Review of Economic Studies (1983) L, 639-646
0034-6527/83/00430639$00.50

Credit Rationing and Payment Incentives

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A model of borrowing for production is presented where default leads to exclusion from the capital market. This means contracts are enforceable, provided the current payment is less than or equal to the value of future access to the capital market. The main result of the paper is to show that if this constraint binds then credit is rationed.

1. INTRODUCTION

One of the features of credit markets that differentiates them from other markets, is that people are sometimes rationed in the amount that they can borrow: at the going rate of interest they would like to borrow more, but lenders prevent them from doing this. This contrasts with most other markets, where such a situation simply leads to a rise in price, which equates demand and supply.

The feature of credit markets which leads to this difference, would appear to be the two stage nature of the transactions. In most markets the payment for and transfer of the good occur at the same time. This enables sellers to ensure buyers actually make the payment: if they do not, sellers can simply refuse to deliver the goods. However, in credit markets the borrower receives the loan at one point in time and then does not have to make payments until some time later. This raises two problems which the literature on credit rationing has focused on. The first is that when payments are due, the project may have been unsuccessful, so that the borrower does not have the financial means to make the payment. In this case the borrower defaults and the lender becomes the owner of the investment. Keeton (1979) and Stiglitz and Weiss (1981) have suggested models based on this problem, where borrowers and lenders have asymmetric information on the distribution of returns to investments, which results in adverse selection and incentive effects in choosing projects. These can lead to credit rationing because a higher interest rate leads to more defaults, which may eventually reduce the lender's expected return.

The second problem is that even if a borrower has the means, he may have no incentive to actually make the payment to the lender. This forms the basis of a theory developed by Jaffee and Russell (1976). In their model people have an exogenously specified, unobservable, fixed cost of default which differs across individuals. Borrowers default if these costs are less than the loan and interest payments combined. It is clearly possible to stop people defaulting by rationing credit until the loan and interest payments are less than the fixed costs of default. If the distribution of the costs of default is such that when all demands for loans are met, some borrowers default, then in order for lenders to recover their opportunity costs, honest borrowers who repay their loans must subsidize the defaulters. Credit rationing may make honest borrowers better off because fewer people default, so that the subsidy they have to pay for lenders to recoup their opportunity costs is reduced.

The problem with this model is the exogeneity of the costs of default. This assumption would be justified if there were criminal penalties, such as debtors' prison, for defaulters. However, in most modern capital markets this assumption is not justified. Instead it would appear that in many cases, the main penalties for default are imposed by the market itself: borrowers who do not make payments on their loans, are excluded from the market. The assumption of exogenous fixed costs of default is thus not appropriate.

Eaton and Gersovitz (1981) have suggested a model with endogenous default costs in the context of international credit markets, where countries borrow to smooth consumption. They assume that default leads to an embargo on future borrowing, so that the cost depends on the penalty of not being able to smooth consumption.

The purpose of this paper is to provide a theory, in the context of individuals borrowing for production, where default costs are also endogenous. In the model used interest on old debts has to be paid before new debt can be serviced. This implies borrowers contract with only one lender: they cannot default and go to another lender because the latter knows the previous interest must be paid before he receives anything. Lenders specify that they will deny further credit to borrowers if they default. Provided this threat is credible the effective penalty for default is then exclusion from the loan market. Given this, a contract is enforceable provided each payment is less than the value of future access to the market. It can be shown that, if this enforcement constraint binds, then credit is rationed. This is because the benefit of default, namely the payment, is linear in the amount borrowed, but since the returns to the investment of the loan are concave, the value of future access to the capital market is less than linearly increasing in the amount borrowed. By reducing credit it is, therefore, possible to increase the value of future access relative to the payment and prevent default.

The model is described in Section 2. In Section 3 the equilibrium where there are no possibilities for saving and using collateral, is considered; in Section 4 this assumption is relaxed. Finally, Section 5 contains a summary and some concluding remarks.

2. THE MODEL

There is an initial, exogenous distribution of wealth, which determines whether a person is a lender or a borrower. For simplicity, all borrowers are taken to have no initial wealth of their own. Lenders have an opportunity cost for their funds of r.

Loans are used to buy an infinitely durable capital good denoted by K. This cannot be removed or damaged; it can be thought of as something like land. If the borrower defaults on any of his payments, the lender gains possession of the capital. The principal of the loan is therefore secure, and it is only possible to default on the interest.

Borrowers are taken to differ in their ability A which can be observed at zero cost. There is a continuum of different ability groups with lower bound 0. Each person's output Y depends on his ability and the amount of capital he uses.

$$Y = Ay(K) \tag{1}$$

The function y has the usual properties

$$y' > 0;\ y'' < 0. \tag{2}$$

Time is divided into discrete production periods with inputs being supplied at the beginning and outputs produced at the end.

At time 0 lenders and borrowers agree on a long term contract. Lenders are forced to keep their side of the contract by pledging their loan as security. Competition among lenders ensures that they only earn their opportunity cost r per period on their funds.

At the end of each period the output is produced by the borrower and is initially observed only by him. He then decides whether to fulfil the contract with the lender by making the necessary payment. If he does honour the contract, then he continues next period. If he defaults, then he removes the output and goes away. It is taken to be prohibitively costly for the lender to physically prevent him from doing this.

There are no criminal or other legal penalities for the default. However the creditor of any debtor who has defaulted has a prior right to any interest that is paid. Thus interest payments on old debts must be completed before any more recent debt can be serviced.

People are risk neutral, infinitely long lived and their utility depends only on their discounted consumption: there is no disutility from controlling wealth.

$$U=\sum_{i=1}^{\infty}\rho^i X_i \tag{3}$$

where X_i is the consumption in the ith period and $\rho(<1)$ is the discount factor.

3. EQUILIBRIUM WITHOUT SAVING

In this section there are assumed to be no possibilities for saving or the use of collateral to guarantee payments.

The requirement that the interest on old debts must be serviced before any new interest is paid means that each borrower deals with only one lender. It is not possible for a borrower to default and then try to borrow from another lender, since the latter knows the previous lender's interest must be repaid before he receives anything.

At time 0 lender and borrower draw up a contract which specifies, among other things, that if the borrower defaults the lender will not provide him with any more loans. It is important that this threat to withdraw credit is credible. If it is not then the contract will not be time consistent. To see this suppose the borrower defaults, goes to his original lender and offers a contract whose rate of return exceeds r, protected by the same threat of no additional credit as the original loan. Given that the lender accepted the original contract he should also accept the new one. Knowing this the borrower will always default.

The essential reason this problem arises is that bygones are bygones and once the borrower has defaulted the situation is the same as the original one. In order to make the threat credible it is necessary for the actions of the lender after the default, to be affected by the default. The standard way in which this can be achieved is for the lender to sign a contract with a third party which requires him to make a large payment should he ever recontract with a defaulter (Schelling (1963)). The threat of withdrawing credit in the original contract is then credible. This solution may not be necessary if there are many cohorts of debtors and the actions of the lender with one group affects the behaviour of other groups. In what follows one of these solutions is taken to be applicable and the threat to withdraw credit, credible.

Anybody who defaults will therefore be excluded from the credit market. They will then not default if the current payment is less than the value of the possibility of borrowing in the future. If contracts to people with ability A involve a payment $\phi_i(A)$ for $K_i(A)$ of capital in period i contracts are enforceable provided

$$\phi_i(A)\leqq\sum_{j=i+1}^{\infty}\rho^{j-i}(Ay(K_j(A))-\phi_j(A)), \qquad i=1,\ldots,\infty \tag{4}$$

(It is assumed that when people are indifferent between defaulting and not defaulting they do not.)

In order for a person to be able to make a payment, it must also be the case that it is less than his output for that period.

$$\phi_i(A)\leqq Ay(K_i(A)), \qquad i=1,\ldots,\infty. \tag{5}$$

Contracts must be such that their expected return is just sufficient to cover the lender's opportunity cost of capital. It follows that

$$\phi_i(A) = rK_i(A), \qquad i = 1, \ldots, \infty. \tag{6}$$

Loans must be nonnegative, since borrowers have no wealth

$$K_i(A) \geqq 0, \qquad i = 1, \ldots, \infty. \tag{7}$$

Feasible Pareto optimal contracts are therefore ones where $\phi_i(A)$ and $K_i(A)$ are such that the utility of the tenant is maximised and the constraints (4)–(7) are satisfied. For each level of ability, ϕ_i and K_i can be found by solving

$$\max_{\phi_i, K_i} \sum_{i=1}^{\infty} \rho^i (Ay(K_i) - \phi_i), \qquad i = 1, \ldots, \infty \tag{8}$$

subject to (4)–(7).

Using (6) to eliminate ϕ_i, (8) can be rewritten in the following dynamic programming formulation.

$$V_i = \max_{K_i} \rho(Ay(K_i) - rK_i + V_{i+1}), \qquad i = 1, \ldots, \infty \tag{9}$$

subject to

$$rK_i \leqq V_{i+1} \tag{10}$$

$$rK_i \leqq Ay(K_i) \tag{11}$$

$$K_i \geqq 0. \tag{12}$$

In can be seen that the constraint (11) bounds the problem, so that the solution must be stationary. The problem can therefore be rewritten dropping the i subscripts with $V_i = V$ and $K_i = K (i = 1, \ldots, \infty)$. If λ, μ and ν are the Lagrange multipliers for the constraints (10)–(12) respectively, it follows that the first order conditions for the determination of K are

$$\rho(Ay'(K) - r) - \lambda r + \mu(Ay'(K) - r) + \nu = 0 \tag{13}$$

$$\lambda(V - rK) = 0 \tag{14}$$

$$\mu(Ay(K) - rK) = 0 \tag{15}$$

$$\nu K = 0. \tag{16}$$

The constraint (11) cannot bind at the optimum, since it implies $V = 0$. Thus $\mu = 0$ and

$$K = \max\left\{\min\left[g\left(\frac{r}{A}\right), h\left(\frac{r}{\rho A}\right)\right], 0\right\} \tag{17}$$

where g is the inverse of the marginal product function $K \to y'(K)$ and h is the inverse of the average product function $K \to y(K)/K$.

There are thus three types of solution to the problem, depending on productivity A. The first, where K is such that the marginal product is equated to r, is the standard case which occurs when the no-default constraint (10) fails to bind. This is illustrated in Figure 1 where marginal product is plotted against K so that the area under the curve represents output.

In the second type of solution, K is such that r is equated to a proportion ρ of average product. It arises when the no-default constraint (10) binds. It occurs because at the level of K where marginal product is equated to r, the total payment rK is above the value V of future access to the capital market, so that borrowers would default. In order to prevent this, K is rationed: this reduces the payment rK linearly but reduces $V = \rho Ay(K)$ less than linearly, because of the concavity of y. In this way the ratio of

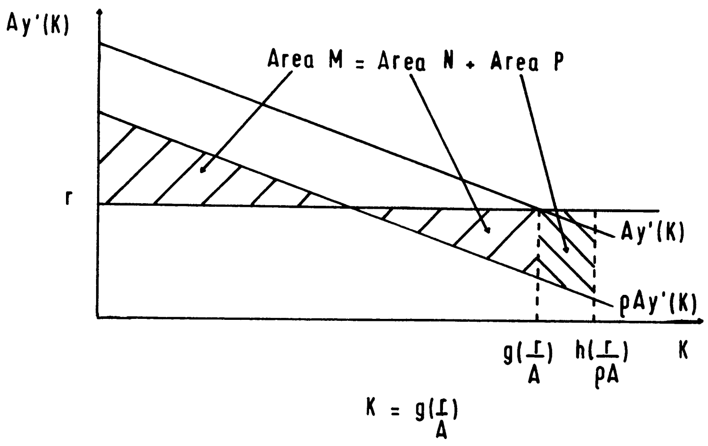

FIGURE 1

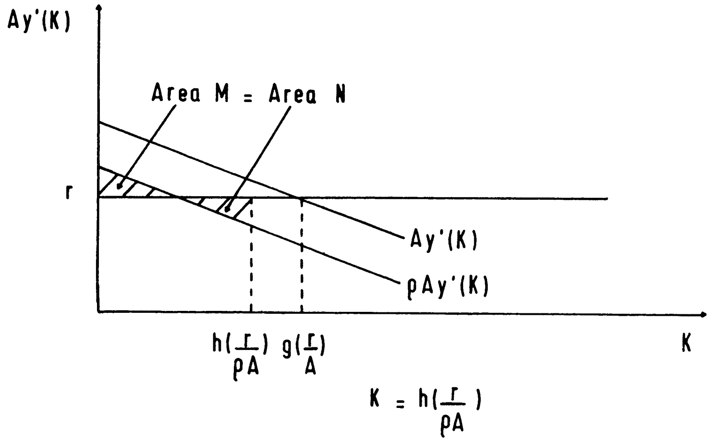

FIGURE 2

the benefits to the costs can be increased until default is no longer attractive. A situation of this type is illustrated in Figure 2.

The final type of solution, where $K=0$, occurs when A is so low that it is not worthwhile to make any payments: people are better off to default and be excluded from the market in the future. This is illustrated in Figure 3.

It can be shown that all three types of solution are possible with different values of A for almost all forms of y satisfying (2). It follows from the no-default constraint (10) that the first type of solution will occur when A is such that

$$f(A)=rg\left(\frac{r}{A}\right)\left(\frac{1}{1-\rho}\right)-Ay\left[g\left(\frac{r}{A}\right)\right]\left(\frac{\rho}{1-\rho}\right)<0 \tag{18}$$

Using the fact that g is the inverse of the marginal product function, it can be shown that

$$f'(A)=\frac{-r^2}{A^2y''[g(r/A)]}-y\left[g\left(\frac{r}{A}\right)\right]\left(\frac{\rho}{1-\rho}\right). \tag{19}$$

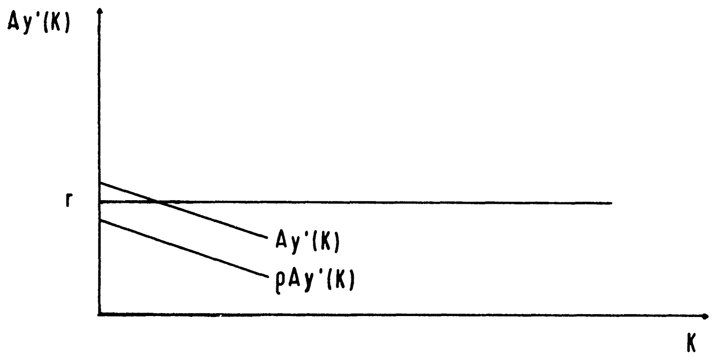

FIGURE 3

Except in perverse cases where $A^2y''(g(r/A)) \to 0$ as $A \to \infty$ or $y(g(r/A))$ is bounded above, $f'(A)$ will be negative for sufficiently high A, since $dg(r/A)/dA > 0$ and hence (18) will also be satisfied for sufficiently high A.

As A falls, it can be seen that there eventually exist values of A such that

$$f(A) \geqq 0 \tag{20}$$

so that constraint (10) binds and borrowers are rationed. In general, there may be a number of regions where (20) is satisfied, interspersed with regions where (18) is satisfied. It is not the case, therefore, that rationed borrowers necessarily have a lower productivity A than unrationed borrowers.

Finally for A such that

$$\rho A y'(0) \leqq r \tag{21}$$

it will not be worthwhile for borrowers to make payments. It can be seen that, provided $y'(0) \neq \infty$, values of A will always exist such that (21) is satisfied.

The main results of this section are summarised by the following proposition.

Propositon 1. *Each borrower deals with one lender throughout since interest on old debts has priority over that on new debts. Contracts contain a crediblè clause specifying that if the borrower defaults the lender will not supply any more loans. The penalty for default is therefore exclusion from the capital market. This means contracts are enforceable provided the current payment is less than or equal to the value of future access to the market. If this constraint binds credit is rationed. If it binds tightly enough then the credit rationing may be so strict that no borrowing is possible.*

4. EQUILIBRIUM WITH SAVING

In the previous section, it was assumed that people could not save and acquire collateral to guarantee their payments. In this section the effect of allowing for this possibility is considered.

Instead of consuming all their earnings, borrowers can now save S_i in period i and acquire assets which can be used as collateral. These assets are assumed to earn a rate of interest of $(1/\rho - 1)$. People put up collateral of C_i at the beginning of period i, to guarantee their payments. At the end of each period they make a payment, ϕ_i, given by (6) as before, and their collateral is returned to them with interest. If they do not make a payment, they forfeit their collateral and its interest.

The dynamic programming problem that must be solved in this case to find the Pareto optimal contract for each level of ability, is then

$$V_i = \max_{K_i, S_i} \rho\left(Ay(K_i) - rK_i + \left(\frac{1}{\rho} - 1\right)C_i - S_i + V_{i+1}\right), \qquad i = 1, \dots, \infty \tag{22}$$

subject to

$$rK_i - \frac{1}{\rho} C_i \leqq V_{i+1} \tag{23}$$

$$rK_i \leqq Ay(K_i) + \left(\frac{1}{\rho} - 1\right)C_i - S_i \tag{24}$$

$$C_i = C_{i-1} + S_i \tag{25}$$

$$\frac{1}{\rho} C_i \leqq \max\left[0, rg\left(\frac{r}{A}\right) - V_{i+1}\right] \tag{26}$$

$$C_1 = 0 \tag{27}$$

$$S_i, K_i \geqq 0. \tag{28}$$

Apart from the obvious changes to allow for the inclusion of savings and collateral, the constraint (26) is added to ensure a determinate solution. This is needed because there are two types of saving in the model. The first is the saving to provide collateral for productive purposes, which is the one of interest here. The second is saving to transfer consumption from one period to another. With risk neutrality and a rate of interest equal to the rate at which utility is discounted, the latter type of saving leads to indeterminacy: somebody who forgoes S_i now, receives consumption in the future whose discounted value is also S_i. The constraint (26) is included to rule out this problem; it allows saving up to the point where collateral ceases to be useful as far as production is concerned, but not farther.

As in Section 3, it follows from the first order conditions for the problem (22)–(28) that

$$K_i = \max\left\{\min\left[g\left(\frac{r}{A}\right), \frac{V_{i+1} + C_i/\rho}{r}\right], 0\right\}, i = 1, \dots, \infty. \tag{29}$$

For periods where $g(r/A)$ is greater than $(V_{i+1} + C_i/\rho)/r$, people save all their income because the discounted value of the interest is the same as the utility of immediate consumption, but saving increases C_{i+1} and hence K_{i+1}, so that they can earn more the next period. Eventually $g(r/A)$ equals $(V_{i+1} + C_i/\rho)/r$, and saving for collateral is no longer necessary. Thus the essential difference between the solution to (22)–(28) and that of (9)–(12) is that whereas before, if $g(r/A)$ was greater than $h(r/\rho A)$, borrowers were permanently rationed, now they are only temporarily rationed during the initial periods.

The main result of this section is summarised by the following proposition.

Proposition 2. *If savings and the use of collateral are feasible, credit is again rationed when the no-default constraint binds, but this is only temporary until borrowers acquire enough assets to guarantee payment on the amount of capital required to equate marginal product to the opportunity cost of capital.*

5. SUMMARY AND CONCLUDING REMARKS

This paper has presented a model which focuses on the incentives people have not to default where the costs of default are endogenous. If interest on old debts must be serviced before that on new ones, borrowers will deal with only one lender throughout. Lenders specify that if the borrower defaults then he can obtain further credit. Provided this threat is credible contracts are enforceable when the current payment is less than the future value of access to the market: if this constraint binds, then credit is rationed.

Many of the assumptions of the model, such as permanent capital, an infinite time horizon, a constant opportunity cost of capital and so on, were made to simplify the derivation. However, at least three assumptions were crucial for the result. First, it was necessary that defaulters were excluded from the capital market. The second was that rationing increases the benefits relative to the costs of default. Finally, people must not be able to abscond with the entire principal and interest of the loan, since in this case, exclusion from the capital market is no penalty: defaulters already have the capital they need. Provided these requirements are satisfied, then credit may be rationed if the no-default constraint binds.

First version received May 1981; *final version accepted November* 1982 (*Eds.*).

This is a development of part of Chapter 2 of my D.Phil. thesis, Allen (1980). I am very grateful to J. A. Mirrlees, who supervised this, and to A. K. Dixit, M. Harris, K. W. S. Roberts and J. E. Stiglitz for many useful comments. I am also indebted to O.D. Hart and two anonymous referees, one of whom has been particularly helpful, for their remarks on later revisions of the work.

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