Endogenous Gentrification and Housing Price Dynamics

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Abstract

In this paper, we explore differential changes in house prices across neighborhoods within a city to better understand the nature of house price dynamics across cities. We do so by proposing a previously unexplored mechanism where individual utility is increasing in the income of one's neighbors. Instead of proximity to jobs, it is proximity to "richer" people that drive differences in land prices within and across cities. In the model, segregation by income occurs: richer households are concentrated together with poorer households living in the periphery. In response to a positive increase in housing demand (e.g. a decrease in the interest rate, an increase in city-wide income, an influx of richer households), richer households expand into adjacent poorer neighborhoods. This is what we term "endogenous gentrification". As the richer households expand into poorer neighborhoods, the land value increases due to the externality, driving house prices up. The model also predicts that the city-wide responsiveness of house prices to a given demand shock will depend on the income distribution within the city. As a result, richer cities are predicted to respond more to the housing demand shock, even if housing supply is perfectly elastic, because they experience a higher degree of gentrification. Using a variety of different data sources, we show that the data are consistent with many predictions of our model. In particular, we find that those neighborhoods whose house values increase the most during city wide housing booms are the poor neighborhoods that are in close proximity to the richer neighborhoods. This pattern is robust to controlling for distance to jobs. Additionally, we find that these neighborhoods that experience the highest price increases also show strong evidence of gentrification (large increases in income, large reductions in the poverty rate, and an influx of new residents). We formally assess the mechanism of the model by showing that house prices increase substantially in a poorer neighborhood when the surrounding neighborhoods receive a positive shock to income. Lastly, we assess how much of cross city differences in price appreciation rates during the 1990s and the 2000s can be explained by our mechanism.

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1 Introduction

According to the Case-Shiller index, real residential property prices appreciated by over 80 percent for the United States as a whole between the late 1990s and the mid 2000s. Since that time, they have dropped by roughly 40 percent nationwide. However, the national data mask a large degree of heterogeneity at more localized levels. For example, real property prices increased by over 100 percent in Washington DC, Miami, and Los Angeles between 2000 and 2006 while property prices appreciated by less than 10 percent in Charlotte, Denver, and Detroit during the same time period. Similarly, during the 1990s, real house prices in Denver and Portland increased by 40 percent while house prices in Chicago remained roughly constant.

Two prominent classes of models have been put forth to explain cross city differences in house prices. First, cities differ with respect to their long run housing supply elasticities.[1] These differences can be driven by differences in regulation that restrict building (Glaeser, Gyourko, and Saks (2005b) and Glaeser, Gyourko, and Saks (2005a)) or in natural geographical features (Saiz (2009)). With differences in long run housing supply elasticities, a given housing demand shock (such as rising incomes or low interest rates) can have persistent differential effects on housing prices within a given location because housing supply is restricted from fully adjusting. Second, cities can experience a shock to the productivity of the firms within in the city. The local productivity shock will result in the in-migration of workers from other areas. Given that space around the firms in the city is limited, the inflow of workers from other areas bids up land prices in the city causing housing prices to rise. Such a model has been explored by many including in the seminal works of Rosen (1979) and Roback (1982) and recent work by Van Nieuwerburgh and Weill (2009) and Moretti (2009).[2] A key implication of these latter models is that differences in commuting times within a city will lead to differences in house prices within the city. Locations farther away from the firms will be cheaper than locations closer to the firm.[3]

Our goals in this paper are fourfold. First, we present a model that emphasizes a very different mechanism that can simultaneously help to explain both within city and cross city differences in housing price dynamics. Instead of focusing on the fact that differences in land

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[1] For example, this is the approach taken by Glaeser, Gyourko, and Saks (2005b) and Glaeser, Gyourko, and Saks (2005a), Glaeser and Gyourko (2007), and Saiz (2009). In these papers, regulatory constraints and land gradients are posited to be the drivers of differences in housing supply elasticities across areas. In his paper, Saiz goes further and explores the political economy of land use regulations.

[2] For a survey of some of these papers, see Glaeser (2007).

[3] See the classic works by Alonso (1964), Mills (1967), and Muth (1969) for a discussion of how the relationship between the location of jobs and the location of workers can determine differences in land prices within a metropolitan area.

prices across locations are being driven by differences in medium-to-long run supply elasticities or by the proximity to desirable jobs, we present a spatial equilibrium model where individual utility is increasing in the income of one's neighbors. Instead of proximity to jobs, it is proximity to "richer" people that drive differences in land prices within and across cities. Although, we do not explicitly model the direct mechanism for the externality, we have many potential channels in mind. For example, crime rates are lower in rich neighborhoods. If households value low crime, they will prefer to live amongst wealthier neighbors as opposed to poorer neighborhoods. Likewise, the quality and extent of public goods may be correlated with the income of neighborhood residences. For example, school quality - via peer effects, parental monitoring or direct expenditures - tends to increase with neighborhood income. Additionally, if there are increasing returns to scale in the production of desired neighborhood amenities (number and variety of restaurants, easier access to service industries such as dry cleaners, movie theaters, etc.) such amenities will be more common as the richness of one's neighbors increase. Although we do not take an official stand on which mechanism is driving the externality, our preference structure is general enough to allow for any story that makes higher income neighborhoods more desirable. The importance of urban density in facilitating local consumption externalities has been recently emphasized in the work of Glaeser, Kolko, and Saiz (2001) and in Becker and Murphy (2003). Our innovation is to embed these consumption externalities into a model of neighborhood development within a city and show how such preferences affect the reaction of house prices to housing demand shocks both across various types of neighborhoods and at the aggregate level. As far as we know, we are the first paper to theoretically explore within city and cross city housing price dynamics in a world where there are residential externalities generated by the richness of one's neighbors.

We show that under relatively general conditions, our model generates an equilibrium where households are segregated based on their incomes. Given the externality, households are willing to pay more to live closer to richer neighbors. Poorer households who benefit less from the externality are less willing to pay high rents to live in the richer neighborhoods, so in equilibrium they live farther from the rich. House prices achieve their maximum in the richer neighborhoods and decline as we move away from them, to compensate for the lower level of the externality. At the margin of the city (i.e., far from the rich), there is no externality and house prices are equal to the marginal cost of construction.

Our model predicts that permanent shocks to housing demand lead to permanent increases in house prices within the city as a whole – even when the size of the city is completely elastic.

In other words, our model generates cross city differences in house prices and house price appreciation rates that do not rely on traditional supply constraints (regulation that prevents building, the steepness of the land gradient, natural barriers, etc.). The main implication of this simple model is that a housing demand shock (including an influx of richer residents) will cause richer households to expand into areas previously occupied by the poor. We refer to this phenomenon as gentrification. As this happens, house prices in the richest neighborhoods are not affected, while house prices in gentrified neighborhoods are driven up due to the neighborhood externality. Furthermore, our model shows that aggregate (city wide) house prices can respond differently in different cities even if supply constraints are not binding. We show that average price growth is affected both by the size of the demand shock and by the particular shape of preferences, technology, and the income distribution within the city. For example, given a large housing demand shock (such as an interest rate decline or a proportional shock to income), a city with a larger fraction of richer households will experience a larger average price response and more gentrification than a city with a smaller fraction of richer households. Moreover, we show that shocks to income of residents of a city will lead to increased house prices within the city even if there were no changes in commuting times within the city.

Our second goal of this paper is to document a series of new stylized facts about the movement of house prices across neighborhoods within a city during city-wide housing price booms and busts. As far as we can tell, no one has ever systematically explored the movement of house prices across neighborhoods within a city for a large cross section of cities during many different time periods. Regardless of the empirical strength of the mechanism at the heart of our model, we think that any model of housing price dynamics designed to explain cross city house price dynamics should also be able to explain the within city house price dynamics that we have documented.

Using a variety of different data sources including the zip code level data that underlies the Case-Shiller indices, U.S. Census data, and deed data that we have compiled for certain cities, we find that three stable relationships emerge. First, we find that initially low priced neighborhoods systematically experience higher appreciation (depreciation) rates during city-wide housing booms (busts) than do initially higher priced neighborhoods. In other words, our results show that it is low housing price neighborhoods - not high housing price neighborhoods - that are the most price elastic during city-wide property price booms and busts. We also find that the extent of the difference in appreciation (depreciation) rates between low and high price neighborhoods within the city is greater the larger the price appreciation (depreciation) rate

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for the city as a whole. Second, while low property price neighborhoods are likely to appreciate (depreciate) more on average, there is a large degree of heterogeneity in the price responsiveness during property price booms (busts) among the low price neighborhoods. Specifically, we show that it is the low price neighborhoods that are in close proximity to the high price neighborhoods that are the most price elastic. Lastly, we show that these neighborhoods that experienced high growth rate in property prices - particularly during the 1980s and 1990s - showed signs of gentrification. In particular, these initially low priced neighborhoods that appreciated the most experienced large increases in median income, substantial declines in the poverty rate, and an influx of new residents. We show that such results held during the recent property price boom as well as local property price booms that occurred during the 1980s and the local property price booms and busts that occurred during the 1990s.

Our third goal in this paper is to use the within city house price data (discussed above) to more formally evaluate the existence of the consumption externality at the core of our model. We ask the question of how shocks to income in a given neighborhood affect property prices and gentrification in the surrounding neighborhoods, all else equal. To do this, we follow others in the literature by using the initial industry mix of residents in the neighborhood to predict how incomes in a given neighborhood should have evolved going forward. Using data from the U.S. Census IPUMS data, we calculate the percentage change in income at the national level for all workers in two digit SIC industries between 1990 and 2000. We then use the industry shares of working residents in each zip code or census tract in 1990 to predict how income should have evolved in those zip codes between 1990 and 2000. Our predicted income growth measure at the neighborhood level does in fact correlate strongly with actual income growth at the neighborhood level over the same time period.

We show that house prices in a given neighborhood respond strongly to shocks to income of nearby neighborhoods, even after controlling for the expected income shock in their own neighborhood. Income shocks experienced by far away neighbors have no effect on house prices in a given neighborhood. It is only the income shocks of close neighbors that affect housing prices. Moreover, we show that neighborhoods whose neighbors experienced big income shocks actually gentrified. Not only did their house prices increase, the neighborhoods in close proximity to those experiencing income shocks also experienced actual income increases, declines in the poverty rate, and an increase of new residents relative to other zip codes in the city. Finally, we show that initial differences in commuting times or changes in commuting time between 1990 and 2000 (using the U.S. Census data) do not explain the patterns in the

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data. In other words, it is not that proximity to jobs or the mobility of jobs are explaining the patterns of changing house prices that we document.

Our fourth goal in this paper is to ask how much of the cross city variation in house price appreciation experienced between 1990 and 2000 and between 2000 and 2009 can be explained by changing incomes within the city and how much can be explained by traditional measures of supply constraints or by changes in commuting times. We find that changes in commuting time explain little of the cross city variation in house price growth between either period. We also find that changes in income broadly explain much of the variation in house price growth across cities during the 1990s and little of the variation in house price growth across cities during the 2000s. Conversely, traditional measures of supply constraints (regulation, differences in topography) explain essentially none of the variation in house prices across cities in the 1990s but much of the variation in house prices across cities in the 2000s. The results suggest that during the 1990s, income shocks were an important driver of cross city differences in prices even in places where supply constraints were non-binding and commuting times remained constant.

These results, coupled with the within city results discussed above, provide strong evidence that neighborhood externalities are quantitatively important for explaining cross city differences in prices during certain time periods. The results also suggest that our mechanism was active during the 2000 period, but this mechanism was small relative to other shocks that are more likely to result in price increases when supply constraints are limited (like an extension of credit to low income households as discussed in Mian and Sufi (2009)).

Before continuing, it might be useful to address why one would care whether changes in house prices were driven by the mechanism highlighted in our paper or one of the other mechanisms traditionally used to explain within city and cross city differences in house prices. First, in terms of making welfare inferences associated with the run up in wages and the associated run up in house prices, it matters greatly why house prices are increasing. In our mechanism, shocks to local productivity which increases wages also increases house prices. But, the increase in house prices is driven by the fact that the flow utility from living on a piece of land has increased when that land is populated by richer households. Such a mechanism will change the welfare calculations of local productivity booms as done by researchers such as Moretti (2009). Additionally, the existence of the mechanism documented in this paper definitely complicates the analysis when trying to infer the consumption response to local changes in house prices. Some researchers have examined whether changes in house prices lead to increases in consumption using local variation in house prices (see Campbell and Cocco (2007) and Case

and Quigley). The estimation of consumption effects in a world where there are externalities from living around richer neighbors complicates the estimation of the response of non-housing consumption to changes in housing prices. Lastly, in terms of local developers or city planners, the existence of neighborhood externalities expands the set of policies they may choose to adopt when pursuing urban development plans.

Finally, our paper contributes to many literatures. First, our paper speaks to the large literature on the gentrification of urban areas. Recent work has discussed the role of the following in explaining gentrification: the increased consumption benefits from living in a city (Glaeser, Kolko, and Saiz (2001)), the age of a city's housing stock (Rosenthal (2008) and Brueckner and Rosenthal (2008)) and direct public policy initiatives via community redevelopment programs (Busso and Kline (2007), Rossi-Hansberg, Sartre, and III (2009)).[4] Despite the broad literature on gentrification, very little work emphasizes the importance of spatial dependence - either theoretically or empirically - in predicting the spatial patterns of gentrification. Our work adds to this literature by showing how generic shocks to housing demand within a city can result in the gentrification of neighborhoods.[5] Second, our work is related to spatial segregation resulting from discrimination (see, for example, Schelling (1969) and Card, Mas, and Rothstein (2008)). Given that race and income are correlated, it complicates the identification of preferences for racial sorting from preferences for income sorting. We return to a discussion of this in the conclusion.

In summary, our paper shows that the existence of neighborhood externalities has important implications for the nature of real estate price dynamics across neighborhoods within a city and across cities. We conclude that such externalities need to be embedded in both theoretical and empirical models designed to explain both time series and cross sectional housing price dynamics.

2   Model

In this section, we develop a spacial equilibrium model of housing prices across neighborhoods within a city. The key ingredient of the model is a positive neighborhood externality: people

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[4]For a recent survey of the theoretical and empirical work on gentrification, see Kolko (2007)

[5]There are two notable exceptions. Brueckner (1977) finds that urban neighborhoods in the 1960s that were in close proximity to rich neighborhoods got relatively poorer between 1960 and 1970 (as measured by income growth). Conversely, Kolko (2007) finds the opposite pattern. Kolko finds that 1990 neighborhoods bordering richer neighborhoods grew faster between 1990 and 2000 than 1990 neighborhoods bordering poorer neighborhoods. Our addition to this literature is that we propose a model that explains both of these facts and then formally tests the model's predictions. During periods of declining housing demand in urban areas (like the suburbanization movement during the 1960s), the richer neighborhoods on border of the rich areas will be the first to contract. Conversely, during urban renewals (like what was witnessed during the 1990s), the poor neighborhoods bordering the richer neighborhoods will be the first to gentrify.

like to live next to rich people.

2.1 Baseline model

2.1.1 Set up

Time is discrete and runs forever. We consider a city populated by two types of infinitely-lived households: a continuum of rich households of measure N^R and a continuum of poor households of measure N^P.

The city is represented by the real line and each point on the line $i \in (-\infty, +\infty)$ is a different location. Agents are fully mobile and can choose to live in any location i. Denote by $n_t^s(i)$ the measure of households of type s, for $s = R, P$, who live in location i at time t and by $h_t^s(i)$ the size of the house they choose. In each location, there is a maximum space that can be occupied by houses normalized to 1, that is,

$$n_t^R(i) h_t^R(i) + n_t^P(i) h_t^P(i) \leq 1 \text{ for all } i, t.$$

Moreover, market clearing requires

$$\int_{-\infty}^{+\infty} n_t^s(i) \, di = N^s \text{ for } s = R, P. \tag{1}$$

The key ingredient of the model is that there is a positive location externality: households like to live in areas where more rich households live. Each location i has an associated neighborhood, given by the interval centered at i of radius γ. Let $H_t(i)$ denote the total space occupied by houses of rich households in the neighborhood around location i, that is,

$$H_t(i) = \int_{i-\gamma}^{i+\gamma} h_t^R(j) \, n_t^R(j) \, dj. \tag{2}$$

Households have separable utility in non-durable consumption c and housing services h. The location externality is captured by the fact that households enjoy more to live in locations with higher $H_t(i)$. The utility of an household of type s located in location i at time t is given by

$$u(c) + v^s(h, H_t(i)),$$

where $u(.)$ and $v^s(.)$ are weakly concave functions. For tractability, we assume that u is linear, so that we abstract from wealth effects, and that v^s takes the following functional form: $v^s(h, H) = \phi^s h^\alpha (A + H)^{\beta^s}$, where ϕ^s, α^s, and β^s are non negative scalar. The parameter ϕ^s captures the willingness of the households to pay for housing services and we let $\phi^P < \phi^R$. The fact that $\phi^P < \phi^R$ implies that poor agents have smaller willingness to pay for housing than

rich ones, or, equivalently, that they have a smaller marginal value of money.[6] Moreover, we assume that that $\beta^R \geq \beta^P$, so that the rich households who generate the externality are also the ones who benefit the most from it. Also, each period households of type s receive an exogenous endowment of consumption goods equal to y^s, with $y^P < y^R$.

On the supply side, there is a representative firm who can build housing in any location $i \in (-\infty, +\infty)$. There are two types of housing: rich houses (type R) and poor houses (type P). Each type of household only demands houses of his own type. The marginal cost of building houses of type s is equal to C^s, with $C^R \geq C^P$. If the firm wants to convert houses of type $\tilde{s}$ into houses of type s, he has to pay $C^s - C^{\tilde{s}}$.

The (per square foot) price of a house for household of type s in location i at time t is equal to $p_t^s(i)$. Hence there is going to be construction in any empty location i as long as $p_t^s(i) \geq C^s$. Moreover, if the firm wants to construct a house of type s in a location occupied by a house of type $\tilde{s}$, he has to pay the converting cost and the additional cost of convincing households of type $\tilde{s}$ to leave. Hence, there is going to be construction of houses of type s in any location occupied by agents of type $\tilde{s}$ if $p_t^s(i) \geq C^s - C^{\tilde{s}} + p_t^{\tilde{s}}(i)$.

Finally, there is a continuum of competitive intermediaries who own the houses and rent them to the households. The intermediaries are only introduced for ease of exposition and nothing would change if we allowed the households to own.[7] The (per square foot) rent for a house of type s in location i at time t is denoted by $R_t^s(i)$. As long as the rent in location i at time t is positive, the intermediaries find it optimal to rent all the houses in that location. Also, for simplicity, assume that houses do not depreciate. Competition among intermediaries requires that for each location i the following arbitrage equation holds:

$$p_t^s(i) = R_t^s(i) + \left(\frac{1}{1+r}\right) p_{t+1}^s(i) \text{ for all } t, i, s. \tag{3}$$

2.1.2 Equilibrium

An equilibrium is a sequence of rent and price schedules $\left\{R_t^R(i), R_t^P(i), p_t^R(i), p_t^P(i)\right\}_{i \in R}$ and of allocations $\left\{n_t^R(i), n_t^P(i), h_t^R(i), h_t^P(i)\right\}_{i \in R}$ such that households maximize utility, the representative firm maximizes profits, intermediaries maximize profits, and markets clear.

At each time, households decide their non-durable consumption, in which location to live, and the size of the house they want to live in, taking as given the rental price and the neighborhoods characteristics. Because of full mobility, the household's maximization problem reduces to a series of static problems.

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[6]This would be an endogenous outcome due to $y^P < y^R$ if utility from consumption was strictly concave.
[7]This is thanks to the assumption of linear utility in consumption.

The problem of a household of type s at time t is simply

$$\max_{c,h,i\in\mathcal{I}_t^s} c + \phi^s h^\alpha \left[A + H_t(i)\right]^{\beta_s},$$

$$s.t. \ c + hR_t^s(i) \leq y^s,$$

where $\mathcal{I}_t^s$ is the set of locations where houses for poor households are available and the household takes as given the function $H_t(i)$ and the rent schedule $R_t^R(i)$. Hence, conditional on choosing to live in location i at time t, the optimal house size is

$$h_t^s(i) = \left(\frac{\alpha\phi^s\left(A + H_t(i)\right)^{\beta_s}}{R_t^s(i)}\right)^{\frac{1}{1-\alpha}}. \tag{4}$$

Households choose to live in bigger houses in neighborhoods where the rental price is lower and the externality is stronger. Moreover, conditional on a location, richer households choose bigger houses both because of their higher willingness to pay for them and because they benefit more from the externality. Given that households are fully mobile, it must be that at each point in time, the equilibrium rents in different locations make them indifferent. In particular, agents of type s have to be indifferent among living in different locations where houses of their type are available at time t, that is, in all $i \in \mathcal{I}_t^s$.[8] Then it must be that

$$U^s(i) \equiv y^s - \left(\alpha^{\frac{1}{1-\alpha}} - \alpha^{\frac{\alpha}{1-\alpha}}\right)\left(\frac{\phi^s\left(A + H_t(i)\right)^{\beta_s}}{R_t^s(i)^\alpha}\right)^{\frac{1}{1-\alpha}} = \bar{U}^s \text{ for all } i \in \mathcal{I}_t^s. \tag{5}$$

This, in turns, requires that

$$R_t^s(i) = K^s\left[A + H_t(i)\right]^{\frac{\beta_s}{\alpha}} \text{ for all } i \in \mathcal{I}_t^s, \tag{6}$$

for some constant K^s. This expression is intuitive, as rents must be higher in locations with a stronger externality. Moreover, rich households who are more affected by the location externality, given $\beta^R \geq \beta^P$, are willing to pay higher rents for the same location.

Proposition 1. *If $\beta^R \geq \beta^P$, there exists an equilibrium with full segregation.*

Let us construct an equilibrium with full segregation, where the rich households are concentrated in the city center, while the poor households live at the periphery of the city. As a normalization, let us choose point 0 as the center of the city. It follows that $\mathcal{I}_t^R = [-I_t, I_t]$ and $\mathcal{I}_t^P = [-\bar{I}_t, -I_t,) \cup (I_t, \bar{I}_t]$, for some $\bar{I}_t > I_t > 0$. In this model, both the size of rich neighborhoods, I_t, and the size of the city, $\bar{I}_t$, are equilibrium objects. Given that such an equilibrium is symmetric in i, from now on, we can restrict attention to $i \geq 0$.

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[8]If there was a location with construction of type s and no type s households living there, the intermediaries would be willing to decrease the rent to 0 inducing households of type s to move into that location.

Given that rich households live in locations where there are no poor, it must be that $h_t^R(i)\, n_t^R(i)$ is either equal to 1 or to 0 and is equal to 1 for all $i \in [0, I_t]$. Then, we can easily derive the function $H_t(.)$ as follows:

$$H_t(i) = \begin{cases} 2\gamma & \text{for } i \in [0, I_t - \gamma] \\ \max\{\gamma + I_t - i, 0\} & \text{for } i \in (I_t - \gamma, \bar{I}_t] \end{cases}, \tag{7}$$

so that the neighborhoods close to the city center are fully developed and enjoy the maximum externality degree, while the farther a location is from the center the smaller the strength of the externality. If $\bar{I}_t > I_t + \gamma$, there are going to be location at the margin of the city where the externality has zero effect. From now on, we assume that the measure of poor households, N^P, is sufficiently large so that $\bar{I}_t > I_t + \gamma$. Using (6), we obtain

$$K^R = R_t^R(I_t)(A + \gamma)^{-\frac{\beta_R}{\alpha}} \quad \text{and} \quad K^P = R_t^P(\bar{I}) A^{-\frac{\beta_P}{\alpha}}, \tag{8}$$

so that we can rewrite the rent schedules as

$$R_t^R(i) = R_t^R(I_t)\left(1 + \frac{\min\{\gamma, I_t - i\}}{A + \gamma}\right)^{\frac{\beta_R}{\alpha}} \quad \text{for } i \in [0, I_t], \tag{9}$$

$$R_t^P(i) = R_t^P(\bar{I}_t)\left(1 + \frac{\max\{\gamma + I_t - i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}} \quad \text{for } i \in (I_t, \bar{I}_t]. \tag{10}$$

From the optimizing behavior of the representative firm, it must be that the price of a poor house at the boundary of the city is equal to the marginal cost C^P, while the price of a rich house at the boundary of the rich neighborhoods must be equal to the price of a poor house, which is the compensation needed to vacate poor households living there, plus the additional cost of transforming a poor house in a rich one. This implies that

$$p_t^P(\bar{I}_t) = C^P \quad \text{and} \quad p_t^R(I_t) = p_t^P(I_t) + C^R - C^P.$$

In equilibrium prices are constant over time and hence arbitrage conditions (3) require that for each location $i \in \mathcal{I}_t^s$ prices satisfy

$$p_t^s(i) = \frac{1+r}{r} R_t^s(i) \quad \text{for all } t, i, s. \tag{11}$$

Combining this conditions we obtain

$$R_t^P(\bar{I}_t) = \frac{r}{1+r} C^P \quad \text{and} \quad R_t^R(I_t) = R_t^P(I_t) + \frac{r}{1+r}\left(C^R - C^P\right), \tag{12}$$

where, from (6) and (8), we have

$$R_t^P(I_t) = \frac{r}{1+r} C^P \left(\frac{A + \gamma}{A + \max\{\gamma + I_t - \bar{I}_t, 0\}}\right)^{\frac{\beta_P}{\alpha}}. \tag{13}$$

Combining these last two expressions with (9), (10), and (11) allows us to determine the rent and the price schedules as a function of I_t and $\bar{I}_t$ only.

To complete the characterization of the equilibrium, we need to determine the size of the city, $\bar{I}_t$, and the size of the rich neighborhoods, I_t. Using market clearing (1) together with the optimal housing size (4) and the fact that $\mathcal{I}_t^R = [0, I_t]$ and $\mathcal{I}_t^P = [I_t, \bar{I}_t]$, we obtain the following expressions for I_t and $\bar{I}_t$:

$$I_t = \gamma + (A + 2\gamma)^{-\frac{\beta_R}{\alpha}} \left\{ \left(\frac{\phi^R \alpha}{K_R}\right)^{\frac{1}{1-\alpha}} \frac{N^R}{2} - \frac{\alpha}{\alpha + \beta_R} \left[(A + 2\gamma)^{\frac{\alpha + \beta_R}{\alpha}} - (A + \gamma)^{\frac{\alpha + \beta_R}{\alpha}} \right] \right\} \quad (14)$$

$$\bar{I}_t = I_t + \gamma + A^{-\frac{\beta_P}{\alpha}} \left\{ \left(\frac{\phi^P \alpha}{K_P}\right)^{\frac{1}{1-\alpha}} \frac{N^P}{2} - \frac{\alpha}{\alpha + \beta_P} \left[(A + \gamma)^{\frac{\alpha + \beta_P}{\alpha}} - A^{\frac{\alpha + \beta_P}{\alpha}} \right] \right\}. \quad (15)$$

As intuition suggests, the rich neighborhoods are more developed when there are more rich households N^R and when the marginal cost of construction C^R or the interest rate r are lower. Moreover, the city overall is bigger when the rich neighborhoods are more developed, when there are more poor households, lower N^P, and when the marginal cost of construction C^P or the interest rate are lower.

Finally, to complete the construction of the equilibrium, we have to check that the households choose their location optimally, that is, we have to check that the rich would not prefer to move to a poor neighborhood and vice versa. In particular, we need to prove that

$$U^R(i) \leq \bar{U}^R \text{ for all } i \in \left[I_t, \bar{I}_t\right]$$
$$U^P(i) \leq \bar{U}^P \text{ for all } i \in [0, I_t]$$

where $U^s(i)$ is defined in expression (6). In the Appendix, we show that both these conditions are satisfied if $\beta^R \geq \beta^P$, completing the proof of the Proposition.

In our full segregation equilibrium, the rich households are concentrated in the city center, while the poor are located at the boundary of the city. Moreover, equilibrium prices reflect the fact that locations that are further away from the city center and closer to space occupied by poor households are less appealing. In particular, prices are the highest in the city center where the rich neighborhoods are fully developed and there is the maximum concentration of rich households. As we move away from the center, prices start declining because the space in the neighborhood occupied by rich households goes down. This segregation equilibrium is sustained by the fact that the externality is more beneficial for the rich who are the ones who create the externality itself.

2.1.3 Demand shock

We are now interested in analyzing how house prices, both at an aggregate and at a disaggregate level, react to shocks to the demand for housing. We will do so, by focusing on the equilibrium with full segregation we have constructed in the previous section.

In equilibrium, the aggregate price level is given by

$$P_t = \frac{2}{I_t} \int_0^{I_t} p_t^R(i)\, di + \frac{2}{\bar{I}_t - I_t} \int_{I_t}^{\bar{I}_t} p_t^P(i)\, di,$$

where, from the analysis in the previous section,

$$p_t^R(i) = \left[C^P \left(1 + \frac{\gamma}{A} \right)^{\frac{\beta_P}{\alpha}} + C^R - C^P \right] \left(1 + \frac{\min\{\gamma, I_t - i\}}{A + \gamma} \right)^{\frac{\beta_R}{\alpha}} \text{ for } i \in [0, I_t], \quad (16)$$

$$p_t^P(i) = C^P \left(1 + \frac{\max\{\gamma + I_t - i, 0\}}{A} \right)^{\frac{\beta_P}{\alpha}} \text{ for } i \in (I_t, \bar{I}_t], \quad (17)$$

with I_t and $\bar{I}_t$ given by (14) and (15).

For concreteness, we analyze the economy's reaction to an interest rate shock. Let us assume that at time t the interest rate is $r_t = r^H$. At $t+1$ the economy is hit by an unexpected and permanent decrease in the interest rate so that $r_{t+1} = r^L < r^H$. We now show that in reaction to this positive demand shock, the aggregate level of house prices permanently increases and prices in locations with higher initial price level typically react less than prices in locations where houses are cheaper to start with. This implies that our model is consistent with our first stylized fact. The mechanism is driven by the externality at the core of our model. As a demand shock hits the economy, the city starts expanding, that is, $\bar{I}_t$ increases, and the rich households start expanding in poor neighborhoods, that is, I_t increases. This is what we refer to as endogenous gentrification. The house prices in gentrified neighborhood are driven up due to our externality. This generates the counterintuitive effect that prices react more in neighborhoods where the house supply is more elastic.

Let us define the function $g(.): \left[C^P, \bar{p} \right] \mapsto [1, \infty)$, where $g(p)$ denotes the average gross growth rate in locations where the initial price is equal to p, that is,

$$g(p) = E_{t+1} \left[\frac{p_{t+1}(i)}{p_t(i)} | p_t(i) = p \right].$$

Next proposition shows that after an unexpected permanent demand shock, the aggregate price level permanently increases and the price growth rate is higher in locations with initial level of prices, whenever prices are higher than the minimum level. This is consistent with our first fact.

Proposition 2. *If at time $t+1$ the economy is hit by an unexpected and permanent decrease in r, then there is a permanent increase in the aggregate price level P_t, and*

$$E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)}|p_t(i)=\bar{p}\right] < E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)}|p_t(i)<\bar{p}\right].$$

Moreover, if the shock is large enough, $g(p)$ is non-increasing in p for all $p > C^P$.

At the initial equilibrium, the city size $\bar{I}^H$, the rich neighborhood size I^H, and the price schedule for rich and poor households are given by conditions (14), (15), (16), and (17) with $r_t = r^H$. When the economy is hit by the shock, it immediately reaches a new equilibrium, with I^L and $\bar{I}^L$ and the price schedules given by the same conditions with $r_{t+1} = r^L$. From these conditions, it is easy to see that: (1) the city expands, $\bar{I}^L > \bar{I}^H$, (2) the rich neighborhoods expand, $I^L > I^H$, (3) prices remain constant in the central locations, $i \in [0, I^H - \gamma]$, (3) prices strictly increase at the boundary with the rich neighborhoods, $i \in [I^H - \gamma, \min\{I^L + \gamma, \bar{I}^H\}]$, (4) prices may remain constant far enough from the center, $i \in [I^L + \gamma, \bar{I}^H]$ if $\bar{I}^H > I^L + \gamma$. Notice that initial prices are not well defined for locations $i \in [\bar{I}^H, \bar{I}^L]$, given that these locations were not developed before the shock, hence we drop these locations from the calculation of average price growth. Clearly, the aggregate level of prices is going to increase permanently.

Figure 1 illustrates the reaction of house prices to a positive demand shock (a decrease in the interest rate) in different locations. Given that the city is symmetric, the picture shows only the positive portion of the real line. Figure 2 shows the price growth rate as a function of the initial price level in different locations together with the OLS regression that corresponds to the regressions run in the data. Our model typically delivers a negative slope of the regression line as in our first stylized fact.[9]

Next, we want to use our simple model to explore our second stylized fact, that is, that, across cities, the slope of this regression line is steeper the higher is the average growth rate of prices. We consider two different stories which can rationalize this relationship within our model. First, it could be that different cities are hit by demand shocks of different size. Second, it could be that different cities are hit by a common demand shock, but differ in preferences and/or technology (e.g. different income). In both cases, our model is able to generate higher gentrification associated with stronger price boom.

Proposition 3. *If at time $t+1$ the economy is hit by an unexpected and permanent decrease in r, then the growth rate in the aggregate price level is larger the larger is the decrease in r and,*

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[9]There is a lot of volatility in the growth rate for locations where the initial price is equal to C^P. In theory, this may revert the slope of the regression line when the measure of locations in poor neighborhoods, where prices do not change, is large enough.

if the shock is large enough,

$$\frac{d^2 g(p)}{dpdr} \geq 0$$

for all $p > C^P$ where the derivative is well-defined.

This proposition shows that if two identical cities are hit by demand shocks of different sizes, the one hit by the larger shock is going to feature both higher aggregate price growth rate and more price convergence due to a higher degree of gentrification. Figure 3 shows the reaction of house prices in different neighborhoods after demand shocks of different sizes. In particular, the figure shows the house price growth rate in different locations after a decrease of the interest rate from 3.5% to 3% (blue dots) and from 3.5% to 2.8% (red dots). The figure also shows the corresponding OLS regression lines (solid lines). After a larger demand shock average prices increase more and, at the same time, the regression coefficient is larger, consistently with our second stylized fact.

Proposition 4. *Consider two cities, A and B, where $\phi_R^A \geq \phi_R^B$ and $\phi_P^A \geq \phi_P^B$ with at least one strict inequality. If at time $t+1$ they are both hit by an unexpected and permanent decrease in r of the same size and large enough, then the growth rate in the aggregate price level is larger in city A and $g_A'(p) \leq g_B'(p)$ for all $p > C^P$ where the derivative is well-defined.*

This proposition shows that in two cities at a different stage of development hit by the same demand shock, house prices react differently. In particular, if the shock is large enough, the more developed city is the one that features both higher aggregate price growth rate and higher convergence. Figure 4 shows the reaction of house prices to the same demand shock in cities with different preferences. In particular, the figure shows the house price growth rate in different location after a decrease of the interest rate from 3.5% to 3% in a city with $\phi_R = 1$ (blue dots) and in a city with $\phi_R = 1.1$ (red dots). Clearly the city with a lower ϕ_R is less developed for the same initial interest rate. The figure also shows the associated OLS regression lines (solid lines), showing that the regression line is steeper in the city where the externality is stronger (higher ϕ_R), which is also the city where prices grow more on average, delivering again our second empirical fact.

Both these stories may be relevant for different cross sections in different time periods. However, we believe the second story may be more reasonable to understand, for example, why Charlotte did not react as Chicago or Boston did to the recent aggregate demand shock.

2.2 Model with Adjustment Costs

In this section, we extend the previous model, by introducing standard adjustment costs on the supply side. This extension is interesting for two reasons. First, adjustment costs make the dynamics of the model richer. Second, we can compare our model to a standard adjustment cost model and show that the aggregate implications are substantially different.

2.2.1 Equilibrium

The set up of the model is exactly the same as in Section (2.1.1), except that the representative firm faces now a construction cost that increases with the amount of construction at each time t. Also, we make two simplifying assumptions. First, we assume that $\beta_P = 0 < \beta_R \equiv \beta$. Second, in order to get rid of an extra state variable, we assume that the houses for rich and poor are the same, that is, $C^R = C^P = C$. Then, by market clearing, the amount of construction of new houses at time t needs to be equal to the increase in the size of the city $\bar{I}_t - \bar{I}_{t-1}$. Then, the construction cost is a convex function of $\bar{I}_t - \bar{I}_{t-1}$.

The analysis in Section (2.1.2) still goes through, except that the price of the rich houses in the marginal location I_t is now equal to the the price of poor houses and

$$p_t^R(I_t) = p_t^P = c'\left(\bar{I}_t - \bar{I}_{t-1}\right). \tag{18}$$

Also, the equalization of rents for poor households and for rich households living in the marginal location I_t implicitly defines I_t as a function of $\bar{I}_t$ as follows:

$$\left(\bar{I}_t - I_t\right)^{1-\alpha}\gamma^\beta\frac{\phi^R}{\phi^P}\left(\frac{N^R}{N^P}\right)^{1-\alpha} = \left[2^{\frac{\beta}{\alpha}}\left(I_t - \gamma\right) + \frac{\alpha\gamma}{\alpha+\beta}\left(2^{\frac{\alpha+\beta}{\alpha}} - 1\right)\right]^{1-\alpha}. \tag{19}$$

In our numerical examples, we set the cost function to be $c(x) = C_1 x + C_2 x^\psi/\psi$. Moreover, the arbitrage equation (3) together with the expression for the rents (9) for any location $i \leq I_t$ yields

$$p_t^R(i) = \sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j R_t^R(I_{t+j})\min\left\{\left(1 + \frac{I_{t+j} - i}{\gamma}\right)^{\frac{\beta}{\alpha}}, 2^{\frac{\beta}{\alpha}}\right\}, \tag{20}$$

where, combining the market clearing condition (1) with the optimality condition (4) and the expressions (7) and (9), one obtains

$$R_t^R(I_{t+j}) = \rho(I_{t+j}) \equiv \gamma^\beta\phi^R\alpha\left(\frac{N^R}{2}\right)^{1-\alpha}\left[2^{\frac{\beta}{\alpha}}\left(I_{t+j} - \gamma\right) + \frac{\alpha\gamma}{\alpha+\beta}\left(2^{\frac{\alpha+\beta}{\alpha}} - 1\right)\right]^{\alpha-1}.$$

Letting $\tilde{p}_t$ denote the price at the boundary location at time t, $p_t(I_t)$, equation (18) and equation (20) evaluated at $i = I_t$ together with expression (19) give us a dynamic system that characterizes the equilibrium sequence $\{\bar{I}_t, I_t, \tilde{p}_t\}_{t=0}^{\infty}$. Once we have solved for the sequence

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$\left\{ \bar{I}_t, I_t, \tilde{p}_t \right\}_{t=0}^{\infty}$, we can solve for the sequence of rents at any location i, $\left\{ R\left(i\right) \right\}_{t=0}^{\infty}$, using expression (6), and for the sequence of prices at any location i, $\left\{ p_t\left(i\right) \right\}_{t=0}^{\infty}$, using the same equation (20).

2.2.2 Demand shock

We now analyze the same interest rate shock studied in the baseline model. Imagine the economy starts with an interest rate $r_t = r^H$ and at time $t+1$ is hit by an unexpected permanent decrease in the interest rate $r_{t+1} = r^L < r^H$. We show that, after the shock, the city starts growing slowly and the aggregate price overshoots in the short run and then, in contrast with the standard adjustment cost model, converges to a permanently higher level.

In our model, price dynamics are affected by the combination of adjustment costs and gentrification. The price in the marginal location I_t, which is changing over time as the city grows, behaves similarly to the standard adjustment cost model. It overshoots on impact and then declines back to the marginal cost C, once the steady state is reached and the city does not grow anymore. However, price dynamics in each given location are richer as Figure 5 shows.

Figure 5 illustrates the price behavior in different locations over time. Panel (a) shows a sample of locations, including some developed after the shock, as the city expanded. The dynamics differ across neighborhoods. Developed neighborhoods experience price dynamics that are qualitatively similar to the standard adjustment cost model, with an initial spike in prices followed by a gradual return to the initial price level. Neighborhoods that develop late in the process do not experience overshooting at all, and their prices display a gradual increasing path towards their long run level. Finally, intermediate neighborhoods show hump-shaped dynamics, with a gradual increase followed by a gradual decrease towards the long run level. Initially these neighborhoods grow and the local externality makes them more desirable, but eventually the city expands reducing the pressure of housing demand and driving prices down. Panel (b) focuses on locations which used to exist already in the initial steady state, and shows that prices in neighborhoods originally father from the city center react more to the shock than prices in neighborhoods already fully developed. This is due to our gentrification mechanism and it is what drives the amplification result. Also, this implies that, after a city-wide demand shock, richer neighborhoods experience a more cyclical behavior than poorer neighborhoods which feature a trend increase.

Let us now compare the aggregate house price behavior in our model with a standard adjustment cost model with no externalities. In the benchmark adjustment cost model, where $\beta = 0$, the rent is going to be the same in all locations and equal to $\alpha \left(N/2\right)^{1-\alpha} I_t^{\alpha-1}$. The

equilibrium is then simply characterized by equations (18) together with

$$p_t = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \alpha \left(\frac{N}{2}\right)^{1-\alpha} I_t^{\alpha-1}.$$

After a decrease in the interest rate, construction slowly increases to the new steady state level and aggregate price increase on impact and then decline to the same constant initial level. In our model, instead, the price in the marginal neighborhood goes back to the marginal cost $c'(0)$, but the aggregate price level of prices is going to be permanently higher. This is due to the fact that, after the shock, the city starts expanding, so that neighborhoods that were not fully developed are going to grow, reaching permanently higher price levels. In the short run, prices in the less developed neighborhoods grow faster than in the standard model, because agents living there expect their neighborhood to grow and the neighborhood externality to be stronger. This generates the amplification effect.

Figure 6 shows the behavior of aggregate prices, after a decline in the interest rate from 3.5% to 3%, both in our model and in the standard adjustment cost model, showing both the level effect and the amplification effect on aggregate prices.

3 Data

To examine house price appreciation across neighborhoods within a city during different time periods, we use a variety of different data sets. In particular, we use (1) zip code level housing price indices computed as part of the Case-Shiller index, (2) transaction level data on the universe (or near universe) of residential housing transactions for Chicago, New York, and Charlotte, and (3) micro data from the 1980, 1990 and 2000 U.S. Censuses. As we show in the following section, all three data sources yield similar results about how housing prices evolve within a city during a given time period.[10]

The bulk of our results use the Case-Shiller zip code-level price indices.[11] The Case-Shiller indices are calculated from data on repeat sales of single-family homes. The benefit of the Case-Shiller index is that it provides consistent constant-quality price indices for localized areas within a city or metropolitan area over long periods of time. Most of the zip code-level price indices go back in time through the late 1980s or the early 1990s. The data was provided to us at the quarterly frequency and our most recent data is for the fourth quarter of 2008. As a

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[10]For a complete discussion of the data sources - including a link to all of our online documentation - see the Data Appendix.

[11]The zip code indices are not publicly available. Fiserv, the company overseeing the Case-Shiller index, provided them to us for the purpose of this research project. The data are the same as the data provided to other researchers studying very local movements in housing prices. See, for example, Mian and Sufi (2009).

result, for each metro area, we have quarterly price indices on selected zip codes within selected metropolitan areas going back roughly 20 years.

There are four limitations to the Case-Shiller data. First, the Case-Shiller index only covers approximately 30 metropolitan areas.[12] Second, as noted above, for almost all cities the Case-Shiller index does not start until late 1980s or early 1990s. This limits the analysis we can do for cities that experienced housing price booms in the early to mid 1980s. Third, the Case-Shiller index only covers single-family homes (as opposed to also including condos or multi-family buildings). To the extent that condos and multi-family buildings evolve differently during housing price booms and to the extent that different neighborhoods have different compositions of housing structures, the Case-Shiller data could yield a biased picture of within city house prices changes. Finally, given its focus on repeat sales of single-family homes, there is not enough data to compute reliable price indices for all zip codes within a metropolitan area. As a result, many zip codes - particularly those in urban city centers - have no reported Case-Shiller price indices. Appendix Figure A1 illustrates this point. In this Figure we show the zip codes within three cities: Chicago (panel A), New York City (panel B), and Charlotte (panel C). The darkened zip codes on the city maps are the ones for which a Case-Shiller index exists. Notice, that a Case-Shiller zip code index exists for less than 50 percent of the zip codes in Chicago, less than 10 percent of the zip codes in New York City, and essentially all of the zip codes in Charlotte.[13] Notice, for New York City proper, only some zip codes in Staten Island are included.

To overcome some of the shortcomings of the Case-Shiller zip code indices, we supplement our analysis using data from two additional sources. First, for three cities, we were able to get data for the universe (or near universe) of residential housing transactions. The three cities were Chicago, Charlotte, and New York. These data allow us to explore whether our results change using data that include condos and multi-family buildings (as well as single-family homes) and allows us to examine price behavior in all zip codes within a city. As we show below, the results in these more detailed data sets mirror the results from the Case-Shiller indices giving us confidence that these limitations of the Case-Shiller indices are not biasing our results.

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[12]We were provided with zip code level data for at least 20 zip codes at the city level or at least 65 zip codes at the metro area level for the following cities/metropolitan areas: Atlanta, Baltimore, Boston, Charlotte, Chicago, Cincinnati, Columbus (OH), Dayton, Denver, Detroit, Hartford, Jacksonville, Las Vegas, Los Angeles, Memphis, Miami, Minneapolis, New York, Orlando, Philadelphia, Phoenix, Portland (OR), Sacramento, San Bernardino, San Diego, San Francisco, San Jose, Seattle, Tampa, and Washington DC.

[13]Coverage in the metro areas are much higher given that most zip codes outside the center city have a sufficiently large number of single-family home transactions to compute reliable price indices. For example, there are a total of 384 zip codes covered in the Case-Shiller data from the New York metro area (compared on only 9 in New York City proper). The reason that zip codes in center cities tend to have lower coverage is that most homes sales are condos or multi-family buildings which are excluded from the Case-Shiller index.

For Chicago and Charlotte, we compiled the data on all residential real estate transactions ourselves. Using the Chicago Tribune website, we downloaded all residential real estate transaction data for the City of Chicago. The data we were able to download include all residential real estate transactions from 2000 through 2008 (inclusive). We believe this to be the universe of residential real estate transactions. We merged the data from the Chicago Tribune on Chicago real estate transactions with information from the Cook County Tax Assessor which included information on the age of the structure, building type, etc.[14]. For Charlotte, the procedure was much easier. We downloaded all real estate transactions back to 1990 using the Mecklenburg County Real Estate Lookup System. These data include an extensive list of structural characteristics. Like the data from the Chicago Tribune, we believe the Charlotte Deed data to be the universe of all residential real estate transactions.

Given that we had some attributes of the structure in the Chicago data that we merged in from the Cook County Tax Assessor, we made a simple price index for each Chicago neighborhood.[15] The real estate transactions in the Chicago Tribune are mapped to Chicago neighborhoods (all within the city of Chicago). Chicago neighborhoods are slightly smaller than Chicago zip codes. For example, there are 77 Chicago neighborhoods and only 60 Chicago zip codes. In Appendix Figure A2, we show the Chicago neighborhood map. We have kept the Chicago Tribune price data at the level of Chicago neighborhood (instead of converting them to zip codes) so as to match our building permit data - which is at the neighborhood level - discussed in Section 6. To make the simple price index, we regressed all Chicago residential real estate transaction (log) price data on dummies for building type (multi-family, single-family, or condo), dummies for the age of the building (1-5 year old, 6-10, 11-20, 21-30, etc.), and dummies for neighborhood interacted with year. We evaluated the estimating equation using the mean structural characteristics for the entire city and added this component to coefficients for the neighborhood-year variables to form the neighborhood price indices over time. We used the same procedure to construct a hedonic index for Charlotte zip codes over time. The only difference was that we had a much richer set of building characteristics for each property in Charlotte.

For New York City, we use the Furman Center repeat sales index which covers all of NYC. The Furman data uses NYC community districts as its level of aggregation.[16] There are 59

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[14]In the Data Appendix, we discuss our methodology in much greater detail

[15]For Chicago, the only structural characteristics that are available for all property types are the age of the building and the property type. A much richer set of structural characteristics are available for all Charlotte properties. See the data appendix for further details.

[16]See `http://furmancenter.org/`.

community districts in New York City which represent clusters of several neighborhoods. The Furman data for New York City extends back to 1974. The benefit of the Furman data is that it gives us extensive coverage of New York neighborhoods over a long time period and covers all residential real estate transactions in New York City (not just single-family homes). Additionally, this is a repeat sales index measuring the change in price for constant quality housing units. A map of the NYC community districts can be found in the second panel of Appendix Figure A2.

To help examine neighborhood trends within a broad set of cities during the 1980s and the 1990s, we augment our analysis using data from the 1980, 1990 and 2000 U.S. census. We restricted our analysis of the U.S. Census data to census tracts within a metro area. So, when we use the census data, our unit of analysis - a census tract - is much smaller than zip code or community area. We only focus on census tracts whose boundaries remained constant between the two adjacent census years. For each census tract within the metro area, we computed the growth rate in median home prices. When computing the growth rate in median house prices by census tract, we did not hedonically adjust the series for changing housing characteristics. As a result, using this data, we can explore changes in house prices within a neighborhood that are both due to the fact that the price of a constant-quality unit of housing may be changing and due to fact the quality of housing in the neighborhood may be changing. Again, the Data Appendix gives a complete description of all of our data sources and our sampling restrictions.

Lastly, when comparing differences in house price appreciation across cities, we use both the Case-Shiller metro level price indices and the OFHEO metro level price indices. The Case-Shiller metro indices are limited to only 30 cities while the OFHEO metro indices cover over 170 cities. As noted above, the Case-Shiller index is a repeat sales index that only includes single family homes regardless of the type of financing used to purchase the home. The OFHEO index is also a repeat sales index but it includes properties of all different types (single family homes, condos, town homes, etc.) but restricts the properties in their index to only ones that are purchased with conventional mortgages. Despite the difference in coverage, the time series path of the OFHEO and Case Shiller indices are nearly identical for the metro areas where both indices exist.[17]

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[17]See online documentation for a discussion of how the Case Shiller index is computed. See http://www.fhfa.gov/Default.aspx?Page=14 for a similar discussion for the OFHEO index. Lastly, see http://faculty.chicagobooth.edu/erik.hurst/research/ for a comparison of the Case Shiller index and the OFHEO index for various MSAs.

4   New Facts About Within City House Price Movements

In this section, we systematically explore the nature of housing price movements within a city or metro area during different time periods. Two facts emerge. First, we show that neighborhoods that are initially low priced within a city are more price elastic (during both housing booms and busts) than higher priced neighborhoods. Second, we show that the low priced neighborhoods in close proximity to the high priced neighborhoods have higher property price appreciation rates during housing price booms than low priced neighborhoods farther from the high priced neighborhoods. These within city facts persist during the 1980s, the 1990s, and the 2000s and are found using a variety of house price measures. As far as we can tell, these facts are novel to the literature.[18] While these facts are broadly consistent with the model presented in the prior section, they may also be interpreted through the lens of many different models - particularly fact 1. In the next section, we try to test for our mechanism more directly.

4.1   Fact 1:   Initially Low Priced Neighborhoods Are More Price Elastic

We begin our analysis by estimating the relationship between the initial level of house prices across neighborhoods within a city and the subsequent growth rate in house prices for those neighborhoods during periods of city-wide housing booms and busts. Specifically, we estimate the following relationship of within city house price dynamics:

$$\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}} = \alpha_{t,t+k} + \beta_{t,t+k}^j \ln(P_t^{i,j}) + \epsilon_{t,t+k}^{i,j} \tag{21}$$

where $P_t^{i,j}$ is the level of housing prices in neighborhood i, within city (metro area) j, in year t and $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$ is the growth in housing prices in neighborhood i, within city (metro area) j, between years t and $t + k$. We estimate these relationships separately within each city (metro area). $\beta_{t,t+k}^j$ is an estimate of the relationship between the initial level of house prices (in logs) and the subsequent house price appreciation within city (metro area) j between t and $t+k$. We will examine neighborhood price movements within both cities and broader metro areas. As a

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[18]Very few existing studies have examined within city (or metropolitan area) price movements. Of those that exist, most perform their within city analysis by comparing the appreciation rates of "high end" properties to the appreciation rate of "low end" properties. Moreover, these studies tend to focus their analysis on a given market (or a very small set of markets) during a given time period. See, for example, Mayer (1993) and Poterba (1991) who analyzed Oakland, Dallas, Chicago, and Atlanta between 1970 and the mid 1980s, Case and Shiller (1994) who analyzed Boston and Los Angeles during the 1980s, and Smith and Tesarek (1991) who analyzed Houston during the 1970s and 1980s. Two papers, however, look specifically at differences in house prices appreciation across zip codes within a metropolitan area. Case and Mayer (1996) look at differential movements in prices within cities of the Boston metro area between 1982 and 1992 while Case and Marynchenko (2002) look at differential trends in prices across different zip codes within the Boston, Chicago, and Los Angeles metro areas during the 1983 to 1993 period. No systematic relationships emerged from these studies.

result, sometimes j will index a city and sometimes it will index the broader metro area that contains that city.

Figure 7 shows the estimates of (21) where $j =$ Chicago (Panel A), New York City (Panel B), and Charlotte (Panel C) over the 2000-2006 time period. Along with the actual estimated regression line, we plot the underlying neighborhood data. The house price growth data (on the y-axis) is the growth rate in the neighborhood level house prices indices (which were discussed in the previous section). The initial level of house prices (on the x-axis) is the median house price in the neighborhood as reported by 2000 census.[19] We chose these three metro areas to start our analysis with for two reasons. First, as noted in the previous section, we have house price measures from multiple sources for these three cities. Second, the cities provide a nice contrast with each other given that the New York metro area as a whole experienced a substantial housing price boom between 2000 and 2006 (of 75 percent), while the Chicago metro area experienced a medium sized housing price boom during that period (of 40 percent) and the Charlotte metro area experienced a very small housing price boom during that period (of 8 percent).[20]

For each of the three cities in Figure 7, we provide three panels that shed light on the relationship between house price growth across neighborhoods and the initial level of prices in the neighborhood using different data sources and levels of aggregation. In the left most panel, we use our alternate data sets for each city. For Chicago, we use the Chicago Tribune data. For New York, we use the Furman data for Manhattan. And for Charlotte, we use the universe of deed data. In the middle panel, we use the zip code level data from Case-Shiller. As noted above, for New York City, the Case-Shiller data only covers some zip codes in Staten Island (see Appendix Figure A1). The left and middle panels focus on only neighborhoods within the city of Chicago, New York or Charlotte. In the right most panel, we expand our analysis to include all the zip code data from Case-Shiller for the entire metro area.

Focusing on the Chicago panel of Figure 7 (Panel A), we see that both the Chicago Tribune data and the Case-Shiller data show that lower priced neighborhoods, on average, appreciated at much greater rates than higher price neighborhoods during the 2000-2006 period. Focusing on the Tribune data (which, as discussed in the previous section, only control for limited property

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[19]Figures with zip code-level observations use 2000 Census summary file (SF3) tabulations of median value of owner-occupied housing units by zip code, which are available from `http://factfinder.census.gov/`. The left-most figure of Panel B also uses 2000 Census tabulations of median value of owner-occupied housing units, but these are specially tabulated for NYC community districts and are are available from NYC's GIS department at `http://gis.nyc.gov`. Finally, the x-axis for the left-most figure of panel A uses the mean owner-occupied housing value calculated from census 2000 tabulations by census tract and described in the data appendix.

[20]To compute the change in the metro area price indices as a whole, we use the Case-Shiller metro area price indices.

characteristics in the hedonic relationship), we find that prices of transacted properties in the initially low priced Chicago neighborhoods grew at a rate that was 2 to 3 times higher than the prices of transacted properties in the initially high priced Chicago neighborhoods. For example, the average transacted property grew at about 25 percent or less in Chicago's high priced neighborhoods of Lincoln Park (community 7), Lakeview (community 6) and The Loop (community 32). Conversely, some of the neighborhoods that had initially low price levels experienced an increase of 75 percent or more for the average value of transacted property prices.

The left hand panel of Figure 7 also includes estimates of (21). The estimated β from (21) for Chicago using the Chicago Tribune data during the 2000-2006 period is -0.33 with a robust standard error of 0.05. In other words, a 100 percent increase in the initial level of housing prices reduces the growth rate in housing prices by 33 percentage points. The simple R-squared of the scatter plot is 0.29.

The data in the left-hand panel of Figure 7A includes the universe of transacted properties within the city of Chicago during the 2000-2006 period. The drawback of this data is that it is not based on repeat sales transactions. To explore the movement of prices using a better neighborhood repeat sales price index, we use the Case-Shiller zip code data for Chicago. As noted above, the Case-Shiller data only cover about 45 percent of Chicago's 60 or so zip codes. Moreover, the Case-Shiller data only focus on the price movements of single-family homes. Despite the differences in coverage, the results in the middle panel of Figure 7A (using the Case-Shiller data) are very similar to the results in the left panel of 7A (using the Chicago Tribune data). A one-hundred percent increase in initial housing prices reduces the growth rate in housing prices by 33 percentage points (with a standard error of 4 percentage points). The simple R-squared of the scatter plot is 0.78. The right hand panel of Figure 7A shows the results using the Case-Shiller zip code data for the entire Chicago metro area (as opposed to just the city of Chicago). Again, a similar pattern emerges. High price neighborhoods experienced lower appreciation rates than lower priced neighborhoods (β = -0.23 with a standard error or 0.04).

In Figure 7B, similar results are shown for New York. Using the Furman data for community districts in Manhattan (left-hand panel) or using Case-Shiller zip codes from Staten Island (middle panel), we find that a 100 percent increase in initial housing prices reduced the subsequent growth rate in housing prices between 2000 and 2006 by 39 percentage points or 32 percentage points, respectively. Both estimated slope coefficients are significant at the 1 percent level. For example, the Harlem area of Manhattan appreciated at twice the rate of midtown

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Manhattan. As seen in the right hand panel of Figure 7B, the results also hold broadly for the New York metro area as whole (β = -0.35 with a standard error of 0.02).

In Figure 7C, we see similar graphical representations for Charlotte. The results, however, for Charlotte are very different. No matter what level of aggregation and no matter what measure of housing prices, there is either no systematic relationship between the initial level of housing prices and the subsequent growth in housing prices or evidence in the opposite direction. Across the three figures, β equals 0.14, 0.07 and 0.08 - only the first coefficient is significant at standard levels. As we will discuss below, the results

Figure 8 shows similar patterns for different cities and across different time periods. In the top panel of Figure 8, we show the relationship between initial neighborhood level of prices and subsequent growth rate in prices for other metropolitan areas during the 2000-2006 period (using the Case-Shiller data). These metro areas include Boston, Los Angeles, San Francisco, and Washington DC. These pictures are analogous to the right hand panels of Figure 8. In every one of these metro areas, property prices as a whole increased by at least 50 percent throughout the metro area. Also, in every one of these metro areas, the neighborhoods with initial low levels of housing prices increased by a substantially higher amount than higher price neighborhoods.

In the middle panel of Figure 8, we present similar plots for metro areas during the 1990-1997 period. We start in 1990 so we can anchor the initial house price to the data from the 1990 census. We stop in 1997 because that is considered the start of the next housing boom cycle. Again, we only use the Case Shiller data for our measures of within city price movements. The metro areas include in the middle panel are: Denver, Portland, San Francisco, and Boston. Denver and Portland both experienced substantial real housing price booms (between 30 and 40 percent) while San Francisco and Boston experienced non-trivial metro area real housing price busts (roughly 20 percent). In Denver and Portland we see that the poor neighborhoods appreciated much more the richer neighborhoods (estimated β's of -0.48 and -0.35, respectively - both statistically significant at the 1 percent level). In San Francisco and Boston we see that the poorer neighborhoods had property prices that declined much more dramatically compared to the property prices in the richer neighborhoods during the city-wide property price busts (estimated β's of 0.09 and 0.22, respectively - both statistically significant at the 1 percent level).

In the bottom panel of Figure 8, we examine the relationship between initial level and subsequent property price appreciation for neighborhoods within cities during the 1980s. We

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start by focusing on New York and Boston - two cities that experienced substantial housing price booms during the 1980s. Average real housing prices increased in these two cities between 1984 and 1989 by well over fifty percent. For New York City, we data from the Furman Center to explore the within city growth rates (left panel). For Boston, we use both the Case-Shiller data which, for Boston, extends back into the early 1980s (middle panel) and then U.S. Census data (right panel). We do this to show that the Census data yields similar patterns as the Case-Shiller data. As noted above, however, the Census data is at the level of the Census tract. Like the results for the 1990s and 2000s, New York and Boston saw sharp convergence in housing prices across neighborhoods during their 1980s property price booms. The estimated β's from the three panels are -0.38, -0.13, and -1.43, respectively. All convergence estimates are significant at the 1 percent level. Given that the Census data is not holding the quality of the housing stock constant in the price estimates, the magnitude of the housing price increases are much higher. That is why the estimated β from the Census data is so much higher than the estimated β's from the other two specifications.

Figure 9 formalizes the relationship between the level of housing price growth within the metro area as a whole and the amount of cross-neighborhood convergence that takes place within the city. In particular, we estimate the following:

$$\beta_{t,t+k}^j = \theta_0 + \theta_1 \frac{\Delta P_{t,t+k}^j}{P_t^{i,j}} + \eta_{t,t+k}^j \tag{22}$$

where the β^j's are estimated for each metro area as described in (21) and $\frac{\Delta P_{t,t+k}^j}{P_t^{i,j}}$ is house price appreciation in the entire metro area j between t and $t+k$. For panels A and B of Figure 9 we restrict our analysis to the 30 metro areas where we have zip code level price indices from Case-Shiller when estimating (22). Panel A, restricts our analysis to the 2000-2006 period while Panel B restricts our analysis to the 1990-1997 period. In Panel C, we examine the 1980-1990 period using U.S. Census data.

In all the different time periods (shown in panels A, B, and C of Figure 9), we find that the larger the price increase (decrease) in the metropolitan area as a whole, the more that poor neighborhoods appreciate (depreciate) relative to richer neighborhoods within the metropolitan area. For example, in all time periods, high positive price appreciation at the metro level are correlated with a more negative β while larger metro wide price depreciations are associated with a more positive β. This latter result can be seen from Panels B and C of Figure 9. The simple correlation between metro area price changes and the estimated β's from equation (21) for the data shown in Panels A, B and C, respectively, are 0.50, 0.58, and 0.28.

Figures 7, 8, and 9 show our first fact. We find that neighborhoods with initially low housing prices are more price elastic than neighborhoods with initially high housing prices. Specifically, during city-wide housing price booms, housing prices appreciate more in initially low priced neighborhoods than in initially high priced neighborhoods. Conversely, during city-wide housing price declines, housing prices fall more in initially low priced neighborhoods than in initially high priced neighborhoods. These facts are robust across time periods and across measures of house price appreciation.

4.2 Fact 2: The Low Price Neighborhoods That Appreciate Most Are Spatially Close to High Price Neighborhoods

While, on average, low priced neighborhoods appreciate more during city-wide housing booms, there is a large amount of heterogeneity within low priced neighborhoods. Some low priced neighborhoods appreciate by a very large amount and other low priced neighborhoods do not appreciate much at all. This fact can be illustrated formally. In particular, we can take the absolute value of the residuals from our estimation of (21) and regress them on the initial level of log house prices in the neighborhood. Systematically, we find that the variance of the residuals is higher for initially low priced neighborhoods relative to high priced neighborhoods. For example, using the Chicago Tribune Data (left hand panel of 7A), we find that the absolute value of the residuals decline by 6.5 percentage points as the initial level of house prices increases by 100 percent during the 2000-2006 period. These results are consistent within all of the metro areas we studied. Even though initially low priced neighborhoods appreciated more on average, the results were not uniform across all low priced neighborhoods. Some low priced neighborhoods increased by a lot, while others increased by less. For our last set of results, we examine the spatial characteristics of the low priced neighborhoods that appreciated most during city-wide house price booms.

The top panel of Figure 10 shows a map of the neighborhoods within the city of Chicago. The underlying map is the same as Panel A of Appendix Figure A2 (discussed above). Using the complete data from the Chicago Tribune, we identify two types of neighborhoods within the city of Chicago. The first set of neighborhoods are the high property price neighborhoods in year 2000. Specifically, these neighborhoods were in the top quartile of all Chicago neighborhoods with respect to average housing prices in year 2000 (as measured by the 2000 U.S. Census). These initially high priced neighborhoods are shown in Panel A of Figure 10 with darker shading. The second set of neighborhoods includes those neighborhoods that experienced the highest growth in property prices between 2000 and 2006 (as measured by our Chicago Tribune Price

Index). Again, we highlight only those neighborhoods in the top quartile of all Chicago neighborhoods with respect to the growth in property prices. On Figure 10, these neighborhoods are shaded light grey. The single neighborhood that falls into both categories, Logan Square, is shaded in black.

The results in Panel A of Figure 10 show the spatial analog to the results shown in Panel A of Figure 7. In this figure, we show that the areas that grew fastest in Chicago were the low priced neighborhoods that directly neighbored the initially high priced neighborhoods. Notice, the extreme south side of Chicago (neighborhoods 50-55 from Figure A1) did not grow as fast as the neighborhoods directly to the west and south of the initially high priced areas. Yet, the neighborhoods on the extreme south side of Chicago (the non-shaded areas) were similar in terms of demographics in 2000 to the neighborhoods that experienced rapid price appreciation from 2000-2006 (the grey shaded areas). For example, using Census data, the mean income of the 32 south side neighborhoods that did not experience rapid growth from 2000 - 2006 was about $49,000 while the mean income of the 14 southside neighborhoods that did was about $36,000. High growth neighborhoods on the southside had an average poverty rate of 34%, while other neighborhoods on the southside had an average poverty rate of 19%. The high growth southside neighborhoods were on average 65% African American, while the other southside neighborhoods were on average 54% African American. In summary, the demographics of the southside neighborhoods that experienced rapid growth from 2000 to 2006 were not wildly different than the demographics of the southside neighborhoods that did not grow rapidly. If anything the neighborhoods that grew were a little bit poorer and more heavily African American. Collectively, for Chicago, our results show that the poor neighborhoods that were close to the rich neighborhoods were much more likely to experience house price gains than the equally poor (or just slightly wealthier) neighborhoods that were not close to the rich neighborhoods.

Our results for Chicago are not unique. The spatial proximity of high price growth neighborhoods to initially high price level neighborhoods is found in nearly all cities where convergence was pronounced. In Panel B of Figure 10, we show similar patterns for New York City (using the Furman Center data) while in Panel C of Figure 10 we show the same type of results for the city of Charlotte (using the detailed Charlotte transaction data). Similar to Chicago, New York City's and Charlotte's high housing price growth areas are adjacent to its initial high price areas.[21]

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[21]In an eventual online appendix that accompanies this paper (but is currently attached to the end), we show similar pictures for many cities during the 1990-2000 and the 2000-2006 periods. All pictures illustrate the same

Table 1 systematically summarizes the results of the spatial patterns in price appreciation across all cities. In particular, we estimate:

$$\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}} = \eta_0 + \eta_1 \ln(P_t^{i,j}) + \eta_2 \ln(P_t^{i,j})\frac{\Delta P_{t,t+k}^{j}}{P_t^{i,j}} + \eta_3 \ln(Dist_t^{i,j}) + \eta_4 \ln(Commute_t^{i,j}) +$$

$$\eta_5 \frac{\Delta Commute_{t,t+k}^{i,j}}{Commute_t^{i,j}} + \mu_j + \epsilon_{t,t+k}^{i,j}$$

where $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$, $\frac{\Delta P_{t,t+k}^{j}}{P_t^{i,j}}$ and $P_t^{i,j}$ are defined as above. $\ln(Dist_t^{i,j})$ is the log of the distance of the centroid of neighborhood i in city j (in miles) to the centroid of the nearest neighborhood in j that is in the top quartile (out of all neighborhoods within j) with respect to average housing price during period t. $ln(Commute_t^{i,j})$ is the average time spent commuting to work by residents in neighborhood i within city j during time t while $\Delta Commute_{t,t+k}^{i,j}$ is the change in mean commuting time by residents in neighborhood i of city j from t to $t+k$. The regression results are robust to the inclusion of a more extensive vector of controls. This is not surprising given that initial house prices - which are a summary of neighborhood characteristics - is already included in the regression.

The first two columns show our results for the 2000 - 2006 period ($t = 2000$ and $t + k = 2006$) using the growth in house prices from our Case-Shiller data. We restrict our analysis to only the primary cities in the data. In other words, we exclude information from the zip codes in the suburbs. All of our variation is from differences in price appreciation across zip codes within a city. We also restrict our analysis to those zip codes in the bottom half of the initial housing price distribution within the city in period t (i.e., 2000) as measured by the U.S. Census. Our experiment is to ask "Among low priced neighborhoods, do the ones in close proximity to the high priced neighborhoods have housing prices that appreciate faster?" For the results in the first two columns of Table 1, our analysis includes information from 277 zip codes in 30 cities. The first columns show the results of the regression without the controls for commuting time. In the second column, we include controls for commuting time. For the 2000-2006 regressions, we cannot include the change in commuting time because we do not have detailed measures of commuting time at the zip code level in 2006.

The results are analogous to the pictures shown in Figure 10. Poorer neighborhoods closer to high priced neighborhoods systematically had greater house price appreciation than otherwise similar neighborhoods that resided farther from the high priced neighborhoods. Specifically,

patterns. Those low priced neighborhoods in close proximity to the high priced neighborhoods are the ones that appreciated the most during the period when the prices were increasing in the city.

a doubling of distance from high priced neighborhoods reduced the appreciation rate of properties between 2000 and 2006 by 9 percent. In other words, properties 1 mile from a high priced neighborhood increased by 18 percent more than properties 4 miles from high priced neighborhoods between 2000 and 2006 (the mean distance to high priced neighborhood for the zip codes in our sample was 3.9 miles).

One potential explanation for the results in column 1 of Table 1 is that people do not care about living around rich people, but instead prefer to live close to the jobs. As noted in the introduction, most of the literature has used this as an explanation for the differences in housing prices within a city or metro area. If the rich people lived in the exact same areas as the jobs, it would be impossible to tell the two stories apart. However, in the data, rich people do not live in the same area as the jobs. For example, in Chicago, most of the jobs are in the "loop" (e.g. zip codes 60601 - 60606 on Appendix Figure A1). Yet, the richer households in Chicago live on the North side of the loop. Equal distance from the loop (in terms of travel times) on the South side are populated by lower income individuals. Similar patterns are evident in most U.S. cities. The Lower East Side of Manhattan is equally close to Wall Street as the lower West side, yet the lower West side has higher property prices and is populated by residents with higher incomes compared to the lower East side. Such variation within cities allows us to distinguish proximity to "richer" households separately from proximity to jobs. In column 2 of Table 1, we also control for how close the neighborhood is to the jobs. We measure this by average commuting time to work by working individuals in the neighborhood. There is some evidence that poorer neighborhoods closer to the jobs appreciated more; the coefficient on log commuting time is -0.18 with a standard error of 0.11. However, the key result from this column is that even after controlling for commuting times, the magnitude and significance of proximity to high priced neighborhoods remains unchanged.

The same results are shown in columns 3 and 4 (using Case-Shiller data for the 1990-2000 period) and shown in columns 5 and 6 (using Census data for the 1990-2000 period). The Case-Shiller data restricts the sample to only those zip codes and cities where we have Case-Shiller data. Again, as above, we further restrict our analysis to only those zip codes in the bottom half of the house price distribution. Similar patterns emerge. Low price neighborhoods in close proximity to the high price neighborhoods were more likely to appreciate between 1990 and 2000. Using the Case Shiller repeat sales index, low price properties 1 mile from a high priced neighborhood increased by 14 percent more than properties 4 miles from high priced neighborhoods between 2000 and 2006.

4.3 Summary of Fact

In this section we have shown that there is a tremendous amount of heterogeneity in house price movements within a city/metro area during housing price booms and busts. We document two new facts about within city property price movements. First, we show that initially low priced neighborhoods are much more price elastic than initially high priced neighborhoods. During city-wide housing price booms, low price neighborhoods - on average - appreciate more and during city-wide housing price busts, low price neighborhoods - on average - depreciate more. Second, while low property price neighborhoods are likely to appreciate (depreciate) more on average, there is a large degree of heterogeneity among the low property price neighborhoods. We show that it is the low priced neighborhoods that are in close proximity to the high priced neighborhoods that are the most price elastic. This latter effect persists even after controlling for proximity to jobs. These facts are consistent with the model of neighborhood externalities based on the income of one's neighbors discussed above.

5 Within City House Price Movements and Signs of Gentrification

Within the model, the mechanism by which initially lower priced neighborhoods experience a rapid increase in home prices is through gentrification. As part of our descriptive analysis, we examine whether the neighborhoods that experienced rapidly growing home prices showed signs of gentrification. To do this, we use very detailed data on income, poverty rates and migration from the 1980, 1990, and 2000 U.S. Census. In this section, we are just showing the correlation between house price growth and measures of gentrification. In the next section, we will try to test for a more causal relationship.

To analyze whether a potential neighborhood that experienced a rapid growth in prices also experienced signs of gentrification, we estimate the following regression:

$$y_{t,t+k}^{i,j} = \omega_0 + \omega_1 \frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}} + \mu^j + \nu_{t,t+k}^{i,j}$$

where $y_{t,t+k}^{i,j}$ is some measure of gentrification in neighborhood i of city/metro area j during the period t to $t+k$, $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$ is the growth rate in house prices within the neighborhoods during t and $t+k$ (as defined above) and μ^j is a vector of city/metro area fixed effects.

In Table 2, we estimate the above equations on potentially gentrifying neighborhoods from the 1990-2000 period and from the 1980-1990 period. The three gentrification measures explored in this table (i.e., our measures of $y_{t,t+k}^{i,j}$) are: (1) the percentage change in median income, (2)

the percentage point change in the poverty rate, and (3) the percentage change in the mean tenure of residents in the neighborhood. In essence, we are asking whether the areas that had high house price appreciation got richer at the same time and whether new residents moved into those neighborhoods. In row (1), our measure of $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$ is the percentage change in the Case-Shiller index between 1990 and 2000 within the zip code. As a result, we again restrict our analysis to only those zip codes covered by Case-Shiller. For this regression, we examine all metro areas covered by Case-Shiller during the 1990s. In particular, we estimate the above regressions by pooling together all the metro areas and including metro area fixed effects. Consistent with the model's predictions, neighborhoods that appreciated the fastest seemed to gentrify. Relative to other areas within the city, high house price appreciation neighborhoods experienced higher growth in median income, a bigger decline in the poverty rate, and a greater influx of new residents in that median tenure of residents fell. The results are sizable. For example, a 10 percent increase in housing prices within the zip code was associated a 1.25 percent increase in the median income of the zip code. This effect is large given that the average zip code in our sample only increased their median income by 6.4 percent between 1990 and 2000.

Columns (2)-(3) are analogous to column (1) except our measure of $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$ is now the percentage change in median census tract house prices (using Census data). No hedonics were done in this regression to adjust the house price change. We only restrict our analysis to those census tracts that did not change their boundaries between the beginning and end years and only included metro areas where there was at least 50 of such consistently defined census tracks. In column (2), we examine the relationship between our measures of gentrification and the change in median housing prices between 1990 and 2000 within census tracts of a metropolitan area. In column (3), we restrict our analysis to the 1980-1990 period. As with the results in Figure 9, we only include metro areas that have at least 50 consistently measured census tracts between 1980 (1990) and 1990 (2000) with non-missing house price information. Again, we find that neighborhood price appreciation within an area is correlated with observable measures of gentrification. For all results in Table 2, we re-estimated all the specification including the initial level of the variable being examined as a control. For example, we would include the initial level of median neighborhood income as a regressor when estimating the results in column (1). The results were very similar to those reported in the table. For this reason, we just simply reported the bi-variate relationship.[22]

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[22]We also explored the extent that house price growth was associated with gentrification in the 2000-2006 period. Given the lack detailed neighborhood characteristics in 2006 prevented us from doing an analysis similar

6 Industry Wage Shocks, Neighborhood House Price Appreciation, and Gentrification

One problem with the descriptive results above is that we have simply shown correlations and not anything related to causation. We have shown that, on average, low priced neighborhoods are more price elastic, that low priced neighborhoods in close proximity to high priced neighborhoods are the ones that the appreciate the most, and that these low price neighborhoods that appreciate the most show signs of gentrification (particularly prior to the recent cycle). In this section, we try to measure directly whether an exogenous shock to income in one neighborhood affects property prices and measures of gentrification in neighboring areas. As before, we also rule out that these effects are being driven by changes in commuting times (the common alternate explanation of within-city price movements). As with most of our results above, we are going to focus on within-city price movements and ignore the price dynamics in the suburbs. We are doing this so as to focus only on movements in prices within municipalities, allowing us to ignore the potential fiscal responses in different areas (in terms of taxing and spending) to shocks to income. All of our results will focus on within-city variation where tax and spending rates are held constant across neighborhoods. However, we wish to note that our results go through if we expand our analysis to include the suburbs.

6.1 Industry Wage Shocks and Neighborhood House Price Appreciation

To measure exogenous shocks to income for each neighborhood, we are going to use variation in national earnings by industry between 1990 and 2000. Using the national variation in earnings growth by industry between 1990 and 2000, we will predict the expected change in neighborhood income for each neighborhood in our sample based upon the neighborhood's industry mix in 1990. Our identifying assumption is that the change to earnings for each industry at the national level is orthogonal to anything else that would drive house prices in the local neighborhoods included in our analysis. This approach to imputing exogenous income shocks for local economies has been used extensively by others in the literature. For example,

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to what we did for the 1990 - 2000 period and for the 1980 - 1990 period (shown in Table 2). We were, however, able to use IRS data at the zip code level which tracks average AGI over time. We found that high zip code neighborhoods experienced a sharp decline in households with income less than $10,000 and less than $25,000 relative to other areas. These were two of the cutoffs that the IRS reported descriptive data. These results were robust to the inclusion of detailed controls for neighborhood characteristics in 2000. However, we found no systematic evidence in the difference in the change in mean AGI in high appreciation neighborhoods for this period relative to other neighborhoods. Some appreciating neighborhoods experienced large increases in AGI while others experienced declines in AGI. Similar results were reported in Mian and Sufi (2009). The fact that the mechanism highlighted in our paper explains better price movements in the 1980s and 1990s is consistent with additional results we discuss below.

see Blanchard et al. (1992).

Specifically, we will estimate the following regression:

$$
\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}} = \delta_0 + \delta_1 \ln(P_t^{i,j}) + \delta_2 \ln(Inc_t^{i,j}) + \delta_3 OwnIncShock_{t,t+k}^{i,j} + \delta_4 NeighborIncShock_{t,t+k}^{i,j} +
$$

$$
\eta_4 \ln(Commute_t^{i,j}) + \eta_5 \frac{\Delta Commute_{t,t+k}^{i,j}}{Commute_t^{i,j}} + \mu_j + \epsilon_{t,t+k}^{i,j}
$$

where $\frac{\Delta P_{t,t+k}^{i,j}}{P_t^{i,j}}$, $P_t^{i,j}$, $Commute_t^{i,j}$, $\frac{\Delta Commute_{t,t+k}^{i,j}}{Commute_t^{i,j}}$, and μ_j are defined as above. $Inc_t^{i,j}$ is the actual level of median income in neighborhood i of city j in period t (from the 1990 Census). $OwnIncShock_{t,t+k}^{i,j}$ is the predicted income growth for neighborhood i in city j between t (1990) and $t+k$ (2000) based on the industry mix in neighborhood i in 1990 while $NeighborIncShock_{t,t+k}^{i,j}$ is the predicted income growth of the neighbors of i between 1990 and 2000 based on the industry mix of their neighbors.

Before proceeding, an additional discussion of how we computed own and neighbor income shocks is needed. Specifically, using the 1 percent samples from the 1990 and 2000 IPUMS data, we caculated income growth between 1990 and 2000 for the 17 industry groupings (based on 2-digit SIC codes) that are available for the 1990 Census Summary File 3 at the zip code level. The only restriction we placed on the data was that the individual had to be employed and over the age of 16. Our measure of income was individual earnings. These two restrictions were necessary given that the industry breakdown for the Census zip code aggregates are for employed individuals over the age of 16. There is a large degree of variation in income growths between 1990 and 2000 across the industries. For example, the Personal Services industry had a real appreciation of annual earnings of 35.4% (followed by Business and Repair Services and Finance, Insurance and Real Estate at 30.7% and 27.2%, respectively) while the Transportation industry had a real appreciation of annual earnings of only 6.5%. The average industry had a real appreciation in earnings of 17.3% between 1990 and 2000 (with a standard deviation of 7.9%).

Using the growth rate in industry earnings over the 1990s, we can compute the predicted increase in earnings for each zip code between 1990 and 2000 based on their industry mix in 1990. Specifically, we multiply the industry growth rate in earnings by the fraction of people in each neighborhood working in those industries. Given the Case-Shiller indices are at the zip code level, we start our analysis by defining zip codes as our unit of analysis. Most zip codes contain individuals working in most industries. However, despite that, there is still variation in predicted incomes across neighborhoods. Within the zip codes we analyzed (the 572 zip

codes in major cities covered by Case-Shiller), the median predicted income change was 18.0 percent with a standard deviation of 1.0 percent. The 5th percentile of the predicted income change across zip codes was 16.0 percent while the 95th percentile was 19.3 percent. While the variation is not large when we aggregate to the level of zip code, some variation does exist.

Furthermore, for the zip codes in our data, predicted income changes based on the industry mix does predict actual income changes. Regressing actual income change in the zip code between 1990 and 2000 on the predicted income change and city fixed effects yields a coefficient on actual income changes of 3.97 with a standard error of 0.93 and an adjusted R-squared of 0.46. The incremental adjusted R-squared for the predicted income change above and beyond the city fixed effects is 0.04. Again, while there are lots of differences in income changes across zip codes, the predicted income change measure based on industry mix does have some predictive power.

For the change in income of one's neighbors, the key is defining which are the neighboring zip codes. We do this in a few ways. Throughout all of our analysis, we define neighbors as being those other zip codes that are spatially close to a given zip code. We measure "spatially close" in terms of distance to the centroid of the zip code. In the results below, we define tiers of neighbors: the 10 closest zip codes, the next 10 closest zip codes, and the 10 closest zip codes after that. We also have done everything in rings of the 5 closest zip codes. For the base results, the point estimates from being the 1-5 closest zip codes was similar to being the 6-10 closest zip codes, although the standard errors were larger. To increase the power of our analysis, we do everything - in this section - in rings of the 10 closest zip codes.

The results of estimating the above equation are shown in Table 3. In column 1 of Table 3, we estimate the equation only including the initial house price in the neighborhood, the initial income in the neighborhood, city fixed effects, and the own predicted income shock of the neighborhood. The regression indicates that an exogenous shock to the earnings of the residents in one's zip code is associated with an increase in house prices in the neighborhood. For example, going from the shock experienced by the 5th percentile of the zip codes to the 95th percentile of the zip codes (a 0.033 increase) will result in an increase in house prices of 17.9 percent (0.033 * 5.43).

In the second column, we include both the own income shock and the average income shock experienced by one's 10 closest neighbors (for now, we just simply average over the neighborhoods weighting each equally - we will change this with future iterations). The results show that a shock to one's neighbors' income increases property prices in one's own neighborhood -

even controlling for the predicted income shock in one's own neighborhood. This effect is also large. Moving from the 5th percentile of the average neighbor's income shock (17.1 percent) to the 95th percentile of the average neighbor's income shock (19.2%) increases house prices in one's neighborhood by 11.4 percent (0.021 * 5.42). A shock to income of neighboring zip codes has a large effect on the house prices in one's own zip code. This is very much consistent with the results of the model set forth above. Notice, in column 3, including the average income shocks of those zip codes that are the 11-20 and the 21-30 nearest zip codes have no effect on house prices in one's own neighborhood. It is only the shock to income for those zip codes that are spatially close to own's own zip code that affect property prices.

In the fourth and fifth columns, we show that the results are relatively unchanged if we include variables that measure how close the residents in the neighborhood are to the jobs in the city. Notice, that the inclusion of initial commuting times and the change in commuting times (column 4) have no effect on the spatially equilibrium results pertaining to the income shocks of neighbors. In other words, neighborhood house prices are not changing because the proximity to jobs is changing. In the fifth column, the dependent variable is the percentage change in commuting time in the neighborhood (instead of the percentage change in house prices). There are not statistically significant results on how shocks to income in a given neighborhood affect the distribution of commuting time in that neighborhood or in surrounding neighborhoods.

The results above show that income shocks tend to raise housing prices in both the neighborhood experiencing the income shock and in neighboring areas that did not experience the income shock. These results are not driven by changes in commuting times within the city. These results are consistent with the model described above. An increase in the income of one's neighbors will increase house prices in the neighboring areas via gentrification.

6.2 Industry Wage Shocks and Neighborhood Gentrification

Do house prices increase in the neighboring areas because those neighboring areas gentrified? In our model, that is the mechanism which leads house prices in neighboring areas to increase. Those who want to increase their housing demand (those in the neighborhood experiencing the income shock who want to buy bigger houses or those migrants into the city who want to work at the firms paying the higher wages), will do so by moving into the poorer neighborhoods in close proximity to the richer neighborhoods. In this last subsection, we explore whether the increases in income in one neighborhood actually lead to the gentrification of surrounding

neighborhoods.

To do this, we estimate:

$$y_{t,t+k}^{i,j} = \delta_0 + \delta_1 \ln(P_t^{i,j}) + \delta_2 \ln(Inc_t^{i,j}) + \delta_3 OwnIncShock_{t,t+k}^{i,j} + \delta_4 NeighborIncShock_{t,t+k}^{i,j} + \mu_j + \epsilon_{t,t+k}^{i,j}$$

where all the variables are defined as above. The results of this regression are shown in Table 4. Again our three measures of gentrification are percent change in median income in the neighborhood (column 1), percentage point decline in the poverty rate in the neighborhood (column 2) and percentage change in the median tenure of residents in the neighborhood (column 3). A predicted shock to income of residents in the neighborhood leads to increased income in the zip code, but has no statistically significant effect on either the poverty rate or the change in median tenure. However, the results are much stronger when one's neighbors receive a positive income shock. Receiving a positive income shock to neighboring zip codes causes one's own zip code to experience a large and statistically significant increase in median income, a statistically significant decline in the poverty rate, and a statistically significant decline the tenure of residents. On net, an income shock in one area causes richer households to move into neighboring neighborhoods. Their moving in is also associated with housing prices increasing dramatically. This is exactly the mechanism highlighted in the paper.

7 Gentrification and Cross City Differences in Price Appreciation

In this last section, we examine how much of the cross city variation in house prices between 1990 and 2000 and then again between 2000 and 2009 can be explained by differences in the changes in income across the cities (holding commuting times constant) and how much can be explained by differential supply elasticities across the cities. For this analysis, we use the repeat sales data from Federal Housing Finance Agency (formally OFHEO). The reason we do this is to expand the number of cities we analyze.[23] Figure 11 shows that there is a strong correlation between metro area house price changes and metro area income changes between 1990 and 2000. In Panel A of Figure 11, we include all metro areas where the population was greater than 125,000. There were 204 such metro areas. In Panel B of Figure 11, we

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[23]For this analysis, we are not interested in within city house price movements. The FHFA provide house price indices for nearly all metro areas within the U.S. extending back through the late 1980s. Given that the FHFA data are repeat sales data, they are very comparable to the Case Shiller data. For all metro areas which have an overlap between the FHFA data and the Case Shiller data, the patterns of house price appreciation are nearly identical period by period since 1990. The reason that the FHFA aggregate house price series shows a difference in appreciation rates relative to the Case Shiller data is due to differences in geographic coverage and not differences within a given geographic area. See Hurst's web page for a discussion.

show the patterns are nearly identical if we restrict the analysis to cities where density is very low. In particular, we only graph the relationship for those cities in the bottom third of the 1990 density distribution (out of all cities included in Panel A). Even in low density places, there is an equally strong relationship between house prices and income growth. For example, consider two low density cities in 1990. Austin, Texas experienced a 35 percent increase in income between 1990 and 2000 and a 45 percent increase in real house prices. Beaumont, Texas experienced only a 10 percent increase in income and only experienced a 10 percent increase in house prices.

In Tables 5 and 6, we formally examine the difference in house prices across cities between 1990 and 2000 (Table 5) and 2000 and 2009 (Table 6). For the 2000-2009 period, we use the change in income from the IRS where income is measured as adjusted gross income (AGI). The IRS data comes at the zip code level. We aggregate up to cities using both the zip code aggregates in AGI and the number of tax forms filed in each zip code. We do this by mapping zip codes to MSA.[24]

Specifically, for the results in Tables 5 and 6, we estimate:

$$
\frac{\Delta P_{t,t+k}^j}{P_t^j} = \delta_0 + \delta_1 \frac{\Delta Inc_{t,t+k}^j}{Inc_t^j} + \delta_2 SupplyConstraints_t^j + \delta_3 \frac{\Delta Inc_{t,t+k}^j}{Inc_t^j} * SupplyConstraints_t^j
$$
$$
+ \delta_4 \frac{\Delta Commute_{t,t+k}^j}{Commute_t^j} + \epsilon_{t,t+k}^j
$$

where $\frac{\Delta P_{t,t+k}^j}{P_t^j}$, $\frac{\Delta Inc_{t,t+k}^j}{Inc_t^j}$, and $\frac{\Delta Commute_{t,t+k}^j}{Commute_t^j}$ is the percentage growth in housing prices, income, and commuting time, respectively, for city j between t and t+k. $SupplyConstraints_t^j$ is vector of measures of the extent to which supply can adjust in the metro area. We use three measures for our analysis. The first is initial density within the metro area (measured as people per square mile). Again, low density may be a sign of binding supply constraints (which limit building). For our two other measures of supply constraints, we use standard measures in the literature. First, we use the Wharton Residential Land Use Regulatory Index. This index is available for about 100 metro areas. The index measures how hard it is to build in a metro area incorporating such regulatory constraints as minimum lot size and the time it takes to get approval for a building permit (see Gyourko, Saiz, and Summers (2008) for details). Second, we use the developable land measure within a metro area as computed by Saiz (2009). This measure is used as the basis to compute long run differences in housing supply elasticities across

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[24]The IRS data is not available at the zip code level in 2000. Instead we use the 2001 data (which pertains to income earned in 2000). The last year of data we have available is the 2008 data.

locations. The basis for this measure is differences in land gradients (which increase the cost of building) and water coverage within a metro area. Given the two supply constraint measures are not computed for all cities, we restrict our analysis to include only the 185 cities for which we have complete measures.

As seen from column 1 of Table 5, changes in income alone explain 35 percent of the variation in house prices growth across cities between 1990 and 2000s. This is consistent with the results seen in Figure 11. In column 2 of Table 5, we include only the supply constraint measures. Supply constraints explain only 7 percent of the variation across cities. Column 3, we include both the change in income and the supply constraints, the incremental adjusted R-squared only increases by 2 percentage points (from 0.35 to 0.37). In essence, changes in income explain most of the variation in house prices across cities in the 1990s (relative to supply constraints). The interesting results are in columns 4 - 6. Most of the relationship between house price growth and income growth is found even in cities where supply constraints are not binding. This is seen from the fact that the coefficient on the change in income is large, positive and statistically significant in all specifications. The results suggest, as theory predicts, that the correlation between income growth and housing price growth gets slightly stronger when supply constraints are binding. However, even where supply constraints are not binding, changes in income have large effects on house prices. This is exactly what is predicted by a model where there are positive externalities from living around rich people. An increase in income induces gentrification and increases house values. In the last four columns, we control for changes in commuting time within the city. This allows us to conclude that house prices are not increasing because people are bidding up land prices near to jobs which, by implication, would increase the average commute time in the city. As another test (not reported), we find that the change in income within a city is not systematically related to changes in average commuting time within the city.

The results are quite different for the 2000-2009 period (Table 6). The specifications presented in the table are analogous to those shown in Table 5. During the recent period, differences in income growth across neighborhoods only explain 7 percent of the variation in house prices across cities. Again, like the earlier periods, the change in income is positively associated with the increase in house prices. Supply constraints, however, by themselves explain roughly 29 percent of the variation in house prices across locations. This seems to suggest that there may have been a large demand shock for housing that was orthogonal to changes in income (or was larger for low income households than high income households). A reduction in interest rates

and an extension of credit to low income borrowers would be consistent with these findings. But, as seen from Table 6 and from the within city results above, our mechanism was also active during the current cycle. It is just that our mechanism during this period is small relative to other stories that explain cross city housing price dynamics.

8 Conclusion

In this paper, we have focused our attention on within city price movements to learn about housing price dynamics across cities. We have presented a spatial equilibrium model of property price movements across neighborhoods within a city to understand how such a mechanism will explain price movements both within and across cities. There are two key features of our model. First, we analyze a linear city with rich and poor households. Second, we assume that there is a positive neighborhood externality: households like to live closer to rich neighbors. One story behind this assumptions is that the more rich people live in a neighborhood, the greater is the number and the variation in services and public goods (restaurants, museums, coffee shops, dry cleaners, etc.) that are going to be provided. The importance of urban density in facilitating neighborhood externalities has been recently emphasized in the work of Glaeser, Kolko, and Saiz (2001) and in Becker and Murphy (2003). Our innovation is to embed these externalities into a model of neighborhood development within a city and show how they affect the evolution of house prices across neighborhoods in reaction to housing demand shocks.

The existence of such externalities affects the nature of housing price dynamics in important ways. Our analysis focuses on an equilibrium where there is full segregation: the rich households live together and the poor live in the periphery of the rich. Hence, after a demand shock, there is going to be endogenous gentrification. The rich households are going to expand into the adjacent poor neighborhoods (perhaps through the in-migration of new residents), driving the house prices up, due to the externality. Our model predicts that, even in the presence of an elastic housing supply at the city level, house prices increase permanently in reaction to an unexpected and permanent demand shock. Moreover, our model puts some structure on which factors may drive different house price dynamics across different cities. In particular, we show that richer cities gentrify more in reaction to the same shock, and hence experience higher average price increases. Also, higher gentrification leads to higher convergence of prices across neighborhoods. This suggests not only that different cities may react differently to a common demand shock, but also that the same city, at different stages of its development, may experience differential housing price responses to housing demand shocks of similar size.

We start by using a variety of data sources to show two new empirical facts. First, during housing price booms (busts), the neighborhoods with lower initial house prices experience larger appreciation (depreciation) than neighborhoods with higher initial prices. This fact is robust to the use of different housing price series and across different time periods. Second, we find that the low priced neighborhoods which appreciate the most are the ones spatially closer to the high priced neighborhoods. We also show that these neighborhoods that experience high price growth also show strong signs of gentrification. We measure gentrification as being an increase in median income, a fall in the poverty rate and a decline in the average tenure of residence (relative to other neighborhoods within the city). The first two facts that we have documented are persistent in all time periods while the latter gentrification results are most pronounced in the 1980s and 1990s, relative to the current cycle.

We then examine what happens to property prices and measures of gentrification in one's own neighborhood and in surrounding neighborhoods when the neighborhood receives an exogenous (at least to local property markets) shock to income. Our measure of the income shock is the predicted change in earnings of residents in a given neighborhood based on the industry mix of the residents. We use national data to compute changes in earnings by industry. Using this approach, we find strong support for our mechanism. A shock to income in one neighborhood results in both gentrification and increases in house prices of surrounding neighborhoods. The results are very strong for close neighbors and almost non-existent for neighbors farther away. Moreover, these results persist even after controlling for initial commuting times and changes in commuting times within the neighborhoods.

This latter result is important. Most of the literature explains within neighborhood price differences being driven by differences in commuting costs across neighborhoods. Those neighborhoods spatially far from jobs are less desirable than those that are spatially close to jobs. In this paper we propose a new mechanism that says that it is not only proximity to jobs that matters but also proximity to certain types of neighbors that drive differences in land prices within a city. Given that our results are very robust to controls for commuting time, we show that proximity to rich people matters when determining land prices above and beyond proximity to jobs.

This paper has lots of implications for both urban development policies and for explaining both cross city and aggregate house price dynamics. The model predicts that even in a model where firms are mobile, an increase in the skill premium in the U.S. will lead to increased land prices. The existence of either more rich people or richer rich people will cause land prices to

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increase. If our mechanism is true, land is more valuable when it is populated by richer people. Furthermore, if our mechanism is true, it sheds light on how cities can change the nature of development. Is it more important to subsidize firms or subsidize local amenities?

In the last part of the paper, we assess how much traditional supply constraints or changes in income can explain cross city differences in price appreciation rates. We find that during the 1990s, changes in income explain much of the differences in house prices across cities while the role for traditional supply constraints was quite limited. However, during the 2000s, changes in income explained little of the change in house prices across neighborhoods. Instead, traditional supply constraints had large predictive power for explaining cross city price differences.

One area on which our paper is silent is the welfare implications of our model. A full model – potentially incorporating commuting costs – could be used to quantitatively assess the welfare effects of a housing demand shock. Although we did not attempt such an analysis in this paper, we feel this is an important area for future research.

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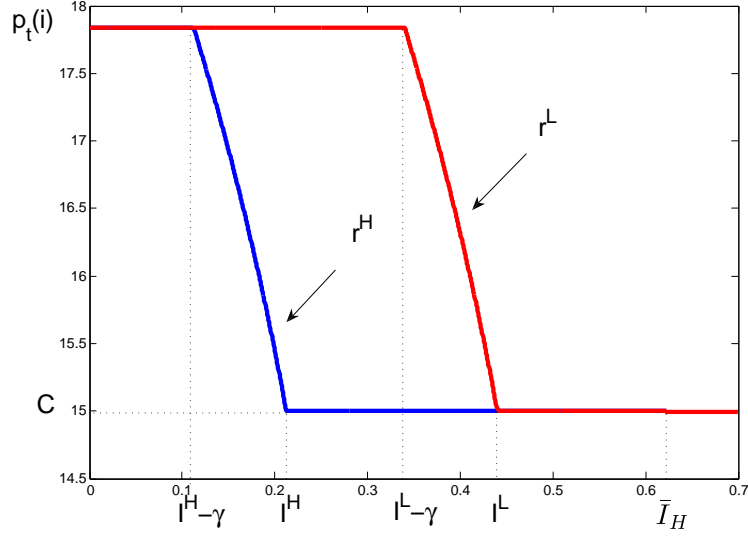
Figure 1: Reaction of the price schedule to a decline in the interest rate. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
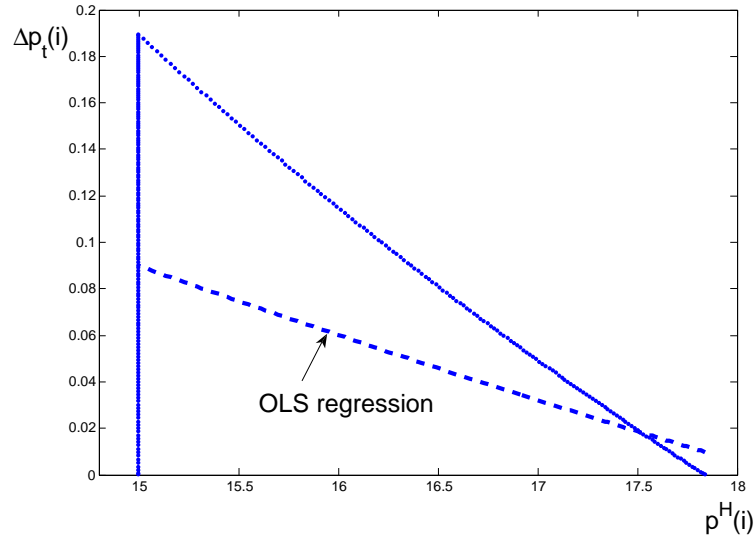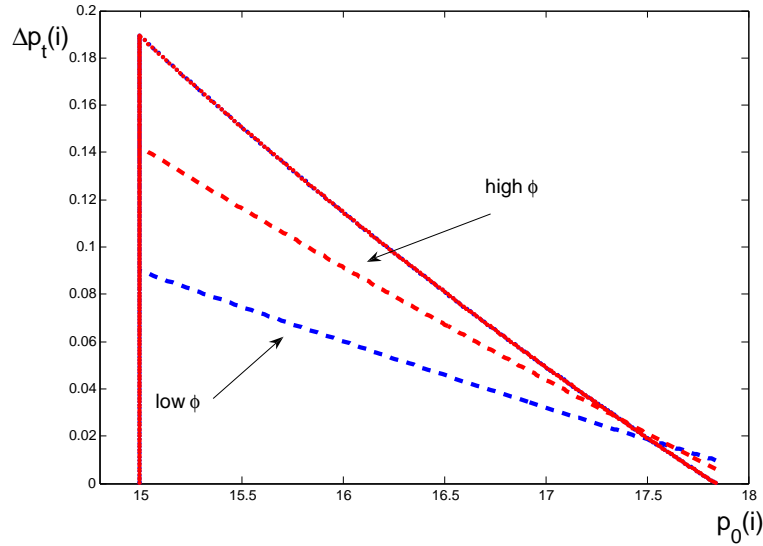

Figure 2: Price growth rate across locations as a function of the initial price level and OLS regression. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
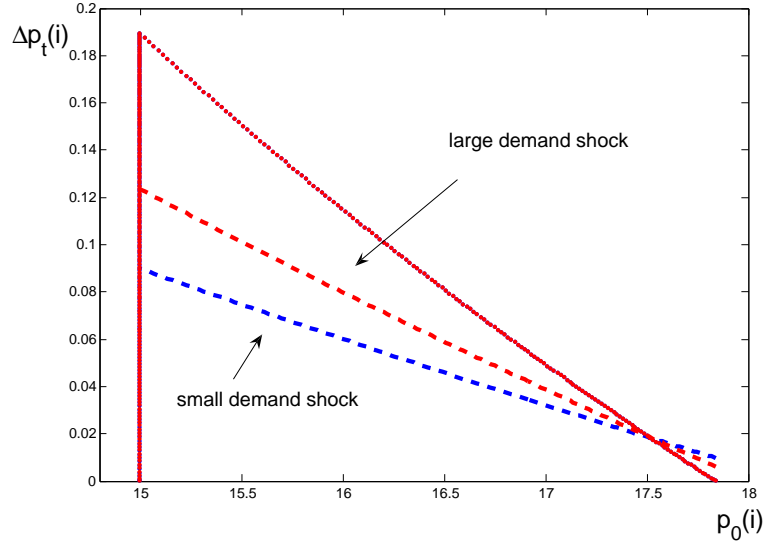
Figure 3: Price growth rate across locations as a function of the initial price level in reaction to a decline of the interest rate from .035 to .03 (blue dots) and from .035 to .028 (red dots). The associated solid lines represent the OLS regressions. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$.

Figure 4: Price growth rate across locations as a function of the initial price level in reaction to a decline of the interest rate from .035 to .03 in a city with $\phi^R = 1$ (blue dots) and one with $\phi^R = 1.1$ (red dots). The associated solid lines represent the OLS regressions. We set $\alpha = .8$, $\beta_R = .2$, $\beta_P = 0$, $A = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C^P = C^R = 15$, $r^L = .03$, and $r^H = .035$.
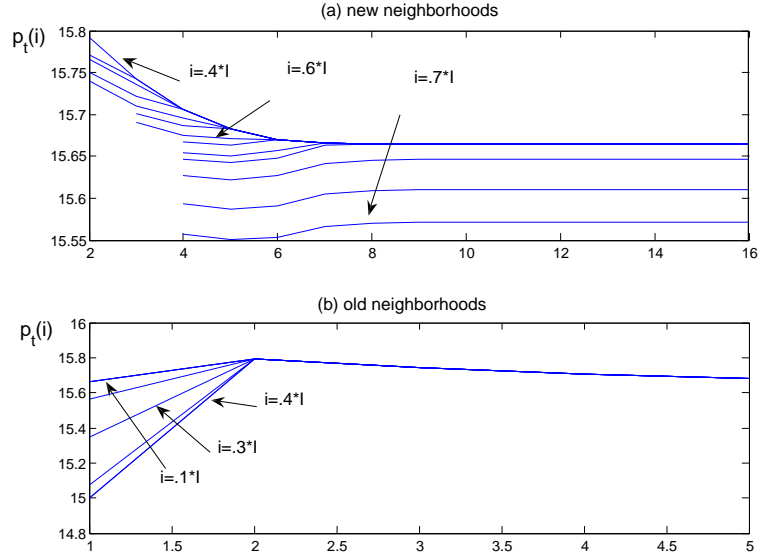
Figure 5: The figure shows the price behavior in different locations in reaction to a decline in the interest rate. Panel (a) shows neighborhoods that are born as a result of the shock, while panel (b) shows neighborhoods that existed at the initial steady state. We set $\alpha = .8$, $\beta = .05$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C_1^= 15$, $C_2 = 2$, $\psi = 2$, $r^L = .03$, and $r^H = .035$.
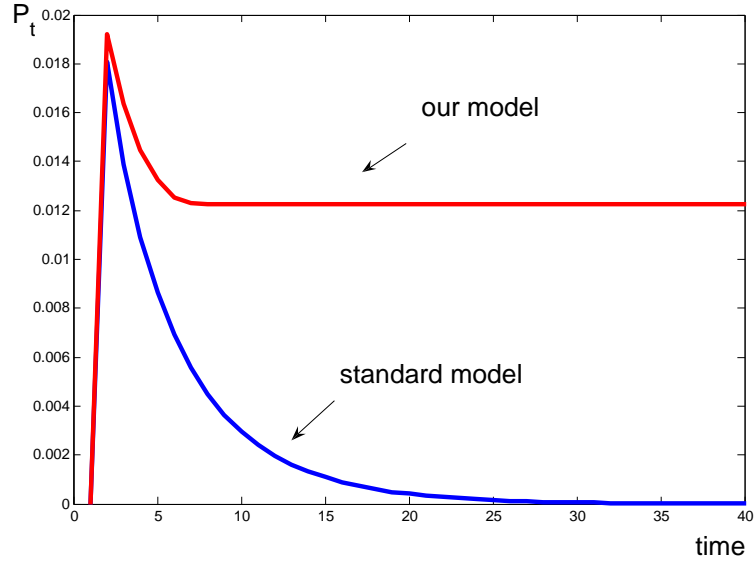
Figure 6: The figure shows the behavior of aggregate house prices after a positive demand shock in our model (red line) and in the standard adjustment cost model (blue line). We set $\alpha = .8$, $\beta = .05$, $\phi^R = 1$, $\phi^P = .7$, $\gamma = .1$, $N^R = N^P = .5$, $C_1^= 15$, $C_2 = 2$, $\psi = 2$, $r^L = .03$, and $r^H = .035$. In the standard adjustment cost model we set all the same common parameters.
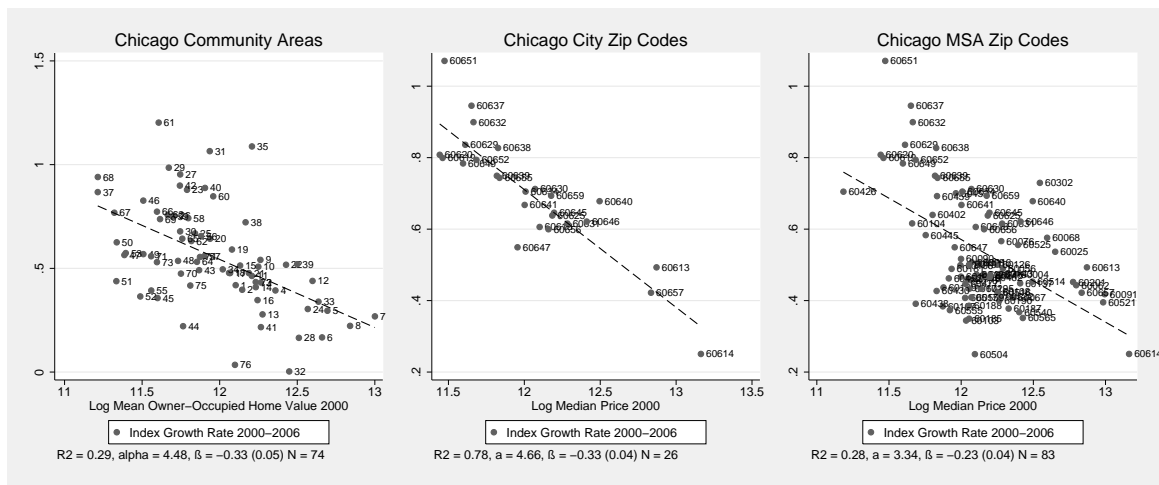
A: Chicago
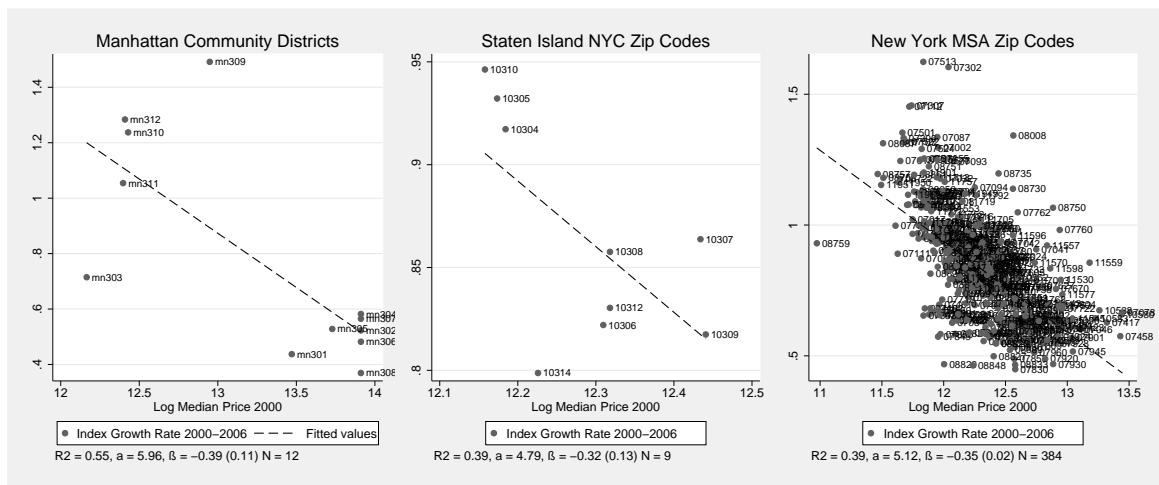
B: New York
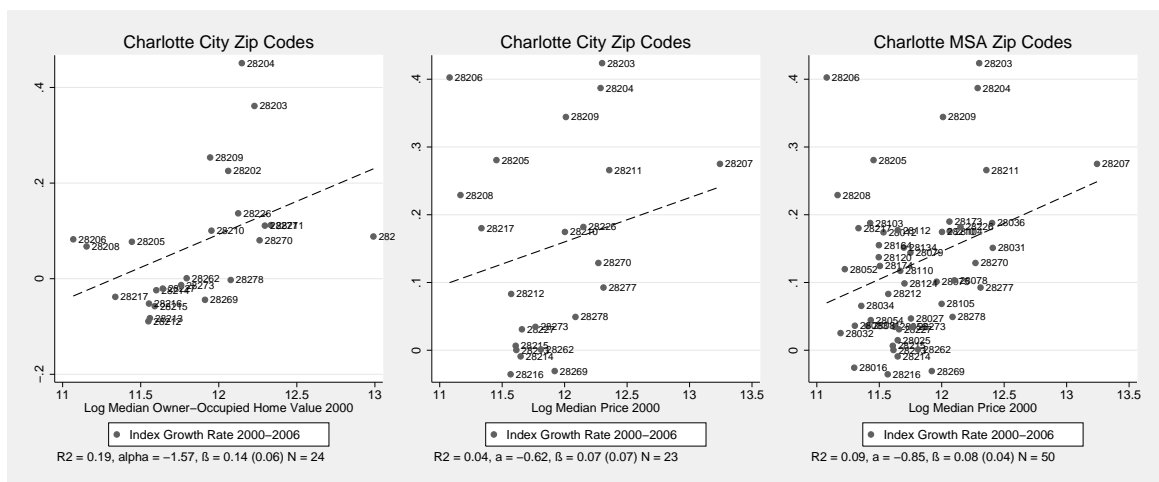
C: Charlotte

Figure 7: Price Growth vs. Initial Price 2000-2006 for Chicago, NYC, and Charlotte.
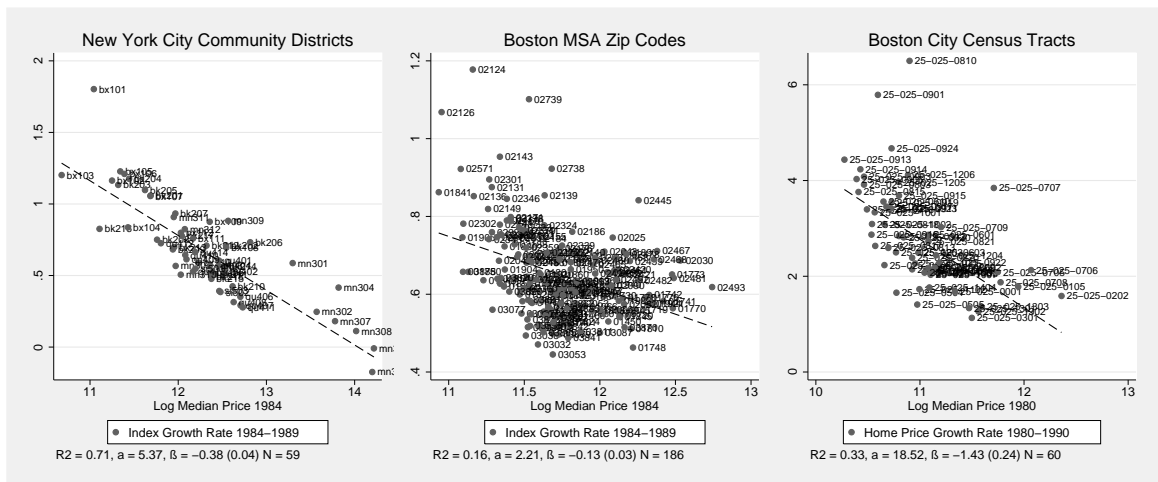
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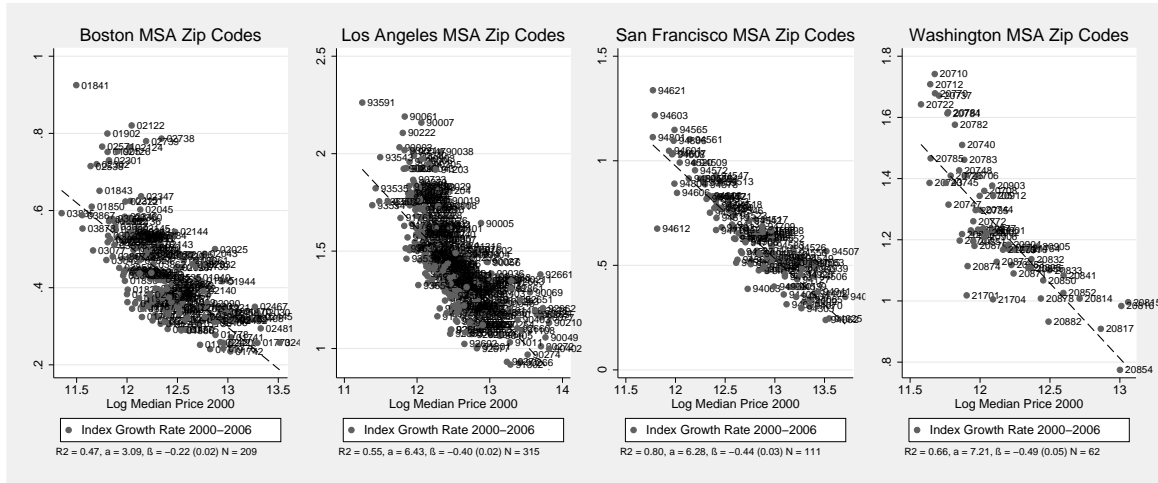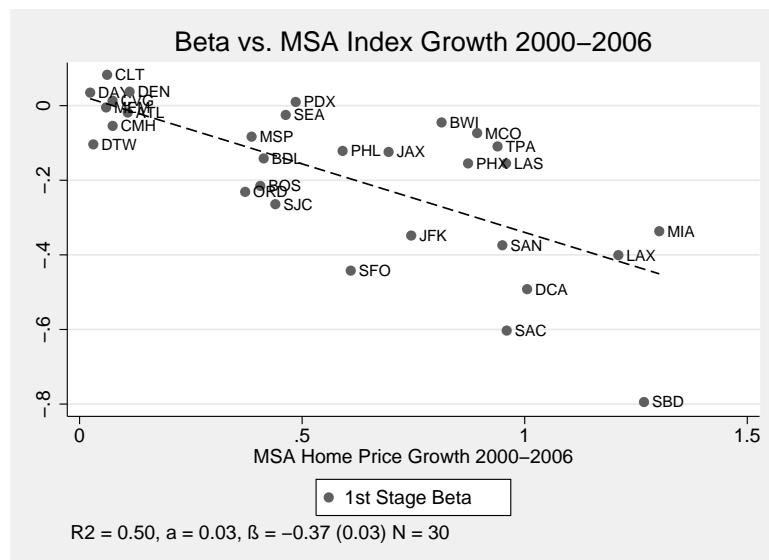
A: Other cities 2000-2006.
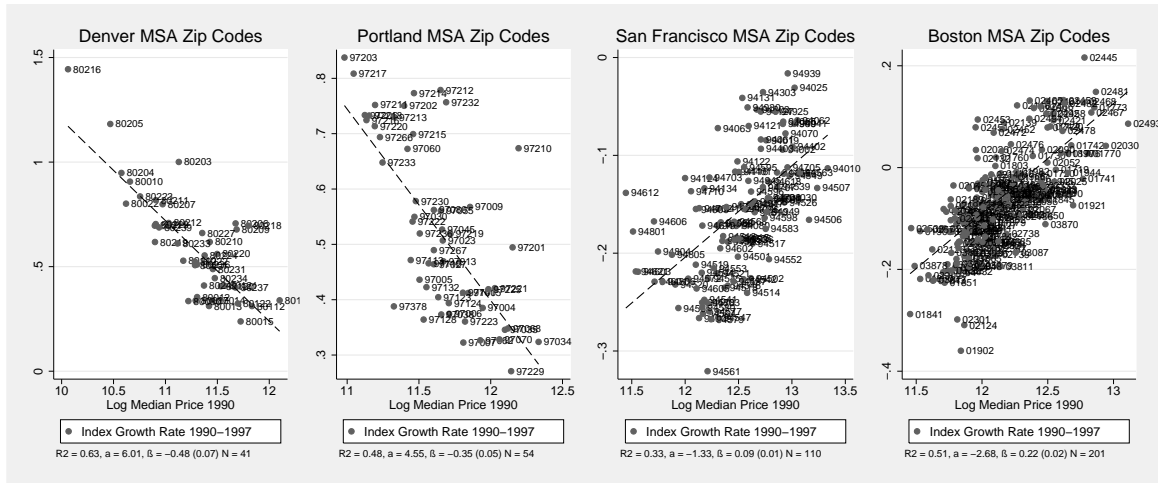
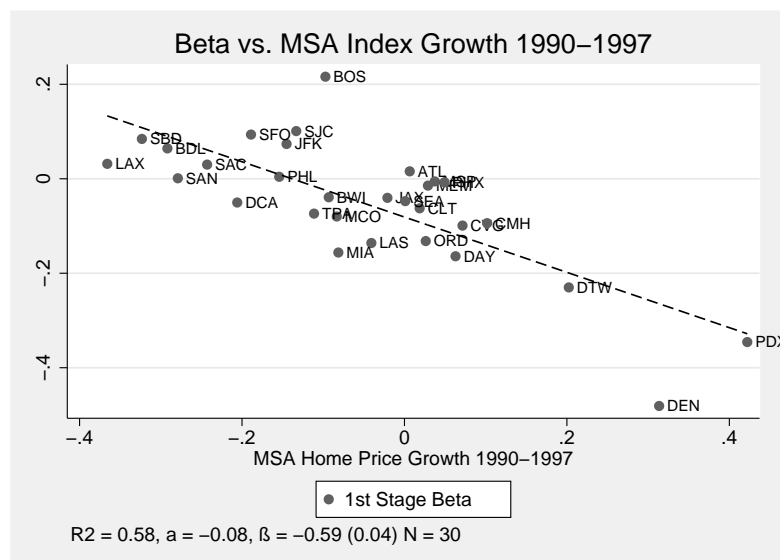

B: Other cities 1990-1997.
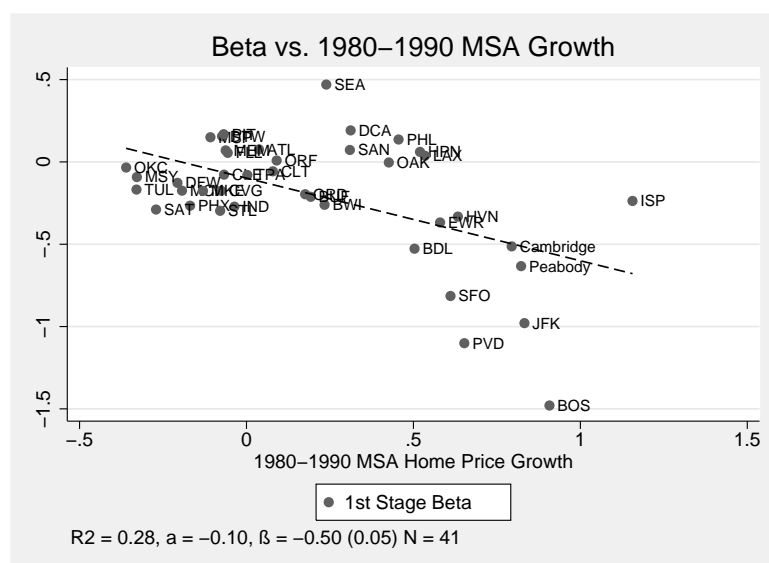

C: Other cities 1984-1989.

Figure 8: Price Growth vs. Initial Price during several periods for several other cities.
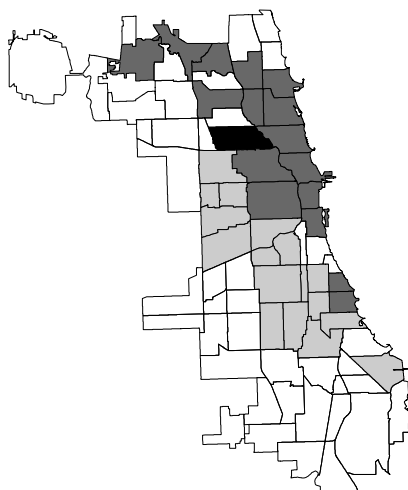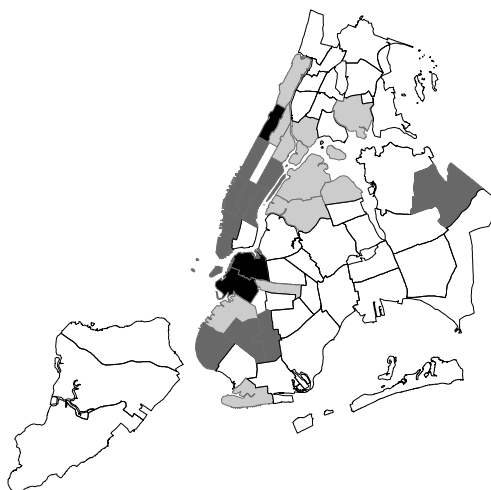
A: 2000-2006.

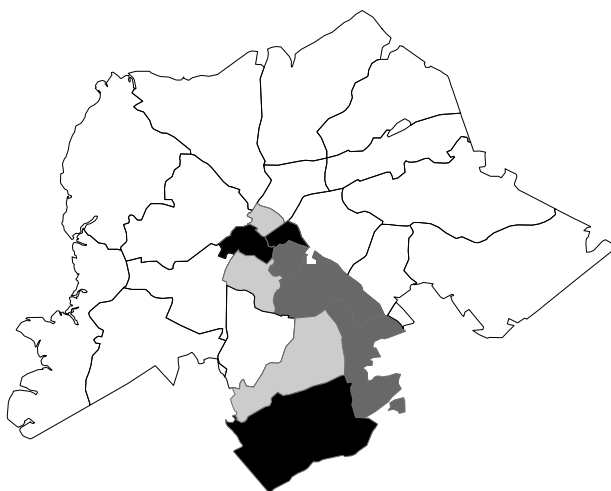B: 1990-1997.

C: 1980-1990.

Figure 9: Beta vs. MSA Price Growth
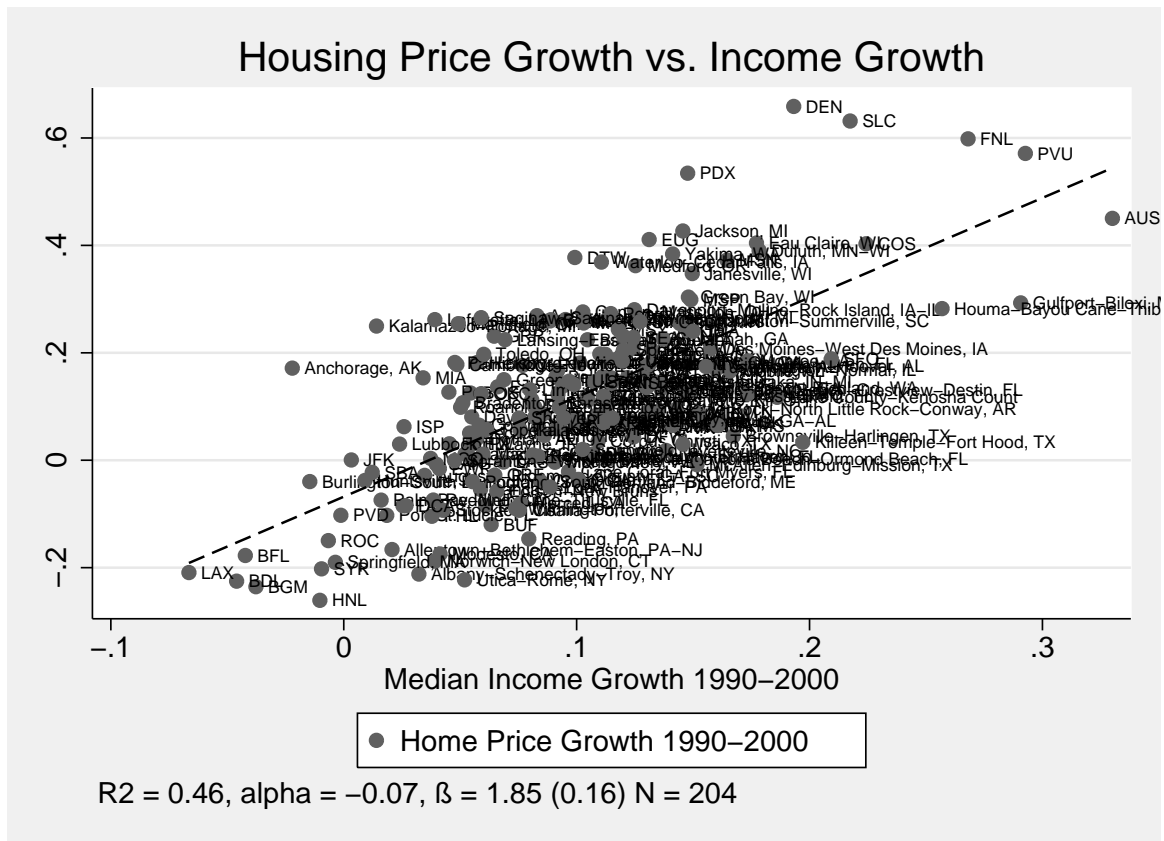
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A: Chicago City
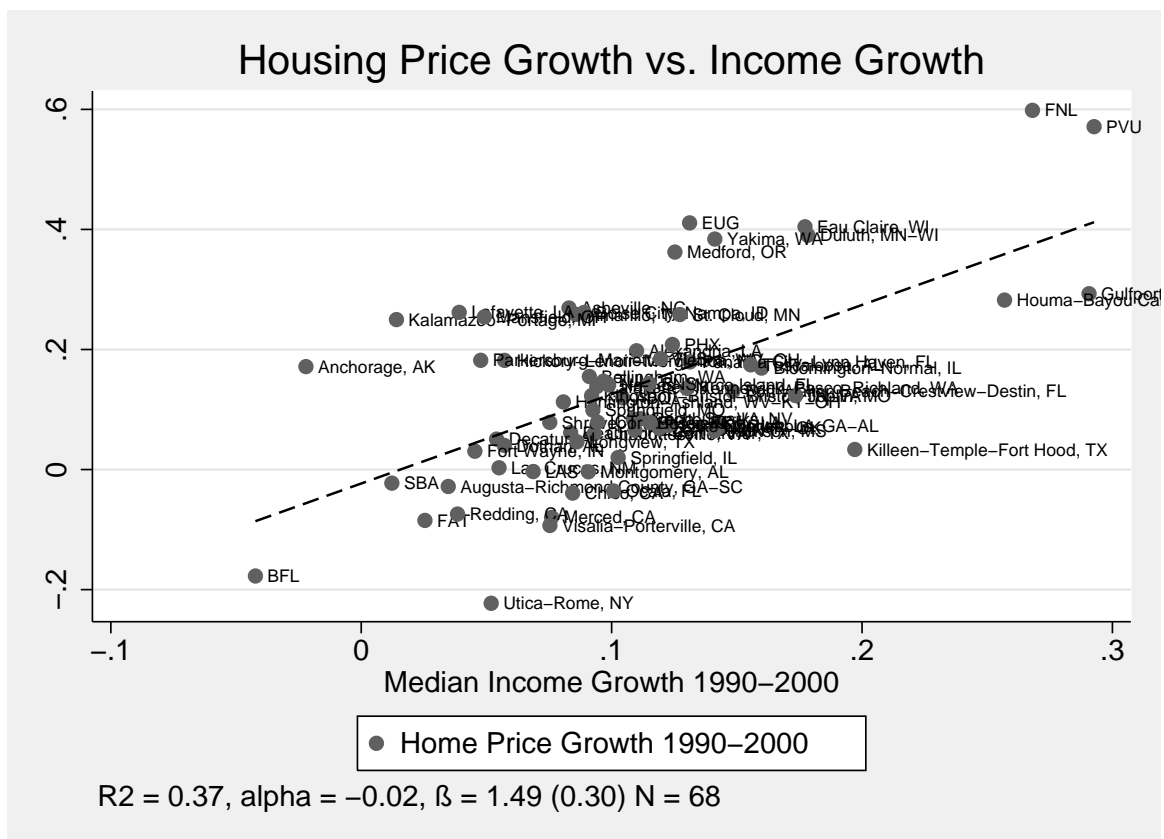

B: New York City

C: Charlotte City

Figure 10: Diffusion Maps. Dark shaded areas are high price in 2000, light shaded ares are high growth 2000 - 2006, black regions are both.

A: All MSAs over 125K in Population

B: Bottom Tercile of Population Density Distribution for MSAs over 125K in Population

Figure 11: Housing Price Growth vs. Income Growth 1990 - 2000.

Table 1: The Relationship Between House Price Growth Among Low Price Neighborhoods and Distance to High Price Neighborhoods

	(1)	(2)	(3)	(4)	(5)	(6)
	Case-Shiller Zips (277 obs.) 2000-2006		Case-Shiller Zips (281 obs.) 1990-2000		Census Zips (500 obs.) 1990-2000	
$\log HP_t^{i,j}$	-0.19	-0.22	-0.17	-0.20	-1.21	-1.26
	(0.09)	(0.09)	(0.06)	(0.06)	(0.34)	(0.32)
$\log HP_t^{i,j} * \overset{g}{H}\overset{j}{P}_{t,t+k}$	-0.57	-0.55	-2.25	-2.19	-7.44	-6.74
	(0.12)	(0.13)	(0.61)	(0.58)	(3.46)	(3.34)
Log Dist. to Nearest High Price Neighborhood$_t$	-0.09	-0.08	-0.07	-0.07	-0.28	-0.17
	(0.02)	(0.02)	(0.02)	(0.02)	(0.10)	(0.09)
Log Mean Commuting Time$_t$		-0.18		-0.36		-2.16
		(0.11)		(0.12)		(0.58)
Percent Change in Commuting Time$_{t,t+k}$				-0.36		-0.35
				(0.22)		(0.79)
City Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Mean Dist. to High Price	3.9 miles		3.9 miles		3.5 miles	
Mean Commuting Time	28.9 minutes		27.4 minutes		29.3 minutes	
Mean Gr. Commute Time			10.0%		9.3%	

The samples in columns (1) and (2) contain zip codes in the bottom two quartiles of median owner-occupied home value from the 2000 census. The samples in columns (3) through (6) contain zip codes in the bottom two quartiles of median owner-occupied home value from the 1990 census. The dependent variable, $\overset{g}{H}\overset{i,j}{P}_{t,t+k}$, is Case-Shiller zip code home price growth from 2000-2006 for columns (1) and (2), Case-Shiller zip code home price growth from 1990-2000 for columns (3) and (4), and Census zip code home price growth from 1990-2000 for columns (5) and (6)

Table 2: Relationship Between House Price Growth and Measures of Gentrification

	(1) Case-Shiller Zip Codes 1990-2000	(2) Census Zip Codes 1990-2000	(3) Census Tracts 1980-1990
Percent Change in Neighborhood Median Income	0.181 (0.018) Mean = 0.060	0.184 (0.029) Mean = 0.129	0.080 (0.012) Mean = 0.219
Percentage Point Change in Neighborhood Poverty Rate	-0.042 (0.005) Mean = 0.008	-0.024 (0.005) Mean = -0.009	-0.010 (0.002) Mean = 0.016
Percent Change in Mean Tenure in Home	-0.18 (0.02) Mean = 0.050	-0.08 (0.01) Mean = 0.023	-0.33 (0.06) Mean = 0.92

Table shows the coefficient on housing price growth in a regression of measures of gentrification on house price growth and metro area fixed effects. The variation in the table is across neighborhoods within a metro area. Each cell in the table is a separate regression. In column 1, our sample includes all zip codes (our measure of neighborhood) covered within the Case-Shiller data between 1990 and 2000. The measure of house price growth in these regressions is the percentage increase in house prices at the zip code level using the Case-Shiller index between 1990 and 2000. In column 2, we include zip codes from major metropolitan areas broadly. For these regressions, the measure of house price change is the change in median house price in the zip code (without any hedonic adjustments). The third column is analogous to the second column, except for the 1980-1990 period and for census tracts rather than zip codes. Our measures of gentrification are from zip code and census tract data from the Censuses.

Table 3: House Price and Commuting Time Response to Industry Earnings Shocks of Own and Adjacent Neighborhoods

| Dependent Variable: | (1) | (2) | (3) | (4) | (5) |
	Neighborhood House Price Growth				Neighborhood Commute Time Growth
Predicted Income Shock	5.43	3.74	4.04	4.21	-5.20
	(1.47)	(1.67)	(1.68)	(1.62)	(17.3)
Avg. Neighborhood Shock: 1-10		5.42	4.71	4.89	26.2
		(1.80)	(1.77)	(1.68)	(19.4)
Avg. Neighborhood Shock: 11-20			-0.25	2.15	-9.80
			(0.07)	(1.66)	(18.3)
Avg. Neighborhood Shock: 21-30			-0.18	-2.36	5.28
			(0.03)	(1.61)	(17.80)
Log Commuting Time				-0.34	
				(0.07)	
Percentage Change in Commuting Time				-0.43	
				(0.13)	
Init. Inc. and Init. House Price Controls	Yes	Yes	Yes	Yes	Yes

Columns (1)-(4) of this tables reports the coefficients of different variables from a regression of house price change between 1990 and 2000 (measured by growth in the Case-Shiller index) in neighborhood i of city j on the predicted income change in neighborhood i of city j during the same period as well as the average predicted income shock of neighborhood of various degrees of proximity to neighborhood i of city j. Our measure of neighborhood in this regression is zip code. In the regression, we partition proximity to neighbors based on the 10 zip codes that are spatially closest (measured in distance to midpoint) to the reference zip code. Avg. Neighborhood Shock: 11-20 is a simple average of the income shock over the zip codes that were the 11th to the 20th closest to the reference zip code. Neighborhood income shocks are predicted using the industry mix of residents in the neighborhood. See text for details. The regression also includes controls for the initial house price in the zip code (from the 1990 census), average median income of the zip code (from the 1990 census), log commuting time of residents in the zip code (from the 1990 census), the change in commuting time of residents in the zip code (between the 1990 and 2000 census), and city fixed effects. The regression is restricted to 583 zip codes in the Case Shiller data residing within the main sample cities. In the last column, the dependent variable is the growth in commuting time of the residents in neighborhood i of city j between 1990 and 2000.

Table 4: House Price and Commuting Time Response to Industry Earnings Shocks of Own and Adjacent Neighborhoods

Dependent Variable	(1) Percentage Change in Median Income	(2) Percentage Point Change in Poverty Rate	(3) Percentage Change in Tenure
Predicted Income Shock	2.61 (1.20)	-0.17 (0.36)	-1.87 (1.18)
Avg. Neighborhood Shock: 1-10	4.34 (1.52)	-1.34 (0.44)	-2.79 (1.43)
Avg. Neighborhood Shock: 11-20	-0.06 (1.30)	0.17 (0.38)	-1.09 (1.45)
Avg. Neighborhood Shock: 21-30	-3.04 (1.27)	0.81 (0.37)	0.83 (1.60)

This tables reports the coefficients of different variables from a regression of measures of neighborhood gentrification between 1990 and 2000 on measures of predicted neighborhood income shock between 1990 and 2000 and the average predicted income shock of adjacent neighborhoods between 1990 and 2000. See the note to Table 3 for details. The regression also includes city fixed effects. Our three measures of gentrification are as defined in the note to Table 2. As in Table 3, the sample for this regression are the Case Shiller zip codes (restricting the sample to only those in the main city of the metro areas sampled by Case-Shiller).

Table 5: Income Growth, Supply Constraints, and Cross City Differences in House Price Appreciation: 1990-2000

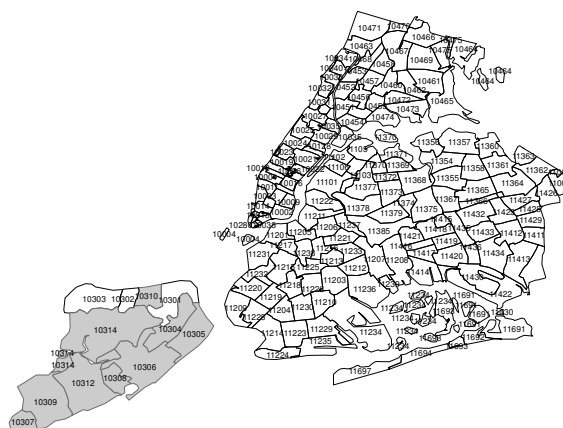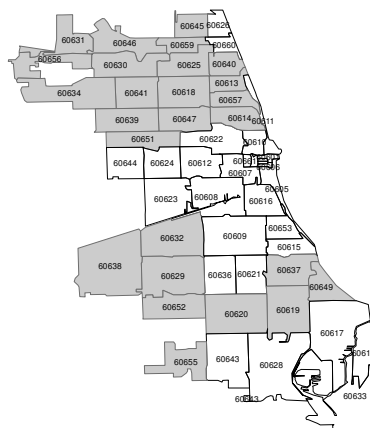|	(1)	(2)	(3)	(4)	(5)	(6)
Percentage Change in Income	1.37 (0.15)		1.35 (0.16)	1.17 (0.18)	1.13 (0.33)	1.17 (1.53)
Log Initial Density		-0.02 (0.02)	0.01 (0.02)	0.00 (0.02)	0.01 (0.01)	0.01 (0.01)
Wharton Residential Land Use Regulation Index (WRLURI)		0.02 (0.02)	0.00 (0.01)	-0.01 (0.01)	0.00 (0.01)	0.00 (0.01)
Saiz Undevelopable Land Measure		-0.20 (0.07)	-0.13 (0.06)	-0.16 (0.06)	-0.17 (0.07)	-0.13 (0.06)
WRLURI * % Change in Income				0.55 (0.26)		
Saiz Measure * % Change in Income					0.54 (0.84)	
Log Density * % Change in Income						0.02 (0.18)
Percent Change in Median Commute			-0.03 (0.10)	-0.06 (0.10)	-0.03 (0.10)	-0.03 (0.10)
Adjusted R^2	0.35	0.07	0.37	0.38	0.38	0.38

This table reports the coefficient of a regression of changes in house prices at the metropolitan area level between 1990 and 2000 against the change in median income at the metropolitan area level between 1990 and 2000, measures of supply constraints, and the change in commuting time for residents in the metro area between 1990 and 2000. The change in house prices at the metro level came from the FHFA repeat sales price index. Log density, median income and median commuting times come from the Census. See text for details of the WRLURI and the Saiz measure of undevelopable land.
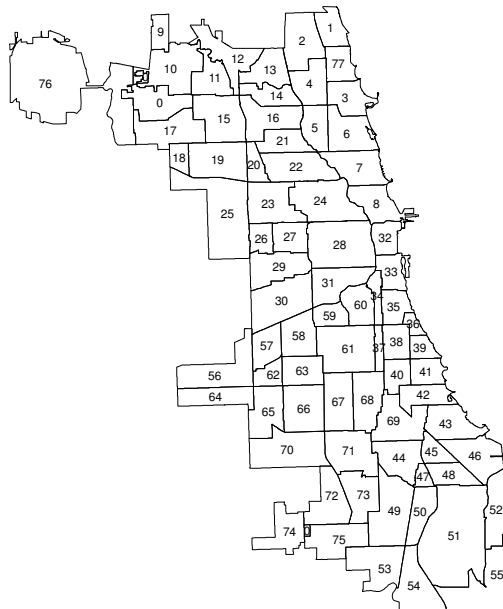
Table 6: Income Growth, Supply Constraints, and Cross City Differences in House Price Appreciation: 2000-2009

	(1)	(2)	(3)	(4)	(5)	(6)
Percentage Change in Income	0.51		0.29	0.14	0.14	0.43
	(0.15)		(0.11)	(0.08)	(0.10)	(0.68)
Log Initial Density		0.04	0.04	0.04	0.04	0.05
		(0.03)	(0.03)	(0.01)	(0.03)	(0.03)
Wharton Residential Land Use Regulation Index (WRLURI)		0.02	0.02	0.02	0.02	0.02
		(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
Saiz Undevelopable Land Measure		0.67	0.64	0.65	0.65	0.65
		(0.10)	(0.10)	(0.10)	(0.10)	(0.10)
WRLURI * % Change in Income				0.07		
				(0.13)		
Saiz Measure * % Change in Income					0.02	
					(0.22)	
Log Density * % Change in Income						-0.04
						(0.09)
Adjusted R^2	0.07	0.29	0.31	0.31	0.31	0.31

This table reports the coefficient of a regression of changes in house prices at the metropolitan area level between 1990 and 2000 against the change in median income at the metropolitan area level between 1990 and 2000, measures of supply constraints, and the change in commuting time for residents in the metro area between 1990 and 2000. The change in house prices at the metro level came from the FHFA repeat sales price index. Log density, median income and median commuting times come from the Census. See text for details of the WRLURI and the Saiz measure of undevelopable land.

A: Chicago

B: New York City

C: Charlotte

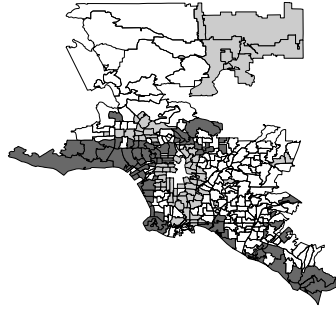Figure A1: Shaded Zip codes are Covered by Case-Shiller Indices in 2005
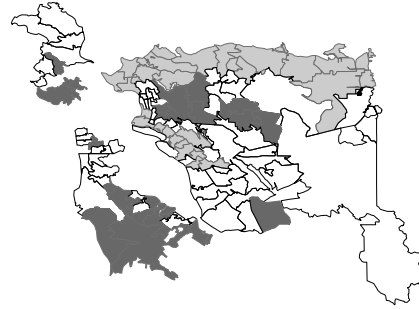
A: Chicago
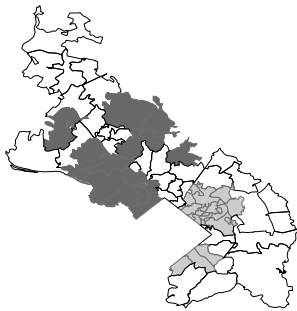

B: New York City

Figure A2: Chicago Community Areas and NYC Community Districts
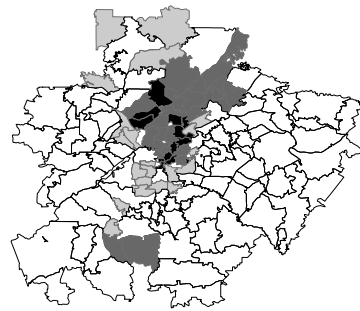
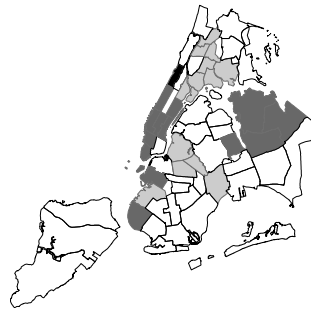A: Los Angeles Metro 2000 - 2006   B: San Francisco Metro 2000 - 2006

C: Washington DC Metro (MD) 2000 - 2006   D: Atlanta Metro 2000 - 2006

E: New York City 1984-1989

Figure A3: Diffusion Maps: dark shaded areas are high price in initial year, light shaded ares are high growth over period listed, black regions are both.

D1 Appendix

D1.1 Proof of Proposition 1

Most of the proof of Proposition 1 is in the text. As we argue in the text, we are left to check only that

$$U^R(i) \leq \bar{U}^R \text{ for all } i \in \left[I_t, \bar{I}_t\right],$$

$$U^P(i) \leq \bar{U}^P \text{ for all } i \in [0, I_t],$$

where $U^s(i)$ is defined in expression (5). Using expression (6), these two conditions can be rewritten as

$$K_R(A + H_t(i))^{\frac{\beta_R}{\alpha}} \leq K_P^{\frac{\beta_P}{\alpha}}(A + H_t(i)) + \frac{r}{1+r}\left(C^R - C^P\right) \text{ for all } i \in \left[I_t, \bar{I}_t\right], \quad (23)$$

$$K_P^{\frac{\beta_P}{\alpha}}(A + H_t(i)) \leq K_R^{\frac{\beta_R}{\alpha}}(A + H_t(i)) - \frac{r}{1+r}\left(C^R - C^P\right) \text{ for all } i \in [0, I_t]. \quad (24)$$

Combining (8) with (12) and (13) we obtain

$$K_P = \frac{r}{1+r}C^P A^{-\frac{\beta_P}{\alpha}},$$

$$K_R = \frac{r}{1+r}\left[C^P\left(\frac{A}{A+\gamma}\right)^{-\frac{\beta_P}{\alpha}} + \left(C^R - C^P\right)\right](A+\gamma)^{-\frac{\beta_R}{\alpha}}.$$

Using these expressions, condition (23) can be rewritten as

$$\left[\frac{A + H_t(i)}{A+\gamma}\right]^{\frac{\beta_R - \beta_P}{\alpha}} \leq \frac{1 + \left(\frac{C^R - C^P}{C^P}\right)\left(\frac{A}{A+H_t(i)}\right)^{\frac{\beta_P}{\alpha}}}{1 + \left(\frac{C^R - C^P}{C^P}\right)\left(\frac{A}{A+\gamma}\right)^{\frac{\beta_P}{\alpha}}}.$$

for all $i \in \left[I_t, \bar{I}_t\right]$. This implies that $H_t(i) < \gamma$ and hence the RHS is not smaller than 1 and that, if $\beta_R \geq \beta_P$, the LHS is not bigger than 1. Hence, $\beta_R \geq \beta_P$ is a sufficient condition for this condition to be satisfied. Notice that if $C^R = C^P$, this is also a necessary condition.

Next, condition (24) can be rewritten as

$$\left[\frac{A + H(i)}{A+\gamma}\right]^{\frac{\beta_P - \beta_R}{\alpha}} \leq 1 + \frac{C^R - C^P}{C^P}\left(\frac{A}{A+\gamma}\right)^{\frac{\beta_P}{\alpha}}\left[1 - \left(\frac{A+\gamma}{A+H_t(i)}\right)^{\frac{\beta_R}{\alpha}}\right]$$

for all $i \in [0, I_t]$. In these locations, by construction, $H_t(i) > \gamma$, which implies that the RHS is not smaller than 1 and that, if $\beta_R \geq \beta_P$ the LHS is not bigger than 1. Hence, $\beta_R \geq \beta_P$ is also a sufficient condition for this equation to hold. Again, it is also a necessary condition if $C^R = C^P$. Hence, this completes the proof that a fully segregated equilibrium exists if $\beta^P \leq \beta^R$.

D1.2 Proof of Proposition 2

Denote by r^H the initial level of r, and by r^L the level of r after the shock, with $r^L > r^H$. The price schedule before $(s = H)$ and after the shock $(s = L)$ are:

$$
p^s(i) = \begin{cases}
\left[C^P \left(1 + \frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P \right] \left(1 + \frac{\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}} & \text{for } i \in [0, I^s - \gamma] \\
\left[C^P \left(1 + \frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P \right] \left(1 + \frac{I^s - i}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}} & \text{for } i \in [0, I^s] \\
C^P \left(1 + \frac{\gamma + I^s - i}{A}\right)^{\frac{\beta_P}{\alpha}} & \text{for } i \in [I^s, I^s + \gamma] \\
C^P & \text{for } i \in [I^s + \gamma, \bar{I}^s]
\end{cases}
\tag{25}
$$

First, notice that if $i \geq I^H + \gamma$, then $p^H(i) = C^P$, and if $i < I^H + \gamma$, then $p^H(i) > C^P$. Also, if $i < I^H - \gamma$, then $p^H(i) = \bar{p}$, where

$$
\bar{p} \equiv \left[C^P \left(1 + \frac{\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + C^R - C^P \right] \left(1 + \frac{\gamma}{A + \gamma}\right)^{\frac{\beta_R}{\alpha}} .
$$

Next, we obtain

$$
\frac{p^L(i)}{p^H(i)} = \begin{cases}
\dfrac{\left(1 + \frac{\min\{\gamma, I^L - i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(1 + \frac{\min\{\gamma, I^H - i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}} & \text{for } i \in [0, I^H] \\
\left[\left(\frac{A+\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + \frac{C^R - C^P}{C^P} \right] \dfrac{\left(1 + \frac{\min\{\gamma, I^L - i\}}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(1 + \frac{\max\{\gamma + I^H - i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}} & \text{for } i \in [I^H, I^L] \\
\dfrac{\left(1 + \frac{\max\{\gamma + I^L - i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}}{\left(1 + \frac{\max\{\gamma + I^H - i, 0\}}{A}\right)^{\frac{\beta_P}{\alpha}}} & \text{for } i \in [I^L, \bar{I}^H]
\end{cases}
\tag{26}
$$

Also, from equations (14) and (15), we obtain $I^L > I^H$ and $\bar{I}^L > \bar{I}^H$. Then, if $i < I^H - \gamma$, it must be that $p^L(i)/p^H(i) = 1$, which implies that

$$
E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)} | p_t(i) = \bar{p} \right] = 1.
$$

Moreover, $I^L > I^H$, together with expression (26), immediately implies that $p^L(i)/p^H(i) \geq 1$ for $i > I^H - \gamma$, and hence

$$
E_{t+1}\left[\frac{p_{t+1}(i)}{p_t(i)} | p_t(i) < \bar{p} \right] > 1,
$$

which proves the first statement or the proposition.

We now want to prove the second statement of the proposition, that is, that the price ratio $p_{t+1}(i)/p_t(i)$ is non-increasing in $p_t(i)$, where $p_t(i) = p^H(i)$ and $p_t(i) = p^L(i)$. First, notice that $p^H(i)$ is non-increasing in i, so proving that $p^L(i)/p^H(i)$ is non-increasing in $p^H(i)$ is

equivalent to prove that $p^L(i)/p^H(i)$ is non-decreasing in i. The ratio $p^L(i)/p^H(i)$ is continuous and differentiable except at a finite number of points. Hence, in order to prove that it is non-decreasing in i, it is enough to show that $d\left[p^L(i)/p^H(i)\right]/di$ is non-negative, for all i where this derivative exists. Let us show that.

For $i \in \left[0, I^H - \gamma\right]$, $p^L(i)/p^H(i) = 1$ and hence $p^L(i)/p^H(i)$ is constant in i. For $i \in \left[I^H - \gamma, I^H\right]$, we have that

1. if $I^H - \gamma < i < I^L - \gamma$, then

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_R}{\alpha(A+\gamma)}\left(\frac{A+2\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}\left(\frac{A+\gamma+I^H-i}{A+\gamma}\right)^{-\frac{\beta_R}{\alpha}-1} > 0 \qquad (27)$$

2. if $I^H - \gamma < i < \min\left\{I^L - \gamma, I^H\right\}$, then

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_R}{\alpha}\left(\frac{A+\gamma+I^L-i}{A+\gamma+I^H-i}\right)^{\frac{\beta_R}{\alpha}}\left[\frac{1}{A+\gamma+I^H-i} - \frac{1}{A+\gamma+I^L-i}\right] > 0 \qquad (28)$$

given that $I^H < I^L$.

For $i \in [I^H, I^H + \gamma]$ we have that

1. if $I^H < i < \min\left\{I^H + \gamma, I^L - \gamma\right\}$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\tilde{C}}{A}\left(\frac{A+2\gamma}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}\left(\frac{A+\gamma+I^H-i}{A}\right)^{-\frac{\beta_P}{\alpha}-1} > 0 \qquad (29)$$

where

$$\tilde{C} \equiv \left[\left(\frac{A+\gamma}{A}\right)^{\frac{\beta_P}{\alpha}} + \frac{C^R - C^P}{C^P}\right]$$

2. if $\max\left\{I^H, I^L - \gamma\right\} < i < \min\left\{I^L, I^H - \gamma\right\}$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\tilde{C}}{\alpha}\frac{\left(\frac{A+\gamma+I^L-i}{A+\gamma}\right)^{\frac{\beta_R}{\alpha}}}{\left(\frac{A+\gamma+I^H-i}{A}\right)^{\frac{\beta_P}{\alpha}}}\left[\frac{\beta_P}{A+\gamma+I^H-i} - \frac{\beta_R}{A+\gamma+I^L-i}\right] \qquad (30)$$

hence

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} > 0 \text{ iff } \frac{\beta_R}{\beta_P} < \frac{A+\gamma+I^L-i}{A+\gamma+I^H-i},$$

which is true if the shock is big enough and $I^L - I^H$ is big enough;

3. if $\max\left\{I^H, I^L\right\} < i < I^H + \gamma$

$$\frac{d\left(\frac{p^L(i)}{p^H(i)}\right)}{di} = \frac{\beta_P}{\alpha}\left(\frac{A+\gamma+I^L-i}{A+\gamma+I^H-i}\right)^{\frac{\beta_P}{\alpha}}\left[\frac{1}{A+\gamma+I^H-i} - \frac{1}{A+\gamma+I^L-i}\right] > 0. \qquad (31)$$

This proves that, if the shock is big enough, the second statement of the proposition holds.

D1.3 Proof of Proposition 3

First, notice that at time t, each location i may lie in four possible intervals that implies different pricing behavior: $[0, I_t - \gamma]$, $[I_t - \gamma, I_t]$, $[I_t, I_t + \gamma]$, and $[I_t, \bar{I}_t]$. From expression (25), it is immediate that prices at time $t+1$ in each location i are weakly increasing in I_{t+1}, whenever i is in the same type of interval at t and $t + 1$. From expression (14), I_{t+1} is non-decreasing in r and hence prices are weakly decreasing in r for all i which remain in the same type of interval. Let us consider any $r_{t+1}^A < r_{t+1}^B < r_t$, with $I_{t+1}^A > I_{t+1}^B$. Then all $i \in [0, I_{t+1}^B - \gamma]$ are also in $[0, I_{t+1}^B - \gamma]$, but some $i \in [I_{t+1}^B - \gamma, I_{t+1}^B + \gamma]$ may be in $[0, I_{t+1}^A - \gamma]$ or some $i \in [I_{t+1}^B, I_{t+1}^B + \gamma]$ may be in $[I_{t+1}^A - \gamma, I_{t+1}^A]$. Given that, from inspection of expression (25), $p_{t+1}(i)$ is non-increasing in i, this implies that aggregate prices P_{t+1} must be non-increasing in r. Hence, if at time $t + 1$ the economy is hit by an unexpected and permanent decrease in r, then P_{t+1} is going to be higher, the larger is the decrease in r. Given that P_t is given, this immediately proves the first statement of the proposition that the percentage increase in aggregate price is higher the larger is the decrease in r.

Second, we want to prove the second statement of the proposition, that

$$\frac{d^2 \left(p_{t+1}(i) / p_t(i) \right)}{dp_t(i) \, dr} \geq 0$$

for all $p_t(i) > C^P$ where the derivative is well-defined. Equations (27)-(31) in the proof of Proposition (2) define $d \left[p_{t+1}(i) / p_t(i) \right] / di$ for all i where this derivative is well-defined and $p_t(i) > C^P$. If the decrease in r is big enough, $d \left[p_{t+1}(i) / p_t(i) \right] / di > 0$ for all $p_t(i) > C^P$. Moreover, by inspection, it is easy to see that $d \left[p_{t+1}(i) / p_t(i) \right] / di$ is increasing in I_L, and hence decreasing in r, whenever i is in the same type of interval after a small or a large shock, say r^A or r^B. Moreover, given that $I_A^L > I_B^L$, i may lie in different types of interval in the two cases. In particular, it could be that $\min \left\{ I_B^L - \gamma, I^H \right\} < i < I^H$ but $I^H - \gamma < i < \min \left\{ I_A^L - \gamma, I^H \right\}$, or that $\max \left\{ I^H, I_B^L - \gamma \right\} < i < I^H + \gamma$ and $I^H < i < \min \left\{ I^H + \gamma, I_A^L - \gamma \right\}$, or that $I_B^L < i < I^H + \gamma$ but $\max \left\{ I^H, I_A^L - \gamma \right\} < i < I^H + \gamma$. It is easy to see that expression (27) is not smaller than expression (28) and that expression (29) is not smaller than expression (30). Finally expression (30) is bigger than expression (31) iff

$$\left(\frac{A + \gamma + I^L - i}{A + \gamma} \right)^{\frac{\beta_R - \beta_P}{\alpha}} \left[1 - \frac{(\beta_R - \beta_P) \left(A + \gamma + I^H - i \right)}{\beta_P \left(I^L - I^H \right)} \right] > 1,$$

which is true if the shock is large enough so that $I^L - I^H$ is big enough, as we assumed. This proves that $d^2 \left[p_{t+1}(i) / p_t(i) \right] / di dr$ is positive for all i such that the derivative exists and $p_t(i) > C^P$. Given that $p_t(i)$ is non-increasing in i, this completes the proof of the second claim of the proposition.

D1.4 Proof of Proposition 4

Consider two cities, A and B, with both $\phi_R^A > \phi_R^B$ and $\phi_P^A > \phi_P^B$. First, we want to prove the claim that if at time $t+1$ they are both hit by an unexpected and permanent decrease in r of the same size, with $r_t = r^H > r^L = r_{t+1}$, the percentage increase in the aggregate price level P_t is larger in city A. The two cities are exactly the same except for the size of the city and of the rich neighborhoods, which, from expressions (14) and (15), are so that $I_A^h > I_B^h$ and $\bar{I}_A^h > \bar{I}_B^h$ for both $h = H$ and $h = L$. Hence city A has a larger center and a larger size overall. After the decrease in r, prices do not change for all $i < I_j^H - \gamma$ and for all $i > I_j^L + \gamma$ if $I_j^L + \gamma < \bar{I}_j^H$, for both $j = A$ and $j = B$. Moreover, given that $I_A^L > I_B^L$, the growth rate $p_A^L(i)/p_A^H(i)$ is weakly higher than $p_B^L(i + I_B^H - I_A^H)/p_B^H(i + I_B^H - I_A^H)$ for all $i \in [I_A^H - \gamma, \bar{I}_A^H]$. This implies that the gross growth rate in aggregate prices is also higher in city A than in city B if the shock is big enough that the higher expansion in city A dominates the zero growth rate in the rich neighborhoods in the center.

To prove the second claim notice that the price in city A at time t for $i \in [I_A^H - \gamma, I_A^H + \gamma]$ are exactly the same as in city B for $i \in [I_B^H - \gamma, I_B^H + \gamma]$. However, $I_A^H > I_B^H$, so that the interval $[0, I_A^H]$ is larger than $[0, I_B^H]$. When a decrease in r hits both cities, expressions (14) and (15) give that both I and $\bar{I}$ increase more in city A. Hence, the expression for $d(p_{t+1}(i)/p_t(i))/di$ in city A for all $i \in [I_A^H - \gamma, I_A^H + \gamma]$ is equivalent to $d(p_{t+1}(i)/p_t(i))/di$ in city B for all $i \in [I_B^H - \gamma, I_B^H + \gamma]$ if city B was facing a larger decrease in r, and, in particular, if r_{t+1} was equal to

$$\tilde{r}^L = \left[\left(1 + \frac{1}{r^L} \right) \frac{\phi_R^A}{\phi_R^B} - 1 \right]^{-1} < r^L.$$

From the proof of Proposition 3, this immediately implies that $d(p_{t+1}^A(i)/p_t^A(i))/di > d(p_{t+1}^B(i + I_A^H - I_B^H)/p_t^B$ that is, $g_A'(p) > g_B'(p)$ for all $p \in (\bar{p}, C^P)$. Finally, the gross growth rate of prices in all locations where the initial price was $\bar{p}$ is everywhere equal to 1 so that is not sensitive to i and $g_A'(\bar{p}) = g_B'(\bar{p})$. This completes the proof that $g_A'(p) \geq g_B'(p)$ for all $p > C^P$ whenever this derivative is well defined.