Appendix 16A Some Useful Formulas of Financial Structure

Definitions:

$$E(EBIT) = \text{A perpetual expectation of cash operating income before interest and taxes.}$$

V_U = Value of an unlevered firm.

V_L = Value of levered firm.

B = Present value of debt.

S = Present value of equity.

R_S = Cost of equity.

R_B = Cost of debt capital.

R_0 = Cost of capital to an all-equity firm. In a world of no corporate taxes, the weighted average cost of capital to a levered firm, R_{WACC}, is also equal to R_0. However, with corporate taxes, R_0 is above R_{WACC} for a levered firm.

Model I (No Tax):

$$V_L = V_U = \frac{E(EBIT)}{R_0}$$

$$R_S = R_0 + (R_0 - R_B) \times B/S$$

Model II (Corporate Tax, $t_C > 0$; No Personal Taxes, $t_S = t_B = 0$):

$$V_L = \frac{E[EBIT] \times (1 - t_C)}{R_0} + \frac{t_C R_B B}{R_B} = V_U + t_C B$$

$$R_S = R_0 + (1 - t_C) \times (R_0 - R_B) \times B/S$$

Model III (Corporate Tax, $t_C > 0$; Personal Tax, $t_B > 0$; $t_S > 0$):

$$V_L = V_U + \left[1 - \frac{(1 - t_C) \times (1 - t_S)}{(1 - t_B)} \right] \times B$$

Appendix 16B The Miller Model and the Graduated Income Tax

In Section 16.9, we assumed a flat personal income tax on interest income. In other words, we assumed that all individuals are subject to the same personal tax rate on interest income. Merton Miller derived the results of this section in a classic paper.[1] However, the genius of his paper was to consider the implications of personal taxes when tax rates differ across individuals.

This *graduated* income tax is consistent with the real world. For example, individuals are currently taxed at rates from 0 to 35 percent in the United States, depending on income. In addition, other entities, such as corporate pension funds, individual retirement accounts (IRAs), and universities, are tax exempt.

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[1]M. Miller, "Debt and Taxes," *Journal of Finance* (May 1977). Yes, this is the same Miller of MM.

To illustrate Miller's model with graduated taxes, we consider a world where *all* firms initially only issue equity. We assume that $t_C = 35$ percent and $t_S = 0$.[2] The required return on stock, R_S, is 10 percent. In addition, we posit a graduated personal income tax, where tax rates vary between 0 and 50 percent. All individuals are risk-neutral.

Now consider a courageous firm contemplating a $1,000 issue of debt. What is the interest rate that the firm can pay and still be as well off as if it issued equity? Because debt is tax deductible, the after–corporate tax cost of debt is $(1 - t_C) \times R_B$. However, equity is not deductible at the corporate level, so the aftertax cost of equity is R_S. Thus, the firm is indifferent to whether it issues debt or equity when

$$(1 - t_C) \times R_B = R_S \tag{16.1}$$

Because $t_C = 35$ percent and $R_S = 10$ percent, the firm could afford to pay a rate on debt as high as 15.38 percent.

Miller argues that those in the lowest tax brackets (tax exempt in our example) will buy the debt because they pay the least personal tax on interest. These tax-exempt investors will be indifferent to whether they buy the stock or purchase bonds also yielding 10 percent. Thus, if this firm is the *only* one issuing debt, it can pay an interest rate well below its break-even rate of 15.38 percent.

Noticing the gain to the first firm, many other firms are likely to issue debt. However, if there are only a fixed number of tax-exempt investors, new debt issues must attract people in higher brackets. Because these individuals are taxed on interest at a higher rate than they are taxed on equity distributions, they will buy debt only if its yield is greater than 10 percent. For example, an individual in the 15 percent bracket has an interest rate after personal tax of $R_B \times (1 - 0.15)$. He will be indifferent to whether he buys bonds or stock if $R_B = 11.765$ percent because $0.11765 \times 0.85 = 10$ percent. Because 11.765 percent is less than the 15.38 percent rate of Equation 16.1, corporations gain by issuing debt to investors in the 15 percent bracket.

Now consider investors in the 35 percent bracket. A return on bonds of 15.38 percent provides them with a 10 percent $= 15.38$ percent $\times (1 - 0.35)$ interest rate after personal tax. Thus, they are indifferent to whether they earn a 15.38 percent return on bonds or a 10 percent return on stock. Miller argues that in equilibrium, corporations will issue enough debt so that investors with personal tax brackets up to and including 35 percent will hold debt.[3] Additional debt will not be issued because the interest rate needed to attract investors in higher tax brackets is above the 15.38 percent rate that corporations can afford to pay.

The beauty of competition is that other companies can so capitalize on someone's innovation that all value to the courageous first entrant is eliminated. According to the Miller model, firms will issue enough debt so that individuals up to and including the 35 percent bracket hold it. To induce these investors to hold bonds, the competitive interest rate becomes 15.38 percent. No firm profits from issuing debt in equilibrium. Rather, all firms are indifferent to whether they issue debt or equity in equilibrium.

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[2]The assumption that $t_S = 0$ is perhaps an extreme one. However, it is commonly made in the literature, justified by the investor's ability to defer realization of capital gains indefinitely. Besides, the same qualitative conclusions hold if $t_S > 0$, though the explanation would be more involved.

[3]All investors with $t_B < 35$ percent hold bonds. Because investors with $t_B = 35$ percent are indifferent to whether they hold stocks or bonds, only some of them are likely to choose bonds.

Miller's work produces three results:

1. In aggregate, the corporate sector will issue just enough debt so that individuals with tax brackets equal to and below the corporate tax rate, t_C, will hold debt, and individuals with higher tax brackets will not hold debt. Thus, individuals in these higher brackets will hold stock.

2. Because people in tax brackets equal to the corporate rate hold debt, there is no gain or loss to corporate leverage. Therefore, the capital structure decision is a matter of indifference to an individual firm. Though the Miller model is quite sophisticated, this conclusion is identical to that reached by MM in a world without any taxes.

3. As given in Equation 16.1, the return on bonds will be higher than the return on stocks of comparable risk. [An adjustment to Equation 16.4 must be made to reflect the greater risk of stocks in the real world.]

EXAMPLE 16A.1

Miller's Model   Consider an economy in which there are four groups of investors and no others:

Group	Marginal Tax Rate (%) on Bonds (t_B)	Personal Wealth (in $ millions)
Finance majors	50%	$1,200
Accounting majors	35	300
Marketing majors	20	150
Management majors	0	50

We assume that investors are risk-neutral and that equity income is untaxed at the personal level for all investors (i.e., $t_S = 0$). All investors can earn a tax-free return of 5.4 percent by investing in foreign real estate; therefore, this is the return on equity. The corporate tax rate is 35 percent. Interest payments are tax deductible at the corporate level and taxable at the individual level. Corporations receive a total of $120 million in cash flow before tax and interest. There are no growth opportunities, and every year is the same in perpetuity. What is the range of possible debt–equity ratios?

The return on equity, r_S, will be set equal to the return on foreign real estate, which is 0.054. In a Miller equilibrium, $R_S = (1 - t_C) \times R_B$. Therefore,

$$R_B = \frac{0.054}{1 - 0.35} = 0.0831$$

Given the tax brackets of the different groups of investors, we would expect that finance majors would hold equity and foreign real estate, and accounting majors would be indifferent to whether they held equity or debt. Marketing and management majors would hold bonds because their personal tax rates are below 0.35. Because accounting majors are indifferent to whether they hold bonds or stocks, we must learn what happens if they invest in bonds or equity. If accounting majors use their $300 to buy bonds, $B = \$300 + \$150 + \$50 = \$500$. Then the following calculations can be made:

$$S = \frac{(\text{EBIT} - R_B B) \times (1 - t_C)}{R_S}$$

$$= \frac{[\$120 - (0.0831 \times \$500)] \times (1 - 0.35)}{0.054}$$

$$= \$944$$

(continued)

$$B = \frac{R_B B}{R_B} = \$500$$

$$V_L = S + B = \$944 + \$500 = \$1,444$$

$$\frac{B}{S} = \frac{\$500}{\$944} = 0.530$$

If accounting majors buy stocks and foreign real estate ($B = \$150 + \$50 = \$200$),

$$S = \frac{(\text{EBIT} - R_B B) \times (1 - t_C)}{R_S}$$

$$= \frac{[\$120 - (0.0831 \times \$200)] \times (1 - 0.35)}{0.054}$$

$$= \$1,244$$

$$B = \$200$$

$$V_L = S + B = \$1,244 + \$200 = \$1,444$$

$$\frac{B}{S} = \frac{\$200}{\$1,244} = 0.161$$

Thus, depending on the amount of bonds held by accounting majors, the debt–equity ratio in the economy can lie in the range of 0.161 to 0.530.