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OPTIMAL LINEAR INCOME TAXATION WITH GENERAL EQUILIBRIUM EFFECTS ON WAGES

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The paper argues that the endogeneity of wages may make a considerable difference to optimal linear tax rates. In this case there is not only the usual direct redistribution via the fiscal system but also the general equilibrium redistribution due to the change in wages. Depending on the elasticity of substitution of the production function and the labour supply elasticities these may be reinforcing or opposing. It is shown that the general equilibrium effect may plausibly outweigh the direct effect so that net redistribution to the lower paid involves a wage subsidy financed by a lump-sum tax.

1. Introduction

Income taxation has been the focus of a large and growing literature in recent years. Mirrlees (1971) formulated a model for the investigation of the distortion on the choice between labour and leisure which has been the basis of much of this discussion [see, for example, Sheshinski (1972), Atkinson (1973), Stern (1976) and Helpman and Sadka (1978)]. One of the main assumptions of this approach is that there is only one type of labour: differences in wages are due to differences in the amount of this that can be supplied in a given period of time which depends on the ability of an individual. This ensures that there is only redistribution through the fiscal system.

Feldstein (1973) considered a model in which production requires two types of labour and wages are determined endogenously. There is then also redistribution resulting from the change in wages caused by a tax. Solving the model numerically he found that the optimal linear tax rates were little changed from those obtained with wages held constant.

Using an analytic approach, it is argued below that these results are due to the use of a Cobb–Douglas production function since this ensures that the

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redistribution via the fiscal system and due to the change in wages are reinforcing. In general this is not the case: in plausible situations they can be offsetting and it is possible for the general equilibrium effect to dominate so that the worst-off people are helped by a linear tax consisting of a wage subsidy and a uniform lump-sum tax, which redistributes from poor to rich.

In section 2 the model is described. In section 3 the effect of local changes in the linear tax at the market solution is considered and in section 4 the sign of the tax required to maximise Rawlsian social welfare is investigated. Finally, section 6 contains a summary and a few concluding remarks.

2. The model

Attention is confined to the simplest case where two types of labour are required to produce a single homogeneous output and the tax function is linear.

The two types of labour can be regarded as, for example, skilled and unskilled or entrepreneurial and raw labour. They are denoted by the subscripts 1 and 2. There are two ability groups, A_1 and A_2, whose relative size is one to n. The only difference between them is that the A_1's are better at supplying the type 1 labour. Provided they are better off doing so, which is taken to be the case, they supply all of this. The A_2's supply the type 2 labour.

The earnings of the A_1's are taken to be the higher of the two:

$$X = \frac{w_1 L_1}{w_2 L_2} > 1, \tag{1}$$

where w_i and L_i are the wage and labour supply of the members of group A_i.

There are constant returns to scale and the production possibilities are represented by a cost function $g(w_1, nw_2)$. Equilibrium in the output market requires

$$g(w_1, nw_2) = 1. \tag{2}$$

Individuals choose their consumption and labour supply to maximise their utility subject to their budget constraint. This gives rise to indirect utility and labour supply functions:

$$V_i = V^i(w, I), \qquad i = 1,2 \tag{3}$$

$$L_i = L^i(w, I), \qquad i = 1,2, \tag{4}$$

where w is the net wage and I is lump sum income.

It follows from the theory of indirect utility functions that (3) and (4) are related by

$$L_i = \frac{V_w^i}{V_I^i}, \qquad i = 1,2, \tag{5}$$

where $V_a^i = \partial V^i/\partial a$.

If the proportionate component of the linear tax is denoted $(1-\beta)$ and the lump-sum grant α, then from the theory of cost functions equilibrium in each labour market requires

$$L^i(\beta w_i,\alpha) = Yg_i(w_1,nw_2), \qquad i = 1,2, \tag{6}$$

where $g_1 = \partial g/\partial w_1$, $g_2 = \partial g/\partial(nw_2)$ and Y is total output.

Finally, overall equilibrium requires that the government's budget constraint is satisfied so that

$$\alpha = (1-\beta)k, \tag{7}$$

where k is per capita income

$$k = \frac{w_1L_1 + nw_2L_2}{n+1}. \tag{8}$$

3. Local changes in the tax at the market solution

The equilibrium conditions (2), (6) and (7) can be used to investigate the effect of a change in the proportionate tax rate $(1-\beta)$ while altering the lump-sum grant α to keep the government's budget constraint satisfied.

The change in utilities is given by

$$\frac{\mathrm{d}V_i}{\mathrm{d}\beta} = w_iV_w^i + \beta V_w^i\frac{\mathrm{d}w_i}{\mathrm{d}\beta} + V_I^i\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}, \qquad i = 1,2. \tag{9}$$

Using (5) this simplifies to

$$\frac{\mathrm{d}V_i}{\mathrm{d}\beta} = V_I^i\left[w_iL_i + \frac{\mathrm{d}\alpha}{\mathrm{d}\beta} + \beta w_iL_i\frac{\mathrm{d}\log w_i}{\mathrm{d}\beta}\right], \qquad i = 1,2. \tag{10}$$

The term in brackets can be split into two components. The first, $w_iL_i + \mathrm{d}\alpha/\mathrm{d}\beta$, is the direct redistribution via the fiscal system resulting from a reduction in the marginal tax rate. The second, $\beta w_iL_i\,\mathrm{d}\log w_i/\mathrm{d}\beta$, is the change in net wages which is the general equilibrium effect.

The term $\mathrm{d}\log w_i/\mathrm{d}\beta$ can be found using (6). Dividing the equation with $i = 1$ by that with $i = 2$ and differentiating logarithmically with respect to β, gives

$$\left[\frac{\varepsilon_1}{\beta}+\varepsilon_1\frac{\mathrm{d}\log w_1}{\mathrm{d}\beta}+\zeta_1\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right]-\left[\frac{\varepsilon_2}{\beta}+\varepsilon_2\frac{\mathrm{d}\log w_2}{\mathrm{d}\beta}+\zeta_2\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right]=\frac{\mathrm{d}\log(g_1/g_2)}{\mathrm{d}\beta}, \tag{11}$$

where $\varepsilon_i=\beta w_i L^i_w/L_i$ and $\zeta_i=L^i_I/L^i$.

Using the definition of the elasticity of substitution of the production function σ and (6) to eliminate $\mathrm{d}\log(g_1/g_2)/\mathrm{d}\beta$ and rearranging, (11) becomes

$$(\sigma+\varepsilon_1)\frac{\mathrm{d}\log w_1}{\mathrm{d}\beta}-(\sigma+\varepsilon_2)\frac{\mathrm{d}\log w_2}{\mathrm{d}\beta}=\frac{(\varepsilon_2-\varepsilon_1)}{\beta}+\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}(\zeta_2-\zeta_1). \tag{12}$$

Differentiating (2) logarithmically with respect to β gives

$$\theta_1\frac{\mathrm{d}\log w_1}{\mathrm{d}\beta}+\theta_2\frac{\mathrm{d}\log w_2}{\mathrm{d}\beta}=0, \tag{13}$$

where $\theta_1=w_1L_1/Y$ and $\theta_2=nw_2L_2/Y$.

Combining (12) and (13) yields

$$\frac{\mathrm{d}\log w_i}{\mathrm{d}\beta}=\frac{(-1)^j\theta_j N(\mathrm{d}\alpha/\mathrm{d}\beta)}{D},\qquad i,j=1,2;\quad i\neq j, \tag{14}$$

where

$$N\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}\right)=\frac{\varepsilon_2-\varepsilon_1}{\beta}+\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}(\zeta_2-\zeta_1) \tag{15}$$

and

$$D=\sigma+\varepsilon_1\theta_2+\varepsilon_2\theta_1. \tag{16}$$

Differentiating (7)

$$\frac{\mathrm{d}\alpha}{\mathrm{d}\beta}=-k+(1-\beta)\frac{\mathrm{d}k}{\mathrm{d}\beta}. \tag{17}$$

In this section a change in the tax at the market solution with $\beta=1$ is considered. In this case (17) simplifies to

$$\mathrm{d}\alpha/\mathrm{d}\beta=-k. \tag{18}$$

Using this together with (10) and (14) gives

$$\left(\frac{\mathrm{d}V_i}{\mathrm{d}\beta}\right)_{\beta=1}=\frac{(-1)^j\theta_j w_i L_i V^i_I}{\theta_1}Z,\qquad i,j=1,2;\quad i\neq j, \tag{19}$$

where

$$Z = \frac{X-1}{n+1} + \frac{\theta_1 N(-k)}{D}. \tag{20}$$

Eq. (19) can be used to find the effect of a change in the tax on Rawlsian and utilitarian social welfare, S and S_u, which are given respectively by

$$S = V_2 \tag{21}$$

and

$$S_u = V_1 + nV_2. \tag{22}$$

It can be seen directly from (19) and (21) that the effect of the tax on Rawlsian social welfare depends on the sign of Z.

Differentiating (22) and using (19) it follows that

$$\left(\frac{\mathrm{d}S_u}{\mathrm{d}\beta}\right)_{\beta=1} = nw_2L_2(V_I^1 - V_I^2)Z. \tag{23}$$

It is usual to assume that the marginal utility of income of the more highly paid people is less than that of the lower paid so that

$$V_I^1 < V_I^2. \tag{24}$$

Theorem 1 then follows.

Theorem 1. $(\mathrm{d}S/\mathrm{d}\beta)_{\beta=1}$ *and* $(\mathrm{d}S_u/\mathrm{d}\beta)_{\beta=1}$ *have the same sign which is negative or positive as*

$$Z \gtrless 0. \tag{25}$$

The first term in Z, $(X-1)/(n+1)$, is the direct effect and it follows from (1) that this is always positive: the redistribution through the fiscal system caused by a decrease in the marginal tax rate makes the higher paid better off and decreases the utility of the lower paid, and Rawlsian and utilitarian social welfare.

The second term, $\theta_1 N(-k)/D$, is the general equilibrium effect. The sign of this depends on the labour supply functions, the income shares of the two groups and the elasticity of substitution of the production function.

In considering possible cases a large number of assumptions concerning the labour supply of the two groups are conceivable. The enjoyment of the two types of work may be different, leisure opportunities may not be the same because of different abilities, and so on. The simplest case where both types of labour are regarded as the same as far as the person supplying them is concerned is considered here.

With a conventional type of labour supply curve a higher wage corresponds to a lower elasticity so that $(\varepsilon_2 - \varepsilon_1)$ will be positive. Provided leisure is a normal good the ζ_i terms will be negative. Since a uniform lump-sum grant is a smaller component of income the higher is a person's earnings, it seems likely that ζ will increase with income so that $(\zeta_2-\zeta_1)$ will be negative. Overall $N(-k)$ will then be positive. It is shown in Allen (1979) that with a CES utility function in consumption and leisure of the type Feldstein (1973) used, $N(-k)$ is positive except in extreme cases. $N(-k)$ is thus taken to be positive in the examples below.

Even with this restriction it is possible to show that the general equilibrium effect can operate in both directions because D can plausibly have both signs. A positive elasticity of substitution between consumption and leisure in the utility function, which is the usual assumption, implies $\varepsilon_i>-1$. If $\sigma \geqq 1$ it follows that D must be positive. Hence with a Cobb–Douglas production function of the type Feldstein (1973) used, D is always positive and the direct and general equilibrium effects are reinforcing. However, if $\sigma<1$ and either or both of the labour supply elasticities are negative, D may also be.

The following examples can be used with theorem 1 to give a corollary which demonstrates that the effects can be reinforcing or opposing, and that in the latter case the general equilibrium effect can dominate so that a decrease in the proportionate tax at the market solution which redistributes from the lower to the higher paid can actually make the former better off.

Examples

(a) ε_1, ε_2, $(\varepsilon_2-\varepsilon_1)>0$; ζ_1, ζ_2, $(\zeta_2-\zeta_1)<0$.

(b) As in (a) except that $\varepsilon_1<0$ and the magnitudes of the terms are such that $D<0$.

These can both arise if, for example, the types of labour have a low elasticity of substitution in the production function and the labour supply curve is backward bending. All that is required to go from (a) to (b) is that the higher paid move from the part of the curve with positive labour supply elasticities to that with negative ones.

Corollary. If the conditions in (i) (a) and (ii) (b) are satisfied at $\beta=1$, then $(\mathrm{d}S/\mathrm{d}\beta)_{\beta=1}$ and $(\mathrm{d}S_{\mathrm{u}}/\mathrm{d}\beta)_{\beta=1}$ are (i) negative and (ii) positive.

Proof

(i) The result follows directly from theorem 1 since (1) and the conditions in (a) imply that $Z>0$.

(ii) Using the fact that $n\theta_1=X\theta_2$ it can be shown that

$$Z=\frac{1}{(n+1)D}\{(\sigma+\varepsilon_2\theta_1)(X-1)-\varepsilon_1-(n+1)\theta_1(k(\zeta_2-\zeta_1)-\varepsilon_2)\}. \tag{26}$$

Hence (1) and the conditions in (b) imply $Z<0$. The result then follows from theorem 1.

4. The Rawlsian optimal tax

The first-order condition for the maximisation of Rawlsian social welfare requires

$$\frac{\mathrm{d}S}{\mathrm{d}\beta}=\frac{\mathrm{d}V_2}{\mathrm{d}\beta}=0. \tag{27}$$

Solving (13), (14) and (17) simultaneously it is possible to derive an expression for $\mathrm{d}\alpha/\mathrm{d}\beta$ in terms of the elasticities. Using this, (10) and (14) in (27) and taking $V_1^2\neq 0$, the following result can be proved:

Theorem 2. For all $(1-\beta)$ *such that* $\mathrm{d}S/d\beta=0$

$$A=(1-\beta)B, \tag{28}$$

where

$$A=(\sigma+\varepsilon_2\theta_1)(X-1)-\varepsilon_1-(n+1)\theta_1\{\beta k(\zeta_2-\zeta_1)-\varepsilon_2\}, \tag{29}$$

$$\begin{aligned}B=&2n\varepsilon_1\varepsilon_2\theta_1/\beta-nw_2L_2\theta_1\varepsilon_1(\zeta_2-\zeta_1)+(\sigma+\varepsilon_2\theta_1)X\varepsilon_1/\beta\\&+(\sigma+\varepsilon_1\theta_2)n\varepsilon_2/\beta-w_2L_2\theta_1(\zeta_2-\zeta_1)X\varepsilon_1\\&-w_2L_2(\sigma+\varepsilon_1)(X\zeta_1+n\zeta_2).\end{aligned} \tag{30}$$

Examples (a) and (b) can be combined with theorem 2 to give the following corollary which follows directly given that $\beta>0$ and $n\theta_1=X\theta_2$.

Corollary 1. For all (i) negative and (ii) positive values of $(1-\beta)$ *such that the conditions in (i) (a) and (ii) (b) are satisfied there are no values of S such that* $\mathrm{d}S/\mathrm{d}\beta=0$.

If the conditions in example (a) are satisfied for $\beta=1$ and all feasible negative $(1-\beta)$ then the Rawlsian optimal tax is positive since corollary 1(i) of theorem 2 rules out negative interior maxima and corollary (i) of theorem 1 rules out negative corner maxima. Similarly, if the conditions in example (b) are satisfied, the optimal tax is negative.

Corollary 2. If the conditions in (i) (a) and (ii) (b) are satisfied for $\beta=1$ *and all feasible (i) negative and (ii) positive values of* $(1-\beta)$, *the Rawlsian optimal value of* $(1-\beta)$ *is (i) positive and (ii) negative.*

The utilitarian case is similar, the difference being that there are extra terms to take into account the utility of the higher paid. It is omitted here because of the algebraic complexity of the expressions but is given in Allen (1979).

5. Summary and concluding remarks

It has been argued that a serious deficiency of many of the recent models that have been used to investigate the taxation of income from labour has been their production assumptions. An exception was Feldstein (1973) but his use of a CES utility function and a Cobb–Douglas production function was a special case. The former ensures $N(-k)$ will be positive, except in extreme cases, and the latter combined with a positive elasticity of substitution between consumption and leisure implies D must be positive. The general equilibrium effect at the market solution will then always reinforce the direct effect. It is therefore not surprising that, although one cannot rule out negative optimal taxes in his model, they do not occur in his calculations.

It was demonstrated not only that the general equilibrium effect at the market solution may have both signs, but that this could outweigh the direct effect. It was also shown that it was possible for the optimal tax to have either sign. A case was described in which the general equilibrium effect is predominant, characterised by a low elasticity of substitution between the types of labour and a negative labour supply elasticity for the highly paid. It seems by no means implausible. In this case all that is required for the sign to change is for the high earners' labour supply elasticity to switch from a positive to a negative value.

Stern (1982) has derived a similar result for the optimal non-linear income tax. He showed analytically that, with a production function in two types of labour, this involves a positive marginal tax rate for the lowest paid workers if leisure is a normal good and a negative marginal rate for the highest paid if the marginal social utility of their consumption is lower than that of the other group.

There may, however, be difficulties in implementing a wage subsidy if the optimal linear tax is negative. One of the problems in enforcing a positive tax is that people tend to conceal their true incomes and declare their earnings to be lower than they actually are. However, the problems of doing this on a large scale are usually such that it is worthwhile only for those with high incomes to do it; and even then there is an upper bound to the amount of tax that can be avoided. With a negative tax, where the proportionate component takes the form of a subsidy, everybody has an unlimited incentive to conceal their true incomes and inflate their declared earnings. It might be very difficult to enforce such a system since artificial jobs paying large amounts can usually be created without problems and there is no upper bound to the amount that can be claimed, because the purpose of the system is to subsidise higher incomes the most. A negative linear tax is equivalent to a proportionate subsidy on consumption and it might be more feasible to implement the tax in this way though it may still be fairly easy to create false transactions and exploit the system.

In conclusion, because of changes in wages, attempts to alleviate poverty by progressive income taxation may not be very successful and may even exacerbate the problem, especially if labour supply elasticities are negative and different types of labour are complements rather than substitutes.

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