# The Macroeconomics of Delegated Management

by

**Jean-Pierre Danthine** Université de Lausanne, CEPR and FAME

and

John B. Donaldson Columbia University

First draft: November 2002 This version: November 19, 2004

This paper is dedicated to David Cass, our former teacher, on the occasion of the celebration of his 30 years as professor at the University of Pennsylvania. David Cass did much of the original work on the optimal growth paradigm which has become a workhorse of macroeconomics and which is given a new interpretation in the present article. A preliminary treatment of the question posed in this paper was undertaken in Danthine and Donaldson (2002b). We thank workshop participants at Heidelberg, Lausanne, Mannheim, Rochester, Toulouse, Columbia Business School, UQAM (Montréal), the New York Fed, UCLA, CERGE (Prague), Athens School of Economics, IIES- Stockholm, McGill (Montréal), Tinbergen Institute (Rotterdam), the 2004 CEPR-ESSIM and the 2004 SED meeting in Florence for useful comments. Donaldson's work has benefited from financial support of the Faculty Research Fund, Graduate School of Business, Columbia University. Danthine's research is carried out within the National Center of Competence in Research "Financial Valuation and Risk Management." The National Centers of Competence in Research are managed by the Swiss National Science Foundation on behalf of the Federal authorities.

# Abstract

We are interested in the macroeconomic implications of the separation of ownership and control. A new decentralized interpretation of the stochastic growth model is proposed, one where shareholders hire a self-interested manager who is in charge of the firm's hiring and investment decisions. Delegation is seen to give rise to a generic conflict of interests between shareholders and managers. This conflict fundamentally results from the different income base of the two types of agents, once aggregate market clearing conditions are taken into account. An optimal contract exists resulting in an observational equivalence between the delegated management economy and the standard representative agent business cycle model. The optimal contract has two components: a performance-based component that must be proportional to free-cash-flow and a variable 'salary' component indexed to the aggregate wage bill and to aggregate dividends. The incentive component is akin to a non-tradable equity position in the firm. In our context it is not sufficient to resolve the 'micro' level agency issues raised by delegation. Failure to properly index the 'salary' component of the manager's overall compensation may result in severe distortions in the investment policy of the firm with significant macroeconomic consequences. Specifically, if the 'salary' component is too smooth or too small (as seems inevitable in economies where agency issues are non-negligible), the manager adopts an excessively passive investment policy resulting in a very smooth economy. We take this observation as providing general equilibrium support for the quiet life hypothesis and explore the possibility of remedying the problem by introducing convex performance-based remuneration. When the manager is less risk averse than log, we show that the optimal contract can be closely approximated by a remuneration package composed of a convex performance-based component in conjunction with a smallerthan-optimal constant salary feature.

JEL : E32, E44

Keywords: delegated management, optimal contracts, quiet life hypothesis

### 1. Introduction

Standard dynamic macroeconomics has avoided issues raised by the separation of ownership and control. It implicitly assumes either that there is no such separation or that all problems arising from it are entirely resolved by the complete monitoring of managers' decisions. As a result the crucial intertemporal decisions (and pricing) are all in accord with the intertemporal marginal rate of substitution of the representative shareholder-worker-consumer.

In the present paper we question the abstraction behind the standard framework. In reality, separation of ownership and control is the rule, at least for the all-important publicly traded companies, and it is all too clear that the degree of monitoring exercised by shareholders can be very loose. For the IMRS of the shareholder-worker-consumer to be represented, let alone be predominant in the dayto-day operations of the firm, a contracting framework must be put in place that aligns the interests of managers and shareholders. In this paper we delineate the characteristics of an optimal contract and prove an equivalence theorem under the hypotheses of which the dynamics of the standard macroeconomic model is indeed descriptive of a world where management is delegated to an unmonitored manager.

We also show, however, that the characteristics of the optimal contract severely limit the scope of performance based remuneration and that, as a result, it may not be implementable in situations where the agency problem between firm owners and managers is significant. We go on to show that plausible deviations from the first best contract provide the foundations for the quiet life hypothesis in the sense that they lead to situations where managers' investment policies will be too prudent and fail to exploit all profitable opportunities. Finally we discuss the extent to which

1

convex performance-based contracts can substitute for the optimal contract in situations where the salary component of the manager's remuneration deviates from the optimal prescription.

While macroeconomics has typically ignored corporate governance problems, corporate finance has largely been developed in partial equilibrium. Our inquiry can be viewed as providing a general equilibrium perspective on the corporate governance nexus. Our main result stresses that the partial equilibrium solution to agency conflicts may itself be the source of a new conflict that arises because it implies properties for the manager's stochastic discount factor that are inherently at variance with those of the representative agent's discount factor. Correcting this bias imposes strict requirements on the characteristics of a manager's remuneration package.

In the micro literature, incentive issues can take a variety of forms, e.g., shirking of effort, empire building, and/or the pursuit of private benefits. In this paper we observe that, in a macro general equilibrium context with delegated management, a generic conflict of interests may arise even when shareholders and managers have identical preferences defined over consumption alone. If this conflict cannot be resolved through appropriate contracting, self-interested managers will make intertemporal decisions that will not be those favored by shareholders. Imperfect control thus implies that the dynamics at the heart of the standard business cycle model based on the representative agent IMRS is invalidated<sup>1</sup>.

The neo-classical stochastic growth model was originally conceived as a summary of the problem faced by a benevolent macroeconomic central planner (Cass, 1965). Not until the seminal work of Prescott and Mehra (1980) and Brock (1982) did the model become eligible for use as a vehicle for analyzing data from actual

<sup>&</sup>lt;sup>1</sup> This criticism also applies to less standard representative agent models such as those in the younger New Neo-classical Synthesis tradition.

competitive economies. These authors provided a decentralization scheme; that is, a formulation of the model under which its optimal allocations can be interpreted as the market allocations of a competitive economy in recursive equilibrium.

The models of Prescott and Mehra (1980) and Brock (1982) share a number of essential features: both interpretations postulate infinitely lived consumer-workerinvestors who rent capital and labor to a succession of identical one period firms. It is these consumer-worker-investors who undertake the economy's intertemporal investment decision. Subsequent, more realistic interpretations admit an infinitely lived firm responsible for the investment decision, usually under the added assumption either than the firm issues and maximizes the value of a complete set of state claims, or that it issues and maximizes the value of a single equity share while otherwise being supplied with the representative shareholder's marginal rates of substitution (see Danthine and Donaldson (2002b) for an elaboration). Here, we relax the complete market hypothesis and discuss the extent to which the stochastic growth model can be viewed as describing the time series properties of a decentralized economy in which firms' management is delegated to better-informed managers who cannot be perfectly monitored by firm owners<sup>2</sup>.

An outline of the paper is as follows: Section 2 proposes the framework of our inquiry and discusses a number of modeling options. Section 3 focuses on the sources of conflict between firm owners and the manager and identifies the form of the optimal contract. The optimal contract requires not only endowing the manager with a non-tradable equity share of the firm but also ensuring that the time series properties of the manager's stochastic discount factor, and thus his consumption, are identical to those of the firm owners. This latter condition in turn requires that the manager's

 $<sup>^2</sup>$  Another extension in the same spirit is provided by Shorish and Spear (1996) who propose an agency theoretic extension of the Lucas (1978) asset pricing model.

remuneration includes a time-varying salary-like component whose properties are indexed to the aggregate wage bill. Section 4 generalizes the set-up to a world with multiple firms. The salary component of a manager's remuneration must then include a share in the aggregate economy-wide dividend payment but rather strikingly a manager's equity stake in the firm under management cannot exceed his share of the market portfolio. We argue that this extreme restriction on the performance-based portion of a manager's remuneration is likely to justify deviations from the optimal contract in situations where agency problems are non-negligible. Section 5 looks at the consequences of such plausible deviations from the optimal contract and at the possibility that convex contracts can correct the managerial timidity that appears to result. Section 6 concludes the paper.

### 2. The framework and modeling issues

For ease of exposition we start with the assumption that the entire economy's output is produced by a single perfectly competitive firm, a stand-in for a continuum of identical competitive firms. Section 4 discusses the extension to many firms. There is a continuum of identical agents, a subset of which – of measure  $\mu$  - are selected at the beginning of time to permanently manage the firm. We view these managers as acting collegially and thus refer to them collectively as "the manager".<sup>3</sup> The rest acts as workers and shareholders. The manager is self-interested and he is assumed to make all the relevant decisions in view of maximizing his own intertemporal utility.

The main motive for delegation is, realistically, to relieve shareholders of the day-to-day operation of the firm and the information requirements it entails. This means that shareholders delegate to the manager the hiring and investment decisions

<sup>&</sup>lt;sup>3</sup> Nothing would be lost with the assumption of a single manager (of measure zero) managing the firm and we will adopt it later on, particularly when our goal is to compare the delegated management economy with the standard representative agent business cycle model. Our approach is meant to make clear where the measure zero assumption turns out to matter.

and all that goes with them (human resource management, project evaluation, etc.) but that, as a by-product, they lose the informational base upon which to evaluate and monitor the manager's performance and to write complete contracts with him. Here we portray shareholders as detached firm owners, keeping informed of the main results of the firm's activities but not of the "details" of its operations such as the current level of, and future perspectives on, total factor productivity (which is stochastic), its capital stock level, and the level of the investment expenses decided by the manager. In particular, they lack sufficient information to compute optimal employment and investment levels themselves, and to issue contracts that would deter the manager from deviating from their preferred decisions.<sup>4, 5</sup>

The manager could, in principle, use his informational advantage for several purposes. One particular hypothesis, emphasized in the corporate finance literature, asserts that managers are empire builders (Jensen, 1986) who tend to over-invest and possibly over-hire rather than return cash to shareholders. Philippon (2003) and Dow, Gorton and Krishnamurthy (2003) explore some of the general equilibrium implications of this hypothesis in related contexts. By contrast, we purposefully refrain from postulating "external" conflicts of interests, that is, our managers have standard preferences defined over consumption and their innate risk tolerance is identical to that of shareholders. We rather concentrate on those conflicts that could arise endogenously as a result of the fact that, by the very nature of delegation, the

<sup>&</sup>lt;sup>4</sup> Strictly speaking, in the one-firm economy shareholders know output net of investment (they can infer this quantity from their dividend payment and their wage compensation) but not either quantity separately. In the many-firm economy it may be assumed that they observe aggregate output and invesment, but they have no way to keep track of firm level quantities given that as workers they are affiliated with a single firm. In line with some of the literature (e.g., Morellec, 2004), one could equally assume that all the relevant quantities are in fact known by shareholders, but the relevant actions to monitor, penalize or fire the manager are costly to them.

<sup>&</sup>lt;sup>5</sup> Importantly, managers' superior information makes Arrow-Debreu markets non-viable. We are in a world where assuming the manager maximizes Arrow-Debreu profits is not an option. The same reason justifies that managers are not allowed to trade the equity of their own firm.

manager's <u>marginal</u> risk preferences may differ from those of shareholders or, for that matter, those of the representative agent of the standard stochastic growth paradigm.

Telling a simple and consistent story requires resolving the following two modeling issues. First and least importantly, we assume that managers are <u>not</u> paid an hourly wage and that consequently the labor-leisure trade-off becomes irrelevant for them the day they accept a managerial position<sup>6</sup>.

The second and more difficult problem is the issue of the managers' outside income. Outside income influences the marginal attitude toward risk and is relevant in the contracting problem between shareholders and managers as will be obvious from what follows. Clearly, the spirit of our analysis is one of incomplete risk exchange opportunities between the manager and the shareholders. We naturally assume that the manager cannot trade the equity issued by the firm he manages. This rule is realistic because it protects shareholders from trading with insiders. And it is one where managers cannot use the financial markets to "undo" the characteristics of their incentive remuneration. Restrictions on the ability of managers to take (short) positions in the stock of their own firm or to adjust their long positions at specific times are common.

It is more controversial (although customary in the partial equilibrium contracting literature) to assume that the manager is also prevented from taking a position in the risk free asset. It turns out to be convenient to characterize the optimal contract under this assumption, however. Since the optimal contract attains the firstbest, the no borrowing and lending constraint is not binding and they are no welfare consequences attached to it.

<sup>&</sup>lt;sup>6</sup> This is a minor point in the sense that the optimal contract would lead to a first best labor supply decision on the part of the manager.

Besides choosing their optimal consumption and portfolio investment streams, worker-shareholders are in charge of defining the form of the manager's compensation function, g<sup>m</sup>(.). Managers are offered renewable one-period contracts limiting to the maximum the shareholders' need to collect reliable accounting information on the performance of the firm. In line with much of the contracting literature, we assume that the base contract is made of two parts, a fixed ("salary") component that is potentially time-varying but is not dependent on variables influenced by the manager's decisions, and an incentive component that is a (linear or non-linear) function of some measure of the firm's performance. The latter is clearly affected by the manager's decisions. In general terms,

$$g^{m}(x_{t}) = A_{t} + g(x_{t})$$

where  $A_t$  represents the manager's salary and  $x_t$  denotes an appropriate measure of the firm's performance that is a function of the manager's actions. Given these considerations, the manager's problem can be written:

(1)  

$$V^{m}(k_{0},\lambda_{0}) = \max_{\{n_{t},i_{t}\}} E\left(\sum_{t=0}^{\infty}\beta^{t}u(c_{t}^{m})\right)$$
s.t.  

$$c_{t}^{m} \leq g^{m}(x_{t}) = A_{t} + g(x_{t})$$

$$x_{t} = x(i_{t},n_{t};k_{t},\lambda_{t})$$

$$k_{t+1} = (1-\Omega)k_{t} + i_{t}; k_{0} \text{ given.}$$

$$c_{t}^{m}, i_{t}, n_{t} \geq 0$$

$$(A_{t+1},\lambda_{t+1}) \sim dF(A_{t+1},\lambda_{t+1};A_{t},\lambda_{t}).$$

In problem (1) the manager's (homogeneous) period utility function is denoted u();  $\beta$  is his discount factor and E is the expectations operator (we assume rational expectations). The manager's decision variables are i<sub>t</sub>, the portion of current output invested at date t, and n<sub>t</sub>, the level of employment. The date t state variable vector contains k<sub>t</sub>, the beginning of period t capital stock,  $\lambda_t$ , the current productivity level and  $A_t$ , the current value of the "salary" component of the manager's remuneration;  $(A_t, \lambda_t)$  follows a Markov process whose characteristics are summarized in the transition density function F. The law of motion of capital is standard with  $\Omega$  being the constant depreciation rate of physical capital. For later reference, the dividend paid by the firm,  $d_t$ , takes the form

$$d_t = f(k_t, n_t)\lambda_t - n_t w_t - \mu g^m(x_t) - i_t$$
,

where  $f(.) = f(k_t, n_t)\lambda_t$  is the aggregate production function,  $w_t$ , the market determined wage payment, or free-cash-flow,  $\mu g^m$ , the aggregate contractual payment to the managers.<sup>7</sup> There is no dividend smoothing in our model and the dividend and free cash flow are thus identical; we use the terms interchangeably.

The form of the representative shareholder-worker's problem is standard although we want to be specific as to the content of his information set. We do not assume shareholder-workers are aware of the aggregate state variables  $(k_t, \lambda_t)$  per se. We rather view them as statisticians able to correctly infer the transition probability functions of the variables that they take as market or firm determined:  $w_t$ ,  $q_t$  (the equilibrium share price) and  $d_t$ .<sup>8</sup> The representative shareholder-worker's problem reads:

(2)  

$$V^{s}(z_{0}, d_{0}, q_{0}, w_{0}) = \max_{\{z_{t+1}, n_{t}^{s}\}} E\left(\sum_{t=0}^{\infty} \beta^{t} \left[u(c_{t}^{s}) + H(1 - n_{t}^{s})\right]\right)$$
s.t.  

$$c_{t}^{s} + q_{t} z_{t+1} \le (q_{t} + d_{t}) z_{t} + w_{t} n_{t}^{s}$$

$$(d_{t+1}, q_{t+1}, w_{t+1}) \sim dG(d_{t+1}, q_{t+1}, w_{t+1}; d_{t}, q_{t}, w_{t}),$$

<sup>&</sup>lt;sup>7</sup> Nothing would change materially if we included a fixed amount of managerial input as an additional productive factor with the overall production function being constant returns to scale. This would make comparisons with the standard business cycle model more difficult, however. In the present version of the model, if the manager is not of measure zero, his remuneration decreases the return to stock holding.

<sup>&</sup>lt;sup>8</sup> They can be viewed as the shareholders of a Lucas-tree economy: the firm is a fruit-producing tree. They observe the net output after the labor necessary to shake the trees has been paid and the fruits composted for fertilizing purposes have been set aside.

where  $u(\cdot)$  is the consumer-worker-investor's (homogeneous) period utility of consumption - note that we assume both agent types have the same period preferences over consumption-, H(.) his utility for leisure;  $c_t^s$  his period t consumption,  $n_t^s$  his period t labor supply,  $z_t$  the fraction of the single equity share held by him in period t, and G(.) describes the transition probabilities for the indicated variables. The period utility function is purposefully assumed to be separable in consumption and leisure to permit comparison with a set-up where the relevant intertemporal decision is made by an agent whose utility for leisure is not specified.

## **3.** The optimal contract

Problem (2) has the following recursive representation

$$V^{s}(z_{t}, d_{t}, q_{t}, w_{t}) = \max_{\{z_{t+1}, n_{t}^{s}\}} \{u(z_{t}(q_{t} + d_{t}) + w_{t}n_{t}^{s} - q_{t}z_{t+1}) - H(1 - n_{t}^{s}) + \beta \int V^{s}(z_{t+1}, d_{t+1}, q_{t+1}, w_{t+1}) dG(d_{t+1}, q_{t+1}, w_{t+1}; d_{t}, q_{t}, w_{t}) \}$$

whose solution is characterized by the following relationships:

(3) 
$$u_1(c_t^s)w_t = H_1(l-n_t^s),$$

(4) 
$$u_1(c_t^s)q_t = \beta \int u_1(c_{t+1}^s)[q_{t+1} + d_{t+1}] dG(d_{t+1}, q_{t+1}, w_{t+1}; d_t, q_t, w_t).$$

Note from (3) that worker-shareholders' (static) labor supply decisions are independent of the probability distribution summarizing their information. From (4), the non-explosive equilibrium ex-dividend stock price takes the form:

(5) 
$$q_{t} = E_{t}^{G} \left( \sum_{j=1}^{\infty} \beta^{j} \frac{u_{1}(c_{t+j}^{s})}{u_{1}(c_{t}^{s})} \right) d_{t+j},$$

 $<sup>^{9}</sup>$  It follows from Blackwell's (1965) Theorem and the results in Benveniste and Scheinkman (1979) that a continuous, bounded V<sup>s</sup>() exists and has a unique solution characterized by (3) and (4) provided u() and H() are increasing, continuously differentiable and concave, q() and w() are continuous, and that dG() has the property that it is continuous and whenever h(d,q,w) is continuous,

 $<sup>\</sup>int h(d',q',w')dG(d',q',w';d,q,w)$  is continuous as a function of (d,q,w). The continuity of q() and w() is then confirmed in equilibrium.

where  $E^{G}$  refers to the expectations operator based on the information contained in the probability transition function G. From (4) or (5) it is clear that the pricing kernel relevant for security pricing is the shareholders' IMRS.

Under appropriate conditions, the manager's problem has recursive representation:

(6) 
$$\mathbf{V}^{m}(\mathbf{k}_{t}, \lambda_{t}, \mathbf{A}_{t}) = \max_{\{i_{t}, n_{t}\}} \left\{ u\left(\mathbf{c}_{t}^{m}\right) + \beta^{m} \int \mathbf{V}^{m}(\mathbf{k}_{t+1}, \lambda_{t+1}) dF(\mathbf{A}_{t+1}, \lambda_{t+1}; \mathbf{A}_{t}, \lambda_{t}) \right\}.^{10}$$

The necessary and sufficient first order conditions to problem (6) can be written

(7) 
$$u_1(c_t^m)g_1^m(x_t)\frac{\partial x_t}{\partial n_t} = 0,$$

(8)  
$$u_{1}(c_{t}^{m})g_{1}^{m}(x_{t})\frac{\partial x_{t}}{\partial i_{t}} = \beta\int u_{1}(c_{t+1}^{m})g_{1}^{m}(x_{t+1})\left[f_{1}(k_{t+1},n_{t+1})\lambda_{t+1}+(1-\Omega)\right]dF(A_{t+1},\lambda_{t+1};A_{t},\lambda_{t}),$$

where this latter representation is obtained using a standard application of the envelope theorem.

In equilibrium, at all dates t,

$$(9) \qquad (1-\mu)n_t^s = n_t$$

(10) 
$$z_t = 1$$
, and

(11) 
$$y_t \equiv f(k_t, n_t)\lambda_t = (1-\mu)c_t^s + \mu c_t^m + i_t$$

At this stage, it is useful for comparison purposes to spell out the equations that characterize the equilibrium in the standard stochastic growth model where the central planner solves

<sup>&</sup>lt;sup>10</sup> It again follows from Blackwell's (1965) Theorem and the results in Benveniste and Scheinkman (1979) that a continuous, bounded  $V^m()$  exists that solves (6) provided u() and f() are increasing, continuous and bounded, and that  $g^m()$  is itself continuous and that  $dF(A',\lambda';A,\lambda)$  is continuous with the property that for any continuous  $h(k',A',\lambda')$ ,  $\int h(k',A',\lambda')dF(A',\lambda';A,\lambda)$  is also continuous in k and  $\lambda$ . In order for (7) and (8) to characterize the unique solution, the differentiability of u(),  $g^m()$  and f() is required and  $u(g^m())$  must be concave. The assumptions made in this and the preceding footnote are maintained throughout the paper.

(12)  

$$\begin{aligned}
\max_{\{n_t, i_t\}} E\left(\sum_{t=0}^{\infty} \beta^t [u(c_t) + H(1 - n_t)]\right) \\
s.t. \\
c_t + i_t \le f(k_t, n_t)\lambda_t \\
k_{t+1} = (1 - \Omega)k_t + i_t; k_0 \text{ given.} \\
c_t, i_t, n_t \ge 0 \\
\lambda_t \sim d\hat{F}(\lambda_{t+1}; \lambda_t),
\end{aligned}$$

and  $c_t$ ,  $n_t$ ,  $k_t$ , and  $i_t$  have interpretations entirely consistent with problems (1) and (2); e.g.,  $c_t$  denotes the consumption of the representative agent,  $i_t$  his period t investment, etc, and  $d\hat{F}(.;.)$  describes the transition density for  $\lambda_t$  alone. In this economy,  $n_t$  and  $i_t$ are fully characterized by, respectively,

(13) 
$$u_1(y_t - i_t)f_2(k_t, n_t)\lambda_t = H_1(1 - n_t),$$

(14) 
$$u_1(y_t - i_t) = \beta \int u_1(y_{t+1} - i_{t+1}) \left[ f_1(k_{t+1}, n_{t+1}) \lambda_{t+1} + (1 - \Omega) \right] d\hat{F}(\lambda_{t+1}, \lambda_t), \text{ where}$$

(15) 
$$\mathbf{c}_t + \mathbf{i}_t = \mathbf{f}(\mathbf{k}_t, \mathbf{n}_t) \boldsymbol{\lambda}_t \equiv \mathbf{y}_t.$$

Let us focus on contracts for which  $g^m()$  is linear in  $x_t$ . A comparison of equation (13) with (3) and (7) makes clear that for the standard optimality condition for employment to obtain, the measure of firm performance  $x_t$  must satisfy

$$\frac{\partial \mathbf{x}_{t}}{\partial \mathbf{n}_{t}} = \left[ \mathbf{f}_{2} \left( \mathbf{k}_{t}, \mathbf{n}_{t} \right) \boldsymbol{\lambda}_{t} - \mathbf{w}_{t} \right]$$

Similarly, for equation (14) to obtain from (8) it is necessary and sufficient

that 
$$\frac{\partial \mathbf{x}_t}{\partial \mathbf{i}_t} = -1$$
.

Integrating these two conditions with respect to  $n_t$  and  $i_t$ , respectively, yields (up to a constant term):

$$\mathbf{x}_{t} = \mathbf{f}(\mathbf{k}_{t}, \mathbf{n}_{t})\boldsymbol{\lambda}_{t} - \mathbf{w}_{t}\mathbf{n}_{t} - \mathbf{i}_{t} \equiv \mathbf{d}_{t}$$

In other words, if there is to be no first-order distortion, that would be manifest even in the steady state of this economy, the only appropriate measure of firm performance in our economy is free-cash-flow or dividend.

The intuition for this result is clear. Absent strong extraneous sources of conflicts of interest, it is sensible, in order to align the interests of managers and shareholders, to endow the former with a non-tradable equity position, hence to a claim to a fraction of present and future dividends. For the rest of the paper we adopt this identification which is also consistent with the minimal information requirement we may want to impose on worker- shareholders.

Therefore, (3), (7) and (11) together yield, in equilibrium,

(16) 
$$u_{1}\left(\frac{y_{t}-i_{t}-\mu c_{t}^{m}}{1-\mu}\right) f_{2}(k_{t},n_{t})\lambda_{t} = H_{1}(1-n_{t})$$

With the *form* of the leisure-labor trade-off unaffected by the delegation of management, the labor supply decision will be the same in the delegated management economy as in the standard model *provided that the investment and capital stock levels <u>and</u> the level of consumption of the representative worker-shareholder are all the same. The assumption that the manager is of measure zero is designed to guarantee that the latter condition holds, i.e., \frac{y\_t - \mu c\_t^m - i\_t}{1 - \mu} \approx y\_t - i\_t, \forall t.* 

The same sort of assessment cannot be made for the dynamics of investment. Indeed, equation (14) can be written as

(17) 
$$1 = \beta \int \frac{u_1(y_{t+1} - i_{t+1})}{u_1(y_t - i_t)} \left[ f_1(k_{t+1}, n_{t+1}) \lambda_{t+1} + (1 - \Omega) \right] d\hat{F}(\lambda_{t+1}; \lambda_t)$$

while, together with (11), equation (8) yields

(18) 
$$1 = \beta \int \frac{u_1(c_{t+1}^m)g_1^m(d_{t+1})}{u_1(c_t^m)g_1^m(d_t)} \left[ f_1(k_{t+1}, n_{t+1})\lambda_{t+1} + (1-\Omega) \right] dF(A_{t+1}, \lambda_{t+1}; A_t, \lambda_t).$$

Again equations (17) and (18) have a similar *form* and they yield the same steady state levels of investment and capital stock. In this sense we can assert that with the proposed contract there are no "micro"- incentive issues: the manager perceives the key trade-offs within the firm in the same way as do the firm owners. But, while equations (17) and (18) have a similar form, they effectively differ in that the relevant IMRS need not be the same. In (17) the argument in the utility function of the representative agent is, <u>of necessity</u>, the result of market clearing restrictions and equal to output net of investment, i.e., to aggregate consumption. No such "discipline" is necessarily imposed in the case of the consumption of the manager in a delegated management economy. There is an additional distortion or "correction" to the manager's IMRS in the case of a non-linear incentive contract. All this suggests that it is unlikely, except by design of his contract, that the manager's preferred consumption stream will possess the same time series properties as the representative shareholder's. This is the source of a generic conflict of interests between the agent and the principal.<sup>11</sup>

The preceding discussion has placed us in position to complete the characteristics of an optimal contract. We know that the incentive component of the contract should be based on  $d_t$ , while the 'salary' component should be designed to achieve the equality

$$\frac{u_1(g^m(d_{t+1}))}{u_1(g^m(d_t))}\frac{g_1^m(d_{t+1})}{g_1^m(d_t)} = \frac{u_1(y_{t+1} - i_{t+1})}{u_1(y_t - i_t)}$$

<sup>&</sup>lt;sup>11</sup> At this stage, one might speculate that conditioning the manager's remuneration on the pre-dividend market value of the firm,  $q_t$ , might mitigate the problem at hand, and it does. This is because with his remuneration depending on  $q_t$ , the self-interested manager is lead to use a weighted average of his own and the shareholder-workers' IMRS to guide his investment decisions. But  $q_t$  is not the appropriate measure of performance in our context. Specifically it induces a propensity for overinvestment that is manifest even in the steady state.

It is easy to see that these conditions are satisfied with a linear contract of the form  $g^{m}(d_{t}) = A_{t} + \psi d_{t}$ , where the above condition, together with the homogeneity of the utility function, imposes

$$\mathbf{A}_{t} + \psi \mathbf{d}_{t} = \varphi(\mathbf{y}_{t} - \mathbf{i}_{t}) = \varphi(\mathbf{w}_{t}\mathbf{n}_{t} + \mathbf{d}_{t}), \forall t \text{, for some } 0 < \varphi \ll 1.$$

This is satisfied for

$$\varphi = \psi$$
, and  $A_t = \varphi W_t n_t$ 

In other words, the link between the "fixed" salary component of the manager's contract and the aggregate wage bill must be given by the power of the incentive component, that is, the fraction of free-cash-flow allocated to the manager. We thus obtain the following

<u>Theorem 3.1</u>: A contract  $g^m(d_t) = A_t + \varphi d_t$  with  $A_t = \varphi w_t n_t$  is necessary and sufficient for a Pareto optimal allocation of labor and capital.<sup>12</sup>

Proof: see the Appendix

Theorem 3.1 has the immediate following corollary:

<u>Theorem 3.2 (Equivalence Theorem</u>). Suppose the manager is of measure  $\mu =$ 

0. Then under the linear contract  $g^{m}(d_{t}) = A_{t} + \varphi d_{t}$  with  $A_{t} = \varphi w_{t}n_{t}$  the delegated management economy exhibits the same time series properties as, and is thus observationally equivalent to, the representative agent business cycle model.

This result is important since it extends the realm of application of the standard business cycle model. The measure zero assumption is made for convenience only to facilitate comparison with the standard representative agent model.<sup>13</sup> With a positive measure of managers, it would be necessary to increase the productivity of

 $<sup>^{12}</sup>$  The reader is reminded of the (entirely standard) maintained assumptions of the paper which may be found in footnotes (9) and (10).

<sup>&</sup>lt;sup>13</sup> The measure zero assumption also eliminates any need to consider a possible participation constraint for the manager. There is no problem in making the manager at least as well off being a manager as the typical worker-shareholder.

factors to make up for the consumption of the manager in such a way that the consumption level of shareholder-workers, and consequently their labor supply decision, remain unchanged in equilibrium.

In concluding this section it is worth stressing that the optimal contract must be understood as one where the incentive component depends on firm level performance as measured by free-cash-flow while the 'salary' component depends on the <u>aggregate</u> wage bill. In the next section we formalize this distinction in a more realistic economy with many firms each with a separate manager.

## 4. Many firms

Let us assume the existence of J competitive firms, each of which is managed by a single manager of measure zero. The total measure of managers remains  $\mu$ =0. Each manager is offered a linear contract based on d<sup>j</sup><sub>t</sub> as the measure of firm j's performance.

The representative manager j solves

$$V^{j}(k_{0}^{j},\lambda_{0}^{j},A_{0}^{j};w_{t}) = \max_{\{n_{t}^{j},i_{t}^{j}\}} E\left(\sum_{t=0}^{\infty}\beta^{t}u(c_{t}^{j})\right)$$
  
s.t.  
$$c_{t}^{j} \leq g(d_{t}^{j}) = A_{t}^{j} + \varphi d_{t}^{j}$$
  
$$d_{t}^{j} = f(k_{t}^{j},n_{t}^{j})\lambda_{t}^{j} - n_{t}^{j}w_{t} - i_{t}^{j}$$
  
$$k_{t+1}^{j} = (1-\Omega)k_{t}^{j} + i_{t}^{j}; k_{0}^{j} \text{ given.}$$
  
$$c_{t}^{j},d_{t}^{j},i_{t}^{j},n_{t}^{j} \geq 0$$
  
$$(A_{t+1}^{j},\lambda_{t+1}^{j}) \sim dF^{j}(A_{t+1}^{j},\lambda_{t+1}^{j};A_{t}^{j},\lambda_{t}^{j})$$

Worker-shareholders are perfectly diversified. They collectively hold the market whose total value is measured by  $q_t$  and are thus entitled to the aggregate dividend that we continue to identify as  $d_t$ . In addition, to the extent that individual firms do not go bankrupt (we do not assume limited liability) and that there is a competitive aggregate labor market, their income is not tied to the specific firm for

which they work (alternatively we could make the standard assumption that they share their working time across all firms). Under these assumptions, problem (2) still perfectly represents the problem of the representative worker-shareholder.

We use the same strategy as before to derive the optimal contract. That is, we derive the necessary and sufficient first-order conditions for a solution to the representative agent problem and then ensure that, under the optimal contract, these conditions are reproduced in the context of many firms with delegated management.

The representative agent problem now reads<sup>14</sup>

$$\begin{split} \max_{\{n_{t}^{j}, i_{t}^{j}\}} & E\left(\sum_{t=0}^{\infty} \beta^{t}[u(c_{t}) + H(1 - n_{t})]\right) \\ \text{s.t.} \\ & c_{t} \leq \sum_{j=1}^{J} c_{t}^{j}, \quad n_{t} \leq \sum_{j=1}^{J} n_{t}^{j}, \quad i_{t} \leq \sum_{j=1}^{J} i_{t}^{j}, \text{ and } \forall j: \end{split}$$

$$(20) \quad & c_{t}^{j} + i_{t}^{j} \leq f(k_{t}^{j}, n_{t}^{j})\lambda_{t}^{j} \\ & k_{t+1}^{j} = (1 - \Omega)k_{t}^{j} + i_{t}^{j}; \quad k_{0}^{j} \text{ given.} \\ & c_{t}^{j}, i_{t}^{j}, n_{t}^{j} \geq 0 \\ & (\lambda_{t}^{1}, \lambda_{t}^{2}, ..., \lambda_{t}^{J}) \sim d\tilde{F}(\lambda_{t+1}^{1}, \lambda_{t+1}^{2}, ..., \lambda_{t+1}^{J}; \lambda_{t}^{1}, \lambda_{t}^{2}, ..., \lambda_{t}^{J}). \end{split}$$

Problem (20) yields the following first-order conditions:

(21) 
$$u_1(y_t - i_t)f_2(k_t^j, n_t^j)\lambda_t^j = H_1(1-n_t)$$

(22) 
$$u_1(y_t - i_t) = \beta \int u_1(y_{t+1} - i_{t+1}) \left[ f_1(k_{t+1}^j, n_{t+1}^j) \lambda_{t+1}^j + (1 - \Omega) \right] d\tilde{F}(.;.)$$

By contrast, in recursive form problem (19) can be written as

$$V(k_{t}^{j},\lambda_{t}^{j};A_{t}^{j},w_{t}) = \max_{i_{t+1}^{j},n_{t}^{j}} \left\{ u(A_{t}^{j}+\varphi d_{t}^{j}) + \beta \int V((1-\Omega)k_{t}^{j},\lambda_{t+1}^{j},A_{t+1}^{j}) dF^{j}(.) \right\}, j=1,2,.,J.$$

where we assume that  $dF^{j}(.)$  contains enough information to permit the manager j 's expectation of the future value of  $\lambda^{j}$  to be as precise as the expectation of the

<sup>&</sup>lt;sup>14</sup> This central planning representation implicitly assumes that capital cannot be reallocated across firms once it has been installed. It is also implicit in the manager's problem (19).

representative agent in (20). It includes as well the relevant information on the statistical process for  $A_t^j$ .

The FOC's for problem (19) are as follows:

$$f_2(k_t^J, n_t^J)\lambda_t^J = w_t$$
, which in conjunction with (3) yields (21) (provided the

total measure of managers  $\mu=0$  and thus no consumption is "lost" for the shareholderworker). As to the investment decision, one obtains:

$$u_1(c_t^{j}) = \beta \int u_1(c_{t+1}^{j}) \left[ f_1(k_{t+1}^{j}, n_{t+1}^{j}) \lambda_{t+1}^{j} + (1 - \Omega) \right] dF^{j}(.)$$

For the latter to correspond to (22), one must have

$$\begin{aligned} \mathbf{c}_{t}^{j} &= \boldsymbol{\phi}^{j} \mathbf{c}_{t} = \boldsymbol{\phi}^{j} \left[ \mathbf{w}_{t} \sum_{j=1}^{J} n_{t}^{j} + \sum_{j=1}^{J} d_{t}^{j} \right], \text{ or } \\ \mathbf{A}_{t}^{j} &+ \boldsymbol{\phi}^{j} d_{t}^{j} = \boldsymbol{\phi}^{j} \left[ \mathbf{w}_{t} \sum_{j=1}^{J} n_{t}^{j} + \sum_{j=1}^{J} d_{t}^{j} \right], \text{ which requires } \\ \mathbf{A}_{t}^{j} &= \boldsymbol{\phi}^{j} \left[ \mathbf{w}_{t} n_{t} + \sum_{i \neq j} d_{t}^{i} \right] \end{aligned}$$

That is, the total compensation of manager j effectively has 3 elements:

$$g^{m}(d_{t}) = \varphi^{j} w_{t} n_{t} + \varphi^{j} \sum_{j \neq i} d_{t}^{i} + \varphi^{j} d_{t}^{j}.$$

In a world with a large number of equal-size firms, the first two elements add up approximately to  $\overline{\phi}(y_t - i_t)$ , where  $\overline{\phi}$  stands for the average of the  $\phi^j$ .

Two interesting facts stand out. First, the "salary" component of the manager's remuneration, although time-varying, is indeed independent of variables under his control. Second and most striking, the macro-incentive problem identified in this paper imposes the requirement that the manager's compensation be <u>equally</u> sensitive to the aggregate wage bill <u>and</u> to the aggregate dividend payment made by other firms (or, by approximation, to the economy's total GDP net of aggregate investment) as it is to the measure of performance of the firm he or she manages. In a world with a

large number of firms this implies that the incentive component of managers' must be extremely small: in an economy with 100 firms where managers are given the right to 1% of their firm's dividends, the entire national income would be absorbed by managerial remunerations under the optimal contract! The optimal contract can thus only characterize an economy where incentive problems can be resolved with a very weak link between the manager's income and the firm's performance. Were external incentive problems, not modeled here, be sufficiently severe, forcing shareholders to increase the power of the incentive component of the manager's remuneration, the optimal contract would require them to simultaneously and proportionately increase the size of the "salary" component of his remuneration, a requirement that can easily become an impossibility if the number of firms is large.

Another way to stress the same point is to observe that the optimal contract stipulates that *a manager's (direct or indirect) equity stake in the firm he manages should not exceed his share of the world market portfolio*. Again in a world where the number of firms is large this means that the performance-based portion of a manager's remuneration must be very small. All this is clearly at variance with what is usually meant by incentive contracts. The presence of strong conflicts of interests is thus likely to prevent the emergence of first-best optimal contracts and to result in a conflict between, on the one hand, the objective of making sure the trade-offs determining optimal investment and employment are seen in the same way by managers and their shareholders and, on the other hand, the need to equate the marginal risk tolerance of the two classes of agents. In the next section we look at the implications of sub-optimal contracting. For ease of exposition this is accomplished in the one-firm economy context of Section 3.

#### 5. The macroeconomic consequences of suboptimal contracting

We concluded the previous section with the view that actual contracting may plausibly deviate from the optimal contract of Theorem 3.1 by overemphasizing the performance-based component and correspondingly placing smaller weight on the salary part of a manager's remuneration. In this section we aim to understand what the consequences of such deviations from the optimal contract are likely to be and to propose some possible remedies. The natural representative agent benchmark is the real business cycle model of Hansen (1985) because under the optimal contract our DM economy has the identical equilibrium characterization. Accordingly, we specialize our model as per Hansen (1985):  $f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$ , with  $\alpha = .36$  (note that  $w_t n_t = (1-\alpha)y_t$  under the competitive labor market assumption) and  $\lambda_{t+1} = \rho \lambda_t + \tilde{\epsilon}_{t+1}$ , where  $\tilde{\epsilon}_t \sim N(0, \sigma_t^2), \sigma_t = .00712$ ,  $H(1-n_t) = -Bn_t$  with B = 2.85,  $\Omega = .025$ , and

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$
, with  $\gamma = 1$  corresponding to log  $c_t$ . Subject to the considerations

emphasized at the conclusion of Section 4, it is sufficient to explore contracts of the form

(23) 
$$g^{m}(d_{t}) = \kappa \varphi(1-\alpha) y^{ss} + \varphi d_{t}, \text{ and}$$

(24) 
$$g^{m}(d_{t}) = \kappa \varphi(1-\alpha)y_{t} + \varphi d_{t}$$

for various values of  $\kappa \leq 1$ . Here y<sup>ss</sup> denotes the identical steady state output of the Hansen (1985) and DM economies under the above specifications.<sup>15</sup> It is instructive to start the discussion with the case of no salary component, that is, where the remuneration of the manager is purely performance based.

<sup>&</sup>lt;sup>15</sup> The quantitative computations underlying Figure 2 and Tables 1 and 2 are performed for  $\varphi = .01$  while maintaining the hypothesis that the manager is of measure zero. The characteristics of the economy are absolutely identical when this number is 2% or  $\frac{1}{2}$ % instead of 1%, that is, if the two components of the managers' income are increased or decreased simultaneously.

# 5.1. Dynamics with a pure performance-based contract $(A_t \equiv 0)$

Here we assume the manager's contract has a salary component that is constant at zero,  $A_t = 0$ . In that situation (and more generally whenever the salary component is a constant), the manager's IMRS essentially shares the time series properties of free cash flows or dividends, rather than those of aggregate consumption. This may be expected to have an impact on the investment decision and consequently on the dynamics of the economy for at least two reasons. First, operating leverage, that is, the quantitatively large priority payment to wage earners, makes the residual free cash flow a much more volatile variable than aggregate consumption.<sup>16</sup> This in turn implies that, ceteris paribus, the manager will tend to be excessively prudent in his investment decisions. Second, in the same model the free cash flow is a countercyclical variable. This results almost mechanically from calibrating properly the relative size of investment expenses, of the wage bill, and generating an aggregate investment series that is significantly more variable than output<sup>17</sup>. But this can be expected to have an important impact on investment. Indeed, in the standard RBC model, a positive productivity shock has both a push and a pull effect on investment. On the one hand, shock persistence implies that the return to investment between today and tomorrow is expected to be unusually high. This is the pull effect. On the other hand, the high current productivity implies that output and consumption are relatively high today. The latter signifies that the cost of a marginal consumption sacrifice is small. This is the push effect. While the pull effect is unchanged in the delegated management model with  $A_t = 0$ , the push effect would be absent, or even

 $<sup>^{16}</sup>$  In the standard Hansen (1985) RBC model the non-filtered quarterly standard deviation of the former is about 14% vs. 3.3% for the latter.

<sup>&</sup>lt;sup>17</sup> With  $d_t = y_t - w_t n_t - i_t = \alpha y_t - i_t$  and  $\alpha = .36$ , if investment is about 20% of output on average, an investment series that is twice as volatile as output will make  $d_t$  countercyclical.

negative if the free cash flow variable were to remain countercyclical. This should make for a much weaker reaction of investment to a positive productivity shock.<sup>18</sup>

Another way to express this is to note that, as a rational risk averse individual, the manager wants to increase his consumption upon learning of a positive productivity shock realization since the latter is indicative of an increase in his permanent income. But, for the manager with a fixed salary contract, such a consumption increase necessitates an increase in dividends, which obtains only if the response of investment to the shock is moderate enough.





See note under Figure 2

<sup>&</sup>lt;sup>18</sup> It is not possible to resolve this conflict by conditioning the manager's incentive compensation on the wage bill as excessive hiring would result.





Note: Same functional forms and parameters for both economies: u() = log(); H(1n<sub>t</sub>)= Bn<sub>t</sub>, B = 2.85;  $\alpha$  = .36,  $\Omega$ =.025,  $\lambda_{t+1} = \rho\lambda_t + \tilde{\epsilon}_t$ ;  $\rho$  = .95,  $\tilde{\epsilon}_t \sim N(0; \sigma_{\epsilon}^2)$ ;  $\sigma_{\epsilon} = .00712$ ;  $\phi$  = .01; y<sup>ss</sup> = 1.14.

Figures 1 and 2 display the Impulse Response Functions of the Hansen (1985) indivisible labor model (or, equivalently, of the delegated management economy under the optimal contract) and the delegated management model with  $A_t \equiv 0$ , respectively.<sup>19</sup> In both cases,  $u(c_t) = \log c_t$ . This deviation from the optimal contract is seen to alter profoundly the dynamics of the economy. The starting point is the much more sober reaction of investment to the productivity shock yielding, as expected, a much smoother behavior for the investment series (Table 1 shows that the relative SD(i) is less than one third of its value in the reference Hansen (1985) economy). The natural consequence of this fact is to make aggregate (shareholders') consumption absorb a larger proportion of the shock and be more variable. This in turn means that

<sup>&</sup>lt;sup>19</sup> These are the products of computing the dynamic equilibria of the model with the help of the algorithm provided by Harald Uhlig

<sup>(</sup>http://www.wiwi.hu-berlin.de/wpol/html/toolkit/version4\_1.html).

the marginal utility of consumption is very responsive to the exogenous shock implying that the reaction of labor supply required to maintain the equality in (16) is smaller. That is, the reactivity of employment to the shock is significantly smaller, yielding a weaker propagation mechanism and a smoother output series.<sup>20</sup>

From this experiment one can conclude that the 'macro' conflict of interests, originating solely in the specific income position of the manager, is potentially extremely significant for economic dynamics.<sup>21</sup>

# 5.2. More general implications of sub-optimal contracting $(A_t \neq 0)$

Keeping with the one-firm economy, we now implement contracts (23) and (24) with  $0 < \kappa < 1$ . The intuition developed in the preceding sub-section leads us to anticipate that increasing the size of the fixed component of the manager's remuneration makes the manager effectively less risk averse at the margin, or, in other words, more willing to substitute consumption across time. This makes him more willing to accept a counter-cyclical consumption pattern consistent with the first best investment policy. Similarly, linking appropriately the salary component to the aggregate wage bill improves the compatibility of pro-cyclical consumption with the reality of counter-cyclical free-cask-flows, thus making it easier for the manager to adopt an investment policy that is more responsive to productivity shocks.

<sup>&</sup>lt;sup>20</sup> These results stand in sharp contrast to the implications of models built upon the Jensen (1986) hypothesis that managers will invest all available free cash flow to build empires, a feature that tends to accentuate the volatility of investment, to enhance its procyclicity and to strengthen the propagation mechanism. The Dow et al (2003) model, in particular, replicates quite well a limited set of business cycle stylized facts, and most especially the volatility of investment. It is a model, however, in which the manager does not undertake an actual investment decision except in the most trivial sense. In addition, the shareholder-owners are presumed to retain a detailed knowledge of the firm's production process, a hypothesis we have, realistically we believe, proposed to relax.
<sup>21</sup> Allowing the manager to borrow or lend might mitigate but not resolve the problem raised by

<sup>&</sup>lt;sup>21</sup> Allowing the manager to borrow or lend might mitigate but not resolve the problem raised by suboptimal calibration of the salary component of his remuneration. In particular, if as we argue is likely, the salary component is too small relative to the incentive feature, the managers' wealth position will become progressively negative and large, tending to  $-\infty$ , if he undertakes risk free market trades aimed at generating the first best consumption path. Even if the salary component is, on average, of the correct magnitude, attempting to reach the first best consumption path would lead to the manager's wealth position being large and negative with positive probability over extended periods of time. Under standard hypotheses on the functioning of financial markets, such trades are not be feasible.

Table 1 reports results for a broad set of macroeconomic variables obtained from simulating the dynamic equilibria of the same economy under various hypotheses on the manager's contract. The economy with the optimal contract is used as a benchmark. This purely descriptive approach permits visualizing the massive impact of the contract characteristics along the entire set of macroeconomic dimensions.

Columns 1-3 and 7-9 of Table 1 report the results obtained for a constant salary component starting with the case of  $\kappa = 0$ , i.e., the case discussed in Section 5.1. The parameter  $\kappa$  is then increased to .5 and then to 1. The latter case corresponds to the situation where the manager receives an equal percentage of the economy's free cash flow and of the steady state wage bill. Here the salary component has the right size on average but it is not time varying as in the optimal contract.

Table 1 confirms the role played by the natural counter-cyclicity of dividends. Without fixed remuneration ( $\kappa = 0$  as in Figure 2), the manager decides on investment expenses compatible with *his* consumption being pro-cyclical. This leads to a very smooth behavior of investment. When  $\kappa$  increases from 0 to  $\frac{1}{2}$ , the variability of investment increases by 24%, and dividends move from being positively correlated with output to a correlation with output of -.81. Yet even for  $\kappa = 1$ , SD(y) is still only 1.14 %, remaining far short of the 1.8 % typically reported in business cycle studies of the US economy. The standard deviation of the exogenous shock process would have to be increased by about 62% to restore the aggregate volatility of the economy to its observed level.<sup>22</sup>

<sup>&</sup>lt;sup>22</sup> In order for the economy's dynamics to approximate those of the optimal contract (indivisible labor) model, one needs to increase the relative weight of the fixed component of the manager's remuneration to 8 times the weight of the variable incentive component ( $\kappa = 8$ ). With this sort of contract, the manager is indeed willing to accept a countercyclical consumption path and to adopt an investment policy that is as responsive to productivity shocks as the investment policy of the representative agent

|                           | Standard Deviation in % |                   |                  |        |                 |        | Correlation with output |                   |                  |         |         |         |
|---------------------------|-------------------------|-------------------|------------------|--------|-----------------|--------|-------------------------|-------------------|------------------|---------|---------|---------|
|                           | Col. 1                  | Col. 2            | Col. 3           | Col. 4 | Col. 5          | Col. 6 | Col. 7                  | Col. 8            | Col. 9           | Col. 10 | Col. 11 | Col. 12 |
| κ                         | 0                       | .5y <sup>ss</sup> | 1y <sup>ss</sup> | .5yt   | 1y <sub>t</sub> | OC*    | 0                       | .5y <sup>ss</sup> | 1y <sup>ss</sup> | .5yt    | 1yt     | OC*     |
| у                         | 1.01                    | 1.07              | 1.14             | 1.33   | 1.79            | 1.80   | 1.00                    | 1.00              | 1.00             | 1.00    | 1.00    | 1.00    |
| $\mathbf{c}^{\mathrm{m}}$ | .13                     | .17               | .19              | .29    | .52             | .52    | .26                     | 81                | 87               | .79     | .87     | .87     |
| d                         | .13                     | .69               | 1.38             | 3.23   | 8.05            | 8.08   | .26                     | 81                | 87               | 97      | 97      | 97      |
| c <sup>s</sup>            | .88                     | .85               | .82              | .71    | .52             | .52    | 1.00                    | 1.00              | .99              | .99     | .87     | .87     |
| i                         | 1.41                    | 1.75              | 2.11             | 3.15   | 5.74            | 5.74   | 1.00                    | 1.00              | .99              | 1.00    | .99     | .99     |
| k                         | .13                     | .15               | .18              | .28    | .49             | .49    | .26                     | .32               | .36              | .32     | .35     | .35     |
| n                         | .14                     | .24               | .34              | .63    | 1.37            | 1.37   | .99                     | .97               | .96              | .99     | .98     | .98     |
| W                         | .88                     | .85               | .82              | .71    | .52             | .52    | 1.00                    | 1.00              | .99              | .99     | .87     | .87     |

Table 1: Delegated Management Economy with suboptimal contracts

 $\gamma = 1$ ;  $\varphi = .01$ ;  $y^{ss} = 1.14$  - Other parameter values as in Figure 2.

\* Delegated management economy with optimal contract or, equivalently, the indivisible labor economy with log utility (Hansen, 1985).

Columns 4, 5, 10,11 report the results obtained when the contract takes the form (24), but with  $\kappa < 1$ , that is, with an overemphasis on the performance-based component. On the basis of our intuition thus far, we would expect the movement of the salary component to encourage the manager to adopt both a more pro-cyclical and a more volatile investment policy. This is indeed the case. With the parameter  $\kappa = .5$ , one observes that the results better approximate the OC economy than for the analogous case with constant salary component. Consistent with Theorem 3.2, when  $\kappa = 1$ , the DM economy replicates the OC economy.

The lesson of this and the preceding section is that the most likely deviations from the optimal contract, those that could be viewed as a result of a second-best analysis in a context where agency problems would be more severe than in the present model, are likely to induce the manager to adopt an excessively passive investment policy. This can be viewed as an alternative explanation for, and a confirmation in a general equilibrium context of, the "quiet life" hypothesis. According to this view, first expounded by Smith and Stulz (1985), risk averse managers forgo positive net

in the indivisible labor model. This follows from the fact that such an enormous salary component places the manager on a portion of his utility surface where he is nearly risk neutral.

present value projects because they are unable to diversify risk specific to their claims on the corporation.<sup>23</sup> Our analysis shows that the requirements of the optimal contract leading managers to possess the same marginal risk tolerance as shareholders and thus adopt the same investment policy are indeed quite severe. Deviations from the optimal contract, most plausibly inducing an excessively timid investment policy, are thus likely. Our analysis also shows that the problem is as much related to the business cycle properties of free-cash-flows and the intertemporal elasticity of substitution of managers than it is to their risk position per se.

It may be argued, however, that in situations where the salary component of the manager's remuneration possesses sub-optimal characteristics, it would be second best also to deviate from the <u>linear</u> properties of the optimal performance-based component. In particular, part of the literature on executive compensation has attempted to justify convex performance-based remunerations as a way to provide incentives for managers to increase their risk taking. This hypothesis is reviewed in the next section.

### **5.3 Convex performance-based contracts**

Are convex performance-based contracts the solution to excessive managerial timidity whenever the salary component of his remuneration does not have the appropriate size or cyclical properties? To answer this question, we relax the assumption of a linear g<sup>m</sup>. It turns out the intuition can be made sharper when the manager's contract has a fixed salary component that is set to zero. We thus start with an analysis of contracts of the form

(25) 
$$g^{m}(d_{t}) = \varphi(\overline{d})^{1-\theta}(d_{t})^{\theta},$$

<sup>&</sup>lt;sup>23</sup> Bertrand and Mullainathan (2003) provides recent support for an enlarged definition of the quiet life hypothesis, one that includes a desire for peaceful labor relations in addition to a calmer-than-optimal investment policy.

where  $\overline{d}$  is the average free-cash-flow level when  $\theta = 1$ . The constant term is designed to insure that the average manager's remuneration is little affected by changes in  $\theta$ , the curvature of the function. With a contract specified as per (25) and a

CES utility function for the manager,  $u(c_t^m) = \frac{(c_t^m)^{1-\gamma}}{1-\gamma}$ ,  $\gamma > 0$ , the marginal utility term

in the RHS of (8) takes the form:

$$u_1(g^m(d_t))g_1^m(d_t) = \theta[M(\overline{d})^{1-\theta}]^{1-\gamma}(d_t)^{\theta(1-\gamma)-1}$$

and the effective IMRS of the manager becomes:

(26) 
$$\beta^{m} \frac{u_{1}(g^{m}(d_{t+1}))g_{1}^{m}(d_{t+1})}{u_{1}(g^{m}(d_{t}))g_{1}^{m}(d_{t})} = \beta^{m} \left(\frac{d_{t+1}}{d_{t}}\right)^{\theta(1-\gamma)-1}$$

Expression (26) provides the basis for the following observations:

<u>Theorem 5.1</u>. Under contract (25), the manager's effective risk aversion results from a combination of his subjective coefficient of risk aversion and the curvature of the contract. It is given by the expression:  $1 - \theta(1 - \gamma)$ .

In practice this result implies that an economy with  $\gamma = 3$  and a linear contract  $(1-\theta(1-\gamma)=3)$  is observationally equivalent (except for the volatility of the manager's consumption and its correlation with output) to one where  $\gamma = 2$  and  $\theta = 2$  or  $\gamma = 4$  and  $\theta = 2/3$ , etc.

Theorem 5.1 has the following corollary implications:

<u>Corollary 5.1</u>. If the manager has logarithmic utility ( $\gamma = 1$ ), then his investment decision cannot be influenced by the curvature of the remuneration contract.

<u>Corollary 5.2</u>. If the manager is less risk averse than the log  $((1-\gamma) > 0)$ , then a convex contract  $\theta > 1$  makes the manager's effective rate of risk aversion smaller than his subjective rate of risk aversion, thus leading to a more aggressive investment policy. For the FOC on investment to be necessary and sufficient, the effective

measure of risk aversion must be larger than unity, however, requiring that  $\theta$  be

strictly smaller than  $\frac{1}{1-\gamma}$ .

<u>Corollary 5.3</u>. If the manager is more risk averse than the log,  $(1-\gamma) < 0$ , then the larger  $\theta$ , the *more effectively risk averse* the manager becomes.

In the context of corollary 5.3, if one wants the manager to behave more aggressively, that is, for his effective measure of risk aversion to be larger than his subjective rate of risk aversion, one would rather propose a concave contract ( $\theta < 1$ )! Note that if the manager's  $\gamma$  is larger than 1, there is no way to make him effectively less risk averse than the log short of proposing a contract with  $\theta < 0$ ! For the

exponent of the effective IMRS to be negative, one needs  $\theta > \frac{1}{1-\gamma}$ .<sup>24</sup>

|                | Standa | rd Devia | ations in | %     | Correlation with output |      |      |      |       |      |
|----------------|--------|----------|-----------|-------|-------------------------|------|------|------|-------|------|
| θ              | 1.5    | 1.95     | 1.96      | 1.054 | OC*                     | 1.5  | 1.95 | 1.96 | 1.055 | OC*  |
|                | к=0    | к=0      | к=0       | κ=1   |                         | к=0  | к=0  | к=0  | κ=1   |      |
| у              | 1.07   | 1.65     | 1.77      | 1.79  | 1.80                    | 1.00 | 1.00 | 1.00 | 1.00  | 1.00 |
| $c^m$          | 1.01   | 14.02    | 16.78     | 1.29  | .52                     | 81   | 89   | 89   | 89    | .87  |
| d              | .67    | 7.19     | 8.56      | 8.79  | 8.08                    | 81   | 89   | 89   | 89    | 97   |
| c <sup>s</sup> | .85    | .69      | .70       | .70   | .52                     | 1.00 | .76  | .65  | .63   | .87  |
| i              | 1.73   | 5.09     | 5.79      | 5.91  | 5.74                    | 1.00 | .97  | .96  | .96   | .99  |
| k              | .15    | .38      | .42       | .42   | .49                     | .32  | .48  | .50  | .50   | .35  |
| n              | .23    | 1.21     | 1.42      | 1.45  | 1.37                    | .97  | .93  | .93  | .93   | .98  |
| W              | .85    | .69      | .70       | .70   | .52                     | 1.00 | .76  | .65  | .63   | .87  |

Table 2: Delegated Management Economy: convex contracts (various  $\theta$ )

 $\gamma = \frac{1}{2}$ ; other parameter values as in Table 1

\* Delegated management with optimal contract and log utility;

The upshot of these results is that the only plausible case where a short run non-

linear contract is likely to have the desired effect is the case where the manager is less

<sup>&</sup>lt;sup>24</sup> This discussion suggests that, if an additional source of conflict between the manager and the shareholder is heterogeneity in their attitude toward risk, then that specific source of conflict can be resolved by appropriately (that is, with the right curvature  $\theta$ ) designing a short term contract of the form (25). This is true, however, only if the manager's utility function is not logarithmic. This statement also ignores the effects of a non-zero fixed salary feature.

risk averse than log and he is offered a convex contract. Table 2 displays the results obtained for several convex contracts when the manager's rate of risk aversion is  $\frac{1}{2}$ .

Table 2 shows that it is possible to get very close to the time series properties of the indivisible labor economy, but to obtain that result when  $\kappa = 0$ , we have to make the manager effectively nearly risk neutral.<sup>25</sup> With  $\theta = 1.96$  and  $\gamma = \frac{1}{2}$ , the exponent of dividend growth in the IMRS is  $\theta(1-\gamma)-1=-.04$ . Note that with these parameter values, the variability of manager's consumption becomes quite extreme<sup>26</sup>. Moreover the manager's consumption is then highly countercyclical.

Essentially what these results stress once again is the importance of operating leverage naturally translating into countercyclical free-cash-flows. The incentive dimension of the manager's contract then has the natural property of inducing a countercyclical consumption path. To avoid this undesirable characteristic, a risk averse manager is led to moderate the response of investment to a favorable productivity shock. The more risk averse, that is the lower the elasticity of intertemporal substitution, the more pronounced is this effect. On the contrary, if the manager is almost risk neutral or if his contract makes him effectively close to risk neutral relative to changes in dividends, then he becomes again freer to react to the pull effect on investment of a positive productivity shock.

<sup>&</sup>lt;sup>25</sup> Here, as in Table 1, we rely on the intuition that similar time series are the outcome of equally similar investment policies. Under footnotes (9) and (10) and provided  $u(g^{m}())$  is concave, the policy functions of the manager's problem are unique. Confronted by the same shocks, the policies will be the same, hence the resulting statistics. Note that the capital stock process in the case of  $\theta = 1.96$  is

 $k_t = .8023k_{t-1} + .1674\lambda_t$ , while it is  $k_t = .7986k_{t-1} + .1706\lambda_t$  when  $\kappa = 1$ ;  $\theta = 1.054$ .

<sup>&</sup>lt;sup>26</sup> As an application of Theorem 5.1, let us observe that the same macroeconomic dynamics would be obtained in an economy where the manager's risk aversion is  $\gamma = 2$  and the contract curvature is  $\theta = -$ .98. The only (important) difference is that with such a contract the manager's consumption would turn pro-cyclical:  $\rho(y,c^m)=+.89$  instead of -.89.

Finally we check the possibility of combining the two dimensions discussed so far, a positive fixed 'salary' component and a convex incentive component.<sup>27</sup> The last set of results in Table 2 show that, with a rate of risk aversion of  $\gamma = \frac{1}{2}$ , the impact of a high fixed component in the manager's remuneration reinforces the effects of a convex contract. With a constant salary component of the right size, i.e.,  $\kappa = 1$ , a very close match with the time series of the indivisible labor model is obtained with a contract curvature of only  $\theta = 1.054$ . In the case of a less-risk-averse-than-log manager, a remuneration combining appropriately a fixed salary component with an incentive element that is a convex function of free cash flow thus appears as a possible alternative to the optimal contract.

## 6. Conclusions

In this paper we have shown that in the general equilibrium of an economy where shareholders delegate the management of the firm, the key decision maker, the manager, inherits an income position that naturally leads him to make very different investment decisions than firm owners, or the representative agent of the standard business cycle model, would make. The conflict of interests is endogenous, that is, it does not result from postulated behavioral properties of the manager; it is generic, that is, it characterizes the situation of the "average" manager as a necessary implication of market clearing conditions; and, it is severe in the sense that, if it is unmitigated by appropriate contracting or monitoring, it results in macro dynamics very distant from the Pareto Optimum.

<sup>&</sup>lt;sup>27</sup> The contract then takes the form :  $g^{m}(d_{t}) = \kappa \varphi(1-\alpha)y^{ss} + \varphi(\overline{d})^{1-\theta}(d_{t})^{\theta}$ . We can observe again that if the manager is less risk averse than log ( $\gamma = \frac{1}{2}$ ), it is easier to have him adopt a pro-cyclical investment policy. This translates into the fact that a linear contract with  $\kappa = 4$  now assures an almost

We derive the properties of an optimal contract. This contract attains the first best and it results in an observational equivalence between the delegated management economy and the standard representative agent business cycle model. The optimal contract has two main components: an incentive component that must be proportional to free-cash-flow and a variable 'salary' component indexed to the aggregate wage bill and to aggregate dividends. The incentive component is akin to a non-tradable equity position in the firm. In our general equilibrium context it is thus not sufficient to resolve the 'micro' level agency issues raised by delegation. We show that a failure to properly index the 'salary' component of the manager's may result in severe distortions in the investment policy of the firm with significant macroeconomic consequences. We also show intuitively that, in circumstances where agency problems are more severe than those considered here, the optimal contract will not be attainable. We argue that the most plausible deviations from the optimal contract may help rationalize observations made on managers' behavior under the quiet life hypothesis.

Specifically if the 'salary' component is too small or too smooth, the manager adopts an excessively passive investment policy resulting in a very smooth economy. This is because, in these circumstances, the consumption path of the manager under the optimal investment policy is too variable and too counter-cyclical. In order to align the interests of a manager, so remunerated, with those of firm owners, one must make him highly willing to substitute consumption across time. If this is the case, he will be prepared to sacrifice his consumption in good times (choosing to delay dividend payments in order to finance large investment expenses) and he will respond sufficiently vigorously to favorable investment opportunities.

We explore the potential of convex contracts to resolve the incentive problem of an excessively timid manager and conclude that they are no panacea. This is true

31

first because a logarithmic manager is insensitive to the curvature of the contract. Second, if the manager is more risk averse than log, there is no solution but to propose an unconventional remuneration that is inversely related to the firm's results, paying high compensation when free cash flows are low and conversely. In the case of a lessrisk-averse-than-log manager, however, it is possible to closely approximate the optimal contract by a remuneration package composed of a convex performancebased component in conjunction with a smaller-than-optimal constant salary feature.

#### References

- Benveniste, L. and J. Scheinkman, "On the Differentiability of the Value Function in Dynamic Models of Economics," <u>Econometrica</u>, 47, 727-732, 1979
- Bertrand, M. and Mullainathan, S., "Enjoying the Quiet Life? Corporate Governance and Managerial Preferences", Journal of Political Economy, 111, 1043-1075, 2003
- Blackwell, D., "Discounted Dynamic Programming," <u>Annals of Mathematical</u> <u>Statistics</u>, 36, 226-235, 1965
- Blanchard, O., "Debt, Deficits, and Finite Horizons", <u>The Journal of Political</u> <u>Economy</u>, 93, 223-247, 1985
- Brock, W.A., "Asset Prices in a Production Economy," in J.J. McCall, ed., <u>The</u> <u>Economics of Information and Uncertainty</u> (University of Chicago Press, Chicago, 1982
- Cass, D., "Optimum Growth in an Aggregative Model of Capital Accumulation," <u>Review of Economic Studies</u>, 32, 233-240, 1965
- Danthine J. P., and J. B. Donaldson, "Labor Relations and Asset Returns," <u>Review of</u> <u>Economic Studies</u>, 69, 41-64, 2002a
- Danthine J. P., and J. B. Donaldson, "Decentralizing the Stochastic Growth Model", Cahier de recherche du DEEP 02.05, University of Lausanne, 2002b
- Dow, J., Gorton G., and A. Krishnamurthy, "Equilibrium Asset Prices under Imperfect Corporate Control," NBER Working Paper 9758, June 2003
- Guvenen, F., "A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross- Sectional Heterogeneity?", mimeo, U. of Rochester, 2003

- Hansen, G., "Indivisible Labor and the Business Cycle", Journal of Monetary Economics, 16, 1985, 309-327
- Jensen, M., "The Agency Costs of Free Cash Flow: Corporate Finance and Takeovers," <u>American Economic Review</u>, 76, 2, 323-330, 1986
- Lucas, R.E.Jr., "Asset Pricing in an Exchange Economy", <u>Econometrica</u>, 46, 1429-1444, 1978
- Morellec, E., "Can Managerial Discretion Explain Observed Leverage Ratios", <u>Review of Financial Studies</u>, 17, 257-294, 2004
- Philippon, Th., "Corporate Governance and Aggregate Volatility", mimeo MIT, 2003
- Prescott, E.C. and R. Mehra, "Recursive Competitive Equilibrium: The Case of Homogeneous Households," <u>Econometrica</u>, 48, 1365-1379, 1980
- Shorish, S., and S. Spear, "Shaking the Tree: An Agency-Theoretic Model of Asset Prices", working paper, Carnegie-Mellon University, 1996
- Smith, C.W., and R.M. Stulz, "The determinants of Firms' Hedging Policies," Journal of Financial and Quantitative Analysis, 20, 391-405, 1985

### Appendix: Proof of Theorems 3.1 and 3.2

$$\Rightarrow \text{ Suppose the contract is of the form } g^{m}(d_{t}) = A_{t} + \varphi d_{t}$$
  
Since  $d_{t} = f(k_{t}, n_{t}^{f})\lambda_{t} - w_{t}n_{t}^{f} - i_{t} - \mu(A_{t} + \varphi d_{t})$   
 $(1 + \mu\varphi)d_{t} = f(k_{t}, n_{t}^{f})\lambda_{t} - w_{t}n_{t}^{f} - i_{t} - \mu A_{t}$   
(27)  $d_{t} = \frac{1}{1 + \mu\varphi} [f(k_{t}, n_{t}^{f})\lambda_{t} - w_{t}n_{t}^{f} - i_{t} - \mu A_{t}].$   
Given this definition, the N/S first order conditions for problem (1)-(6) solve

$$W^{m}(k_{t},\lambda_{t},A_{t}) = \max_{i_{t},n_{t}^{f}} \left\{ \begin{aligned} & u\{A_{t} + \frac{\phi}{1+\mu\phi}[f(k_{t},n_{t}^{f})\lambda_{t} - w_{t}n_{t}^{f} - i_{t} - \mu A_{t}]\} \\ & +\beta\int W^{m}[(1-\Omega)k_{t} + i_{t},\lambda_{t+1},A_{t+1}]dF(A_{t+1},\lambda_{t+1};A_{t},\lambda_{t}) \end{aligned} \right\}.$$

They are

(28) 
$$\left(\frac{\phi}{1+\mu\phi}\right) u_1 \{A_t + \frac{\phi}{1+\mu\phi} [f(k_t, n_t^f)\lambda_t - w_t n_t^f - i_t - \mu A_t]\}$$
  
=  $\beta \int W_1^m [(1-\Omega)k_t + i_t, \lambda_{t+1}, A_{t+1}] dF(A_{t+1}, \lambda_{t+1}; A_t, \lambda_t)$   
(29)  $u_1 \{\} \frac{\phi}{t} [f_2(k_t, n_t^f)\lambda_t - w_t] = 0$ 

where 
$$W_1^m(k_t, \lambda_t, A_t) =$$
  
 $u_1\{A_t + \frac{\phi}{1+\mu\phi}[f(k_t, n_t^f)\lambda_t - w_t n_t^f - i_t - \mu A_t]\} \frac{\phi}{1+\mu\phi}[f_1(k_t, n_t^f)\lambda_t + (1-\mu)A_t]\}$ 

 $-\Omega$ ].

Thus (28) becomes

$$u_1\{A_t + \frac{\phi}{1+\mu\phi}[f(k_t, n_t^f)\lambda_t - w_t n_t^f - i_t - \mu A_t]\} =$$

(30) 
$$\beta \int u_1 \{ A_{t+1} + \frac{\phi}{1+\mu\phi} [f(k_{t+1}, n_{t+1}^f)\lambda_{t+1} - w_{t+1}n_{t+1}^f - i_{t+1} - \mu A_{t+1}] \}$$

$$[f_{1}(k_{t+1}, n_{t+1}^{f})\lambda_{t+1} + (1 - \Omega)])dF(A_{t+1}, \lambda_{t+1}; A_{t}, \lambda_{t})$$

The corresponding first order condition for labor supply by the shareholder worker is (31)  $u_1(w_t n_t^s + d_t)w_t = H_1(1-n_t^s).$ 

In equilibrium  $z_t = 1$  and  $n_t^f = n_t^s$ ; thus (29) and (31) yield

(32) 
$$u_1(w_t n_t + d_t) f_2(k_t, n_t) \lambda_t = H_1(1 - n_t)$$

Substituting for the equilibrium value of  $A_t = \varphi w_t n_t$ , equations (30) and (32) become

$$(33) \quad u_{1}\{\frac{\phi}{1+\mu\phi}w_{t}n_{t} + \frac{\phi}{1+\mu\phi}[f(k_{t},n_{t}^{f})\lambda_{t} - w_{t}n_{t} - i_{t}]\} = u_{1}\{\frac{\phi}{1+\mu\phi}(y_{t} - i_{t})\} = \beta\int u_{1}\{\frac{\phi}{1+\mu\phi}(y_{t+1} - i_{t+1})\}[f_{1}(k_{t+1},n_{t+1}^{f})\lambda_{t+1} + (1-\Omega)]dF(A_{t+1},\lambda_{t+1};A_{t},\lambda_{t}), \text{ and} \\ (34) \quad u_{1}[\frac{1}{1+\mu\phi}(y_{t} - i_{t})]f_{2}(k_{t},n_{t})\lambda_{t} = H_{1}(1-n_{t}).$$

The homogeneity of u() implies that (33) can equivalently be written

(35) 
$$u_1(y_t - i_t) = \beta \int u_1(y_{t+1} - i_{t+1}) [f_1(k_{t+1}, n_{t+1})\lambda_{t+1} + (1 - \Omega)] dF(.;.), \text{ or}$$
  
(36)  $u_1[\frac{1}{1 + \mu\phi}(y_t - i_t)] = \beta \int u_1[\frac{1}{1 + \mu\phi}(y_{t+1} - i_{t+1})] [f_1(k_{t+1}, n_{t+1})\lambda_{t+1} + (1 - \Omega)] dF(.;.).$ 

It is well known that (34) and (35) are the necessary and sufficient conditions for the unique solution to problem (12) if  $\mu = 0$ . Theorem 3.2 follows. If  $\mu \neq 0$ , then the labor supply decision of the shareholder-worker resulting from condition (34) will not be identical to the one obtained in the Hansen (1985) RBC model; however, equations (34) and (36) together imply that the intertemporal marginal rates of substitution of the two agents are identical as required for Pareto optimality. The sufficiency part of Theorem 3.1 follows.

<= We start by demonstrating the following

**Lemma**: Suppose the investment and consumption allocations in the Delegated Management economy define a Pareto optimum. Then the manager's compensation function must be of the form  $g^m(.) = \varphi(y_t - i_t)$ , for some  $0 < \varphi < 1$ .

**Proof**: Since the consumption allocations define a Pareto optimum,  $u_{i}(e^{m}) = uu_{i}(e^{s})$  for some constant w > 0 By the homogeneity property of u(.), this

$$u_1(c_t^{-1}) = \psi u_1(c_t^{-1})$$
 for some constant  $\psi > 0$ . By the homogeneity property of u(.), this

implies 
$$u_1(c_t^m) = u_1(\psi^{\overline{\Delta}} c_t^s)$$
, for some  $\Delta$ .

Since  $u_1()$  is continuous and monotone decreasing, it has an inverse. We may then write

$$u_1^{-1}(u_1(c_t^m)) = u_1^{-1}(u_1(\psi^{\frac{1}{\Delta}}c_t^s)).$$

Therefore,

$$\mathbf{c}_t^m = \boldsymbol{\psi}^{\underline{1}} \mathbf{c}_t^s.$$

Since

$$\mu c_{t}^{m} + (1-\mu)c_{t}^{s} = y_{t} - i_{t},$$

$$\mu \psi^{\frac{1}{\Delta}} c_{t}^{s} + (1-\mu)c_{t}^{s} = y_{t} - i_{t}$$

$$c_{t}^{s} (\mu \psi^{\frac{1}{\Delta}} + (1-\mu)) = y_{t} - i_{t}$$

$$c_{t}^{s} = \frac{1}{(\mu \psi^{\frac{1}{\Delta}} + (1-\mu))} (y_{t} - i_{t})$$

Thus we identify

$$1 - \varphi = \frac{1}{(\mu \psi^{\frac{1}{\Delta}} + (1 - \mu))}, \text{ and } \varphi = 1 - \frac{1}{(\mu \psi^{\frac{1}{\Delta}} + (1 - \mu))}.$$

Suppose now that the joint DM-SWI equilibrium investment and labor service functions are P.O. choices from the perspective of problem (12), and assume  $g_1^m(x_t) > 0$ , for some variable  $x_t$  on which the manager's compensation is based. We need to identify  $x_t$ . From the manager's problem (6), the necessary and sufficient first order condition with respect to labor is:

$$u_1(c_t^m)g_1^m(x_t)\frac{\partial x_t}{\partial n_t^f}=0.$$

In order for labor to be allocated optimally is must be that

(37) 
$$\frac{\partial \mathbf{x}_{t}}{\partial \mathbf{n}_{t}^{f}} = \mathbf{f}_{2}(\mathbf{k}_{t}, \mathbf{n}_{t}^{f})\boldsymbol{\lambda}_{t} - \mathbf{w}_{t}$$

Furthermore, since the manager chooses the optimal investment function from the perspective of problem (12), it must be that

(38) 
$$u_1(y_t - i_t) = Lu_1(c_t^m)g_1^m(x_t)\frac{\partial x_t}{\partial i_t}$$

for any L>0. Finally, in order for the optimal allocation of consumption to be P.O., we know from the Lemma below that

(39) 
$$g^{m}(x_{t}) = \varphi(y_{t} - i_{t})$$

for some  $\phi > 0$ . Since u() is homogeneous, (38) and (39) imply

$$u_{1}(y_{t} - \dot{i}_{t}) = Lu_{1}[\phi(y_{t} - \dot{i}_{t})]\phi \frac{\partial x_{t}}{\partial \dot{i}_{t}}$$
$$= Lu_{1}(y_{t} - \dot{i}_{t})\phi^{1+\Delta} \frac{\partial x_{t}}{\partial \dot{i}_{t}},$$

where  $\Delta$  is the degree of homogeneity of  $u_1()$ . Without loss of generality, choose  $L = \frac{1}{\varphi^{1+\Delta}}$ , and thus (40)  $\frac{\partial x_t}{\partial i_t} = -1$ . Integrating (37) and (40),  $x_t = x(i_t, n_t; k_t, \lambda_t) = f(k_t, n_t)\lambda_t - w_t n_t - i_t + B_t$ , where  $B_t$  is unrelated to  $n_t$  and  $i_t$ . Yet from (39),  $x_t = y_t - i_t$ . Thus  $y_t - i_t = y_t - w_t n_t - i_t + B_t$ , and  $B_t = w_t n_t$ . Thus the contract is of the form  $g^m(x_t) = \phi(y_t - i_t) = \phi d_t + A_t$ , where  $A_t = \phi w_t n_t$ .