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Journal of Economic Theory 149 (2014) 100–127

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Money, financial stability and efficiency ☆

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Received 27 December 2011; final version received 1 October 2012; accepted 20 October 2012

Available online 19 February 2013

Abstract

Most analyses of banking crises assume that banks use real contracts but in practice contracts are nominal. We consider a standard banking model with aggregate return risk, aggregate liquidity risk and idiosyncratic liquidity shocks. With non-contingent nominal deposit contracts, a decentralized banking system can achieve the first-best efficient allocation if the central bank accommodates the demands of the private sector for fiat money. Price level variations allow full sharing of aggregate risks. An interbank market allows the sharing of idiosyncratic liquidity risk. In contrast, idiosyncratic (bank-specific) return risks cannot be shared using monetary policy alone as real transfers are needed.

JEL classification: G01; G21; G28

☆ We are grateful to Todd Keister, Michael Woodford, Luigi Zingales, participants at the Bank of Portugal Conference on Financial Intermediation in Faro in June 2009, at workshops at ESSET 2011 in Gerzensee, the Becker–Friedman Conference on Macroeconomic Fragility 2012, FIRS 2012 in Minneapolis, the NBER Finance and Macroeconomics Summer Institute Workshop 2012, the Federal Reserve Banks of Chicago and New York, the Sveriges Riksbank, the University of Maryland, the University of North Carolina, the University of Pennsylvania and particularly to the Associate Editor for helpful comments. This paper is produced as part of the project 'Politics, Economics and Global Governance: The European Dimensions' (PEGGED) funded by the Theme Socio-economic Sciences and Humanities of the European Commission's 7th Framework Programme for Research, Grant Agreement No. 217559. We are also grateful to the Wharton Financial Institutions Center for financial support.

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0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2013.02.002

Keywords: Central bank; Commercial banks; Risk sharing

1. Introduction

Most models in the banking literature (e.g., Diamond and Dybvig [15]; Chari and Jagannathan [12]; Jacklin and Bhattacharya [21]; Calomiris and Kahn [9]; Allen and Gale [2,3]; Diamond and Rajan [16,17]) treat banking as a real activity with no role for fiat money. Following Diamond and Dybvig [15], consumers' liquidity preference is modeled as uncertainty about their time preference for consumption. Liquid assets are modeled as a storage technology. A deposit contract promises a depositor a fixed amount of consumption depending on the date of withdrawal. Thus, a crisis can arise when a large number of consumers decide to withdraw their deposits from the banking system, because the demand for goods is greater than the banks' limited stock of liquid assets.

While "real" models have provided valuable insights into the nature of financial fragility, they do not capture important aspects of reality, such as the role of fiat money in the financial system. In practice, financial contracts are almost always written in terms of money. This fact has important consequences for the theory. Because the central bank can costlessly create fiat money in a crisis, there is no reason why the banking system should find itself unable to meet its commitments to depositors (see, e.g., Buiter [8]).

In this paper, we develop a model, based on Allen, Carletti and Gale [1], henceforth ACG, in which fiat money is issued by the central bank. Deposit contracts and loan contracts are denominated in terms of money and money is used in transactions. In other words, money is both a unit of account and a medium of exchange. In contrast to most of the banking literature, which is reviewed below, we show that the combination of nominal contracts and a central bank policy of accommodating commercial banks' demand for money leads to first-best efficiency. This result holds when there are aggregate liquidity and asset return shocks and also when there are idiosyncratic liquidity shocks.

There are three dates and, at each date, there is a single good that can be used for consumption or investment. Assets are represented by constant returns to scale technologies that allow the consumers' initial endowment of the good to be transformed into consumption at the second and third dates. The short-term asset is a storage technology. The long-term asset requires an investment at the initial date and yields a random return at the final date. There is a large number of ex ante identical consumers, each of whom is endowed with one unit of the good at the initial date. At the second date, a random fraction of consumers discover they are early type and want to consume only at the second date while the remaining consumers are late type and want to consume only at the third date.

We start by characterizing the first-best allocation as the solution to a planner's problem. The planner invests the consumers' endowments in a portfolio of short- and long-term assets and distributes the asset returns to the early and late consumers at the second and third dates. The portfolio is chosen before the realization of the aggregate state, which consists of the fraction of early consumers and the return on the risky asset. The consumption allocation is determined after the state is realized and is therefore state contingent.

Our first main result is to show that the planner's allocation can be implemented in a decentralized economy, where there are three types of institutions, a central bank, commercial banks, and firms. At the initial date, the central bank makes money available to the commercial banks on an intraday basis at a zero interest rate. The banks make loans to the firms, which in turn use the money to buy the consumers' endowments and invest them in the short- and long-term assets. At the intermediate and final dates, the central bank again makes intraday loans to the banks, which they use to meet depositors' withdrawals. The depositors in turn use the money to purchase goods from the firms. Then the firms use the same money to repay their loans to the banks and the banks use it to repay the central bank. The central bank's policy is passive: at each date it supplies the amount of money demanded by the commercial banks. The banks and firms are assumed to be profit maximizing and in a competitive equilibrium they earn zero profit. Consumers maximize expected utility. In equilibrium they deposit the money received in exchange for the sale of their endowments to the firms at the first date, and withdraw and spend all their money at the second or third date, depending on whether they are early or late consumers.

The reason that a competitive equilibrium implements the same state-contingent allocation as the planner's problem despite the debt-like nature of the deposit contract, is that deposit contracts are written in terms of money. Regardless of the liquidity and asset return shocks, banks are able to meet their commitments as long as the central bank supplies them with sufficient amounts of fiat money. The price level adjusts in response to aggregate shocks in order to clear markets. When the number of early consumers is high, the amount of money withdrawn from the banks is also high and this increases the price level. When the returns on the long asset are low, the supply of goods is also low and this increases the price level. The adjustments in the price level ensure that early and late consumers receive the efficient, state-contingent levels of consumption.

The quantity theory of money holds in equilibrium since the price level at each date is proportional to the supply of money extended to the commercial banks by the central bank. This result follows from the market-clearing condition in the goods market at each date. The central bank can control the nominal interest rate and the expected inflation rate, but it has no effect on the equilibrium allocation of goods. Deposits and loans are denominated in terms of money, but they are also interest bearing, so any change in the expected inflation rate is compensated by a change in the nominal interest rate. Thus, money is not merely neutral, it is super-neutral.

Central to our results is full flexibility of prices. This ensures that even if contracts are non-contingent, optimal risk sharing can be achieved. In this context, such contracts emerge optimally in equilibrium even if banks are given the possibility to offer contingent contracts. We show that contingent contracts and price flexibility are alternative ways of obtaining the first-best allocation.

The basic model is then extended in a number of ways that allow the efficiency result to remain valid. We first introduce idiosyncratic (bank-specific) liquidity shocks and an interbank market. This allows banks to reshuffle money between those that receive high and low liquidity shocks at the second date so that each bank can meet the required level of withdrawal by its depositors, without being subject to distress. Second, we extend the efficiency result to a multi-period setting.

We next turn to the case of idiosyncratic (bank-specific) asset return risk. If the banks' asset-specific returns are observable, efficiency can be restored by introducing a public or a private insurance scheme whereby banks can securitize the assets and effectively hold a diversified portfolio of asset-backed securities. However, such schemes are vulnerable to moral hazard if there is asymmetric information about asset returns. Insuring low returns gives banks an incentive to engage in asset substitution and to misrepresent the realized returns of the assets.

The results obtained from our monetary model of banking stand in stark contrast to those obtained from models with real contracts such as ACG. In their model, banks need to sell the long-term asset to deal with aggregate liquidity shocks and may thus be unable to meet their commitments to their customers. The introduction of a central bank that fixes the price of the long asset through open market operations allows banks to implement the constrained efficient allocation provided there is no bankruptcy. In contrast, here a central bank policy of passively accommodating the demands of the commercial banks for money is sufficient to eliminate financial crises and achieve the first-best.

The rest of the paper proceeds as follows. The remainder of this section considers the related literature. Section 2 describes the primitives of the real economy. The efficient allocation is derived in Section 3. In Section 4, we introduce a financial system with money as a medium of exchange and define the equilibrium of this economy. The main results are found in Section 5, where we show that the efficient allocation can be decentralized as an equilibrium and where we discuss the optimality of contingent contracts and the importance of price flexibility for the results. A number of extensions are considered in Section 6. Finally, Section 7 contains some concluding remarks.

1.1. Related literature

As we have noted, most of the literature on banking crises has assumed contracts are written in real terms. The papers that have considered fiat money and banking crises can be divided into two strands. The first introduces banks into models of fiat currency. The second considers financial systems where the vast majority of transactions involve the transfer of money from one bank account to another without introducing fiat currency.

An important contribution to the first strand is Champ, Smith and Williamson [11] where banks hold currency and issue notes to meet the relocation shocks of their younger consumers. When banks accommodate these shocks freely, there exists a stationary Pareto optimal equilibrium in which currency and bank notes are perfect substitutes and the nominal interest rate is zero. In contrast, when the bank note issuance is fixed, there will be a banking crisis if the shock is large enough to exhaust the banks' currency reserves. Antinolfi, Huybens and Keister [4] extend this model by introducing a lender of last resort that lends freely at a zero nominal interest rate, and show that under certain conditions only a stationary Pareto optimal equilibrium exists. Smith [24] considers a similar framework but without an elastic money supply. He shows that the lower the inflation rate and nominal interest rate, the lower is the probability of a banking crisis. Cooper and Corbae [14] consider a model with increasing returns to scale in the intermediation process and show the existence of multiple equilibria corresponding, in their interpretation, to different levels of confidence.

Diamond and Rajan [16] develop a model where banks have special skills to ensure that loans are repaid and where liquidity shortages may arise. Using a variant of this model in Diamond and Rajan [18], they show that the use of money and nominal contracts can improve risk sharing, since price adjustments introduce a form of state contingency to contracts, but the variations in the transaction value of money can lead to bank failures. Monetary intervention in the form of bond acquisitions helps ease this problem. Allen and Gale [2] develop a model of banking crises caused by asset return uncertainty and real contracts. They subsequently show that the use of contracts denominated in terms of money allows the implementation of the incentive-efficient allocation. Finally, Cao and Illing [10] extend this framework to consider the case where a central bank creating money creates a moral hazard problem. They show that the optimal second-best contract can be implemented by imposing ex ante liquidity requirements and having the central bank create money ex post.

The second strand of papers starts with Skeie [23], who develops a standard banking model as in Diamond and Dybvig [15] but with nominal contracts and fiat money and shows the existence of a unique efficient equilibrium. In a follow-up paper with aggregate liquidity risk, Freixas, Martin and Skeie [19] show that the central bank's interest policy can directly improve liquidity conditions in the interbank market and select the efficient equilibrium.

The current paper belongs in this second strand of literature. All payments are made with fiat money and money can be created costlessly by the central bank. In contrast to the other papers surveyed here, it is shown that first-best efficiency, rather than just incentive or constrained efficiency, can be achieved in a wide range of situations. Risk sharing is achieved because prices are flexible and thus the real value of debt varies. This resembles the result in Bohn [7], where it is shown that governments can improve the allocation of resources by issuing nominal debt. In Bohn's model, taxes are distortionary and nominal debt combined with price variations provides valuable insurance against the budgetary effect of economic fluctuations. Another related paper is Krueger and Lustig [22]. They offer conditions under which an incomplete market endowment economy with idiosyncratic and aggregate risks achieves efficient risk sharing with a single aggregate security offering the "right" state contingencies. They provide conditions under which this is true generally for a claim on aggregate capital. Here, the insights are similar, with the difference that the "right" state contingencies emerge endogenously from price adjustment.

2. The real economy

In this section we describe the primitives of the real economy. The model is based on ACG. There are three dates $t = 0, 1, 2$ and a single good that can be used for consumption or investment at each date.

There are two assets, a short-term asset that we refer to as the *short asset* and a long-term asset that we refer to as the *long asset*. The short asset is represented by a riskless storage technology, where one unit of the good invested at date t produces one unit of the good at date $t + 1$, for $t = 0, 1$. The long asset is a constant-returns-to-scale investment technology that takes two periods to mature: one unit of the good invested in the long asset at date 0 produces a random return equal to R units of the good at date 2.

There is a large number (strictly, a continuum with unit measure) of identical consumers. All consumers have an endowment of one unit of the good at date 0 and nothing at dates 1 and 2. Consumers are uncertain about their future time preferences. With probability λ they are *early consumers*, who only value the good at date 1, and with probability $1-\lambda$ they are *late consumers*, who only value the good at date 2. The fraction of early consumers λ is a random variable. The utility of consumption is denoted by $u(c)$ where $u(\cdot)$ is a von Neumann Morgenstern utility function with the usual properties.

We assume that the random variables λ and R have a joint cumulative distribution function F with support in the interval $[0, 1] \times [0, R^{\max}]$. The mean of R is denoted by $\bar{R} > 1$ and the mean of λ is denoted by $0 < \bar{\lambda} < 1$.

Uncertainty about time preferences generates a preference for liquidity and a role for intermediaries as providers of liquidity insurance. The expected utility of a consumption profile (c_1, c_2) is given by

$$\lambda u(c_1) + (1 - \lambda)u(c_2),$$

where $c_t \geqslant 0$ denotes consumption at date $t = 1, 2$.

All uncertainty is resolved at the beginning of date 1. In particular, the state (λ, R) is revealed and depositors learn whether they are early or late consumers. While each depositor's realization of liquidity demand is private information, the state (λ, R) is publicly observed.

3. The efficient allocation

Suppose that a central planner were to make all the investment and consumption decisions in order to maximize the expected utility of the representative consumer. At the first date, the planner would invest the representative consumer's endowment of 1 unit of the good in a portfolio consisting of $0 \leqslant y \leqslant 1$ units of the short asset and $1 - y$ units of the long asset. Then, at the second date, once the aggregate state of nature (λ, R) is known, the planner would assign $c_1(\lambda, R)$ units of the good to the representative early consumer and $c_2(\lambda, R)$ units of the good to the representative late consumer. The total amount of consumption available at date 1 is given by the amount y invested in the short asset. Given the fraction of early consumers λ, the planner's allocation will be feasible at date 1 if and only if

$$\lambda c_1(\lambda, R) \leqslant y, \tag{1}$$

for every aggregate state (λ, R). The left hand side of (1) is the total amount consumed at date 1 and the right hand side is the total supply of goods. If the amount consumed, $\lambda c_1(\lambda, R)$, is less than the total supply, y, the difference,

$$S(\lambda, y) = y - \lambda c_1(\lambda, R),$$

is stored until the last period. At date 2, the fraction of late consumers is $1 - \lambda$ so the planner's allocation will be feasible if and only if

$$(1 - \lambda)c_2(\lambda, R) = (1 - y)R + S(\lambda, R) = (1 - y)R + y - \lambda c_1(\lambda, R), \tag{2}$$

for every aggregate state (λ, R). The left hand side of (2) is total consumption at date 2 and the right hand side is the total supply of the good at date 2. We assume the two sides are equal since all of the good must be used up at the last date. Rearranging the terms in the equation above, we can rewrite this condition in terms of total consumption at dates 1 and 2 and the total returns of the two assets as

$$\lambda c_1(\lambda, R) + (1 - \lambda)c_2(\lambda, R) = y + (1 - y)R. \tag{3}$$

The planner's task is to maximize the expected utility of the representative consumer subject to the feasibility constraints (1) and (3). A necessary condition for maximizing the expected utility of the representative consumer is that, given the portfolio y chosen at the first date, the expected utility of the representative consumer is maximized in each aggregate state (λ, R). This problem can be written as

$$\begin{aligned} \max \quad & \lambda u(c_1) + (1 - \lambda)u(c_2) \\ \text{s.t.} \quad & \lambda c_1 \leqslant y, \quad \lambda c_1 + (1 - \lambda)c_2 = y + (1 - y)R. \end{aligned} \tag{4}$$

Problem (4) has a very simple yet elegant solution. Either there is no storage, in which case $\lambda c_1 = y$ and $(1 - \lambda)c_2 = (1 - y)R$, or there is positive storage between the two dates, in which case $c_1 = c_2 = y + (1 - y)R$. This solution can be summarized by the following two "consumption functions,"

$$c_1(\lambda, R) = \min\left\{\frac{y}{\lambda}, y + (1 - y)R\right\}, \tag{5}$$

$$c_2(\lambda, R) = \max\left\{\frac{(1 - y)R}{1 - \lambda}, y + (1 - y)R\right\}, \tag{6}$$

which are illustrated in Fig. 1.

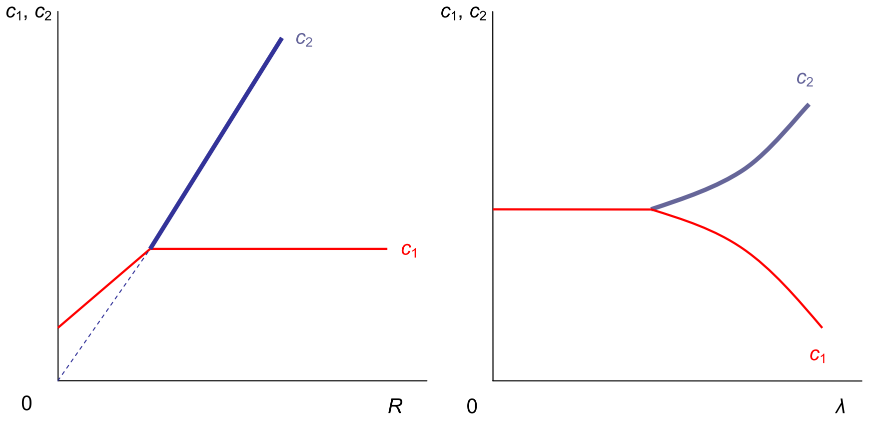

Fig. 1. Consumption functions at dates 1 and 2. The left hand panel shows the consumption of an individual at each date as a function of R holding λ constant. The right hand panel shows the consumption of an individual at each date as a function of λ holding R constant.

The left hand panel illustrates the relationship between consumption and R, holding λ constant. For very small values of R, the late consumers would receive less than the early consumers if there were no storage. This cannot be optimal, so some of the returns of the short asset will be re-invested up to the point where consumption is equalized between early and late consumers. At some critical value of R, the long asset provides just enough to equalize the consumption of early and late consumers without storage. For higher values of R, early consumers consume the entire output of the short asset and there is no storage. Late consumers consume the output of the long asset and their per capita consumption is increasing in R.

The right hand panel illustrates the relationship between consumption and λ, holding R constant. For small values of λ, the short asset provides more consumption than is needed by early consumers, so some is stored and given to late consumers. At the margin, the rate of exchange between early and late consumption is one for one, so optimality requires that early and late consumers receive the same consumption. For some critical value of λ, there is just enough of the short asset to provide early consumers the same amount of consumption as late consumers, i.e., $\frac{y}{\lambda} = \frac{(1-y)R}{1-\lambda}$. For higher values of λ, early consumers continue to consume the entire output of the short asset but their per capita consumption is declining in λ. The late consumers, by contrast, receive the entire output of the long asset and their per capita consumption is increasing in λ.

Note that the consumption functions in (5) and (6) are determined by the choice of y and the exogenous shocks (λ, R), so the planner's problem can be reduced to maximizing the expected utility of the representative consumer with respect to y. The optimal portfolio choice problem is:

$$\max_{y} \quad E\big[\lambda u\big(c_1(\lambda, R)\big) + (1-\lambda)u\big(c_2(\lambda, R)\big)\big], \tag{7}$$

where $c_1(\lambda, R)$ and $c_2(\lambda, R)$ are defined in (5) and (6). The solution to the planner's problem is summarized in the following proposition.

Proposition 1. *The unique solution to the planner's problem consists of a portfolio choice y^* and a pair of consumption functions $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ such that y^* solves the portfolio choice problem (7) and $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ satisfy (5) and (6), respectively.*

Proof. See Appendix A. □

4. Money and exchange

In this section we describe a decentralized economy consisting of four groups of actors: a central bank, a banking sector, a productive sector, and a consumption sector.

The central bank's only function is to provide money that the private sector needs to facilitate transactions. It lends to banks on an intraday basis and charges zero interest. The central bank's policy is passive in the sense that it provides whatever amounts of money the banks demand.

Banks compete for deposits by offering contracts that in exchange for a current deposit, provide a future payment of D_1 units of money at date 1, or D_2 units of money at date 2. Consumers respond by choosing the most attractive of the contracts offered. Free entry ensures that banks offer deposit contracts that maximize consumers' utility and earn zero profits in equilibrium. There is no loss of generality in assuming that consumers deposit all their money in a bank at date 0 since the bank can do anything the consumers can do.

There is free entry to the productive sector, which ensures that in equilibrium firms earn zero profits. Firms take out one-period loans from banks in the first period and use the money to purchase goods from the consumers. These goods are invested in the short and long assets. Some of the returns from these assets are sold at date 1 and used to repay part of the firm's debt. The rest of the debt is rolled over and repaid at date 2 using the proceeds from selling the remaining asset returns at date 2.

Consumers have an initial endowment of goods which they sell to firms in exchange for money at the first date. This money is deposited in the consumers' bank accounts and provides income that can be used for consumption in future periods.

We assume that all transactions are mediated by money. Money is exchanged for goods and goods for money and loans are made and repaid in terms of money. Only banks have access to loans from the central bank and only firms have access to loans from the banks. Consumers can save using deposit accounts at the banks. Given the timing of consumption, there is no need for consumers to borrow.[3] Consumers have the option to purchase goods and store them. Banks can lend to one another on an interbank market, but for the moment there is no need for this activity. There are no forward markets. These assumptions give rise to a particular flow of funds at each date.

The flow of funds at date 0 is illustrated in Fig. 2. Initially, banks borrow funds from the central bank. Banks give loans to firms, which then purchase goods from the consumers. They deposit the proceeds from the sale of goods in their bank accounts. Banks repay their intraday loans to the central bank. The money supply M_0 created by the central bank follows a circuit from the central bank to banks to firms to consumers to banks and, finally, back to the central bank. At each stage the same amount of money changes hands so that the net demand for money is zero at the end of the period.

At the beginning of the second date, all uncertainty is resolved and the aggregate state (λ, R) is realized. Transactions occur in the same order at date 1 and date 2, as illustrated in Fig. 3. Initially banks borrow from the central bank. Then consumers withdraw their savings from their bank accounts and use these funds to purchase goods from the firms. Firms use this money to repay part of their loans to the banks, which then repay their intraday loans to the central bank.

[3] As Cone [13] and Jacklin [20] showed, consumers must be excluded from the market for borrowing and lending at date 1, otherwise they will undermine the ability of the banks to provide them with liquidity insurance.

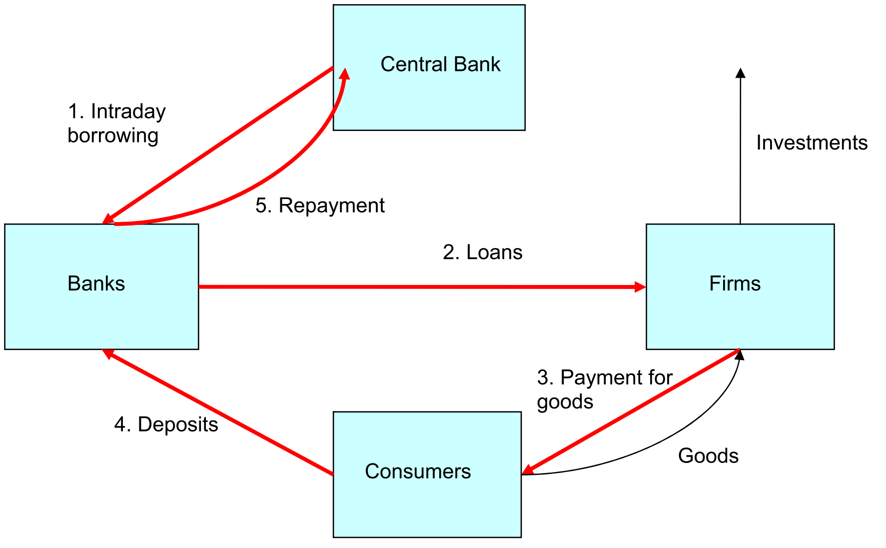

Fig. 2. Flow of funds at date 0. 1. Banks borrow cash from the central bank. 2. Firms borrow cash from the banks. 3. Firms purchase goods from the consumers. 4. Consumers deposit cash with the banks. 5. Banks repay their intraday loans to the central bank.

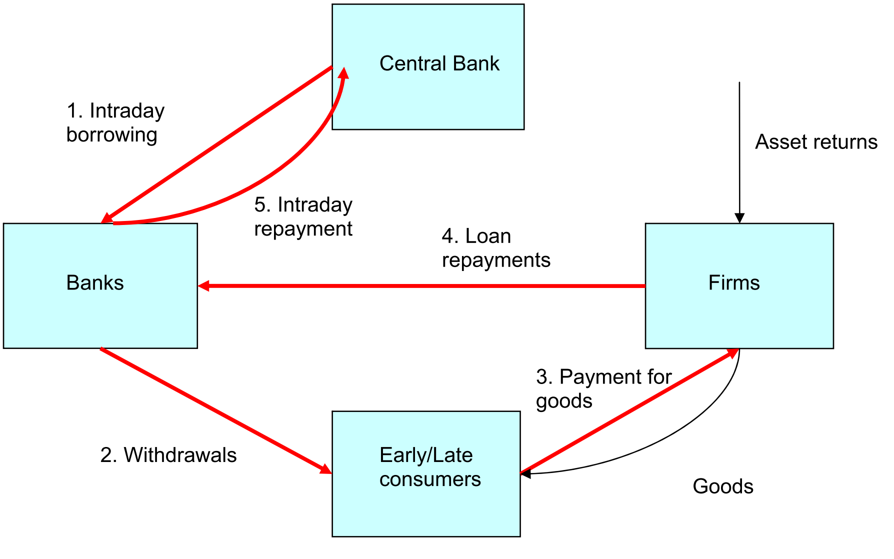

Fig. 3. Flow of funds at dates 1 and 2. 1. Banks borrow cash from the central bank. 2. Early consumers withdraw cash from the banks. 3. Consumers purchase goods from the firms. 4. Firms repay part of their loans to the banks. 5. Banks repay their intraday loans to the central bank.

4.1. Market clearing and the price level

The central bank does not charge interest on intraday balances. The (nominal) interest rate on loans between periods t and $t+1$ is denoted by r_t. That is, one dollar borrowed at date t requires a repayment of $1 + r_t$ dollars at date $t + 1$. Without essential loss of generality we can set interest rates to zero: $r_0 = r_1 = 0$ (this assumption is relaxed in Section 6.1).

The standard homogeneity property of excess demands with respect to prices allows us to normalize the price level at date 0 to unity:

$$P_0 = 1.$$

At date 0 the demand for money comes from firms to buy goods from consumers. Since there is one unit of the good (per capita), firms will borrow one unit of money from the banks in order to purchase the goods. The banks demand this amount of money from the central bank, which therefore must supply the amount

$$M_0 = P_0 = 1 \tag{8}$$

to meet the banks' demand.

At date 1, early consumers withdraw their deposit D_1 from the bank and supply it inelastically in exchange for consumption goods. The amount needed by banks is therefore λD_1 and this is the amount supplied by the central bank in state (λ, R):

$$M_1(\lambda, R) = \lambda D_1. \tag{9}$$

The firms supply either y if the price level at date 1 exceeds that at date 2, i.e., if $P_1(\lambda, R) > P_2(\lambda, R)$, or an amount less than or equal to y, if $P_1(\lambda, R) = P_2(\lambda, R)$. In the latter case, firms are indifferent about whether to sell or store the goods so, in equilibrium, they supply the amount demanded by consumers. Thus, the goods market clears[4] if

$$\lambda c_1(\lambda, R) \leqslant y. \tag{10}$$

The firms return their revenue to the banks in partial payment of their debts and the remaining debt is rolled over.

At date 2, late consumers use their deposit D_2 in the bank and supply it inelastically in exchange for consumption goods. The amount needed by banks is therefore $(1 - \lambda)D_2$ and this is the amount supplied by the central bank:

$$M_2(\lambda, R) = (1 - \lambda)D_2. \tag{11}$$

The firms supply all their goods inelastically, that is, the return from the long asset, $(1 - y)R$, plus the amount stored from the previous period, $y - \lambda c_1(\lambda, R)$. Thus, the goods market clears if

$$\lambda c_1(\lambda, R) + (1 - \lambda)c_2(\lambda, R) = (1 - y)R + y. \tag{12}$$

The firms use the proceeds from their sales of the consumption good to repay their remaining debt to the banks.

4.2. *The bank's decision*

The representative bank's decision problem is quite simple. At the first date, the bank lends money to firms and accepts the money as deposits from consumers. In order to satisfy its budget

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[4] Strictly speaking, we also need to assume that the prices $P_1(\lambda, R)$ and $P_2(\lambda, R)$ satisfy the complementary conditions, $P_1(\lambda, R) \geqslant P_2(\lambda, R)$ and the inequality holds as an equation if there is positive storage $\lambda c_1(\lambda, R) < y$.

constraint, the outflow of loans must equal the inflow of deposits. Without loss of generality, consider the case of a bank that makes loans of one dollar and receives an equal amount of deposits. Since the nominal interest rate has been normalized to zero, the repayment of the loan will yield a stream of payments equal to one dollar spread across the last two dates. The deposit contract (D_1, D_2) is feasible for the bank if $\lambda D_1 + (1-\lambda)D_2 \leqslant 1$ for every (λ, R). In case the repayment of loans does not coincide with the withdrawal of deposits, the bank will plan to use the interbank market to obtain money as needed. In equilibrium, the two flows will be perfectly matched. Competition among banks will cause them to offer depositors the most attractive deposit contracts. This implies that

$$\lambda D_1 + (1-\lambda)D_2 = 1.$$

Since a non-degenerate distribution of λ is assumed, this condition must be satisfied for multiple values of λ and this is only possible if D_1 and D_2 are equal and, hence, equal to 1. The bank will earn zero profits and there is no possibility of doing better.

4.3. The firm's decision

Now consider the representative firm's decision problem. Since the firm's technology exhibits constant returns to scale, there is no loss of generality in restricting attention to a firm that borrows one unit of money at date 0. The firm can obtain one unit of the good with the money it has borrowed, since $P_0 = 1$. Suppose it invests y units in the short asset and $1 - y$ units in the long asset. This will produce y units of the good at date 1 and $(1-y)R$ units of the good at date 2. In equilibrium, it must be optimal to hold the long asset between dates 1 and 2 in every state (λ, R). It may be also optimal to store the good between dates 1 and 2. These conditions require that $P_1(\lambda, R) \geqslant P_2(\lambda, R)$—otherwise the short asset would dominate the long asset at date 1—and $P_1(\lambda, R) = P_2(\lambda, R)$ in any state in which the good is stored between dates 1 and 2. Then, in any case, it will be optimal for the firm to set storage equal to zero in calculating the optimal profit. Since the nominal interest rate is zero, the firm's total revenue is $P_1(\lambda, R)y + P_2(\lambda, R)(1-y)R$ in state (λ, R). Then the firm's budget constraint requires that

$$P_1(\lambda, R)y + P_2(\lambda, R)(1-y)R \geqslant 1, \quad \forall(\lambda, R),$$

and the profit will be zero in equilibrium if and only if the equality holds as an equation for every value of (λ, R).[5]

To sum up, the firm's choice of y^* is optimal if it yields zero profit in every state and there is no alternative plan that yields non-negative profit everywhere and positive profit with positive probability. More formally, the zero-profit condition for y^* can be written

$$P_1(\lambda, R)y^* + P_2(\lambda, R)\left(1-y^*\right)R = 1, \quad \forall(\lambda, R),$$

while the requirement that no feasible y yields positive profit means that if there exists a state (λ, R),

$$P_1(\lambda, R)y + P_2(\lambda, R)(1-y)R > 1,$$

[5] The zero-profit condition at date 0 implies that the firm must earn zero profits in a set of states that occurs with probability one. Since the price functions are continuous in (λ, R), the continuity of prices in (λ, R) implies that the zero-profit condition holds for every state in the support of the distribution.

then there exists a state (λ', R') such that

$$P_1(\lambda', R')y + P_2(\lambda', R')(1-y)R < 1.$$

In other words, a production plan y' that produces positive profits in some state must produce negative profits in another state.

The assumption that firms must satisfy their budget constraints with probability one is obviously restrictive. This kind of assumption is standard in general equilibrium theory. One interpretation is that the bank making the loan imposes covenants that prevent the firm from undertaking any production plan that carries a risk of default. In practice, banks have limited information about the actions chosen by firms. It is well known that asymmetric information gives rise to moral hazard and the possibility of default and there is a vast literature dealing with these problems. We ignore these issues in order to provide a set of sufficient conditions in which monetary policy can achieve the first-best. In other words, this has to be regarded as a benchmark model.

4.4. The consumer's decision

The consumer's decision is straightforward. Consumers deposit the proceeds from selling their endowment of goods to firms. If they turn out to be early consumers, they use the withdrawals from their bank accounts to purchase consumption goods at date 1. If they are late consumers, they will keep their funds in the bank at date 1 provided

$$c_1(\lambda, R) \leqslant c_2(\lambda, R).$$

At date 2 they will use their savings to purchase goods from the firms.

4.5. Equilibrium

An equilibrium consists of the price functions $(P_0^*, P_1^*(\cdot), P_2^*(\cdot))$, the money supply functions $(M_0^*, M_1^*(\cdot), M_2^*(\cdot))$, the portfolio choice y^*, the consumption functions $(c_1^*(\cdot), c_2^*(\cdot))$ and the deposit contract (D_1^*, D_2^*) such that the following conditions are satisfied.

Market clearing The market-clearing conditions (8) through (12) are satisfied.

Optimal bank behavior The representative bank lends to firms and accepts deposits at the first date. It offers a deposit contract $(D_1^*, D_2^*) = (1, 1)$ to depositors.

Optimal firm behavior The representative firm buys one unit of the good at date 0 and chooses a portfolio y^* such that $P_1^*(\lambda, R)y^* + P_2^*(\lambda, R)(1-y^*)R = 1$ for every (λ, R) and, for any y,

$$\begin{aligned} &P_1^*(\lambda, R)y + P_2^*(\lambda, R)(1-y)R > 1, \quad \exists(\lambda, R) \\ &\quad \text{implies} \quad P_1^*(\lambda', R')y + P_2^*(\lambda', R')(1-y)R' < 1, \quad \exists(\lambda', R'). \end{aligned}$$

Optimal consumer behavior Each consumer supplies his endowment inelastically at date 0 and deposits the money he receives in exchange in his bank account. He uses this one unit of money at date 1 if he is an early consumer to purchase

$$c_1^*(\lambda, R) = \frac{1}{P_1^*(\lambda, R)}$$

units of the good. Similarly, if he is a late consumer, he uses the one unit of money at date 2 to enable him to consume

$$c_2^*(\lambda, R) = \frac{1}{P_2^*(\lambda, R)}$$

units of the good.

It is interesting to note that the equilibrium defined above satisfies the Quantity Theory of Money. If the total income (equals total expenditure) at date $t = 1, 2$ is denoted by $Y_t(\lambda, R)$ and defined by

$$Y_t(\lambda, R) = \begin{cases} \lambda c_1(\lambda, R) & \text{if } t = 1, \\ (1-\lambda)c_2(\lambda, R) & \text{if } t = 2, \end{cases}$$

then the market-clearing conditions imply that

$$M_t(\lambda, R) = P_t(\lambda, R)Y_t(\lambda, R),$$

for every state (λ, R) and each date $t = 1, 2$. The price level at each date is proportional to the amount of money supplied by the central bank, as claimed by the Quantity Theory of Money.

5. Decentralization

In this section we show the existence of an efficient equilibrium. Our approach is constructive. We assume that the equilibrium allocation is efficient, that is, the amount invested in the short asset, y^*, and the consumption functions, $(c_1^*(\cdot), c_2^*(\cdot))$, are taken from the solution to the planner's problem discussed in Section 3. Then the goods-market-clearing conditions, (10) and (12), are satisfied by construction. Next we show that the money supply, prices, and deposit contracts can be defined to satisfy the remaining equilibrium conditions.

We set the deposit contracts $(D_1^*, D_2^*) = (1, 1)$ and then use the consumers' budget constraints to define the price functions $P_t^*(\lambda, R)$ for $t = 1$,

$$P_1^*(\lambda, R) = \frac{1}{c_1^*(\lambda, R)} \tag{13}$$

and for $t = 2$,

$$P_2^*(\lambda, R) = \frac{1}{c_2^*(\lambda, R)}. \tag{14}$$

The money supply by the central bank responds passively to the commercial banks' demand at each date so we can use the money-market-clearing equations (9) and (11) to define the central bank's money supply functions:

$$M_1^*(\lambda, R) = \lambda$$

and

$$M_2^*(\lambda, R) = 1 - \lambda.$$

The banks' total liabilities (deposits) at date 0 are equal to their assets (loans). They lend $P_0^* = 1$ to firms and receive deposits of $P_0^* = 1$. In state (λ, R) at date 1, withdrawals equal λ and repayments by firms also equal λ. In state (λ, R) at date 2, withdrawals equal $1 - \lambda$. Since interest rates are zero, the total repayment of the loans will equal the original loan amount and the bank makes zero profits on the loan. Similarly, the withdrawals equal the original deposit amount and the bank makes zero profits on the deposits.

Finally, consider the firm's problem. As we have shown, the firm will make zero profits since the amount of money it receives for its output, $\lambda + 1 - \lambda = 1$, is equal to the amount of money it originally borrows from the bank. It is feasible for the firm to supply the optimal levels of consumption, $\lambda c_1^*(\lambda, R)$ and $(1-\lambda)c_2^*(\lambda, R)$, at dates 1 and 2 respectively. To see that this is optimal, we have to check that it is optimal for the firm to store the good in states where $\lambda c_1^*(\lambda, R) < y^*$. But from the planner's problem, we know that $\lambda c_1^*(\lambda, R) < y^*$ implies that $c_1^*(\lambda, R) = c_2^*(\lambda, R)$, in which case the definition of price functions in Eqs. (13) and (14) implies that $P_1^*(\lambda, R) = P_2^*(\lambda, R)$. Thus, storage is optimal.

To complete our demonstration of the optimality of the firm's behavior, we have to show that the firm cannot profitably deviate from the specified production plan without being unable to repay its loan in some states. Without loss of generality, we can assume the firm borrows one unit of cash from a bank at date 0. The firm must choose a value of y so that it can repay this debt in every state (λ, R). Let (λ_0, R_0) be a state satisfying $\lambda_0 = y^*$ and $R_0 > 1$. Then

$$\frac{y^*}{\lambda_0} = 1 < \frac{1-y^*}{1-\lambda_0} R_0,$$

which implies that $c_1^*(\lambda_0, R_0) < c_2^*(\lambda_0, R_0)$. In fact, the continuity of the feasibility conditions implies that $c_1^*(\lambda, R) < c_2^*(\lambda, R)$ for any state (λ, R) sufficiently close to (λ_0, R_0). The firms' total revenue, TR, in state (λ, R) is

$$TR = P_1^*(\lambda, R)y + P_2^*(\lambda, R)(1-y)R = \frac{y}{c_1^*(\lambda, R)} + \frac{(1-y)R}{c_2^*(\lambda, R)} \tag{15}$$

and, for all states sufficiently close to (λ_0, R_0), this simplifies to

$$TR = \frac{\lambda}{y^*} y + \frac{(1-\lambda)}{(1-y^*)}(1-y).$$

so

$$\frac{dTR}{dy} = \frac{(\lambda - y^*)}{y^*(1-y^*)}.$$

Note that $0 < y^* < 1$ since $u'(c) \to \infty$ as $c \to 0$ and $\bar{R} > 1$. Thus, $\frac{dTR}{dy} > 0$ for $\lambda > y^*$ and $\frac{dTR}{dy} < 0$ for $\lambda < y^*$. Since $TR = 1$ for all values of λ when $y = y^*$, it follows that if $y < y^*$ then $TR < 1$ for some $\lambda > y^*$ sufficiently close to λ_0 and, similarly, if $y > y^*$ then $TR < 1$ for some $\lambda < y^*$ sufficiently close to λ_0. Hence the firm cannot deviate from $y = y^*$ and still repay its loan for all (λ, R).

We have the following result.

Proposition 2. *The unique solution to the planner's problem can be supported as an equilibrium* $e = (P_0^*, P_1^*(\cdot), P_2^*(\cdot), M_0^*, M_1^*(\cdot), M_2^*(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^*, D_2^*)$.

The reason that the first-best efficient allocation can be supported as an equilibrium in our model is that the consumer price level adjusts to provide risk sharing. One issue concerns the interpretation of this mechanism. We have been using the terminology that commercial banks borrow and repay the central bank intraday. Of course, the consumer price level does not literally adjust on a daily basis. However, the model is very simple. There is a banking sector, firms and consumers, without any additional components of an economy or a financial system. In this case the only possible impact of money is an immediate impact on consumer prices. Our three-date framework exaggerates the changes in λ and R as well as prices. In continuous time, all three might be continuous and slow moving. Also, in practice prices can be sticky for many reasons (see, e.g., Blinder [6]) and the impact on consumer prices may be more long run.

5.1. Non-contingent contracts, price flexibility and the interbank market

So far, we have assumed that deposit contracts are fixed in nominal terms. In this section, we consider a version of the model in which banks can offer contingent contracts to depositors. This sheds light on two interesting aspects of the model: it highlights the trade-off between contingent contracts and price flexibility and it shows that an additional market-clearing condition is required.

When contracts are assumed to be non-contingent, the bank's decision is trivial. First, there is a unique (non-contingent) deposit contract that satisfies the feasibility condition $\lambda D_1 + (1 - \lambda)D_2 = 1$, namely, the contract defined by $D_1 = D_2 = 1$. So the bank has no choice about the deposit contract. Second, since the interest rate is zero and banks earn no profits in equilibrium, the bank is indifferent about the timing of the firms' repayments. The bank's behavior is determined by technology and by the decisions of firms and consumers. If banks are allowed to offer contingent contracts, the bank's options are greatly increased.

In order to describe the bank's decision problem with contingent contracts, we have to introduce some additional notation. Suppose that a deposit contract promises $D_1(\lambda, R)$ (respectively, $D_2(\lambda, R)$) units of money to a depositor who withdraws at date 1 (respectively, at date 2) in state (λ, R). Let $L_1(\lambda, R)$ (respectively, $L_2(\lambda, R)$) denote the money repaid by firms at date 1 (respectively, date 2) in state (λ, R). Because there may be a mismatch between withdrawals $(D_1(\lambda, R), D_2(\lambda, R))$ and the bank's revenue stream $(L_1(\lambda, R), L_2(\lambda, R))$, the bank may need to access the interbank market. The bank is assumed to be able to borrow and lend at the interest rate $i(\lambda, R)$. Using this notation, we can define the bank's decision problem. The bank chooses $(D_1(\lambda, R), D_2(\lambda, R))$ to maximize

$$E\left[\lambda U\left(\frac{D_1(\lambda, R)}{P_1(\lambda, R)}\right) + (1-\lambda)U\left(\frac{D_2(\lambda, R)}{P_2(\lambda, R)}\right)\right]$$

subject to the budget constraint

$$\lambda D_1(\lambda, R) + (1-\lambda)\frac{D_2(\lambda, R)}{1+i(\lambda, R)} \leqslant L_1(\lambda, R) + \frac{L_2(\lambda, R)}{1+i(\lambda, R)},$$

for every state (λ, R), where the left hand side is the present value of withdrawals and the right hand side is the present value of loan repayments, measured at date 1 in state (λ, R). This is equivalent to choosing (c_1, c_2) to maximize

$$E\big[\lambda U\big(c_1(\lambda, R)\big) + (1-\lambda)U\big(c_2(\lambda, R)\big)\big]$$

subject to

$$\lambda P_1(\lambda, R)c_1(\lambda, R) + \frac{(1-\lambda)P_2(\lambda, R)c_2(\lambda, R)}{1+i(\lambda, R)} \leqslant L_1(\lambda, R) + \frac{L_2(\lambda, R)}{1+i(\lambda, R)},$$

for any state (λ, R). The first-order condition for this problem for any state (λ, R) is

$$\frac{U'(c_1(\lambda, R))}{P_1(\lambda, R)} = \big(1 + i(\lambda, R)\big)\frac{U'(c_2(\lambda, R))}{P_2(\lambda, R)}.$$

Suppose that we want to implement the optimal consumption profile $(c_1(\lambda, R), c_2(\lambda, R))$. We can define the equilibrium price levels by setting

$$\big(P_1(\lambda, R), P_2(\lambda, R)\big) = \left(\frac{1}{c_1(\lambda, R)}, \frac{1}{c_2(\lambda, R)}\right).$$

These prices will clear the markets at dates 1 and 2 if and only if

$$\big(D_1(\lambda, R), D_2(\lambda, R)\big) = (D_1, D_2) = (1, 1),$$

for every state (λ, R). The bank's budget constraint will be satisfied because, as we have seen before,

$$\lambda D_1 = L_1(\lambda, R) \quad \text{and} \quad (1 - \lambda)D_2 = L_2(\lambda, R)$$

in every state (λ, R). Finally, with the prices defined above, the first-order conditions reduce to

$$U'\big(c_1(\lambda, R)\big)c_1(\lambda, R) = \big(1 + i(\lambda, R)\big)U'\big(c_2(\lambda, R)\big)c_2(\lambda, R),$$

for every state (λ, R). We can solve these equations for the equilibrium interbank interest rate:

$$1 + i(\lambda, R) = \frac{U'(c_1(\lambda, R))c_1(\lambda, R)}{U'(c_2(\lambda, R))c_2(\lambda, R)} = \max\left\{1, \frac{U'(\frac{y}{\lambda})\frac{y}{\lambda}}{U'(\frac{(1-y)R}{1-\lambda})\frac{(1-y)R}{1-\lambda}}\right\},$$

for any state (λ, R). So,

$$i(\lambda, R) = \max\left\{0, \frac{U'(\frac{y}{\lambda})\frac{y}{\lambda}}{U'(\frac{(1-y)R}{1-\lambda})\frac{(1-y)R}{1-\lambda}} - 1\right\} \tag{16}$$

is the interest rate at which the interbank market clears. At this rate, it is optimal for every bank neither to borrow nor lend. The rest of the equilibrium definition remains unchanged.

An interesting feature of this equilibrium is that, even though we allow for contingent contracts, it is *optimal* for the bank to use a non-contingent deposit contract, both in the usual sense that it achieves the first-best, and in the sense that the bank perceives no advantage in choosing a non-contingent contract from the expanded budget set. Optimality in this second sense requires the interest rate $i(\lambda, R)$ to satisfy the interbank market-clearing condition (16). This condition was absent from our previous definition of equilibrium because it was not needed. When $D_1 = D_2 = 1$ is the *only* feasible choice for banks, borrowing and lending on the interbank market are ruled out and the market clears at any interest rate.

It is important to note that the interbank interest rate $i(\lambda, R)$ is not the same as the interest rate paid on deposits and loans. In fact, it is crucial for the risk sharing function of banks that the interest rates be different. The interest rate on loans and deposits allows firms and consumers to balance their budgets, whereas the interbank rate is a signal that guides banks to make the right trade-off between consumption at date 1 and consumption at date 2.

There is an interesting connection between the result that the interbank rate is not the same as the rates on loans and deposits and the results of Cone [13] and Jacklin [20] that banks could not offer-welfare improving risk sharing services if depositors had access to a credit market at date 1. The reason was that a credit market allows depositors to transfer wealth between periods, so all they care about is the present value of their withdrawal. Then early and late consumers have the same preferences, which prevents the bank from separating the two types. This makes it impossible to offer an incentive-compatible risk sharing contract that improves welfare.

In the present setting, allowing a bank to borrow and lend at the same rate as its own customers gives the bank an incentive to arbitrage against the other banks. For example, if we assume that $i(\lambda, R) = 0$ for all (λ, R), then the bank will try to maximize

$$E\left[\lambda U\left(\frac{D_1(\lambda, R)}{P_1(\lambda, R)}\right) + (1-\lambda)U\left(\frac{D_2(\lambda, R)}{P_2(\lambda, R)}\right)\right]$$

subject to the budget constraint

$$\lambda D_1(\lambda, R) + (1-\lambda)D_2(\lambda, R) \leqslant L_1(\lambda, R) + L_2(\lambda, R),$$

for every (λ, R). The first-order condition becomes

$$U'\big(c_1(\lambda, R)\big)c_1(\lambda, R) = U'\big(c_2(\lambda, R)\big)c_2(\lambda, R),$$

for every (λ, R). For example, if $U(\cdot)$ exhibits constant relative risk aversion $\rho \neq 1$, then the first-order condition can only be satisfied if $c_1(\lambda, R) = c_2(\lambda, R)$, for every (λ, R), which is not optimal in general.

We have shown that the first-best allocation can be implemented as an equilibrium with non-contingent deposit contracts, even in the extended framework where banks are allowed to choose contingent contracts. This is only possible, of course, because we chose the appropriate prices and accompanying monetary policy. A different specification of prices and monetary policy could make contingent contracts essential. Suppose, for example, that we wanted to have price stability in equilibrium:

$$P_1(\lambda, R) = P_2(\lambda, R), \tag{17}$$

for every state (λ, R). As before, the interbank interest rate $i(\lambda, R)$ will have to satisfy (16) in order for banks to choose the optimal deposit contract

$$\big(D_1(\lambda, R), D_2(\lambda, R)\big) = \big(P_1(\lambda, R)c_1(\lambda, r), P_2(\lambda, R)c_2(\lambda, R)\big).$$

Again, the bank's budget constraint is satisfied since

$$\lambda D_1(\lambda, R) = L_1(\lambda, R) \quad \text{and} \quad (1-\lambda)D_2(\lambda, R) = L_2(\lambda, R).$$

Note that (17) implies that

$$D_1(\lambda, R) = P_1(\lambda, R)c_1(\lambda, r) \leqslant P_2(\lambda, R)c_2(\lambda, R) = D_2(\lambda, R),$$

so the incentive constraint is also satisfied for all (λ, R).

Finally, we have a degree of freedom in specifying the equilibrium deposit contract. Suppose we agree that a deposit contract should offer a fixed payment at date 1, but may offer a variable interest rate, depending on the state of nature (λ, R), on withdrawals at date 2. Then we can set $D_1(\lambda, R) = 1$ and

$$D_2(\lambda, R) = \big(1 + r(\lambda, R)\big)D_1(\lambda, R),$$

where $r(\lambda, R)$ is the deposit rate offered to late withdrawers at date 1. Then

$$P_1(\lambda, R) = P_2(\lambda, R) = \frac{1}{c_1(\lambda, R)}$$

and

$$1 + r(\lambda, R) = \frac{c_2(\lambda, R)}{c_1(\lambda, R)}$$

for every (λ, R). Note again that the deposit interest rate $r(\lambda, R)$ is not the same as the interbank rate $i(\lambda, R)$.

This example shows that flexibility in the interest rate paid on deposits $r(\lambda, R)$ is a substitute for flexibility of *relative* prices $\frac{P_2(\lambda,R)}{P_1(\lambda,R)}$. It remains true, however, that flexibility in the price *level* $P_1(\lambda, R) = P_2(\lambda, R)$ is required when new information becomes available. The real value of deposits at date 1 has to fluctuate in response to liquidity shocks and rate of return shocks. This can only come about through adjustments in the date-1 price level. A special feature of this model is that all uncertainty is resolved at the beginning of date 1. If instead new information were to become available at date 2, for example, information about the return R on the long asset, then flexibility in the date-2 price level *relative* to the date-1 price level would in general be required.

6. Extensions

In this section we consider various extensions of the basic model. We first relax the assumption of zero nominal interest rates. Then we consider the case of idiosyncratic liquidity shocks and the role of the interbank market in shuffling liquid balances around the system. Then we extend the analysis to a multi-period framework. In all these cases, we show that our main efficiency result remains valid. Finally, we turn to the case of idiosyncratic asset return risk and show that monetary policy is no longer sufficient for the efficiency result to hold. A different level of intervention, involving real transfers among banks, is required to obtain the first-best.

6.1. Nominal interest rates

We have claimed that we can set nominal interest rates equal to zero without loss of generality. This is because the real rates of interest are all that matter when money is not held as a store of value outside the banking system between periods. The real rates are independent of the nominal rate as long as the price levels are adjusted appropriately. Suppose that

$$e = \left(P_0^*, P_1^*(\cdot), P_2^*(\cdot), M_0^*, M_1^*(\cdot), M_2^*(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^*, D_2^*, r_0^*, r_1^*\right)$$

is an equilibrium with interest rates normalized to $r_0^* = r_1^* = 0$ and suppose that we choose some arbitrary nominal interest rates $r_0^{**} > 0$ and $r_1^{**} > 0$. Then we claim that there exists an equilibrium

$$e' = \left(P_0^{**}, P_1^{**}(\cdot), P_2^{**}(\cdot), M_0^{**}, M_1^{**}(\cdot), M_2^{**}(\cdot), c_1^*(\cdot), c_2^*(\cdot), y^*, D_1^{**}, D_2^{**}, r_0^{**}, r_1^{**}\right),$$

with the same allocation $(c_1^*(\cdot), c_2^*(\cdot), y^*)$, where

$$P_0^{**} = 1,\ P_1^{**}(\lambda, R) = \frac{(1 + r_0^{**})}{c_1^*(\lambda, R)}, \quad \text{and} \quad P_2^{**}(\lambda, R) = \frac{(1 + r_0^{**})(1 + r_1^{**})}{c_2^*(\lambda, R)},$$

$$M_0^{**} = 1,\ M_1^{**}(\lambda, R) = \lambda\left(1 + r_0^{**}\right), \quad \text{and} \quad M_2^{**}(\lambda, R) = \left(1 + r_0^{**}\right)\left(1 + r_1^{**}\right)$$

and

$$D_1^{**} = \left(1 + r_0^{**}\right) \quad \text{and} \quad D_2^{**} = \left(1 + r_0^{**}\right)\left(1 + r_1^{**}\right).$$

Clearly, the money-market-clearing conditions (9) and (11) and the goods-market-clearing conditions (10) and (12) are satisfied. The banks continue to earn zero profits since

$$1 = \frac{\lambda(1+r_0^{**})}{(1+r_0^{**})} + \frac{(1-\lambda)(1+r_0^{**})(1+r_1^{**})}{(1+r_0^{**})(1+r_1^{**})}.$$

This equation says that the present value of repayments (respectively, withdrawals) equals the value of the initial loan amount (respectively, the initial deposit amount). It is equally easy to see that the bank cannot profitably deviate from this strategy.

For firms, the zero-profit condition follows immediately from the fact that (a) expenditures at date 0 equal the loan at date 0 and (b) the revenues at date $t = 1, 2$ equal the repayments at dates $t = 1, 2$, respectively. The banks' zero-profit condition implies that the present value of the firm's repayments equals the value of the loan, so the firm makes zero profit on borrowing and lending. The optimality of storage in states where $\lambda c_1^*(\lambda, R) < y^*$ follows from the fact that $c_1^*(\lambda, R) = c_2^*(\lambda, R)$ implies that $(1 + r_1^{**})P_1^{**}(\lambda, R) = P_2^{**}(\lambda, R)$. The argument given in Appendix B can be used to show that there is no profitable deviation for firms from the efficient production plan.

6.2. *Idiosyncratic liquidity shocks and the interbank market*

The decentralization result in Section 5 can easily be extended to deal with heterogeneity in the liquidity shocks received by individual banks. Suppose that banks are identified with points on the unit interval and let $\lambda_i = \theta_i\lambda$ be the fraction of early consumers among bank i's depositors, where $\{\theta_i\}$ are i.i.d. random variables with $E[\theta_i] = 1$ for all i. The efficient allocation is the same as before and the market-clearing prices will also be the same as before since the shocks $\{\theta_i\}$ are idiosyncratic and do not affect the aggregate proportion of early consumers. Since in equilibrium the banks offer deposit contracts that satisfy $D_1 = D_2 = D^* = 1$, the shock θ_i has no effect on the bank's ability to meet the withdrawals of its depositors. More precisely,

$$\begin{aligned} P_1^*(\lambda, R)y^* + P_2^*(\lambda, R)\big(1 - y^*\big)R &= 1 \\ &= \theta_i\lambda + (1 - \theta_i\lambda) \\ &= \theta_i\lambda P_1^*(\lambda, R)c_1^*(\lambda, R) + (1 - \theta_i\lambda)P_2^*(\lambda, R)c_2^*(\lambda, R), \end{aligned}$$

since $P_1^*(\lambda, R)c_1^*(\lambda, R) = P_2^*(\lambda, R)c_2^*(\lambda, R) = D^* = 1$.

The banks with $\theta_i > 1$ borrow from banks with $\theta_i < 1$ at date 1 and repay the loan at date 2 when the number of late consumers will be correspondingly lower. The interbank market clears because the Law of Large Numbers implies that

$$\int_0^1 \theta_i \, di = 1.$$

Let $B_1(\theta_i, \lambda, R)$ denote the net interbank borrowing at date 1 by a bank with shock θ_i and let $B_2(\theta_i, \lambda, R)$ denote the repayment at date 2. Then, for every (λ, R) and θ_i,

$$\begin{aligned} B_1(\theta_i, \lambda, R) &= \theta_i\lambda D^* - P_1^*(\lambda, R)\lambda c_1^*(\lambda, R) \\ &= \theta_i\lambda D^* - \frac{\lambda D^*}{\lambda c_1^*(\lambda, R)}\lambda c_1^*(\lambda, R) \\ &= (\theta_i - 1)\lambda D^* = (\theta_i - 1)\lambda, \end{aligned}$$

for every (λ, R) and θ_i. Thus,

$$\int_0^1 B_1(\theta_i, \lambda, R)\,di = \int_0^1 (\theta_i - 1)\lambda\,di = 0,$$

for every (λ, R). Similarly, at date 2,

$$\begin{aligned} B_2(\theta_i, \lambda, R) &= (1 - \theta_i\lambda)D^* - P_2^*(\lambda, R)(1-\lambda)c_2^*(\lambda, R) \\ &= (1 - \theta_i\lambda)D^* - \frac{(1-\lambda)D^*}{(1-\lambda)c_2^*(\lambda, R)}(1-\lambda)c_2^*(\lambda, R) \\ &= (\lambda - \theta_i\lambda)D^* = (1-\theta_i)\lambda, \end{aligned}$$

for every (λ, R) and θ_i. Then

$$\int_0^1 B_2(\theta_i, \lambda, R)\,di = \int_0^1 (1 - \theta_i)\lambda\,di = 0,$$

for every (λ, R).

To sum up, the interbank market allows banks to shuffle liquid balances around the system so that, as long as there is no aggregate excess demand, each bank is able to pay out the appropriate amount of money to its depositors. In order for the bank's budget to balance, however, it is necessary for the interest rate on these interbank loans to be set equal to zero. It will be recalled from the discussion in Section 5.1 that this may not be the "market-clearing" interest rate. This is not a problem as long as the distribution of λ is non-degenerate, in which case the only feasible non-contingent deposit contract satisfies $D_1 = D_2 = 1$. Given the fixed contract, there is no gain to the bank from trying to exploit its ability to borrow and lend at a zero interest rate. With contingent contracts, however, there might be a conflict between the market-clearing interbank interest rate (needed to support the first-best consumption allocation) and the budget-balancing interbank interest rate (needed to allow banks to obtain the liquidity needed to meet withdrawals at each date). In fact, it may not be possible in general for a simple interbank market to meet the liquidity needs of the banking system and, at the same time, provide the price signals that support optimal risk sharing. Bhattacharya and Gale [5] showed that, when bank portfolios and liquidity shocks are private information, the first-best may not be implementable and the incentive-efficient mechanism may entail rationing of banks' access to liquidity. We face a similar problem here since, in order to achieve the first-best, it may be necessary to restrict the banks' ability to trade at the zero interest rate.

6.3. The multi-period case

Suppose now that, instead of three dates, we have a finite sequence of dates $t = 0, 1, \dots, T$. The random variable λ_t is the fraction of consumers that wish to consume only at date t. There is a long asset for each date $t = 1, \dots, T$. The random variable R_t is the return on one unit of the good invested at date 0 in the asset that pays off at date t. We also allow for investment in the short asset at each date $t = 1, \dots, T$. As usual, all uncertainty is resolved at date 1, when the random vectors $\boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_T)$ and $\mathbf{R} = (R_1, \dots, R_T)$ are realized. We assume that the support of $\boldsymbol{\lambda}$ is $\{\boldsymbol{\lambda} \geqslant \mathbf{0}\colon \sum_{t=1}^T \lambda_t = 1\}$ and the support of $\mathbf{R}$ is $[1, R_{\max}]^T$, for some $1 < R_{\max} < \infty$. Since the long asset dominates the short asset, there will be no investment in the short asset at date 0. However, the short asset may be used at subsequent dates to smooth consumption intertemporally, for some realizations of $(\boldsymbol{\lambda}, \mathbf{R})$. For example, when the realization of R_t is high, some of the output can be carried forward to offset high liquidity shocks or low asset returns in subsequent periods.

Let $c_t(\boldsymbol{\lambda}, \mathbf{R})$ denote the consumption at date t in state $(\boldsymbol{\lambda}, \boldsymbol{R})$ and let $\mathbf{x} = (x_1, \ldots, x_T)$ denote the portfolio of long assets. The planner will choose a portfolio $\mathbf{x}$ and a sequence of consumption functions $\{c_t(\boldsymbol{\lambda}, \mathbf{R})\}$ in order to maximize

$$\mathbf{E}\left[\sum_{t=1}^{T} \lambda_t u\big(c_t(\boldsymbol{\lambda}, \mathbf{R})\big)\right]$$

subject to

$$\sum_{t=1}^{T} x_t = 1,$$

$$\sum_{s=1}^{t} \lambda_s c_t(\boldsymbol{\lambda}, \mathbf{R}) \leqslant \sum_{s=1}^{t} R_s x_s, \quad \forall t,\ \forall(\boldsymbol{\lambda}, \mathbf{R}).$$

The first constraint ensures that the investments in the long assets exhaust the endowment. The second constraint ensures that, at every date, the cumulative consumption at that date is less than or equal to the cumulative output at that date. These constraints incorporate the possibility of storage. Let $(\mathbf{x}^*, c_1^*(\cdot), \ldots, c_T^*(\cdot))$ denote the solution to this problem.

This solution has a simple form. If consumption could be carried forward and backward through time without restriction, it would be optimal to equate per capita consumption each period. However, consumption can only be carried forward through time. When R_t is high or λ_t is low, it is optimal to carry forward output using the short-term asset until there is another high R_t or low λ_t. Thus the dates $t = 1, \ldots, T$ can be partitioned into n intervals $\{1, \ldots, t_1\}, \{t_1 + 1, \ldots, t_2\}, \ldots, \{t_n + 1, \ldots, T\}$, where $t_0 = 0$, $t_{n+1} = T$ and $i = 0, \ldots, n$. Each of these intervals $\{t_i + 1, \ldots, t_{i+1}\}$ corresponds to a sequence of dates where a positive amount of the good is being stored at each date $t_i + 1, \ldots, t_{i+1} - 1$ and none of the good is stored at the last date t_{i+1}. Then the first-order conditions for the planner's problem imply that consumption is equalized across every date in the interval $\{t_i + 1, \ldots, t_{i+1}\}$ and the feasibility conditions hold exactly at the end dates $t_1, \ldots, t_{n+1}$, that is,

$$\sum_{s=1}^{t_i} \lambda_s c_s^*(\boldsymbol{\lambda}, \mathbf{R}) = \sum_{s=1}^{t_i} R_s x_s^*, \quad \text{for } i = 1, \ldots, n.$$

These feasibility conditions clearly imply that

$$\sum_{s=t_i+1}^{t_{i+1}} \lambda_s c_s^*(\boldsymbol{\lambda}, \mathbf{R}) = \sum_{s=t_i+1}^{t_{i+1}} R_s x_s^*, \quad \text{for } i = 0, \ldots, n.$$

Using the fact that consumption is constant throughout the interval $\{t_i + 1, \ldots, t_{i+1}\}$, we can solve this equation for the value of $c_t^*(\boldsymbol{\lambda}, \mathbf{R})$ for any $t \in \{t_i + 1, \ldots, t_{i+1}\}$ and the result is

$$c_t^*(\boldsymbol{\lambda}, \mathbf{R}) = \frac{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s}. \tag{18}$$

We next show that the optimal solution to the planner's problem can be decentralized as an equilibrium. At date 0, we normalize $M_0^* = P_0^* = 1$. Firms borrow one unit of money, purchase the consumers' endowments, and invest them in a portfolio $\mathbf{x}^*$ of the long assets. We assume that the nominal interest rate on loans to the firms is zero. Consumers deposit the money in the bank in exchange for a deposit contract that will offer $D_t = 1$ unit of money to any consumer who withdraws at date t.

To ensure the goods market clears, we set

$$P_t^*(\boldsymbol{\lambda}, \mathbf{R}) = \frac{1}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})},$$

for every date $t = 1, \ldots, T$ and every state $(\boldsymbol{\lambda}, \mathbf{R})$. To ensure that the demand for money equals the supply, we define the money supply functions recursively by setting

$$M_t(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t D_t = \lambda_t,$$

for every date $t = 1, \ldots, T$ and every state $(\boldsymbol{\lambda}, \mathbf{R})$.

It can be shown that firms make zero profits and can repay their loans and that banks break even on their loans and deposits. The argument for the banks is the usual one. At each date t, the amount of money the representative bank borrows from the central bank, $M_t(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t$, is returned to it as the consumers pay $\lambda_t P_t^*(\boldsymbol{\lambda}, \mathbf{R}) c_t^*(\boldsymbol{\lambda}, \mathbf{R}) = \lambda_t$ to the firms in exchange for consumption and the firms use this revenue to repay (part of) their debt to the banks. Over the course of the dates $t = 1, \ldots, T$, the representative bank is paid one unit of money, the amount of the initial loan. Note that it is optimal for the banks to choose $D_t = 1$. They can't afford to pay $D_t > 1$ and competition ensures $D_t < 1$ will not attract any customers. Since loans make zero profits, the banks cannot increase profits by changing the amount of loans.

The analysis of the firm's problem is contained in Appendix B, where we show that firms make zero profits if they choose the portfolio $\mathbf{x}^*$ and that they make losses with positive probability if they choose any feasible $\mathbf{x} \neq \mathbf{x}^*$.

6.4. Idiosyncratic return risk

One source of uncertainty that cannot be dealt with by monetary policy alone is idiosyncratic or bank-specific asset return risk. In order to achieve efficient sharing of idiosyncratic asset return risk, it is necessary to introduce new markets or institutions. To analyze this case, we assume without loss of generality that there is no aggregate uncertainty and that there are no idiosyncratic liquidity shocks. These risks can be shared efficiently through the interbank market and adjustments in the price level, as we have shown in Section 5. We assume that the probability of being an early consumer is a constant $\bar{\lambda}$ and that the expected return on the long asset is a constant $\bar{R} > 1$, but that each bank receives a random return $\tilde{R}_i$ on its holding of the long asset. The returns are i.i.d. across banks and the mean return is $E[\tilde{R}_i] = \bar{R}$. Then the Law of Large Numbers implies that

$$\int_0^1 \tilde{R}_i \, di = \bar{R}$$

with probability one. To simplify the analysis further, we can assume that banks hold the long and short assets directly, thus eliminating any reference to firms and their need to borrow. In other respects the model remains the same as before.

From the point of view of the central planner, idiosyncratic risk is irrelevant. Since the planner can redistribute returns in any way he pleases, only the aggregate (mean) return matters. Since the mean return on the long asset is a constant, $\bar{R}$, the central planner's problem is essentially a decision problem under certainty so we can optimize with respect to y, c_1 and c_2 at date 0:

$$\begin{aligned} \max \quad & \bar{\lambda} u(c_1) + (1-\bar{\lambda}) u(c_2) \\ \text{s.t.} \quad & \bar{\lambda} c_1 \leqslant y, \\ & \bar{\lambda} c_1 + (1-\bar{\lambda}) c_2 = y + (1-y)\bar{R}. \end{aligned}$$

The short asset is used to satisfy the demands of early consumers and the long asset is used to satisfy the demands of the late consumers. The optimum is characterized by the first-order condition

$$u'(c_1) = \bar{R} u'(c_2),$$

where

$$c_1 = \frac{y}{\bar{\lambda}},$$
$$c_2 = \frac{(1-y)\bar{R}}{1-\bar{\lambda}}.$$

Now suppose that y^* is the efficient portfolio and (c_1^*, c_2^*) is the efficient consumption profile and consider how we can implement this outcome. As usual, we can assume without loss of generality that at date 0 the price of goods is $P_0^* = 1$ and the money supply is $M_0^* = 1$. The central bank supplies money passively to the commercial banks that use it to buy goods from consumers. The banks invest the goods in the short and long asset in the same proportions, y^* and $1 - y^*$, as the efficient allocation. The consumers then deposit their money in the banks in exchange for a demand deposit contract promising them $D_1^* = D_2^* = 1$ given the assumption that the nominal interest rates r_0^* and r_1^* are both zero.

At dates 1 and 2, in order to satisfy the market-clearing condition, the price levels must be

$$P_1^* = \frac{1}{c_1^*} \quad \text{and} \quad P_2^* = \frac{1}{c_2^*}.$$

At these prices, the average bank (i.e., one that earns a return of $\bar{R}$ on the long asset) will be just solvent as it requires one unit of money to repay its depositors and its portfolio will yield

$$P_1^* y^* + P_2^* \left(1-y^*\right)\bar{R} = \frac{1}{c_1^*} y^* + \frac{1}{c_2^*}\left(1-y^*\right)\bar{R} = \bar{\lambda} + (1-\bar{\lambda}) = 1.$$

However, any bank that makes a return $R_i < \bar{R}$ on the long asset will have too little money to meet the withdrawals of its depositors at dates 1 and 2 and any bank that makes a return $R_i > \bar{R}$ will have more than enough money to meet its withdrawals.

This problem can be solved by providing banks with deposit insurance of the following form: each bank that has a deficit $(R - \bar{R})(1 - y^*) < 0$ is paid an amount of money equal to $(\bar{R} - R)(1 - y^*) > 0$, whereas any bank that has a surplus $(R - \bar{R})(1 - y^*) > 0$ must pay a tax equal to that amount. Then every bank has exactly the right nominal value of assets to pay back its depositors and by construction the money market and goods markets will clear. More precisely, if we set $M_1^* = \lambda$, then banks will have enough cash to pay the early consumers one dollar each at date 0, the consumers will have enough cash to purchase the efficient amount of the good, since

$P_1^* c_1^* = 1$, and the goods market will clear because $\lambda c_1^* = y^*$. Similarly, if we set $M_2^* = 1 - \lambda$, the banks will have just enough money to pay the late consumers one dollar each at date 2, the consumers will have enough cash to purchase the efficient amount of the good, since $P_2^* c_2^* = 1$, and the goods market will clear because $(1 + \bar{\lambda})c_2^* = (1 - y^*)\bar{R}$. Thanks to the interbank market, the precise timing of the taxes and the insurance payments does not matter.

This brief sketch suggests how the efficient allocation can be implemented. There are, of course, many other ways to achieve the same end. Any solution such as securitization or insurance contracts that allow banks to pool their assets to get rid of the idiosyncratic risk would implement the same allocation. However, none of these methods can avoid the necessity of making real transfers among banks and these transfers cannot be achieved through price level adjustments alone. In practice such transfers may be difficult to accomplish as, in the presence of asymmetric information, banks with high returns will have an incentive to hide them from both government and private schemes. Also, if a bank's portfolio is opaque, moral hazard problems can emerge (cf., Bhattacharya and Gale [5]). In such situations, monetary policy alone is not sufficient to eliminate idiosyncratic return risk and it may not be possible to eliminate all of this risk in an incentive-compatible way.

7. Conclusion

This paper has developed a model of banking with nominal contracts and money. We introduce a wide range of different types of uncertainty, including aggregate return uncertainty, aggregate liquidity shocks, and idiosyncratic (bank-specific) liquidity shocks. With deposit contracts specified in real terms, as most of the literature assumes, these risks would lead to banking crises. We have shown, however, that with nominal contracts and a central bank, it is possible to eliminate financial instability. More importantly, it is possible to achieve the first-best allocation. This does not require heavy intervention by the central bank or the government. All that is required is that the central bank accommodates the commercial banks' liquidity needs. Moreover, because the central bank can set the nominal interest rate, it can also control the expected rate of inflation. The one type of risk that cannot easily be dealt with is idiosyncratic return shocks. This requires that the government or a private institution make transfers between banks with high and low returns to achieve the first-best. Implementing this type of scheme is problematic as it may be subject to moral hazard and other incentive problems.

Since flexible prices are crucial to our argument, the possibility of sticky prices poses a challenge. We have suggested that there is a trade-off between contingent contracts and price flexibility, insofar as we can implement the efficient outcome as an equilibrium with stable prices if contracts are allowed to be contingent on the state (λ, R). The stability referred to is limited, however. Prices are stable between date 1 and date 2, but not between date 0 and date 1. More importantly, there is a difference between an economy in which prices are flexible but equilibrium prices are stable and an economy in which prices are sticky. In a sticky price economy, the prices may be "stuck" at the wrong level. This topic goes far beyond the scope of the present study. Extending our results to a neo-Keynesian framework poses many challenges.

Another feature of our model is the special technology, which is typical in the banking literature. Investment decisions made at date 0 cannot be revised at date 1 when the liquidity and asset return shocks are realized. The only investment decision made at date 1 is whether or not to store the good. Fortunately, this simplifying assumption is easily relaxed and the qualitative features of the model are unchanged if we allow for varying degrees of ex post substitutability. Instead of assuming that investment in short and long assets is fixed at date 0, we can assume that a decision variable $0 \leqslant z \leqslant 1$ chosen at date 0 determines a convex production possibility set $Y(z) \subset \mathbf{R}_+^2$ at date 1.[6] Once the state (λ, R) is realized, firms choose a vector $(y_1, y_2) \in Y(z)$ that determines an output y_1 at date 1 and $y_2 R$ at date 2. The technology assumed in this paper corresponds to the special case $Y(z) = \{(y_1, y_2)\colon\ y_1 \leqslant z,\ y_2 \leqslant 1 - z\}$. With a flexible technology, the planner's problem still results in an optimal consumption profile $(c_1(\lambda, R), c_2(\lambda, R))$ that can be decentralized in the usual way. While the flexibility allowed by ex post investment decisions may offset some fluctuations in prices, it cannot eliminate them as long as non-contingent deposit contracts are used.

Appendix A

To characterize the efficient allocation, we assume that a planner invests in a portfolio of the short and long assets and distributes the proceeds directly to the early and late consumers. The portfolio of investments, expressed in per capita terms, consists of y units of the short asset and $1 - y$ units of the long asset. The allocation of consumption will depend on the random vector (λ, R), so we write the consumption profile of the typical consumer as $(c_1(\lambda, R), c_2(\lambda, R))$. Then the planner's problem is to

$$\begin{aligned} \max_{(c_1, c_2, y)} \quad & E\big[\lambda u\big(c_1(\lambda, R)\big) + (1-\lambda) u\big(c_2(\lambda, R)\big)\big] \\ \text{s.t.} \quad & \lambda c_1(\lambda, R) \leqslant y \quad \text{and} \quad \lambda c_1(\lambda, R) + (1-\lambda) c_2(\lambda, R) \leqslant y + (1-y)R. \end{aligned} \tag{19}$$

Note that the problem contains an infinite number of constraints, one pair for each value of (λ, R). The first constraint says that the total consumption given to the early consumers must not exceed the supply of the short asset at date 1. The second constraint says that total consumption summed over the two dates cannot exceed the total returns of the two assets. The constraints are expressed this way to take account of the possibility of storage between date 1 and date 2.

For given values of y and (λ, R), the consumption profile $(c_1(\lambda, R), c_2(\lambda, R))$ must maximize $\lambda u(c_1(\lambda, R)) + (1-\lambda) u(c_2(\lambda, R))$ subject to the two feasibility constraints. The first-order conditions for this problem, which are necessary and sufficient, can be written as

$$u'\big(c_1(\lambda, R)\big) - u'\big(c_2(\lambda, R)\big) \geqslant 0$$

with the complementary slackness condition

$$\big[u'\big(c_1(\lambda, R)\big) - u'\big(c_2(\lambda, R)\big)\big]\big[y - \lambda c_1(\lambda, R)\big] = 0.$$

Note that $u'(c_1(\lambda, R)) = u'(c_2(\lambda, R))$ implies that $c_1(\lambda, R) \leqslant c_2(\lambda, R)$ so the incentive constraint is automatically satisfied. In other words, the first-best (Pareto-efficient) allocation is the same as the second-best (incentive-efficient) allocation.

Since $u'(c_1(\lambda, R)) = u'(c_2(\lambda, R))$ implies that

$$c_1(\lambda, R) = c_2(\lambda, R) = y + (1-y)R,$$

the optimal consumption functions can be written as

$$c_1^*(\lambda, R) = \min\left\{\frac{y}{\lambda}, y + (1-y)R\right\} \quad \text{and} \quad c_2^*(\lambda, R) = \max\left\{\frac{(1-y)R}{1-\lambda}, y + (1-y)R\right\}.$$

[6] $Y(z)$ represents the production possibilities associated with a unit investment at date 0. Constant returns to scale are assumed, so an investment of I units results in a production possibility set $I \cdot Y(z)$, and this implies zero profits as usual.

Substituting these values for $c_1^*(\lambda, R)$ and $c_2^*(\lambda, R)$ into the planner's problem, we obtain the optimal portfolio choice problem:

$$\max_{0\leqslant y\leqslant 1} \quad \mathbf{E}\left[\lambda u\left(\min\left\{\frac{y}{\lambda}, y+(1-y)R\right\}\right)+(1-\lambda)u\left(\max\left\{\frac{(1-y)R}{1-\lambda}, y+(1-y)R\right\}\right)\right].$$

The objective function is continuous in y and hence attains a maximum. Since the function u is strictly concave, the maximizer y^* is unique.

Appendix B

The total revenue of a representative firm is

$$\sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R})x_t^* R_t = \sum_{t=1}^{T} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})}.$$

Now consider the expression for the firm's revenue during the interval $\{t_i + 1, \dots, t_{i+1}\}$. Since there is no storage in the last period of each interval, (18) implies that

$$\begin{aligned}\sum_{t=t_i+1}^{t_{i+1}} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})} &= \sum_{t=t_i+1}^{t_{i+1}} \frac{x_t^* R_t}{(\frac{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s})} \\ &= \left(\frac{\sum_{s=t_i+1}^{t_{i+1}} \lambda_s}{\sum_{s=t_i+1}^{t_{i+1}} x_s^* R_s}\right)\sum_{t=t_i+1}^{t_{i+1}} x_t^* R_t \\ &= \sum_{s=t_i+1}^{t_{i+1}} \lambda_s\end{aligned}$$

and hence

$$\sum_{t=1}^{T} \frac{x_t^* R_t}{c_t^*(\boldsymbol{\lambda}, \mathbf{R})} = \sum_{i=0}^{n} \sum_{s=t_i+1}^{t_{i+1}} \lambda_s = 1.$$

This proves that each firm earns zero profits and can repay its debt to the bank.

Can the firm make a positive profit by deviating from the equilibrium portfolio $\mathbf{x}^*$? Under the assumptions we have made, we can show that it is not possible for the firm to deviate at all without violating its budget constraint in some non-negligible set of states. The proof is by contradiction. Suppose that $\mathbf{x} \neq \mathbf{x}^*$ is a feasible portfolio, i.e., $\mathbf{x} \geqslant \mathbf{0}$ and $\sum x_t = 1$, and satisfies the budget constraint

$$\sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R})x_t R_t \geqslant 1$$

for every $(\boldsymbol{\lambda}, \mathbf{R})$. Note that we do not have to assume that $\mathbf{x}$ is profitable in any state, just that it at least breaks even. Now fix some $\mathbf{R}$ and suppose that $\boldsymbol{\lambda}^0 = (0, \dots, 0, 1, 0, \dots, 0)$, where the 1 is in the τ-th place. Recall that by assumption $\boldsymbol{\lambda}^0$ belongs to the support of $\boldsymbol{\lambda}$ and hence is the limit of a sequence of random vectors $\boldsymbol{\lambda}^q \to \boldsymbol{\lambda}^0$ that have a small but positive measure of consumers at each date $s = 1, \dots, \tau$. Then it is clear that, in the limit, assuming the consumption functions are continuous in $\boldsymbol{\lambda}$ and $\mathbf{R}$, consumption will have to be equalized at the dates $1, \ldots, \tau$ and will become unboundedly large at the dates $\tau + 1, \ldots, T$. Thus,

$$c_s^*(\boldsymbol{\lambda}, \mathbf{R}) = \begin{cases} c_\tau^*(\boldsymbol{\lambda}, \mathbf{R}) & \text{for } s = 1, \ldots, \tau, \\ \infty & \text{for } s = \tau + 1, \ldots, T \end{cases}$$

and, therefore,

$$\begin{aligned} \sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t R_t &= \sum_{s=1}^{\tau} \frac{x_s R_s}{c_\tau^*(\boldsymbol{\lambda}, \mathbf{R})} \\ &\geqslant \sum_{t=1}^{T} P_t^*(\boldsymbol{\lambda}, \mathbf{R}) x_t^* R_t \\ &= \sum_{s=1}^{\tau} \frac{x_s^* R_s}{c_\tau^*(\boldsymbol{\lambda}, \mathbf{R})}. \end{aligned}$$

Thus, for any $\tau = 1, \ldots, T$ and any $\mathbf{R}$,

$$\sum_{s=1}^{\tau} x_s R_s \geqslant \sum_{s=1}^{\tau} x_s^* R_s.$$

This means that $\mathbf{x}$ provides at least as much output as $\mathbf{x}^*$ at every date and, since $\mathbf{x} \neq \mathbf{x}^*$, there must be some date at which the inequality is strict. In other words, $\mathbf{x}$ produces strictly more than $\mathbf{x}^*$ at some dates. But this contradicts the optimality of $\mathbf{x}^*$. Thus, the only optimal choice of portfolio is $\mathbf{x} = \mathbf{x}^*$.

References

[1] F. Allen, E. Carletti, D. Gale, Interbank market liquidity and central bank intervention, J. Monet. Econ. 56 (2009) 639–652.
[2] F. Allen, D. Gale, Optimal financial crises, J. Finance 53 (1998) 1245–1284.
[3] F. Allen, D. Gale, Financial contagion, J. Polit. Economy 108 (2000) 1–33.
[4] G. Antinolfi, E. Huybens, T. Keister, Monetary stability and liquidity crises: the role of the lender of last resort, J. Econ. Theory 99 (2001) 187–219.
[5] S. Bhattacharya, D. Gale, Preference shocks, liquidity and central bank policy, in: W. Barnett, K. Singleton (Eds.), New Approaches to Monetary Economics, Cambridge University Press, Cambridge, 1987, pp. 69–88.
[6] A. Blinder, On sticky prices: academic theories meet the real world, in: N.G. Mankiw (Ed.), Monetary Policy, in: NBER Studies in Business Cycles, vol. 29, University of Chicago Press, Chicago, 1994, pp. 117–150.
[7] H. Bohn, Why do we have nominal government debt, J. Monet. Econ. 21 (1988) 127–140.
[8] W. Buiter, Maverecon, Financial times, December 13, 2007, as quoted in Cao and Illing, 2011.
[9] C. Calomiris, C. Kahn, The role of demandable debt in structuring optimal banking arrangements, Amer. Econ. Rev. 81 (1991) 497–513.
[10] J. Cao, G. Illing, Endogenous exposure to systemic liquidity risk, Int. J. Cent. Bank. (2011) 173–216.
[11] B. Champ, B. Smith, S. Williamson, Currency elasticity and banking panics: theory and evidence, Can. J. Econ. 29 (1996) 828–864.
[12] V. Chari, R. Jagannathan, Banking panics, information, and rational expectations equilibrium, J. Finance 43 (1988) 749–760.
[13] K. Cone, Regulation of depository institutions, Ph.D. dissertation, Stanford University, 1983.
[14] R. Cooper, D. Corbae, Financial collapse: a lesson from the great depression, J. Econ. Theory 107 (2002) 159–190.
[15] D. Diamond, P. Dybvig, Bank runs, deposit insurance, and liquidity, J. Polit. Economy 91 (1983) 401–419.
[16] D. Diamond, R. Rajan, Liquidity risk, liquidity creation and financial fragility: a theory of banking, J. Polit. Economy 109 (2001) 2431–2465.

[17] D. Diamond, R. Rajan, Liquidity shortages and banking crises, J. Finance 60 (2005) 615–647.
[18] D. Diamond, R. Rajan, Money in a theory of banking, Amer. Econ. Rev. 96 (2006) 30–53.
[19] X. Freixas, A. Martin, D. Skeie, Bank liquidity, interbank markets and monetary policy, Rev. Finan. Stud. 24 (2011) 2657–2692.
[20] C. Jacklin, Demand deposits, trading restrictions, and risk sharing, in: E. Prescott, N. Wallace (Eds.), Contractual Arrangements for Intertemporal Trade, University of Minnesota Press, Minneapolis, MN, 1987, pp. 26–47.
[21] C. Jacklin, S. Bhattacharya, Distinguishing panics and information-based bank runs: welfare and policy implications, J. Polit. Economy 96 (1988) 568–592.
[22] D. Krueger, H. Lustig, When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)?, J. Econ. Theory 145 (2010) 1–41.
[23] D. Skeie, Banking with nominal deposits and inside money, J. Finan. Intermediation 17 (2008) 562–584.
[24] B. Smith, Monetary policy, banking crises, and the Friedman rule, Amer. Econ. Rev. 92 (2002) 128–134.