Can Real Options Explain Financing Behavior?*

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Abstract

Dynamic structural trade-off models commonly invoke financial transactions costs in order to explain observed leverage fluctuations. This paper offers an alternative explanation for this pattern: real options. In the model, the only financial friction is a tax advantage to debt. However, the model incorporates two investment frictions: irreversibility and fixed costs of investment. Despite its parsimony, the model is broadly consistent with observed financing patterns. First, market leverage ratios are negatively related to profitability in the cross section. Second, leverage ratios in the simulated firms are mean-reverting and depend on past stock returns. Third, gradual and lumpy leverage adjustments occur in the absence of financial transactions costs. Fourth, debt tends to be the primary source of external financing for new investment. The predictive power of the model highlights the necessity of incorporating real frictions into structural models of corporate financing decisions.

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Introduction

The techniques of real options have fundamentally altered the way economists think about firms' real investment decisions. Leaving theoretical elegance aside, the appeal of real options techniques is found in its ability to explain empirical investment regularities. It has long been noted that theories of investment in which the buy and sell price of capital are equalized, e.g., Tobin's (1969) Q-theory, cannot explain investment behavior. Early attempts to remedy this weakness focused on the convex costs of adjusting the capital stock, e.g., Hayashi (1982). However, even this framework cannot replicate observed investment patterns. A recent paper by Cooper and Haltiwanger (2005) shows that in order to replicate observed investment patterns, it is necessary to introduce real frictions, such as wedges between the buy and sell prices of capital and/or fixed costs of adjustment, as in the model of Abel and Eberly (1994).

Since Modigliani and Miller (MM hereafter) (1958) published their financial irrelevance result, theorists have focused their attention on violations of the MM assumptions in order to understand corporate financing choices. Motivated by this literature, empiricists have looked to taxes and financial frictions for an explanation of observed financial behavior. For example, it is commonly argued that transactions costs are responsible for wide fluctuations in leverage ratios over time. This is because firms will adjust financing variables infrequently if doing so forces them to incur costs, e.g., underwriting fees on new debt flotations.

In this paper, I show that the same technological factors (real frictions) that explain the investment decisions of firms can also explain their financial behavior. In particular, there is no need to appeal to large financial frictions in order to explain observed financing patterns.¹ Rather, a much simpler explanation is available: real frictions. The attractive feature of my model is that it can successfully explain *both* financing behavior and real investment decisions. In contrast, theories relying on transactions costs cannot explain investment behavior. In particular, theories relying exclusively upon transactions costs cannot explain why surges in financing activity tend to be correlated with surges in real investment. Empirically, firms tend to issue debt as they invest. For example, Shyam-Sunder and Myers (1999) show that a financing gap is matched almost dollar-for-dollar by new debt in their sample.

To illustrate the main argument, I develop a dynamic model of optimal fi-

 $^{^{1}}$ Transaction costs in most structural models are high. This is because the cost is levied on the full value of new debt rather than on the change in the value of debt.

nancial structure in the presence of real frictions. The model is parsimonious on the financial side, featuring: zero transactions costs, convex corporate income taxes, and a linear tax on interest income. As such, the model features a *single* violation of the MM assumptions: there is a tax advantage to debt. Setting transactions costs to zero well illustrates the main argument in this paper. Anticipating, I will show that real frictions are sufficient to replicate observed financing patterns. To illustrate the implications for statistical inference regarding financial frictions, note that lumpy adjustments of leverage ratios are commonly attributed to the high fixed costs of debt flotations. However, I show that lumpy financial behavior can also be explained by the fixed costs of real investment.

The model can explain what Myers termed "long drifts away from target" absent any transactions costs. The argument relies on two real-world considerations in addition to the existence of real options. First, Graham (2000) shows that loss limitations in the tax code cause the expected marginal corporate tax rate to increase with taxable income. For a healthy company, there is a tax advantage to debt. However, when taxable income is sufficiently low, there is a tax disadvantage to debt, as the tax rate on interest income at the individual level exceeds the expected marginal corporate rate. The second important consideration is that only installed capital (assets in place) generates taxable revenue for the firm. Future growth options increase the value of equity but do not yet generate taxable revenues.

With these facts in mind, consider a firm that experiences a positive demand shock. Real frictions, such as a wedge between the buy-sell price of capital, cause the firm to delay installing additional capital. Consequently, the value of growth options increases at a faster rate than taxable revenues. Under the optimal tax planning strategy, the firm increases debt service in proportion to increases in taxable revenues. Hence, debt value increases more slowly than equity value. That is, while the firm is in the region of "optimal investment inaction," the market leverage ratio falls in response to positive shocks. Note that transactions costs are not necessary to produce this effect.

Despite its parsimony, the model is able to replicate a broad set of stylized facts. First, I show that the model replicates the empirically observed inverse relationship between profitability and leverage. This effect is commonly attributed to firms delaying action due to reluctance to incur financial transactions costs. In my model, the market leverage ratio falls with positive shocks due to increases in the value of growth options. The relationship reverses when growth options are finally exercised. However, the infrequent lumpy investment in my simulated cross section of firms causes the first effect to dominate.

Second, the simulated firms exhibit mean reverting leverage ratios, which is also consistent with empirical observation. Mean reversion is a reflection of the investment cycle. Intuitively, leverage ratios decline when the firm is in the investment inertia region. Low leverage firms have valuable growth options and are likely to invest in the near future. However, they do not reflexively add more debt simply because equity value is relatively high. The leverage ratio only spikes upwards at the time that the firm exploits its growth options. The reason is simple. It is only optimal to increase debt once new productive capacity comes on-line and starts generating taxable revenues. In contrast, equity value increases continuously in the underlying state variable– reflecting the capitalized value of future growth options. This increase in equity value is not met with a proportional increase in debt within the inertia region. The leverage ratio only reverts to the mean at the time the real option is exploited.

Third, I find that, consistent with empirical observation, the market leverage ratio depends on the path of past stock returns. For example, a firm that experiences a run-up in stock price followed by a decline is predicted to have a high leverage ratio. This is because the firm operates with excessive capacity that cannot be shed. For such a "cash cow" firm, debt value is high because taxable revenues are high. At the same time, the value of growth options is low since the firm is far away from the endogenous investment threshold. The result is that firms in sectors where the state variable (demand) has declined significantly relative to historic highs are predicted to have high leverage ratios. This leads to a path-dependent leverage ratio.

Aside from shedding light on empirical observation, the model also casts doubt on a popular "folk-theorem" of corporate finance. Based upon MM (1963), it is commonly argued that, in a world where the only financing friction is a tax advantage of debt, the optimal financial structure entails 100% debt finance. In my model, the tax advantage accorded to debt *is* the only financing friction. However, the market leverage ratio generally falls between 43% and 73%. The intuition is based on two observations. First, debt is proportional to current revenues from assets in place, while equity value incorporates expectations regarding *growth* in revenues from assets in place. This effect has already been noted by Ross et al. (2002) and Berens and Cuny (1995). In addition to this previously noted effect, my model illustrates the importance of growth options. In particular, the fact that growth option value is capitalized into equity value, but not debt value, causes the leverage ratio to fall below that predicted in MM (1963).

The primary contribution of my paper is to illustrate the necessity of incorporating real investment decisions and real frictions into dynamic structural models of corporate financial decisions. Table 1 provides an overview. This paper builds on extensive literature on dynamic capital structure. Fisher et al. (1989) and Goldstein et al. (2001) develop tractable multi-period models incorporating dynamic debt restructuring. However, neither model incorporates real investment decisions. Leary and Roberts (2005) show that the model of Fisher et al. can be reconciled with observed financing patterns under a suitable parameterization of transactions costs. Similarly, Strebulaev (2005) shows that a variant of the model of Goldstein et al. can also be reconciled with a number of observed financing regularities. All of these models share a reliance upon financial transactions costs to explain the stylized facts. My paper offers an alternative explanation.

Three closely related papers by Titman and Tsyplakov (2002) and Hennessy and Whited (2005a, 2005b) use dynamic programming to analyze a broad variety of capital structure effects. The central factor differentiating the models is that my framework strips away all financing frictions, aside from taxes, and augments these models with irreversible real investment. Results presented in Hennessy and Whited (2005b) support the argument made in this paper. In particular, they find that their structural model can fit financing moments reasonably well, but tends to overshoot the variance of real investment. The types of real frictions discussed in my paper offer a plausible resolution of this problem.

The empirical literature largely focuses on cross-sectional leverage determinants and simple tests aimed at distinguishing the trade-off theory from alternatives. The stylized facts I focus on are as follows. Titman and Wessels (1988) and Rajan and Zingales (1995) document an inverse relationship between profitability and leverage. Shyam-Sunder and Myers (1999) and Fama and French (2002) document mean-reversion in leverage ratios. Welch (2004) documents the dependence of leverage ratios on past stock returns.

I now provide some detail on the model and simulation methodology. I first consider a simple setting in which the firm holds a single growth option. The reader can gain from this model much intuition about the causal mechanics of the general model with multiple growth options. In subsequent sections, I relax this assumption and consider a firm holding an infinite basket of growth options. This extended model is used for generating the "simulated data." I consider two *real* frictions: irreversibility and fixed costs of investment. "Fixed costs of investment" refer to costs that are a function of the size of the *existing* capital stock rather than the size of the new investment. For example, if a new investment project disrupts current business operations, fixed costs are incurred.

The model with infinite growth options is solved analytically for two particular cases: irreversibility only, and irreversibility with maximum fixed costs. The solution for the zero fixed costs case closely follows analysis in Pindyck (1988), modified to accommodate debt financing and taxation. In this case, firms invest continuously along a boundary and have long periods of inactivity away from the boundary. The empirical section shows that irreversibility alone (without fixed costs) is sufficient to replicate most empirical results, except for lumpy adjustment. I also derive a solution for the model with fixed costs using a scaling property.² Investment in the presence of fixed costs is lumpy and infrequent. This leads to lumpy adjustment in the firm's leverage ratio.

The calibrated model is used to generate artificial panels of data. Using these panels, I perform standard cross-sectional tests regarding leverage ratios. Each simulation results in an artificial panel of data for 1000 firms over 300 quarters. The simulations are performed multiple times to achieve a high degree of consistency. I calculate Fama-MacBeth t-statistics using the panel data and average the results across simulations.

The model replicates qualitatively, and in some cases quantitatively, the results of several regressions commonly found in the empirical capital structure literature. In particular, the simulated firms exhibit mean reversion in market leverage ratios, a negative profitability-leverage relationship, and a sensitivity of leverage to historic stock returns. Finally, I establish that the simulated firms rely upon debt to fill their financing gaps.

The rest of this paper is organized as follows. Section 1 states the assumptions and works out a single option example. Extending the analysis to multiple options, Sections 2 and 3 present the model with irreversible investment, without and with fixed costs of investment. Section 4 describes the simulation procedure and the empirical tests on the simulated data. The final section offers concluding remarks.

²Novy-Marx (2005) applies a scaling property to solve the equilibrium investment model in competitive economy. Goldstein, Ju, and Leland (2001) use a scaling approach to describe the capital structure adjustments while keeping investment fixed. Hennessy and Tserlukevich (2004) use scaling to solve a convertible/callable bond model allowing for upward and downward debt adjustment.

1 Single Growth Option Setting

In order to highlight the basic framework, I first consider a simple setting in which the firm holds a single growth option. As in the remainder of the paper, I consider two *real* frictions: irreversibility, and fixed costs of investment. To reiterate, "irreversibility" refers to the existence of a wedge between the price at which capital can be purchased and the price at which capital can be sold. For example, physical capital, once installed, may become firm-specific and hence has little or no resale value. "Fixed costs of investment" refer to costs that are a function of the size of the existing capital stock rather than the size of the new investment. For example, if a new investment project disrupts current business operations, fixed costs are incurred.

Although the single growth option model is rather simple, it still generates the main results that I derive below for the more general model with infinite growth options. In particular, in the simple model, the market leverage ratio is declining in profitability and is mean reverting. Further, adjustments in capital structure are lumpy whenever there are fixed costs of investment. Finally, in both the simple and general models, debt constitutes the primary source of external funds when the firm exploits new growth options.

The assumptions used in the framework are highlighted below.

• **TECHNOLOGY**: Assets in place (K) costlessly produce one unit of a nonstorable good at each instant. The price p of the output satisfies an inverse demand function of constant elasticity form. Demand is $D = (p/X_t)^{1/\alpha-1}$ with $\alpha \in [0,1)$. The firm is a monopolist, implying $p = X_t \cdot K^{\alpha-1}$.³ Revenue is equal to output price times output: $p \cdot K = X_t \cdot K^{\alpha}$. The demand shock X_t follows a Geometric Brownian Motion (GBM):

$$\frac{\partial X_t}{X_t} = \mu dt + \beta_i \sigma_m dM_t + \sigma_i dB_t \tag{1}$$

Here $\sigma_m dM_t$ is a common random component (market influence) and $\sigma_i dB_t$ is an idiosyncratic shock. The variables M_t and B_t denote uncorrelated Brownian motions. The total variance of the shock is $\sigma^2 \equiv (\beta_i^2 \sigma_m^2 + \sigma_i^2)$.

• **REBALANCING**: Firms can adjust leverage costlessly. There is no

 $^{^{3}\}mathrm{It}$ is possible to present the same argument with the firm being a price taker and with stochastic input costs.

agency conflict between debt and equity, and the firm implements the first-best capital structure.⁴ Costless debt adjustment is possible, for example, when debt takes the form of private bank loans or is closely-held. Under symmetric information, closely held debt allows for continuous renegotiation of the terms of the loan and rules out hold-out problems or coordination issues. As stated in the introduction, this assumption is adopted for the sake of logical clarity and analytical tractability.

- TAXATION: I assume a simple tax structure that creates a tax advantage to debt as long as revenue is larger than interest payment. The corporate tax rate τ_c^+ is levied on positive taxable income, while τ_c^- is levied on negative taxable income. I assume that $\tau_c^- < \tau_c^+$ to approximate the effect of loss limitations. Individuals pay tax on interest income at the rate $\tau_i > \tau_c^-$. This assumption ensures that there is a tax advantage to debt if and only if corporate taxable income is positive. Dividend and capital gains taxes are set to zero. This is done to exclude the "tax wedge" effect in Hennessy and Whited (2005a).
- **INVESTMENT**: The firm has an option to increase its capital stock K by paying fixed and/or proportional costs of investment.⁵ The investment cost structure resembles that used by Abel and Eberly (1994). The cost of changing the capital stock from K_0 to K_1 is:

$$COST(K_1, K_0) = P \cdot \max(K_1 - K_0, 0) - P^- \cdot \max(K_0 - K_1, 0) + F \cdot P \cdot K_0$$
(2)

The first term in this expression is the cost of buying the capital. The second term represents the amount that can be recovered when assets are sold. "Irreversibility" entails $P^- < P$. For simplicity, I assume $P^- = 0$, meaning that investment is "perfectly irreversible." Finally, the last term is the "fixed cost" of investment, i.e., a cost that depends on the initial capital stock, not on the size of the investment per se.

The following lemma highlights that, when financial rebalancing is costless, the optimal financing policy entails setting the debt coupon equal to revenue $(X \cdot K^{\alpha})$, shielding all corporate income from taxation.⁶

⁴From a private perspective, not socially.

⁵It is straightforward to extend this model to allow for convex costs instead of proportional.

⁶Brennan (1986) describes the debt policy in the case of free adjustment.

Lemma 1 Under the optimal financing policy, instantaneous interest expense is equal to $X_t \cdot K^{\alpha}$.

Proof. Consider a firm with revenue XK^{α} that optimizes its debt service strategy by choosing its interest payment *i*. If $XK^{\alpha} > i$, then the flow of tax savings is $i(\tau_c^+ - \tau_i) > 0$. If $XK^{\alpha} < i$, then the flow of tax savings is $i(\tau_c^- - \tau_i) < 0$.

It is worth noting that in the model there are no bankruptcy or agency costs of debt. This includes asset substitution (Jensen and Meckling (1976)), free cash flow problems (Jensen (1986)), and underinvestment problems (Myers (1977)). For example, Barclay, Morellec, and Smith (2004) present a capital structure model with debt balancing under- and overinvestment problems. They find that growth options have negative debt capacity; that is, firms anticipating investment decrease their book leverage. The causation in their model differs from that presented here in that the firm in their model takes on less debt in order to mitigate the underinvestment problem. Such effects are necessarily absent from my model since the firm implements the first-best financing policy.

Note that in the setting considered, it is optimal to distribute all cash. To see this, note that there is a tax disadvantage to corporate savings in this model. In addition, since there are no financial frictions in my model, there are no precautionary motives to offset this tax disadvantage. Auerbach (2001) demonstrates the tax disadvantage to holding cash.⁷

As a first step, consider a firm with capital K_0 that does not have any growth options. Define $V(X, K_0)$ as the total (after-tax) value of the firm. It is equal to the present value of after-tax revenues $(1 - \tau_i)XK_0^{\alpha}$ that grow at the rate μ and are discounted at the tax adjusted risk free rate $r(1 - \tau_i)$:

$$V(X, K_0) = \frac{XK_0^{\alpha}(1 - \tau_i)}{r(1 - \tau_i) - \mu}$$
(3)

According to Lemma 1, the firm optimally sets an instantaneous interest payment equal to revenue XK_0^{α} . The owner of each unit of debt is entitled by the contract to receive a fixed debt coupon payment. But the firm can adjust the total interest payment in response to a change in revenue by issuing (or repurchasing) additional debt. The debtholders who purchase corporate debt when the shock is X will be receiving a (fixed) after-tax interest payment $(1-\tau_i)XK_0^{\alpha}$.

⁷Additionally, Leland (1994) shows that accumulating cash is equivalent to reducing debt for tax purposes. Mauer and Triantis (1994) argue that holding cash increases financial flexibility when external financing is costly.

The value of debt D is equal to the perpetuity of expected interest payments.

$$D(X, K_0) = \frac{(1 - \tau_i)XK_0^{\alpha}}{(1 - \tau_i)r} = \frac{XK_0^{\alpha}}{r}$$
(4)

The value of the residual equity claim is the difference between the value of the assets and the debt.

$$E(X, K_0) = V(X, K_0) - D(X, K_0) = \frac{\mu X K_0^{\alpha}}{r[r(1 - \tau_i) - \mu]}$$
(5)

When there is a positive demand shock, the firm issues new debt and distributes the proceeds to shareholders. Negative shocks lead to retirement of debt. I assume that shareholders can finance the retirement by selling more equity or issuing rights. Note that if $\mu = 0$, debt value is equal to the value of assets in place, and the equity value [5] is zero. This is intuitive because the expected proceeds from selling new debt (if a shock is positive) are equal to the expected costs of retiring debt (if a shock is negative).

The leverage ratio is equal to the debt [4] divided by the total value of the assets [3]:

$$LR = \frac{r(1 - \tau_i) - \mu}{r(1 - \tau_i)}$$
(6)

Evidently, the leverage ratio of the firm without growth options is constant, independent of the revenue. This is consistent, for example, with Strebulaev (2005), who finds that the leverage is the same at each refinancing point.

Further, observe that the leverage ratio in [5] is below one if $\mu > 0$. This may first seem surprising given that the firm issues debt to shield *all* revenues from tax. The reason is that the value of debt is proportional to the current revenues from assets in place, while equity value incorporates expectations regarding *growth* in revenues from assets in place. This effect has already been noted by Ross et al. (2002) and Berens and Cuny (1995). They show that the leverage ratio falls below the 100% figure predicted by Modigliani and Miller (1963).

Next, I allow for a single option to increase capital from K_0 to K_1 by paying proportional and fixed costs $P(K_1-K_0)+F \cdot P \cdot K_0$. In this example, I assume that $F = 1.^8$ The correct valuation of the option requires knowing the optimal exercise strategy. Note that the problem is time independent. Therefore, investment is triggered when the demand shock reaches some critical value X_* .

 $^{^{8}}$ My assumptions about fixed costs for this example are not important. I have already placed a constraint on the investment policy, similar to the fixed costs, by assuming that firm can invest only once.

The following proposition establishes the optimal investment threshold X_* and the value of the growth option $GO(X, K_0)$ using standard real option tools.⁹

Proposition 1 Denote

$$b(\sigma, \mu, r) \equiv -2\mu + \sigma^2 + \sqrt{\sigma^4 + 4\mu^2 + 8r\sigma^2 - 4\sigma^2\mu}$$
(7)

The option is exercised at the first passage of the demand shock to the threshold value X_*

$$X_{*} = \frac{P(r(1-\tau_{i})-\mu)}{(1-\tau_{i})\alpha} K_{0}^{1-\alpha} \left[1-\frac{\alpha b}{b-1}\right]^{\frac{\alpha-1}{\alpha}}$$
(8)

The value of this option is:

$$GO(X, K_0) = \frac{(1 - \tau_i)\alpha K_0^{\alpha}}{(r(1 - \tau_i) - \mu)((b - 1) - \alpha b)} X_*^{1 - b} X^b$$
(9)

Proof.

Conjecture that the firm invests at some shock X_* , and calculate the optimal capital $K_1(X_*)$ that the firm chooses to install. At the optimal $K_1(X_*)$, the firm maximizes total firm value less costs:

$$K_{1}(X_{*}) = \arg\max_{K_{1}} \left(\frac{(1-\tau_{i})X_{*}K_{1}^{\alpha}}{r(1-\tau_{i})-\mu} - PK_{1} \right) = \left[\frac{(1-\tau_{i})\alpha X_{*}}{P(r(1-\tau_{i})-\mu)} \right]^{\frac{1}{1-\alpha}}$$
(10)

The value of the equity is given by the solution to the following differential equation:¹⁰

$$\frac{1}{2}\sigma^2 X^2 S''(X,K) + \mu X S'(X,K) - r(1-\tau_i)S(X,K) + \frac{\mu X K_0^{\alpha}}{r} = 0$$
(11)

The particular solution is (see Appendix A for more details):

$$S(X,K) = AX^{a} + BX^{b} + \frac{\mu X K_{0}^{\alpha}}{r(r(1-\tau_{i})-\mu)}$$
(12)

where a < 0 and b > 1 solve the fundamental quadratic equation w.r.t. Y:

$$\frac{1}{2}\sigma^2 Y^2 + \left(\mu - \frac{1}{2}\sigma^2\right)Y - r(1 - \tau_i) = 0.$$
(13)

The constant A in the first term in [12] must be equal to zero because S(0) = 0. The second term is a value of the investment option $GO(X, K_0) = BX^b$. Constant B and the threshold X_* have to be determined by the boundary conditions. The first boundary condition requires that the value equity be continuous at the exercise threshold (value

⁹This exercise policy is consistent with real option literature that predicts a specific linear relationship between the benefits and costs of irreversible investment. It can be shown in our case that $Benefit = \frac{b}{b-1}Cost$, where $\frac{b}{b-1} > 1$ (see Appendix for details). ¹⁰See Merton (1973) and Black and Scholes (1973) on the option pricing, and Leland (1994)

on valuing equity as a call option.

matching condition):

$$S(X_*, K_0) = V(X_*, K_1) - \frac{X_* K_0^{\alpha}}{r} - PK_1$$
(14)

The second boundary condition is that first derivatives of the equity value must be the same before and after the exercise. This is commonly labeled the smooth pasting condition¹¹. This condition ensures that the exercise threshold X_* is indeed optimal:

$$\frac{\partial S(X,K_0)}{\partial X}|_{X_*} = \frac{\partial}{\partial X}|_{X_*} \left(V(X,K_1) - \frac{XK_0^{\alpha}}{r} - PK_1 \right)$$
(15)

Substituting equity [12] and capital [10] into the boundary conditions [14] and [15] produces a system of linear equations for X_* and B. Solving this system produces [8] and [9].

The equity value in this case is equal to the equity value without options [5] plus the value of the growth option. Using [9], write the value of equity for $X < X_*$ as

$$S(X, K_0) = \frac{\mu X K_0^{\alpha}}{r(r(1 - \tau_i) - \mu)} + \frac{(1 - \tau_i)\alpha K_0^{\alpha} X_*}{(r(1 - \tau_i) - \mu)\left((b - 1) - \alpha b\right)} \cdot \left(\frac{X}{X_*}\right)^b \quad (16)$$

The second term in [16] represents the value of the growth option. One can immediately see that this value is positive and is homogeneous of degree b > 1in X. That is, the growth option value grows faster than the value of assets in place and debt. Since debt value increases more slowly than firm value, the leverage ratio declines with positive shocks until the growth option is exercised.

The example illustrates an important feature of my modeling framework. Under the equilibrium investment strategy, total firm value has the same degree of homogeneity in shock X_* at every "capital adjustment" point. To see this, observe that revenue $X_*K_1^{\alpha}$, capital expenditure PK_1 (from [10]) and the value of future growth options BX_*^b (from [8] and [16]) are homogeneous of the same degree $1/(1 - \alpha)$ in shock X_* . This property will be later used in building the model with multiple investment options.

Turning to Figure 1, I briefly discuss the implication of the financing policy for the firm with an investment option. The figure plots the graphs of equity, leverage ratio, debt, and firm value against the demand shock.¹² The critical value of the shock X_* approximately corresponds to the middle of the horizontal scale. The market leverage ratio (panel A of Figure 1) falls in profitability on

¹¹For details, see Dumas (1991) and Dixit (1993).

 $^{^{12}\,\}mathrm{In}$ other words, this is what the observer would see if the demand grew predictably and constantly with time.

the left side of the graph, i.e., as long as the growth option is not exercised. However, at X_* the firm exploits its growth option and issues debt. As a result of this debt issuance, the leverage ratio spikes upwards.

Next, as Figure 1 C shows, the firm issues debt at the investment to shield the higher revenue from tax. In fact, the proceeds from selling this debt in this example are sufficient to pay the investment cost. When debt covers the financing needs, the remaining amount is simply paid as dividend to the shareholders. Figure 1 B shows that ex-dividend value of equity falls at X_* by the amount of the positive dividend paid to the shareholders. On the other hand, the total firm value (Figure 1 D) increases at X_* . This is because the difference between the new debt issued and the dividend is positive.

Finally, notice that the corporate claim values have different dynamic properties for $(X < X_*)$ and $(X > X_*)$, that is, before and after the investment. For example, debt tends to increase faster after investment because larger capital makes the revenues more sensitive to the demand shocks. The equity of the firm grows exponentially when the option is outstanding. Once the single growth option is exploited, equity value for $(X > X_*)$ is linear in X, and the leverage ratio is constant.

For the empiricist who investigates the data properties, the behavior of the leverage ratio seems to be consistent with the presence of refinancing costs. In my model, the leverage ratio changes because of the change in the growth option value. The jumps in the leverage ratio exhibited by my model are commonly attributed to fixed financing costs. I explain it within the model by the fixed costs of investment. That is, the explanation relies on real costs as opposed to financial frictions.

Next, I proceed to develop a multiple investment model capable of generating a uniform panel of data.

2 Irreversible Incremental Investment

I now extend the analysis to consider a firm holding an infinite basket of growth options. In this section, I assume that there are no fixed costs of investment. The section that follows considers the optimal investment and financing program for a firm facing irreversibility and fixed costs. The independent analyses are interesting in that they illustrate the powerful effect that alternative real costs have on financial policies. The cost of capital accumulation is proportional to the change in capital $P\Delta K$. Pindyk (1988) and Dixit and Pindyk (1994) show that the solution in this case is given by a "barrier policy." The barrier x(K) is the critical value of the shock that justifies installation of an additional unit of the capital. The region in (K, X) space where X < x(K) is termed an "inaction region" since no new investment is made there. However, the firm never crosses the barrier: when X reaches x(K), new investment keeps the firm from crossing it. The Bellman equation for the total value of the firm (debt plus equity) A(K, X) is:

$$A(K,X) = \max_{K'} [(1-\tau_i)K^{\alpha}X + \exp(-r(1-\tau_i)dt)(A(K',X+dX) - P(K'-K))]$$
(17)

Irreversibility places an additional constraint on the solution, i.e., $K' \ge K$. The maximization is described by Kuhn-Tucker condition at the barrier when the firm invests:

$$A_K(K,X) = P \quad s.t. \ K' \ge K. \tag{18}$$

Similar to the single option case, one can write the solution in the inaction region as a sum of the value of assets in place and the option value to increase capacity upon reaching the barrier:

$$A(K,X) = \frac{(1-\tau_i)K^{\alpha}X}{r(1-\tau_i) - \mu} + B(K)X^b$$
(19)

Value matching and smooth pasting conditions¹³ simultaneously determine the boundary and constant B(K):

$$A_K(K, x(K)) = P$$

$$A_{KX}(K, x(K)) = 0$$
(20)

Therefore (see Appendix for details):

$$B'_{K}(K) = -\left(\frac{b-1}{P}\right)^{b-1} \left(\frac{(1-\tau_{i})\alpha K^{\alpha-1}}{b(r(1-\tau_{i})-\mu)}\right)^{b}$$
(21)

And the "barrier" is given by:

$$x(K) = \left(\frac{b}{b-1}\right) \frac{(r(1-\tau_i)-\mu)P}{(1-\tau_i)\alpha K^{\alpha-1}}$$
(22)

 $^{^{13}}$ See, Dumas (1991) and Dixit (1993), p.42, for a discussion of the Super Contact Condition and barrier control policies.

Value of the equity can be found by integrating [21], substituting the result into the total firm value [19] and subtracting the value of the debt:

$$S(K,X) = \frac{\mu X K_0^{\alpha}}{r(r(1-\tau_i)-\mu)} + \left(\frac{b-1}{P}\right)^{b-1} \left(\frac{\alpha(1-\tau_i)}{b(r(1-\tau_i)-\mu)}\right)^b \frac{K^{(\alpha-1)b+1}}{(1-\alpha)b-1} X^b$$
(23)

Investment is contingent on X and is done in a series of small steps. The option value $B(K)X^b$ is, naturally, positive, but its derivative $B'_K(K)X^b$ is negative, indicating that the option becomes less valuable when the amount of capital is increased.

Figure 2 provides an illustration for the investment policy under the zero fixed costs case. All graphs are plotted against time; that is, they incorporate the noise from random shocks. The graphs in Figure 2 are based on a single simulation and one randomly selected firm. For better illustration, I choose to keep the full (without truncation) time series of the data observed over 400 quarters.

Exhibit A is a sample random path of X_t . For this particular path, I plot the "desired" capital stock $K_*(X_t)$ (Exhibit B), which is an inverted function [22]. It is desired in the sense that it features downward adjustments not feasible under irreversibility. In particular, the "desired capital" is defined as the optimal amount of irreversibly installed capital as justified by the current value of X_t . In contrast, Exhibit C shows that the "actual" amount of installed capital is non-decreasing. The firm increases its actual capital incrementally when shock X_t reaches its new historical high. Mathematically, the relationship between the desired and actual capital is given by:

$$K_{t+1} = K_t + \max(0, K^*(X_t) - K_t)$$

Exhibit D displays the market leverage ratio corresponding to the path X_t .

Interestingly, in this particular simulation, the path increments are, on average, positive in the beginning. Therefore, the leverage ratio is relatively low due to the valuable growth options. In the second half, the shocks are, on average, negative. Consequently, the leverage ratio sharply increases. As explained in the introduction, the market leverage ratio is expected to depend on the path of past stock returns. It is easy to see from the example that the firm that experiences a run-up in stock price followed by a decline has a high leverage ratio. This is because the firm operates with an excessive capacity that cannot be shed. Debt value is high because taxable revenues are high. At the same time, the value of growth options is low since the firm is far away from the investment threshold. The result is that firms in sectors where the state variable (demand) has declined significantly relative to historic highs are predicted to have high leverage ratios.

3 Irreversible Investment with Fixed Costs

In this section, I assume that the firm faces two "real frictions": irreversibility, and fixed costs of investment. In the presence of fixed costs, continuous investment is infinitely expensive. Consequently, the firm invests in lumps, with bursts of investment spaced in time. Solving the dynamic model with multiple growth options is complicated. However, the analysis in this section is greatly simplified by the use of a scaling property to describe the optimal investment strategy. In the model, all claims at the time of investment retain the same degree of homogeneity in the underlying demand process X. Consequently, at the time of each investment, the firm is simply an appropriately scaled replica of itself. The financing and investment rules of the firm in each round are identical in form. Indeed, this scaling feature allows me to obtain closed-form solutions.¹⁴

Proposition 2 Let $K_0(X)$ denote the initial choice of capital. Then the optimal investment strategy entails increasing capital to $K_n = \gamma^{\frac{n}{1-\alpha}} K_{n-1}$ upon first passage of the X_t to $X_n = \gamma^n X_0$, $n = 1...\infty$. The scaling parameter $\gamma > 1$ is determined by the equity maximization problem.

Proof: (see Appendix B)

The significance of *Proposition* 2 is that it reduces the optimization problem to essentially a static optimization. This is because the firm simply optimizes over a single parameter γ that fully characterizes the optimal investment thresholds. This may be contrasted with a non-homogeneous setting in which the firm must optimize over an infinite sequence $\{\gamma_t\}_{t>1}$.

Much intuition for the proof of *Proposition* 2 can be gained by considering the following argument. Starting at X_0 , assume that the optimal capital stock is K_0 , and the optimal threshold for next investment is X_1 . Define the scaling

 $^{^{14}}$ Frictions such as fixed costs are not the only way to generate investment lumpiness. For example, Guo, Miao and Morellec (2002) show that if underlying processes exogenously shift between different states, investment is intermittent but may exhibit the spurts of growth.

factor $\gamma_1 = X_1/X_0$. That is, γ_1 is a factor by which one has to multiply the initial value of the shock to obtain the first investment threshold. For example, if $\gamma_1 = 2$, the firm invests as the shock value doubles.

With this definition in mind, I show that at X_1 the optimal capital, cost of investment and revenue are all scalar multiples of the original capital, cost, and revenue at X_0 . To see this, observe that optimal capital is a scalar multiple of X to power $\frac{1}{1-\alpha}$.

$$K_0 = \left(\frac{(1-\tau_i)X_0\alpha}{P(r(1-\tau_i)-\mu)}(1-\gamma_1^{1-b})\right)^{\frac{1}{1-\alpha}}$$
(24)

It is similar to the single option case [10], except for the term $(1 - \gamma_1^{1-b})$ that accounts for possible future increase in the current capital stock. Therefore, optimal capital increases between X_0 and X_1 as:

$$K_1 = \gamma^{\frac{1}{1-\alpha}} \cdot K_0$$

Next, observe that the revenue $X_t \cdot K^{\alpha}$ also increases by factor $\gamma^1 \cdot \gamma^{\frac{\alpha}{1-\alpha}} = \gamma^{\frac{1}{1-\alpha}}$. The same applies to the cost of investment, which is proportional to new capital stock. Therefore the maximization problem for new threshold X_2 is just a scalar multiple of original problem. The next round of investment will occur when X_t increases once more by the factor γ , that is, $X_2 = \gamma X_1$, and so on.

The next step is to determine the optimal scaling factor γ . According to *Proposition* 2, the value of γ that maximizes the initial equity value is "regret free", meaning that the firm will find the same γ optimal in each round. Therefore γ can be found by maximizing equity at X_0 under assumption that γ will not change. *Proposition* 3 defines optimal γ as a solution to single-argument non-linear maximization problem:

Proposition 3 The optimal parameter γ that determines the timing and scale of capital adjustment is given by the solution to the following maximization problem:

$$\max_{\gamma} \left[\frac{(1-\gamma^{1-b})^{\frac{1}{1-\alpha}}}{1-\gamma^{\frac{1}{1-\alpha}-b}} (\mu - \gamma^{1-b}r(1-\tau_i) + (r(1-\tau_i) - \mu)\gamma^{\frac{1}{1-\alpha}-b} - \frac{\alpha r}{1}(1-\tau_i)(1-\gamma^{1-b})\gamma^{\frac{1}{1-\alpha}-b}) \right]$$
(25)

Proof. See the Appendix.

Once γ is known, the optimal investment times are defined contingent on the value of the shock X. In particular $T_N = \min(t, X_t \ge \gamma^N X_0)$, for N = 1, 2... It follows from the *Proposition* 3 that γ is independent of the cost P. Intuitively, the proportional cost P decreases the initial investment and all subsequent investments, but it does not change the ratio between them.

The following corollary gives the value of Equity and Debt at each point of time. I use the number of completed investments N as a space variable because it defines the capital $K = K_0 \gamma^N$.

Corollary 1 The initial value of the equity $S(X_0)$ is given by:

$$S(X_0) = \frac{\frac{X_0 K_0^{\alpha}}{(r(1-\tau_i)-\mu)} \left(\frac{\mu}{r} - \gamma^{1-b}(1-\tau_i)\right) + \gamma^{\frac{1}{1-\alpha}-b} \frac{X_0 K_0^{\alpha}}{r} - \gamma^{\frac{1}{1-\alpha}-b} P K_0}{1 - \gamma^{\frac{1}{1-\alpha}-b}}$$
(26)

The value of equity for the firm that has completed N investments S(X, N) is given by:

$$S(X,N) = \frac{\mu X (K_0 \gamma^{\frac{N}{1-\alpha}})^{\alpha}}{r(r(1-\tau_i)-\mu)} + \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \cdot$$
(27)
$$\left[\gamma^{\frac{N+1}{1-\alpha}} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right) - \frac{(\gamma^{N+1} X_0) (K_0 \gamma^{\frac{N}{1-\alpha}})^{\alpha} (1-\tau_i)}{(r(1-\tau_i)-\mu)}\right]$$

The value of debt for the firm that has completed N investments D(X, N) is given by:

$$D(X,N) = \frac{X(K_0\gamma^{\frac{N}{1-\alpha}})^{\alpha}}{r}$$
(28)

Parameters are chosen such that the equity value is bounded above.¹⁵ Figure 3 illustrates the case of irreversible investment with fixed costs. The graphs for Figure 3 are plotted against time, using a single realization of the stochastic demand shock (Exhibit A). Exhibit B shows the amount of the capital stock corresponding to this path. The investment is triggered when demand shock

$$\left(1 - \gamma^{\frac{1}{1 - \alpha} - b}\right) > 0 \quad \Rightarrow \quad b > \frac{1}{1 - \alpha}$$

¹⁵For the solution to converge, it must be that:

Here $b(r, \mu, \sigma^2)$ is increasing in r and decreasing in μ and σ^2 . The condition above ensures that the value of the future growth options is finite. The direct analogy is a Gordon's growth formula $V = \frac{X}{r-\mu}$ that requires $\mu < r$ to be correctly interpreted.

reaches the set of critical levels defined by *Proposition* 2. For this particular example, the investment is completed in quarters 91, 110, 113, and 137. It is visible on the graph as a series of nondifferentiable jumps in capital stock value. *Exhibit* C shows the plot of the debt value corresponding to the path realization. Debt continuously increases in revenue. However, debt increases in lumps when new debt issued at the time of investment. Finally, the market leverage ratio that corresponds to this path is plotted on *Exhibit* D. The market leverage ratio increases (decreases) with decreasing (increasing) profitability, and the relationship reverses at the exercise. The leverage ratio is significantly higher in the second half; the intuition is the same as provided for the case without fixed costs.

The general observation is that the investment patterns, investment amount, and behavior of the leverage ratios are similar for two cases.¹⁶ Intuitively, irreversibility is in a sense similar to fixed costs in my framework since they both present a constraint to the investment. Including fixed costs in the model simply exacerbates the investment delays. However, the model without fixed costs cannot generate the lumps in leverage adjustment, contradicting the empirically observed stylized facts.

Next, I use the model to generate the data and perform the empirical tests.

4 Data and Empirical Results

The non-linear nature of the equations in the model prevents me from making a direct estimation. Therefore, I take a simulation approach used in previous studies. For example, Hennessy and Whited (2005a), Strebulaev (2005), and Leary and Roberts (2005) use simulations to study capital structure. Berk, Naik, and Green (1999) employ this method to explain cross-sectional returns. Shürhoff (2004) uses simulations to highlight the effect of capital gains tax policy.¹⁷ Although I borrow some elements from all of these studies, I most closely follow Strebulaev (2005) in the simulation and estimation method.

First, I calibrate the technology parameters and simulate random price

 $^{^{16}\,{\}rm The}$ quantitative comparison for two models is provided when I examine the generated samples in the empirical section.

¹⁷For example, Hennessy and Whited use the solution to their model to generate the simulated panel of the firms and minimize the distance between interesting moments from the actual data and the corresponding moments from the simulated data. Leary and Roberts use the reduced-form model for equity returns to generate the leverage path between exogenously specified upper and lower bounds. Strebulaev (2003) studies how the cross-sectional properties of the data generated by the firms is affected by adjustment costs.

paths. Then, I use them to create artificial panels of equity, debt, and leverage for the economy of firms. Next, I perform a number of tests commonly employed in the empirical literature. The focus of the tests is on commonly discussed properties of the leverage ratio, such as the correlation between leverage and profitability, the mean-reversion and the dependence of leverage on past stock returns. I document that my model produces results that are qualitatively, and in most cases quantitatively, consistent with existing empirical findings.

4.1 Simulation Procedure

This section briefly explains the data generating process and outlines the main assumptions and definitions. Further details on the simulation procedure can be found in the Appendix.

The firms within the economy are affected by common market conditions, but there is no competition, and the firms do not interact with each other through the equilibrium price. Given that the scope of my paper is on capital structure, I assume that the firms produce highly specialized products, and an increase in one firm's output has no influence on the price that another firm receives.¹⁸

I assume that the total demand shock consists of two parts: the idiosyncratic part and the economy-wide shock that leads to a correlation between cash flows of different firms. The economy-wide (systematic) shocks have no pricing implications under a risk-adjusted measure. But the common component is a source of cross-sectional correlation in residuals. As Fama and French (2002) explain, the cross-sectional correlation is important to correctly understand the tests used. Ito's Lemma applied to the logarithm of Brownian Motion produces discrete-time representation of the shock dynamics defined by Equation [1]:

$$X_{t+dt} = X_t \exp(\left(\mu - \sigma^2/2\right) dt + \sigma_i dW(t) + \beta_i \sigma_m dM(t))$$
(29)

Here W(t) and M(t) are uncorrelated BM processes; $\sigma = \sqrt{\beta_i^2 \sigma_m^2 + \sigma_i^2}$ is combined volatility; and market "beta" β_i is a measure of systematic risk of the firm.

I use the following guidelines when making assumptions about technology and economy parameters. Some parameters, as I explain below, have a trivial effect on the statistics of interest and are simply normalized. The second

 $^{^{18}}$ Kogan (2001) and Novy-Marx (2004) build the equilibrium asset pricing models with investment.

group of parameters (for example, volatility) is calibrated outside of the model with empirical data.¹⁹ The remaining parameters are adopted from the studies as explained below. As a caveat, I acknowledge that some of the parameters are selected *ad hoc*. However, as a simple robustness check, I vary each of the parameters within a reasonable range. This ensures that the results remain significant as long as real options constitute a nontrivial part of the firm value. Simulation parameters are summarized in the table below.

Parameter	Notation	Determination	Value
Drift of the demand shock	μ	adopted	1%
Initial value of the shock	X_0	normalized	10
Proportional cost of investment	P	adopted	$U \ [10, \ 100]$
Demand elasticity	α	adopted	$U \; [.1, \; .2]$
Risk-free rate	r	empirical	5%
Individual tax rate	$ au_i$	adopted	1%
Corporate tax rate	τ_c	normalized	35%
Volatility (idiosyncratic)	σ_i	empirical	empirical
Volatility (market)	$\beta_i \sigma_m$	empirical	empirical
Fixed Costs of Investment	F	min. or max.	$\{0, 1\}$
Time horizon, in quarters	Т	normalized	300

I use empirical data from COMPUSTAT and CRSP to calibrate the volatility parameters. To be included in a sample, the firm must have Assets (item 6), Tangible Assets (8), Long Term Debt (9), Interest Expense (15), Income (18), Share Price (24), Number of Shares (25), Adjustment Factor (27), Current Liabilities (34), Retained Earnings (36), Invested Capital (37), Extraordinary Items (48), Capital Expenditures (128), Total Liabilities (181), Fiscal Year Closing Price (199), and SIC code (324). I exclude utilities (SIC: 4900-4949) and financial institutions (6000-6999). Year-end market value is computed as the price per share times the number of shares; the book value of debt is computed as Long term debt plus Current Liabilities.

The resulting sample consists of 18,7501 observations for 20,975 firms. The typical firm in this sample has a market leverage ratio with mean 0.292, minimum 0 and maximum .999. Annual time series standard deviations vary greatly across firms, averaging 0.133. Matching the large size of this sample in the

 $^{^{19}{\}rm I}$ do not use, of course, the calibrated parameters, such as the volatility of cash flows, to gauge how well the calibrated model links with empirical data.

simulations is problematic because of limited computing power. Therefore, I simulate the smaller economies of 1000 firms that have the attributes of the empirical sample. To ensure that this sample is representative, I randomly pick the volatilities for the simulations from the larger pool of the empirically observed volatilities.

The distribution of the firms' systematic risk (represented by β) is obtained by running OLS one factor regression for all firms in the empirical sample on the value weighted index as a proxy for market. Distribution of the cash flow volatilities is constructed using standard deviations of the equity returns. I adopt the assumption that the volatility of the debt returns is negligible compared to the volatility of the equity returns, and that the correlation between the two is close to zero²⁰. The standard deviation of the cash flows (as opposed to equity) is estimated as $\sigma = (1 - L)\sigma_{equity}$, where L is a mean leverage ratio (over all quarters and all firms) from the COMPUSTAT sample. Finally, the idiosyncratic volatility is calculated by subtracting the market component from the total volatility:

$$\sigma_i = \sqrt{\sigma^2 - \beta_m^2 \sigma_m^2} \tag{30}$$

As a robustness check, I verify that the results are independent (at least qualitatively) of particular method of selecting the heterogeneous parameters. For example, I obtain similar results by using normally distributed total volatilities.

The initial shock value X_0 that has a trivial proportional effect on the valuation is normalized to 10 for all firms. This translates into the initial value of assets $V_0 = \frac{X_0}{r-\mu} = 250$. Following Hennessy and Tserlukevich (2005), I assume that cash flows grow at the rate (net of depreciation) $\mu = 1\%$ and the risk-free rate is equal to r = 5%. I require the tax rate on the individual interest income to be positive (as explained in the assumptions in Section 1). However, I select a very small value $\tau_i = 1\%$ for the simulations and check that the results are robust to the higher rate. This is done for two reasons. First, I follow many studies that use a very small or zero rate. Second, by using small individual income tax rate I prove that the tax assumptions play no other role in the model than to create a benefit to debt. I assume that the proportional investment cost and the elasticity of demand are drawn from the uniform distributions on the domains [10, 100] and [.1, .2], respectively.²¹

For the benchmark model, I simulate 400 quarters of data for firms, remov-

²⁰This is consistent with the model assumption that debt is riskless.

²¹In this, I loosely follow the heterogeneity assumptions used in Strebulaev (2005).

ing the first 100 quarters to minimize the impact of the initial conditions on the results. During each simulation, I run the set of cross-sectional tests on the resulting panel and save the coefficients and Fama-MacBeth T-statistics. Simulations are repeated 1000 times, producing a sampling distribution for the statistics of interest. Following Berk, Naik, and Green (1999) and Strebulaev (2005), I average the resulting statistics over the simulations (population mean), and examine the distribution of the coefficients across simulated economies. By observing this distribution, one can gauge whether a model with real frictions can give rise to the coefficients commonly found in empirical work.

In regressions I follow the literature and use the market (as opposed to book) leverage ratios. The book value of equity is a poor reflection of the firm's profits and asset value. Welch (2004) emphasizes that book equity value has little meaning, calling it a plug number used to balance the right- and lefthand sides of the balance sheet. Myers (1977) considers market leverage ratios "more pertinent".²².

Definition 1 The Market Leverage Ratio is defined as a ratio of the book value of debt to the sum of book value of debt and the market value of equity:

$$LR_t = \frac{D_t}{D_t + S_t} \tag{31}$$

Descriptive statistics for the leverage ratios in simulated economies are summarized in the *Table* 2. The leverage ratio is 45% in the case without fixed costs and 73% in the case with fixed costs. This result is higher than the typical average US leverage ratio of about 30%.²³ This is not a surprising result because my assumptions rule out distress costs. As an example, imagine that firms (e.g., due to possibility of bankruptcy) adopt the strategy of keeping the interest payments capped at half of the revenues. In this case, the model will produce leverage ratio is larger in the case with fixed costs. This is also expected because the growth options are, on average, less valuable when the real frictions are larger.

In addition to the leverage ratio properties, Table 2 also provides statistics

 $^{^{22}}$ Myers (1977) says: "Anyone familiar with modern finance theory considers ratios based on market values much more pertinent. Yet there is an element of sense in (using book leverage). It is not that the book values are more accurate than stock market values, but simply that they refer to assets already in place."

 $^{^{23}}$ The leverage ratios reported by different studies vary due to the difference in definitions and the sample selection. Most common estimates lie between 2.9 and 3.5.

on the average investment. For each firm, the average investment is calculated as the total investment divided by the number of quarters with non-zero investment. The average investment figures for zero and non-zero fixed costs cases are 0.703 and 0.454, respectively. While in the former case, firms invests in small increments, the average aggregate amount invested per quarter exceeds the aggregate investment in the latter case. This is because aggregate investment is larger in the firm with more valuable growth options. The fixed costs are creating infrequent bursts of investment while reducing the total amount invested in each period.

4.2 Cross-Sectional Regressions

In this section, I investigate the cross-sectional properties of the model. That is, I use the model to generate an artificial panel of firms. I then compare the model-generated moments and regression coefficients with those observed in real-world data.

In all of the regressions, I employ the Fama-MacBeth (1973) method to calculate the robust standard errors. Fama and French (2002) highlight a serious problem in the traditional empirical leverage literature: understated standard errors that may cloud statistical inference. The problem arises due to correlations in residuals across firms and across years. Following their method, I use the average slopes from year-by year cross-sectional regressions and use the time series of the standard errors to draw inferences. Average slopes obtained by this method are identical to the slopes from a single panel regression commonly used in the literature. The t-statistics are average slopes divided by the standard error, defined as the time series standard deviation divided by the square root of the number of observations. To account for the autocorrelation in residuals, an additional problem noted by Fama and French (2002), I use the higher cut-off to gauge the statistical significance of t-statistics. I average the slope coefficients and t-statistics obtained by this method over 1000 simulations and provide the distribution statistics of this sample.

The simple theory of capital structure predicts a positive relationship between earnings and leverage, contradictory to the empirical evidence. Strebulaev (2005) and Leary and Roberts (2005) find that this negative relationship can result from infrequent leverage adjustment in the presence of financial transactions costs. I run the same regression for the model where transaction costs are *explicitly* set to zero. Below is a definition of the profitability of the firm used in the cross-sectional tests:²⁴

Definition 2 Profitability π_t is defined as a sum of earnings and an increase in book value of assets divided by the book value of assets:

$$\pi_t = \frac{earnings_t + dB_t}{B_{t-1}} \tag{32}$$

I run the following cross-sectional regression of market leverage on profitability and control variables.

$$LR = \beta_0 + \beta_1 \pi + \beta_2 \sigma + \beta_3 \alpha + \varepsilon \tag{33}$$

Results are summarized in *Table* 3. The first column gives the coefficients and t-statistics calculated with the Fama-MacBeth method and averaged using 1000 simulations. The columns 2 through 5 describe the distribution of these statistics across the simulations, including the standard deviation and percentiles. The coefficients on the profitability are negative and significant for two cases of the model even after Fama-French adjustment for autocorrelation. Of particular interest is the fact that the average estimated coefficient on the profitability variable is -0.6, with the average t-statistic of -8.7 for the base case with no fixed costs. Similarly, I document the average coefficient of -0.13 and the average t-statistic of -13.3 in the case with fixed costs. Intuitively, the leverageprofitability correlation is larger for the case without fixed costs since investment options are more valuable and leverage fluctuations are more significant in the "inaction" region.

These results are broadly consistent with existing empirical evidence. For example, Fama and French (2002) and Baker and Wurgler (2002) report coefficients around -0.6. Figure 4A displays the histogram with the distribution of coefficients in the simulation sample. It is easy to see from the histogram that the empirical coefficient of -0.6 can be generated by the model with irreversible investment. It is less likely, however, that a coefficient of this size can be generated by the model with fixed costs. For example, as *Table 3* shows, less than 1% percent of simulations produce coefficients below -0.2.

It is easy to see the source of the negative correlation in my model. Observe that the firm value is a sum of assets in place V and the value of growth options

 $^{^{24}}$ This definition is used, for example, by Strebulaev (2005). The term dB_t in the numerator is suppose to account for the difference between earnings and free cash flows. I have also checked my results using cash flows adjusted by assets.

GO. The leverage ratio is a ratio of debt to the total value of assets:

$$LR_t = \frac{D_t}{V_t + GO_t} \tag{34}$$

Let us see how the leverage ratio is affected by positive profitability shock $X \rightarrow \xi X$, $\xi > 1$. The shock will have a proportional effect on the revenues from the assets in place and on the debt value that depends on revenues. However, the value of the option is more sensitive to the shock and will grow at a faster (exponential) rate, as was demonstrated in Section 1. In particular, it follows from equations [3] and [16] that:

Installed Capital Value
$$V_t(\xi X) = \xi V_t(X)$$
(35)Debt Value $D_t(\xi X) = \xi D_t(X)$ Growth Options Value $GO_t(\xi X) \sim (\xi X)^b = \xi^b X^b$

Consequently, the leverage ratio declines with positive shock to the profitability as long as the option is not exercised:

$$LR_t(\xi X) = \frac{\xi D_t}{\xi V_t + \xi^b GO_t} < LR_t(X)$$
(36)

When the firm invests (exercising the option), it issues new debt, and the relationship between profitability and leverage reverses. However, since most firms do not invest at the same time, the first effect dominates in the cross section. It is important to stress that the amount of debt in my model is instantaneously adjusted and is always optimal. In this sense, my explanation for cross-sectional effects is different from the one offered by the transaction cost models. There, debt adjustment is constrained while the firm value is allowed to change with profitability.

Next, I follow Fama and French (2002) and Shyam-Sunder and Myers (1999) and investigate whether leverage is mean reverting in the simulated panel of firms. This empirical regularity is often seen as evidence that management sporadically attempts to move the leverage ratio back to some "target" once the benefit exceeds the financing costs. However, there are no financial frictions in my model since I allow for instantaneous and free leverage adjustment. I run a cross-sectional regression similar to one used by Fama and French (2002),

using average leverage ratio as a proxy for the "target."

$$LR^{t+k} - LR^t = \beta_0 + \beta_1 LR^t + \beta_2 \overline{LR} + \beta_3 \sigma + \beta_4 \alpha + \varepsilon$$
(37)

Here $\overline{LR_i}$ is a time series mean of the leverage, and $LR_i^{t+k}-LR_i^t$ is a change in leverage from period t to period t + k. The negative coefficient β_1 and positive coefficient β_2 indicate a mean reversion. Table 4 summarizes the findings: the first column is for k = 10 (two and a half years), and the last two columns are for k = 5 and k = 20. The regression coefficients consistently show the evidence of mean reversion. I find that the mean coefficient on the lagged leverage is -0.18 and -0.16 for the model without and with fixed costs. respectively. The coefficient on $\overline{LR_i}$ for all cases and horizons is similar in magnitude and opposite in sign. I confirm that the simplified regression on the difference $LR^t - \overline{LR}$ produces a very close estimate of the coefficient. This means that the difference between the past leverage and its "target" has the most explanatory power for the change in leverage. Next, I investigate the mean reversion of the leverage at different horizons. The absolute value of the coefficient decreases with horizon. For example, I document coefficients -0.09 (no fixed costs) and -0.07 (fixed costs) for k = 10.

The magnitude of the mean reversion coefficient falls within the bounds documented in empirical studies. Fama and French (2002) find that the coefficients range from -0.07 to -0.18 for different samples. *Panel B* of *Figure* 4 provides the distribution of the coefficients in simulated economies. I find that both variations of my model can provide the estimates that are reasonably close to the empirical values.

The mean reversion arises naturally because of the fluctuations in the value of the growth options. For example, low leverage is an indicator of the high ratio of the growth options to the assets in place. Therefore, I argue that the leverage ratio is more likely to increase than decrease, regardless of whether subsequent demand shocks are positive or negative. This is because the positive shocks lead to the exercise of the growth options and to additional debt issuance, while the negative shocks make the growth options less valuable and the leverage ratio larger.

Finally, I document that in the simulated panel of firms the leverage depends on the history of firm's profitability. To investigate the relationship between leverage and past stock returns, I run a test similar to the one described in Welch (2004). The independent variables are Implied Debt Ratio (or IDR), one quarter lagged leverage ratio as well as two control variables: cash flows volatility (σ) and elasticity of demand (α):

$$LR_i^{t+l} = \beta_0 + \beta_1^l (IDR^l) + \beta_2 LR^{t+l-1} + \beta_3 \sigma + \beta_4 \alpha + \varepsilon$$
(38)

Implied Debt Ratio $IDR_{t,t+l}$ is defined similar to Welch (2004) as a market leverage of the firm that makes no attempts to adjust in response to equity shocks over l periods:

$$IDR_{t,t+l} = \frac{D_t}{D_t + S_t(1 + r_{t,t+l})}$$
(39)

Here, $r_{t,t+l}$ is equity return for the period (t, t + l). In the cross-sectional regression that controls for the last quarter leverage, the $IDR_{t,t+l}$ variable should not have any impact on the firm's present leverage ratio provided that the firm adjusts often. However, I find that the coefficient on the implied debt ratio is statistically significant in the regressions even if the last quarter leverage is included to pick up the leverage persistence. The (untabulated) results are very similar for the investments model with and without fixed costs. Using the data from the model without fixed costs, I document the *IDR* coefficient of 7% for a 4 quarter horizon (1 year), and about 5% for 10 quarters. Welch (2004) reports a coefficient in the order of 1.0 for a one year horizon.

Intuitively, the lagged profitability affects the number of growth options that are exercised and, therefore, determines current production capacity. For example, a formerly profitable, large firm experiencing a negative shock will have a high leverage ratio since it operates with excessive capacities and the growth options are nearly worthless. It is not surprising that the size of coefficients is different in magnitude. Firms in my model do adjust their leverage in response to the changes in equity value. But the difference is that the optimal debt increases (decreases) to match the increasing (decreasing) revenues from assets in place, ignoring the growth options. Therefore, the *IDR* variable can explain some of the variation in current leverage, but cannot explain it all.

4.3 Sources of External Financing

When firms go external for funds, debt tends to be the primary source. Leary and Roberts (2004) document that 22.5% of firms in his sample relied on debt,

compared to 9.7% and 5.5% that relied on equity or dual issues, respectively. Shyam-Sunder and Myers (1999) show that financing gap is matched almost dollar-for-dollar by new debt in their sample of large firms during the 1970s and 1980s. Chen and Zhao (2005) investigate the active issuance decisions and find that firms primarily issue debt.

This is not to say that corporate borrowing is equal to capital expenditures. In recent data, capital expenditures far exceed borrowing. In 2004, for example, U.S. non-farm, non-financial corporations had \$900 billion in capital expenditures and obtained \$231 billion through credit markets (Board of Governors 2005, Table F.102). Net equity issues are negative for the same year at -\$157billion²⁵. This implies that the largest source of financing investment in U.S. data is retained earnings.

These stylized facts are often attributed to informational asymmetries, as in Myers and Majluf (1984) and Myers (1984). However, I hypothesize that real frictions can explain the firms' reliance on debt as their primary source of external funds. Intuitively, expanding firms issue more debt to shield the increasing cash flows, and use the proceeds to finance expansion. In fact, the debt flotations often exceed the financial gap.

Just as in pecking order literature, when investment exceeds internal resources, firms turn to external financing; and when investment exceeds the proceeds from selling new debt, firms turn to equity (assuming it is cheap). Therefore, my model implies that firms may issue equity in the absence of debt capacity constraints or need for financial slack.²⁶ Fama and French (2005) and Leary and Roberts (2004) document that year-by-year equity decisions of more than half of their samples violate the pecking order.

However I find that in the model, under the selected parameters, the proceeds from issuing new debt at investment are always sufficient to cover the cost of investment. This is partially because the model does not allow for the "time to build." Since the capital is installed instantaneously and immediately produces cash flows, the firm sells significant quantity of debt instantaneously. Modeling the extension that allows for the delay between investment and increase in cash flows would result in the investment financed, in the model, with both debt and equity.

 $^{^{25}}$ Fama and French (2005) point out that firms often issue equity through channels that do not affect the cash flow statements (e.g., employee option grants) and, as such, may be excluded from the net issues measure.

 $^{^{26}}$ See Lemmon and Zender (2002) and Leary and Roberts (2004) for a discussion of the financial slack and its role in the optimal capital structure.

My results allow me to speculate on the reason why debt is used more then equity; however, the model is ill-suited to make a clear prediction for the choice between internal and external financing. In my model, where savings can be viewed as a "negative debt", it would be suboptimal to accumulate cash. Precautionary savings have no value since debt financing is cheaply available and, as Auerbach (2001) demonstrates, cash reserves are tax disadvantageous. Therefore, the extent of my claim based on the above considerations is that the firms have a preference for debt when turning to external sources to finance the investment.

5 Conclusion

This paper developed a dynamic model of optimal financial structure in the presence of real frictions: fixed costs and irreversibility of capital investment. In order to abstract from existing theories resting upon transactions costs, the model explicitly assumes free instantaneous capital structure rebalancing. The only MM assumption violated in the model is the existence of a tax benefit to debt. Despite its parsimony, the model generates predictions consistent with empirical facts. In the simulated economy, leverage is mean reverting, with gradual and lumpy adjustments. Profitability is negatively correlated with leverage, and leverage depends on the path of past stock returns.

I confirm this by running a number of capital structure tests using modelgenerated data. I find that estimated coefficients are similar to results documented in the literature. It appears that the model without fixed costs of investment (as opposed to the model with fixed costs) does a better job of explaining the empirical data since the size of the coefficients is closer to the empirical estimates. Additionally, I find that the model is broadly consistent with both the timing and mix of external securities issued by firms. In particular, when firms go external for funds, debt tends to be the primary source.

The framework can be naturally extended in a number of ways. Most importantly, the model can be extended to incorporate transactions costs, which are surely features of the real-world environment facing firms. The results in my paper, taken in conjunction with models emphasizing transactions costs, suggest that structural models of financing decisions can indeed have significant predictive power. By ignoring real frictions, the literature has perhaps understated the power of structural models to resolve the "capital structure puzzle."

Appendix A: Valuation of Claims

A contingent claim generating an instantaneous linear "dividend" equal to mX + k, where X follows Geometric Brownian Motion, m and k constants, satisfies the following ODE.

$$\frac{1}{2}\sigma^2 X^2 V''(X) + \mu X V'(X) - r(1 - \tau_i) V(X) + mX + k = 0.$$
(40)

The particular solution is:

$$V(X) = AX^{a} + BX^{b} + \frac{mX}{r(1-\tau_{i}) - \mu} + \frac{k}{r(1-\tau_{i})},$$
(41)

where a < 0 and b > 1 solve the quadratic equation:

$$\frac{1}{2}\sigma^2 Y^2 + \left(\mu - \frac{1}{2}\sigma^2\right)Y - r(1 - \tau_i) = 0.$$
(42)

Unknown constants (A, B) are determined by appropriate boundary conditions. For all contingent claims that are considered in this paper A = 0 because a < 0 and the value of the claims is required to be finite at X = 0. For example, equity S(X) is a claim on instantaneous dividend:

$$E(\frac{K^{\alpha}dX}{r}) = \frac{\mu K^{\alpha}X}{r}$$

$$S(X) = BX^{b} + \frac{\mu XK^{\alpha}}{r\left(r(1-\tau_{i})-\mu\right)}$$
(43)

S

Therefore:

Similarly, for the Debt claim that pays constant coupon C:

$$S(X) = \frac{C}{r} \tag{44}$$

The upward hitting claim $G_u(X)$ (state price) that pays nothing continuously pays \$1 upon X_t first passage to $X_1 > X_0$ is subject to the following condition $G_u(X_1) = BX_1^b = 1$. Therefore:

$$G_u(X) = \left(\frac{X_1}{X}\right)^{-b}$$

 $G_u(X)$ takes a particular simple form when $X_* = \gamma X_0$:

$$G_u(X_0 \mid X_* = \gamma X_0) = \gamma^{-b}$$

Using this result, I can write the value to equity dividend received until X reaches a particular threshold. For example, the value of dividends accumulated in $X < X_1$ can be found as a difference between the value of perpetual dividend at X_0 and the value of perpetual dividend at X_1 times the state price:

$$S(X_0|X_0 < X_1) = \frac{\mu X_0 K^{\alpha}}{r(r(1-\tau_i) - \mu)} - G_u(X_0|X_1) \frac{\mu X_1 K^{\alpha}}{r(r(1-\tau_i) - \mu)}$$
(45)

In particular, when $X_1 = \gamma X_0$ this is simply:

$$S(X_0|X_1 = \gamma X_0) = \frac{\mu X K^{\alpha}}{r \left(r(1 - \tau_i) - \mu\right)} (1 - \gamma^{1-b})$$
(46)

These results are used throughout the paper and in the proof of Proposition 1.

Appendix B: Calculations for single option case:.

First, rewrite the equity value at the investment threshold as (value to shareholders from assets in place)+(value of the additional capital installed at X_*)-(cost of installing this capital):

$$S(X_*.K) = \frac{\mu X_*K^{\alpha}}{r(r(1-\tau_i)-\mu)} + \frac{X_*(K_*^{\alpha}-K^{\alpha})(1-\tau_i)}{(r(1-\tau_i)-\mu)} - K_*P$$
(47)

Substitute into value matching [14] and smooth pasting [15] conditions. Then, simplify:

$$BX_{*}^{b} = \frac{X_{*}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)} \left[\frac{(1-\tau_{i})\alpha X_{*}}{P(r(1-\tau_{i})-\mu)} \right]^{\frac{1}{1-\alpha}}$$
(48)
$$-\frac{X_{*}K^{\alpha}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)} - P \left[\frac{(1-\tau_{i})\alpha X_{*}}{P(r(1-\tau_{i})-\mu)} \right]^{\frac{1}{1-\alpha}}$$
$$BbX_{*}^{b} = \frac{X_{*}(1-\tau_{i})}{r(1-\tau_{i})-\mu} \left(\frac{1}{1-\alpha} \right) \left[\frac{(1-\tau_{i})\alpha X_{*}}{P(r(1-\tau_{i})-\mu)} \right]^{\frac{\alpha}{1-\alpha}}$$
$$-\frac{X_{*}K^{\alpha}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)} - \left(\frac{P}{1-\alpha} \right) \left[\frac{(1-\tau_{i})\alpha X_{*}}{P(r(1-\tau_{i})-\mu)} \right]^{\frac{1}{1-\alpha}}$$

System [48] can be solved for two unknowns: X_* and B. The critical shock value X_* is found by multiplying the first equation times b and subtracting the second equation.

$$X_* = \frac{P(r(1-\tau_i)-\mu)}{(1-\tau_i)\alpha} K^{1-\alpha} \left[1 - \frac{b}{b-1}\frac{\alpha}{1}\right]^{\frac{\alpha-1}{\alpha}}$$
(49)

Next, multiply the first equation times $\frac{1}{1-\alpha}$ and subtract the second equation, this gives expression for constant B:

$$B = \frac{(1 - \tau_i)\alpha K^{\alpha}}{(r(1 - \tau_i) - \mu)((b - 1) - \alpha b)} \cdot$$

$$\left[\frac{P(r(1 - \tau_i) - \mu) K^{1 - \alpha}}{(1 - \tau_i)\alpha} \left(1 - \frac{\alpha b}{(b - 1)}\right)^{\frac{\alpha - 1}{\alpha}}\right]^{1 - b}$$
(50)

Then growth option value (see Appendix A) is equal to $GO(X, K) = BX^b$; substituting here constant B from above produces value of the option in [9].

The last step is to show that the exercise policy is consistent with real option literature that predicts a specific linear relationship between the benefits and costs of irreversible investment. In the above case, the benefits from exercise are derived from an increase in capital from K to K_* :

$$Value = \frac{X(K_*^{\alpha} - K^{\alpha})(1 - \tau_i)}{r(1 - \tau_i) - \mu} = \frac{(1 - \tau_i)X}{r(1 - \tau_i) - \mu} \left(\left[\frac{(1 - \tau_i)\alpha X_*}{P(r(1 - \tau_i) - \mu)} \right]^{\frac{\alpha}{1 - \alpha}} - K^{\alpha} \right)$$
(51)

While the cost of purchasing new capital is:

$$Cost = \left[\frac{(1-\tau_i)\alpha X_*}{P(r(1-\tau_i)-\mu)}\right]^{\frac{1}{1-\alpha}}P$$
(52)

Given optimal threshold value X_* found above, this implies that:

$$Value = \left(\frac{b}{b-1}\right)Cost \quad \text{where} \quad \left(\frac{b}{b-1}\right) > 1 \tag{53}$$

Appendix C: Incremental investment case.

Substitute the value of the firm [19] into the system [20]:

$$\frac{\alpha(1-\tau_i)K^{\alpha-1}x}{r(1-\tau_i)-\mu} + B'(K)x^b = P$$

$$\frac{\alpha(1-\tau_i)K^{\alpha-1}}{r(1-\tau_i)-\mu} + bB'(K)x^{b-1} = 0$$
(54)

Solve this system for x(K) and B'(K). This is done, for example, by multiplying the second equation times x and subtracting the first equation:

$$B'_{K}(K) = -\left(\frac{b-1}{P}\right)^{b-1} \left(\frac{(1-\tau_{i})\alpha K^{\alpha-1}}{b(r(1-\tau_{i})-\mu)}\right)^{b}$$
(55)

$$x(K) = \left(\frac{b}{b-1}\right) \frac{(r(1-\tau_i)-\mu)P}{(1-\tau_i)\alpha K^{\alpha-1}}$$
(56)

B(K) found by integration on $[-\infty, K]$ or, equivalently, by integration on $[K, \infty]$ with a negative sign:

$$B(K) = -\int_{K}^{\infty} B'_{K}(K) dK = \left(\frac{\alpha(1-\tau_{i})}{b(r(1-\tau_{i})-\mu)}\right)^{b} \frac{K^{(\alpha-1)b+1}}{(1-\alpha)b-1} \left(\frac{b-1}{P}\right)^{b-1}$$

In the integral, $(1 - \alpha)b - 1 > 0$ is required for convergence. Substituting B(K) back into [19] gives:

$$A(K,X) = \frac{(1-\tau_i)K^{\alpha}X}{r(1-\tau_i)-\mu} + \left(\frac{b-1}{P}\right)^{b-1} \left(\frac{\alpha(1-\tau_i)}{b(r(1-\tau_i)-\mu)}\right)^b \frac{K^{(\alpha-1)b+1}}{(1-\alpha)b-1}X^b \quad (57)$$

Finally, I write equity and debt claim values separately. Value of the debt is the same as in the single option case. The value of equity is found as a difference between the total firm value and the debt value:

$$D(K,X) = \frac{K^{\alpha}X}{r}$$
(58)

$$E(K,X) = \frac{\mu X K^{\alpha}}{r(r(1-\tau_i)-\mu)} + \left(\frac{b-1}{P}\right)^{b-1} \left(\frac{\alpha(1-\tau_i)}{b(r(1-\tau_i)-\mu)}\right)^{b} \frac{K^{(\alpha-1)b+1}}{(1-\alpha)b-1} X^{b-1}$$

When X reaches the barrier x(K), the new (larger) capital is installed. It is also referred to in the text as a "desired" capital because this is the optimal amount of the irreversible investment that the firm would like to make conditional on current value of X. The optimal amount of the capital at the barrier is found by inverting the expression for barrier x(K):

$$K^{*}(x) = \left(x\frac{(b-1)\alpha(1-\tau_{i})}{b(r(1-\tau_{i})-\mu)P}\right)^{\frac{1}{1-\alpha}}$$
(59)

For the simulations, I first compute the capital [59] and then find the actual amount of the capital by requiring the adjustments in the capital to be positive:

$$K_{t+1} = K_t + \max(0, K^*(X_t) - K_t)$$
(60)

K(X) is not, in general, differentiable function.

Appendix D: Case with Fixed Costs

Proof. Proposition 2:

The proof builds on the induction argument. Assume first that the strategy is followed from period N, that is $K_{n+1} = \delta K_n$ and $X_{n+1} = \gamma X_n$ for $\forall n \geq N$, and γ and δ are solutions to equity maximization problem. The assumption that parameters γ and δ remain the same from T_{N+1} makes no difference if N is large enough, $N \gg 1$. At date T_N , the firm chooses optimal capital by maximizing the value of issued claims minus the investment costs:

$$K_{N} = \arg \max \left(S(X_{N}, K_{N}) + \frac{X_{N}K_{N}^{\alpha}}{r} - PK_{N} \right)$$

$$= \arg \max \left(\begin{array}{c} \frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - PK_{N} + \\ \gamma^{-b} \left(\frac{X_{N+1}K_{N+1}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \frac{X_{N+1}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - PK_{N+1} \right) \\ + \gamma^{-2b} \left(\frac{X_{N+2}K_{N+2}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \frac{X_{N+2}K_{N+1}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - PK_{N+2} \right) + \dots \right) = \\ = \arg \max \left(\begin{array}{c} \frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \frac{X_{N+2}K_{N+1}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - PK_{N} + \\ \gamma^{-b} \left(\gamma\delta^{\alpha} \frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \gamma\frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \delta PK_{N} \right) \\ + \gamma^{-2b} \left(\gamma^{2}\delta^{2\alpha} \frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \gamma^{2}\delta^{\alpha} \frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} - \delta^{2}PK_{N} \right) + \dots \right) = \\ = \arg \max \left(\frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{r(1-\tau_{i})-\mu} R(\gamma,\delta) - PK_{N}C(\gamma,\delta) \right)$$

Here I used conjectures about the continuation strategy at n > N. $R(X_N, \gamma, \delta)$ and $C(X_N, \gamma, \delta)$ are Revenue and Cost constants independent of K_N , in particular:

$$R(\gamma, \delta) = 1 + \gamma^{1-b} \left(\delta^{\alpha} - 1\right) + \gamma^{2-2b} \delta^{\alpha} \left(\delta^{\alpha} - 1\right) + \dots = 1 + \frac{(\delta^{\alpha} - 1)\gamma^{1-b}}{1 - \gamma^{1-b}\delta^{\alpha}}$$
(62)
$$C(\gamma, \delta) = 1 + \gamma^{-b} \delta^{2} + \gamma^{-2b} \delta^{4} + \dots = \frac{1}{1 - \gamma^{-b}\delta}$$

Via FOC, optimal capital solves:

$$K_N = \left[\frac{X_N R(\gamma, \delta) \alpha (1 - \tau_i)}{(r(1 - \tau_i) - \mu) PC(\gamma, \delta)}\right]^{\frac{1}{1 - \alpha}}$$
(63)

Similarly, the optimal amount of capital installed in the next period is:

$$K_{N+1} = \left[\frac{X_{N+1}R(\gamma,\delta)\alpha(1-\tau_i)}{(r(1-\tau_i)-\mu)\beta PC(\gamma,\delta)}\right]^{\frac{1}{1-\alpha}}$$
(64)

It immediately follows that $\delta = \gamma^{\frac{1}{1-\alpha}}$ since $X_{N+1} = \gamma X_N$ by assumption. What is left to show is that $X_N = \gamma X_{N-1}$ for $\forall n < N$. Recall that stochastic stopping time X_N maximizes equity claim value at T_{N-1} (after debt has been sold already). Define γ_* through $X_N \equiv \gamma_* X_{N-1}$, and write the maximized expression as:

$$S(X_{N}, K_{N}, \gamma_{*}) = \frac{\mu X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{r(r(1 - \tau_{i}) - \mu)} +$$

$$\gamma_{*}^{-b} \left(\gamma_{*}^{\frac{1}{1 - \alpha}} \frac{X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{r(1 - \tau_{i}) - \mu} - \gamma_{*} \frac{X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{r(1 - \tau_{i}) - \mu} - \gamma_{*}^{\frac{1}{1 - \alpha}} PK_{N} \right) +$$

$$\gamma_{*}^{-b} \gamma_{*}^{-b} \gamma_{*}^{\frac{1}{1 - \alpha}} \left(\gamma_{1 - \alpha}^{\frac{1}{1 - \alpha}} \frac{X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{r(1 - \tau_{i}) - \mu} - \gamma_{*} \frac{X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{r(1 - \tau_{i}) - \mu} - \gamma_{1 - \alpha}^{\frac{1}{1 - \alpha}} PK_{N}^{1} \right) + \dots$$

$$= \frac{X_{N} K_{N}^{\alpha} (1 - \tau_{i})}{(r(1 - \tau_{i}) - \mu)} R^{*}(\gamma, \delta) - PK_{N}^{1} C^{*}(\gamma, \delta)$$
(65)

with notation:

$$R^*(\gamma,\delta) = \frac{\mu}{r} + \gamma_*^{-b} \left(\gamma_*^{\frac{1}{1-\alpha}} - \gamma_* \right) + (\gamma_*\gamma)^{-b} \gamma_*^{\frac{1}{1-\alpha}} \left(\gamma_*^{\frac{1}{1-\alpha}} - \gamma \right) + \dots$$
(66)

and

$$C^*(\gamma,\delta) = \gamma_*^{-b} \left(\gamma_*^{\frac{1}{1-\alpha}}\right) + (\gamma_*\gamma)^{-b} \gamma_*^{\frac{1}{1-\alpha}} \left(\gamma_*^{\frac{1}{1-\alpha}}\right) + \dots$$
(67)

Similarly, for the next period:

$$S(X_{N}, K_{N}, \gamma_{*}) = \frac{X_{N+1}K_{N+1}^{\alpha}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)}R^{*}(\gamma, \delta) - PK_{N+1}C^{*}(\gamma, \delta) = (68)$$
$$= \gamma_{**}^{\frac{1}{1-\alpha}} \left[\frac{X_{N}K_{N}^{\alpha}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)}R^{*}(\gamma, \delta) - PK_{N}C^{*}(\gamma, \delta)\right]$$

Where γ_{**} is a result of maximization from the previous round and is treated as fixed. The last expression produces identical maximization problem, which is what we wanted to show. I conclude that $\gamma_* = \gamma$ for all periods before N, that is, $X_{n+1} = \gamma X_n$ for $\forall n \in \{1, 2, ...\infty\}$.

Proof. Proposition 3.

The optimal capital K_0 is found by maximizing the initial value of the firm (equity plus debt) minus costs of installing the capital. It is easy to see that this is equivalent to maximizing the value of assets in place net of the investment costs, ignoring the future growth options value

$$K_0 = \arg_{K_0} \max\left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right) = \arg\max\left(V(X_0) - PK_0\right)$$
(69)

The present value of the payout is given by (see Appendix A):

$$V(X_0) = \frac{X_0 K_0^{\alpha} (1 - \tau_i)}{(r(1 - \tau_i) - \mu)} \left(1 - \gamma^{1-b}\right)$$
(70)

Applying the F.O.C. to the expression above, the optimal capital that is installed at data zero is:

$$K_0 = \left(\frac{(1-\tau_i)X_0\alpha}{P(r(1-\tau_i)-\mu)}(1-\gamma^{1-b})\right)^{\frac{1}{1-\alpha}}$$
(71)

The initial investment in the perpetual investment case is smaller by the factor $(1 - \gamma^{1-b}) < 1$ than the initial investment in the single option case. Intuitively, the firm now has an opportunity to increase the capital in the future as needed.

It follows from the proof of Scaling theorem that equity value in the next round is related to the corresponding value in the previous round as: $S(X_1) = \gamma^{\frac{1}{1-\alpha}} S(X_0)$. This gives a value of initial equity as a solution to the recursive equation; however, the same solution is obtained by direct method, such as evaluation of the infinite series used in proof of *Prop.* 1. Equity $S(X_0)$ is a value of all dividends to shareholders on before X reaches X_1 plus discounted new equity and special dividend (the latter is positive if the proceeds from debt sale exceed the investment needs, and negative otherwise) :

$$S(X_0) = (\text{dividend stream before } X_1) + G_u(X)(\gamma^{\frac{1}{1-\alpha}}S(X_0) + \text{special dividend})$$
(72)

Denoted $G_u(X)$ is the value of the claim at X that pays \$1 when the shock reaches the new level $X_* = \gamma X_0$. I have shown in Appendix A that $G_u(X_0) = \gamma^{-b}$. Then the present value of dividends received before process X_t hits X_1 is :

(dividends before
$$X_1$$
) = $\frac{\mu X_0 K_0^{\alpha}}{r(r(1-\tau_i)-\mu)} \left(1-\gamma^{1-b}\right)$ (73)

The "special dividend" is paid when the proceeds from new debt issued at expansion exceeds the cost:

special dividend=
$$\left(\gamma^{\frac{1}{1-\alpha}} - \gamma\right) \frac{X_0 K_0^{\alpha}}{r} - P K_0$$
 (74)

When put together, this produces the recursive equation for $S(X_0)$:

$$S(X_{0}) = \left(\frac{\mu X_{0} K_{0}^{\alpha}}{r(r(1-\tau_{i})-\mu)} - G_{u}(X_{0})\gamma \frac{X_{0} K_{0}^{\alpha}(1-\tau_{i})}{(r(1-\tau_{i})-\mu)}\right) + G_{u}(X_{0}) \left(\gamma \frac{1}{1-\alpha} S(X_{0}) + (\gamma \frac{1}{1-\alpha}-\gamma) \frac{X_{0} K_{0}^{\alpha}}{r} - PK_{0}\right)$$
(75)

and solving yields:

$$S(X_0) = \frac{\frac{\mu X_0 K_0^{\alpha}}{r(r(1-\tau_i)-\mu)} + \gamma^{-b} \left[\gamma^{\frac{1}{1-\alpha}} \frac{X_0 K_0^{\alpha}}{r} - \gamma \frac{X_0 K_0^{\alpha}}{(r(1-\tau_i)-\mu)} (1-\tau_i) - \gamma^{\frac{1}{1-\alpha}} P K_0 \right]}{1 - \gamma^{\frac{1}{1-\alpha}-b}}$$
(76)

The scaling parameter γ is determined by maximizing the equity claim once the debt is already in place. The usual method of finding γ involves writing the value matching and smooth pasting conditions at the optimal stopping time. However, the direct maximization is equivalent in this case because the firm is engaged in first-best maximization. Rewrite $S(X_0, \gamma)$:

$$S(X_0) = X_0 K_0^{\alpha} \left(\begin{array}{c} \frac{\mu}{r(r(1-\tau_i)-\mu)} + \frac{\gamma^{\frac{1}{1-\alpha}-b}}{r} - \\ \frac{\gamma^{1-b}}{(r(1-\tau_i)-\mu)} (1-\tau_i) - \gamma^{\frac{1}{1-\alpha}-b} \frac{PK_0^{1-\alpha}}{X_0} \end{array} \right) \left(1 - \gamma^{\frac{1}{1-\alpha}-b} \right)^{-1}$$
(77)

Substitute the optimal capital expression:

$$K_0 = \left(\frac{(1-\tau_i)X_0\alpha}{P(r(1-\tau_i)-\mu)}(1-\gamma^{1-b})\right)^{\frac{1}{1-\alpha}}$$
(78)

then:

$$\gamma = \arg\max\frac{\left(\mu + \gamma^{\frac{1}{1-\alpha}-b}(r(1-\tau_i)-\mu) - \gamma^{1-b}(1-\tau_i)r - \gamma^{\frac{1}{1-\alpha}-b}\frac{(1-\tau_i)\alpha r}{1}(1-\gamma^{1-b})\right)}{\left(1-\gamma^{\frac{1}{1-\alpha}-b}\right)(1-\gamma^{1-b})^{-\frac{\alpha}{1-\alpha}}}$$
(79)

Proof. Corollary 1.

The initial value of the equity $S(X_0)$ is derived as a part of the proof for the previous proposition. Generalized expression for the equity value at any point X after N increases in capital can be found from the following observations. After N adjustments, the amount of capital is $K_0 \gamma^{\frac{N}{1-\alpha}}$. The capital will be increased next time the firm invests at $\gamma^{N+1}X_0$ to new level of $K_0 \gamma^{\frac{N+1}{1-\alpha}}$. Values of equity and debt at this point can be written as $\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1}$ multiples of the initial claim value.

$$S(X,N) = \left[\frac{\mu X (K_0 \gamma^{\frac{N}{1-\alpha}})^{\alpha}}{r(r(1-\tau_i)-\mu)} - \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \frac{\left(\gamma^{N+1} X_0\right) (K_0 \gamma^{\frac{N}{1-\alpha}})^{\alpha} (1-\tau_i)}{(r(1-\tau_i)-\mu)}\right] \otimes (1-\tau_i) = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{Y_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{Y_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\frac{\gamma^{N+1} X_0}{T}\right)^{-b} \left(S(X_0) + \frac{Y_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{1}{1-\alpha}}\right)^{N+1} \left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{-b} \left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{-b} \left(S(X_0) + \frac{Y_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{N+1} \left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{-b} \left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{-b} \left(S(X_0) + \frac{Y_0 K_0^{\alpha}}{r} - PK_0\right)\right] = \left[\left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{N+1} \left(\gamma^{\frac{N+1}{1-\alpha}}\right)^{-b} \left(\gamma^{\frac{N+1}{1-\alpha}}\right$$

[assets in place] + [value of unexercised options]

That is, simplified:

$$S(X,N) = \frac{\mu X (K_0 \gamma^{\frac{1}{1-\alpha}})^{\alpha}}{r(r(1-\tau_i)-\mu)} +$$

$$\left(\frac{\gamma^{N+1} X_0}{X}\right)^{-b} \left[\gamma^{\frac{N+1}{1-\alpha}} \left(S(X_0) + \frac{X_0 K_0^{\alpha}}{r} - PK_0\right) - \frac{\left(\gamma^{N+1} X_0\right) \left(K_0 \gamma^{\frac{N}{1-\alpha}}\right)^{\alpha} (1-\tau_i)}{(r(1-\tau_i)-\mu)}\right] =$$
(81)

Value of debt after N expansions is:

$$D(X,N) = \frac{X(K_0 \gamma^{\frac{N}{1-\alpha}})^{\alpha}}{r}$$
(82)

The expressions above for debt and equity value are used in the simulations (case with fixed costs only) \blacksquare

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Notational Key

- X =product demand shock
- K =installed capital
- $\alpha =$ elasticity of demand
- $\mu = \text{drift of the demand shock}$

 $\sigma_m = \text{market volatility}$

- σ_i = demand shock idiosyncratic volatility
- σ = demand shock total volatility
- $\beta =$ market sensitivity "beta"
- $\tau_c = \text{corporate tax rate}$
- $\tau_i =$ tax rate on interest at individual level
- r =pre-tax rate of return on risk free asset
- ϕ = flotation cost as percentage of bond value
- P =proportional investment cost
- F =fixed cost parameter
- I =interest payment
- V(X) = value of assets in place
- D(X) =value of debt
- S(X) = value of equity
- A(X) =total value of the firm
- GO(X) = value of the growth options
- A(X) =total value of assets
- x(K) = investment barrier
- b = constant, solution to quadratic equation
- B = valuation constant determined by the boundary conditions
- $\gamma = \text{scaling constant}$
- $\pi = \text{profitability}$

Table 1 Classification of assumptions of the selected research on dynamic capital structure and investment	$he\ contingent\ claims\ framework.$ The table highlights differences in assumptions and features employed in each model. " Y/N "	ns"Yes/No." In general, "Yes" enriches the model by relaxing some of the limitations. "R" means "restricted" explicitly by author(s).	und/or P" refers to the presence of the fixed and/or proportional costs. In some cases, classification is not available due to other	mptions; this is denoted by " n/a ". For the papers that feature endogenous investment, the table specifies whether it is reversible and	t costs assumptions are made. Tax rate assumptions are sorted into four categories: corporate τ_c^+ , convex corporate τ_c^- , individual	and dividend τ_d (only one paper allows for endogenous distribution and assumes dividend tax). For the recapitalization costs, "N"	ns that zero leverage adjustment costs are assumed. The last column specifies whether the solution is in the closed form.	
Table 1 Class	in the contingen	means"Yes/No." In	"F and/or P" refer	assumptions; this is	what costs assumpt	$ au_i$, and dividend $ au_i$	means that zero lev	

Features:		Investm	lent		\mathbf{T}_{a}	x		\mathbf{Recaps}	Dividend	Solution
Papers	endog.	revers.	cost struct.	τ_c^+	τ_c^-	τ_i	τ_d	cost struct.	endog.	analytic
$Brennan\ and\ Schwartz\ (1984)$	Υ	Υ	F&P	Y	Z	Ζ	Ζ	Ν	Ν	N
Mauer and Triantis (1994)	Υ	Υ	Ч	Υ	Υ	Ζ	Z	F&P	Ν	Ν
$Titman \ and \ Tsyplakov, \ (2002)$	Υ	N	Р	Y	Y	Z	Z	F&P	Ν	Ν
Hennessy and $Whited~(2005)$	Υ	Υ	Р	Y	Y	Х	Y	Р	Υ	Ν
Tserlukevich~(2005)	Υ	Ν	F&P	Υ	Υ	Υ	Ζ	Ν	Ν	Υ
Kane, Markus, McDonald (1984)	Z	n/a	n/a	Υ	Z	Υ	Ζ	R	Ν	Ν
Fischer, Heinkel, Zechner (1989)	Z	n/a	n/a	Υ	Ζ	Υ	Ζ	Р	Ν	Υ
Goldstein, Ju, Leland (2001)	Z	n/a	n/a	Υ	Ζ	Υ	Ζ	Ь	Ν	Υ
Strebulaev (2005) (GJL)	Z	n/a	n/a	Υ	Ч	Υ	Ζ	Ь	Ν	Υ
Leary and Roberts (2005) (FHZ)	Z	n/a	n/a	Х	Z	Υ	Z	Р	Ν	Υ

Table 2: Descriptive Statistics for the Generated Samples. The statistics are calculated from 1000 independent simulations. For each simulation, the values are computed in cross section and averaged over time. The leverage ratio data includes mean, standard deviation, and 1%, 25%, 50%, 75%, 99% percentiles for the average mean and maximum leverage ratios in the cross section. Investment statistics include the number of quarters with investment (Quarters) and average investment (Investment), calculated as the total investment over lifetime divided by the number of quarters where investment took place. For the zero cost model all quarters with non-zero intermittent investment are included. Panel A gives results for the case without fixed costs, and Panel B offers corresponding results for the case with fixed cost.

	percentiles				
Mean	1%	25%	50%	75%	99%

Panel A - Zero Fixed Costs Model

Average Leverage Ratio	0.453	0.242	0.382	0.451	0.524	0.676
Minimum Leverage Ratio	0.002	0	0.001	0.001	0.003	0.017
Maximum Leverage Ratio	0.775	0.728	0.767	0.778	0.786	0.796
Investment / Quarter	0.703	0	0.054	0.144	0.421	9.435
Quarters w. Investment	24.28	0	6	19.5	39	81.5

Panel B - Fixed Costs Model

Average Leverage Ratio	0.730	0.673	0.716	0.734	0.748	0.754
Minimum Leverage Ratio	0.032	0.003	0.012	0.027	0.046	0.113
Maximum Leverage Ratio	0.777	0.776	0.777	0.777	0.777	0.777
Investment / Quarter	0.584	0	0.091	0.258	0.606	4.927
Quarters w. Investment	0.58	0	1	2	7	12

Table 3: Leverage Cross-Sectional Regression. LR is a leverage of the firm defined as as a ratio of the book value of debt to the sum of book value of debt and market value of equity. Profitability π_t is defined as the increase in assets adjusted by the book value of assets in place. Independent variables are profitability (π), volatility of cash flows (σ), and demand elasticity (α). T-statistics for each simulations are calculated using the Fama-MacBeth method as a mean (across years) of regression intercepts and slopes adjusted by the standard error (time series standard deviation of the regression coefficient divided by \sqrt{T}). Coefficients and t-statistics are means over 1000 independent simulations. Other columns display standard deviations for these values as well as 1%, 50% and 99% percentiles.

	summ	ary	percentiles			
	mean coeff	std dev	1%	50%	99%	
Panel A - Zero	o Fixed Costs	Model				
Constant	0.691	0.166	0.317	0.696	1.02	
	(140)	(66.7)	(49.2)	(123)	(358)	
D C 1. '1' +	-0.591	0.256	-1.417	-0.550	-0.199	
1 Iontability /	(-8.70)	(2.79)	(-16.2)	(-8.4)	(-3.80)	
Volatility of	0.601	0.324	-0.242	0.659	1.100	
volatility o	(48.9)	(27.9)	(-33.0)	(52.0)	(95.8)	
Flagtigity of	-2.342	0.277	-2.90	-2.36	-1.55	
Elasticity α	(-228)	(107)	(-511)	(-213)	(-51.4)	

Panel B - Fixed Costs Model

Constant	0.977	0.023	0.925	0.976	1.03
Constant	(357)	(72.1)	(249.3)	(343)	(584)
Duofitability of	-0.127	0.033	-0.215	-0.125	-0.063
i iontability <i>x</i>	(-13.3)	(3.0)	(-20.0)	(-13.3)	(-7.1)
Volatility a	-0.565	0.060	-0.709	-0.566	-0.435
volatility 0	(-59.3)	(11.0)	(-95.4)	(-57.0)	(-40.2)
Flagtigity of	-0.747	0.146	-1.11	-0.732	-0.443
Elasticity α	(-99.0)	(48.1)	(-264)	(-84.2)	(-41.8)

Table 4: Mean Reversion Tests. Leverage change is defined as a difference between leverage ratios at time t + k and t, $LR^{t+k} - LR^t$. Independent variables are: past leverage ratio (LR^t) , mean leverage ratio (\overline{LR}) , volatility of the cash flows (σ) , and elasticity of demand (α) . The base case is for k = 10 (two and a half years); the last two columns display the results for alternatives: k = 5 and k = 20. The tstatistics for simulations are calculated using the Fama-MacBeth method. Coefficients and t-statistics are means over 1000 independent simulations.

	summary	k = 10	alternative horizons	
	mean coeff	std dev	k = 5	k = 20
Panel A - Zero Fixed Cost	ts Model			
Constant	-0.005	0.004	-0.003	-0.012
	(-3.31)	(2.41)	(-2.39)	(-5.69)
Dest I serve as Dette I Dt	-0.181	0.010	-0.090	-0.343
rast Leverage Ratio LR	(-22.8)	(2.14)	(20.7)	(-22.9)
Tangat Lawana ga Datio ID	0.188	0.010	0.094	0.350
Target Leverage Ratio LR	(22.0)	(2.49)	(2.05)	(22.9)
Volotilitar a	0.005	0.007	0.003	0.016
volatility o	(1.97)	(2.38)	(1.76)	(4.33)
Flagticity o	0.028	0.007	0.013	0.070
Elasticity α	(8.04)	(2.62)	(4.61)	(16.4)

Panel B - Fixed Costs Model

Constant	-0.006	0.002	0.0001	-0.013
Constant	(-4.54)	(1.98)	(0.346)	(-8.08)
Past Loverage Patio LDt	-0.160	0.013	-0.070	-0.290
I ast Leverage Ratio LR	(-30.8)	(3.73)	(-24.2)	(-27.8)
Target Leverage Patie IP	0.163	0.015	0.068	0.300
Target Leverage Ratio LR	(28.1)	(3.88)	(21.2)	(26.1)
Volatility o	0.012	0.002	0.004	0.020
Volatility 0	(12.8)	(3.52)	(12.1)	(16.8)
Flasticity o	0.009	0.004	0.005	0.007
Elasticity α	(4.80)	(2.21)	(5.75)	(3.34)



(A) Market Leverage Ratio as a Function of Demand Shock



(C) Debt Value as a Function of Demand Shock



(B) Ex-Dividend Equity Value as a Function of Demand Shock



(D) Firm Value as a Function of Demand Shock

Figure 1: Firm with Single Irreversible Investment Option. All graphs are plotted against the value of demand shock. The critical value of the shock X_* that triggers investment corresponds, approximately, to the middle of the horizontal scale. The example assumes fixed investment costs (F = 1) and also adopts a requirement that a firm has a single option. Parameters used in this example: $\sigma = .2$, I = 10, $\alpha = 0.2$, K = 1. Each of the exhibits plots the value of one of the contingent claims: (A) Market Leverage Ratio, (B) Ex-dividend value of Equity, (C) Debt Value, and (D) Total Firm Value (Debt plus Equity).



Figure 2: Incremental Investment Without Fixed Costs. All graphs are plotted against time and based on one randomly selected economy simulation for a randomly selected firm. This case is for zero fixed costs (F = 0). Figures generated for a sample path of the demand shock are shown in Exhibit A. "Desired Capital Stock" on Exhibit B is found as the optimal amount of irreversible investment corresponding to the current value of the shock. Exhibit C shows the actual amount of capital stock after irreversibility constraint has been applied. (See [60]). Exhibit D is the market leverage ratio corresponding to this path.



Figure 3: Irreversible Investment With Fixed Costs. All graphs are plotted against time and based on one randomly selected economy simulation for a randomly selected firm. The simulation assumes maximium fixed costs (F = 1). Figures are generated for a sample path of the demand shock that is shown on Exhibit A. The capital stock dynamics corresponding to this sample path is depicted on Exhibit B. Firm invests (in this particular simulation) at quarters 91, 110, 113 and 137, visible as jumps in the amount of capital. Exhibit C is the market value of Debt corresponding to this path. Exhibit D is the Market Leverage Ratio corresponding to this path.



(B) Investment with Fixed costs

(A) Investment with Zero Fixed Costs

Figure 4: Frequency Distribution of the Coefficients for Profitability and Mean-Reversion Regressions. The coefficients are obtained from 1000 independent observations by running cross-sectional regressions and averaging the result over time. Panel A (left side of the exhibit) is for zero fixed costs case (F = 0); Panel B (right side) is for the fixed costs case (F = 1). The upper graphs give the distribution of the profitability coefficient β_1 over simulations. The lower graphs give the distribution of the mean reversion coefficients.