Real Exchange Rates, International Trade and Macroeconomic Fundamentals

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Abstract

Many studies on real exchange rates have found little relationship between macroeconomic fundamentals and real exchange rates in the short- to medium-run. This is widely-known as the 'exchange rate disconnect puzzle'. This paper derives a new equilibrium condition between real exchange rates, international trade and macroeconomic fundamentals for a wide class of general equilibrium models, allowing for goods market frictions with proportional transport costs and non-traded goods, and a wide variety of asset market structures. The key is to link the price indices through prices of traded goods. If consumption bundle is a constant-elasticity-of-substitution bundle between the home traded good, foreign imports and the non-traded good, then there is an equilibrium relationship between real exchange rates and relative composite-good consumptions plus two other factors: the ratio of bilateral trade flows and the ratio of domestic traded good consumptions. These additional trade factors arise from bilateral intratemporal allocations. The intratemporal elasticity of substitution between goods plays a key role in real exchange rate determination.

I present empirical evidence that this trade-based representation of real exchange rates significantly improves on the standard consumption-ratio formula in understanding actual real exchange rates movements. In particular, it identifies preference shocks or incomplete markets as possible explanations for the Backus-Smith (1993) puzzle by breaking the tight relationship between real exchange rates and relative consumptions.

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1 Introduction

A large number of studies over the past twenty years have discovered virtually no relationship between real exchange rates and macroeconomic fundamentals, such as relative consumptions, money supplies, GDPs, etc. This is widely known as the 'exchange rate disconnect puzzle'. Obstfeld and Rogoff (2000) describe the situation as, '*Exchange rates are remarkably volatile relative to any models we have of underlying fundamentals, such as interest rates, outputs, money supplies and no model seems to be very good at explaining exchange rates even ex post*'. Frankel and Rose (1995) also state, *"We, like much of the profession, are doubtful of the value of further time series modeling of exchange rates at high or medium frequencies using macroeconomic models"*. In addition, many empirical studies have documented the purchasing power parity puzzles. Real exchange rates are extremely volatile compared with macroeconomic variables. Real exchange rates are also highly persistent. Consensus half-lives of real exchange rates are between three and five years, implying a long time for innovations to be arbitraged away.

This paper studies real exchange rate determination for a wide class of general equilibrium models. I show that real exchange rate can be expressed as a function of international trade flows, relative domestic traded goods consumptions, and relative composite goods consumptions. I call this equilibrium condition the *trade-based representation* of real exchange rates. I demonstrate empirically that for a wide variety of cross-country pairs, actual real exchange rates are highly correlated with the trade-based representation. In particular, for the major trading partners with the U.S, the correlations between actual real exchange rates and their trade-based representations are over 0.8. Thus, I find that in the data, real exchange rates are in fact closely connected to international trade flows and macroeconomic fundamentals, as predicted by economic theory.

I derive the trade-based representation of real exchange rates as follows. The real exchange rate is an intra-temporal relationship between national price levels. The price indices across two countries can be quite different, due to non-traded goods or different compositions of goods within the consumption bundle. I relate these two price indices through the prices of traded goods across countries. I set up a class of general equilibrium models of international trade with three basic assumptions. (i) There are multiple goods. Each country is only endowed with one of the traded goods. (ii) Utility is strictly increasing and strictly concave in consumption. The consumption aggregator is homogeneous of degree 1 with respect to the goods within the consumption bundle, strictly concave, time-separable and satisfies Inada conditions. (iii) Prices are perfectly flexible. All countries take prices as given in competitive markets.

A key result for any general equilibrium models satisfying the above assumptions is a no-arbitrage pricing condition for each traded good. Each country i is indifferent between selling its own traded good i at home at the domestic price p_{ii}, or selling the same good i at another country

j at a price p_{ji} that takes into account the proportional transport costs. Assuming only η fraction per unit of goods shipped is delivered at the foreign border, the no-arbitrage condition is $p_{ii} = p_{ji}\eta$. Without any transport costs, this no-arbitrage condition is the 'Law of One Price'.

From the no-arbitrage pricing condition, the real exchange rate can then be expressed as a ratio of two relative prices: the price of a traded good to price index in country i versus the price of the same traded good to price index in country j. These relative prices are related to the marginal utilities of that traded good to the relative marginal utilities of the consumption bundles. I then derive the theoretical equilibrium condition between real exchange rates, international trade and macroeconomic variables. If the country i's composite consumption c_i is a constant-elasticity-of-substitution aggregator with respect to consumptions of the home traded good d_i, imports from country j m_{ij} and non-traded good n_i with $\frac{1}{\rho}$ as the intratemporal elasticity of substitution between all goods, then the real exchange rate is

$$\ln e = \rho \left(\frac{1}{2} \ln \frac{d_j}{d_i} + \frac{1}{2} \ln \frac{m_{ji}}{m_{ij}} + \ln \frac{c_i}{c_j} \right) \tag{1}$$

Equation (1) is an equilibrium condition between real exchange rates and relative composite good consumptions $\frac{c_i}{c_j}$ plus two other factors: the ratio of bilateral trade flows $\frac{m_{ji}}{m_{ij}}$ and the ratio of domestic good consumptions across country pairs $\frac{d_j}{d_i}$. The additional trade factors enter from bilateral intratemporal allocations of traded goods to their respective consumption bundles. The relative compositions of the consumption bundles and the intra-temporal elasticity of substitution between goods $\frac{1}{\rho}$ are crucial for real exchange rate determination. I call the right-hand-side of Equation (1) the trade-based representation of real exchange rate.

The trade-based representation in Equation (1) is valid for any economy that satisfies the three key assumptions. Hence it is valid for any specification of intertemporal preferences, intertemporal trade and asset markets, goods market frictions of proportional transport costs or non-traded goods. It is also robust for any specification of production possibilities and the sources of shocks. These other features of the economy determine *how* real exchange rates and real quantities move over time. My theoretical analysis is that no matter what are the sources of the fluctuations, real exchange rates and real quantities should co-move together according to the equilibrium condition in Equation (1).

I evaluate the fit of equilibrium condition (1) using data from 1980 to 1998 for 13 major industrialized economies. I find that the trade-based representation fit the data well. For close trading partners with the U.S, such as Canada, Japan, U.K, France and Germany, the raw data correlations for the trade-based representation with the actual real exchange rates in log levels are over 0.8. For all 78 bilateral pairs in the sample, over 50% of them have over 0.7 raw data correlations for actual real exchange rates and the trade-based representation. These correlations remain high even when

I filter out the long-run components in the data using HP-filter or band-pass-filter.

It is useful to contrast my trade-based representation with the consumption-based representation derived by Backus and Smith (1993). They show that if asset markets are complete and if there are no preference shocks, then real exchange rates and relative consumptions must satisfy the equilibrium condition:

$$\ln e = \gamma \ln \frac{c_i}{c_j} + \text{constant} \tag{2}$$

where γ is the coefficient of risk aversion in the utility function, and the constant term represents the ratio of initial Pareto-Negishi weights of the social planner across countries i and j. I call the right-hand-side of Equation (2) the *consumption-based representation* of real exchange rates.

I show that the consumption-based representation have a low, often negative correlation with actual real exchange rates for most bilateral country pairs in the OECD[1], while the trade-based representation have positive correlations with actual real exchange rates for most bilateral pairs in the sample. I also show that the consumption-based representation is not as volatile as actual real exchange rates unless γ is above 2.5, whereas the trade-based representation matches the volatility of actual real exchange rates if the elasticity of substitution between goods ρ equals 1. This is because additional trade factors add to the volatility of the trade-based representation of real exchange rates. Finally, I show that the consumption-based representation is not cointegrated with real exchange rates, while the trade-based representation is cointegrated with real exchange rates with a long-run relationship.

Why are the empirical results for the trade-based representation a significant improve compared to the consumption-based representation in Backus and Smith (1993)? I demonstrate that under complete markets and no preference shocks, the trade-based representation of real exchange rate is the same as the consumption-based representation. It is because Pareto optimality requires each traded good to be allocated such that the ratio of marginal utilities of each traded good to all countries equals to a constant that corresponds to the initial social planner's weights. Therefore, the empirical failure of the consumption-based representation (i.e. the Backus-Smith (1993) puzzle) relative to the trade-based representation indicates that either asset markets are incomplete or there are different preference shocks across countries. With incomplete markets or preference shocks, the trade-based representation can help explain the low correlation between real exchange rates and relative consumptions as the additional trade factors are negatively-covaried with relative consumptions.

[1] Backus and Smith (1993) provide empirical evidence that the correlations between real exchange rates and relative consumptions are close to zero on average and even quite negative for certain countries. This empirical evidence of low correlations between relative consumptions and real exchange rates is also documented in other studies such as Chari, Kehoe and McGrattan (2002), Ravn (2001), etc. This is known as the Backus-Smith puzzle.

These results indicate that theoretically and empirically, real exchange rates are closely connected to international trade flows and macroeconomic fundamentals. However, while the analysis shows that intra-temporal trade in goods market is related to real exchange rate determination, there are still open questions about the true source of real exchange rate fluctuations and the inter-temporal properties of real exchange rates. In particular, we need better understanding of the asset markets and the dynamics of international trade to enhance our understanding for the inter-temporal properties of real exchange rates.

Most studies of real exchange rate determination in general equilibrium models have found a close theoretical link between real exchange rates and relative consumptions. This arises from the first order conditions for agents choosing between domestic and foreign goods, or choosing between domestic and foreign assets (e.g. Backus and Smith 1993, Chari, Kehoe and McGrattan (2002), Atkeson Alvarez and Kehoe (2002), Sercu and Uppal (2003), etc). Other research papers that study deviations from purchasing power parity usually assume some type of goods market frictions or asset market frictions. For example, Dumas (1992) and Obstfeld and Rogoff (2000) suggest that transport costs in international trade plays a key role for volatile and persistent deviations from parity. Betts and Devereux (2000) and Chari, Kehoe and McGrattan (2002) suggest that monetary shocks interacting with sticky goods prices can generate volatile and persistent real exchange rate fluctuations. Alvarez, Atkeson and Kehoe (2002) suggest that endogenously segmented asset markets leads to volatile and persistent exchange rates. There are fewer studies that have offered theoretical explanations to the Backus-Smith puzzle. Chari, Kehoe and McGrattan (2002) suggest that some forms of asset market frictions are required to break the link between real exchange rates and relative consumptions.

Section 2 presents a class of general equilibrium models of international trade. The natural implication from the model is a no-arbitrage pricing condition for each traded good. I derive the trade-based representation for real exchange rate from this no-arbitrage pricing condition. Section 3 empirically evaluates how this trade-based representation performs in understanding actual real exchange rate movements, especially the PPP volatility and persistence puzzles and the Backus-Smith puzzle. Section 4 concludes.

2 General Equilibrium Model of International Trade and Real Exchange Rates

This section sets up a class of general equilibrium models of international trade. With three basic assumptions of (i) complete specialization of traded goods endowments, (ii) standard assumption on preferences and (iii) flexible prices, a natural implication is a no-arbitrage pricing condition for each traded good. The individual goods prices can aggregate up to form the price index for each country. I derive the real exchange rates as the ratio of national price indices of composite good consumptions.

2.1 The Model: Key Assumptions

Time is discrete $t = 1, ..., T$.[2] At time t, the state s_t is realized and it can take on any element from the finite set S. Let $s^t = (s_1, ..., s_t)$ denote the history for state s_t in time t. There are I countries, $i = 1...I$, located in I different geographical locations. There is a representative agent in each country.

Multiple Goods. Complete Specialization of Traded Good Endowment. There are I perishable traded consumption goods. Country i is only endowed with one traded good $y_i(s^t) > 0$. The traded good endowment process $y_i(s^t)$ is drawn on a positive finite set Y_i.

There can also be a non-traded good $y_{iN}(s^t) \geq 0$ endowed in each country i. The non-traded good endowment process is drawn on a non-negative finite set Y_{iN}. Each country consumes a composite good which is a consumption bundle of the I traded goods and the non-traded good (the composite good comprises of $I + 1$ goods). To consume the other non-endowed traded goods, country i would need to purchase the other traded goods from abroad.

Preferences The utility function $U_i(c_i(s^t)) : \Re_+ \to \Re$ of the representative agent in country i at date t state s^t can be country-specific and can take on the most general form given composite good consumption $c_i \geq 0$. Assume U_i is strictly increasing and strictly concave in c_i (i.e. $U_i'(c_i) > 0$, $U_i''(c_i) < 0$). This general form of utility can include the standard time-separable CRRA utility functions, non-time separable utilities such as habit persistence, non-state separable utilities such as Kreps-Porteus of Epstein-Zin preferences, etc[3].

The consumption aggregator $c_i(s^t) = c_i(d_i(s^t), \{m_{ij}(s^t)\}_{j\neq i, j\in\{1,...,I\}}, n_i(s^t)) : \Re_+ \times \Re_+^{I-1} \times \Re_+ \to \Re_+$ consists of date t state s^t consumption of the traded good endowed in their own country ($d_i(s^t) \geq 0$), consumption of the foreign traded good endowed by country j ($m_{ij}(s^t) \geq 0$), consumption of the non-traded good endowed in country i ($n_i(s^t) \geq 0$). Assume the consumption

[2]It can be a static economy with $T = 1$ or dynamic economy with $T > 1$ with asset market trading. I focus the analysis of dynamic economies in this paper. If asset markets are complete or exogenously incomplete, T can be finite or infinite. If asset markets are endogenously incomplete subject to solvency constraints similar to Alvarez-Jermann (2000), I require T to be infinite for reputation to play a role in determining allocations.

[3]This general form of preference also includes utility functions with more arguments such as utility with leisure or money-in-the-utility functions.

bundle is homogeneous of degree 1[4]. Assume $c_i(s^t)$ is strictly concave with respect to each of its components[5], twice differentiable and satisfies Inada conditions with respect to the imported good (i.e. $\lim_{m_{ij}\to 0} \frac{\partial c_i(s^t)}{\partial m_{ij}(s^t)} = \infty$,$\lim_{m_{ij}\to\infty} \frac{\partial c_i(s^t)}{\partial m_{ij}(s^t)} = 0$). Assume the consumption bundle $c_i(s^t)$ is time-separable with respect to $\{d_i(s^t), \{m_{ij}(s^t)\}_{j\neq i}, n_i(s^t)\}$ and does not depend on past consumptions of each of the components $\{d_i(s^{t-\tau}), \{m_{ij}(s^{t-\tau})\}_{j\neq i}, n_i(s^{t-\tau})\}$ for $\tau > 0$.

Flexible Prices. Goods prices are perfectly flexible. All countries take prices as given in competitive markets.

2.2 Goods Market and Asset Market

In each country, there are $I+1$ goods market open for each I traded goods and the non-traded goods[6]. Goods market shipping can be subject to a proportional (iceberg) transport costs. For each unit of good shipped, only $\eta(s^t) \in (0,1]$ fraction of the good is delivered at the foreign border. Country i can buy a certain traded good j either in the country i (home) market after the good is shipped to home country, or country i can buy it directly in country j's (foreign) market and ship the good back home itself.[7]

Let $x_{ij}(s^t) \geq 0$ be the export of traded good i from country i to country j. The consumptions of each good within the consumption bundle of country i is as follows

$$\begin{aligned} d_i(s^t) &= y_i(s^t) - \sum_{j\neq i} x_{ij}(s^t), \qquad i \neq j & (3) \\ m_{ij}(s^t) &= \eta(s^t) x_{ji}(s^t), \qquad i \neq j & (4) \\ n_i(s^t) &= y_{iN}(s^t) & (5) \end{aligned}$$

where (3) is the market clearing condition of traded good i, (4) is import-export relationship for traded good j with transport costs and (5) is the market clearing condition for non-traded good i.

[4] For scalar value ξ,

$$c_i(\xi d_i(s^t), \xi\{m_{ij}(s^t)\}_{j\neq i}, \xi n_i(s^t)) = \xi c_i(d_i(s^t), \{m_{ij}(s^t)\}_{j\neq i}, n_i(s^t))$$

This assumption is required so that the expenditure on the consumption bundle is the same as the sum of expenditure on the individual goods $p_i(s^t)c_i(s^t) = p_{ii}(s^t)d_i(s^t) + \sum_{j\neq i} p_{ij}(s^t)m_{ij}(s^t) + p_{iN}(s^t)n_i(s^t)$. Differentiate with respect to ξ and evaluate the derivative at $\xi = 1$:

$$c_i(s^t) = \frac{\partial c_i(s^t)}{\partial d_i(s^t)} d_i(s^t) + \sum_{j\neq i} \frac{\partial c_i(s^t)}{\partial m_{ij}(s^t)} m_{ij}(s^t) + \frac{\partial c_i(s^t)}{\partial n_i(s^t)} n_i(s^t)$$

[5] The strict concavity for $c_i(s^t)$ with respect to $d_i(s^t), m_{ij}(s^t), n_i(s^t)$ is to guarantee that there is always a positive amounts of exports and imports. If $c_i(s^t)$ is linear (e.g. $c_i(s^t) = d_i(s^t) + \sum_{j\neq i} m_{ij}(s^t) + n_i(s^t)$), there exists a cone of no shipping similar to Dumas (1992) and the real exchange rates would fluctuate between $\eta(s^t)$ and $\frac{1}{\eta(s^t)}$.

[6] There are a total of $I(I+1)$ markets in the world.

[7] Apart from the iceberg transport costs, there are no further limitations to arbitrage for traded goods.

The notations for goods prices are as follows. Let $p_{ij}(s^t)$ be the price of traded good j in the country i market. Let $p_{iN}(s^t)$ be the price of non-traded good i in country i.

There can be a wide variety of asset market structures. I assume the net asset holdings in state s^t are summarized by the wealth accumulated $W_i(s^t)$.[8] This can encompass complete markets, endogenously incomplete markets or exogenously incomplete markets[9].

Each country i solves the following maximization problem in time t state s^t

$$\max_{\{d_i, m_{ij}, n_i\}_{i \neq j}} U_i(c_i(s^t)) \tag{6}$$

where

$$\begin{aligned} c_i(s^t) &= c_i(d_i(s^t), \{m_{ij}(s^t)\}_{j \neq i}, n_i(s^t)) \\ m_{ij}(s^t) &\geq 0 \end{aligned}$$

subject to the sequential budget constraints for country i

$$p_{ii}(s^t)d_i(s^t) + \sum_{j \neq i} p_{ij}(s^t)m_{ij}(s^t) + p_{iN}(s^t)n_i(s^t) = p_{ii}(s^t)y_i(s^t) + p_{iN}(s^t)y_{iN}(s^t) + W_i(s^t) \tag{7}$$

Assume existence of equilibrium. The definition of competitive equilibrium is as follows.

Definition of Equilibrium: An equilibrium is a sequence of allocations $\{c_i(s^t), d_i(s^t), m_{ij}(s^t), n_i(s^t)\}_{i \neq j, i,j=1....I}$, a sequence of goods prices $\{p_{ij}(s^t),\, p_{iN}(s^t)\}_{i,j=1....I}$ such that
(i) Each country i solves the maximization problem (6).
(ii) Goods market clearing is satisfied for each traded good and non-traded good (i.e. Equations (3) to (5)).

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8 The details of the asset markets are as follows. Assume there are H securities available. There is no cost in trading securities in the asset markets. Let $q_h(s^t)$ be the price of the security h in terms of consumption bundle c_i. at time t state s^t with a payoff of $a_h(s^{t+1})$ in terms of consumption bundle c_i at time $t+1$ state s^{t+1}. Let $b_{ih}(s^t)$ be country i's holding of security h at time t state s^t. The net asset holdings or wealth accumulated $W_i(s^t)$ equals $\sum_{h=1}^{H}[b_{ih}(s^{t-1})a_h(s^t) - b_{ih}(s^t)q_h(s^t)]$. The assets can also be in terms of other bundles (e.g. composite good j: c_j) or in terms of specific goods within the consumption bundle. In a static economy, there are no asset market trades and $W_i = 0$.

9 Asset holdings are subject to a general form for $K \leq H$ borrowing limits $\phi_{ik}(b_{i1}(s^t), ..., b_{iH}(s^t)) \geq 0$ for $k = 1...K$ depending on the asset market structures, where $\phi_{ik} : \Re^H \rightarrow R_+$ is a linear function. If asset markets are complete, there is a full set of state-contingent securities $H = S$. Asset holdings are subject to natural borrowing limits that never bind in equilibrium. If asset markets are endogenously incomplete similar to Alvarez and Jermann (2000), there are still $H = S$ securities available for trading, but asset holdings are subject to state-contingent endogenous borrowing constraints $B_i(s^t)$: $\phi_{ik}(b_{i1}(s^t), ..., b_{iH}(s^t)) = \sum_{h=1}^{H} \bar{b}_{ih}(s^t) - B_i(s^t) \geq 0$[10] and $K = 1$. If asset markets are exogenously incomplete, $H < S$. If there are $K \leq H$ additional borrowing or short-sale constraints of $\underline{B}_k$ asset holdings are restricted by the K constraints of $\phi_{ik}(b_{i1}(s^t), ..., b_{iH}(s^t)) = b_{ik}(s^t) - \underline{B}_k \geq 0$ for $k = 1...K$.

2.3 No-arbitrage Pricing Condition and Trade-based Representation of Real Exchange Rates

In this section, I focus on the subset of equilibrium conditions from goods market optimization.

Proposition 1: In an equilibrium with strictly positive trade flows at all states (i.e. $m_{ij}(s^t) > 0$), the no-arbitrage condition $p_{ii}(s^t) = p_{ji}(s^t)\eta(s^t)$ holds.

Proposition 1 is a no-arbitrage pricing condition for any traded good i. This no-arbitrage condition implies all countries face common prices for the same good adjusted for transport costs; and country i is indifferent between selling traded good i at home or selling traded good i in country j at a price that takes into account the transport costs.

The intuition for Proposition 1 is as follows. Suppose $p_{ii}(s^t) < p_{ji}(s^t)\eta(s^t)$, then country i or country j would have an incentive to buy the traded good i in country i and sell traded good i in the country j market and make a profit. In this case, demand for traded good i increases and the price of good traded i in country i increases until it equilibrates to $p_{ji}(s^t)\eta(s^t)$. A similar argument holds for the opposite case $p_{ii}(s^t) > p_{ji}(s^t)\eta(s^t)$. If there is no transport cost ($\eta(s^t) = 1$), this no-arbitrage goods market pricing condition is the 'Law of One Price'.

Since our goal is to understand real exchange rate movements as the ratio of price indices of the composite goods of two countries, I shall construct below the price index $p_i(s^t)$ for each country i from individual goods prices. The consumption-based price index for country i $p_i(s^t)$ is defined as the minimum expenditure for the unit consumption bundle c_i, given individual goods prices $\{p_{ii}(s^t), p_{ij}(s^t), p_{iN}(s^t)\}$.[11] Since the consumption bundle $c_i(s^t)$ is CES with respect to $\{d_i(s^t), \{m_{ij}(s^t)\}_{j\neq i}, n_i(s^t)\}$, I can express the price index for country i as follows.

$$p_i(s^t) \equiv \frac{p_{ii}(s^t)d_i(s^t) + \sum_{j\neq i} p_{ij}(s^t)m_{ij}(s^t) + p_{iN}(s^t)n_i(s^t)}{c_i(s^t)}$$

From this definition of the price index, the sequential budget constraint (7) can be rewritten as

$$p_i(s^t)c_i(s^t) = p_{ii}(s^t)y_i(s^t) + p_{iN}(s^t)y_{iN}(s^t) + W_i(s^t) \tag{8}$$

The price of a consumption bundle can be found from the first order condition with respect to $c_i(s^t)$ in the new budget constraint (8).

$$p_i(s^t)\sigma_i(s^t) = U_i'(c_i(s^t)) \tag{9}$$

The real exchange rate between countries i and j is defined as the ratio of price indices across

[11] Definition from Obstfeld and Rogoff (1996).

countries[12]

$$e_{ij}(s^t) \equiv \frac{p_j(s^t)}{p_i(s^t)} = \frac{U_j'(c_j(s^t))}{U_i'(c_i(s^t))}\frac{\sigma_i(s^t)}{\sigma_j(s^t)} \tag{10}$$

To derive our equilibrium relationship between real exchange rates and allocations, our goal is to express the ratio of the Lagrange Multipliers of country i and country j's budget constraints $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ in terms of allocations. The two national price indices can be very different due to non-traded goods or different preferences within the consumption bundle. However, it is possible to link the two national price indices through prices of traded goods. The prices of traded good i sold in country i and country j can be found from the first order conditions of country i's problem with respect to d_i in budget constraint (7) and country j's problem with respect to m_{ji}

$$p_{ii}(s^t) = \frac{U_i'(c_i(s^t))}{\sigma_i(s^t)}\frac{\partial c_i(s^t)}{\partial d_i(s^t)}, \qquad \frac{p_{ii}(s^t)}{p_i(s^t)} = \frac{\partial c_i(s^t)}{\partial d_i(s^t)} \tag{11}$$

$$p_{ji}(s^t) = \frac{U_j'(c_j(s^t))}{\sigma_j(s^t)}\frac{\partial c_j(s^t)}{\partial m_{ji}(s^t)}, \qquad \frac{p_{ji}(s^t)}{p_j(s^t)} = \frac{\partial c_j(s^t)}{\partial m_{ji}(s^t)} \tag{12}$$

Equation (11) shows that if more traded good i is allocated to country i's bundle, then the relative price of traded good i to country i's price level decreases. Similarly, Equation (12) shows that if more traded good i is allocated to country j's bundle, then the relative price of traded good i to country j's price level decreases. I can then calculate the real exchange rate by applying the no-arbitrage pricing condition in Proposition 1 to (11) and (12). I arrive at the main proposition of this paper.

Proposition 2: The equilibrium condition between real exchange rates and allocations is

$$e_{ij}(s^t) = \left(\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)}\right)^{\frac{1}{2}} \left(\frac{\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)}\right)^{\frac{1}{2}} \tag{13}$$

$$= \left(\frac{p_{ii}(s^t)/p_i(s^t)}{\eta(s^t)p_{ji}(s^t)/p_j(s^t)}\right)^{\frac{1}{2}} \left(\frac{\eta(s^t)p_{ij}(s^t)/p_i(s^t)}{p_{jj}(s^t)/p_j(s^t)}\right)^{\frac{1}{2}} \tag{14}$$

I denote the right hand side of Proposition 2 as the trade-based representation of real exchange rates $\ln e^T$. It can be decomposed into two parts.

$$e^T_{ij}(s^t) = \underbrace{(\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)})^{\frac{1}{2}}}_{\text{alloc. of good } i} \underbrace{(\frac{\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)})^{\frac{1}{2}}}_{\text{alloc. of good } j}$$

The first part indicates how country i allocates traded good i intra-temporally between $d_i(s^t)$ and

[12]Suppose we have a model with nominal exchange rates ε_{ij} (price of currency i in terms of currency j), the real exchange rate between countries i and j is defined as $e_{ij} = \varepsilon_{ij}\frac{p_j}{p_i}$. In this model, I assume there is no money for any countries or they use the same cash for transactions.

$m_{ji}(s^t)$. The second part in Proposition 2 indicates how country j allocates traded good j intra-temporally between $m_{ij}(s^t)$ and $d_j(s^t)$. The '1/2' power is due to our assumption that the transport cost from country i to country j is the same as the transport cost from country j to country i (i.e. $\eta_{ij}(s^t) = \eta_{ji}(s^t) = \eta(s^t)$).

The key insight for Proposition 2 is to relate price indices across countries by price of traded goods.

$$e_{ij}(s^t) \equiv \frac{p_j(s^t)}{p_i(s^t)} = \frac{p_{ii}(s^t)/p_i(s^t)}{\eta(s^t)p_{ji}(s^t)/p_j(s^t)} = \frac{\eta(s^t)p_{ij}(s^t)/p_i(s^t)}{p_{jj}(s^t)/p_j(s^t)}$$

The second equality is the ratio of the price of traded good i relative to price index in country i versus the price of the same traded good i relative to the price index in country j, adjusted for transport cost. Similarly, the third equality is the ratio of the price of traded good j relative to price index in country i adjusted for transport cost, versus the price of the same traded good i relative to the price index in country j. Equation (14) in Proposition 2 links the prices of both traded goods to the price indices by substituting out the transport cost.

Existing studies on real exchange rates focus mostly on the relative price indices of consumption bundles $\frac{p_j(s^t)}{p_i(s^t)}$ and ignore the effects from price components for the specific goods (i.e. $\frac{p_{ii}(s^t)}{\eta(s^t)p_{ji}(s^t)}$ and $\frac{\eta(s^t)p_{ij}(s^t)}{p_{jj}(s^t)}$). While these price components for specific goods are equal to 1 in equilibrium from Proposition 1, they have implications in relating real exchange rates and the allocations of specific goods within the bundle.

The trade-based representation of real exchange rate in Proposition 2 is valid for any economy that satisfies the three key assumptions in Section 2.1. The composition of specific goods $\{d_i, m_{ij}, n_i\}$ within the consumption bundle c_i is crucial in the determination of real exchange rates. While real exchange rate is still defined as the ratio of marginal utilities of consumption bundles, the form of utility function U_i does not enter directly in Proposition 2. It affects real exchange rates only indirectly through the allocations. Hence the trade-based representation is robust to a wide class of time-consistent preferences, such as the HARA class of utility functions, non-time separable utilities (e.g. external or internal habit persistence), recursive utilities or non-state separable utilities. It is also robust to utility functions with non-separability with leisure or money-in-the-utility functions. Both countries can indeed have very different utility functions and the trade-based representation in Proposition 2 still holds.

This trade-based representation is robust to more general frictions of goods market of country-specific, time-varying proportional transport costs. It is also robust to different asset market structures such as complete markets, endogenously incomplete or exogenously incomplete markets. I shall explore in the next section how asset market structures relate to real exchange rates determination.

The trade-based representation also holds in a production economy with capital and labor because it is mainly a spot relationship from the intra-temporal optimal allocations in state s^t. It also holds in an economy with money and flexible prices. The no-arbitrage pricing condition would be $p_{ii} = \varepsilon_{ij} p_{ji} \eta$ and $\varepsilon_{ij} p_{jj} = p_{ij} \eta$ where ε_{ij} is the nominal exchange rate of currency i in terms of currency j. The real exchange rate e_{ij} is $e_{ij} = \varepsilon_{ij} \frac{p_j}{p_i}$. It is easy to verify the same equilibrium condition for real exchange rates in (13) from this no-arbitrage pricing condition. These other features of the economy determine *how* real exchange rates and real quantities move over time. My theoretical analysis is that no matter what are the sources of the fluctuations, real exchange rates and real quantities should co-move together according to the trade-based representation in Proposition 2.

2.4 Real Exchange Rates, Asset Market Structures and Preference Shocks

The derivation for the trade-based representation of real exchange rate in Proposition 2 does not rely on the first order conditions on asset holdings. This explains its robustness across a wide variety of asset market structures. Asset markets, however, affect real exchange rates indirectly through the allocations.

Complete Markets, No Preference Shocks. If markets are complete, there exists a social planner for optimal allocations. Let α_i be the social planner's initial weight on country i. The consumption-based representation of real exchange rate is

$$e^C = \frac{\alpha_j}{\alpha_i} \frac{U_j'(c_j)}{U_i'(c_i)}$$

By the first and second welfare theorems, the allocations in the planner's problem are the same as the decentralized market problem if the planner's weight is the inverse of the Lagrange Multiplier of the sequential budget constraint $\alpha_i = \frac{1}{\sigma_i(s^t)}$. In complete markets, the ratio $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ would correspond to the initial ratios of social planner's weights $\frac{\alpha_j}{\alpha_i}$. Pareto optimality requires each traded good to be allocated such that the marginal utilities of each traded good to countries i and j equal to a constant that corresponds to the initial ratio of planner's weights.

$$\frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))} \frac{\partial c_i(s^t)/\partial d_i(s^t)}{\eta(s^t)\partial c_j(s^t)/\partial m_{ji}(s^t)} = \frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))} \frac{\eta(s^t)\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)} = \frac{\sigma_i(s^t)}{\sigma_j(s^t)} = \frac{\alpha_j}{\alpha_i} = \text{constant}$$

Therefore if markets are complete and if there are no preference shocks, the trade-based representation e^T has the same value as the consumption-based representation of real exchange rate e^C

in Backus and Smith (1993).

$$e^C = \frac{\alpha_j}{\alpha_i}\frac{U_j'(c_j(s^t))}{U_i'(c_i(s^t))} = \frac{\partial c_i(s^t)/\partial d_i(s^t)}{\eta(s^t)\partial c_j(s^t)/\partial m_{ji}(s^t)} = \frac{\eta(s^t)\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)} = e^T$$

Under complete markets, there is complete consumption smoothing across traded goods and real exchange rate fluctuations should be due to non-traded goods. This confirms Balassa and Samuelson's (1964) proposition that if country i has a higher shock to traded good relative to non-traded goods and the prices of traded goods equalize across countries, the relative price of non-traded goods is higher and country i's real exchange rate appreciates.

Incomplete Markets, Preference Shocks. I shall demonstrate that preference shocks or incomplete markets can be possible explanations for the Backus-Smith's puzzle. Suppose the utility for country i in state s^t is $\delta_i(s^t)\frac{c_i(s^t)^{1-\gamma}}{1-\gamma}$ where $\delta_i(s^t)$ is the preference shock to country i's consumption bundle, then the real exchange rate is

$$e_{ij}(s^t) = \frac{\delta_j(s^t)}{\delta_i(s^t)}\frac{\sigma_i(s^t)}{\sigma_j(s^t)}\left(\frac{c_i(s^t)}{c_j(s^t)}\right)^{\gamma} \tag{15}$$

Empirically, the correlations between real exchange rates and relative consumptions are very low, even negative for many country-pairs. Equation (15) shows that the low correlation can be due to either different preference shocks across countries $\frac{\delta_j(s^t)}{\delta_i(s^t)}$ or incomplete asset markets for time-varying ratio of Lagrange Multipliers of the budget constraints $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$.[13] Combining (15) and Proposition 2 imply the following equilibrium condition

$$\frac{\delta_j(s^t)}{\delta_i(s^t)}\frac{\sigma_i(s^t)}{\sigma_j(s^t)} = \left(\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)}\right)^{\frac{1}{2}}\left(\frac{\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)}\right)^{\frac{1}{2}}\left(\frac{c_j(s^t)}{c_i(s^t)}\right)^{\gamma} \tag{16}$$

The higher the relative preference shocks $\frac{\delta_j(s^t)}{\delta_i(s^t)}$ for country j versus country i, the more country j desires to consume compared to country i and the higher the allocations for the traded goods i and j to country j's bundle versus to country i's bundle. This would be reflected in a increase in the relative ratios of $\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)}$ and $\frac{\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)}$.

If markets are endogenously incomplete, there also exists a social planner and the welfare theorems still hold (Alvarez and Jermann (2000)). However, the social planner's weights can be time-varying according to changes in promised utilities. Applying the no-arbitrage pricing condition

[13] Kehoe and Perri (2002) suggest that endogenously incomplete markets help to explain international business cycles in a single-good model with production. Corsetti, Dedola and Leduc (2002) demonstrate numerically that incomplete market with goods market frictions may explain the low correlation of real exchange rates and relative consumptions. Kollman (1995) shows that the fluctuations of consumption and real exchange rates are consistent with incomplete asset markets.

to (11) and (12),

$$\frac{\sigma_i(s^t)}{\sigma_j(s^t)} = \frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))} \left(\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\eta(s^t)\partial c_j(s^t)/\partial m_{ji}(s^t)} \right)$$

If country i has a good shock such that its enforcement constraint binds, its promised utility increases accordingly. Country i enjoys more consumption (i.e. $d_i(s^t), m_{ij}(s^t), n_i(s^t)$ increases) which lowers the marginal utility of consumption with respect to domestic traded goods (i.e. $U_i'(c_i(s^t))\frac{\partial c_i(s^t)}{\partial d_i(s^t)}$ decreases). $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ decreases. Therefore when country i has a good shock in y_i and y_{iN}, (1) country i increases its composite good consumption relative to country j, the marginal utility of composite good consumption decreases relative to that of country j (i.e. $\frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))}$ decreases) and price index for country i $p_i(s^t)$ decreases; (2) the price level $p_i(s^t)$ in country i can increase relative to country j because of a higher promised utility (i.e. $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ decreases). Since real exchange rates relate to both components of $\frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))}$ and $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ and these two forces work in opposite directions, real exchange rate for country i can either appreciate or depreciate. The optimal allocations would be such that that the ratio of marginal utilities of traded good i and traded good j for country i and country j reflect the changes of promised utilities for the countries.

If markets are exogenously incomplete, the optimal allocations would be such that the marginal utilities for traded good i and traded good j for country i and country j changes according to the wealth accumulated for each country. The higher the wealth accumulated for country i, the lower the ratio of both $\frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))}$ and $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$. There can be either real exchange rates appreciation or depreciation. Under exogenously incomplete markets, $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ would also be time-varying and the trade-based representation would differ in value from the consumption-based representation of real exchange rate (i.e. $\ln e^T \neq \ln e^C$). Time-varying ratio of Lagrange Multipliers of the budget constraint $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ due to incomplete markets can be a possible explanation for the low correlation between real exchange rates and relative consumptions across countries

2.5 Examples: Preliminaries for Empirical Analysis

We consider a special case of the consumption aggregator for our empirical analysis in the next section. Given a consumption bundle $c_i(s^t)$, we allow for arbitrary strictly increasing and strictly concave utility function $U_i(c_i)$. Suppose the composite consumption good is constant elasticity of substitution with the same elasticity of substitution between goods $\frac{1}{\rho}$ for both countries, i.e.

$$c_i(s^t) = [\omega_1 d_i(s^t)^{1-\rho} + \sum_{j\neq i, j\in\{1..I\}} \omega_2 m_{ij}(s^t)^{1-\rho} + \omega_3 n_i(s^t)^{1-\rho}]^{\frac{1}{1-\rho}} \tag{17}$$

where $\omega_1, \omega_2, \omega_3 > 0$ indicate the bias in the preference in consuming the domestically endowed traded good $d_i(s^t)$, versus the imported foreign good $m_{ij}(s^t)$ versus the non-traded good $n_i(s^t)$.

The trade-based representation of real exchange rate from (13) is

$$e^T(s^t) = \underbrace{\frac{(\omega_1 c_i(s^t)/d_i(s^t))^{\frac{\rho}{2}}}{(\omega_2 c_j(s^t)/m_{ji}(s^t))^{\frac{\rho}{2}}}}_{\text{good } i}\underbrace{\frac{(\omega_2 c_i(s^t)/m_{ij}(s^t))^{\frac{\rho}{2}}}{(\omega_1 c_j(s^t)/d_j(s^t))^{\frac{\rho}{2}}}}_{\text{good } j} \tag{18}$$

$$e^T(s^t) = \left(\frac{d_j(s^t)}{d_i(s^t)}\right)^{\frac{\rho}{2}}\left(\frac{m_{ji}(s^t)}{m_{ij}(s^t)}\right)^{\frac{\rho}{2}}\left(\frac{c_i(s^t)}{c_j(s^t)}\right)^{\rho} \tag{19}$$

There is an equilibrium condition between real exchange rates and relative composite good consumptions plus two other factors: the ratio of bilateral trade flows $\frac{m_{ji}(s^t)}{m_{ij}(s^t)}$ and the ratio of consumptions of domestically-endowed traded goods $\frac{d_j(s^t)}{d_i(s^t)}$. The elasticity of substitution between goods $\frac{1}{\rho}$ within the bundle plays a key role in real exchange rates determination.

Since the additional trade factors are negatively correlated with relative consumptions, the trade-based representation of real exchange rate has the potential to explain the Backus-Smith puzzle that real exchange rates and relative consumptions have low or even negative correlation in the data.

$$cov(\ln e^T(s^t), \ln\frac{c_i(s^t)}{c_j(s^t)}) = \frac{\rho}{2}\underbrace{Cov(\ln\frac{d_j(s^t)}{d_i(s^t)}, \ln\frac{c_i(s^t)}{c_j(s^t)})}_{<0} + \frac{\rho}{2}\underbrace{Cov(\ln\frac{m_{ji}(s^t)}{m_{ij}(s^t)}, \ln\frac{c_i(s^t)}{c_j(s^t)})}_{<0} + \rho Var(\ln\frac{c_i(s^t)}{c_j(s^t)})$$

3 Empirical Analysis

3.1 Data

This section performs empirical analysis for understanding actual real exchange rate movements. I obtain quarterly data from 13 major industrialized countries between 1980 to 1998: Australia, Canada, Japan, Switzerland, United Kingdom, Austria, Finland, France, Germany, Italy, Portugal, Spain and the U.S. There are a total of 78 bilateral country-pairs. The data sources are from International Financial Statistics, OECD Quarterly National Accounts, Direction of Trade Statistics and Datastream. A detailed description of the data sources and construction of variables are in the Data Appendix.

3.2 Actual Real Exchange Rates, Consumption-based and Trade-based Representations of Exchange Rates

Let $\ln e^A_{ijt}$ be the log of actual real exchange rate from the data. Let $\ln e^C_{ijt}$ be the log consumption-based representation of real exchange rates. If the utility function is CRRA with γ as the coefficient of relative risk aversion, the consumption-based representation is the ratio of relative real consump-

Table 1: Raw Data: $Corr(\ln e^A; \ln e^C)$ and $Corr(\ln e^A; \ln e^T)$

	Raw Data	Canada	Japan	U.K.	France	Germany
U.S.	$Corr(\ln e^A; \ln e^C)$	-0.367	-0.651	-0.431	-0.073	-0.526
	$Corr(\ln e^A; \ln e^T)$	0.887	0.874	0.801	0.864	0.907

tions,

$$\ln e^C_{ijt} = \gamma \ln \frac{c_{it}}{c_{jt}} + \text{constant}$$

Let $\ln e^T_{ijt}$ be the trade-based representation of real exchange rates derived in Proposition 2. If the elasticity of substitution between all goods in the consumption bundle is $\frac{1}{\rho}$

$$\ln e^T_{ijt} \quad = \quad \rho \left(\frac{1}{2} \ln \frac{d_{jt}}{d_{it}} + \frac{1}{2} \ln \frac{m_{jit}}{m_{ijt}} + \ln \frac{c_{it}}{c_{jt}} \right)$$

To highlight the results of this paper, I focus on five major trading partner countries against the U.S.: Canada, U.K., Japan, France and Germany. Table 1 shows these correlations in the raw data[14]. Most of the correlations for $\ln e^A$ and $\ln e^C$ are very low and negative in many country pairs. This confirms the 'Backus-Smith' puzzle that the correlations of real exchange rates and relative consumptions are very low. On the contrary, the correlations for $\ln e^A$ and $\ln e^T$ are much higher. For the major trading partners against the U.S, the raw data $Corr(\ln e^A, \ln e^T)$ are over 0.8 . This higher correlation is because of the higher correlation of actual real exchange rate and the two other trade factors: the ratio of consumption in domestically-endowed good ($\ln \frac{d_j}{d_i}$) and the ratio of bilateral trade flows ($\ln \frac{m_{ji}}{m_{ij}}$).

The details for the correlations for all bilateral pairs are in the Appendix. For 50% of all 78 bilateral pairs, the raw data $Corr(\ln e^A, \ln e^T)$ are over 0.7. This is a significant improvement over $Corr(\ln e^A, \ln e^C)$ in which only 1% have correlations over 0.7.

I also show the correlations with filters of different frequencies. I select the first-difference filter to focus on the short-term correlations, the band-pass filter for the medium-term correlations and the HP-filter for the long-term correlations. For the HP-filter, the smoothing parameter is 1600 for quarterly data. I focus on the cyclical component correlations after detrending. The band pass filter admits frequencies between 6 and 32 quarters. The moving average parameter for the band pass filter has 12 leads/lags.

Table 2 shows the correlations of Corr($\ln e^A, \ln e^C$) and Corr($\ln e^A, \ln e^T$) with different filters for

[14]Notice that these correlations do not depend on the parameter values of γ for $\ln e^C$ and ρ for $\ln e^T$.

Table 2: First-difference filtered data, Band-pass filtered data, HP-filtered data: $Corr(\ln e^A; \ln e^C)$ and $Corr(\ln e^A; \ln e^T)$

	Base Country: U.S.	Canada	Japan	U.K.	France	Germany
First Differences	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.021	0.056	0.008	-0.190	0.019
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.495	0.618	0.250	0.496	0.596
Band-pass Filtered	$Corr(\ln e^A; \ln e^C)$	0.008	0.196	-0.150	-0.425	-0.105
	$Corr(\ln e^A; \ln e^T)$	0.908	0.755	0.681	0.883	0.802
HP-Filtered Data	$Corr(\ln e^A; \ln e^C)$	-0.056	0.340	-0.109	-0.122	0.060
	$Corr(\ln e^A; \ln e^T)$	0.756	0.733	0.475	0.721	0.767

major trading partners against the U.S. The correlation in first differences for $Corr(\Delta \ln e^A; \Delta \ln e^C)$ is almost zero for most trading partners of U.S. In general, $Corr(\Delta \ln e^A, \Delta \ln e^T)$ are lower than in log levels, but they are positive and higher than $Corr(\Delta \ln e^A, \Delta \ln e^C)$.

The correlations with the band-pass filter and the HP-filter are similar to Table 1. The correlations between the band-passed-filtered actual real exchange rates and the band-passed-filtered trade-based representation are over 0.68 for these countries. The correlations between the HP-filtered actual real exchange rates and HP-filtered trade-based representation are over 0.47 for these countries.

Figure 1 shows the histograms for the densities of $Corr(\ln e^A, \ln e^C)$ (left figures) and $Corr(\ln e^A, \ln e^T)$ (right figures) for the raw data, first-differenced data, HP-filtered and band-pass-filtered data for all 78 bilateral pairs in the sample. The figures show that $Corr(\ln e^A, \ln e^T)$ has much higher correlations in general than $Corr(\ln e^A, \ln e^C)$.

3.3 Real Exchange Rate Puzzles

3.3.1 The Volatility Puzzle

Empirically real exchange rates are much more volatile than relative consumptions. The volatility of the standard consumption-based log real exchange rates is the volatility of relative consumptions adjusted for the coefficient of relative risk aversion.

$$Var(\ln e^C_{ijt}) = \gamma^2 Var(\ln \frac{c_{it}}{c_{jt}})$$

The variance of the trade-based log real exchange rate is

$$\begin{aligned} Var(\ln e^T_{ijt}) &= \rho^2[\frac{1}{4}Var(\ln \frac{d_{jt}}{d_{it}}) + \frac{1}{4}Var(\ln \frac{m_{jit}}{m_{ijt}}) + Var(\ln \frac{c_{it}}{c_{jt}}) + \frac{1}{2}Cov(\ln \frac{d_{jt}}{d_{it}}, \ln \frac{m_{jit}}{m_{ijt}}) \\ &+ Cov(\ln \frac{d_j}{d_i}, \ln \frac{c_{it}}{c_{jt}}) + Cov(\ln \frac{m_{jit}}{m_{ijt}}, \ln \frac{c_{it}}{c_{jt}})] \end{aligned}$$

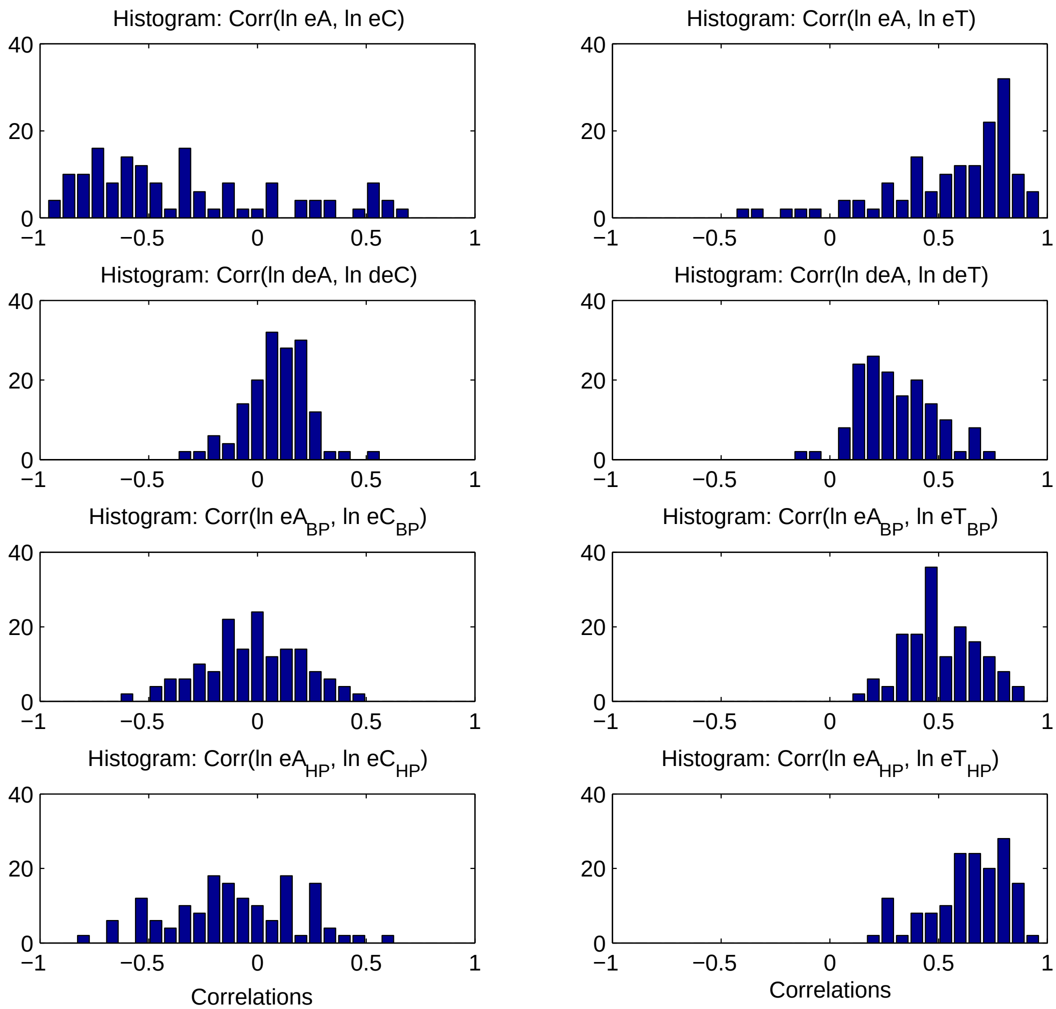

Figure 1: Histogram for the Corr$(\ln e^A, \ln e^C)$ (left figures) and Corr$(\ln e^A, \ln e^T)$ (right figures). First row of figures: raw data. Second row of figures: First-differenced data. Third row of figures: Band-pass filtered data. Fourth row of figures: HP-filtered data.

	$\gamma = 2, \rho = 1$	Canada	Japan	U.K.	France	Germany
U.S.	$\text{var}(\ln e^A)$	0.008	0.052	0.020	0.026	0.028
	$\text{var}(\ln e^C)$	0.010	0.011	0.014	0.005	0.042
	$\text{var}(\ln e^T)$	0.033	0.032	0.031	0.027	0.018

Table 3: Volatility of real exchange rates: $\text{var}(\ln e^A)$, $\text{var}(\ln e^C)$ and $\text{var}(\ln e^T)$.

$\gamma = 2, \rho = 1$	$\text{Var}(\ln(\frac{d_j}{d_i}))$	$\text{Var}(\ln(\frac{m_{ji}}{m_{ij}}))$	$\text{Var}(\ln(\frac{c_i}{c_j}))$
Canada	0.118	0.017	0.002
Japan	0.009	0.153	0.001
U.K.	0.047	0.073	0.001
France	0.006	0.125	0.003
Germany	0.020	0.082	0.007

Table 4: Breakdown of volatility of real exchange rates.

Table 3 compares $var(\ln e^A), var(\ln e^C)$ and $var(\ln e^T)$. Assume that the coefficient of risk aversion is the same for all countries and $\gamma = 2$. Assume the inverse of the elasticity of substitution is the same for all countries and $\rho = 1$. Although the value of ρ is below γ, the volatility of the trade-based representation matches the high volatility of actual real exchange rates quite well[15]. The breakdown of the variance of the trade-based representation is shown in Table 4. The variance of the components $var(\ln \frac{d_{US}}{d_i})$ and $var(\ln \frac{m_{US,i}}{m_{i,US}})$ are much higher than $var(\ln \frac{c_i}{c_{US}})$.

3.3.2 The Persistence Puzzle

Real exchange rates are highly persistent. Consensus half-lives of real exchange rates are about three to five years[16]. For the consumption-based representation, the correlation of real exchange rates today and tomorrow is equal to the correlation of relative consumptions between today and tomorrow: $Corr(\ln e^C_{ijt}, \ln e^C_{ijt+1}) = Corr(\ln \frac{c_{ijt}}{c_{jt}}, \ln \frac{c_{it+1}}{c_{jt+1}})$. There are nine covariance components

[15]The details for $\text{var}(\ln e^A)$, $\text{var}(\ln e^C)$ and $\text{var}(\ln e^T)$ for all bilateral pairs are listed in Table 14 in the Appendix.

[16]Taylor (2001) points out that many PPP tests in the literature may be subject to temporal aggregation and non-linearity biases. The half-life estimates would tend to bias upwards.

		Canada	Japan	U.K.	France	Germany
U.S.	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.968	0.955	0.913	0.930	0.927
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.978	0.961	0.985	0.959	0.989
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.983	0.930	0.900	0.908	0.949

Table 5: Persistence of real exchange rates: $\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$, $\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$and $\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$.

for the persistence of the trade-based representation of real exchange rates $Corr(\ln e_{ijt}^T, \ln e_{ijt+1}^T)$. [17]

Table 5 shows the results for the persistence of $\ln e^A, \ln e^C$ and $\ln e^T$ for the major trading partners with the U.S[18]. The persistence of actual real exchange rates is quite high and above 0.9 for many bilateral pairs. The relative consumptions are in general more persistent than the actual real exchange rates. The trade-based representation are usually less persistent than actual real exchange rates, but persistent enough that we cannot reject $\ln e^T$ as unit root processes.

3.3.3 Backus-Smith Puzzle

Backus and Smith (1993) state that in theory there should be a close relationship between fluctuations in consumption ratios and bilateral real exchange rates, but they find little evidence for this relation in the time-series data for 8 OECD countries. They find that the rank correlation of Δe_{ijt} and $\Delta \ln \frac{c_{it}}{c_{jt}}$ is almost zero, and negative for certain countries [19].

The benchmark consumption-based representation predicts a perfect correlation of one between real exchange rates and relative consumption (i.e. $\mathrm{Corr}(\ln \frac{c_i(s^{t+1})}{c_j(s^{t+1})}, \ln e(s^{t+1})) = 1$). On the other hand, the covariance between the trade-based representation of real exchange rate and relative

[17] The nine covariance components for the persistence for $\ln e^T$

$$
\begin{aligned}
& Cov(\ln e_{ijt}^T, \ln e_{ijt+1}^T) \\
= \; & Cov\left(\frac{\rho}{2}\ln\frac{d_{jt}}{d_{it}} + \frac{\rho}{2}\ln\frac{m_{jit}}{m_{ijt}} + \rho\ln\frac{c_{it}}{c_{jt}}, \frac{\rho}{2}\ln\frac{d_{jt+1}}{d_{it+1}} + \frac{\rho}{2}\ln\frac{m_{jit+1}}{m_{ijt+1}} + \rho\ln\frac{c_{it+1}}{c_{jt+1}}\right) \\
= \; & \rho^2[\frac{1}{4}Cov(\ln\frac{d_{jt}}{d_{it}}, \ln\frac{d_{jt+1}}{d_{it+1}}) + \frac{1}{4}Cov(\ln\frac{d_{jt}}{d_{it}}, \ln\frac{m_{jit+1}}{m_{ijt+1}}) + \frac{1}{2}Cov(\ln\frac{d_j}{d_i}, \ln\frac{c_{it+1}}{c_{jt+1}})... \\
+ \; & \frac{1}{4}Cov(\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{d_{jt+1}}{d_{it+1}}) + \frac{1}{4}Cov(\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{m_{jit+1}}{m_{ijt+1}}) + \frac{1}{2}Cov(\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{c_{it+1}}{c_{jt+1}})... \\
+ \; & \frac{1}{2}Cov(\ln\frac{c_{it}}{c_{jt}}, \ln\frac{d_{jt+1}}{d_{it+1}}) + \frac{1}{2}Cov(\ln\frac{c_{it}}{c_{jt}}, \ln\frac{m_{jit+1}}{m_{ijt+1}}) + Cov(\ln\frac{c_{it}}{c_{jt}}, \ln\frac{c_{it+1}}{c_{jt+1}})]
\end{aligned}
$$

[18] The results for the persistence of $\ln e^A, \ln e^C, \ln e^T$ for all bilateral pairs are listed in Table 16 in the Appendix.

[19] Backus-Smith (1993) find that the rank correlations of $(\mathrm{Std}(\Delta \ln \frac{c_i}{c_j}), \mathrm{Std}(\Delta \ln e_{ij}))$ is -0.263; the rank correlations of $(\mathrm{autocorr}(\Delta \ln \frac{c_i}{c_j}), \mathrm{autocorr}(\Delta \ln e_{ij}))$ is -0.466 and the rank correlations of $(\mathrm{mean}(\Delta \ln \frac{c_i}{c_j})\ \mathrm{mean}(\Delta \ln e_{ij}))$=0.074.

		Canada	Japan	U.K.	France	Germany
U.S.	corr($\ln e^A; \ln c_i/c_j$)	-0.367	-0.651	-0.431	-0.073	-0.526
	corr($\ln e^C; \ln c_i/c_j$)	1	1	1	1	1
	corr($\ln e^T; \ln c_i/c_j$)	-0.645	-0.772	-0.765	-0.186	-0.617

Table 6: Correlation between $\ln e^A, \ln e^C, \ln e^T$ and relative consumptions.

consumptions is

$$\begin{aligned} Cov(\ln e^T_{it}, \ln \frac{c_{it}}{c_{jt}}) &= Cov(\frac{\rho}{2}\ln\frac{d_{jt}}{d_{it}} + \frac{\rho}{2}\ln\frac{m_{jit}}{m_{ijt}} + \rho\ln\frac{c_{it}}{c_{jt}}, \ln\frac{c_{it}}{c_{jt}}) \\ &= \frac{\rho}{2}\underbrace{Cov(\ln\frac{d_{jt}}{d_{it}}, \ln\frac{c_{it}}{c_{jt}})}_{<0} + \frac{\rho}{2}\underbrace{Cov(\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{c_{it}}{c_{jt}})}_{<0} + \rho Var(\ln\frac{c_{it}}{c_{jt}}) \end{aligned}$$

Since c_{it} includes $\{d_{it}, m_{ijt}\}$ as components in the bundle, the two covariance terms Cov($\ln\frac{d_{jt}}{d_{it}}, \ln\frac{c_{it}}{c_{jt}}$) and Cov($\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{c_{it}}{c_{jt}}$) are negative in theory and also negative in the data. The intuition for the negative covariance is due to the fact that both countries allocate their traded goods intratemporally relative to country i and country j's bundles. This relative allocations of specific goods to consumption bundles lead to the negative covariances for Cov($\ln\frac{d_{jt}}{d_{it}}, \ln\frac{c_{it}}{c_{jt}}$) and Cov($\ln\frac{m_{jit}}{m_{ijt}}, \ln\frac{c_{it}}{c_{jt}}$). The comparison for corr($\ln e^A, \ln\frac{c_{it}}{c_{jt}}$), corr($\ln e^C, \ln\frac{c_{it}}{c_{jt}}$) and corr($\ln e^T, \ln\frac{c_{it}}{c_{jt}}$) for major trading partners against the U.S. is reported in Table 6.[20] It is clear from the table that the trade-based representation is much better in matching the low correlation between actual real exchange rates and relative consumptions.

3.4 Panel Estimation

This section estimates the coefficient of relative risk aversion $\hat{\gamma}$ from the consumption-based representation of real exchange rates $\ln e^C$ and inverse of elasticity of substitution between goods $\hat{\rho}$ from the trade-based representation $\ln e^T$. Most international business cycle models parametrize γ to be between 2 and 5[21]. For the studies estimating the elasticity of substitution between goods, the general conclusion is that the elasticity of substitution between traded goods is higher than 1 ($\rho < 1$)[22], but the elasticity of substitution between traded and non-traded goods is lower than 1

[20]The details for the correlations between $\ln e^A, \ln e^C, \ln e^T$ and relative consumptions for all bilateral pairs are listed in the Appendix (Table 17).

[21]Backus, Kehoe and Kydland (1992) use a value of 2 for γ for their international real business cycle model. Alvarez Atkeson and Kehoe (2002) parametrize γ to be 2 to illustrate the interest rate and exchange rate dynamics. Chari, Kehoe and McGrattan (2002) use a value of 5 to match up the volatility of real exchange rates and volatility of relative consumptions.

[22]Obstfeld and Rogoff (2000) summarize from recent trade studies that elasticity of import demand with respect to price (relative to the overall domestic consumption basket) is around 5 to 6. Chari, Kehoe and McGrattan (2002) state the most reliable studies in the literature for the elasticity of substitution between home and foreign good is

($\rho > 1$)[23]. As the consumption bundle c_i in our model include both traded goods and non-traded goods, I expect that the estimated inverse of elasticity of substitution ρ between 0.15 to 2.3 to be consistent with other studies in the literature.

Since our model requires that the elasticity of substitution $\frac{1}{\rho}$ be a constant such that the consumption aggregator c_i is CES with respect to $\{d_i, \{m_{ij}\}_{j\neq i}, n_i\}$, using U.S. as the base country, I estimate the inverse of the elasticity of substitution ρ to be equal for all the countries in our sample. I perform the estimation under a balanced panel for the five major trading partners against the U.S. ($N = 5$) with T=76 observations for each country-pair between 1980:1-1998:4.
The panel regressions on the consumption-based and the trade-based representations of real exchange rates are

$$\ln e^A_{iUSt} = \gamma \left(\ln \frac{c_{it}}{c_{USt}} \right) + \delta_C D_i + \varepsilon_{Cit},\ E(\varepsilon_{Cit}\varepsilon'_{Cit}) = \Omega_{\varepsilon_C} \tag{20}$$

$$\ln e^A_{iUSt} = \rho \left(\left[\frac{1}{2} \ln \frac{d_{USt}}{d_{it}} + \frac{1}{2} \ln \frac{m_{USit}}{m_{iUSt}} + \ln \frac{c_{it}}{c_{USt}} \right] \right) + \delta_T D_i + \varepsilon_{Tit},\ E(\varepsilon_{Tit}\varepsilon'_{Tit}) = \Omega_{\varepsilon_T} \tag{21}$$

The dependent variable is the actual real exchange rate $\ln e^A_{iUSt}$ of country i at time t where $i = 1..N$, $t = 1...T$. The explanatory variable is $\ln \frac{c_i}{c_{US}}$ for the consumption-based representation of real exchange rates, and $\frac{1}{2} \ln \frac{d_{US}}{d_i} + \frac{1}{2} \ln \frac{m_{USi}}{m_{iUS}} + \ln \frac{c_i}{c_{US}}$ for the trade-based representation of real exchange rates. D_i represents a matrix of variables that vary across countries but for each country are constant across periods. This represents the time-invariant country-specific (fixed) effect[24]. δ_C, δ_T represents the vector of coefficients for the dummy variables D_i. ρ and γ are our coefficients of interest. $\varepsilon_{Cit}, \varepsilon_{Tit}$ are the error structures of the disturbance terms. The standard errors are Newey-West (1987) heteroscedasticity and autocorrelation consistent with four lags using quarterly data.

The results of the panel regressions for log levels with country dummies for quarterly raw data are reported in Table 7. The estimate $\hat{\gamma}$ estimated from relative consumptions is negatively significant at -1.09. This is again inconsistent with the basic assumption of a positive coefficient of relative risk aversion. The estimate $\hat{\rho}$ from the trade-based representation is 0.97. It is quite close to the Cobb-Douglas case for the unit elasticity of substitution between all goods. The R^2 is higher for the trade-based representation of real exchange rate at 0.697.

Figure 2 shows the comparison for $\{\ln e^A, \ln e^C\}$ (left graphs) and $\{\ln e^A, \ln e^T\}$ (right graphs) for two major trading partners against the U.S.: Canada and Japan. The upper graphs is for

between 1 to 2.

[23]Tesar (1993) and Stockman and Tesar (1995) estimate that the elasticity of substitution between traded and non-traded goods is 0.44.

[24]I need to control for the fixed effects because the numeraires for the country bundle versus the U.S. bundle are different.

Quarterly Data	$\hat{\gamma}$	R^2
Consumption-based Representation	-1.085 (0.152)	0.183
	$\hat{\rho}$	R^2
Trade-based Representation	0.970 (0.044)	0.697

Table 7: Panel regressions for log levels with country dummies using quarterly raw data assuming all countries have the same γ and the same ρ. Top panel: Explanatory variable is the consumption-based representation of real exchange rates $\ln e^A_{iUSt} = \gamma(\ln \frac{c_{it}}{c_{USt}}) + \delta_C D_i + \varepsilon_{Cit}$. Bottom panel: Explanatory variable is the the trade-based representation of real exchange rate: $\ln e^A_{iUSt} = \rho([\frac{1}{2}\ln\frac{d_{USt}}{d_{it}} + \frac{1}{2}\ln\frac{m_{USit}}{m_{iUSt}} + \ln\frac{c_{it}}{c_{USt}}]) + \delta_T D_i + \varepsilon_{it}$. Let $\tilde{X}_{it}$ be the explanatory variables on the right-hand-side. Total number of observations: 380, where $N = 5$ and $T = 76$. The coefficient estimate for $(\hat{\gamma}, \hat{\rho})$ is $\frac{\sum_{i=1}^N \sum_{t=1}^T (X_{it}-\bar{X}_i)(Y_{it}-\bar{Y}_i)}{\sum_{i=1}^N \sum_{t=1}^T (X_{it}-\bar{X}_i)^2}$ where $\bar{Y}_i = \frac{1}{T}\sum_{t=1}^T Y_{it}, \bar{X}_i = \frac{1}{T}\sum_{t=1}^T X_{it}$. The variance for $(\hat{\gamma}, \hat{\rho})$ is $(\sum_{i=1}^N \sum_{t=1}^T (X_{it} - \bar{X}_i)^2)^{-1}\hat{\Omega}(\sum_{i=1}^N \sum_{t=1}^T (X_{it} - \bar{X}_i)^2)^{-1}$ where $\hat{\Omega}$ is Newey-West (1987) heteroscedasticity and autocorrelation consistent matrix with 4 lags. $\hat{\Omega} = \hat{\Omega}_0 + \sum_{j=1}^p (1-\frac{j}{p+1})(\hat{\Omega}_j + \hat{\Omega}_j')$, $\hat{\Omega}_0 = \frac{1}{NT}\sum_{i=1}^N \sum_{t=1}^T (\varepsilon_{it}\otimes(X_{it}-\bar{X}_i))^2$ and $\hat{\Omega}_j = \frac{1}{N}\sum_{i=1}^N \frac{1}{T}\sum_{t=j+1}^T (\varepsilon_{it}\otimes(X_{it}-\bar{X}_i))(\varepsilon_{i,t-j}\otimes(X_{i,t-j}-\bar{X}_i))'$. R^2 is calculated as $1 - \frac{\sum_{i=1}^N \sum_{t=1}^T u_{it}^2}{\sum_{i=1}^N \sum_{t=1}^T (y_{it}-\bar{y}_i)^2}$

Canada/U.S. and the lower graphs are for Japan/U.S. It can be seen that there is a much more positive correlation between $\{\ln e^A, \ln e^T\}$ than $\{\ln e^A, \ln e^C\}$. This result generalizes to many other countries. Figure 3 illustrates graphically the actual real exchange rates against the consumption-based representation of real exchange rates for all countries in our sample against the U.S. Each cluster of points correspond to each country in our sample. From the almost-vertical plots for each country-pair, we observe graphically that actual real exchange rates have low correlations with the relative consumptions and real exchange rates are much more volatile compared to relative consumptions. On the other hand, figure 4 illustrates graphically that the actual real exchange rates have a positive correlation with the trade-based representation.

Using the estimate of $\hat{\rho}$=1 from the panel regression in Table 7, I plot the time-series of $\ln e^A, \ln e^C$ and $\ln e^T$ with U.S. as the base country using quarterly data in Figure 5. The smooth line is the actual real exchange rate $\ln e^A_{iUSt}$. The dotted line is benchmark consumption-based representation of real exchange rate $\ln e^C_{iUSt} = \gamma \ln \frac{c_{it}}{c_{USt}}$. The line with '+' sign is trade-based representation of real exchange rate in this paper $\ln e^T_{iUSt} = \frac{\rho}{2}\ln\frac{d_{USt}}{d_{it}} + \frac{\rho}{2}\ln\frac{m_{USit}}{m_{iUSt}} + \rho\ln\frac{c_{it}}{c_{USt}}$. Assume $\gamma = 2$ and $\rho = 1$ for all countries. We observe graphically the trade-based representation (the line with '+' sign) are more correlated with the actual real exchange rates; while the consumption-based representations have lower correlations with the actual real exchange rate. I also plot the band-pass-filtered time series for $\ln e^A, \ln e^C$ and $\ln e^T$ in Figure 6.

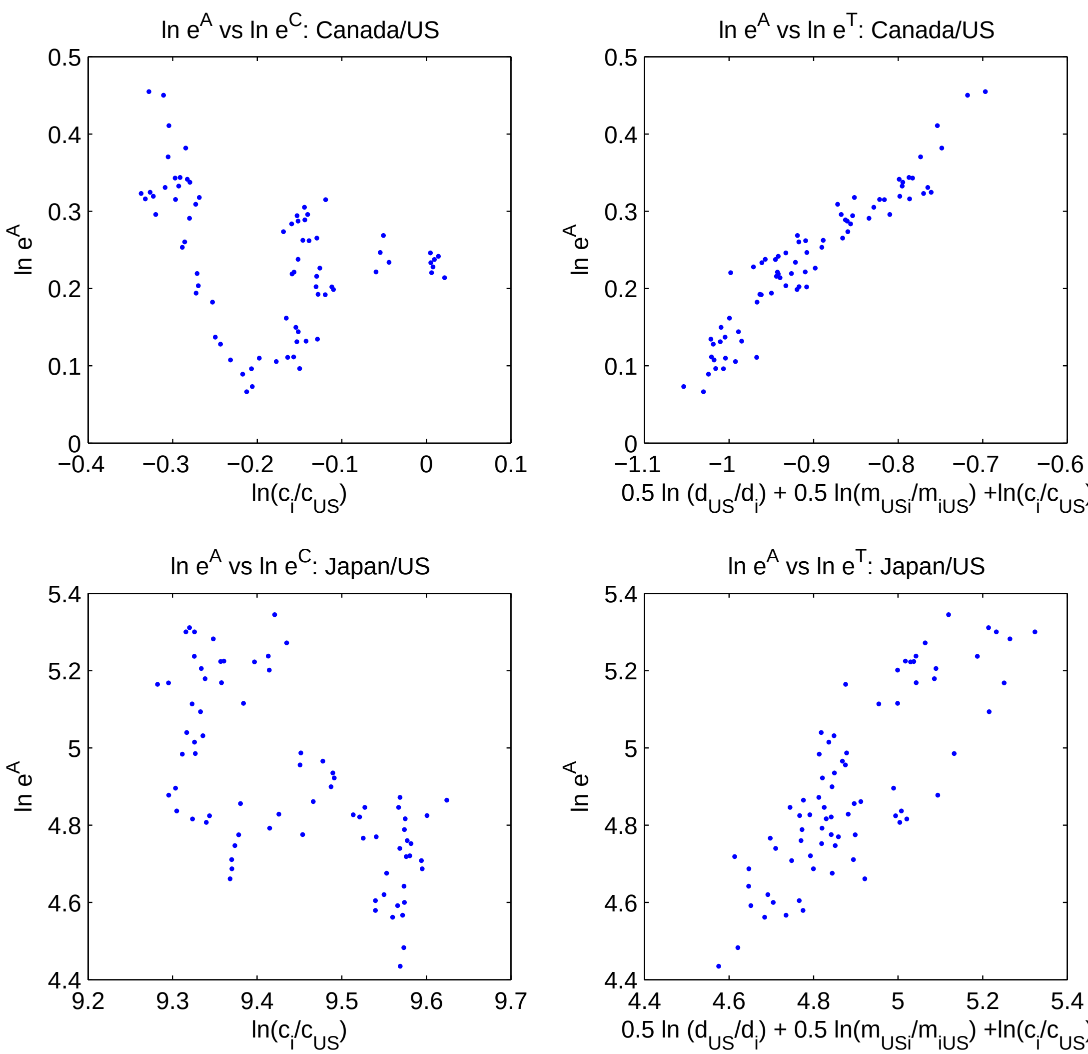

Figure 2: Real exchange rates $\ln e^A, \ln e^C, \ln e^T$ for the Canada/U.S. and Japan/U.S. pairs. Left figures: $\ln e^A$ versus $\ln \frac{c_i}{c_{US}}$. Right Figures: $\ln e^A$ versus $\frac{1}{2} \ln \frac{d_{US}}{d_i} + \frac{1}{2} \ln \frac{m_{USi}}{m_{iUS}} + \ln \frac{c_i}{c_{US}}$.

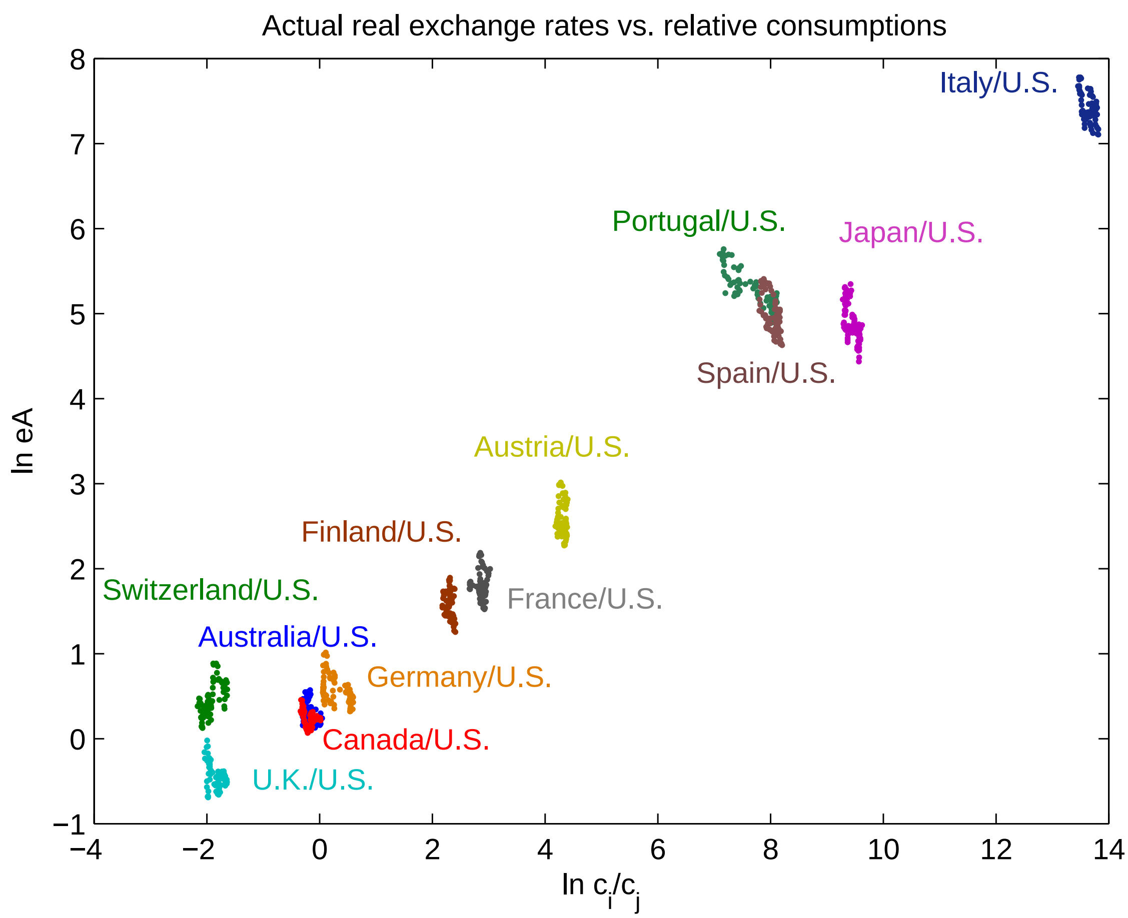

Figure 3: Actual real exchange rates $\ln e^A$ versus consumption-based representation $\ln \frac{c_i}{c_{US}}$ for all sample country pairs against the U.S.

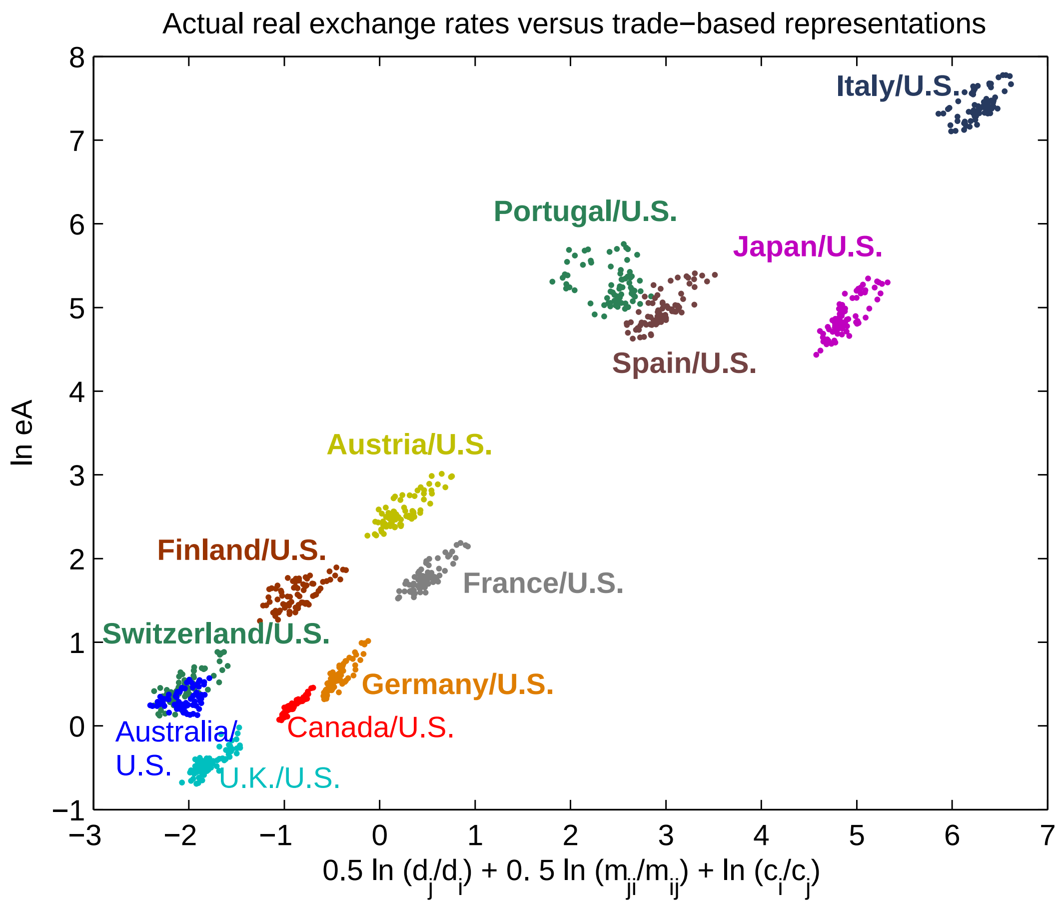

Figure 4: Actual real exchange rates $\ln e^A$ versus trade-based representation $\frac{1}{\rho}\ln e^T_{iUS} = \frac{1}{2}\ln\frac{d_{US}}{d_i} + \frac{1}{2}\ln\frac{m_{USi}}{m_{iUS}} + \ln\frac{c_i}{c_{US}}$ for all sample country pairs against the U.S.

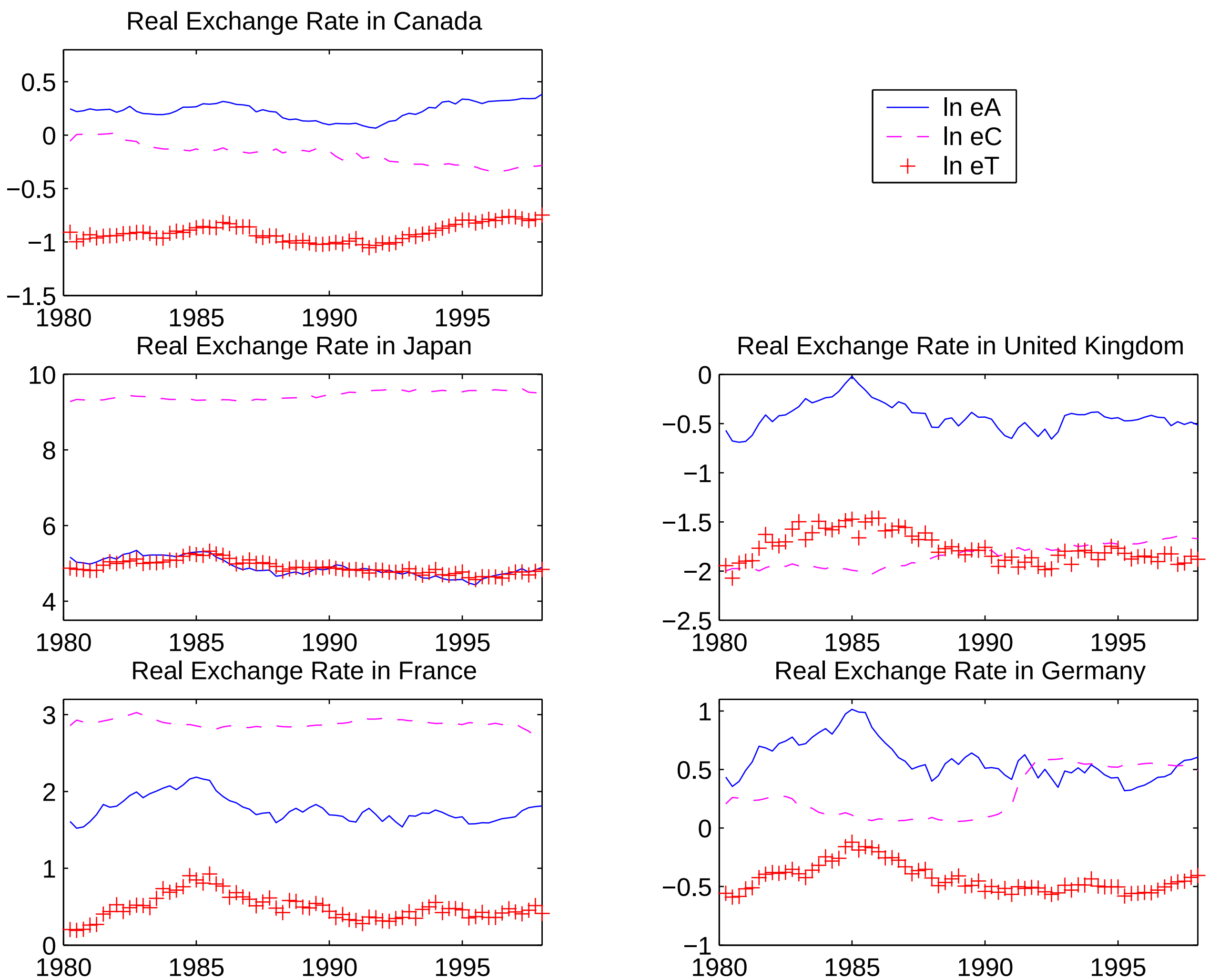

Figure 5: Smooth line is the actual real exchange rates $\ln e^{A}_{iUS}$. Dotted line is the predicted benchmark consumption-based representation $\ln e^{C}_{iUS} = \gamma \ln \frac{c_i}{c_{US}}$. Line with '+' sign is the trade-based representation $\ln e^{T}_{iUS} = \rho[\frac{1}{2} \ln \frac{d_{US}}{d_i} + \frac{1}{2} \ln \frac{m_{USi}}{m_{iUS}} + \ln \frac{c_i}{c_{US}}]$. Dates for all countries are from 1980:1-1998:4. $\gamma = 2$ and $\rho = 1$ for all countries.

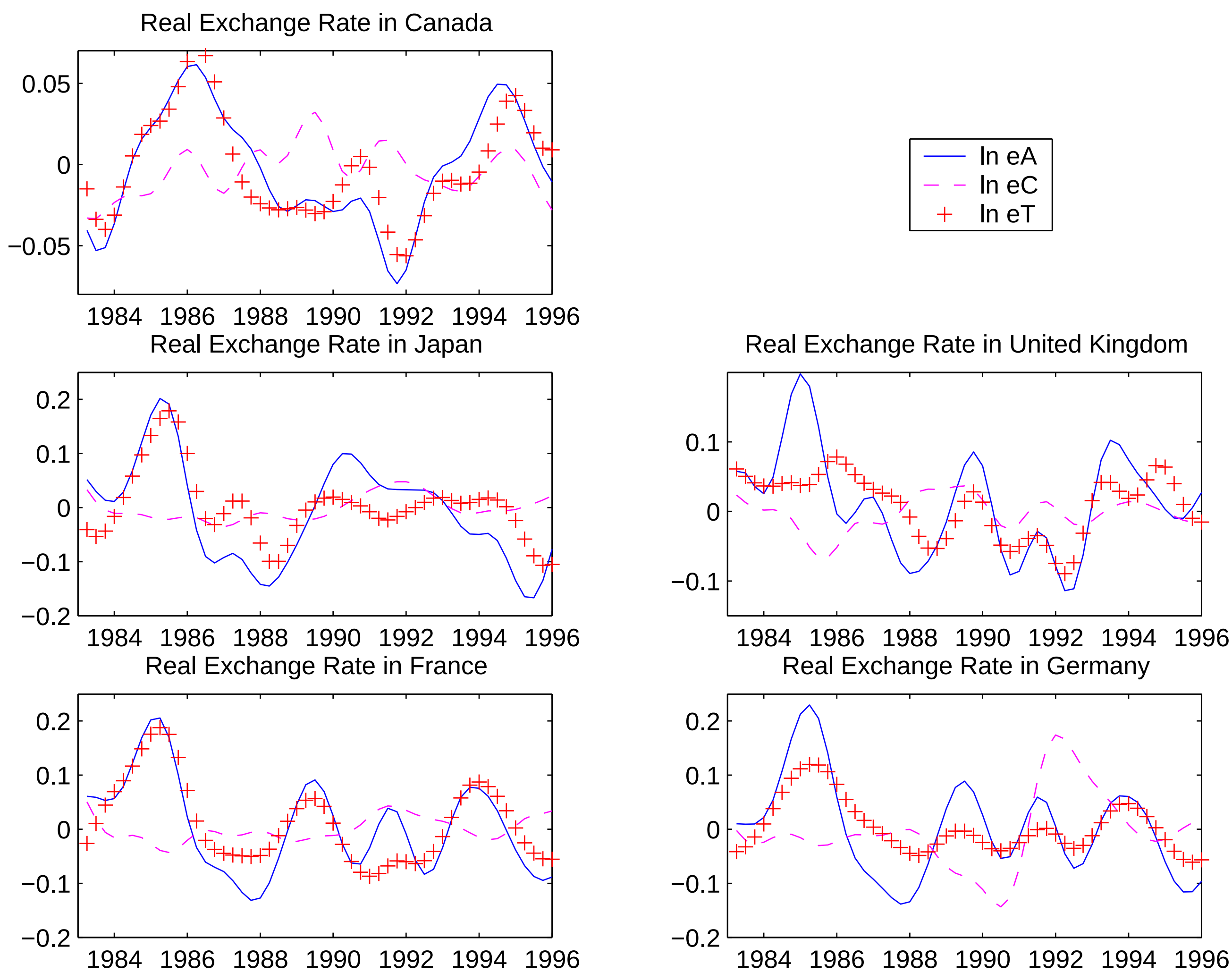

Figure 6: Smooth line is the actual band-pass-filtered real exchange rates $\ln e^{A}_{iUS}$. Dotted line is the band-pass-filtered consumption-based representation $\ln e^{C}_{iUS} = \gamma \ln \frac{c_i}{c_{US}}$. Line with '+' sign is the band-pass-filtered trade-based representation in this paper $\ln e^{T}_{iUS} = \rho[\frac{1}{2} \ln \frac{d_{US}}{d_i} + \frac{1}{2} \ln \frac{m_{USi}}{m_{iUS}} + \ln \frac{c_i}{c_{US}}]$. Dates for all countries are from 1980:1-1998:4. $\gamma = 2$ and $\rho = 1$ for all countries.

3.5 Unit Root and Cointegration of Real Exchange Rates

Many studies have documented that we cannot reject that real exchange rates are unit root processes (e.g. Meese and Rogoff (1983)). To check whether actual real exchange rates $\ln e^A$, consumption-based representation $\ln \frac{c_i}{c_j}$ and the trade-based representation $\frac{1}{\rho}\ln e_{ij}^T = \frac{1}{2}\ln\frac{d_j}{d_i} + \frac{1}{2}\ln\frac{m_{ji}}{m_{ij}} + \ln\frac{c_i}{c_j}$ are unit root processes, we perform the augmented Dickey-Fuller test and the Phillips-Perron test of unit root[25]. The results for the unit root tests for real exchange rates with U.S. as the base country are reported in Table 8. In general we cannot reject the unit root processes for all $\ln e^A, \ln e^C$ and $\ln e^T$ at 10% significance. This is quite consistent with other studies that it is difficult to beat the random walk hypothesis of real exchange rates.

Since we cannot reject that the dependent variable and the explanatory variables are unit root processes, we need to check whether the results reported in Table 7 are merely spurious regressions or whether they are cointegrated and have a long-run relationship. We employ Kao's (1999) test of cointegration for non-stationary panels. We first obtain the residuals from the panel regressions (20) and (21)

$$\begin{aligned}
\hat{\varepsilon}_{Cit} &= \ln e_{iUSt}^A - \left(\ln\frac{c_{it}}{c_{USt}}\right)\hat{\gamma} + D_i\hat{\delta}_C \\
\hat{\varepsilon}_{Tit} &= \ln e_{iUSt}^A - \left(w_i\left[\frac{1}{2}\ln\frac{d_{USt}}{d_{it}} + \frac{1}{2}\ln\frac{m_{USit}}{m_{iUSt}} + \ln\frac{c_{it}}{c_{USt}}\right]\right)\hat{\rho} + D_i\hat{\delta}_T
\end{aligned}$$

The DF-type test from Kao (1999) can be calculated from the estimated residuals

$$\hat{\varepsilon}_{Cit} = \psi_C\hat{\varepsilon}_{Ci,t-1} + \upsilon_{Cit}, \qquad \hat{\varepsilon}_{Tit} = \psi_T\hat{\varepsilon}_{Ti,t-1} + \upsilon_{Tit}$$

Let ψ represent either ψ_C and ψ_T from the estimated residuals. The null hypothesis of no cointegration is $H_0 : \psi = 1$. The OLS estimate of $\hat{\psi}$ and the t-statistic are given as

$$\hat{\psi} = \frac{\sum_{i=1}^N\sum_{t=2}^T \hat{\varepsilon}_{it}\hat{\varepsilon}_{i,t-1}}{\sum_{i=1}^N\sum_{t=2}^T \hat{\varepsilon}_{it}^2}, \qquad t_\psi = \frac{(\hat{\psi}-1)\sqrt{\sum_{i=1}^N\sum_{t=2}^T \hat{\varepsilon}_{i,t-1}^2}}{S_e}$$

where $S_e^2 = \frac{1}{NT}\sum_{i=1}^N\sum_{t=2}^T(\hat{\varepsilon}_{it} - \hat{\psi}\hat{\varepsilon}_{i,t-1})^2$. The DF tests[26] are

$$DF_\psi = \frac{\sqrt{N}T(\hat{\psi}-1) + 3\sqrt{N}}{\sqrt{10.2}}, \qquad DF_t = \sqrt{1.25}t_\psi + \sqrt{1.875N}$$

The results for Kao's panel cointegration test for quarterly data are reported in Table 9. The

[25] Details are available upon request for the results of other bilateral time-series unit root tests, and Levin and Lin's (1991) panel unit root test. We also cannot reject unit root of $\ln e^A, \ln e^C$ and $\ln e^T$ at 5% significance with the panel unit root test.

[26] Kao (1999) also defines DF_ψ^* and DF_t^* statistics to test for cointegration with endogenous relationship between regressors and errors. For our sample size of $N = 12, T = 76$, the DF_ψ and DF_ψ^* statistics and DF_t and DF_t^* statistics have approximately the same sample size and power at 5%. See Kao (1999).

Augmented Dickey-Fuller Test	$\ln e^A$		$\ln \frac{c_i}{c_j}$		$\frac{1}{\rho} \ln e^T$	
	$\hat{\beta}_1 - 1$	τ_i^{DF}	$\hat{\beta}_1 - 1$	τ_i^{DF}	$\hat{\beta}_1 - 1$	τ_i^{DF}
Canada	0.01	0.29	-0.02	-0.80	0.04	1.71
Japan	-0.05	-1.53	-0.04	-1.26	-0.07	-1.61
United Kingdom	-0.09	-1.90	-0.02	-0.80	-0.10	-1.92
France	-0.08	-1.80	-0.02	-0.51	-0.11	-2.35
Germany	-0.08	-1.79	-0.01	-1.02	-0.06	-1.62
Phillips-Perron Test	$\ln e^A$		$\ln \frac{c_i}{c_j}$		$\frac{1}{\rho} \ln e^T$	
	$\hat{\sigma}_i$	τ_i^{PP}	$\hat{\sigma}_i$	τ_i^{PP}	$\hat{\sigma}_i$	τ_i^{PP}
Canada	0.02	-0.03	0.01	-0.67	0.03	1.85
Japan	0.07	-1.60	0.01	-1.42	0.07	-1.43
United Kingdom	0.06	-2.02	0.01	-0.71	0.08	-1.44
France	0.06	-1.96	0.01	-0.76	0.07	-2.21
Germany	0.06	-1.93	0.01	-1.02	0.04	-1.67

Table 8: Augmented Dickey-Fuller and Phillips-Perron Tests of Unit Root. Test Regression: $\Delta y_{it} = \beta_0 + (\beta_1 - 1)y_{i,t-1} + u_{it}$. Column 1: y_{it} process is the actual real exchange rates $\ln e^A_{iUSt}$. Column 2: y_{it} process is the benchmark consumption-based representation of real exchange rates, $\ln \frac{c_{it}}{c_{USt}}$. Column 3: y_{it} process is the trade-based representation derived in this paper, $\frac{1}{\rho} \ln e^T_{iUSt} = \frac{1}{2} \ln \frac{d_{USt}}{d_{it}} + \frac{1}{2} \ln \frac{m_{USit}}{m_{iUSt}} + \ln \frac{c_{it}}{c_{USt}}$. Upper Panel: Augmented Dickey-Fuller Tests of Unit root. Null hypothesis $H_0 : \hat{\beta}_1 = 1$. Alternative hypothesis: $H_A : \hat{\beta}_1 < 1$. The residuals u_{it} is assumed to follow a stationary AR(1) process: $u_{it} = \rho u_{it-1} + \varepsilon_{it}$ and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. The τ_i^{DF} statistic of Dickey-Fuller is $\tau_i^{DF} = (\hat{\beta}_1 - 1) S_{ei}^{-1} (\sum_{t=2}^{T} y_{i,t-1}^2)^{\frac{1}{2}}$ where $S_{ei}^2 = \frac{1}{T-2} \sum_{t=2}^{T} (y_{i,t} - \hat{\rho} y_{i,t-1})^2$. Lower Panel: Phillips Perron Test of Unit Root. The nonparametric τ_i^{PP} statistic is $\tau_i^{PP} = \frac{\hat{\sigma}_i \tau_i^{DF}}{\hat{\omega}_i} - \frac{n(\hat{\omega}_i^2 - \hat{\sigma}_i^2)}{2\hat{\omega}_i \sum_{t=1}^{T} (y_{it} - \frac{1}{T} \sum_{t=1}^{T} y_{it})}$ where $\hat{\sigma}_i$ is a consistent estimate for σ_i and $\hat{\omega}_i^2 = \frac{1}{T} \left(\sum_{t=1}^{T} \hat{u}_{it}^2 + 2 \sum_{j=1}^{p} (1 - \frac{j}{p-1}) (\sum_{t=j+1}^{T} \hat{u}_{it} \hat{u}_{i,t-j}) \right)$. The τ_i^{DF} and τ_i^{PP} asymptotic critical values are from MacKinnon (1991): -2.567 for 10% significance (*), -2.862 for 5% significance (**) and -3.434 for 1% significance(***).

Kao's (1999) Panel Cointegration Test Quarterly Data	ψ_C	t_{ψ_C}	DF_{ψ_C}	$DF_{t_{\psi_C}}$
Consumption-based Representation	0.927	-3.683	-1.798	-1.056
	ψ_T	t_{ψ_T}	DF_{ψ_T}	$DF_{t_{\psi_T}}$
Trade-based Representation	0.787	-6.661	-9.215	-4.385

Table 9: Kao's (1999) cointegration test for panel regressions with quarterly data. Residuals are from the panel regressions in Table 7. The null hypothesis of no cointegration $H_0 : \psi = 1$. The OLS estimate of ψ and the t-statistic are given as $\hat{\psi} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{\varepsilon}_{it}\hat{\varepsilon}_{i,t-1}}{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{\varepsilon}_{it}^2}$, $t_\psi = \frac{(\hat{\psi}-1)\sqrt{\sum_{i=1}^{N}\sum_{t=2}^{T}\hat{\varepsilon}_{i,t-1}^2}}{S_e}$ where $S_e^2 = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=2}^{T}(\hat{\varepsilon}_{it} - \hat{\psi}\hat{\varepsilon}_{i,t-1})^2$. The DF tests are $DF_\psi = \frac{\sqrt{N}T(\hat{\psi}-1)+3\sqrt{N}}{\sqrt{10.2}}$, $DF_t = \sqrt{1.25}t_\psi + \sqrt{1.875N}$. The DF_ψ and DF_t statistics are asymptotically distributed as $N(0,1)$ if the null hypothesis of no cointegration in the panel is true.

DF_ψ and DF_t statistics are asymptotically distributed as $N(0,1)$ if the null hypothesis H_0 of no cointegration in the panel is true. For the regression on the consumption-based representation $\ln e_{it}^A = \ln e_{it}^C + \varepsilon_{Cit}$, the DF_{ψ_C} value of -1.798 and the $DF_{t_{\psi_C}}$ value of -1.056 indicate that we cannot reject unit root for the ε_{Cit} process at 5% significance. Therefore, actual real exchange rates and their consumption-based representations are not cointegrated.

For the regression on the trade-based representation $\ln e_{it}^A = \ln e_{it}^T + \varepsilon_{Tit}$, both the DF_{ψ_T} (-9.215) and $DF_{t_{\psi_T}}$ (-4.385) statistics indicate that we can reject unit root process for ε_{Tit} at 1% significance. In other words, actual real exchange rates $\ln e_{iUSt}^A$ and the trade-based representations $\ln e_{iUSt}^T$ are cointegrated. The two processes exhibit a long-run relationship and the coefficient $\hat{\rho}$ estimated from (21) is consistent.

3.6 Time-varying Preference Shocks and Lagrange Multipliers of Budget Constraints

From (16), the ratio of time-varying preference shocks and the Lagrange Multipliers of budget constraints in terms of allocation can be expressed as follows:

$$\begin{aligned}\ln\left(\frac{\delta_j(s^t)}{\delta_i(s^t)}\frac{\sigma_i(s^t)}{\sigma_j(s^t)}\right) &= \ln\frac{U_i'(c_i(s^t))}{U_j'(c_j(s^t))} + \frac{1}{2}\left(\ln\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)} + \ln\frac{\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)}\right)\\ &= \frac{\rho}{2}\ln\frac{d_j(s^t)}{d_i(s^t)} + \frac{\rho}{2}\ln\frac{m_{ji}(s^t)}{m_{ij}(s^t)} + (\rho-\gamma)\ln\frac{c_i(s^t)}{c_j(s^t)}\end{aligned}$$

where the second equality is for the special case of CRRA utility $U_i(c_i(s^t)) = \delta_i(s^t)\frac{c_i(s^t)^{1-\gamma}}{1-\gamma}$ with γ as the coefficient of relative risk aversion and taste shock $\delta_i(s^t)$ in state s^t and CES consumption aggregator (17).

The higher the relative preference shocks $\frac{\delta_j(s^t)}{\delta_i(s^t)}$ for country j versus country i, the higher the allocations for the traded goods i and j to country j's bundle versus to country i's bundle. This would be reflected in a increase in the relative ratios of $\frac{d_j(s^t)/c_j(s^t)}{m_{ij}(s^t)/c_i(s^t)}$ and $\frac{m_{ji}(s^t)/c_j(s^t)}{d_i(s^t)/c_i(s^t)}$.

Under complete markets, the ratio of Lagrange Multipliers of budget constraints $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ is a constant because the social planner allocates each traded good such that the marginal utilities of each traded good across countries are a constant that corresponds to the ratio of planner's initial weights. If asset markets are endogenously incomplete, the ratio of Lagrange Multipliers of budget constraints $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ can be time-varying. Moreover, they should move like step functions that this ratio changes only if one of the countries enforcement constraint binds. If asset markets are exogenously incomplete, $\frac{\sigma_i(s^t)}{\sigma_j(s^t)}$ can be time-varying that correspond to the wealth accumulated across countries.[27]

Figure 7 shows the time series of $\ln\left(\frac{\delta_{US}(s^t)}{\delta_i(s^t)}\frac{\sigma_i(s^t)}{\sigma_{US}(s^t)}\right)$ in the raw data. It can be seen that the raw data ratio of $\ln\left(\frac{\delta_{US}(s^t)}{\delta_i(s^t)}\frac{\sigma_i(s^t)}{\sigma_{US}(s^t)}\right)$ drifts around quite a lot. These fluctuations can be due to time-varying preference shocks across countries or incomplete markets. Further research can focus on identifying the major source(s) of real exchange rate fluctuations.

4 Conclusion

In this paper, I examine a class of general equilibrium models of international trade to understand real exchange rate movements. I model a multi-country world with goods market trading with three basic assumptions. (i) There are multiple goods. Each country is endowed with only one of the traded goods. (ii) Utility is increasing in consumption, and the consumption aggregator is homogeneous of degree 1 with respect to the goods within the bundle, strictly concave, time-separable and satisfies Inada conditions with respect to foreign imports. (iii) Goods prices are perfectly flexible. All countries take prices as given in competitive markets. Starting from a no-arbitrage pricing condition for all traded goods, I derive a new equilibrium condition that relates real exchange rates with international trade flows and macroeconomic fundamentals.

Under a simple parametric form of a CES consumption aggregator, real exchange rates can be expressed as a function of relative composite good consumptions plus two other factors: the ratio of bilateral trade flows and the ratio of domestically-endowed traded good consumptions. These two extra factors reflect how the two countries allocate intra-temporally its own traded good between home and foreign. This trade-based representation is valid in any economy that satisfies

[27] It is beyond the scope of this paper to distinguish whether the fluctuations of the additional trade factors are due to preference shocks versus incomplete markets, or whether asset markets are complete, endogenously incomplete or exogenously incomplete. Kehoe and Perri (2002) find that an endogenously incomplete market matches the international real business cycles features better than complete markets or an exogenously incomplete markets in a single-good model with production.

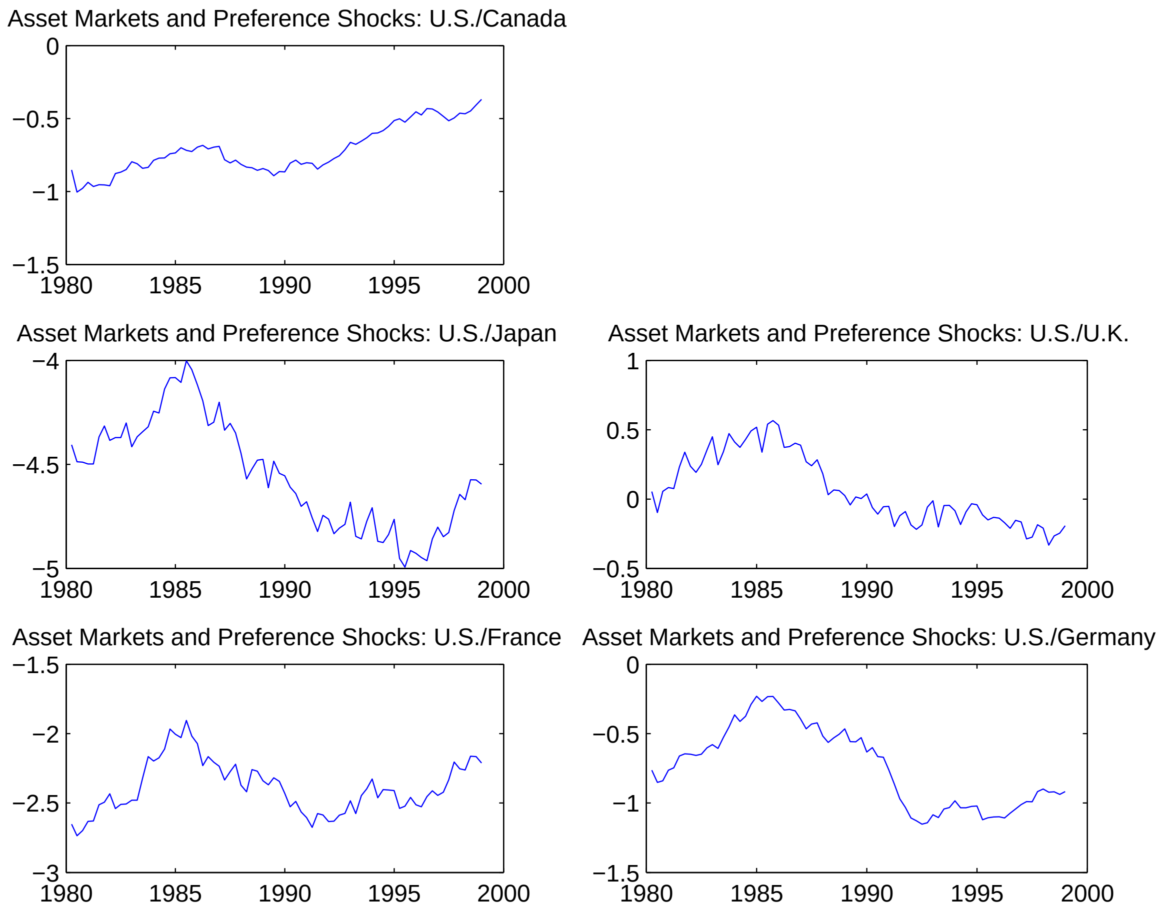

Figure 7: Log of $\frac{\delta_{US}}{\delta_i}\frac{\sigma_i}{\sigma_{US}}$ on raw data for major trading partners against the U.S.

the three key assumptions. Therefore it is robust to economies with goods market frictions such as proportional transport costs and non-traded goods, a wide variety of asset market structures, preferences, endowment versus production economies, monetary versus real economies, etc. I show empirically that this new trade-based representation correlates well with actual real exchange rates. In particular, the major trading partners against the U.S. has correlation of actual real exchange rates and trade-based representation of over 0.8. The volatility of the extra trade factors adds to explain the high volatility of real exchange rates. In addition, it identifies preference shocks or incomplete markets as potential explanations to the Backus-Smith puzzle since the extra trade factors are negatively covaried with relative consumptions. Panel estimation indicates that the intra-temporal elasticity of substitution between goods is around 1, the Cobb-Douglas case.

While the analysis in this paper provides a close intra-temporal link between real exchange rates, international trade and macroeconomic fundamentals, there are still open questions about the source of real exchange rate fluctuations and the inter-temporal properties of real exchange rates. Future research can focus on identifying the key underlying sources for international trade movements and exchange rate fluctuations in the data.

For further research in empirical analysis, the trade-based representation is useful in understanding variations of consumption of specific goods within a consumption bundle. If we assume a much more detailed parametric consumption aggregator which allows for different elasticities of substitutions between traded goods ($\frac{1}{\rho_T}$) and non-traded goods ($\frac{1}{\rho_N}$),

$$\begin{aligned} c_i(s^t) &= \left[c_{iT}(s^t)^{1-\rho_N} + n_i(s^t)^{1-\rho_N}\right]^{\frac{1}{1-\rho_N}} \\ \text{where } \; c_{iT}(s^t) &= [d_i(s^t)^{1-\rho_T} + \sum_{j\neq i} m_{ij}(s^t)^{1-\rho_T}]^{\frac{1}{1-\rho_T}} \end{aligned}$$

then the equilibrium condition between real exchange rate and allocations in (13) becomes

$$e^T = \left(\frac{d_j(s^t)}{d_i(s^t)}\right)^{\frac{\rho_T}{2}} \left(\frac{m_{ji}(s^t)}{m_{ij}(s^t)}\right)^{\frac{\rho_T}{2}} \left(\frac{c_i(s^t)}{c_j(s^t)}\right)^{\rho_N} \left(\frac{c_{iT}(s^t)}{c_{jT}(s^t)}\right)^{\rho_T-\rho_N} \tag{22}$$

Alternatively, if we assume another parametric assumption for the consumption aggregator with country-specific, time-varying bias for home good versus foreign imports versus non-traded goods $\omega_{1i}(s^t), \omega_{2i}(s^t), \omega_{3i}(s^t)$ such that $c_i(s^t) = [\omega_{1i}(s^t)d_i(s^t)^{1-\rho} + \sum_{j\neq i}\omega_{2i}(s^t)m_{ij}(s^t)^{1-\rho} + \omega_{3i}(s^t)n_i(s^t)^{1-\rho}]^{\frac{1}{1-\rho}}$, then the equilibrium condition between real exchange rate and allocations in (13) becomes

$$e^T(s^t) = \left(\frac{\omega_{1i}(s^t)\omega_{2i}(s^t)}{\omega_{1j}(s^t)\omega_{2j}(s^t)}\right)^{\frac{1}{2}} \left(\frac{d_j(s^t)}{d_i(s^t)}\right)^{\frac{\rho}{2}} \left(\frac{m_{ji}(s^t)}{m_{ij}(s^t)}\right)^{\frac{\rho}{2}} \left(\frac{c_i(s^t)}{c_j(s^t)}\right)^{\rho}$$

This paper performs empirical analysis for the special case that all goods have the same elasticity of substitution (i.e. $\rho_T = \rho_N = \rho$) and both countries have the same consumption aggregator without country-specific, time-varying bias in different goods ($\omega_{1i}(s^t) = \omega_1, \omega_{2i}(s^t) = \omega_2, \omega_{3i}(s^t) = \omega_3$). If we allow for a more detailed parametric form of consumption aggregators, additional factors (e.g. $\frac{c_{iT}(s^t)}{c_{jT}(s^t)}$, $\frac{\omega_{1i}(s^t)\omega_{2i}(s^t)}{\omega_{1j}(s^t)\omega_{2j}(s^t)}$) enter in the real exchange rate determination. Further research can focus on how additional factors with a more detailed parametric form for the consumption aggregator can help in understanding real exchange rate movements.

The empirical analysis in this paper is mostly for developed economies with floating exchange rates against the U.S. It would be useful extension to see how this trade-based representation helps in understanding real exchange rate movements for developing countries. In addition, the analysis in this paper can also be applied to study the relative price levels across different states of a country, or different members within a monetary union.

5 Appendix

Proof of Proposition 1: Suppose the contrary that in an equilibrium with positive shipping $m_{ij}(s^t) > 0$, $p_{ii}(s^t) < p_{ji}(s^t)\eta(s^t)$. In this case, country i or country j would have the incentive to purchase traded good i from country i, ship to country j (with transport cost) and sell this good in country j. The can make a profit if $p_{ii}(s^t) < p_{ji}(s^t)\eta(s^t)$ and increases their utility from this profit. Contradiction to the original prices and allocations constituting an equilibrium. QED.

Proof of Proposition 2: From Proposition 1, $p_{ii}(s^t) = p_{ji}(s^t)\eta(s^t)$.

$$p_{ii}(s^t) = \frac{1}{\sigma_i(s^t)} U_i'(c_i(s^t)) \frac{\partial c_i(s^t)}{\partial d_i(s^t)} = \frac{\eta(s^t)}{\sigma_j(s^t)} U_j'(c_j(s^t)) \frac{\partial c_j(s^t)}{\partial m_{ji}(s^t)} = p_{ji}(s^t)\eta(s^t) \tag{23}$$

Rearranging terms, I obtain the following result for real exchange rates.

$$e_{ij}^T(s^t) = \frac{\partial c_i(s^t)/\partial d_i(s^t)}{\eta(s^t)\partial c_j(s^t)/\partial m_{ji}(s^t)} = \frac{p_{ii}(s^t)/p_i(s^t)}{\eta(s^t)p_{ji}(s^t)/p_j(s^t)} \tag{24}$$

Similarly, apply Proposition 1 for traded good j: $p_{jj}(s^t) = p_{ij}(s^t)\eta(s^t)$.

$$p_{jj}(s^t) = \frac{1}{\sigma_j(s^t)} U_j'(c_j(s^t)) \frac{\partial c_j(s^t)}{\partial d_j(s^t)} = \frac{\eta(s^t)}{\sigma_i(s^t)} U_i'(c_i(s^t)) \frac{\partial c_i(s^t)}{\partial m_{ij}(s^t)} = p_{ij}(s^t)\eta(s^t) \tag{25}$$

Rearranging terms, I obtain the following result for real exchange rates.

$$e_{ij}^T(s^t) = \frac{\eta(s^t)\partial c_i(s^t)/\partial m_{ij}(s^t)}{\partial c_j(s^t)/\partial d_j(s^t)} = \frac{\eta(s^t)p_{ij}(s^t)/p_i(s^t)}{p_{jj}(s^t)/p_j(s^t)} \tag{26}$$

From (24) and (26),

$$\eta(s^t) = \left(\frac{\partial c_i(s^t)/\partial d_i(s^t)}{\partial c_j(s^t)/\partial m_{ji}(s^t)} \frac{\partial c_j(s^t)/\partial d_j(s^t)}{\partial c_i(s^t)/\partial m_{ij}(s^t)} \right)^{\frac{1}{2}} \tag{27}$$

Substitute (27) into (24) or (26), I obtain the result for the Proposition. The second equality in (14) can be verified by combining first order conditions with respect to individual goods and first order condition with respect to the consumption bundle. QED.

6 Data Appendix

I obtain quarterly data from 13 major industrialized countries between 1980 to 1998: Australia, Canada, Japan, Switzerland, United Kingdom, Austria, Finland, France, Germany, Italy, Portugal, Spain and the U.S. There are a total of 78 bilateral country-pairs.

I obtain the price data for nominal exchange rate (line AE) and consumer price index (line 64) from International Financial Statistics (IFS). Let $\varepsilon_{ij}(s^t)$ be the nominal exchange rate defined as country i's currency in terms of country j's currency. Let $p_j(s^t)$ and $p_i(s^t)$ be the Consumer Price Indices in country j

and country i. The actual real exchange rate $e^A_{ij}(s^t)$ is constructed from the nominal exchange rate adjusted by the ratio of Consumer Price Indices across the two countries.

$$e^A_{ij}(s^t) = \varepsilon_{ij}(s^t)\frac{p_j(s^t)}{p_i(s^t)} \tag{28}$$

The household consumption expenditure (including NPISHs) data (line 96F) and population data (line 99Z) are from International Financial Statistics. The real consumption of composite good $c_i(s^t)$ is constructed by deflating household consumption expenditure by the CPI.

The consumption-based representation of real exchange rate $\ln e^C$ is the ratio of relative real consumptions

$$\ln e^C_{ij}(s^t) = \gamma \ln \frac{c_i(s^t)}{c_j(s^t)} + \text{constant} \tag{29}$$

The data for consumption expenditures of bilateral imports or exports $p_{ij}(s^t)m_{ij}(s^t)$ is obtained from bilateral trade data from Direction of Trade Statistics. Since there are usually discrepancies between the reported amount of exports from country j to country i and the reported amount of imports of country i from country j in IFS (both reported in U.S. dollars), I take the average of these two numbers as the country i's consumption expenditure on traded good j ($p_{ij}m_{ij}$) in our model.

For the construction of $d_i(s^t)$, we need to subtract consumptions of total imports and non-traded good from the composite consumption expenditure. Similar to Stockman and Tesar (1995), the non-traded good expenditure data is obtained from OECD Quarterly National Accounts for 'private consumption services' for the proxy for non-traded goods for Canada, Japan, U.K., Finland, France, Italy and U.S. For Australia, Austria, Germany, Portugal and Spain, I use data from 'services' from GDP by activity as the proxy for non-traded goods[28]. Non-traded good data is not available for Switzerland and I assume 40% of total expenditure spent for non-traded good consumption[29].

For our construction of the variable $(\sum_{j\neq i} p_{ij}(s^t)m_{ij}(s^t))$, the total imports of goods and services are obtained from data from IFS (line 71.D). Since not all imports are for consumption, we obtain the breakdown of total imports in terms of consumption goods versus capital goods from Datastream[30]. For Australia, Switzerland, Austria, Finland, France, Germany and Spain, I subtract imports of capital goods, intermediate goods and raw materials from total imports to obtain consumption from imported goods. For Canada, Japan, U.K, Italy, Portugal and U.S., I subtract imports of machinery and equipment from total imports to obtain consumption from imported goods.

We calculate expenditure on domestic traded good as the difference between total expenditure less

[28]Data from France and Italy indicate that consumption services are usually 40% of GDP services. We assume non-traded goods consumption are 40% of the 'GDP services' data.

[29]Stockman and Tesar (1990) show that on average countries consume a fraction of 30% to 50% on non-traded goods out of total consumption (Table 9)

[30]The original sources from datastream for the breakdown of imports are listed as follows. Data for Australia is from Australia Bureau of Statistics. Data for Canada is from Cansim - Statistics Canada. Data for Japan are from Ministry of Finance, Japan. Data for Switzerland are from National Bank of Switzerland. Data for U.K. are from Office of National Statistics. Data for Austria are from Statistik Austria. Data for Finland are from Central Statistical Office of Finland. Data for France are from French Customs. Data for Germany are from Statistisches Bundesamt. Data for Italy are from Istituto Nazionale Di Statistica. Data for Portugal are from National Statistics Office, Portugal. Data from Spain are from Ministerio De Economia Y Hacienda. Data for U.S. are from U.S. Census Bureau.

expenditure on non-traded good less expenditure on consumption of imported goods.

$$p_{ii}(s^t)d_i(s^t) = p_i(s^t)c_i(s^t) - \sum_{j\neq i} p_{ij}(s^t)m_{ij}(s^t) - p_{iN}(s^t)n_i(s^t)$$

For the trade-based representation of real exchange rate $\ln e^T$, we use total expenditures (not deflated) for domestic good consumption $p_{ii}(s^t)d_i(s^t)$ and imported good $p_{ij}(s^t)m_{ij}(s^t)$ and real consumption (deflated) for $c_i(s^t)$.

$$\ln e^T_{ij}(s^t) = \rho[\frac{1}{2}\ln\frac{d_j(s^t)}{d_i(s^t)} + \frac{1}{2}\ln\frac{m_{ji}(s^t)}{m_{ij}(s^t)} + \ln\frac{c_i(s^t)}{c_j(s^t)}] = \rho[\frac{1}{2}\ln\frac{p_{jj}(s^t)d_j(s^t)}{p_{ii}(s^t)d_i(s^t)} + \frac{1}{2}\ln\frac{p_{ji}(s^t)m_{ji}(s^t)}{p_{ij}(s^t)m_{ij}(s^t)} + \ln\frac{c_i(s^t)}{c_j(s^t)}]$$

where the second equality is due to the no-arbitrage equilibrium pricing condition $p_{ii}(s^t) = p_{ji}(s^t)\eta(s^t), p_{jj}(s^t) = p_{ij}(s^t)\eta(s^t)$ from Proposition 1.

The trade-based representation of real exchange rate holds for both the per capita variable and the aggregate variables. Let M_i be the population for country i.

$$\begin{aligned}\ln e^T_{ij}(s^t) &= \rho\left(\frac{1}{2}\ln\frac{M_j(s^t)d_j(s^t)}{M_i(s^t)d_i(s^t)} + \frac{1}{2}\ln\frac{M_j(s^t)m_{ji}(s^t)}{M_i(s^t)m_{ij}(s^t)} + \ln\frac{M_i(s^t)c_i(s^t)}{M_j(s^t)c_j(s^t)}\right)\\ &= \rho\left(\frac{1}{2}\ln\frac{d_j(s^t)}{d_i(s^t)} + \frac{1}{2}\ln\frac{m_{ji}(s^t)}{m_{ij}(s^t)} + \ln\frac{c_i(s^t)}{c_j(s^t)}\right)\end{aligned}$$

For simplicity, I have used aggregate variables for our analysis for $\ln e^T$. For $\ln e^C$, I follow Backus and Smith's (1993) method and use per-capita variables.

All data series are quarterly series, except the population series is annual frequency. To facilitate our analysis, real exchange rates and quantity variables are converted to natural logarithms.

7 Appendix: Tables and Figures

This Appendix provides the additional details in the empirical section.

Table 10 compares the correlations of $Corr(\ln e^A, \ln e^C)$ versus $Corr(\ln e^A, \ln e^T)$ for all bilateral pairs in the sample[31]. Most of the correlations for $\ln e^A$ and $\ln e^C$ are very low and negative in many country pairs. On the contrary, the correlations for $\ln e^A$ and $\ln e^T$ are much higher for 75 out of 78 bilateral pairs (96%) in Table 10. Except the a few bilateral country pairs, all the correlations $Corr(\ln e^A, \ln e^T)$ in the table are positive. For most close trading partners with the U.S, such as Canada, Japan, U.K., France and Germany, $Corr(\ln e^A, \ln e^T)$ is over 0.8 for these countries. This higher correlation is because of the higher correlation of actual real exchange rate and the two other trade factors: the ratio of consumption in domestically-endowed good ($\ln\frac{d_j}{d_i}$) and the ratio of bilateral trade flows ($\ln\frac{m_{ji}}{m_{ij}}$).

Table 11 compares the $Corr(\Delta\ln e^A, \Delta\ln e^C)$ versus $Corr(\Delta\ln e^A, \Delta\ln e^T)$ for all bilateral pairs in the sample. In general, $Corr(\Delta\ln e^A, \Delta\ln e^T)$ are lower than in log levels. Nonetheless, except for two pairs (Spain-Australia and Spain-Finland) that have negative $Corr(\Delta\ln e^A, \Delta\ln e^T)$, all the other correlations $Corr(\Delta\ln e^A, \Delta\ln e^T)$ are positive. $Corr(\Delta\ln e^A, \Delta\ln e^T)$ are much higher than $Corr(\Delta\ln e^A, \Delta\ln e^C)$ for

[31] Table 10 is symmetric because $Corr(\ln e^A, \ln e^C) = Corr(-\ln e^A, -\ln e^C)$ and $Corr(\ln e^A, \ln e^T) = Corr(-\ln e^A, -\ln e^T)$.

67 out of 78 bilateral pairs (86%) in Table 11. Table 12 and Table 13 compare the correlations in log levels the HP-filtered and Band-pass-filtered series for $\ln e^A, \ln e^C$ and $\ln e^T$. The smoothing parameter for the HP-filter is 1600 for quarterly data. The band pass filter admits frequencies between 6 and 32 quarters. The moving average parameter for the band pass filter has 12 leads/lags. These correlations are similar to Table 10. The correlations between the HP-filtered $Corr(\ln e^A, \ln e^T)$ are much higher than the HP-filtered $Corr(\ln e^A, \ln e^C)$ for 76 out of 78 bilateral pairs (97%). The correlations between the band-pass-filtered $Corr(\ln e^A, \ln e^T)$ are much higher than the band-pass-filtered $Corr(\ln e^A, \ln e^C)$ for also 77 out of 78 bilateral pairs (99%). All the correlations of the band-pass filtered series are positive.

Table 14 illustrates the variances for $\ln e^A, \ln e^C, \ln e^T$ for all bilateral pairs in the sample. Although the value of ρ is below γ, the volatility of the trade-based representation matches the high volatility of actual real exchange rates better than the consumption-based representations for 68% of the times in Table 14. I also calculate the implied γ and implied ρ from the variance of the actual real exchange rates, the consumption-based and the trade-based representations of real exchange rates.

$$\text{Implied } \gamma = \left(\frac{Var(\ln e^A)}{Var(\ln \frac{c_i}{c_j})}\right)^{\frac{1}{2}}$$

$$\text{Implied } \rho = \left(\frac{Var(\ln e^A)}{Var(\frac{1}{2}\ln \frac{d_j}{d_i} + \frac{1}{2}\ln \frac{m_{ji}}{m_{ij}} + \ln \frac{c_i}{c_j})}\right)^{\frac{1}{2}}$$

The results are in Table 15 in the Appendix. The implied γ is between 0.303 to 8.449 to match the volatilities of real exchange rates and relative consumptions. The implied ρ is between 0.347 and 1.775 to match the volatilities of actual real exchange rates and the trade-based representation. These implied ρ values are also quite consistent with the $\hat{\rho}$ estimated earlier in this paper and other studies in the literature.

Table 16 shows the results of the persistence of $\ln e^A, \ln e^C, \ln e^T$ for all bilateral pairs. The persistence of actual real exchange rates is quite high and above 0.9 for many bilateral pairs. The relative consumptions are in general more persistent than the actual real exchange rates. The trade-based representation is quite persistent. We cannot reject $\ln e^T$ as unit root processes.

Table 17 examines the Backus-Smith (1993) puzzle and compares the correlations between $\ln e^A, \ln e^C, \ln e^T$ and relative consumptions. It is clear from the table that the trade-based representation of real exchange rate is much better in matching the low correlation between real exchange rates and relative consumptions.

Table 10: Correlation of actual $\ln e^A_{ijt}$ and the benchmark consumption-based representation of real exchange rates $\ln e^C = \gamma \ln \frac{c_{it}}{c_{jt}}$ + constant and Correlation of actual $\ln e^A_{ijt}$ and the trade-based representation of real exchange rates $\ln e^T_{ijt} = \frac{\rho}{2} \ln \frac{d_{jt}}{d_{it}} + \frac{\rho}{2} \ln \frac{m_{ijt}}{m_{jit}} + \rho \ln \frac{c_{it}}{c_{jt}}$. Data for all countries are from 1980:1-1998:4.

	Raw Data	Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	$Corr(\ln e^A; \ln e^C)$	-	-0.262	-0.758	0.587	-0.481	-0.686	-0.687	-0.307	-0.678	-0.708	-0.836	-0.767	-0.225
	$Corr(\ln e^A; \ln e^T)$	-	0.569	0.938	0.613	0.758	-0.061	0.477	0.085	0.424	0.587	-0.305	0.333	0.327
Canada	$Corr(\ln e^A; \ln e^C)$	-0.262	-	-0.712	0.524	-0.444	-0.564	-0.255	-0.285	-0.652	-0.681	-0.843	-0.701	-0.367
	$Corr(\ln e^A; \ln e^T)$	0.569	-	0.866	0.846	0.795	0.745	0.285	0.822	0.898	0.648	0.360	0.778	0.887
Japan	$Corr(\ln e^A; \ln e^C)$	-0.758	-0.712	-	-0.473	0.245	-0.482	-0.741	-0.572	0.090	-0.272	-0.173	-0.500	-0.651
	$Corr(\ln e^A; \ln e^T)$	0.938	0.866	-	0.707	0.843	0.465	0.864	0.837	0.954	0.774	0.547	0.568	0.874
Switzerland	$Corr(\ln e^A; \ln e^C)$	0.587	0.524	-0.473	-	0.586	0.071	0.316	0.708	0.567	0.369	-0.589	0.038	0.601
	$Corr(\ln e^A; \ln e^T)$	0.613	0.846	0.707	-	0.427	0.168	0.802	0.611	0.761	0.826	-0.067	0.676	0.778
U.K.	$Corr(\ln e^A; \ln e^C)$	-0.481	-0.444	0.245	0.586	-	0.635	-0.309	0.122	-0.109	0.370	-0.747	0.312	-0.431
	$Corr(\ln e^A; \ln e^T)$	0.758	0.795	0.843	0.427	-	0.773	0.471	0.446	0.542	0.729	0.436	0.828	0.801
Austria	$Corr(\ln e^A; \ln e^C)$	-0.686	-0.564	-0.482	0.071	0.635	-	-0.761	-0.413	0.551	0.228	-0.628	-0.322	-0.077
	$Corr(\ln e^A; \ln e^T)$	-0.061	0.745	0.465	0.168	0.773	-	0.783	0.084	-0.392	0.822	0.227	0.439	0.837
Finland	$Corr(\ln e^A; \ln e^C)$	-0.687	-0.255	-0.741	0.316	-0.309	-0.761	-	-0.326	-0.801	-0.499	-0.854	-0.870	-0.295
	$Corr(\ln e^A; \ln e^T)$	0.477	0.285	0.864	0.802	0.471	0.783	-	0.652	0.881	0.692	0.805	0.763	0.654
France	$Corr(\ln e^A; \ln e^C)$	-0.307	-0.285	-0.572	0.708	0.122	-0.413	-0.326	-	-0.445	0.087	-0.881	-0.299	-0.073
	$Corr(\ln e^A; \ln e^T)$	0.085	0.822	0.837	0.611	0.446	0.084	0.652	-	0.410	0.735	0.332	0.854	0.864
Germany	$Corr(\ln e^A; \ln e^C)$	-0.678	-0.652	0.090	0.567	-0.109	0.551	-0.801	-0.445	-	-0.566	-0.614	-0.130	-0.526
	$Corr(\ln e^A; \ln e^T)$	0.424	0.898	0.954	0.761	0.542	-0.392	0.881	0.410	-	0.945	0.186	0.844	0.907
Italy	$Corr(\ln e^A; \ln e^C)$	-0.708	-0.681	-0.272	0.369	0.370	0.228	-0.499	0.087	-0.566	-	-0.700	-0.052	-0.555
	$Corr(\ln e^A; \ln e^T)$	0.587	0.648	0.774	0.826	0.729	0.822	0.692	0.735	0.945	-	0.705	0.531	0.650
Portugal	$Corr(\ln e^A; \ln e^C)$	-0.836	-0.843	-0.173	-0.589	-0.747	-0.628	-0.854	-0.881	-0.614	-0.700	-	-0.558	-0.860
	$Corr(\ln e^A; \ln e^T)$	-0.305	0.360	0.547	-0.067	0.436	0.227	0.805	0.332	0.186	0.705	-	0.391	-0.134
Spain	$Corr(\ln e^A; \ln e^C)$	-0.767	-0.701	-0.500	0.038	0.312	-0.322	-0.870	-0.299	-0.130	-0.052	-0.558	-	-0.595
	$Corr(\ln e^A; \ln e^T)$	0.333	0.778	0.568	0.676	0.828	0.439	0.763	0.854	0.844	0.531	0.391	-	0.733
U.S.	$Corr(\ln e^A; \ln e^C)$	-0.225	-0.367	-0.651	0.601	-0.431	-0.077	-0.295	-0.073	-0.526	-0.555	-0.860	-0.595	-
	$Corr(\ln e^A; \ln e^T)$	0.327	0.887	0.874	0.778	0.801	0.837	0.654	0.864	0.907	0.650	-0.134	0.733	-
	$Corr(\ln e^A, \ln e^T) >$ $Corr(\ln e^A, \ln e^C)$	150/156	96%											
	$Corr(\ln e^A, \ln e^C) > 0$	36/156	23%											
	$Corr(\ln e^A, \ln e^T) > 0$	146/156	94%											

Table 11: Correlation of actual real exchange rates $\Delta \ln e^A_{ijt}$ and the benchmark consumption-based representation $\Delta \ln e^C = \gamma \Delta \ln \frac{c_{it}}{c_{jt}}$ and Correlation of actual real exchange rates $\Delta \ln e^A_{ijt}$ and the trade-based representation $\Delta \ln e^T_{ijt} = \frac{\rho}{2}\Delta \ln \frac{d_{jt}}{d_{it}} + \frac{\rho}{2}\Delta \ln \frac{m_{ijt}}{m_{jit}} + \rho \Delta \ln \frac{c_{it}}{c_{jt}}$. Data for all countries are from 1980:1-1998:4.

	First Difference	Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	-	0.148	0.187	-0.045	0.168	0.021	0.058	0.041	0.068	0.086	0.258	0.134	0.086
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	-	0.146	0.723	0.240	0.459	0.174	0.205	0.235	0.515	0.251	0.246	-0.090	0.422
Canada	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.148	-	0.266	0.075	0.276	0.173	0.120	0.103	0.203	0.184	0.200	0.114	0.021
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.146	-	0.592	0.293	0.406	0.222	0.135	0.260	0.685	0.355	0.164	0.303	0.495
Japan	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.187	0.266	-	0.123	0.191	0.217	0.064	0.219	0.253	0.072	0.270	0.178	0.056
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.723	0.592	-	0.387	0.443	0.273	0.300	0.307	0.741	0.168	0.288	0.346	0.618
Switzerland	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	-0.045	0.075	0.123	-	0.118	0.077	-0.034	-0.167	-0.054	-0.004	-0.001	-0.069	-0.214
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.240	0.293	0.387	-	0.261	0.354	0.245	0.221	0.521	0.514	0.432	0.297	0.188
U.K.	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.168	0.276	0.191	0.118	-	0.309	0.081	0.137	0.220	0.205	0.274	0.302	0.008
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.459	0.406	0.443	0.261	-	0.499	0.180	0.561	0.692	0.309	0.373	0.195	0.250
Austria	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.021	0.173	0.217	0.077	0.309	-	0.201	0.406	0.566	0.257	0.140	0.018	-0.062
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.174	0.222	0.273	0.354	0.499	-	0.413	0.188	0.368	0.558	0.331	0.114	0.415
Finland	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.058	0.120	0.064	-0.034	0.081	0.201	-	0.105	0.074	0.298	0.090	0.050	-0.278
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.205	0.135	0.300	0.245	0.180	0.413	-	0.114	0.475	0.189	0.077	-0.022	0.185
France	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.041	0.103	0.219	-0.167	0.137	0.406	0.105	-	0.211	0.148	0.209	0.137	-0.190
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.235	0.260	0.307	0.221	0.561	0.188	0.114	-	0.380	0.442	0.236	0.285	0.496
Germany	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.068	0.203	0.253	-0.054	0.220	0.566	0.074	0.211	-	0.122	0.212	0.192	0.019
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.515	0.685	0.741	0.521	0.692	0.368	0.475	0.380	-	0.721	0.453	0.569	0.596
Italy	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.086	0.184	0.072	-0.004	0.205	0.257	0.298	0.148	0.122	-	0.261	0.395	-0.081
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.251	0.355	0.168	0.514	0.309	0.558	0.189	0.442	0.721	-	0.282	0.437	0.343
Portugal	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.258	0.200	0.270	-0.001	0.274	0.140	0.090	0.209	0.212	0.261	-	0.230	-0.029
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.246	0.164	0.288	0.432	0.373	0.331	0.077	0.236	0.453	0.282	-	0.085	0.170
Spain	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.134	0.114	0.178	-0.069	0.302	0.018	0.050	0.137	0.192	0.395	0.230	-	-0.158
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	-0.090	0.303	0.346	0.297	0.195	0.114	-0.022	0.285	0.569	0.437	0.085	-	0.263
U.S.	$Corr(\Delta \ln e^A; \Delta \ln e^C)$	0.086	0.021	0.056	-0.214	0.008	-0.062	-0.278	-0.190	0.019	-0.081	-0.029	-0.158	-
	$Corr(\Delta \ln e^A; \Delta \ln e^T)$	0.422	0.495	0.618	0.188	0.250	0.415	0.185	0.496	0.596	0.343	0.170	0.263	-
	$Corr(\Delta \ln e^A, \Delta \ln e^T) >$ $Corr(\Delta \ln e^A, \Delta \ln e^C)$	134/156	86%											
	$Corr(\Delta \ln e^A, \Delta \ln e^C) > 0$	128/156	82%											
	$Corr(\Delta \ln e^A, \Delta \ln e^T) > 0$	152/156	97%											

Table 12: Correlation of HP-filtered actual real exchange rates $\ln e_{ijt}^A$ and the HP-filtered benchmark consumption-based representation $\ln e^C = \gamma \ln \frac{c_{it}}{c_{jt}}$ and Correlation of HP-filtered actual real exchange rates $\ln e_{ijt}^A$ and the HP-filtered trade-based representation $\ln e_{ijt}^T = \frac{\rho}{2} \ln \frac{d_{jt}}{d_{it}} + \frac{\rho}{2} \ln \frac{m_{ijt}}{m_{jit}} + \rho \ln \frac{c_{it}}{c_{jt}}$. HP filter smoothing parameter is 1600 for quarterly data. Data for all countries are from 1980:1-1998:4.

		Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	Corr($ln_eA; ln_eC$)	-	0.308	0.270	-0.235	0.252	0.022	-0.085	-0.041	-0.219	-0.267	0.281	-0.074	0.057
	Corr($ln_eA; ln_eT$)	-	0.438	0.831	0.247	0.653	0.548	0.261	0.604	0.698	0.497	0.472	0.592	0.654
Canada	Corr($ln_eA; ln_eC$)	0.308	-	0.439	-0.006	0.344	0.224	0.023	0.033	0.063	-0.107	0.177	0.060	-0.056
	Corr($ln_eA; ln_eT$)	0.438	-	0.775	0.411	0.339	0.336	0.356	0.608	0.806	0.619	0.539	0.467	0.756
Japan	Corr($ln_eA; ln_eC$)	0.270	0.439	-	0.281	0.505	0.016	-0.315	0.187	0.227	0.075	0.131	-0.007	0.340
	Corr($ln_eA; ln_eT$)	0.831	0.775	-	0.495	0.669	0.250	0.492	0.520	0.874	0.444	0.270	0.631	0.733
Switzerland	Corr($ln_eA; ln_eC$)	-0.235	-0.006	0.281	-	0.124	0.036	-0.572	0.088	-0.040	-0.229	-0.134	-0.323	-0.102
	Corr($ln_eA; ln_eT$)	0.247	0.411	0.495	-	0.535	0.413	0.472	0.402	0.527	0.672	0.473	0.432	0.435
U.K.	Corr($ln_eA; ln_eC$)	0.252	0.344	0.505	0.124	-	0.240	-0.170	-0.111	0.156	0.165	0.195	0.080	-0.109
	Corr($ln_eA; ln_eT$)	0.653	0.339	0.669	0.535	-	0.764	0.528	0.512	0.835	0.496	0.469	0.397	0.475
Austria	Corr($ln_eA; ln_eC$)	0.022	0.224	0.016	0.036	0.240	-	-0.413	0.232	0.350	0.040	-0.065	-0.170	0.127
	Corr($ln_eA; ln_eT$)	0.548	0.336	0.250	0.413	0.764	-	0.631	0.374	0.489	0.691	0.439	0.346	0.645
Finland	Corr($ln_eA; ln_eC$)	-0.085	0.023	-0.315	-0.572	-0.170	-0.413	-	-0.367	-0.394	-0.073	-0.120	-0.415	-0.372
	Corr($ln_eA; ln_eT$)	0.261	0.356	0.492	0.472	0.528	0.631	-	0.375	0.762	0.553	0.324	0.189	0.604
France	Corr($ln_eA; ln_eC$)	-0.041	0.033	0.187	0.088	-0.111	0.232	-0.367	-	0.225	-0.002	-0.113	-0.203	-0.122
	Corr($ln_eA; ln_eT$)	0.604	0.608	0.520	0.402	0.512	0.374	0.375	-	0.718	0.654	0.513	0.528	0.721
Germany	Corr($ln_eA; ln_eC$)	-0.219	0.063	0.227	-0.040	0.156	0.350	-0.394	0.225	-	0.022	0.178	0.023	0.060
	Corr($ln_eA; ln_eT$)	0.698	0.806	0.874	0.527	0.835	0.489	0.762	0.718	-	0.891	0.779	0.807	0.767
Italy	Corr($ln_eA; ln_eC$)	-0.267	-0.107	0.075	-0.229	0.165	0.040	-0.073	-0.002	0.022	-	0.196	0.426	-0.197
	Corr($ln_eA; ln_eT$)	0.497	0.619	0.444	0.672	0.496	0.691	0.553	0.654	0.891	-	0.458	0.356	0.713
Portugal	Corr($ln_eA; ln_eC$))	0.281	0.177	0.131	-0.134	0.195	-0.065	-0.120	-0.113	0.178	0.196	-	0.203	-0.088
	Corr($ln_eA; ln_eT$)	0.472	0.539	0.270	0.473	0.469	0.439	0.324	0.513	0.779	0.458	-	0.488	0.351
Spain	Corr($ln_eA; ln_eC$))	-0.074	0.060	-0.007	-0.323	0.080	-0.170	-0.415	-0.203	0.023	0.426	0.203	-	-0.222
	Corr($ln_eA; ln_eT$)	0.592	0.467	0.631	0.432	0.397	0.346	0.189	0.528	0.807	0.356	0.488	-	0.575
U.S.	Corr($ln_eA; ln_eC$))	0.057	-0.056	0.340	-0.102	-0.109	0.127	-0.372	-0.122	0.060	-0.197	-0.088	-0.222	-
	Corr($ln_eA; ln_eT$)	0.654	0.756	0.733	0.435	0.475	0.645	0.604	0.721	0.767	0.713	0.351	0.575	-
	$Corr(\ln e^A, \ln e^T) >$ $Corr(\ln e^A, \ln e^C)$	152/156	97%											
	$Corr(\ln e^A, \ln e^C) > 0$	84/156	54%											
	$Corr(\ln e^A, \ln e^T) > 0$	156/156	100%											

Table 13: Correlation of Baxter and King (1999) band-pass filtered actual real exchange rates $\ln e^A_{ijt}$ and the band-pass filtered benchmark consumption-based representation $\ln e^C = \gamma \ln \frac{c_{it}}{c_{jt}}$ and Correlation of band-pass filtered actual real exchange rates $\ln e^A_{ijt}$ and the band-pass-filtered trade-based representation $\ln e^T_{ijt} = \frac{\rho}{2} \ln \frac{d_{jt}}{d_{it}} + \frac{\rho}{2} \ln \frac{m_{ijt}}{m_{jit}} + \rho \ln \frac{c_{it}}{c_{jt}}$. The band pass filter admits frequency components between 6 to 32 quarters, with moving average parameter of 12. Data for all countries are from 1980:1-1998:4.

	Band-pass filtered series	Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	$Corr(\ln e^A; \ln e^C)$	-	0.475	0.193	-0.415	0.331	-0.020	-0.184	-0.243	-0.286	-0.519	0.152	-0.196	0.183
	$Corr(\ln e^A; \ln e^T)$	-	0.525	0.840	0.285	0.656	0.608	0.434	0.760	0.717	0.765	0.726	0.838	0.633
Canada	$Corr(\ln e^A; \ln e^C)$	0.475	-	0.372	-0.175	0.295	0.138	-0.069	-0.114	-0.039	-0.410	0.045	-0.125	0.008
	$Corr(\ln e^A; \ln e^T)$	0.525	-	0.856	0.456	0.303	0.529	0.539	0.726	0.810	0.797	0.761	0.662	0.856
Japan	$Corr(\ln e^A; \ln e^C)$	0.193	0.372	-	0.329	0.622	-0.133	-0.511	0.371	0.153	-0.115	0.113	-0.196	0.196
	$Corr(\ln e^A; \ln e^T)$	0.840	0.856	-	0.613	0.759	0.470	0.715	0.721	0.886	0.764	0.297	0.724	0.817
Switzerland	$Corr(\ln e^A; \ln e^C)$	-0.415	-0.175	0.329	-	0.062	0.290	-0.770	0.286	-0.003	-0.293	-0.188	-0.324	-0.381
	$Corr(\ln e^A; \ln e^T)$	0.285	0.456	0.613	-	0.792	0.304	0.683	0.663	0.420	0.872	0.534	0.629	0.630
U.K.	$Corr(\ln e^A; \ln e^C)$	0.331	0.295	0.622	0.062	-	0.232	-0.293	-0.124	0.139	-0.089	-0.027	-0.222	-0.150
	$Corr(\ln e^A; \ln e^T)$	0.656	0.303	0.759	0.792	-	0.824	0.755	0.276	0.870	0.602	0.568	0.570	0.710
Austria	$Corr(\ln e^A; \ln e^C)$	-0.020	0.138	-0.133	0.290	0.232	-	-0.623	0.157	0.332	0.036	-0.053	-0.195	-0.064
	$Corr(\ln e^A; \ln e^T)$	0.608	0.529	0.470	0.304	0.824	-	0.689	0.748	0.612	0.822	0.380	0.462	0.853
Finland	$Corr(\ln e^A; \ln e^C)$	-0.184	-0.069	-0.511	-0.770	-0.293	-0.623	-	-0.601	-0.467	-0.265	-0.309	-0.650	-0.479
	$Corr(\ln e^A; \ln e^T)$	0.434	0.539	0.715	0.683	0.755	0.689	-	0.524	0.819	0.706	0.613	0.681	0.801
France	$Corr(\ln e^A; \ln e^C)$	-0.243	-0.114	0.371	0.286	-0.124	0.157	-0.601	-	0.181	-0.136	-0.198	-0.384	-0.425
	$Corr(\ln e^A; \ln e^T)$	0.760	0.726	0.721	0.663	0.276	0.748	0.524	-	0.872	0.872	0.632	0.744	0.887
Germany	$Corr(\ln e^A; \ln e^C)$	-0.286	-0.039	0.153	-0.003	0.139	0.332	-0.467	0.181	-	0.054	0.283	0.099	-0.105
	$Corr(\ln e^A; \ln e^T)$	0.717	0.810	0.886	0.420	0.870	0.612	0.819	0.872	-	0.952	0.819	0.877	0.824
Italy	$Corr(\ln e^A; \ln e^C)$	-0.519	-0.410	-0.115	-0.293	-0.089	0.036	-0.265	-0.136	0.054	-	0.116	0.455	-0.511
	$Corr(\ln e^A; \ln e^T)$	0.765	0.797	0.764	0.872	0.602	0.822	0.706	0.872	0.952	-	0.274	0.217	0.876
Portugal	$Corr(\ln e^A; \ln e^C)$	0.152	0.045	0.113	-0.188	-0.027	-0.053	-0.309	-0.198	0.283	0.116	-	0.290	-0.245
	$Corr(\ln e^A; \ln e^T)$	0.726	0.761	0.297	0.534	0.568	0.380	0.613	0.632	0.819	0.274	-	0.667	0.588
Spain	$Corr(\ln e^A; \ln e^C)$	-0.196	-0.125	-0.196	-0.324	-0.222	-0.195	-0.650	-0.384	0.099	0.455	0.290	-	-0.485
	$Corr(\ln e^A; \ln e^T)$	0.838	0.662	0.724	0.629	0.570	0.462	0.681	0.744	0.877	0.217	0.667	-	0.848
U.S.	$Corr(\ln e^A; \ln e^C)$	0.183	0.008	0.196	-0.381	-0.150	-0.064	-0.479	-0.425	-0.105	-0.511	-0.245	-0.485	-
	$Corr(\ln e^A; \ln e^T)$	0.633	0.856	0.817	0.630	0.710	0.853	0.801	0.887	0.824	0.876	0.588	0.848	-
	$Corr(\ln e^A, \ln e^T) >$ $Corr(\ln e^A, \ln e^C)$	154/156	99%											
	$Corr(\ln e^A, \ln e^C) > 0$	62/156	40%											
	$Corr(\ln e^A, \ln e^T) > 0$	156/156	100%											

Table 14: Variance of actual real exchange rates, consumption-based and trade-based representations of real exchange rates. Assume $\gamma = 2$, $\rho = 1$ for all countries.

		Australia	Canada	Japan	Swtizerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	var($\ln e^A$)	-	0.010	0.065	0.038	0.017	0.038	0.019	0.025	0.029	0.029	0.051	0.036	0.013
	var($\ln e^C$)	-	0.006	0.027	0.008	0.035	0.008	0.010	0.008	0.056	0.024	0.167	0.021	0.008
	var($\ln e^T$)	-	0.027	0.025	0.143	0.020	0.055	0.111	0.033	0.009	0.021	0.088	0.041	0.022
Canada	var($\ln e^A$)	0.010	-	0.062	0.041	0.023	0.041	0.017	0.030	0.034	0.023	0.057	0.038	0.008
	var($\ln e^C$)	0.006	-	0.037	0.004	0.045	0.018	0.009	0.009	0.080	0.034	0.195	0.033	0.010
	var($\ln e^T$)	0.027	-	0.074	0.125	0.059	0.113	0.058	0.065	0.061	0.068	0.052	0.131	0.033
Japan	var($\ln e^A$)	0.065	0.062	-	0.011	0.042	0.012	0.040	0.022	0.018	0.027	0.019	0.026	0.052
	var($\ln e^C$)	0.027	0.037	-	0.054	0.004	0.007	0.018	0.015	0.017	0.004	0.071	0.005	0.011
	var($\ln e^T$)	0.025	0.074	-	0.015	0.020	0.032	0.059	0.025	0.008	0.032	0.035	0.039	0.032
Switzerland	var($\ln e^A$)	0.038	0.041	0.011	-	0.017	0.001	0.020	0.005	0.002	0.012	0.008	0.010	0.033
	var($\ln e^C$)	0.008	0.004	0.054	-	0.067	0.027	0.021	0.015	0.097	0.052	0.233	0.047	0.022
	var($\ln e^T$)	0.143	0.125	0.015	-	0.010	0.009	0.019	0.002	0.002	0.011	0.016	0.013	0.034
U.K.	var($\ln e^A$)	0.017	0.023	0.042	0.017	-	0.016	0.014	0.008	0.011	0.012	0.022	0.013	0.020
	var($\ln e^C$)	0.035	0.045	0.004	0.067	-	0.012	0.021	0.026	0.019	0.005	0.061	0.008	0.014
	var($\ln e^T$)	0.020	0.059	0.020	0.010	-	0.020	0.007	0.004	0.004	0.007	0.011	0.013	0.031
Austria	var($\ln e^A$)	0.038	0.041	0.012	0.001	0.016	-	0.019	0.003	0.001	0.010	0.006	0.007	0.034
	var($\ln e^C$)	0.008	0.018	0.007	0.027	0.012	-	0.008	0.007	0.028	0.008	0.111	0.006	0.004
	var($\ln e^T$)	0.055	0.113	0.032	0.009	0.020	-	0.036	0.004	0.001	0.008	0.012	0.013	0.040
Finland	var($\ln e^A$)	0.019	0.017	0.040	0.020	0.014	0.019	-	0.012	0.016	0.007	0.033	0.013	0.025
	var($\ln e^C$)	0.010	0.009	0.018	0.021	0.021	0.008	-	0.008	0.058	0.017	0.142	0.019	0.003
	var($\ln e^T$)	0.111	0.058	0.059	0.019	0.007	0.036	-	0.010	0.005	0.019	0.060	0.044	0.042
France	var($\ln e^A$)	0.025	0.030	0.022	0.005	0.008	0.003	0.012	-	0.001	0.008	0.011	0.006	0.026
	var($\ln e^C$)	0.008	0.009	0.015	0.015	0.026	0.007	0.008	-	0.048	0.018	0.144	0.017	0.005
	var($\ln e^T$)	0.033	0.065	0.025	0.002	0.004	0.004	0.010	-	0.001	0.004	0.010	0.010	0.027
Germany	var($\ln e^A$)	0.029	0.034	0.018	0.002	0.011	0.001	0.016	0.001	-	0.010	0.010	0.008	0.028
	var($\ln e^C$)	0.056	0.080	0.017	0.097	0.019	0.028	0.058	0.048	-	0.016	0.047	0.015	0.042
	var($\ln e^T$)	0.009	0.061	0.008	0.002	0.004	0.001	0.005	0.001	-	0.004	0.006	0.007	0.018
Italy	var($\ln e^A$)	0.029	0.023	0.027	0.012	0.012	0.010	0.007	0.008	0.010	-	0.019	0.006	0.028
	var($\ln e^C$)	0.024	0.034	0.004	0.052	0.005	0.008	0.017	0.018	0.016	-	0.069	0.002	0.012
	var($\ln e^T$)	0.021	0.068	0.032	0.011	0.007	0.008	0.019	0.004	0.004	-	0.030	0.010	0.028
Portugal	var($\ln e^A$)	0.051	0.057	0.019	0.008	0.022	0.006	0.033	0.011	0.010	0.019	-	0.008	0.049
	var($\ln e^C$)	0.167	0.195	0.071	0.233	0.061	0.111	0.142	0.144	0.047	0.069	-	0.073	0.129
	var($\ln e^T$)	0.088	0.052	0.035	0.016	0.011	0.012	0.060	0.010	0.006	0.030	-	0.030	0.057
Spain	var($\ln e^A$)	0.036	0.038	0.026	0.010	0.013	0.007	0.013	0.006	0.008	0.006	0.008	-	0.041
	var($\ln e^C$)	0.021	0.033	0.005	0.047	0.008	0.006	0.019	0.017	0.015	0.002	0.073	-	0.013
	var($\ln e^T$)	0.041	0.131	0.039	0.013	0.013	0.013	0.044	0.010	0.007	0.010	0.030	-	0.041
U.S.	var($\ln e^A$)	0.013	0.008	0.052	0.033	0.020	0.034	0.025	0.026	0.028	0.028	0.049	0.041	-
	var($\ln e^C$)	0.008	0.010	0.011	0.022	0.014	0.004	0.003	0.005	0.042	0.012	0.129	0.013	-
	var($\ln e^T$)	0.022	0.033	0.032	0.034	0.031	0.040	0.042	0.027	0.018	0.028	0.057	0.041	-
												$\|\mathrm{Var}(\ln e^T)$	$-\mathrm{Var}(\ln e^A)\|$	106/156
												$< \|\mathrm{Var}(\ln e^C)$	$-\mathrm{Var}(\ln e^A)\|$	(68%)

Table 15: Implied γ and Implied ρ. Implied γ is calculated as $\sqrt{\frac{Var(\ln e^A)}{Var(\ln \frac{c_i}{c_{US}})}}$. Implied ρ is calculated as $\sqrt{\frac{Var(\ln e^A)}{Var(\frac{1}{2}\ln\frac{d_{US}}{d_i}+\frac{1}{2}\ln\frac{m_{USi}}{m_{iUS}}+\ln\frac{c_i}{c_{US}})}}$.

Implied γ	Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	-	2.273	5.882	2.443	3.391	8.449	2.516	2.737	2.240	4.820	1.712	5.250	3.640
Canada	2.273	-	4.308	3.839	2.760	4.951	4.577	4.538	1.726	3.818	1.525	3.597	2.210
Japan	5.882	4.308	-	1.110	6.650	2.383	3.052	2.645	1.865	4.128	1.220	4.064	6.592
Switzerland	2.443	3.839	1.110	-	1.297	0.491	2.530	1.484	0.349	1.422	0.444	1.149	2.103
U.K.	3.391	2.760	6.650	1.297	-	2.610	1.777	1.293	1.410	2.595	1.348	2.803	5.640
Austria	8.449	4.951	2.383	0.491	2.610	-	2.693	1.251	0.463	2.680	0.565	2.257	4.763
Finland	2.516	4.577	3.052	2.530	1.777	2.693	-	2.467	1.033	1.451	1.093	1.692	3.171
France	2.737	4.538	2.645	1.484	1.293	1.251	2.467	-	0.303	1.725	0.652	1.307	2.810
Germany	2.240	1.726	1.865	0.349	1.410	0.463	1.033	0.303	-	1.319	1.052	1.408	1.941
Italy	4.820	3.818	4.128	1.422	2.595	2.680	1.451	1.725	1.319	-	1.102	3.517	4.015
Portugal	1.712	1.525	1.220	0.444	1.348	0.565	1.093	0.652	1.052	1.102	-	0.803	1.699
Spain	5.250	3.597	4.064	1.149	2.803	2.257	1.692	1.307	1.408	3.517	0.803	-	4.302
U.S.	3.640	2.210	6.592	2.103	5.640	4.763	3.171	2.810	1.941	4.015	1.699	4.302	-
												Max γ	8.449
												Min γ	0.303

Implied ρ	Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	-	0.609	1.619	0.513	0.918	0.826	0.415	0.877	1.745	1.156	0.757	0.929	0.777
Canada	0.609	-	0.915	0.575	0.625	0.602	0.542	0.683	0.746	0.582	1.048	0.537	0.492
Japan	1.619	0.915	-	0.885	1.449	0.605	0.831	0.940	1.501	0.927	0.732	0.813	1.269
Switzerland	0.513	0.575	0.885	-	1.322	0.347	1.031	1.607	0.959	1.050	0.698	0.881	0.992
U.K.	0.918	0.625	1.449	1.322	-	0.891	1.376	1.413	1.775	1.254	1.394	1.028	0.803
Austria	0.826	0.602	0.605	0.347	0.891	-	0.722	0.867	1.491	1.123	0.716	0.724	0.923
Finland	0.415	0.542	0.831	1.031	1.376	0.722	-	1.093	1.705	0.586	0.745	0.545	0.771
France	0.877	0.683	0.940	1.607	1.413	0.867	1.093	-	1.090	1.391	1.082	0.796	0.982
Germany	1.745	0.746	1.501	0.959	1.775	1.491	1.705	1.090	-	1.557	1.292	1.078	1.235
Italy	1.156	0.582	0.927	1.050	1.254	1.123	0.586	1.391	1.557	-	0.794	0.765	0.998
Portugal	0.757	1.048	0.732	0.698	1.394	0.716	0.745	1.082	1.292	0.794	-	0.533	0.925
Spain	0.929	0.537	0.813	0.881	1.028	0.724	0.545	0.796	1.078	0.765	0.533	-	1.008
U.S.	0.777	0.492	1.269	0.992	0.803	0.923	0.771	0.982	1.235	0.998	0.925	1.008	-
												Max ρ	1.775
												Min ρ	0.347

Table 16: Persistence of actual real exchange rates $\ln e^A$, consumption-based representation $\ln e^C$ and the trade-based representation of real exchange rates $\ln e^T$.

		Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	-	0.894	0.957	0.920	0.865	0.928	0.891	0.903	0.903	0.918	0.952	0.936	0.903
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	-	0.948	0.983	0.974	0.988	0.964	0.972	0.963	0.992	0.989	0.996	0.988	0.969
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	-	0.762	0.918	0.863	0.797	0.704	0.808	0.700	0.723	0.470	0.590	0.751	0.899
Canada	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.894	-	0.960	0.940	0.921	0.947	0.914	0.932	0.933	0.920	0.968	0.954	0.968
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.948	-	0.982	0.952	0.991	0.982	0.972	0.961	0.992	0.991	0.997	0.991	0.978
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.762	-	0.947	0.899	0.932	0.728	0.717	0.845	0.975	0.919	0.617	0.896	0.983
Japan	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.957	0.960	-	0.864	0.953	0.862	0.949	0.928	0.907	0.933	0.900	0.919	0.955
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.983	0.982	-	0.992	0.912	0.932	0.977	0.977	0.963	0.883	0.988	0.895	0.961
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.918	0.947	-	0.717	0.821	0.823	0.890	0.852	0.901	0.779	0.799	0.870	0.930
Switzerland	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.920	0.940	0.864	-	0.918	0.668	0.944	0.925	0.846	0.923	0.894	0.900	0.931
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.974	0.952	0.992	-	0.996	0.994	0.991	0.989	0.997	0.997	0.998	0.997	0.992
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.863	0.899	0.717	-	0.596	0.916	0.884	0.735	0.912	0.806	0.702	0.820	0.853
U.K.	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.865	0.921	0.953	0.918	-	0.919	0.941	0.865	0.889	0.914	0.949	0.926	0.913
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.988	0.991	0.912	0.996	-	0.968	0.990	0.991	0.975	0.937	0.989	0.950	0.985
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.797	0.932	0.821	0.596	-	0.937	0.471	0.593	0.818	0.708	0.717	0.781	0.900
Austria	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.928	0.947	0.862	0.668	0.919	-	0.950	0.959	0.963	0.926	0.917	0.919	0.943
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.964	0.982	0.932	0.994	0.968	-	0.973	0.972	0.986	0.962	0.994	0.970	0.940
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.704	0.728	0.823	0.916	0.937	-	0.929	0.599	0.582	0.753	0.597	0.709	0.866
Finland	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.891	0.914	0.949	0.944	0.941	0.950	-	0.930	0.939	0.895	0.975	0.948	0.946
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.972	0.972	0.977	0.991	0.990	0.973	-	0.979	0.991	0.983	0.996	0.986	0.950
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.808	0.717	0.890	0.884	0.471	0.929	-	0.711	0.885	0.844	0.679	0.771	0.815
France	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.903	0.932	0.928	0.925	0.865	0.959	0.930	-	0.886	0.928	0.960	0.914	0.930
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.963	0.961	0.977	0.989	0.991	0.972	0.979	-	0.992	0.986	0.996	0.986	0.959
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.700	0.845	0.852	0.735	0.593	0.599	0.711	-	0.744	0.812	0.682	0.883	0.908
Germany	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.903	0.933	0.907	0.846	0.889	0.963	0.939	0.886	-	0.930	0.944	0.928	0.927
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.992	0.992	0.963	0.997	0.975	0.986	0.991	0.992	-	0.979	0.984	0.974	0.989
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.723	0.975	0.901	0.912	0.818	0.582	0.885	0.744	-	0.870	0.843	0.875	0.949
Italy	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.918	0.920	0.933	0.923	0.914	0.926	0.895	0.928	0.930	-	0.964	0.894	0.939
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.989	0.991	0.883	0.997	0.937	0.962	0.983	0.986	0.979	-	0.993	0.907	0.974
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.470	0.919	0.779	0.806	0.708	0.753	0.844	0.812	0.870	-	0.908	0.763	0.895
Portugal	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.952	0.968	0.900	0.894	0.949	0.917	0.975	0.960	0.944	0.964	-	0.951	0.967
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.996	0.997	0.988	0.998	0.989	0.994	0.996	0.996	0.984	0.993	-	0.995	0.995
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.590	0.617	0.799	0.702	0.717	0.597	0.679	0.682	0.843	0.908	-	0.897	0.868
Spain	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.936	0.954	0.919	0.900	0.926	0.919	0.948	0.914	0.928	0.894	0.951	-	0.962
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.988	0.991	0.895	0.997	0.950	0.970	0.986	0.986	0.974	0.907	0.995	-	0.979
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.751	0.896	0.870	0.820	0.781	0.709	0.771	0.883	0.875	0.763	0.897	-	0.861
U.S.	$\mathrm{corr}(\ln e_t^A; \ln e_{t+1}^A)$	0.903	0.968	0.955	0.931	0.913	0.943	0.946	0.930	0.927	0.939	0.967	0.962	-
	$\mathrm{corr}(\ln e_t^C; \ln e_{t+1}^C)$	0.969	0.978	0.961	0.992	0.985	0.940	0.950	0.959	0.989	0.974	0.995	0.979	-
	$\mathrm{corr}(\ln e_t^T; \ln e_{t+1}^T)$	0.899	0.983	0.930	0.853	0.900	0.866	0.815	0.908	0.949	0.895	0.868	0.861	-

Table 17: Correlation of relative consumptions with actual real exchange rates, consumption-based representation $\ln e^C$ and the trade-based representation of real exchange rates $\ln e^T$.

		Australia	Canada	Japan	Switzerland	U.K.	Austria	Finland	France	Germany	Italy	Portugal	Spain	U.S.
Australia	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-	-0.262	-0.758	0.587	-0.481	-0.686	-0.687	-0.307	-0.678	-0.708	-0.836	-0.767	-0.225
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	-	1	1	1	1	1	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-	-0.257	-0.779	0.462	-0.463	0.509	-0.577	0.088	0.088	-0.317	0.587	0.122	0.375
Canada	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.262	-	-0.712	0.524	-0.444	-0.564	-0.255	-0.285	-0.652	-0.681	-0.843	-0.701	-0.367
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	-	1	1	1	1	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-0.257	-	-0.632	0.635	-0.407	-0.545	0.103	-0.114	-0.731	-0.606	-0.053	-0.786	-0.645
Japan	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.758	-0.712	-	-0.473	0.245	-0.482	-0.741	-0.572	0.090	-0.272	-0.173	-0.500	-0.651
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	-	1	1	1	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-0.779	-0.632	-	-0.439	0.222	-0.509	-0.774	-0.697	0.202	-0.356	-0.033	-0.041	-0.772
Switzerland	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	0.587	0.524	-0.473	-	0.586	0.071	0.316	0.708	0.567	0.369	-0.589	0.038	0.601
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	-	1	1	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.462	0.635	-0.439	-	0.284	0.888	0.602	0.340	0.879	0.370	0.698	0.367	0.478
U.K.	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.481	-0.444	0.245	0.586	-	0.635	-0.309	0.122	-0.109	0.370	-0.747	0.312	-0.431
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	-	1	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-0.463	-0.407	0.222	0.284	-	0.528	-0.120	-0.348	0.017	0.164	-0.394	0.183	-0.765
Austria	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.686	-0.564	-0.482	0.071	0.635	-	-0.761	-0.413	0.551	0.228	-0.628	-0.322	-0.077
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	-	1	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.509	-0.545	-0.509	0.888	0.528	-	-0.438	-0.095	-0.205	-0.009	0.220	0.371	-0.355
Finland	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.687	-0.255	-0.741	0.316	-0.309	-0.761	-	-0.326	-0.801	-0.499	-0.854	-0.870	-0.295
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	-	1	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-0.577	0.103	-0.774	0.602	-0.120	-0.438	-	-0.293	-0.703	-0.737	-0.738	-0.734	-0.311
France	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.307	-0.285	-0.572	0.708	0.122	-0.413	-0.326	-	-0.445	0.087	-0.881	-0.299	-0.073
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	-	1	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.088	-0.114	-0.697	0.340	-0.348	-0.095	-0.293	-	0.412	0.418	-0.194	-0.364	-0.186
Germany	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.678	-0.652	0.090	0.567	-0.109	0.551	-0.801	-0.445	-	-0.566	-0.614	-0.130	-0.526
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	1	-	1	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.088	-0.731	0.202	0.879	0.017	-0.205	-0.703	0.412	-	-0.420	0.366	-0.382	-0.617
Italy	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.708	-0.681	-0.272	0.369	0.370	0.228	-0.499	0.087	-0.566	-	-0.700	-0.052	-0.555
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	1	1	-	1	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	-0.317	-0.606	-0.356	0.370	0.164	-0.009	-0.737	0.418	-0.420	-	-0.759	0.202	-0.400
Portugal	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.836	-0.843	-0.173	-0.589	-0.747	-0.628	-0.854	-0.881	-0.614	-0.700	-	-0.558	-0.860
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	1	1	1	-	1	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.587	-0.053	-0.033	0.698	-0.394	0.220	-0.738	-0.194	0.366	-0.759	-	0.315	0.314
Spain	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.767	-0.701	-0.500	0.038	0.312	-0.322	-0.870	-0.299	-0.130	-0.052	-0.558	-	-0.595
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	1	1	1	1	-	1
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.122	-0.786	-0.041	0.367	0.183	0.371	-0.734	-0.364	-0.382	0.202	0.315	-	-0.639
U.S.	$\mathrm{corr}(\ln e^A; \ln ci/cj)$	-0.225	-0.367	-0.651	0.601	-0.431	-0.077	-0.295	-0.073	-0.526	-0.555	-0.860	-0.595	-
	$\mathrm{corr}(\ln e^C; \ln ci/cj)$	1	1	1	1	1	1	1	1	1	1	1	1	-
	$\mathrm{corr}(\ln e^T; \ln ci/cj)$	0.375	-0.645	-0.772	0.478	-0.765	-0.355	-0.311	-0.186	-0.617	-0.400	0.314	-0.639	-

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