining variables in the economy and continues following an inattentive plan with respect to these. Then, consumption would be contingent on the extraordinary events. With respect to the other income shocks, the problem is like the one studied so far.

solves the boundary value differential equation:

$$\begin{aligned} A'(t) - rA(t)\ln(A(t)) - \delta A(t) &= -A(0)\frac{\delta}{2}e^{\alpha r K - \frac{\alpha^2 r \sigma^2}{4}}\left(e^{-\alpha r u} + e^{\alpha r u}\right)e^{\frac{\alpha^2 r\sigma^2}{4}e^{2rt}} \\ A(d^*) &= A(0)e^{\alpha r K - \frac{\alpha^2 r\sigma^2}{4}}e^{\frac{\alpha^2 r\sigma^2}{4}e^{2rd^*}} \end{aligned}$$

While there is no analytical solution to this differential equation, it can be solved numerically to find d^*. Panel A of Table 2 uses the parameter estimates from Pischke (1995) to find d^* when extraordinary events occur on average every 2, 5, or 10 years, and when they imply a change in income of \$500, \$2500, and \$5000. Panel B shows the probability that an extraordinary event occurs before the planning time is reached.

Table 2 shows that the larger is the size of the extraordinary event, the longer is inattentiveness. I hold the agent's total income variance constant over the different parameters, so as u rises, a larger share of the variance is accounted for by extraordinary events. The variance of the small income shocks to which the agent is inattentive is then lower, so she stays inattentive for longer. Extraordinary events have a modest effect for these parameter values, at most raising inattentiveness by 3 quarters. Note also that as extraordinary events become more infrequent, the planning horizon approaches the solution without extraordinary events. If an extraordinary event occurs on average every 10 years, then the agent who stays inattentive for 2-year periods will adjust before the end of her plan only about 18% of the time.

Considering extraordinary events leads to the following prediction:

Corollary 6 *Individual consumption responds immediately to large extraordinary events, but only with a delay to small recurring news.*

5 Evidence on aggregate consumption

Propositions 2 to 5 stated a series of predictions of the inattentiveness model for the behavior of aggregate consumption. In this Section, I test these predictions using U.S. data. Section 5.1 examines Proposition 4 by studying whether aggregate consumption adjusts gradually to shocks in the data. Section 5.2 tests the prediction in Proposition 2, as refined by Corollary 5, by estimating the fraction of aggregate consumption attributable to hand-to-mouth behavior. This naturally leads to a test of the inattentiveness model against the alternative suggested by Campbell and Mankiw (1989, 1990). Finally, Section 5.3 contrasts the predictions in Proposition 5 on the spectrum and excess smoothness of consumption, with the estimates from the data.

Empirically implementing the model requires setting a value for I, the longest period of inattentiveness over all the planning agents in the economy. Given the previous calibrations, I set $I = 8$ quarters. In the test in Section 5.1, the larger is I, the smaller the power of the tests. In the tests in Section 5.2, if I is too small this may bias the results against the model, since the joint hypothesis that the model is correct and that $I \leq 8$ is being tested. Setting maximum inattentiveness at 2 years strikes a compromise between these two forces.

I use U.S. quarterly time series data which come from the National Income and Product Accounts, and financial data from the Center for Research in Security Prices. Aggregate consumption, C_t, is measured as real consumption of non-durables and services per capita, while Y_t denotes real disposable personal income per capita.[16] Both are deflated using the price deflator for consumption of non-durables and services. Data on real returns, r_t, will be used as a predictor of income growth. It is measured as the nominal return on the value-weighted S&P500 minus the inflation rate using the price deflator for non-durables and services.[17] The sample runs from 1953:1 through 2002:4.[18] One specification issue is whether to measure consumption in levels or logs. Past research has alternatively chosen one or the other, but I opt for log consumption since the series of observations on C_t appears to be closer to log-linear than linear.[19] Note that while the Propositions in Section 3 concerned the level of consumption, they could equally well be stated for log consumption by log-linearizing rather than linearizing the Euler equations.

5.1 Slow adjustment to shocks and slow dissemination of information

Proposition 4 stated that the impulse response of consumption to shocks should be increasing and concave. A simple analysis of the adjustment of aggregate consumption to shocks comes from estimating a structural vector autoregression (VAR) on consumption and income growth. I set the lag length on the VAR at 5, as suggested by the use of the Schwartz's Bayesian information criterion and by examining the significance of the last lag included in the VAR. As discussed in Blanchard and Quah (1989), one can identify permanent shocks to consumption and the adjustment of consumption to these shocks. Figure 2 displays the impulse response of log consumption to a permanent shock together with a 90% point-wise confidence interval generated by a bootstrap.

As shown in Figure 2, aggregate consumption has a delayed adjustment to the shock, as the inattentiveness model predicts. Moreover, while consumption is sluggish, it is only moderately so: most of the adjustment is completed within one year of the shock. This is consistent with an inattentiveness model in which agents update their information approximately once a year. The model also predicts that the impulse response should be concave and this pattern is also present

[16] I have also experimented using two other measures of C_t: consumption only on non-durables, and Parker's (2001) consumption series, which excludes footwear, housing, medical care, education, and personal business expenditures from non-durables and services. Both led to similar results.

[17] I experimented with many alternative measures for r_t. Different assets were used (the New York Stock Exchange index, the 3-month Treasury Bill rate, and municipal bonds), and after-tax returns were computed using different measures of the tax rate (the ratio between the return on tax-free municipal bonds relative to taxable corporate bonds, and a fixed 30%). The results are robust to using these alternatives.

[18] Data are available from 1947, but I follow Blinder and Deaton (1985) and Campbell and Mankiw (1990) in starting the sample in 1953:1 to avoid the effect of the Korean War and the unusual large spike in disposable income in 1950:1 due to the large payment of National Service Life Insurance benefits to World War veterans.

[19] Regressing the change in consumption on the level of consumption gives a coefficient of 0.003 with a t-statistic of 2.51, suggesting that the mean change rises with the level of the series. Regressing the squared change on the level of consumption gives a coefficient of 0.914 with a t-statistic of 4.88, which is a strong sign that the innovation variance of the series also increases with its level. Both point towards log-linearity.

in Figure 2. In response to news that permanently affect consumption, aggregate consumption therefore exhibits the sluggish response that the model associates with slow dissemination of the news over the population.

A sharper test of the slow adjustment of consumption to news comes from examining its response to news on a particularly important variable: income. Obtaining income news requires having a model of what income is expected to be. This can be described as a function $g(X_{t-1})$ mapping realizations of a set of variables X_{t-1} known at $t-1$, to expected realizations of income growth at period t. Surprises in income are then defined as the one-step ahead forecast errors in income. These surprises have mean zero and are uncorrelated by construction, so they satisfy the properties used to define the consumption innovations e_t in Proposition 3. Constructing $y_t = \Delta \ln(Y_t) - g(X_{t-1})$, it then follows that $e_t = \mu y_t + u_t$, where u_t are other surprises affecting consumption growth aside from income, and μ is the marginal propensity to consume out of income. Regressing consumption growth on several lags of y_t and treating the u_t's as the residual of this regression gives a test of the model's predictions in equation (21). Since the residual of this regression is a composite of u_t's at different points in time, the model suggests that the residuals will be serially correlated. Therefore, the standard errors are corrected for serial correlation of up to 8 lags, using the procedure of Newey and West (1987).

The first possibility I consider for $g(X_{t-1})$ is a univariate process. I estimate an AR(5) for income growth to generate income surprises and in Table 3 I show the results from regressing consumption growth on current and lagged income surprises. As predicted by the model, lagged income surprises affect future consumption growth. The F-statistic reported in the Table tests the null hypothesis that lagged income surprises do not affect current consumption growth as predicted by the Hall model. This hypothesis is strongly rejected with a p-value below 0.01%. Moreover, income surprises explain much of the variability of consumption growth as shown by the high adjusted R^2 of the regression (0.33).

The inattentiveness model predicts that the estimates in Table 3 should be non-increasing, and this seems to be approximately the case. The top panel of Figure 3 confirms this impression by plotting $\hat{\Phi}(j)/\sum_{i=0}^{I+1}\hat{\Phi}(i)$ for j from 0 to $I+1$ using the estimates from Table 3, together with 95% confidence intervals. The inattentiveness model predicts a downward sloping locus of points staying in the positive axis. This is consistent with the data. Another way to examine this property is by looking at the cumulative dissemination of the news. This is plotted in Figure 4, which shows the increasing and concave shape that the model predicts, similar to that estimated earlier in Figure 2.

In the regression in Table 3, the null hypothesis corresponding to the inattentiveness model is that the coefficients on income surprises s periods ago (β_s) are declining in s. Estimating the model by least squares subject to this restriction results in the set of estimates presented in the second to last row of Panel A in Table 3. Figures 3 and 4 display the restricted estimates as well. We can see that these are quite close to the unrestricted estimates supporting the validity of the model's restrictions. This null hypothesis can be formally tested, using the procedure suggested by Wolak

(1989). In Table 3, W_{IN} displays the value of the Wald statistic and the p-value associated with the null hypothesis imposed by the inattentiveness model. The model cannot be rejected at the 5% statistical significance level. Table 3 also displays the Wald statistic for the stricter hypothesis that all the coefficients are the same (W_{INU}), which corresponds to an inattentiveness model in which all agents are inattentive for the same length of time and are uniformly staggered in their dates of planning. This hypothesis is statistically rejected at the 1% significance level.

Panel B of Table 3 models income growth instead as depending on 5 lags of both past income growth and the log consumption income ratio. As did Campbell (1987), I find that these new regressors have significant predictive power for income growth: the p-value of an F-test on their significance is below 0.1% and the adjusted R^2 of the first stage regression rises by a factor of 3. The coefficients in the regression of consumption growth on current and past surprises are nevertheless little affected. They are still jointly very statistically significant and they broadly exhibit the non-increasing pattern predicted by the model. This can be visually apprehended in Figures 3 and 4, and is confirmed by the value of the W_{IN} statistic, so the model cannot be rejected at the 5% significance level. Contrary to before, I now cannot reject the hypothesis of uniform staggered adjustment.

Finally, Panel C adds 5 lags of the after-tax real interest rate as predictors of future income growth. These variables help very little in forecasting income growth. The estimates of equation (21) are similar to those obtained before, and so are the inferences on the fit of the model.

The regressions in Table 3 are in the same style as those in the literature which, starting with Barro (1977), examined the effects of money surprises on output. Mishkin (1983) noted that the two-step procedure does not produce efficient estimates for two reasons. First, since the second stage regressions ignore the estimation error in constructing the innovations in the first stage, they potentially underestimate the uncertainty of the estimates. Second, since the procedure is sequential, it ignores information from the second stage when estimating the first stage. Mishkin (1983) suggests estimating both equations simultaneously, using an iterative procedure which converges to the full information maximum likelihood estimates. Table 4 estimates the system in this way. Panel A of Table 4 is identical to panel A in Table 3. This is the surprising result of Pagan (1984) that when a univariate procedure is used in the first-stage, then the two-step procedure is efficient. Panels B and C display the results from the iterative procedure. They are similar to those in Table 3 so the previous inferences are robust to this econometric issue.[20]

[20] Another econometric issue is that the income innovations (y_t) may be correlated with the "other" shocks to consumption (u_t). The only way to address this concern would be to find a valid instrument for y_t, i.e., some variable revealed at t (since $E[y_t X_{t-s}] = 0$ for all $s > 0$) which only affects consumption choices through income. Finding such an instrument is a considerable challenge. I experimented using national defense spending as an arguably exogenous instrument, but it had no predictive power for income news.

5.2 Excess sensitivity and hand-to-mouth behavior

The general inattentiveness model, in which there are either inattentive consumers and savers, or in which hybrid plans are formed implies that:

$$C_{t+1} - C_t = (1 - \lambda)e_{t+1} + \lambda(Y_{t+1} - Y_t), \tag{33}$$

with $E_{t-i}[e_{t+1}] = 0$ for any $i \geq I + 1$. Proposition 2 stated that, if there are only inattentive consumers, then $\lambda = 0$. Corollary 5 added that if there are also rational non-planners in the economy, they will account for a fraction λ of aggregate consumption. Alternatively, if agents form hybrid plans, then λ will refer to the share of income shocks absorbed by consumption.

Table 5 estimates this equation, instrumenting the change in income with variables dated at least $I+1$ periods ago, which are therefore uncorrelated with e_{t+1}. I use as instruments 4 lags of income growth, and then successively add 4 lags of the log income-consumption ratio, and 4 lags of real returns. Since the model predicts that the residuals of this regression should be serially correlated, I compute the Hayashi and Sims (1984) nearly-efficient estimates, rather than the conventional (but inefficient) two-stage least squares estimates.

The estimates of λ are quite low, between 0.05 and 0.15. This confirms the prediction in Corollary 5, that we should expect that the share of aggregate consumption attributable to hand-to-mouth behavior should be quite small. The null hypothesis that $\lambda = 0$ cannot be rejected at conventional significance levels is any of the regressions, supporting the prediction in Proposition 2. The data is consistent with a model in which inattentive consumers account for the bulk of aggregate consumption dynamics.

The instruments used in these regressions are weak, as reflected by the low F-statistics in the second to last column in Panel A of Table 5. Income growth is difficult to forecast 9 quarters in advance. With weak instruments, the IV estimates are biased towards the OLS estimates (Stock, Wright and Yogo, 2002). These are displayed in the second column of Panel B, and are higher than the IV estimates, suggesting that the estimates of λ in Panel A are, if anything, too large. An alternative estimator is limited information maximum likelihood (LIML) and the third column shows that these estimates are slightly lower than those in Panel A. Columns 4 to 6 of Panel B present three tests proposed in the literature on weak instruments to powerfully test the rational expectations model: the Anderson and Rubin statistic, the Moreira (2003) conditional likelihood ratio statistic, and a conditional Lagrange multiplier statistic. All of them still cannot reject the hypothesis that λ is zero. While these tests likely suffer from lack of power, still both the IV and the LIML estimates are consistent (and the over-identifying restrictions are never rejected), and they consistently estimate λ to be small.

One feature of equation (33) is that it is also the equation that describes aggregate consumption dynamics in the model proposed by Campbell and Mankiw (1989, 1990), in which a fraction $1 - \lambda$ of consumption is made by rational expectations agents, while the remaining λ fraction is made by irrational, myopic, hand-to-mouth people. The difference is that in their model, $E_{t-i}[e_{t+1}] = 0$ for any $i \geq 1$. Using variables lagged two periods as instruments for income growth, Campbell and Mankiw (1989, 1990) found that hand-to-mouth agents account for 40-50% of aggregate consumption. According to their model though, it is equally valid to use instruments lagged nine periods. However, doing so leads to estimates of λ that are insignificant and much lower, between 5% and 15%, supporting instead the inattentiveness model.

In Table 5, the low power of the test may bias the test for the significance of λ towards the null hypothesis of the inattentiveness model. It would be desirable to test the Campbell-Mankiw against the inattentiveness model, having both models stated as null hypotheses, to avoid biasing the results towards the model that is stated as a null hypothesis due to lack of power. Note that I can expand the right-hand side of equation (33) to obtain:

$$\begin{aligned} C_{t+1} - C_t &= \lambda (E_t - E_{t-1}) (Y_{t+1} - Y_t) + \lambda (E_{t-1} - E_{t-2}) (Y_{t+1} - Y_t) + ... \\ &\quad + \lambda (E_{t-T+1} - E_{t-T}) (Y_{t+1} - Y_t) + \lambda E_{t-T}(Y_{t+1} - Y_t) + u_{t+1}, \end{aligned}$$

where $u_{t+1} \equiv (1 - \lambda)e_{t+1} + \lambda (Y_{t+1} - E_t(Y_{t+1}))$ is uncorrelated with the other right-hand side variables. Estimating the regression equation

$$C_{t+1} - C_t = \beta_0 + \sum_{s=1}^{T} \beta_s (E_{t-s+1} - E_{t-s}) (Y_{t+1} - Y_t) + \lambda E_{t-T}(Y_{t+1} - Y_t) + u_{t+1}, \tag{34}$$

the null hypothesis describing the Campbell-Mankiw model is[21]

$$H_0^{CM} : \beta_2 = ... = \beta_T = \lambda.$$

With respect to this equation, the Hall (1978) full information rational expectations hypothesis is

$$H_0^{RE} : \beta_2 = ... = \beta_T = \lambda = 0,$$

so that no variable dated t or before predicts consumption growth. As for the inattentiveness model, note that the variables $(E_{t-s+1} - E_{t-s}) (Y_{t+1} - Y_t)$ have zero expectation as of $t - s$, so they fit into the definition of the news e_{t-s+1}. The prediction of the model in Proposition 3 is that

$$H_0^{IN} : \beta_1 \geq \beta_2 \geq ... \geq \beta_T \geq 0, \ \lambda = 0.$$

as long as $T \geq I + 1$. If there are both inattentive savers and consumers, this leads to the weaker null hypothesis:

$$H_0^{ING} : \beta_1 \geq \beta_2 \geq ... \geq \beta_T \geq 0,$$

[21] There is no restriction on β_1 to allow for time aggregation. I do not impose the restriction that $\lambda \geq 0$, which biases the results in favor of the Campbell-Mankiw model.

and the estimate of λ gives the share of inattentive savers. The different models are now expressed as different null hypotheses on the same regression, so a potential lack of power does not bias the inferences towards any one of them in particular.

The variables on the right-hand side of (34) were generated by estimating a VAR with 5 lags on the change in log income, the log consumption-income ratio, and the real interest rate, and then using this VAR to construct s-step ahead forecasts of income growth. Table 6 presents the point estimates of equation (34), which are somewhat discouraging for all four models. The point estimates of the β_s's do not seem to have the pattern described in either H_0^{CM} or H_0^{IN}, and several of the estimated coefficients are individually large and statistically significant contrary to H_0^{RE}. Panel B of Table 6 formally tests the models using Wald tests for H_0^{CM}, H_0^{RE}, H_0^{IN}, and H_0^{ING}.[22] Consistent with the other results in this paper, the full information rational expectations model is decisively rejected even at a 0.01% significance level. The Campbell-Mankiw model is also rejected at the 5% significance level (but not at the 1% level), which is not surprising given the low estimate of λ. The null hypothesis of the general inattentiveness model on the other hand has a p-value of 12.8%, so it is not statistically rejected at conventional significance levels. Moreover, note that it is estimated that only 3.4% of consumption is done by inattentive savers, so hand-to-mouth behavior is economically and statistically insignificant. Consequently, the model with inattentive consumers alone is not statistically rejected at the 5% significance level. The last row in Table 6 tests a stricter inattentiveness model in which there is uniform staggered adjustment so the inequalities in H_0^{IN} are replaced by equalities. This is rejected at the 5% significance level.

These results suggest that the hand-to-mouth behavior detected in aggregate consumption data may be attributable to inattentive consumer behavior, as described in this paper. Moreover, as predicted by the theory, rational non-planning (or hybrid planning) seems to have a small impact on aggregate consumption.

5.3 Excess smoothness and the spectrum of consumption

Proposition 5 makes sharp predictions on the shape of the power spectrum of aggregate consumption changes. Figure 5 plots estimates of the spectrum, constructed using a sample spectral density weighted over a 5-lag Bartlett window. The normalized power spectrum of consumption growth generally declines with the frequency and has a shape close to Granger's typical shape, aside from a slight hump around the $\pi/2$ frequency.

Figure 5 also displays the spectrum for aggregate consumption growth predicted by the theoretical model using the weights $\hat{\Phi}(i)$ estimated in panel A of Table 3.[23] The theory's predicted spectrum matches the empirical spectrum quite well, albeit with somewhat more pronounced swings. Also in Figure 5 is the predicted spectrum using the weights obtained after imposing the theoretical restrictions of the inattentiveness model. This further improves the fit of the model with the data.

[22] The null hypotheses H_0^{IN} and H_0^{ING} are tested using the Wolak (1989) procedure.

[23] Using the estimates from panels B and C produces very similar plots.

Table 7 displays estimates of the excess smoothness ratio defined in Section 3.3, computed using different methods. The estimates of ψ lie between 0.52 and 0.7, and the full information rational expectations null hypothesis that they equal one is always rejected. The inattentiveness model using the weights from Panel A of Figure 3 predicts an excess smoothness ratio of 0.65, well within the range of estimates. Table 7 shows the predictions of the model using different estimates of the weights. For all sets of weights, the inattentiveness model predicts a ratio of excess smoothness close to (or even slightly below) that in the data. Therefore, the pattern of excess sensitivity captured by the $\hat{\Phi}(i)$ produces an excess smoothness ratio that matches the data as well.

6 Microeconomic evidence on inattentiveness

The inattentiveness model makes a series of sharp predictions on the behavior of individual consumption. This Section examines whether the main predictions are consistent with what we know from studies of individual consumption.

6.1 Sensitivity to past information

Over the last decade, there have been many tests of excess sensitivity using individual consumption data. The results so far have been inconclusive: some studies find it, while other do not, and it is unclear what explains the different results. The inattentiveness model suggests an explanation. Proposition 1 established that agents are generally inattentive to small unpredictable events and thus react with a delay to these when they next adjust. If the event is easily predictable though, the agents will have reacted to it when they set their plans in the past. If the event is extraordinary in size and rarity, Corollary 6 established that reaction was instantaneous. The inattentiveness model therefore predicts that consumption is sensitive to past small unpredictable events, but it is not sensitive to predictable or extraordinary events.

Two key papers that have found evidence that small past news on after-tax income affects consumption some time after are Parker (1999) and Souleles (1999). Parker (1999) looks at the patterns of Social Security tax withholding, while Souleles (1999) looks at income tax refunds. In both cases, the news were small relative to income and they were unpredictable. Their findings that consumption is sensitive to these past news supports the inattentiveness model.

In turn, Browning and Collado (2001) and Souleles (2000) look at the response of consumption to large and easily predictable changes in income, and find that consumption does not react to these past news. Browning and Collado (2001) examine the reaction of Spanish households to well-known income fluctuations driven by the timing of bonus payments, while Souleles (2000) examines the impact of the easily predicted college tuition payments on parent's consumption. Hsieh (2003) studies the reaction of Alaskans to the extraordinary payments made to them by the Alaska's Permanent Fund (on average \$1,964 in 2000) associated with oil royalties. He also finds that consumption does not respond to this past extraordinary news.

The inattentiveness model can therefore reconcile the apparently contradictory findings in the literature that has tested for excess sensitivity to past events.

6.2 Inattention

Shapiro and Slemrod (1995) document inattentiveness on the part of people in a relevant situation to this paper. In 1992, President George H. Bush announced a reduction in the standard rates of withholding for income taxes, which lowered employees' tax withholding by about $29 per month. Using a survey of 501 people 1-2 months after the announcement, Shapiro and Slemrod (1995) find that about half of respondents were not aware of any change in withholding.

The inattentiveness model further predicts that news disseminates slowly throughout the population. Carroll (2003) looks at survey data on inflation expectations and finds that the expectations of the public lag those of professional forecasters, supporting the slow dissemination of information. Moreover, he finds that when the number of references to inflation in the newspapers rises, the public updates its expectations faster, which is consistent with the endogenous determination of inattentiveness in this paper. Mankiw, Reis, and Wolfers (2003) examine the dynamics of disagreement in inflation expectations in three U.S. surveys. In the inattentiveness model, information and expectations are only updated infrequently so, at any point in time, agents will have different expectations determined by when they last updated. Mankiw, Reis, and Wolfers (2003) find that a model with exogenous staggered updating of information matches the time-series of disagreement well, and can explain the particularly large increase in disagreement that occurred in the early 1980s during the Volcker disinflation.

6.3 Planning

Proposition 8 established that in an inattentive world we should expect to find some people who do not set consumption plans. Corollaries 3 and 4 showed that these people never make plans and save less than those who do. Lusardi (1999, 2002) uses data from the Health and Retirement Study (HRS), which surveys people over 50 years old on their attitudes towards retirement, and finds that approximately one third have hardly thought about retirement. Moreover, she finds that those who have not planned are more likely to be less educated, self-report lower cognitive abilities, be single, and do not have older siblings to use as a source of information, all of which likely proxy for the costs of planning in the inattentiveness theory. Lusardi (1999, 2002) then finds that those who do not plan are less wealthy. Yakoboski and Dickemperer (1997) find that 36% of respondents in the Retirement Confidence Survey have done little or no planning for retirement. When questioned as to why they had not planned, among other things respondents replied they did not have time to plan, that it was too complicated to do so, and that they did not know how to find help for it, supporting the interpretation of this behavior as the result of costly planning. Hurst (2003) uses the data from the Panel Study of Income Dynamics (PSID) to distinguish between two groups of consumers, according to whether they have high or low wealth when they reach retirement. He finds that the low wealth group suffers a larger drop in consumption at retirement (consistent with inadequate planning for retirement), has consumption growth responding to predictable changes in income, and its behavior cannot be accounted for by liquidity constraints, precautionary savings, or habit formation. Hand-to-mouth behavior as a result of rational non-planning is consistent with his results, providing support for the theory in this paper.

Ameriks, Caplin and Leahy (2003a) use a TIAA-CREF survey in which households were asked "Have you personally gathered together your household's financial information, reviewed it in detail, and formulated a specific financial plan for your household's long-term future?" The households were also asked further questions on their attitudes towards planning, namely whether they are confident with their mathematical skills, and whether they usually plan their vacations. If these responses provide a proxy for the costs of planning, they can be used as instruments for the household's planning decisions in assessing whether planning predicts savings and wealth. Ameriks, Caplin and Leahy (2003a) also find that approximately 25% of households report not having a financial plan, and that these households seem to face higher costs of planning. Moreover, they find that those who do not plan have significantly lower savings and accumulated wealth.

These studies using microeconomic data therefore suggest that between one quarter and one third of the U.S. population does not make plans. Section 5 in turn found that about 5% of aggregate consumption can be attributed to rational non-planners. These two estimates are surprisingly consistent, since the theory predicts (and the data confirms) that the non-planners have lower savings and wealth, and so account for a small share of aggregate consumption.[24]

In addition, in the Ameriks, Caplin, and Leahy (2003a) survey, agents who reported having a plan were further asked for how long they have had their plan in place. Proposition 6 and Corollary 1 stated that planners who update information less frequently due to larger costs of planning will save less. Ameriks, Caplin, and Leahy (2003a) regress accumulated wealth on the length of the agent's plan, using as instruments their proxies for the costs of planning, and find that agents who have had plans in place for longer accumulate significantly more wealth. This finding supports the inattentiveness model. In related work, Alessie, Kapteyn, and Lusardi (1999) use the Dutch CentER data-panel, which asks households for their planning horizon for expenditures and savings. Again in agreement with the inattentiveness model, they find that the longer is the planning horizon, the larger are savings (but the effect is not statistically significant at the 5% level).

The inattentiveness model therefore matches many of the existing findings on individual consumption. The model also generates many novel predictions, which can be tested in future work.

[24] On the other hand, the Campbell and Mankiw (1989, 1990) estimate that hand-to-mouth consumers account for 40-50% of aggregate consumption implies that a large majority of U.S. households live hand-to-mouth, which is difficult to reconcile with the microeconomic evidence.

7 Models of the costs of planning

The inattentiveness model relies on the assumption that whenever the agent acquires, absorbs and processes information to write a plan, she must pay a monetary cost K_t. There are many reasons to expect this to be the case. Whenever the agent plans, she must obtain information on the state of the economy, the state of the industry in which she works, the financial position of her company, the likely future health and education expenditures within her family, house prices in her area of residence, and many other factors, all of which can be obtained with time and money from publications, consultants and experts. Filtering from this information what is relevant to the agent will likely be as hard and time-consuming and it could also involve hiring consultants and financial advisors. Finally, deciding on the optimal consumption plan in response to this information may require great computational ability which in turn takes time and money (for instance, in buying a computer and financial planning software). All of these are costly activities, not just in direct payments but especially in time spent and income foregone.

7.1 The RAM model

One formal model that can be used to express these costs is the RAM, which stands for Random Access Machine. This is the dominant model used in computer science to measure the costs of different algorithms. The computing agent is modelled as a machine which is fed a tape of input, has unlimited memory and can perform the basic arithmetic operations (addition, subtraction, multiplication and division) as well as store results. Each of these operations can be performed at a unit cost of time or physical resources.[25] Imagine then that in order to perform calculations, the consumer must hire the services of a RAM at a cost per operation of π. If the RAM models the complexity of a given task, π times this measure of complexity is the payment by the consumer to financial consultants and advisors or the cost of publications and software. For the sake of illustration, consider the problem of the agent in Section 2, but with the more realistic ARMA(1,1) income process, with transitory shocks following an AR(1) with autoregressive coefficient ϕ.

The consumer must always incur the cost of inputting all the relevant parameters into her memory. Setting consumption with full information or being inattentive lead to different further costs. The full information agent must first obtain all the relevant information on the state of the economy. This will correspond to inputting some zx1 vector Z_t of data at a cost z in RAM operations. Then, she must use this data to infer the current realizations of permanent and transitory income, by say solving a system of linear equations $(y_t^P, y_t^T)' = GZ_t$ where G is a 2xz matrix. This takes

[25] This model was introduced by Cook and Rechkow (1973) as a formalization of the idealized Von Neumann style computer. The other dominant approach in computer science models agents as Turing machines. This formulation is considerably more abstract and general, but it is mostly used to establish the complexity class of the problem; for instance to see whether a problem can be performed at a cost which is a polynomial function of time or an exponential function of time. For the purposes of comparing different algorithms in the same complexity class, the model is less useful. Many results establish a close link between these two models (van Emde Boas, 1990).

$2z$ operations by the RAM. The full information agent must then read her current level of assets (a_t) and calculate her current wealth, $w_t = a_t + y_t^P/(R-1) + y_t^T/(R+\phi)$ at a combined cost of 5 RAM operations. Finally, she calculates optimal consumption, which is given by $c_t = constant + rw_t$ where the constant depends on parameters and is already loaded into the memory. This takes two arithmetic operations and one final operation to store the final result. Overall, to be attentive at a point in time the consumer must incur costs to acquire the information (z operations), process and interpret it ($2z+3$ operations) and use it to compute the optimal action (3 operations), for a total cost of $3z+8$ RAM operations. In comparison, the inattentive agent does not absorb or interpret any new information. Given last period's consumption choice which is already in memory, she computes consumption this period using the deterministic Euler equation: $c_t = c_{t-1}$, which takes only one storing operation. Clearly, being attentive is always more costly. Moreover, since it likely takes many pieces of information to be able to distinguish between permanent and transitory income, then $3z+8 >> 1$, and attentiveness is substantially more costly than inattentiveness. By being inattentive, the consumer avoids incurring these large costs except at the decision dates when she incurs the additional cost $K = \pi(3z+7)$. The RAM model therefore provides a simple justification of the assumption behind the inattentiveness model.

The RAM model can also justify planning costs associated with time lost. Assume that the agent has a separable utility function in consumption and leisure (L_t): $u(c_t) + v(L_t)$. Given her endowment of time, which is normalized to 1, the consumer may spend it enjoying leisure (L_t), working for a wage w_t (H_t), or planning optimal consumption (P_t). Planning time equals the number of RAM operations times π, which is now interpreted as the units of time taken per RAM operation executed by the consumer. The optimal choice of leisure is determined by the first-order condition $v'(L_t^*) = w_t$, and the agent's income is given by the sum of income she might receive from some endowment E_t plus her income from labor supply $w_t H_t^*$.[26] Using the constraint that $L_t^* + H_t^* + P_t = 1$, her income can then be written as $E_t + (1 - L_t^*)w_t - w_t P_t$. Letting $y_t = E_t + (1 - L_t^*)w_t - 2w_t\pi$ and $K_t = w_t\pi(3z+4)$, yields the original formulation of the inattentive agent's budget constraint and the costs of planning.

These simple illustrations are not meant to be a rigorous model of the costs of planning faced by an economic agent. Yet they show that one can easily devise plausible scenarios based on established models from computer science that justify the assumptions of the inattentiveness model. Some current work in behavioral economics (e.g., Gabaix and Laibson, 2002) offers the hope that we can soon build more formal models of planning costs.

7.2 Constraints on the flow of information

In 1948, Shannon introduced the concept of the entropy of a signal, which started the field of information theory. The entropy of a random variable is the number of bits that it will take on

[26] To ensure that there is an interior solution for leisure requires the assumption that if L_t is below some $\underline{L} \geq 0$, $v'(L_t)$ is arbitrarily large.

average to describe realizations of that random variable. For instance, if a random variable may take one of four values with equal probability, its entropy will be 2 – by associating the number 00 with the first possible realization, 01 with the second, and 10 and 11 with the third and fourth, I only need two bits of information to communicate the realization of the variable. If the channel through which I communicate to you allows only for the transmission of one bit, then after receiving my message you will still face uncertainty on which of two possible values the random variable took.[27]

Sims (2003) models economic agents who must obtain information on the state of the world through a limited transmission channel. This could be interpreted literally since human senses have a finite capacity to absorb information, or instead as a metaphor for the limited ability to interpret this information. If the state of the economy is represented by a vector Z_t, then the agent will not be aware of the actual realizations of this random vector but rather she will perceive a signal $\hat{Z}_t$, which differs from Z_t by a random variable u_t. An important result by Shannon (1948), is that if the agent's objective function is quadratic in Z_t, then the optimal coding of the message $\hat{Z}_t$ (in the sense of maximizing the reduction in entropy) is such that u_t is normally distributed with a given variance which depends on the capacity of the channel. Therefore, in Sims' model, agents behave much like agents in the Lucas (1973) islands economy, every period receiving signals on the state of the world and solving signal extraction problems to estimate the value of the state on which to base their decisions. Information theory brings to the imperfect information models a characterization of the properties of the observation error as a function of agents' channel capacity.

Moscarini (2003) studies price setters facing these constraints in continuous time. He assumes that firms pay a cost per bit of information received and shows that agents optimally decide to be inattentive, only periodically turning on their channel to receive information on the state of the world. This result suggests that modelling consumers' costs of information using the Sims-Moscarini approach (having behind it Shannon's information theory) or the approach in this paper (having behind it the RAM model) leads to similar infrequent updating of plans. One significant difference is that when updating occurs, agents in the Sims-Moscarini model only obtain an imperfect signal on the state of the world, whereas consumers in the inattentiveness model obtain full information on this state. This feature of the inattentiveness model makes it significantly more tractable.[28]

8 Alternative models of near-rationality and consumption

Gabaix and Laibson (2001) were among the first to highlight the implications of inattentiveness for individual behavior.[29] They study the problem of a consumer who can allocate her savings between

[27] Cover and Thomas (1991) provide a very accessible introduction to information theory.

[28] Another interesting difference between the two models is that since in the Sims-Moscarini model the costs of adjustment depend on the parameters of the stochastic processes in the model (e.g., on the variances), the comparative statics on the optimal inattentiveness with respect to these parameters may be different.

[29] They have a close precursor in Lynch (1996), and a contemporary in Parker (2001) who also emphasizes that slow adjustment of consumption can go a long way in justifying the equity premium in the data.

a risky asset (equity) and a riskless one (bonds). The focus of their paper was on the equity premium puzzle, and in particular on the correlation between consumption growth and equity returns. The focus of this paper is instead on the dynamics of consumption and its relation to income. Moreover, during most of their paper, Gabaix and Laibson (2001) set the inattentiveness intervals exogenously. In one section, they endogenize them by solving for optimal constant inattentiveness intervals through a series of approximations. The theory in this paper has focussed on determining optimal inattentiveness, and Sections 3.1 and 3.2 set up and solved this problem in great generality. While the inattentiveness intervals may be constant (e.g., as in Section 4), in general, optimal behavior leads to recursive time-contingent adjustment, as discussed before.

Different papers have recently proposed alternative models to address the empirical shortcomings of rational expectations theory. This Section discusses the habit formation model, presents the model of state-contingent consumption adjustment proposed by Caballero (1995), and briefly covers other theories of consumption and bounded rationality.

8.1 Habit formation models

A popular theory of consumption has stressed that consumers may develop habits over consumption. In its simplest form, this is modelled by assuming that utility at time t depends on $c_t - \gamma c_{t-1}$, with $\gamma > 0$, so that higher consumption last period creates a habit that lowers utility this period. In this case, consumption growth becomes: $\Delta c_{t+1} = \gamma \Delta c_t + e_{t+1}$ (see Deaton, 1992, pp. 31-33).

Taken as a model of a representative consumer, the habit theory predicts that aggregate consumption follows an AR(1), and to fit the moderate amount of sluggishness that Section 5.1 found in the data, the auto-regressive coefficient cannot be too large. Since an $AR(1)$ is also an $MA(\infty)$ with declining coefficients, and if γ is not too large the higher-order moving average coefficients are negligible, then a representative consumer with a habit model generates aggregate consumption observations very close to the $MA(I+1)$ with declining coefficients predicted by the inattentiveness model. The two models are almost indistinguishable using only the stochastic properties of aggregate consumption.[30]

Using other information aside from the time-series properties of aggregate consumption, the two models can be distinguished. The habit model predicts that consumption should respond sluggishly to any event. The inattentiveness model on the other hand predicts that in response to an event that is very noticeable and grabs the attention of the population, consumption should respond instantly. One notable such event is the end of hyper- and high-inflations, which usually occurs suddenly with the implementation of drastic and well-publicized stabilization programs. Fischer, Shay, and Vegh (2002) examine 45 such episodes in 25 countries since 1960. They find

[30] With more flexible specifications of habits, the match between the two models may be even closer. Chetty and Szeidl (2003) show that, under some circumstances, a model with time-contingent consumption adjustment exactly mimics the aggregate consumption dynamics that would be chosen by a representative agent with a specific habit formation process.

that these noticeable disinflation programs have a large effect on real variables, and especially that aggregate consumption responds immediately. They write "...per-capita consumption growth: it is essentially zero in the year before stabilization and jumps to around 2 percent in the year of stabilization..." and moreover they find that after this immediate reaction, consumption growth is stable in the following two years. This immediate response of consumption to these noticeable events is consistent with the inattentiveness model, but not with a habit model that predicts a sluggish response with respect to all shocks.

Habit formation at the level of the individual can be more clearly distinguished from inattentiveness. Inattentiveness implies that all the sluggishness in aggregate consumption comes from aggregation of infrequently adjusted individual consumption, whereas habit formation predicts that individual consumption is serially correlated. Dynan (2002) uses data from the PSID to find that individual consumption growth is close to serially uncorrelated. Therefore, when she estimates the optimality conditions imposed by the habit model she finds no evidence for habits. Her findings are consistent with inattentiveness.

8.2 State-contingent adjustment

Imagine that the relevant costs facing the agent are in computing or adjusting consumption plans but that it is costless to obtain or process information. Consumers will then always be attentive, but only infrequently adjust their consumption at some dates, contingent on the state of the economy at those dates. In the inattentiveness model instead, consumption is adjusted at certain dates independent of the state of the economy at those dates (but depending on the state at the date of the last adjustment). In the inattentiveness model, the adjustment of plans is *time-contingent*, while in this alternative model adjustment is *state-contingent*.

Caballero (1995) proposes a model of non-durables consumption along these lines. Appendix H solves the consumer problem from Section 4 when the costs are of adjusting to the state but not of observing it, and relates it to Caballero's (1995) model. Optimal behavior in this model involves adjusting consumption infrequently, whenever the deviation between the current consumption level and the optimal level given the current state exceeds a certain threshold.

How does state-contingent behavior compare to the inattentiveness model? I have argued that it is costly to collect and process information and to compute an optimal solution. With state contingent adjustment though, every instant the agent *is* observing the full state of the economy, *is* processing this information to realize what is her wealth w_t, and *is* performing costly computations to determine whether consumption is in the inaction region or not. In terms of the RAM model of computational costs in Section 7.1, state-contingent behavior is as complicated as simply following the full information rational expectations optimal plan! Thus, it is difficult to rationalize this model as describing "near-rational" behavior. The inattentiveness model seems to be more plausible since it involves behavior with considerably less computational costs.

With state-contingent adjustment, an aggregate shock to all agents' incomes leads all of their deviations of consumption from its optimal level to shift. Some agents will be pushed outside the inaction region and thus will adjust consumption immediately in response to this shock. Others will remain in the inaction region, and as time evolves they will gradually hit one of the boundaries, at which point they adjust consumption to respond to the shock. Aggregate consumption will therefore exhibit a slow adjustment to news as in the inattentiveness model, so it is difficult to distinguish between the two theories using aggregate data. Micro data on consumption would allow us to distinguish the two theories, since individual adjustments depend on the current state of the economy in the state-contingent adjustment model, but they depend on the past state in the inattentiveness model. This test would require overcoming substantial data challenges though, namely in identifying adjustment dates. The empirical evidence on inattentiveness and planning behavior in Section 6 supports the inattentiveness model against the state-contingent adjustment alternative.

8.3 Other bounded rationality models

A few other papers have also recently tried to address the problems with rational expectations theory by relaxing its extreme assumption on available information and the rationality of agents.

Closely related to the inattentiveness theory are the models of Goodfriend (1992) and Pischke (1995) in which they assume that agents cannot contemporaneously distinguish between permanent and transitory income shocks. In these models though, since the consumer perfectly observes her total income, there should be no evidence of excess sensitivity in the micro data. The studies of Shapiro and Slemrod (1995), Shea (1995), Hayashi (1997), Parker (1999) and Souleles (1999) finding excess sensitivity at the household level therefore reject these models.

Ameriks, Caplin and Leahy (2003b) study “absent-minded” consumers who within a time period do not keep track of how much they consume and so make mistakes in choosing their level of consumption. The implications of this model are different from those of the inattentiveness model: whereas absent-minded agents will on average over-consume, inattentive agents consume less than in the full information case. In turn, Mullainathan (2002) and Wilson (2003) model inattentiveness in an opposite direction to the one in this paper: while I assume that agents don’t monitor the present but can recall the past perfectly, they assume that agents know the present well but only recall the past imperfectly given limitations to their memory.

Economic agents have also been modelled as adaptive learners who act as econometricians estimating the parameters of a (potentially mis-specified) model of the economy by successive least squares regressions on past observations to form expectations of the future. This literature, surveyed in Evans and Honkapohja (2001), has derived conditions under which the rational expectations equilibrium is reached asymptotically (and when many such equilibria exist, which of these is reached), while the dynamics during the learning transition are relatively unexplored due to their technical complexity.[31] The inattentiveness approach in this paper differs in that during adjustment periods agents are fully inattentive and learn nothing, but when they do adjust they learn everything about the economy; adaptive learners on the other hand learn a little every period. Models based on inattentiveness always converge to the unique rational expectations equilibrium since eventually all update their plans, and the model is sufficiently tractable to allow the study of transition dynamics, as in Mankiw and Reis (2002, 2003).

Evans and Ramey (1992) assume that agents face a fixed cost of revising their expectations but in each period they observe all the variables in the economy. Agents decide whether to form a new expectation next period rather than stay with this period's expectation by comparing the forecast error from not adjusting with the cost of doing so. This model is similar in spirit to the state-contingent adjustment model. Implicitly, it assumes that there is a cost to calculating new expectations, but no cost to either acquiring information or calculating whether it is worth revising expectations. I have stressed these costs in this paper.

This existing literature shares with the inattentiveness model in this paper the aim of modelling boundedly rational agents in forming expectations, but differs in the specific modelling assumptions. The implications of some of these models are qualitatively similar to the ones of the inattentiveness model, in which case it is difficult to say which is the best approach. One advantage of the model in this paper is its tractability. For the most part, solving inattentiveness models requires using only the powerful tools that have been developed to solve rational expectations models, and this tractability allows a wide applicability of inattentiveness to model different economic problems.

9 Conclusion

In his Nobel lecture, James Tobin (1982, page 189) wrote:

"Some decisions by economic agents are reconsidered daily or hourly, while others are reviewed at intervals of a year or longer except when extraordinary events compel revisions. It would be desirable in principle to allow for differences among variables in frequencies of change and even to make these frequencies endogenous. But at present, models of such realism seem beyond the power of our analytical tools."

In this paper, I developed some of the tools that Tobin called for and examined the implications of modelling behavior in this way for the dynamics of aggregate consumption. I assumed (and justified) the existence of decision costs inducing agents to only sporadically update their decisions and characterized the decisions of these agents on how much to consume and how often to plan. This individual behavior implies that information should be sticky in the aggregate economy, only gradually dissipating throughout the population, so that aggregate consumption adjusts slowly to the arrival of news. I found that this prediction is confirmed in U.S. data and that the model

[31] Sargent (2001) is an important exception, studying the dynamics of of inflation in a learning model.

also generates dynamics for aggregate consumption which have the "excess sensitivity" and "excess smoothness" with respect to income that had been previously identified in the data. For individual consumption, the model predicted that consumption changes should be sensitive to small and unpredictable past shocks, but should not be sensitive to past large or predictable changes. This dichotomy reconciles the disparate findings of the many microeconomic studies which have studied the excess sensitivity of consumption to shocks. The model further predicted that information and expectations are only sporadically updated, which has also been shown to be the case using inflation expectations surveys. Finally, the model predicted that a group of people do not plan and save less than those who plan, and that among planners, those who plan for longer, save more. Again, this has been confirmed in the data.

Beyond passing tests in the data, the set of theoretical results and empirical estimates in this paper offer a clear description of consumption behavior which is consistent with both the micro and the macro data. There are two types of agents in the United States. About one third of people face high costs of planning (e.g., because of lack of education) and so rationally choose to never plan, living hand-to-mouth and consuming their income less a predetermined amount every period. These people save less and accumulate less wealth. Because they are poorer, they account for only a small fraction of aggregate consumption, around 5%. The bulk of aggregate consumption is accounted for instead by the other two thirds of people who form plans for consumption regularly. Because they only sporadically update their plans, these people react to small unexpected income shocks only gradually over time. Aggregate consumption therefore reacts sluggishly to shocks, but not too sluggishly since people do update their plans within a year or so. This paper has argued that this description of the world is consistent with much of the existing evidence on consumption at both the individual and aggregate levels. There are many more new predictions of the model that future research may explore.

Given these results, it seems promising to study the implications of inattentiveness for other decisions. Some of that work is already in progress (but much more remains to be done). Reis (2003a) has inattentive consumers making portfolio choices. His preliminary findings suggest that inattentiveness can generate the level of participation in equity markets, as well as the dynamics of equity and bond returns that we observe. Reis (2003b) studies inattentive producers and endogenously derives the sticky-information Phillips curve of Mankiw and Reis (2002). Reis (2003c) examines the equilibrium interactions between inattentive producers and consumers. These studies are just a first step in studying the inattentiveness approach but they suggest that inattention deserves some attention.

Appendix A - The discrete-time representation of consumption

Proof of Proposition 3

This Appendix establishes the relation between the continuous time representation of aggregate consumption changes in (20) and its discrete time counterpart (21) in Proposition 3. Treating the vector (i, j) as a random variable with distribution $\Psi(i, j)$, equation (20) shows that (up to a constant), aggregate consumption growth is the expected value of $e_{t+1-j,t-i}$. Because (i, j) can only take finitely many values, it is a *simple random variable* (Billingsley, 1995, Section 5) so the integrals in (20) are Riemann integrals and can be represented as sums. Breaking each unit interval into N part, j takes N values from 0 to $1 - 1/N$, all equidistant in the real line, and i takes $IN + 1$ equidistant values from 0 to I. Equation (20) then becomes:

$$\Delta C_{t+1} = \sum_{k=0}^{N-1} \sum_{m=0}^{NI} e_{t+1-k/N,t-m/N} \Psi(m/N, k/N).$$

Recall that $e_{t,t-s}$ is a random variable such that $E_{t-s}[e_{t,t-s}] = 0$. It can be broken into independent increments by writing: $e_{t,t-s} = \int_{t-s}^{t} \varepsilon(v)dv$, where $\varepsilon(v)$ is a continuous time "white noise" process with $E\left[\varepsilon(v)^2\right] = \sigma_\varepsilon^2$ but $E[\varepsilon(v)\varepsilon(v-k)] = 0$ for any $k > 0$.[32] The change in aggregate consumption is then:

$$\Delta C_{t+1} = \sum_{k=0}^{N-1} \sum_{m=0}^{NI} \left[\int_{t-m/N}^{t+1-k/N} \varepsilon(v)dv \right] \Psi(m/N, k/N).$$

Separating the random variables occurring after t from those before t:

$$\begin{aligned}
\Delta C_{t+1} &= \sum_{k=0}^{N-1} \sum_{m=0}^{NI} \left[\int_{t}^{t+1-k/N} \varepsilon(v)dv + \int_{t-m/N}^{t} \varepsilon(v)dv \right] \Psi(m/N, k/N) \\
&= \sum_{k=0}^{N-1} \left[\int_{t}^{t+1-k/N} \varepsilon(v)dv \right] P^j(k/N) + \sum_{m=0}^{NI} \left[\int_{t-m/N}^{t} \varepsilon(v)dv \right] P^i(m/N). \qquad (35)
\end{aligned}$$

The last expression uses $P^j(.)$ to denote the marginal distribution of j, $P^j(k/N) = \sum_{m=0}^{NI} \Psi(m/N, k/N)$, as well as $P^i(.)$ to denote the marginal distribution over the i, $P^i(m/N) = \sum_{k=0}^{N-1} \Psi(m/N, k/N)$).

Breaking the two terms in (35) into independent increments in intervals of length $1/N$:

[32] More rigorously, I mean that $\varepsilon(v)dv = \zeta(dv)$, where $\zeta(dv)$ is a *random measure*, defined on all subsets of the real line such that $E[\zeta(dv)] = 0$, $E[\zeta(dv)^2] = \sigma^2 dv$, and $E[\zeta(\Delta_1)\zeta(\Delta_2)] = 0$ for any disjoint sets Δ_1 and Δ_2. See Rozanov (1967) for a rigorous definition of random measure and of integration with respect to this measure.

$$\sum_{k=0}^{N-1}\left[\int_{t}^{t+1/N}\varepsilon(v)dv+\int_{t+1/N}^{t+2/N}\varepsilon(v)dv+...+\int_{t+1-k/N-1/N}^{t+1-k/N}\varepsilon(v)dv\right]P^{j}(k/N)$$
$$+\sum_{m=0}^{NI}\left[\int_{t-1/N}^{t}\varepsilon(v)dv+\int_{t-2/N}^{t-1/N}\varepsilon(v)dv+...+\int_{t-m/N}^{t-m/N+1/N}\varepsilon(v)dv\right]P^{i}(m/N)$$
$$=\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-k-1}\int_{t+n/N}^{t+n/N+1/N}\varepsilon(v)dv\right]P^{j}(k/N)+\sum_{m=0}^{NI}\left[\sum_{n=0}^{m/N-1/N}\int_{t-n-1/N}^{t-n}\right]P^{i}(m/N).$$

Collecting all the terms corresponding to each $1/N$ length interval gives:

$$\Delta C_{t+1}=\sum_{k=0}^{N-1}\left[\int_{t+1-k/N-1/N}^{t+1-k/N}\varepsilon(v)dv\right]G^{j}(k)+\sum_{m=1}^{NI}\left[\int_{t-m/N}^{t-m/N+1/N}\varepsilon(v)dv\right]G^{i}(m),$$

where I defined:

$$\begin{aligned}G^{j}(k) &\equiv \sum_{p=0}^{k}P^{j}(p/N)\text{, which is increasing in } k,\\ G^{i}(m) &\equiv \sum_{p=m}^{NI}P^{i}(p/N)\text{, which is decreasing in } m.\end{aligned}$$

One can then re-write this expression as an $MA(N+NI)$ process with independent increments:

$$\begin{aligned}\Delta C_{t+1} &= \sum_{k=0}^{N(I+1)-1}u_{t+1-k/N}f(k), \qquad (36)\\ \text{with } u_{t+1-k/N} &\equiv \int_{t+1-k/N}^{t+1-k/N+1/N}\varepsilon(v)dv\\ F(k) &= G^{j}(k)\text{ for } k=0,...,N-1\\ &= G^{i}(k-N)\text{ for } k=N,...,N(I+1)-1.\end{aligned}$$

Clearly, $E_{s-1/N}[u_s]=0$ and $E[u_s u_k]=0$, while $F(k)$ is increasing from $k=0$ to $N-1$, and decreasing from N to $N(I+1)-1$.

Given (36), the process for aggregate consumption changes in discrete time is:

$$\begin{aligned}\Delta C_{t+1} &= \sum_{s=0}^{I}\left(\sum_{k=0}^{N-1}u_{t+1-k/N-s}F(sN+k)\right)\\ &= \Phi(0)e_{t+1}+\Phi(1)e_{t}+...+\Phi(I)e_{t-I+1},\end{aligned}$$

as long as one defines:

$$\Phi(s) \equiv \sqrt{\frac{1}{N}\sum_{k=0}^{N-1} F(sN+k)^2},$$
$$e_{t+1-s} \equiv \frac{1}{\Phi(s)}\sum_{k=0}^{N-1} u_{t+1-k/N-s} F(sN+k).$$

Clearly, $E_{t-s}\left[e_{t+1-s}\right]=0$ and $Var\left[e_{t+1-s}\right]=\sigma_{\varepsilon}^{2}$. This proves the first part of Proposition 3. Since $F(k)$ is decreasing for $k \geq N$, then $\Phi(s)$ is also decreasing for $s=1,2...,I$, which proves the second part of Proposition 3.$\square$

Time Aggregation

In the data we observe:

$$\bar{C}_{t+1}-\bar{C}_{t}=\frac{1}{N}\sum_{p=0}^{N-1}\Delta C_{t+1-p/N},$$

where again I have used the sum representation of the Riemann integral. Using (36):

$$\Delta\bar{C}_{t+1}=\sum_{p=0}^{N-1}\sum_{k=0}^{N(I+1)-1} u_{t+1-p/N-k/N}F(k).$$

As before, I can collect terms to see that $\Delta\bar{C}_{t+1}$ equals:

$$\sum_{p=0}^{N-1}\left[u_{t+1-p/N}\left(\frac{\sum_{v=0}^{p}F(v)}{N}\right)\right]+\sum_{s=1}^{I}\left[\sum_{k=0}^{N-1}u_{t+1-k/N-s}\left(\frac{\sum_{v=N(s-1)+k+1}^{Ns+k}F(v)}{N}\right)\right]$$
$$+\sum_{p=N(I+1)}^{N(I+2)-1}\left[u_{t+1-p/N}\frac{F(p-N)}{N}\right].$$

This can then be written in discrete time as:

$$\Delta\bar{C}_{t+1} = \tilde{\Phi}(0)e_{t+1}+\tilde{\Phi}(1)e_{t}+...+\tilde{\Phi}(I)e_{t-I+1}+\tilde{\Phi}(I+1)e_{t-I},$$
$$\tilde{\Phi}(0) \equiv \sqrt{\frac{1}{N^2}\sum_{p=0}^{N-1}\left(\sum_{v=0}^{p}F(v)\right)^2},$$
$$\tilde{\Phi}(s) \equiv \sqrt{\frac{1}{N^2}\sum_{k=0}^{N-1}\left(\sum_{v=N(s-1)+k+1}^{Ns+k}F(v)\right)^2},\ \text{for } s=1,...,I,$$

$$\tilde{\Phi}(I+1)\equiv\sqrt{\frac{1}{N}\sum_{p=NI}^{N(I+1)-1}F(p)^2}.$$

Time aggregation therefore turns an $MA(I)$ process into an $MA(I+1)$. It is easy to see that the non-increasing pattern of the $\tilde{\Phi}(i)$ is unaltered, and applies up to $I+1$.

Appendix B - Spectrum of consumption

Proof of Proposition 5

Recall four results: (a) De Moivre's formula, $e^{-i\omega j} = \cos(\omega j) - i \cdot \sin(\omega j)$, (b) $\sin(-\omega j) = -\sin(\omega j)$, (c) $\cos(\omega j) = \cos(-\omega j)$, and (d) that since the MA(I) process in equation (21) is stationary, its autocovariance function is symmetric ($\gamma_j = \gamma_{-j}$). Then, the formula for the power spectrum in footnote 9 becomes:

$$h_{\Delta C}(\omega) = \frac{1}{2\pi}\left[\gamma_0 + 2\sum_{j=1}^{\infty}\gamma_j \cos(\omega j)\right]. \tag{37}$$

For the process in (21) the autocovariance function is:

$$\gamma_j = \begin{cases} \sigma^2 \sum_{k=0}^{I-j} \Phi(k)\Phi(k+j), \text{ for } j = 0, 1, 2, ..., I \\ 0, \text{ for } j > I \end{cases}$$

Replacing this into (37) and dividing by γ_0, gives the expression in Proposition 5, which depends only on $\{\Phi(i)/\Phi(0)\}$ for i from 1 to I. Evaluating (23) at frequency zero and rearranging gives the excess smoothness ratio in (22).□

Appendix C - CARA-utility, ARMA-income, consumer problem

Proof of Proposition 6

The problem is stated in (25)-(26). Using the Ramsey rule in equation (18) and the CARA form of the utility function gives:

$$c_t^* = \frac{(r-\rho)t}{\alpha} + c_0^*. \tag{38}$$

Using this solution to substitute out consumption in the budget constraint (26), a little algebra shows that wealth at the next planning period is:

$$w' = e^{rd^*}\left[w - \frac{(r-\rho)(1-e^{-rd^*} - rd^* e^{-rd^*})}{\alpha r^2} + \int_0^{d^*} e^{-rt}\left(\frac{\sigma_P}{r}dz_t^P + \frac{\sigma_T}{r+\phi}dz_t^T\right)\right] - K - \frac{e^{rd^*}-1}{r}c_0^* \tag{39}$$

Since w' is a linear combination of normally distributed variables, it is normally distributed with:

$$E[w'] = e^{rd^*}\left[w - \frac{(r-\rho)(1-e^{-rd^*} - rd^* e^{-rd^*})}{\alpha r^2}\right] - K - \frac{e^{rd^*}-1}{r}c_0^*, \tag{40}$$

$$Var[w'] = \frac{\sigma^2}{2r}(e^{2rd^*}-1), \tag{41}$$

Next, I make the (educated) guess that the value function is exponential: $V(w) = -A\exp(-Bw)$, where A and B are coefficients to be determined. The envelope theorem condition becomes:

$$-Bw = (r-\rho)d^* + \ln\left(E\left[e^{-Bw'}\right]\right). \tag{42}$$

Since w' is normally distributed, from the properties of the log-normal distribution, $\ln\left[E\left[\exp(-Bw')\right]\right]$ equals $-BE\left[w'\right] + B^2 Var\left[w'\right]/2$. Using this result and (40)-(41) in (42), gives the solution for c_0^*:

$$c_0^* = rw - \frac{rK}{e^{rd^*}-1} - \frac{B\sigma^2}{4}(e^{rd^*}+1) - \frac{(r-\rho)}{e^{rd^*}-1}\left[\frac{rd^*}{B} + \frac{e^{rd^*}-1-rd^*}{\alpha r^2}\right]. \tag{43}$$

Combining the first-order condition (13) at $t=0$ with the envelope theorem in (14) gives:

$$e^{-\alpha c_0^*} = ABe^{-Bw}. \tag{44}$$

For the guess of the value function to be valid, (44) must hold for all possible realizations of w. Matching the coefficients including w or not, gives:

$$B = \alpha r, \tag{45}$$

$$A = \frac{e^{-\alpha(c_0^*-rw)}}{\alpha r}. \tag{46}$$

Armed with the solution for B, I can go back to (43) and rearrange to obtain:

$$c_0^* = rw - \frac{rK}{e^{rd^*}-1} - \frac{\alpha r\sigma^2}{4}(e^{rd^*}+1) - \frac{r-\rho}{\alpha r}. \tag{47}$$

The last optimality condition is the first-order condition with respect to d, which is just: $\partial V(w)/\partial d = 0$. Combining the guess for the value function with (45) and (46):

$$V(w) = \max_d\left\{-\frac{e^{-\alpha c_0^*}}{\alpha r}\right\}.$$

The first-order condition therefore is that $\partial c_0^*/\partial d = 0$, which I can evaluate using (47) to obtain:

$$(e^{rd^*}-1)^2 = \frac{4K}{\alpha\sigma^2}. \tag{48}$$

Solving this equation gives (27). Using the solution for d^* in (47) gives the solution for c_0^* in (29).

The final step is to verify that the guess for the value function accords with the Bellman equation and satisfies the transversality condition. This is left to the reader.□

Appendix D - The inattentive saver's problem

Proof of Proposition 7

The problem facing the inattentive agent can be written as:

$$W(w) = \max_{d,\{s_t\}} E\left[\int_0^d e^{-\rho t}u(y_t - s_t)dt + e^{-\rho d}W(w')\right] \tag{49}$$

$$\text{s.t.} \quad da_t = (ra_t + s_t)dt \tag{50}$$

Integrating (50) between two decision dates, using the fact that $w' = w_d - K$, that $y_t = y_t^P + y_t^T$, and the definition $w_t = a_t + y_t^P/r + y_t^T/(r+\phi)$, leads to:

$$w' = e^{rd}\left(w + \int_0^d e^{-rt}s_t dt\right) - K + \frac{y^{P\prime} - e^{rd}y^P}{r} + \frac{y^{T\prime} - e^{rd}y^T}{r+\phi}.$$

Since permanent income follows a Brownian motion, $y^{P\prime} = y^P + \sigma_P(z_d^P - z_0^P)$. Since z_d^P is a Wiener process, it is normally distributed with mean z_0^P and variance equal to d. Thus $y^{P'}$ is also normally distributed with mean y^P and variance $\sigma_P^2 d$. Likewise, since $dy_t^T = -\phi y_t^T dt + \sigma_T dz_t^T$, then transitory income is also normally distributed with mean $y^T \exp(-\phi d)$ and variance $\sigma_T^2(1 - \exp(-2\phi d))/2\phi$. Therefore, w' is also normally distributed with:

$$E_0\left[w'\right] = e^{rd}\left(w + \int_0^d e^{-rt}s_t dt\right) - K + \frac{1 - e^{rd}}{r}y^P + \frac{e^{-\phi d} - e^{rd}}{r+\phi}y^T, \tag{51}$$

$$Var_0\left[w'\right] = \frac{\sigma_P^2}{r^2}d + \frac{\sigma_T^2(1 - e^{-2\phi d})}{2\phi\,(r+\phi)^2} \tag{52}$$

The first-order conditions determining the optimal choices of s_t are:

$$E_0\left[u'(y_t - s_t)\right] = e^{(r-\rho)(d-t)}E_0\left[W_w(w')\right], \quad \text{for } t \in [0, d). \tag{53}$$

Combining this equation for time t and for time 0:

$$\begin{aligned} u'(y_0 - s_0) &= e^{(r-\rho)t}E_0\left[u'(y_t - s_t)\right] \Leftrightarrow \\ -\alpha y_0 + \alpha s_0 &= (r-\rho)t + \alpha s_t + \ln\left(E_0\left[e^{-\alpha y_t}\right]\right). \end{aligned}$$

Using the normality of y_t it takes a few steps to obtain:

$$s_t = s_0 - (1 - e^{-\phi t})y_0^T - \frac{\alpha}{2}\left(\sigma_P^2 t + \frac{\sigma_T^2(1 - e^{-2\phi t})}{2\phi}\right) - \frac{(r-\rho)\,t}{\alpha}. \tag{54}$$

The envelope theorem condition is:

$$W_w(w) = e^{(r-\rho)d}E\left[W_w(w')\right]. \tag{55}$$

Given the previous experience, I guess that the value function is of the form: $W(w) = -Ae^{-\alpha r w}$, where A is a coefficient to be determined. Taking logs of (55):

$$-\alpha r w = (r-\rho)d + \ln E\left[e^{-\alpha r w'}\right]. \tag{56}$$

Using the properties of the log-normal distribution and (51)-(52) gives, after some rearranging::

$$\begin{aligned} w(e^{rd}-1) &= \frac{(r-\rho)d}{\alpha r} - e^{rd}\int_0^d e^{-rt}s_t dt + K + \frac{e^{rd}-1}{r}y^P + \frac{e^{rd}-e^{-\phi d}}{r+\phi}y^T \\ &\quad + \frac{\alpha\sigma_P^2}{2r}d + \frac{\alpha r\sigma_T^2(1-e^{-2\phi d})}{4\phi(r+\phi)^2}. \end{aligned}$$

Using the solution for s_t in (54) to substitute out savings in this equation gives, after rearranging,:

$$s_0 = -rw + y + \frac{r-\rho}{\alpha r} + \frac{\alpha\sigma_P^2}{2r} + \frac{\alpha\sigma_T^2}{2(r+2\phi)} + \frac{rK}{e^{rd}-1} - \frac{\alpha r\phi\sigma_T^2(1-e^{-2\phi d})}{4(r+2\phi)(r+\phi)^2(e^{rd}-1)} \tag{57}$$

In turn, using this solution to substitute s_0 in (54) gives the solution for savings plans at any time t after the last planning date $D(i)$, which is the counterpart to (28) in Proposition 6 for the case of savings plans:

$$\begin{aligned} \hat{s}_t &= -rw_{D(i)} + y^P_{D(i)} + e^{-\phi t}y^T_{D(i)} + \frac{(r-\rho)(1-rt)}{\alpha r} + \frac{\alpha\sigma_P^2(1-rt)}{2r} \\ &\quad + \frac{\alpha\sigma_T^2\left[(r+2\phi)e^{-2\phi t}-r\right]}{4\phi(r+2\phi)} + \frac{rK}{e^{r\hat{d}}-1} - \frac{\alpha r\phi\sigma_T^2(1-e^{-2\phi\hat{d}})}{4(r+2\phi)(r+\phi)^2(e^{r\hat{d}}-1)} \end{aligned} \tag{58}$$

Combining the envelope theorem (55) with (53) gives the condition:

$$u'(y_0 - s_0) = W_w(w).$$

Using the form of the utility function, the guess for the value function, and the expression for s_0 in (57), gives the solution for A:

$$A = \frac{1}{\alpha r}\exp\left\{\frac{r-\rho}{r} + \frac{\alpha^2\sigma_P^2}{2r} + \frac{\alpha^2\sigma_T^2}{2(r+2\phi)} + \frac{\alpha r K}{e^{rd}-1} - \frac{\alpha^2 r\phi\sigma_T^2(1-e^{-2\phi d})}{4(r+2\phi)(r+\phi)^2(e^{rd}-1)}\right\}. \tag{59}$$

Now, given (59) and the guess for the value function, to maximize $W(w)$ with respect to d is equivalent to minimizing A with respect to d, which in turn is equivalent to minimizing:

$$\hat{A}(K,d) \equiv \frac{K}{e^{rd}-1} - \frac{\alpha\phi\sigma_T^2(1-e^{-2\phi\hat{d}})}{4(r+2\phi)(r+\phi)^2(e^{r\hat{d}}-1)}. \tag{60}$$

The first-order necessary condition for an interior minimum is:

$$\hat{A}_d(.) = \frac{e^{(r-2\phi)d}}{\Xi\left(e^{rd}-1\right)^2}\underbrace{\left[re^{2\phi d}(1-K\Xi)+2\phi e^{-rd}-2\phi-r\right]}_{\equiv B(d)} = 0, \tag{61}$$

$$\text{where} : \Xi \equiv \frac{4(r+2\phi)(r+\phi)^2}{\alpha\phi\sigma_T^2}. \tag{62}$$

If $K\Xi > 1$, $B(d)$ is always negative, which implies that $\hat{A}$ falls monotonically with d, and so the optimal $\hat{d}$ is $+\infty$. Otherwise, $\hat{d}$ is the zero of $B(d)$. Straightforward evaluation and differentiation of $B(d)$ shows that with strictly positive costs of planning: $B(0) < 0$, $B_d(0) < 0$, $B_{dd}(.) > 0$, and $\lim_{d\to+\infty} B(D) = +\infty$. Thus, there is a unique solution to $B(d) = 0$, at a point at which $B(d)$ cuts the horizontal axis from below, and therefore there is a unique optimal finite $\hat{d}$.□

Appendix E - Consumption versus savings plans

Proof of Proposition 8

The agent prefers a consumption plan if the value from doing so, $V(w)$, is larger than the value from following a savings plan, $W(w)$. Using the solution in (30) and that for $W(w)$ in (59), $V(w) > W(w)$ becomes:

$$\frac{\alpha^2\sigma_P^2}{2r} + \frac{\alpha^2\sigma_T^2}{2(r+2\phi)} + \frac{\alpha r K}{e^{r\hat{d}}-1} - \frac{\alpha^2 r\phi\sigma_T^2(1-e^{-2\phi\hat{d}})}{4(r+2\phi)(r+\phi)^2(e^{r\hat{d}}-1)} > \frac{\alpha^2 r\sigma^2}{2} + \alpha r\sqrt{\alpha\sigma^2 K}.$$

Using the definition of σ^2 and rearranging, this becomes:

$$H(K) \equiv \frac{rK}{e^{r\hat{d}}-1} - \frac{\alpha r\phi\sigma_T^2(1-e^{-2\phi\hat{d}})}{4(r+2\phi)(r+\phi)^2(e^{r\hat{d}}-1)} - r\sqrt{\alpha\sigma^2 K} + \frac{\alpha\sigma_T^2\phi^2}{2(r+2\phi)(r+\phi)^2} > 0.$$

Since if $K = 0$ then $\hat{d} = 0$, using L'Hopital's rule it is easy to see that $H(0) = 0$: under full information rational expectations, consumption and savings plans are equivalent. Moreover, when $K > 1/\Xi$ and so $\hat{d} = +\infty$, then the first two terms in the definition of $H(K)$ are zero, so clearly $H(K)$ is declining in K tending towards minus infinity. More generally, using the envelope theorem:

$$H_K(.) = \hat{A}_K(K,\hat{d}) - r\sqrt{\frac{\alpha\sigma^2}{4K}} = \frac{r}{e^{r\hat{d}}-1} - \frac{r}{e^{rd^*}-1},$$

where the second equality follows from using (60) and (48). But then, it is clear that $sign\{H_K(.)\} = sign\left\{d^* - \hat{d}\right\}$, so to study $H(.)$ I must compare optimal inattentiveness with consumption and savings plans.

Evaluating $B(d)$ defined in (61), which determines optimal inattentiveness with savings, at the optimal inattentive ness with consumption d^*, replacing for K, gives:

$$F(d^*) \equiv re^{2\phi d^*}\left(1-\frac{\Xi\alpha\sigma^2\left(e^{rd^*}-1\right)^2}{4}\right)+2\phi e^{-rd^*}-2\phi-r.$$

Since I know that if $B(d)$ is negative it is to the left of its zero, and when it is positive it is to the right of its zero, then if $F(d^*)$ is positive for some d^* it must follow that $d^* > \hat{d}$. Conversely, if for some d^*, $F(d^*)$ is negative, then $d^* < \hat{d}$, and at $\hat{d}$, $F(\hat{d}) = 0$.

Straightforward evaluation and differentiation of the function $F(.)$ shows that: $F(0) = 0$, $F_d(0) = 0$, and $F_{dd}(0) = 2r\phi(2\phi + r) - r^3\Xi\alpha\sigma^2/2$. Using the definition of Ξ in (62) shows that as long as the assumption in the Proposition holds, then $F_{dd}(0) > 0$. Thus, close to 0, $F(.)$ is positive and so $d^* > \hat{d}$.

Next, I will show that aside from the trivial intersection at 0, $d^* = \hat{d}$ only once. Note that the derivative of $F(.)$ at a point of intersection is:

$$\begin{aligned}
F_d(\hat{d}) &= 2\phi re^{2\phi\hat{d}}\left(1-\frac{\Xi\alpha\sigma^2\left(e^{r\hat{d}}-1\right)^2}{4}\right)-\frac{r^2e^{(2\phi+r)\hat{d}}\Xi\alpha\sigma^2\left(e^{r\hat{d}}-1\right)}{2}-2\phi re^{-r\hat{d}} \\
&= 2\phi\left(2\phi+r-2\phi e^{-r\hat{d}}\right)-\frac{r^2e^{(2\phi+r)\hat{d}}\alpha\sigma^2\Xi\left(e^{r\hat{d}}-1\right)}{2}-2\phi re^{-r\hat{d}} \\
&= 2\phi\left(r+2\phi\right)\left(1-e^{-r\hat{d}}\right)\left(1-\frac{r^2\alpha\sigma^2\Xi e^{2(\phi+r)\hat{d}}}{4\phi(r+2\phi)}\right),
\end{aligned}$$

where the second line follows from replacing the first term using the condition $F(\hat{d}) = 0$, and the third line follows from rearranging. Then, it is clear that if $\hat{d}$ is small enough, $F_d(\hat{d})$ is positive, but once $\hat{d}$ rises above a certain threshold, it becomes negative forever. Now, since for small K, $F(d^*)$ is positive, this continuous function must intersect the horizontal axis first at a point where $F_d(\hat{d}) < 0$. Towards a contradiction, say that is intersects the horizontal axis again at some higher d. By continuity of the $F(d)$ function, it must cut the axis from below. Yet, we know that at any zero of the $F(d)$ function the slope must be negative since we are already above the threshold, which leads to a contradiction. Therefore, $d^* = \hat{d}$ only once at some value of K, and if the costs of planning exceed this value then $\hat{d} > d^*$.

Returning back to the initial aim of studying when is $H(.)$ positive, we then conclude that starting from 0 when $K = 0$, the function increases up to a certain K (when $d^* = \hat{d}$). Then it starts declining monotonically towards minus infinity, intersecting the horizontal axis at a unique point $\hat{K}$. Therefore, if $K \in (0, \hat{K})$, then $H(K) > 0$, and so consumption plans are preferred. If $K > \hat{K}$, savings plans are preferred.

Finally, note that at $\hat{K}$ where $H(\hat{K}) = 0$, we know that $H_K(\hat{K}) < 0$, and so that $\hat{d} > d^*$; therefore when K passes $\hat{K}$ and the agent shifts from consumption to savings plans, her inattentiveness takes a discountinuous jump from d^* to $\hat{d}$.□

Proof of Corollary 4

I know that if $K > 1/\Xi$, then $\hat{d} = +\infty$. I also know that if $\hat{d} = +\infty$, then consumption plans are preferred as long as:

$$\begin{aligned} H(K) &> 0 \Leftrightarrow \\ r\sqrt{\alpha\sigma^2 K} &< \frac{\alpha\sigma_T^2\phi^2}{2(r+2\phi)(r+\phi)^2} \Leftrightarrow \\ K &< \frac{\alpha\sigma_T^4\phi^4}{4(r+2\phi)^2(r+\phi)^2\left((r+\phi)^2\sigma_P^2 + r^2\sigma_T^2\right)} \equiv \bar{K}. \end{aligned}$$

Moreover, I know that if $K > \hat{K}$, then savings plans are preferred. Combining these three facts, it then follows that if $\bar{K} > 1/\Xi$, then $\hat{K} = \bar{K}$. The condition $\bar{K} > 1/\Xi$ becomes, after using the definitions of $\bar{K}$ and Ξ, the condition in Corollary 4.□

Proof of Proposition 9

Using the solution in (30) and that for $W(w)$ in (59) with $\hat{d} = +\infty$, $V(w) > W(w)$ becomes:

$$c_0^* > rw - \frac{r-\rho}{\alpha r} - \frac{\alpha\sigma_P^2}{2r} - \frac{\alpha\sigma_T^2}{2\,(r+2\phi)}. \tag{63}$$

Using the solution for c_t^* in (28) gives, after cancelling terms:

$$\frac{4K}{e^{rd^*}-1} + \alpha\left(\frac{\sigma_P^2}{r^2} + \frac{\sigma_T^2}{(r+\phi)^2}\right)\left(e^{rd^*}+1\right) < 2\alpha\left(\frac{\sigma_P^2}{r^2} + \frac{\sigma_T^2}{r(r+2\phi)}\right).$$

After rearranging (and especially using (48) to replace for K), this gives the condition in (32).□

Proof of Corollary 3

Using the fact that $\hat{c}_0 = y_0 - \hat{s}_0$ and (57) with $\hat{d} = +\infty$, shows that:

$$\hat{c}_0 = rw_0 - \frac{r-\rho}{\alpha r} - \frac{\alpha}{2}\left(\frac{\sigma_P^2}{r} + \frac{\sigma_T^2}{r+2\phi}\right).$$

Then, for $\hat{s}_0 < s_0^*$, it must be that $\hat{c}_0 > c_0^*$, which using the expressions above is equivalent to condition (63) holding, which is true for the agent who chooses to be an inattentive saver.□

Appendix F - Hybrid consumption-savings plans

The problem to solve is:

$$\begin{aligned} Z(w) &= \max_{d,\{k_t\},\lambda} E\left[\int_0^d e^{-\rho t}u(\lambda y_t + \tilde{c}_t)dt + e^{-\rho d}Z(w')\right] \\ \text{s.t.} \quad w' &= e^{rd}\left(w - \int_0^d e^{-rt}\tilde{c}_t dt\right) - K + e^{rd}(1-\lambda)\int_0^d e^{-rt}y_t dt + \frac{y^{P'} - e^{rd}y^P}{r} + \frac{y^{T'} - e^{rd}y^T}{r+\phi}, \end{aligned}$$

where the constraint is derived by combining the law of motion for assets, the definition of wealth, and the consumption rule $c_t = \lambda y_t + \tilde{c}_t$.

The first-order condition with respect to $\tilde{c}_t$ is:

$$E\left[u'(\lambda y_t + \tilde{c}_t)\right] = e^{(r-\rho)(d-t)} E\left[Z'(w')\right]. \tag{64}$$

Combining this condition at time 0 with that at some $t < d$ gives:

$$\begin{aligned}
u'(\lambda y_0 + \tilde{c}_0) &= e^{(r-\rho)t} E\left[u'(\lambda y_t + \tilde{c}_t)\right] \Leftrightarrow \\
-\alpha\lambda y_0 - \alpha\tilde{c}_0 &= (r-\rho)t - \alpha\tilde{c}_t - \alpha\lambda E[y_t] + \frac{\alpha^2\lambda^2}{2} Var[y_t] \Leftrightarrow \\
\tilde{c}_t &= \tilde{c}_0 + \lambda(1 - e^{-\phi t}) y_0^T + \frac{(r-\rho)t}{\alpha} + \frac{\alpha\lambda^2}{2} Var[y_t]. \qquad (65)
\end{aligned}$$

The second line follows from the CARA form of the utility function and the normality of income, and the third line from rearranging. I guess that the value function has the same exponential form as before: $Z(w) = -A\exp(-\alpha r w)$, with the coefficient A to be determined. The envelope theorem condition is:

$$Z'(w) = e^{(r-\rho)d} E\left[Z(w')\right] \tag{66}$$

The first order condition (64) at time 0, combined with this condition, leads to:

$$\begin{aligned}
e^{-\alpha(\lambda y_0 + \tilde{c}_0)} &= \alpha r A e^{-\alpha r w} \Leftrightarrow \\
\tilde{c}_0 &= -\frac{\ln(\alpha r A)}{\alpha} + rw - \lambda y_0 \qquad (67)
\end{aligned}$$

Now, using the solutions for $\tilde{c}_t$ in (65) and $\tilde{c}_0$ in (67) to substitute for the consumption terms in the budget constraint, much algebra shows that w' is normally distributed with:

$$\begin{aligned}
E[w'] &= w + \frac{\ln(\alpha r A)(e^{rd} - 1)}{\alpha r} + \frac{\left(1 + dr - e^{rd}\right)(r-\rho)}{\alpha r^2} - \frac{\alpha\lambda^2 e^{rd}}{2} \int_0^d e^{-rt} Var[y_t] dt - K \\
Var[w'] &= Var\left[\frac{y^{P'} - e^{rd} y^P}{r} + \frac{y^{T'} - e^{rd} y^T}{r + \phi} + e^{rd}(1-\lambda) \int_0^d e^{-rt}\left(y_t^P + y_t^T\right) dt\right]
\end{aligned}$$

Going back to the envelope theorem condition (66), this becomes, using the guess for the value function and the normality of value function:

$$-\alpha r w = (r-\rho)d - \alpha r E[w'] + \frac{\alpha^2 r^2}{2} Var[w'].$$

Using the result for $E[w']$, this becomes, after rearranging:

$$\ln(\alpha r A) = \frac{r-\rho}{r} + \frac{\alpha^2 r \lambda^2 e^{rd}}{2(e^{rd} - 1)} \int_0^d e^{-rt} Var[y_t] dt + \frac{\alpha r K}{(e^{rd} - 1)} + \frac{\alpha^2 r^2 Var[w']}{2(e^{rd} - 1)}.$$

The fact that A does not depend on the state w_t or on any component of income, validates the guess for the value function. The optimal λ and d are then the solution to the problem:

$$\min_{d,\lambda}\left\{\frac{\alpha\lambda^2 e^{rd}\int_0^d e^{-rt}Var[y_t]dt+2K+\alpha r Var[w']}{(e^{rd}-1)}\right\}$$

Since none of the expressions in this objective function depend on the sate of the economy, the optimal d and λ are independent of the state. Using the properties of the stochastic processes for y^T and y^P, $Var[y_t]$ and $Var[w']$ can be easily (but tediously) evaluated. The results in Table 1 are found by solving this minimization numerically.

Appendix G - Extraordinary events

Proof of Proposition 10

Define accumulated ordinary income shocks as:

$$\varepsilon_t = e^{rt}\int_0^t e^{-rs}\left(\frac{\sigma_P}{r}dz_s^P+\frac{\sigma_T}{r+\phi}dz_s^T\right).$$

It follows from the properties of Wiener processes that $\varepsilon_t \sim N(0,\sigma^2(e^{2rt}-1)/2r)$, where $\sigma^2 = \sigma_P^2/r^2+\sigma_T^2/(r+\phi)^2$ and that $d\varepsilon_t = r\varepsilon_t + \left(\frac{\sigma_P}{r}dz_t^P+\frac{\sigma_T}{r+\phi}dz_t^T\right)$. Then, defining the non-stochastic part of income as $\bar{w}_t = w_t - \varepsilon_t$, the law of motion for total wealth implies that non-stochastic wealth evolves by $d\bar{w}_t = (r\bar{w}_t - c_t)dt$.

Denote the value function in terms of $\bar{w}_{D+t}$, and in terms of how long has elapsed since the last planning date t by $J(\bar{w}_{D+t},t)$. This is an optimal stopping problem. The Bellman equation is:

$$\begin{aligned}(r+\delta)J(\bar{w}_{D+t},t) \quad &= \quad \max_{c_{D+t},d}\{u(c_{D+t})+\frac{\delta}{2}E_0\left[J(\bar{w}_{D+d}+\varepsilon_{D+d}+u-K,0)+J(\bar{w}_{D+d}+\varepsilon_{D+d}-u-K,0)\right]\\ &\quad\quad +J_w(\bar{w}_{D+t},t)(r\bar{w}_{D+t}-c_{D+t})+J_t(\bar{w}_{D+t},t)\},\end{aligned}$$

and the value matching condition at the optimal stopping date is:

$$J(\bar{w}_{D+d^*},d^*) = E_0\left[J(\bar{w}_{D+d^*}+\varepsilon_d-K,0)\right].$$

To solve this problem, I guess that $J(\bar{w},t) = -(A(t)/\alpha r)\exp(-\alpha r\bar{w})$, where $A(t)$ is a time varying function to be determined. The first-order condition for the optimal choice of c_t is:

$$\begin{aligned}u'(c_{D+t}) &= J_w(\bar{w}_{D+t},t) \Leftrightarrow\\ c_{D+t} &= r\bar{w}_{D+t}-\frac{\ln(A(t))}{\alpha}.\end{aligned} \quad (68)$$

The second line follows by using the CARA utility function and the guess for the value function.

The envelope theorem condition with respect to $\bar{w}_t$ is:

$$\begin{aligned}\delta J_w(\bar{w}_{D+t},t) &= \frac{\delta}{2}E_0\left[J_w(\bar{w}_{D+d}+\varepsilon_{D+d}+u-K,0)+J_w(\bar{w}_{D+d}+\varepsilon_{D+d}-u-K,0)\right]\\ &\quad +J_{ww}(\bar{w}_{D+t},t)\frac{\ln(A(t))}{\alpha}+J_{wt}(\bar{w}_{D+t},t),\end{aligned}$$

where I used (68) to replace out consumption. Using the guess for the value function gives the differential equation in Proposition 9. Note that $A(t)$ does not depend on $\bar{w}_{D+t}$, which confirms the guess on the form of the value function. Using the guess to replace for the value function in the value matching condition gives the boundary condition in Proposition 9. Finally, d is chosen at date 0 to solve:

$$J(\bar{w}_D,0)=\max_d\left\{-\frac{A(0)}{\alpha r}e^{-\alpha r\bar{w}_D}\right\}.$$

Thus, d^* can be found by minimizing $A(0)$. Since $A(0)$ does not depend on the state of wealth, neither does d^*.□

Appendix H - State-contingent adjustment

If $B(a_t,y_t)$ denotes the value function for this problem, and T is the stopping time for adjustment, the consumer's problem is

$$B(a,y) = \max_{\{c_t\},T} E_0\left\{\int_0^T e^{-\rho t}u(c_t)dt+e^{-\rho T}B(a',y')\right\} \tag{69}$$

$$\text{s.t.}\quad da_t = (ra_t-c_t+y_t)dt \text{ for } 0\le t\le T, \tag{70}$$

$$a' = a_T-K. \tag{71}$$

While c_t is set conditional on information at time 0, the choice of T depends on the evolution of the state of the economy. To obtain a tractable solution, I make the same assumptions as in Section 5, namely that the felicity function is of the CARA form and that income follows an ARMA(1,1) process. I further assume that $r=\rho$ in order to simplify the algebra, but this is inessential. The proof of the following Proposition is available on request:

Proposition *In the state-contingent adjustment problem with CARA preferences and ARMA(1,1) income, the value function is*

$$B(a,y)=e^{-\alpha c_0^*}b(q)=-\frac{e^{-\alpha c_0^*}}{\alpha r}+e^{-\alpha c_0^*}\sum_{n=0}^{\infty}z_nq^n \tag{72}$$

where $q=w-c_0^*/r$, and the sequence of coefficients z_n in the power series are given by:

$$z_{n+2} = \left[\prod_{i=0}^{\frac{n-2}{2}} \frac{r\,(2i-1)}{\sigma^2(i+1)(2i+1)}\right] z_0, \textit{ if } n \textit{ is even,} \tag{73}$$

$$z_{n+2} = \left[\prod_{i=0}^{\frac{n-3}{2}} \frac{2ri}{\sigma^2(2i+3)\,(i+1)}\right] z_1, \textit{ if } n \textit{ is odd.} \tag{74}$$

Consumption between adjustments is $c_0^* = r(w_0 - q^*)$. *Consumers adjust when* $q_t = w_t - c_0^*/r$ *hits the bounds* $\underline{q}$ *or* $\bar{q}$, *and then they change consumption to* c_T^*. *The optimal choices of* q^*, $\underline{q}$, *and* $\bar{q}$, *and the parameters* z_0 *and* z_1 *are determined by solving the system:*

$$b(\bar{q}) = -e^{-\alpha(r\bar{q}-rK-q^*)}b(q^*) \tag{75}$$

$$b(\underline{q}) = -e^{-\alpha(r\underline{q}-rK-q^*)}b(q^*) \tag{76}$$

$$b'(\bar{q}) = \alpha r e^{-\alpha(r\bar{q}-rK-q^*)}b(q^*) \tag{77}$$

$$b'(\underline{q}) = \alpha r e^{-\alpha(rq-rK-q^*)}b(q^*) \tag{78}$$

$$b(q^*) = -\frac{b'(q^*)}{\alpha}. \tag{79}$$

Equations (75)-(79) are familiar from Ss models. Equations (75) and (76) are the value matching conditions stating that the pre-adjustment value of hitting the boundaries of the inaction region must equal the post-adjustment value function. Conditions (77) and (78) are the smooth pasting conditions ensuring that the value function pastes smoothly at the different values in the range of q, and equation (79) is the first-order condition determining the optimal choice of consumption following an adjustment.

Caballero (1995) assumes that agents only update their individual consumption of nondurables infrequently, whenever the deviation between their current consumption level and the permanent income hypothesis level of consumption exceeds a given value in absolute level. Proposition 10 differs from Caballero's model, but if the utility function were quadratic, the two models would be identical.

References

Abowd, John, and David Card (1989) "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, vol. 57, pp. 411-445.

Akerlof, George A. (1991) "Procastrination and Obedience," *American Economic Review Papers and Proceedings*, vol. 81 (2), pp. 1-19.

Akerlof, George A., and Janet L. Yellen (1985) "A Near Rational Model of the Business Cycle with Wage and Price Inertia," *Quarterly Journal of Economics*, vol. 100, pp. 823-838.

Alessie, Rob, Arie Kapteyn, and Annamaria Lusardi (1999) "Saving after Retirement: Evidence from Three Different Surveys," *Labour Economics*, vol. 6 (2), pp. 277-310.

Ameriks, John, Andrew Caplin and John Leahy (2003a) "Wealth Accumulation and the Propensity to Plan," *Quarterly Journal of Economics*, vol. 118 (3), pp. 1007-1047.

Ameriks, John, Andrew Caplin and John Leahy (2003b) "The Absent-Minded Consumer," unpublished, New York University.

Andrews, Donald W. K., and Christopher J. Monahan (1992) "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica*, vol. 60 (4), pp. 953-966.

Attanasio, Orazio P. and Guglielmo Weber (1995) "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey," *Journal of Political Economy*, vol. 103 (6), pp. 1121-1157.

Bargh, John, and Tania Chartrand (1999) "The Unbearable Automaticity of Being," *American Psychologist*, vol. 54, pp. 462-479.

Barro, Robert J. (1977) "Unanticipated Money Growth and Unemployment in the United States," *American Economic Review*, vol. 67, pp. 101-115.

Billingsley, Patrick (1995) *Probability and Measure*, 3rd edition, New York, Wiley & Sons.

Blanchard, Olivier J. and Danny Quah (1989) "The Dynamic Effects of Aggregate Demand and Aggregate Supply Disturbances," *American Economic Review*, vol. 79, pp. 655-673.

Blinder, Alan S., and Angus Deaton (1985) "The Time Series Consumption Function Revisited," *Brookings Papers on Economic Activity*, vol. 2, pp. 465-511.

Browning, Martin and M. Dolores Collado (2001) "The Response of Expenditures to Anticipated Income Changes: Panel Data Estimates," *American Economic Review*, vol. 91 (3) pp. 681-692.

Caballero, Ricardo J. (1995) "Near-Rationality, Heterogeneity, and Aggregate Consumption" *Journal of Money, Credit and Banking*, vol. 27 (1), pp. 29–48.

Campbell, John Y. (1987) "Does Savings Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica*, vol. 55, pp. 1249-1273.

Campbell, John Y. and Angus Deaton (1989) "Why is Consumption So Smooth?" *Review of Economic Studies*, vol. 56, pp. 357-374.

Campbell, John Y. and N. Gregory Mankiw (1989) "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual*, vol. 4, pp. 185-216.

Campbell, John Y. and N. Gregory Mankiw (1990) “Permanent Income, Current Income and Consumption,” *Journal of Business and Economic Statistics*, vol. 8 (3), pp. 265-279.

Carroll, Christopher D. (2003) “Macroeconomic Expectations of Households and Professional Forecasters,” *Quarterly Journal of Economics*, vol. 118 (1), pp. 269-298.

Carroll, Christopher D., and Martin Sommer (in progress) “Epidemiological Expectations and Consumption Dynamics," Johns Hopkins University, unpublished.

Chetty, Raj, and Adam Szeidl (2003) “Consumption Commitments and Asset Prices,” Harvard University, unpublished.

Christiano, Lawrence J., Martin Eichenbaum and David Marshall (1991) “The Permanent Income Hypothesis Revisited,” *Econometrica*, vol. 59 (2), pp. 397-423.

Cochrane, John H. (1989) “The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives,” *American Economic Review*, vol. 79, pp. 319-337.

Cook, Stephen A., and Robert A. Reckhow (1973) “Time Bounded Random Access Machines,” *Journal of Computer and Systems Sciences*, vol. 7, pp. 354-375.

Cover, Thomas M., and Joy A. Thomas (1991) *Elements of Information theory*, New York, Wiley Press.

Deaton, Angus (1987) “Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?” in *Advances in Econometrics, Fifth World Congress*, vol. 2, ed. Truman Bewley, Cambridge, Cambridge University Press.

Deaton, Angus (1992) *Understanding Consumption*, Oxford, Oxford University Press.

Dynan, Karen (2000) “Habit Formation in Consumer Preferences: Evidence from Panel Data,” *American Economic Review*, vol. 90, pp. 391-406.

Evans, George, and Seppo Honkapohja (2001) *Learning and Expectations in Macroeconomics*, Princeton, Princeton University Press.

Evans, George W., and Garey Ramey (1992) “Expectation Calculation and Macroeconomic Dynamics,” *American Economic Review*, Vol. 82 (1), pp. 207-224.

Fischer, Stanley, Ratna Sahay, and Carlos A. Vegh (2002) “Modern Hyper- and High Inflations,” *Journal of Economic Literature*, vol. 150 (3), pp. 837-880.

Flavin, Marjorie A. (1981) “The Adjustment of Consumption to Changing Expectations about Future Income,” *Journal of Political Economy*, vol. 89, pp. 974-1009.

Friedman, Milton (1957) *A Theory of the Consumption Function*, Princeton, Princeton University Press.

Gabaix, Xavier and David Laibson (2001) “The 6D Bias and the Equity Premium Puzzle,” *NBER Macroeconomics Annual 2001*, pp. 257-311.

Gabaix, Xavier and David Laibson (2002) “Bounded Rationality and Directed Cognition,” Harvard University, unpublished.

Gali, Jordi (1991) “Budget Constraints and Time Series Evidence on Consumption,” *American Economic Review,* vol. 81, pp. 1238-1253.

Goodfriend, Marvin (1992) "Information-Aggregation Bias," *American Economic Review*, vol. 82, pp. 508-519.

Hall, Robert E. (1978) "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, vol. 86, pp. 971-987.

Hall, Robert E. and Frederick S. Mishkin (1982) "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, vol. 50, pp. 461-481.

Hayashi, Fumio (1997) *Understanding Saving: Evidence from the U.S. and Japan*, Cambridge, MIT Press.

Hayashi, Fumio and Christopher A. Sims (1983) "Nearly Efficient Estimation of Time Series Models with Predetermined, but not Exogenous, Instruments," *Econometrica*, vol. 51 (3), pp. 783-798.

Hsieh, Chang-Tai (2003) "Do Consumers React to Anticipated Income Shocks? Evidence from the Alaska Permanent Fund," *American Economic Review*, vol. 93 (1), pp. 397-405.

Hurst, Erik "Grasshoppers, Ants and Pre-Retirement Wealth: A Test of Permanent Income Consumers," University of Chicago, unpublished.

Laibson, David (1997) "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, vol. 62, pp. 443-477

Lucas, Robert E., Jr. (1973) "Some International Evidence on Output-Inflation Trade-Offs," *American Economic Review*, vol. 63, p. 326-334.

Lusardi, Annamaria (1999) "Information, Expectations, and Savings." in *Behavioral Dimensions of Retirement Economics*, ed. Henry Aaron, Brookings Institution Press/Russell Sage Foundation, New York, pp. 81-115.

Lusardi, Annamaria (2002) "Explaining Why So Many Households Do Not Save," Dartmouth College, unpublished.

Lynch, Anthony W. (1996) "Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Return," *Journal of Finance*, vol. 51 (4), pp. 1479-1497.

MaCurdy, Thomas E. (1982) "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," *Journal of Econometrics*, vol. 18, pp. 83-114.

Mankiw, N. Gregory (1985) "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly," *Quarterly Journal of Economics*, vol. 100, pp. 529-537.

Mankiw, N. Gregory and Ricardo Reis (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics*, vol 117 (4).

Mankiw, N. Gregory and Ricardo Reis (2003) "Sticky Information: A Model of Monetary Nonneutrality and Structural Slumps," in *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, eds. Phillipe Aghion, Romain Frydman, Joseph Stiglitz and Michael Woodford, Princeton, Princeton University Press.

Mankiw, N. Gregory, Ricardo Reis and Justin Wolfers (2003) "Disagreement in Inflation Expectations," *NBER Macroeconomics Annual*, forthcoming.

Meghir, Costas, and Luigi Pistaferri (2003) "Income Variance Dynamics and Heterogeneity," *Econometrica*, forthcoming.

Mishkin, Frederick (1983) *A Rational Expectations Approach to Macroeconometrics: Testing Policy Ineffectiveness and Efficient-Markets Models*, Chicago, University of Chicago Press.

Moreira, Marcelo J. (2003) "A Conditional Likelihood Ratio Test for Structural Models," *Econometrica*, vol. 71 (4), pp. 1027-1048.

Moscarini, Giuseppe (2003) "Limited Information Capacity as a Source of Inertia," *Journal of Economic Dynamics and Control*, forthcoming.

Mullainathan, Sendhil (2002) "A Memory-Based Model of Bounded Rationality", *Quarterly Journal of Economics*, vol. 117 (3), pp. 735-774.

Muth, John (1960) "Optimal Properties of Exponentially Weighted Forecasts," *Journal of the American Statistical Association*, vol. 55, pp. 290-306.

Newey, Whitney and Kenneth West (1987) "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, vol. 55, pp. 703-708.

O'Donoghue, Ted and Matthew Rabin (1999) "Doing it Now or Later," *American Economic Review*, vol. 89 (1), pp. 103-124

Pagan, Adrian (1984) "Econometric Issues in the Analysis of Regressions with Generated Regressors," *International Economic Review*, vol. 25 (1), pp. 221-247.

Parker, Jonathan (1999) "The Reaction of Household Consumption to Predictable Changes in Social Security Taxes," *American Economic Review,* vol. 89 (4), pp. 959-973.

Parker, Jonathan (2001) "The Consumption Risk of the Stock Market," *Brookings Papers on Economic Activity*, vol. 2, pp. 279-348.

Pischke, Jorn-Steffen (1995) "Individual Income, Incomplete Information, and Aggregate Consumption," *Econometrica*, vol. 63 (4), pp. 805-840.

Priestley, M. B. (1981) *Spectral Analysis and Time Series*, San Diego, Academic Press.

Ramsey, Frank P. (1928) "A Mathematical Theory of Saving," *Economic Journal*, vol. 38, pp. 543-559.

Reis, Ricardo (2003a) "Inattentive Investors," Harvard University, unpublished.

Reis, Ricardo (2003b) "Inattentive Producers," Harvard University, unpublished.

Reis, Ricardo (2003c) "Inattentiveness in Equilibrium," Harvard University, unpublished.

Rosen, Sherwin (1990) "Comment on: How do the Elderly Form Expectations? An Analysis of Responses to New Information," in *Issues in the Economics of Aging*, David Wise, ed., Chicago, The University of Chicago Press.

Rozanov, Yu A. (1967) *Stationary Random Processes*, San Francisco, Holden Day.

Sargent, Thomas J. (2001) *The Conquest of American Inflation*, Princeton, Princeton University Press.

Shannon, Claude (1948) "A Mathematical Theory of Communication," *Bell Systems Technical Journal*, vol. 27, pp. 379-423 and 623-656.

Shapiro, Matthew D. , and Joel Slemrod (1995) "Consumer Response to the Timing of Income: Evidence from a Change in Tax Withholding," *American Economic Review*, vol. 85 (1), pp. 274-283.

Shea, John (1995) "Union Contracts and the Life-Cycle / Permanent Income Hypothesis," *American Economic Review*, vol. 85 (1), pp. 186-200.

Simon, Herbert A. (1978) "Rationality as Process and as Product of Thought," *American Economic Review Papers and Proceedings*, vol. 68 (2), pp. 1-16.

Sims, Christopher A. (2003) "Implications of Rational Inattention," *Journal of Monetary Economics*, vol. 50 (3), pp. 665-690.

Souleles, Nicholas S. (1999) "The Response of Household Consumption to Income Tax Refunds," *American Economic Review,* vol. 89 (4), pp. 947-958.

Souleles, Nicholas S. (2000) "College Tuition and Household Savings and Consumption," *Journal of Public Economics*, vol. 77 (2), pp. 185-207.

Stock, James H., Jonathan Wright, and Motohiro Yogo (2002), "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, vol. 20, pp. 518-529.

Tobin, James (1982) "Money and Finance in the Macroeconomic Process," *Journal of Money, Credit and Banking*, May, vol. 14 (2), pp. 171-204.

van Emde-Boas, S. (1990) "Machine Models and Simulations," in *Handbook of Theoretical Computer Science*, vol. A, ed. J. Van Leeuwen ed., Camdbrige, MIT Press, pp. 1-66.

Wilson, Andrea (2003) "Bounded Memory and Biases in Information Processing," unpublished, Prnceton University.

Wolak, Frank A. (1989) "Testing Inequality Constraints in Linear Econometric Models," *Journal of Econometrics*, vol. 41 pp. 205-235.

Yakoboski, Paul and Jennifer Dickemperer (1997) "Increased Saving but Little Planning: Results of the 1997 Retirement Confidence Survey," *EBRI Issue Brief* 191.

Table 1: Optimal hybrid consumption-savings plans

Panel A: Inattentiveness(d)

	K = \$30	K = \$100	K = \$250	K = \$500	K = \$1000
r = 0.5%	13	24	36	50	67
r = 1.5%	10	16	23	31	41
r = 4%	6	9	12	15	20

Panel B: Optimal share of income shocks consumed (λ)

	K = \$30	K = \$100	K = \$250	K = \$500	K = \$1000
r = 0.5%	0.02	0.03	0.03	0.04	0.04
r = 1.5%	0.06	0.06	0.07	0.08	0.09
r = 4%	0.13	0.14	0.16	0.17	0.19

Notes: The remaining parameters were set at the benchmark values: ϕ=0.487, α=2/6926, $\sigma_{P=}$45, $\sigma_{T=}$1962.

Table 2: Extraordinary events and the length of inattentiveness

Panel A: Inattentiveness

	u = \$500	u = \$2,500	u = \$5,000
δ = 1/8	10	10	11
δ = 1/20	9	9	9
δ = 1/40	8	9	9

Panel B: Probability of planning in response to an extraordinary event

	u = \$500	u = \$2,500	u = \$5,000
δ = 1/8	71%	71%	75%
δ = 1/20	36%	36%	36%
δ = 1/40	18%	20%	20%

Notes The remaining parameters were set to match the benchmark values: r=1.5%, ϕ=0.487, α=2/6926, $\sigma^2=(45/r)^2+[1962/(r+\phi)]^2-\delta u^2$. The costs of planning K were set at \$30 so that without extraordinary events, the agent plans every 8 quarters.

Table 3: Regressing log consumption growth on news on income growth

	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
Panel A. Predictors of $\Delta\ln(Y_t)$										
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$	$.288^{***}$	$.077^{***}$	$.072^{***}$	$.104^{***}$	.029	-.034	.032	.006	-.035	-.035
	(.042)	(.032)	(.027)	(.029)	(.035)	(.038)	(.032)	(.022)	(.032)	(.028)
	Restricted Least Squares estimates									
	.287	.084	.084	.084	.023	.001	.001	.001	0	0
	F-test: 7.20^{***}	Adj. R^2: .334		F-test 1st stage	3.55^{***}	Adj. R^2: .062 1st stage		W_{IN}: 4.71		W_{INU}: 26.88
	(.000)				(.004)			(.701)		(.000)
Panel B. Predictors of $\Delta\ln(Y_t)$										
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$,	$.279^{***}$	$.082^{***}$	$.050^{*}$	$.102^{***}$	.059	-.019	.054	$.059^{***}$	-.003	-.003
$\ln(C_{t-1}/Y_{t-1}),\ldots,\ln(C_{t-5}/Y_{t-5})$	(.049)	(.034)	(.026)	(.030)	(.037)	(.044)	(.033)	(.033)	(.039)	(.037)
	Restricted Least Squares estimates									
	.278	.080	.079	.079	.055	.032	.032	.032	0	0
	F-test: 5.35^{***}	Adj. R^2: .262		F-test 1st stage	5.69^{***}	Adj. R^2: .196 1st stage		W_{IN}: 7.58		W_{INU}: 9.12
	(.000)				(.000)			(.428)		(.332)
Panel C. Predictors of $\Delta\ln(Y_t)$										
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$,	$.265^{***}$	$.075^{**}$	.044	$.089^{***}$	.059	-.019	$.058^{*}$	$.066^{**}$	-.001	-.005
$\ln(C_{t-1}/Y_{t-1}),\ldots,\ln(C_{t-5}/Y_{t-5})$,	(.050)	(.037)	(.027)	(.033)	(.033)	(.046)	(.033)	(.033)	(.042)	(.038)
$r_{t-1},\ldots,r_{t-5}$										
	Restricted Least Squares estimates									
	.263	.073	.070	.070	.053	.035	.035	.035	.002	0
	F-test: 4.66^{***}	Adj. R^2: .229		F-test 1st stage	3.96^{***}	Adj. R^2: .187 1st stage		W_{IN}: 8.11		W_{INU}: 8.08
	(.000)				(.000)			(.374)		(.426)

Notes: These are the estimates of the system of two equations: (first stage) $y_t = \Delta\ln(Y_t) - E_{t-1}[\Delta\ln(Y_t)]$, and (second stage) $\Delta\ln(C_{t+1}) = \text{const.} + \beta_0 y_{t+1} + \beta_1 y_t + \ldots + \beta_9 y_{t-8} + \tilde{u}_t$. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. In brackets below the estimates are Newey-West standard errors corrected for heteroskedasticity and autocorrelation up to 8 lags. The F-test is on the significance of the regression, W_{IN} tests the inattentive consumers model, and W_{INU} tests the model with uniformly staggered adjustment. In brackets below the test statistics are the p-values.

Table 4: Simultaneous estimation of income forecasts and consumption as a function of income surprises

	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
Panel A. Predictors of $\Delta\ln(Y_t)$										
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$	.288***	.077***	.072***	.104***	.029	-.034	.032	.006	-.035	-.035
	(.042)	(.032)	(.027)	(.029)	(.035)	(.038)	(.032)	(.022)	(.032)	(.028)
	Adj. R^2: .334		Adj. R^2: .062 1st stage							
Panel B. Predictors of $\Delta\ln(Y_t)$	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$,	.298***	.098***	.063**	.108***	.062*	-.015	.039	.046	-.010	-.023
$\ln(C_{t-1}/Y_{t-1}),\ldots,\ln(C_{t-5}/Y_{t-5})$	(.047)	(.034)	(.028)	(.029)	(.037)	(.043)	(.033)	(.030)	(.037)	(.035)
	Adj. R^2: .310		Adj. R^2: .111 1st stage							
Panel C. Predictors of $\Delta\ln(Y_t)$	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
$\Delta\ln(Y_{t-1}),\ldots,\Delta\ln(Y_{t-5})$,	.303***	.104***	.066**	.115***	.057	-.023	.042	.041	-.013	-.017
$\ln(C_{t-1}/Y_{t-1}),\ldots,\ln(C_{t-5}/Y_{t-5})$,	(.046)	(.033)	(.028)	(.026)	(.037)	(.042)	(.032)	(.030)	(.036)	(.035)
$r_{t-1},\ldots,r_{t-5}$										
	Adj. R^2: .333		Adj. R^2: .090 1st stage							

Notes: These are the estimates of the system of two equations: $y_t = \Delta\ln(Y_t) - E_{t-1}[\Delta\ln(Y_t)]$, $\Delta\ln(C_{t+1}) = \text{const.} + \beta_0 y_{t+1} + \beta_1 y_t + \ldots + \beta_9 y_{t-8} + \tilde{u}_t$, using the Mishkin procedure described in Reis (2003d). ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. In brackets below the estimates are Newey-West standard errors corrected for heteroskedasticity and autocorrelation up to 8 lags.

Table 5: Excess Sensitivity and Hand-to-Mouth Behavior in the Inattentiveness Model

Panel A. IV regressions Instruments for $\Delta\ln(Y_{t+1})$:	Estimates (standard errors)	Adj. R^2	F-stat. 1st stage	J-stat. (p-value)
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$	.157	.165	.80	.81
	(.229)		(.525)	(.848)
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$,	.166	.167	.64	2.33
$\mathrm{Ln}(C_{t-9}/Y_{t-9}),\ldots,\ln(C_{t-12}/Y_{t-12})$	(.180)		(.743)	(.940)
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$,	.049	.073	.80	4.53
$\ln(C_{t-9}/Y_{t-9}),\ldots,\ln(C_{t-12}/Y_{t-12})$	(.139)		(.650)	(.952)
$r_{t-9},\ldots,r_{t-12}$				

Panel B. Weak Instruments Instruments for $\Delta\ln(Y_{t+1})$:	Estimates		Test statistics (p-values)		
	OLS	LIML	A-R	Moreira	LM
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$	.226	.147	1.040	.252	.186
			(.904)	(.887)	(.667)
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$,	.226	.148	2.542	.305	.164
$\mathrm{Ln}(C_{t-9}/Y_{t-9}),\ldots,\ln(C_{t-12}/Y_{t-12})$			(.960)	(.950)	(.685)
$\Delta\ln(Y_{t-9}),\ldots,\Delta\ln(Y_{t-12})$,	.226	-.057	4.085	.086	.057
$\ln(C_{t-9}/Y_{t-9}),\ldots,\ln(C_{t-12}/Y_{t-12})$			(.982)	(.930)	(.812)
$r_{t-9},\ldots,r_{t-12}$					

Notes: The dependent variable in all regressions is $\Delta\ln(C_{t+1})$. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. The estimates use the Hayashi and Sims (1983) procedure with an estimated MA(9) to forward-filter the data. In Panel A, the J-stat. refers to the Hansen-Sargan statistic for testing the over-identifying restrictions associated with the validity of the instruments.

Table 6: Rational Expectations vs. Hand-to-mouth vs. Inattentiveness

Panel A. Regression Estimates

$\Delta \ln(Y_{t-1})$

Const.	E_t-E_{t-1}	E_{t-1}-E_{t-2}	E_{t-2}-E_{t-3}	E_{t-3}-E_{t-4}	E_{t-4}-E_{t-5}	E_{t-5}-E_{t-6}	E_{t-6}-E_{t-7}	E_{t-7}-E_{t-8}	E_{t-8}-E_{t-9}	E_{t-9}-E_{t-10}	E_{t-10}-E_{t-11}	E_{t-11}
.005**	.320*	.620**	.521***	.289*	.104	.536	.790	.816***	.680	1.010	-.498	.034
(.002)	(.163)	(.255)	(.179)	(.166)	(.161)	(.367)	(1.023)	(.315)	(.688)	(.543)	(1.063)	(.453)
Restricted Estimates												
.005	.394	.394	.394	.314	.314	.314	.314	.314	.314	.314	0	0

Unrestricted Adjusted R^2: .090 Restricted Adjusted R^2: .055

Panel B. Tests of the alternative models

Model	Test statistics (p-values)	Accept/Reject (5% significance level)
Rational Expectations (Hall):	72.60 (.000)	Reject
Hand-to-mouth (Campbell-Mankiw):	18.80 (.043)	Reject
Inattentive consumers:	18.10 (.080)	Accept
Inattentive consumers and savers:	15.09 (.128)	Accept
Inattentive consumers with uniformly staggered adjustment:	21.67 (.027)	Reject

Notes: ***, ** and * denote statistical significance at the 1%, 5% and 10% levels respectively. All standard errors are corrected for heteroskedasticity and autocorrelation using a Newey-West procedure. Panel B displays Wald test statistics and asymptotic p-values.

Table 7: The excess smoothness ratio

Panel A: Estimates

Method	Lags	ψ	Standard Errors
Bartlett window	5	.704	.065
	10	.662	.088
	20	.671	.129
AR-HAC	2	.679	.088
	5	.651	.115
	10	.643	.159
Andrews-Monahan	5	.515	.047
	10	.559	.073
	20	.584	.107

Panel B: Predictions of the inattentiveness model

Estimates of the weights $\Phi(i)$:	ψ
From news regressions in Table 6, with predictors:	
- lagged income	.660
(restricted coefficients)	.570
- lagged income and savings	.498
(restricted coefficients)	.480
- lagged income, savings and interest rates	.494
(restricted coefficients)	.473

Notes: The estimates of the excess smoothness ratio (ψ) use data on the change of log aggregate consumption from 1954 to 2002. The different methods used to obtain estimates of the spectrum at frequency zero were: a Bartlett kernel estimator with window length 5, 10 and 20; a parametric AR-HAC estimate using an AR with lags 2, 5 and 10; a Andrews-Monahan (1992) estimator which pre-whitens the data using an AR(1) and then uses a Bartlett kernel with window lengths 5, 10 and 20. Standard errors are obtained by the delta method, and using the result that asymptotically $Var(h_{\Delta C}(\omega))=(4/3)*(M/N)*h_{\Delta C}(\omega)$ for the Bartlett kernel, where M is the window length, and N is the number of observations (see Priestley, 1981, pages 457-461).

Figure 1: Optimal inattentiveness with savings plans

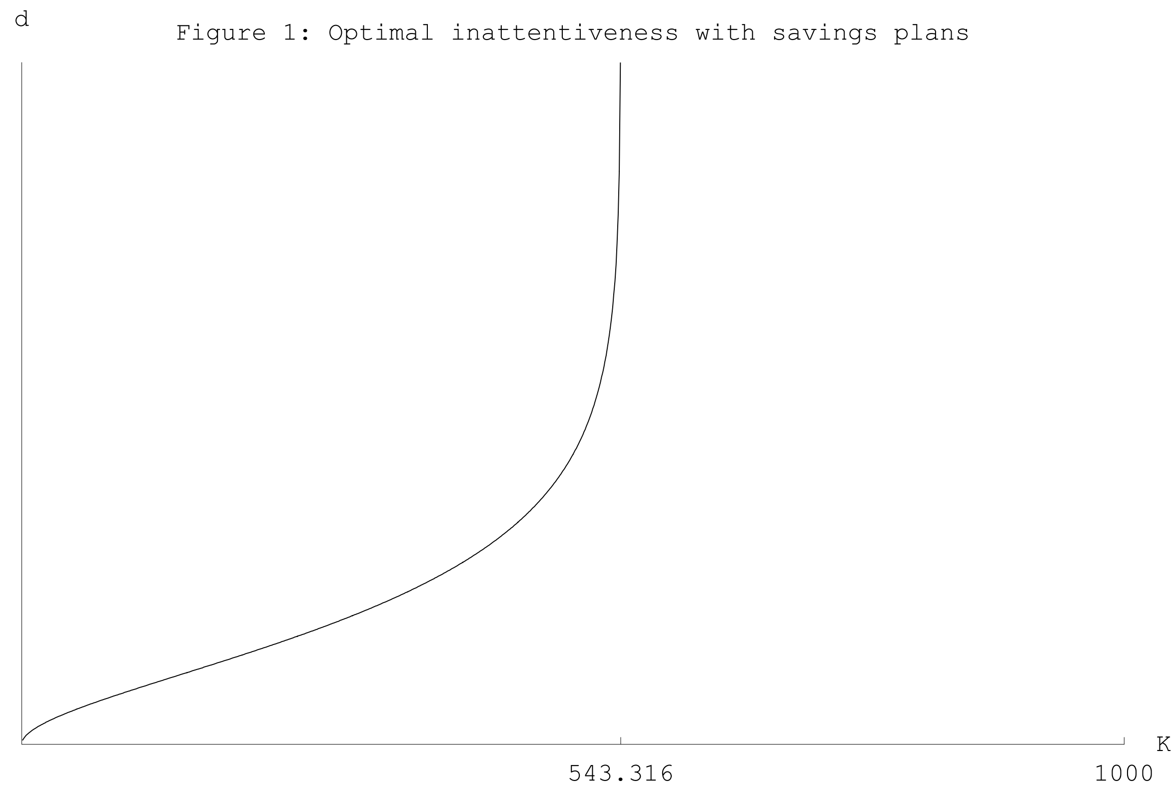

Figure 2. Impulse response of log aggregate consumption to a permanent shock

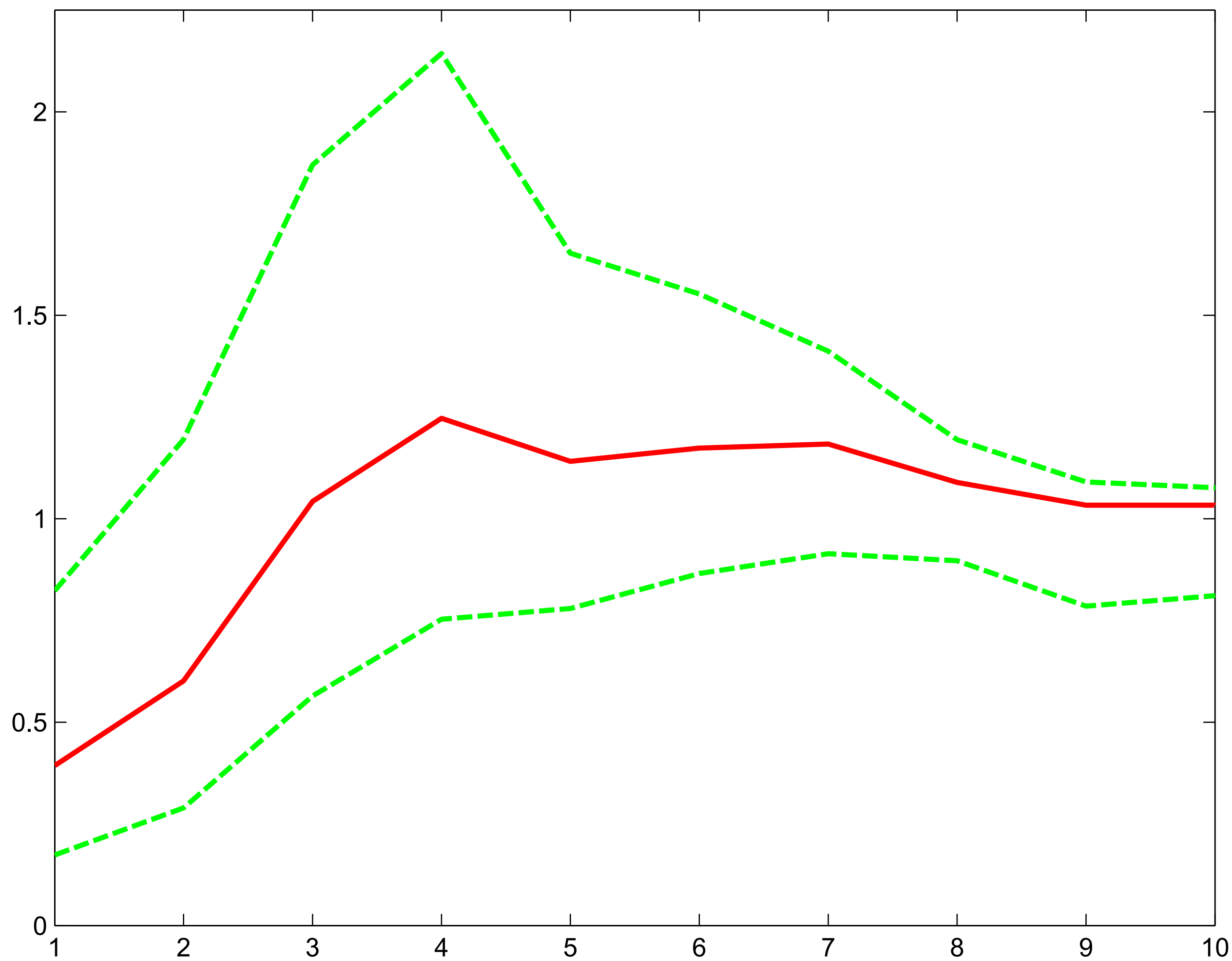

Figure 3. Estimates of the (ratio) inattentiveness weights

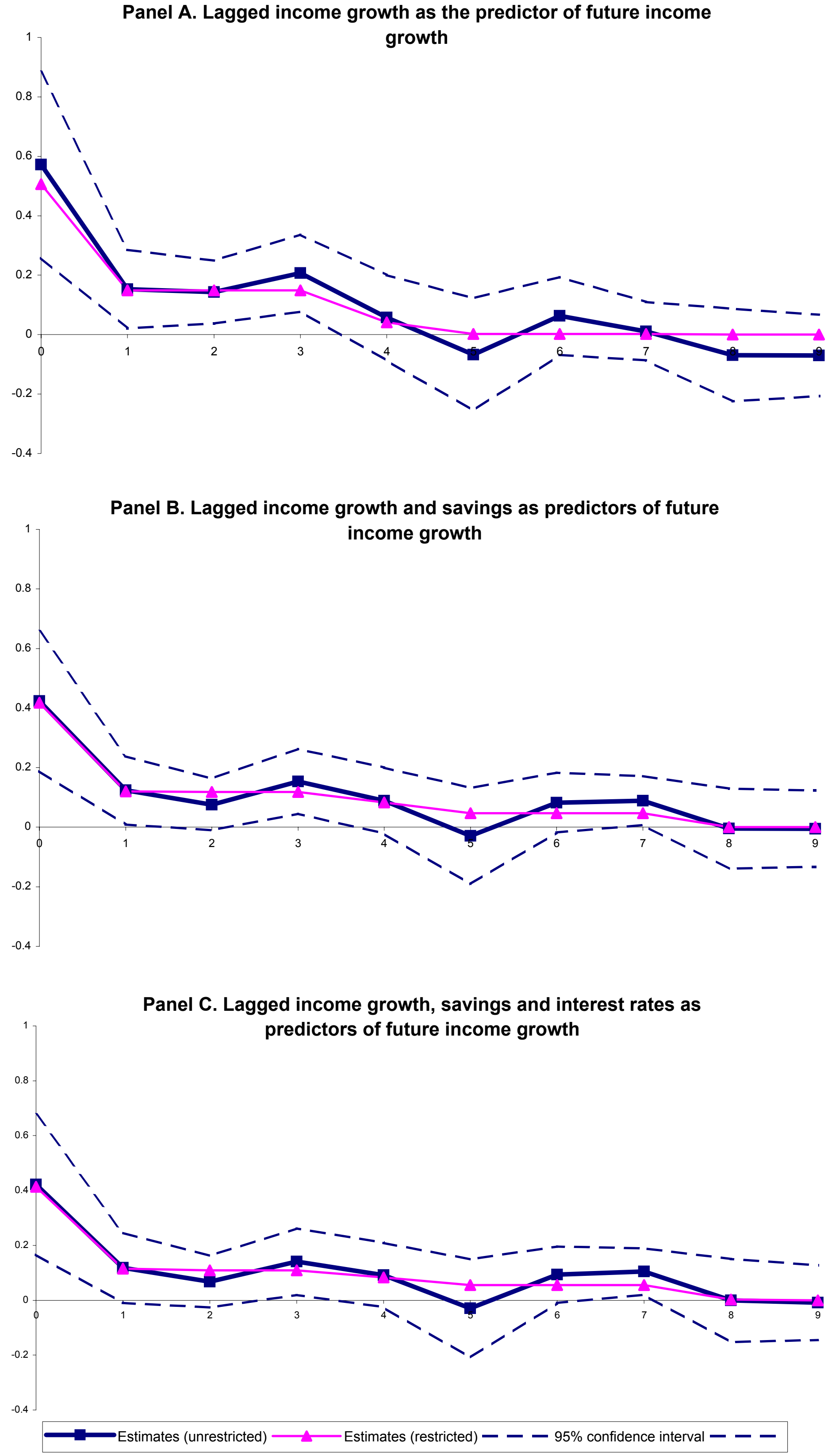

Figure 4. Estimates of the cumulative inattentiveness weights

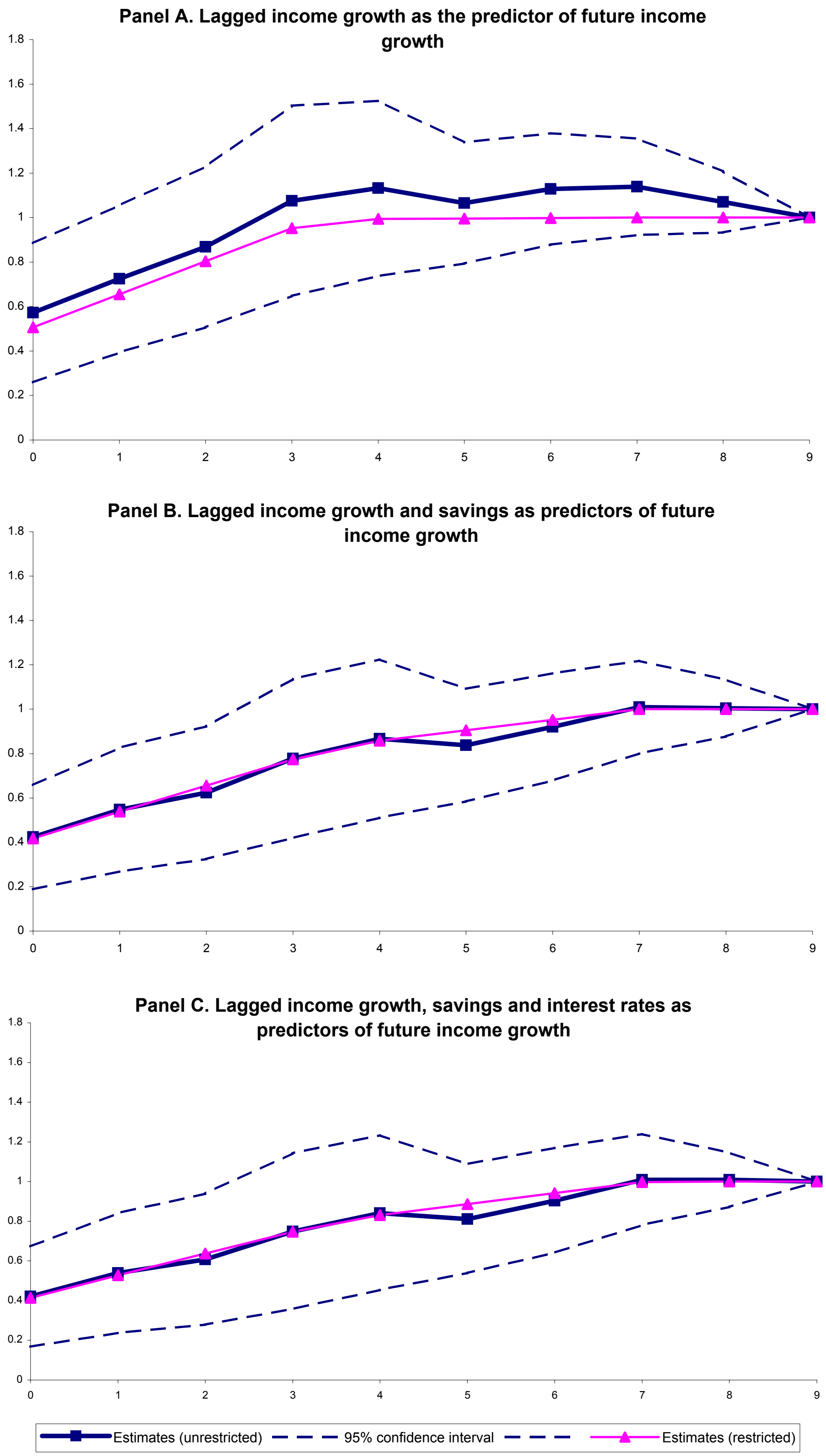

Figure 5. Model-Predicted and Actual Normalised Spectral Densities