

## REPEATED PRINCIPAL–AGENT RELATIONSHIPS WITH LENDING AND BORROWING

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It is shown optimal long-term contracts are strictly better than a series of unrelated short-term contracts for two reasons: long-term contracts allow risk to be spread over time and the agent's intertemporal choices provide information about the unobservable random variable.

### 1. Introduction

One of the characteristics of many interpretations of the principal–agent relationship such as landlord–tenant, shareholder–management, and so on, is that they are repeated. One question that then arises is whether the use of long-term contracts allows an improvement over a series of short-term contracts. As soon as a time element is introduced, the possibility exists for risk to be spread over time. Yaari (1976) has shown that if a person has a risky income each period and the risk is independent over time, then if he can lend and borrow at a zero interest rate his optimal plan as the number of periods goes to infinity is to consume his mean income. It follows immediately from this result that in a repeated principal–agent model where the principal is risk neutral and there is no discounting, the first-best solution can be arbitrarily closely achieved as the number of periods goes to infinity. This is because lending and borrowing at a zero discount rate is a special form of the principal–agent relationship and can be used to eliminate the risk resulting from the unobservable random variable without distorting the agent's incentives.

This result has been proved in another way by Radner (1981), and Rubinstein and Yaari (1980). They adopt a game theoretic formulation and take a very different approach. Their basic notion is the following. If a principal–agent relationship is repeated, then it is possible to statistically test whether the distribution of realized outputs is the same as the distribution of output corresponding to the agent's first-best action and, as the number of repetitions increases, these tests become more powerful. If there is no discounting and the principal is risk-neutral, he can ensure the agent has the correct incentives by threatening a long series of high payments, which is like a large punishment, should the distribution of outputs achieved by the agent ever deviate significantly from what would be expected with the first-best action. It is then no longer necessary to provide incentives by making the agent's current consumption depend on his current output and optimal risk sharing is feasible. Thus, it is again the case that the first-best can be arbitrarily closely achieved as the time horizon

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becomes infinite. However, here it is not borrowing and lending that is responsible for the result but the fact that multiple drawings from a distribution allow something to be said about that distribution.

The model with a risk-neutral principal, no discounting and an infinite time horizon is clearly very special. Moreover, the arguments given are not concerned with the characteristics of second-best contracts, they simply show it is possible to do better in the long run. However, the implication is that both types of argument will characterize second-best contracts in the general case where the principal is risk-averse, there is discounting, and the time horizon is finite.

The purpose of this paper is to consider whether this is in fact the case: is it just implicit lending and borrowing that permits long-term contracts to improve on short-term contracts, or can long-term contracts do better than simple lending and borrowing schemes? If so, why is this: is it because of additional information provided by repeated drawings from the distribution or is it for some other reason?

In order to separate out the role of lending and borrowing it is assumed that the principal and agent both have access to a bond market where they can lend or borrow as much as they like at an exogenously given rate of interest. Thus, if in this model long-term contract is strictly better than a series of short-term contracts, it is not lending and borrowing that is responsible but something else.

A related but less general model has been considered by Townsend (1982). However, his interest was in justifying the assumption adopted in many papers trying to explain macroeconomic phenomena, that transactors use long-term contracts.

The paper proceeds as follows. Section 2 describes the model. Section 3 considers the case where the principal can fully control the agent's intertemporal choice in the contract, and section 4 considers what happens when this is not possible. Finally, section 5 contains conclusions.

## 2. The model

In the general version of the standard principal–agent model, the agent's output is  $Y = Y(s, e)$  where  $s$  is an unobservable random variable and  $e$  is his effort. His consumption is  $c = Y - \omega$  where  $\omega$  is the payment to the principal and his utility is  $V = V(c, e)$ . Here the special case where  $Y = s + e$  and  $V = V(c - e)$  is considered so that

$$V = V(s - \omega). \quad (1)$$

Thus the agent's utility is independent of his effort and the focus of the model is on risk-sharing.

The simplest repeated version where there are two periods is used. In period 1 it is possible to borrow or lend money to be repaid in period 2 with interest  $r$ . The agent borrows  $b_v(s_1)$  and the principal  $b_u(s_1)$ . The agent's two-period utility function is taken to be of the form

$$V = v(s_1 - \omega_1 + b_v(s_1)) + \delta_v v(s_2 - \omega_2 - (1 + r)b_v(s_1)), \quad (2)$$

where subscripts denote the period.

The principal's utility is given by

$$U = u(\omega_1 + b_u(s_1)) + \delta_u u(\omega_2 - (1 + r)b_u(s_1)). \quad (3)$$

Both are taken to have a positive marginal utility of consumption with the agent being risk-averse and the principal being either risk-neutral or risk-averse, so that

$$v', u' > 0, \quad v'' < 0, \quad u'' \leq 0. \quad (4)$$

The  $s_i$  are independent and identically distributed. They lie between 0 and  $\bar{s}$  ( $> 0$ ) and have a density function  $g(s_i)$ , which is continuous for  $0 \leq s_i \leq \bar{s}$ .

### 3. Contracts with controlled lending and borrowing

In this section, optimal contracts are considered assuming that the principal can observe and hence control the borrowing and lending of the agent in the sense that it can be specified in the contract.

It follows from the Revelation Principle [see, e.g., Baron and Myerson (1982)] that without any loss of generality it is possible to assume that the principal requests the agent to report the state of nature and that the schedules are planned so that the agent has no incentive to lie.

In the case of the one-period contract, the agent announces  $\hat{s}_i$  and then makes the principal a payment  $\omega(\hat{s}_i)$ . It can be easily seen that truthtelling requires that the payment be independent of  $\hat{s}_i$  since otherwise the agent would always announce the  $\hat{s}_i$  that gave him the highest payment. This implies the optimal short-term contract does not involve any risk-sharing.

In the first period of two-period contracts, assuming random payments are not feasible, it is only possible to make  $\omega_1$  depend on  $\hat{s}_1$ , since this is the only information available. In the second period, it is conceivable that  $\omega_2$  should be based on both  $\hat{s}_1$  and  $\hat{s}_2$ . However, as with short-term contracts, it is necessary for truthtelling that  $\partial\omega_2/\partial\hat{s}_2 = 0$  so that  $\omega_2$  depends only on  $\hat{s}_1$ . Hence, both  $\omega_1$  and  $\omega_2$  depend on  $\hat{s}_1$ . For simplicity the problem is analyzed as though the agent only lends and borrows through the principal. The total payments paid by the agent to the principal are denoted  $w_i(\hat{s}_1)$  and are given by

$$w_1(\hat{s}_1) = \omega_1(\hat{s}_1) - b_v(\hat{s}_1), \quad (5)$$

$$w_2(\hat{s}_1) = \omega_2(\hat{s}_1) + (1 + r)b_v(\hat{s}_1). \quad (6)$$

If the agent reports  $s_1$  honestly then his utility is given by

$$V(s_1) = v(s_1 - w_1(s_1)) + \delta_v E v(s_2 - w_2(s_1)), \quad (7)$$

where E denotes the expectation operator for the distribution  $g(s_2)$ .

If the agent were to lie and report  $\hat{s}_1$  when  $s_1$  is the true state of nature, then his utility is given by

$$V'(s_1, \hat{s}_1) = v(s_1 - w_1(\hat{s}_1)) + \delta_v E v(s_2 - w_2(\hat{s}_1)). \quad (8)$$

Thus, truthtelling requires that

$$V(s_1) = \max_{\hat{s}_1} V^*(s_1, \hat{s}_1). \quad (9)$$

It is assumed that  $w_1$  and  $w_2$  are continuous and differentiable. It can then be shown that for a two-period contract where lending and borrowing can be controlled the requirements

$$v'(s_1 - w_1(s_1))w_1'(s_1) + \delta_v E v'(s_2 - w_2(s_1))w_2'(s_1) = 0, \quad w_2' \leq 0 \quad (10),(11)$$

are necessary and sufficient for (9) to be satisfied.

If  $\bar{V}$  is the best alternative utility that the agent can obtain elsewhere over the two periods and

$$b(s_1) = b_u(s_1) + b_v(s_1), \quad (12)$$

then it follows that the Pareto optimal contract is given by the solution to

$$\max_{b, w_1, w_2} \int_0^{\bar{s}} [u(w_1(s_1) + b(s_1)) + \delta_u u(w_2(s_1) - (1+r)b(s_1))] g(s_1) ds_1, \quad (13)$$

subject to the truthtelling conditions (10) and (11) and the agent's utility constraint

$$\int_0^{\bar{s}} [v(s_1 - w_1(s_1)) + \delta_v E v(s_2 - w_2(s_1))] g(s_1) ds_1 \geq \bar{V}. \quad (14)$$

Optimal control theory can be used to solve this problem. Using the first-order conditions, it can be shown by contradiction that the optimal long-term contract involves risk-sharing. Since a sequence of short-term contracts with no risk-sharing is a feasible long-term contract but does not satisfy the necessary conditions for an optimal contract, it follows that the optimal long-term contract must strictly dominate a sequence of optimal short-term contracts.

#### 4. Long-term contracts with uncontrolled borrowing and lending

In this section, it is assumed that the agent's borrowing and lending cannot be observed and therefore cannot be controlled. This implies that the agent can evaluate any series of payments to the principal in terms of its discounted present value  $X$ .

$$X = \omega_1(\hat{s}_1) + (1/(1+r))\omega_2(\hat{s}_1). \quad (15)$$

It follows immediately that truthtelling requires

$$\omega_1(s_1) + (1/(1+r))\omega_2(s_1) = h, \quad (16)$$

where  $h$  is a constant, since otherwise the agent would always announce the state of nature that gave him the greatest present value of payments and hence the greatest utility. It follows directly from this that no risk-sharing is possible and the optimal long-term contract is equivalent to a sequence of short-term contracts.

#### 5. Conclusions

The analysis of section 3 demonstrates that optimal long-term contracts specifying the consumption of the agent strictly dominate a series of unrelated optimal short-term contracts not only because of the possibilities for spreading risk through time but also for some other reason. Section 4 shows that this reason is not because multiple drawings provide better information than single drawings as suggested by the work of Radner (1981), and Rubinstein and Yaari (1980), since the number of drawings in this second case is the same as in the first. Instead the reason long-term contracts can be preferable is that they use the intertemporal choice of the agent to gain better information about the unobservable random variable.

Finally, although this paper has focussed on the two-period case, it is clear that similar results will hold for any finite number of periods greater than two.

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