THE SOCIAL VALUE OF ASYMMETRIC INFORMATION

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Abstract

A welfare analysis of a simple noisy rational expectations model is carried out. It is shown that the more information prices convey, the worse off everybody can be. However, the equilibrium where everybody is uninformed may not be Pareto optimal: imposing a tax on information gathering which finances a lump sum grant may allow everybody to be better off when some people are informed.

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*I am grateful to S. Bhattacharya, J. A. Mirrlees, J. A. Ohlson, R. E. Verrecchia, participants of a number of seminars and particularly to A. Admati for many helpful comments and suggestions. I would also like to thank the U.K. SSRC for financial support.

2.26.12

1. Introduction

The question of when information has social value, in the sense that it allows everybody to be made better off, has received considerable attention in recent years. It is widely recognized that if the information is about productive opportunities then this can be socially valuable. In contrast, Hirshleifer (1971) has argued that in exchange economies improved information about the returns to assets is not socially valuable. He was able to give a simple example where improved public information actually makes everybody worse off than with no information because of a reduction in risk sharing opportunities: if people know the realization of a random variable, it is not possible to share the risks associated with the variable. A number of subsequent authors have found similar results (see, e.g., Fama and Laffer (1971), Marshall (1974), Ng (1975) and Wilson (1975)). However, Marshall (1974), Ng (1975) and Jaffe (1975) gave examples of exchange economies where the opposite is true: improved public information does have social value. In a recent paper, Hakansson, Kunkel and Ohlson (1982) have reconciled these results by showing that sufficient conditions for better public information not to have social value are that risk-sharing possibilities are equivalent to those that can be attained with complete markets and investors have homogeneous beliefs. If either of these conditions is not satisfied, as in the Marshall, Ng and Jaffe examples, better public information may lead to a Pareto improvement.

For the case of asymmetric information, Hirshleifer considered a model where the number of people who are informed is determined exogenously and the uninformed do not deduce the information conveyed by prices. In this context, he argued that any gain by the informed must be at the expense of the uninformed so that the market allocation is Pareto noncomparable to the no-information case. It can be further argued that if government intervention is possible the private acquisition of costly information is Pareto inefficient. This is because, if no information is gathered, the uninformed can compensate the informed and the resources previously expended on gathering information can be used to make everybody better off. The Pareto optimal allocation is thus where nobody is informed.

The purpose of this paper is to reconsider the social value of asymmetric information in exchange economies, using Grossman and Stiglitz's (1980) noisy rational expectations model. In this the number of people who are informed in equilibrium is such that everybody is indifferent between either expending resources and becoming fully informed or observing prices and deducing the information imperfectly. It is shown below that, similarly to Hirshleifer's original example, everybody can be strictly worse off with improved public information because of a reduction in risk sharing opportunities. However, Hirshleifer's results concerning the asymmetric information case do not hold in this model. Rather than being Pareto noncomparable, it is shown that, for certain parameter values, equilibria where some people are informed and some are uninformed are Pareto worse than the no-information case and the greater the proportion who are informed the Pareto worse is the equilibrium. The reason for this is similar to the public information results: the more people that are informed the more information prices convey and the less risk sharing there is. Diamond (1985) has obtained related results in the context of the optimal release of information by firms using a variant of the Diamond and Verrecchia (1981) model.

The main result of the paper is to show that the no-information equilibrium may not be Pareto optimal. If the government can tax the acquisition of information and use the proceeds to finance a uniform lump sum grant, examples can be constructed where everybody can be made better off in the equilibrium where some people are informed than in the equilibrium where nobody is informed. The reason for this is that in the no-information equilibrium there is still a risk due to the random supply of the risky asset: in some circumstances taxing information gathering and distributing the proceeds allows this risk to be shared. Thus information can be socially valuable even when improved public information cannot make everybody better off. In considering the social value of information, it is therefore not possible to restrict attention to differing states of public information.

The paper proceeds as follows: Section 2 describes the model and Section 3 considers equilibrium. Section 4 compares equilibria with different costs of information. In Section 5 the effect of taxing information acquisition is investigated. Finally, Section 6 contains concluding remarks.

2. The Model

The model used is a variant of that of Grossman and Stiglitz (1980). There is one period with two points in time: $t = 0, 1$. There are two assets, which are traded in competitive markets at $t = 0$: one safe, yielding R at $t = 1$, and the other risky, with a return u at $t = 1$ where

$$u = \theta + \varepsilon \tag{1}$$

and θ and ε are independent jointly distributed normal variables with mean and variance $(E\theta, \sigma_\theta^2)$ and $(0, \sigma_\varepsilon^2)$ respectively (with $\sigma_\theta^2, \sigma_\varepsilon^2 > 0$). Hence u has mean and variance $(E\theta, \sigma_u^2)$ where

$$\sigma_u^2 = \sigma_\theta^2 + \sigma_\varepsilon^2 . \tag{2}$$

By incurring a cost c it is possible to observe θ. Realizations of ε cannot be observed at any cost.

Traders who consume at $t = 1$ have identical exponential utility functions:

$$V(W_{1i}) = - \exp[-a\ W_{1i}] \tag{3}$$

where a is the degree of absolute risk aversion and W_{1i} is wealth at $t = 1$. The i^{th} trader is endowed at $t = 0$ with stocks of the two types of assets: he originally has $\bar{M}_i$ of the riskless asset and $\bar{X}_i$ of the risky asset. The average endowment of the group who consume at $t = 1$ is $\bar{X}$. At $t = 0$ they buy M_i of the safe asset and X_i of the risky asset. The price of the safe asset is normalized at unity and the price of the risky asset is P. The i^{th} person's budget constraint is therefore

$$W_{0i} = \bar{M}_i + P\bar{X}_i = M_i + PX_i \tag{4}$$

where W_{0i} is initial wealth. At $t = 1$ the person's wealth is then

$$W_{1i} = RM_i + uX_i \tag{5}$$

A crucial feature of the Grossman Stiglitz model is that the aggregate per capita supply of the risky asset, x, is stochastic. This is often justified on the grounds that there is a group of traders who are forced to sell their assets at $t = 0$ for liquidity (life cycle) reasons. This group is not usually explicitly modelled. However, given that the analysis below is normative this is necessary here.

The liquidity traders consume at $t = 0$. For simplicity they are taken to be risk neutral. The results are similar with risk aversion. In either case the effects of changes in informational efficiency can be to either increase or decrease their utility. Their utility function is

$$L(W_{0i}) = \bar{M}_i + P\ell \tag{6}$$

where $\overline{M}_i$ is as before and ℓ is a random endowment of the risky asset which is normally distributed with mean $E\ell$ and variance σ_ℓ^2.

The ratio of the number of liquidity traders to the number in the group that consumes at t = 1, which is normalized at unity, is γ. Hence

$$x = \gamma\ell + \overline{X} . \tag{7}$$

An important feature of the Grossman Stiglitz model is that the endowments of the traders who consume at t = 1, $\overline{X}_i$, are uncorrelated with x. This is achieved here by assuming that $\overline{X}_i$ and $\overline{X}$ are nonstochastic. Thus x is normally distributed with

$$Ex = \gamma E\ell + \overline{X} \; ; \; \sigma_x^2 = \gamma^2\sigma_\ell^2 . \tag{8}$$

A special case of some interest is where $\overline{X}_i = \overline{M}_i = 0$ so that only those who consume at t = 0 are endowed with the assets. All the results below apply to this overlapping generations version of the model.

There is a continuum of people who consume at t = 1 and a continuum of liquidity traders so they are all price-takers.

The sequence of events is as follows. Values of x and θ are realized; people who have paid c observe θ; the safe and risky assets are traded and finally the returns to the assets are received and consumed. Utilities are evaluated taking expectations over both x and θ.

3. Equilibrium

A proportion λ of the group that consumes at t = 1 becomes informed at cost c; they observe θ. The uninformed just observe P. Now the constant absolute risk aversion assumption implies that individuals' demands will depend only on their information and not on their endowment. Thus equilibrium requires

$$\lambda X_I + (1 - \lambda)X_U = x \ , \tag{9}$$

where X_I and X_U are the demands of the informed and uninformed respectively.

It follows from the budget constraint (4) and the moment generating function for the normal distribution that

$$E(V_I(W_{1i})|\theta, x) = -\exp\left[-a\left(R\bar{M}_i + RP\bar{X}_i - c + (\theta - RP)X_I - \frac{a}{2}\sigma_\varepsilon^2 X_I^2\right)\right] \ . \tag{10}$$

Choosing X_I to maximize this gives

$$X_I = \frac{\theta - RP}{a\sigma_\varepsilon^2} \ . \tag{11}$$

Substituting this into (9) and rearranging

$$P = \frac{1}{R}\left[\theta - \frac{a\sigma_\varepsilon^2}{\lambda} x + \frac{(1 - \lambda)}{\lambda} a\sigma_\varepsilon^2 X_U\right] \ . \tag{12}$$

In a rational expectations equilibrium the uninformed will effectively know the parameters λ, a and the means and variances of θ, ε and x. They also know their own demand X_U and R; what they don't know is θ or x. Thus observing P is equivalent to observing

$$w_\lambda = \theta - \frac{a\sigma_\varepsilon^2}{\lambda}(x - Ex) \tag{13}$$

and they should therefore condition their demands on w_λ.

What Grossman and Stiglitz are able to show is that the ratio of the utility of the informed $EV_I(W_{1I})$ to that of the uninformed $EV_U(W_{1i})$ is given by

$$\frac{EV_I(W_{1i})}{EV_U(W_{1i})} = e^{ac}\beta(\lambda)^{\frac{1}{2}} \tag{14}$$

where

$$\beta(\lambda) = \frac{\sigma_\varepsilon^2}{Var(u|w_\lambda)} \ , \tag{15}$$

$$\mathrm{Var}(u|w_\lambda) = \sigma_u^2 - \sigma_\theta^2 \eta(\lambda) \;, \tag{16}$$

$$\eta(\lambda) = \frac{\sigma_\theta^2}{\mathrm{Var}\; w_\lambda} \;, \tag{17}$$

$$\mathrm{Var}\; w_\lambda = \sigma_\theta^2 + \frac{a^2\sigma_\varepsilon^4}{\lambda^2}\sigma_x^2 \;. \tag{18}$$

Properties of $\eta(\lambda)$ and $\beta(\lambda)$ which are important for subsequent proofs are

$$\eta(0) = 0 \quad ; \quad 0 \le \eta(\lambda) < 1 \;, \tag{19}$$

$$\eta'(\lambda) = 2\frac{a^2\sigma_\varepsilon^4}{\lambda^3}\sigma_x^2 \frac{\eta(\lambda)}{\mathrm{Var}\; w_\lambda} \quad \begin{cases} > 0 \text{ for } 0 < \lambda \le 1 \\ = 0 \text{ for } \lambda = 0 \end{cases} \;, \tag{20}$$

$$0 < \beta(\lambda) < 1 \;, \tag{21}$$

$$\beta'(\lambda) = \frac{\beta(\lambda)}{\mathrm{Var}(u|w_\lambda)}\sigma_\theta^2 \eta'(\lambda) \quad \begin{cases} > 0 \text{ for } 0 < \lambda \le 1 \\ = 0 \text{ for } \lambda = 0 \end{cases} \;. \tag{22}$$

There are then three types of equilibrium.

(i) $\underline{e^{ac}\,\beta(0)^{\frac{1}{2}} > 1}$

In this case even if everybody is uninformed $EV_{Ii} < EV_{Ui}$ (since utility is negative) and so it does not pay for anybody to become informed.

(ii) $\underline{e^{ac}\,\beta(\lambda)^{\frac{1}{2}} = 1}$

In this equilibrium there are both informed and uninformed people whose utilities are the same. It follows from (22) that the equilibria of this type are stable. If $\lambda = \lambda^e - \delta\lambda$, where λ^e is an equilibrium value of λ, then $e^{ac}\beta(\lambda^e - \delta\lambda)^{\frac{1}{2}} < 1$ and $EV_{Ii} > EV_{Ui}$; thus more people will become informed until $e^{ac}\beta(\lambda^e)^{\frac{1}{2}} = 1$. Similarly if $\lambda = \lambda^e + \delta\lambda$. The number of people who demand information and become informed adjusts until in equilibrium people are indifferent between paying c and observing θ directly or observing P and hence w_λ, and deducing θ imperfectly.

(iii) $\underline{e^{ac}\beta(1)^{\frac{1}{2}} < 1}$

In this case it pays for everybody to become informed.

A measure of the amount of information conveyed by the price, or in other words the informational efficiency of the price system, is given by the squared correlation coefficient between P and θ which is equal to $\eta(\lambda)$. It follows from (20) that the greater the proportion of the population that are informed, the more informative are prices, or the more efficient in an informational sense, is the market for the risky asset.

The type (ii) equilibrium condition $e^{ac}\beta(\lambda)^{\frac{1}{2}} = 1$ implies that

$$\frac{d\lambda}{dc} < 0 \tag{23}$$

so the lower the cost of information the more who are informed.

Together (20) and (23) imply that as the cost of information is increased the informational efficiency of the market is reduced. In the welfare analysis below, economies with different costs of information and hence different degrees of informational efficiency will be compared.

4. Comparing equilibria with different costs of information

The two extreme cases of informational efficiency in the model are the equilibrium where nobody is informed and the equilibrium where everybody is informed. The first step in the analysis is to compare these (ignoring the cost of information in the latter).

Proposition 1

(a) For every person who consumes at $t = 1$, irrespective of their endowments $\bar{M}_i$ and $\bar{X}_i$, utility in the equilibrium where everybody is uninformed, $EV_{Ui}|_{\lambda=0}$, is greater than utility in the equilibrium where everybody is informed at zero cost, $EV_{Ii}|_{\lambda=1,c=0}$.

(b) For the liquidity traders utility in the former equilibrium may be above or below that in the latter depending on whether

$$E\ell Ex + \frac{\sigma_x^2}{\gamma} \lessgtr 0 . \tag{24}$$

Proof

(a) The first part is demonstrated by evaluating the two levels of utility of those who consume at $t = 1$ and comparing them.

$EV_{Ii}|_{\lambda=1,c=0}$

Putting $c = 0$ and substituting (11) into (10) it follows that the expected utility of the informed in this case is given by

$$E(V_I(W_{1i})|\theta,\ x)|_{c=0} = -\exp[-a(R\bar{M}_i + RP\bar{X}_i + \frac{(\theta - RP)^2}{2a\sigma_\varepsilon^2})] . \tag{25}$$

When $\lambda = 1$ equilibrium requires $X_I = x$ so that using (11)

$$RP = \theta - a\sigma_\varepsilon^2 x . \tag{26}$$

Hence

$$E(V_I(W_{1i})|\theta,\ x)|_{\lambda=1,c=0} = -\exp[-a(R\bar{M}_i + (\theta - a\sigma_\varepsilon^2 x)\bar{X}_i + \frac{a}{2}\sigma_\varepsilon^2 x^2)] . \tag{27}$$

Taking expectations over θ gives

$$E(V_I(W_{1i})|x)\big|_{\lambda=1,c=0} = -\exp[-a(R\overline{M}_i + \overline{X}_i E\theta - \frac{a}{2}\sigma_\theta^2\overline{X}_i^2 - a\sigma_\varepsilon^2 x\overline{X}_i + \frac{a}{2}\sigma_\varepsilon^2 x^2)] \ . \quad (28)$$

$\underline{EV_{Ui}|_{\lambda=0}}$

Similarly to (10) it can be shown that when $\lambda = 0$ and the uninformed people's prior on u has mean and variance $(E\theta, \sigma_u^2)$ then

$$E(V_U(W_{1i})|\theta, x)\big|_{\lambda=0} = -\exp[-a(R\overline{M}_i + RP\overline{X}_i + (E\theta - RP)X_U - \frac{a}{2}\sigma_u^2 X_U^2)] \ . \quad (29)$$

Choosing X_U to maximize this

$$X_U = \frac{(E\theta - RP)}{a\sigma_u^2} \ . \quad (30)$$

Substituting back into (29) gives

$$E(V_U(W_{1i})|\theta, x)\big|_{\lambda=0} = -\exp[-a(R\overline{M}_i + RP\overline{X}_i + \frac{(E\theta - RP)^2}{2a\sigma_u^2})] \ . \quad (31)$$

In the equilibrium with $\lambda = 0$, $X_U = x$ so that

$$RP = E\theta - a\sigma_u^2 x \ . \quad (32)$$

Hence

$$E(V_U(W_{1i})|x)\big|_{\lambda=0} = -\exp[-a(R\overline{M}_i + (E\theta - a\sigma_u^2 x)\overline{X}_i + \frac{a}{2}\sigma_u^2 x^2)] \ . \quad (33)$$

Using the fact that $\sigma_u^2 = \sigma_\theta^2 + \sigma_\varepsilon^2$ it can be shown

$$E(V_U(W_{1i})|x)\big|_{\lambda=0} = E(V_I(W_{1i})|x)\big|_{\lambda=1,c=0}\exp[-\frac{a^2\sigma_\theta^2}{2}(x - \overline{X}_i)^2] \ . \quad (34)$$

Now since $\exp[-a^2\sigma_\theta^2(x - \overline{X}_i)^2/2] \le 1$, x is normally distributed and utility is negative,

$$EV_{Ui}\Big|_{\lambda=0} > EV_{Ii}\Big|_{\lambda=1,c=0} \tag{35}$$

which gives the first part of the proposition.

(b) The second part follows similarly. Using (1), (6) and (7) with (26) and (32)

$$EL(W_{0i})\Big|_{\lambda=0} = \bar{M}_i + \frac{1}{R}\left(E\theta E\ell - a\sigma_u^2[E\ell Ex + \frac{\sigma_x^2}{\gamma}]\right) \tag{36}$$

$$EL(W_{0i})\Big|_{\lambda=1} = \bar{M}_i + \frac{1}{R}\left(E\theta E\ell - a\sigma_\varepsilon^2[E\ell Ex + \frac{\sigma_x^2}{\gamma}]\right) . \tag{37}$$

Hence

$$EL(W_{0i})\Big|_{\lambda=0} - EL(W_{0i})\Big|_{\lambda=1} = - a\sigma_\theta^2\left(E\ell Ex + \frac{\sigma_x^2}{\gamma}\right) \tag{38}$$

which gives the condition in (24). Hence the proposition is demonstrated.

The result in part (a) arises because information is revealed before trade and investors are unable to insure against the distributive risk due to fluctuations in θ and hence in the value of their endowments. However, when information is not revealed they are effectively insured against this risk. This is similar to Hirshleifer's original example. The feature of the model which underlies this result is the fact that there is no trading before θ becomes known. If there were then the information would have no effect: this initial round of trading would allow investors to share the risk of fluctuations in θ. The strict inequality in the proposition is a result of the initial allocation not being a Pareto optimal risk sharing arrangement: otherwise it is well known there will be no trade and hence no change in expected utility when more public information is available.

An important difference between public information and asymmetric information is that in the latter case there cannot be a pre-signal round of trading if equilibrium is to exist. This is because traders could deduce x from the price in the initial pre-signal market and θ from the price in the post-signal market. The Grossman Stiglitz argument for nonexistence when there is costly information, would then hold: if prices reveal information fully nobody will pay anything to discover it; however, if nobody observes it, then there is an incentive for somebody to discover it. Hence, the only case that can be considered with asymmetric information is the one used here where there is no pre-signal trading.

In the second part of the proposition the first term of the condition (24), $E\ell Ex$, arises because the expected price of the risky asset when $\lambda = 1$ is higher than when $\lambda = 0$ (from (26) and (32)). When the liquidity traders have a positive (negative) expected endowment this effect tends to make them better (worse) off than with no information (assuming $Ex > 0$). The second term, σ_x^2/γ, arises because of the effect of information on the covariance of liquidity traders' endowments and prices. High values of ℓ correspond to high values of x and low P. Thus the liquidity traders' endowments and prices have a negative covariance. With full information, it follows from (26) and (32) that this covariance is smaller in absolute value than with no information and hence their expected wealth is higher. Taking these two effects together implies that if average endowments are sufficiently low the liquidity traders are worse off with superior information, otherwise they are better off.

It can be seen from (20) and (23) that having more private information collected is similar to improving public information, since this makes prices better signals. It is therefore natural to suppose that increasing the proportion of the population that is informed could make everybody that consumes at $t = 1$ worse off. This in fact turns out to always be the case.

Proposition 2

(a) An increase in the cost of information and hence a reduction in the equilibrium proportion of people who are informed and the informational efficiency of the market, always leads to an improvement in the welfare of those who consume at $t = 1$. Moreover, their utility is maximized in the equilibrium where nobody is informed and prices convey no information.

(b) For the liquidity traders, utility may rise or fall as the informational efficiency of the market increases.

Proof

(a) Since along the equilibrium path as c is altered, it is an identity that informed and uninformed agents have the same utility levels, they are equally sensitive to the cost of information. Thus to demonstrate the first part of the proposition, it is sufficient to show that $dEV_{Ui}/d\lambda < 0$ for all $\bar{M}_i$ and $\bar{X}_i$.

The uninformed observe P and hence w_λ. Their prior on u thus has mean and variance $(E(u|w_\lambda), \mathrm{Var}\,(u|w_\lambda))$ where the latter is given by (16) and

$$E(u|w_\lambda) = E\theta + (w_\lambda - E\theta)\eta(\lambda) \ . \tag{39}$$

It can be shown in the usual way that

$$X_U = \frac{E(u|w_\lambda) - RP}{a\ \mathrm{Var}(u|w_\lambda)} \tag{40}$$

and

$$E(V_U(W_{1i})|w_\lambda) = -\exp\left[-a\left(R\bar{M}_i + RP\bar{X}_i + \frac{(E(u|w_\lambda) - RP)^2}{2a\ \mathrm{Var}(u|w_\lambda)}\right)\right] \ . \tag{41}$$

In order to evaluate this it is necessary to derive the relationship between P and w_λ. Using (9), (11), (13) and (40) it follows that

$$RP = \frac{\lambda w_\lambda + (1-\lambda)\beta\, E(u|w_\lambda) - a\sigma_\varepsilon^2 Ex}{\lambda + (1-\lambda)\beta} . \tag{42}$$

Substituting for RP in (41), taking expectations over w_λ and rearranging it can be shown (see Appendix)

$$EV_{Ui} = -\left(1 + \frac{a^4\sigma_\varepsilon^6\sigma_x^4}{[\lambda + (1-\lambda)\beta]^2[a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2]}\right)^{-\frac{1}{2}} \tag{43}$$

$$\exp\left[-a\left(R\bar{M}_i + \bar{X}_i E\theta + \frac{a}{2}\sigma_u^2\left(\frac{(\bar{X}_i - Ex)^2}{1 + a^2\sigma_u^2\sigma_x^2 + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\left[1 - (1-\lambda)^2(1-\beta\eta)\right]} - \bar{X}_i^2\right)\right)\right] .$$

Differentiating with respect to λ

$$\frac{dEV_{Ui}}{d\lambda} = EV_{Ui}\Bigg\{\left(\frac{a^4\sigma_\varepsilon^6\sigma_x^4}{[\lambda + (1-\lambda)\beta]^2[a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2] + a^4\sigma_\varepsilon^6\sigma_x^4}\right)$$

$$\left(\frac{1 - \beta + (1-\lambda)\beta'}{\lambda + (1-\lambda)\beta} + \frac{\lambda\sigma_\theta^2}{a^2\sigma_\varepsilon^2\sigma_x^2\sigma_u^2 + \lambda^2\sigma_\theta^2}\right)$$

$$+ \frac{a^2}{2}\sigma_u^2\left(\frac{(\sigma_\theta^2/\sigma_\varepsilon^2)\left[2(1-\lambda)(1-\beta\eta) + (1-\lambda)^2(\beta'\eta + \beta\eta')\right](\bar{X}_i - Ex)^2}{\left[1 + a^2\sigma_u^2\sigma_x^2 + (\sigma_\theta^2/\sigma_\varepsilon^2)[1 - (1-\lambda)^2(1-\beta\eta)]\right]^2}\right)\Bigg\} < 0 . \tag{44}$$

This negative sign results from (19) - (22) and the fact that utility is negative.

It follows from Proposition 1 that for every $\bar{M}_i$ and $\bar{X}_i$

$$EV_{Ui}\big|_{\lambda=0} > EV_{Ii}\big|_{\lambda=1} \qquad \text{for all } c \geq 0 \ . \tag{45}$$

Combining this with the fact that EV_{Ui} has its maximum at $\lambda = 0$ gives part (a) of the proposition.

(b) It follows from (6) and (42) that

$$EL_i = \bar{M}_i + \frac{1}{R}\{E\theta E\ell - \frac{a\sigma_\varepsilon^2}{\lambda + (1-\lambda)\beta}(E\ell Ex + [1 + (\frac{1}{\lambda} - 1)\beta n]\frac{\sigma_x^2}{\gamma})\} \ . \tag{46}$$

Differentiating with respect to λ

$$\frac{dEL_i}{d\lambda} = \frac{1}{R}\{\frac{a\sigma_\varepsilon^2[1 - \beta + (1-\lambda)\beta']}{[\lambda + (1-\lambda)\beta]^2}[E\ell Ex + [1 + (\frac{1}{\lambda} - 1)\beta n]\frac{\sigma_x^2}{\gamma}]$$

$$- \frac{a\sigma_\varepsilon^2\sigma_x^2}{\gamma[\lambda + (1-\lambda)\beta]}(-\beta' n - \beta n' + \frac{a^2\sigma_\varepsilon^6\sigma_x^2\sigma_u^2\sigma_\theta^2 - \lambda^2\sigma_\theta^4\sigma_\varepsilon^4}{[\lambda^2\sigma_\theta^2\sigma_\varepsilon^2 + a^2\sigma_\varepsilon^4\sigma_x^2\sigma_u^2]^2})\} \ . \tag{47}$$

For $\lambda = 0$, $\sigma_\varepsilon^2 = 0.1$, $\sigma_\theta^2 = 0.9$, $\sigma_x^2 = 1$, $a = 1$, $\gamma = 1$, $E\ell = 1$, $\bar{X} = 1$, $Ex = 2$ and $R = 1$ then $dEL_i/d\lambda = 18$. However, if $E\ell = 0$, $Ex = 1$ and $\sigma_x^2 = 0.5$ but otherwise everything is the same then $dEL_i/d\lambda = -4.5$. Hence part (b) is demonstrated.

The first part of the proposition shows that Hirshleifer's result that improved public information can make everybody worse off because of increased distributive risk carries over directly to the asymmetric information case. When the cost of information is lower, so that more agents purchase information, prices are better signals and there are less risk sharing opportunities in the usual way. In addition the uninformed agents now have to trade with more informed agents. Thus it is clear they will be worse off. In equilibrium the informed must have the same utility as the uninformed and hence they are also worse off. Similarly to Proposition 1(b), the liquidity group may be better or worse off depending on whether or not the liquidity traders' average endowments are sufficiently low. If they are sufficiently low the equilibria are Pareto ranked: the lower the cost of information acquisition the greater the informational efficiency of the market and the worse off everybody is.

An important assumption for proving part (a) of the proposition is that $\overline{X}_i$ is independent of x and hence w_λ. In the case where they are correlated it is not possible to always show that $dEV_{Ui}/d\lambda < 0$.

Diamond (1985) has used a similar model to investigate the optimal release of information by firms, by interpreting the risky asset as being shares in a firm. His results provide an interesting contrast to those obtained here. He develops a noisy rational expectations equilibrium of the type considered by Diamond and Verrecchia (1981). The assumptions concerning information acquisition are similar to those of Grossman and Stiglitz (1980). One important difference between his model and the one here is that he does not include a group of liquidity traders with random endowments. Instead he assumes all traders have stochastic endowments of the risky asset which are independent and identically distributed. In order to assure each individual's endowment is uncorrelated with the aggregate per capita endowment he considers the case where the number of traders tends to infinity. He does not consider the case where the number of traders is actually infinite since then the aggregate per capita endowment is no longer stochastic. This would allow the uninformed to deduce the information of the informed from the price of the risky asset at no cost and the usual argument for the nonexistence of equilibrium would hold.

Diamond shows that in situations where without any information release by the firm, the proportion of informed would be positive, the optimal policy is to release information with the maximum precision (i.e., the inverse of variance) such that nobody finds it worthwhile to privately gather information. The savings in resources because people no longer collect information and also an improvement in risk sharing due to the greater homogeneity of information make everybody better off. The firm should not release any information with lower precision than this level because as above the improved public information would destroy risk sharing opportunities and make everybody worse off. By the same reasoning if $\lambda = 0$ when no announcement is made, then the optimal policy is not to release any information.

These results are similar to those obtained above for the group that consumes at $t = 1$: they are always made worse off by an increase in informational efficiency. This is not true for the liquidity traders however. Because of the change in expected price of the asset and the correlation between their endowments and price they can be better off with improved informational efficiency. The precise way in which the randomness of the per capita endowment of the risky asset is introduced into the model is thus important in determining whether everybody is always made worse off by superior information.

5. Taxing information gathering

The comparison of equilibria in the previous section demonstrates that if the government can impose a tax on information gathering then even if they discard the revenue it will be possible to make everybody who consumes at $t = 1$ better off: the tax increases the cost of information and reduces the informational efficiency of the market. If the government used the revenue to pay a uniform lump sum grant then this can further improve people's welfare.

It would seem that there is a possibility the government can use the tax to make everybody better off than in the no-information equilibrium. This in fact turns out to be the case for some configurations of parameters.

Proposition 3

A tax that leads to no information collection may be Pareto suboptimal.

Proof

With a tax t on information the type (ii) equilibrium condition changes to

$$e^{a(c+t)}\beta^{\frac{1}{2}} = 1 \; . \tag{48}$$

Rearranging gives

$$t = -\frac{1}{2a}\log\beta - c \; . \tag{49}$$

Hence

$$\frac{dt}{d\lambda} = -\frac{1}{2a}\frac{\beta'}{\beta} \qquad \begin{cases} < 0 \text{ for } 0 < \lambda \leq 1 \\ = 0 \text{ for } \lambda = 0 \end{cases} \tag{50}$$

from (22). Thus by choosing a particular level of t the government can determine a particular equilibrium level of λ.

The analysis is the same as in Section 4 except it is now possible for the government to use the revenue from the tax to give a lump sum subsidy of λt to everybody who consumes at t = 1. The expected utility of the uninformed then becomes

$$EV_{Ui}^{t} = \exp[-a\lambda t]EV_{Ui} \tag{51}$$

where EV_{Ui} is given by (43). Here

$$\frac{dEV_{Ui}^{t}}{d\lambda} = -EV_{Ui}^{t}\, a(t + \lambda\frac{dt}{d\lambda}) + \exp[-a\lambda t]\frac{dEV_{Ui}}{d\lambda} \; . \tag{52}$$

The proposition is concerned with whether the optimal level of t is such that $\lambda = 0$. It can be shown using (19), (20), (22) and (44)

$$\left.\frac{dEV_{Ui}^{t}}{d\lambda}\right|_{\lambda=0} = -EV_{Ui}^{t}\ \{\frac{1}{2}\log(1 + \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}) - ac$$

$$- a^2\sigma_u^2 \frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\ [\frac{\sigma_x^2}{1 + a^2\sigma_u^2\sigma_x^2} + \frac{(\bar{X}_i - Ex)^2}{(1 + a^2\sigma_u^2\sigma_x^2)^2}]\}\ . \tag{53}$$

Evaluating (47) at $\lambda = 0$

$$\left.\frac{dEL_i}{d\lambda}\right|_{\lambda=0} = \frac{1}{R}\ \{\frac{\sigma_\theta^2}{\sigma_\varepsilon^2}\ [a\sigma_u^2(E\ell Ex + \frac{\sigma_x^2}{\gamma}) - \frac{1}{\gamma a}]\}\ . \tag{54}$$

It is clear that these cannot be signed in general. To see that they can be positive for everybody consider the case where $\sigma_\varepsilon^2 = 0.1$, $\sigma_\theta^2 = 0.9$, $\sigma_x^2 = 0.0001$, $a = 10$, $\gamma = 1$, $E\ell = 0.001$, $\bar{X}_i = \bar{X} = 20.000$, $Ex = 20.001$, $R = 1$ and $c = 0$. Here $dEV_{Ui}^{t}/d\lambda|_{\lambda=0} = (-EV_{ui}^{t})\ 1.06 > 0$ and $dEL_i/d\lambda|_{\lambda=0} = 0.91 > 0$. Hence everybody is better off when some traders are informed and the proposition is demonstrated.

Hirshleifer has argued that in exchange economies the private acquisition of costly information is undesirable since it simply leads to a reallocation of consumption but uses up costly resources: by an appropriate reallocation it would be possible to make everybody better off in the case where no private information is gathered, than in any state in which it is. Proposition 3 shows that this result does not hold in the Grossman Stiglitz model. Taxing the informed group and reallocating to the uninformed allows improved sharing of the risks due to variations in x and everybody including the liquidity traders can be better off.

Although in the equilibrium with no information, transactors are effectively insured against variations in θ, they are not insured against variations in x since P still depends on x. Consider the example in the proposition where $\bar{X}_i = \bar{X}$. It can be shown using (33) that

$$E(V_U(W_{1i})|x)\Big|_{\lambda=0} = -\exp[-a(R\bar{M}_i + E\theta\bar{X} - \frac{a}{2}\sigma_u^2\bar{X}^2 + \frac{a}{2}\sigma_u^2(x - \bar{X})^2)] \ . \qquad (55)$$

Using (6) and (32)

$$E[L(W_{0i})|x]\Big|_{\lambda=0} = \bar{M}_i + \frac{1}{R\gamma}[(E\theta - a\bar{X}\sigma_u^2)(x - \bar{X}) - a\sigma_u^2(x - \bar{X})^2] \ . \qquad (56)$$

It can be seen from (55) and (56) that those who consume at t = 1 have utility which increases with $(x - \bar{X})^2$ while for the liquidity traders the reverse is true. There is therefore scope for sharing the risk associated with x. It is demonstrated below that the effect of taxing information gathering and using the proceeds to finance a uniform lump sum grant can be to allow such risk-sharing.

To see this consider the effect on the expected return (given x) of an uninformed person's portfolio when λ is increased from zero. It can be shown

$$\frac{dE[(uX_U + RM_i)|x]}{d\lambda}\Bigg|_{\lambda=0} = -\frac{\sigma_\theta^2}{\sigma_\varepsilon^2}[\frac{1}{a} - \frac{1}{a\sigma_x^2}(x - Ex)(x - \bar{X}) + a\sigma_u^2 x(2x - \bar{X})] \ . \qquad (57)$$

In the example in the proposition, x is almost certain to lie in the range x = $\bar{X}$ = 20.000 to x = 20.002. For these values (57) is dominated by $a\sigma_u^2 x(2x - \bar{X})$ (for x = 20.000; 20.002, $a\sigma_u^2 x(2x - \bar{X})$ = 4000.0; 4000.8 whereas $1/a - (x - Ex)(x - \bar{X})/a\sigma_x^2$ = 0.100; 0.098 respectively). Hence the greater x is relative to $\bar{X}$ the larger the loss to the uninformed from introducing informed people into the market. However, the profits the informed people earn mean that they are prepared to pay a tax on information which is used to finance a grant of $[\log(1 + \sigma_\theta^2/\sigma_\varepsilon^2)]/2a$ to the uninformed. In the example, the uninformed are therefore better off for low x near $\bar{X} = 20.000$ and worse off for higher x. Thus the effect of having a tax where $\lambda > 0$ can be to smooth their consumption across x states.

For the liquidity traders it can be shown

$$\frac{dE[L(W_{0i})|x]}{d\lambda}\bigg|_{\lambda=0} = (x - \bar{X}) \frac{\sigma_\theta^2}{R\gamma\sigma_\varepsilon^2}[a\sigma_u^2 x - \frac{(x - Ex)}{a\sigma_x^2}] \; . \tag{58}$$

For the relevant range in the example the $a\sigma_u^2 x$ term dominates (for $x = 20.000$; 20.002, $a\sigma_u^2 x = 200.00$; 200.02 whereas $(x - Ex)/a\sigma_x^2 = -1.00$; 1.00 respectively). Hence the higher x the greater the gain for the liquidity traders. This is the opposite of the effect on the informed traders. Thus overall, setting the tax so that $\lambda > 0$ results in improved risk sharing between the uninformed traders and the liquidity traders.

The reason improved public information about θ can make everybody worse off is the absence of possibilities to insure against variations in θ. In contrast, the reason asymmetric information about θ can make everybody better off is the absence of possibilities to insure against variations in x. If markets existed which allowed this, then the asymmetric information about θ would have no value.

Even though in the model used, improved public information does not have social value, it is in fact possible for everybody to be better off with allocations associated with asymmetric information. In considering the social value of information, it is therefore important to consider the allocations associated with asymmetric information as well as those with differing degrees of public information.

2.26.12

6. Concluding remarks

In contrast to the question of the existence of equilibrium in rational expectations models, there has been relatively little work on the welfare properties of rational expectations equilibria (for exceptions see Grossman (1981) and Laffont (1985)). Although the analysis above is concerned with a special example, it has implications for more general models. In rational expectations models, prices reveal information and similarly to Hirshleifer (1971) the more information that is publicly revealed the worse off everybody can be. However, the results of this paper indicate it is not the case that equilibria where no information is publicly revealed are necessarily Pareto optimal. In noisy rational expectations models it may be better to have prices which reveal some information about payoff relevant random variables (here θ), if this is associated with some mechanism such as the taxation of information gathering which permits risk sharing across non-payoff relevant random variables (here x).

Finally it should be pointed out that the welfare comparisons above are very sensitive to the type of ex-ante insurance markets which are available. In particular, information makes prices more variable ex-post. Hence ex-ante markets which allow people to trade claims contingent on the realization of ex-post prices could serve a valuable insurance role.

Appendix

Derivation of Equation (43)

Using (41) and (42) it can be shown

$$E(V_U(W_{1i})|w_\lambda) = -\exp[-a(R\bar{M}_i+(E\theta- \frac{Ex}{D})\bar{X}_i + \frac{1}{D}[\frac{\lambda}{a\sigma_\varepsilon^2} + \frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\eta]\bar{X}_i(w_\lambda-E\theta)$$

$$+ \frac{1}{2aD^2\mathrm{Var}(u|w_\lambda)}(\frac{\lambda}{a\sigma_\varepsilon^2}(1-\eta)(w_\lambda - E\theta) - Ex)^2)] \tag{A1}$$

where
$$D = \frac{\lambda}{a\sigma_\varepsilon^2} + \frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)} . \tag{A2}$$

Taking expectations over w_λ and using the linear transformation

$$w_\lambda^* = w_\lambda - E\theta + \frac{\zeta}{\xi} \tag{A3}$$

where
$$\zeta = \frac{a}{D}(\frac{\lambda}{a\sigma_\varepsilon^2} + \frac{(1-\lambda)}{a\mathrm{Var}(u|w_\lambda)}\eta)\bar{X}_i - \frac{\lambda(1-\eta)Ex}{a\sigma_\varepsilon^2 D^2\mathrm{Var}(u|w_\lambda)} , \tag{A4}$$

$$\xi = \frac{\lambda^2(1-\eta)^2}{a^2\sigma_\varepsilon^4 D^2\mathrm{Var}(u|w_\lambda)} + \frac{1}{\mathrm{Var}\ w_\lambda} \tag{A5}$$

the integral can be eliminated and it follows that

$$EV_U(W_{1i}) = -\frac{\xi^{-\frac{1}{2}}}{\sqrt{\mathrm{Var}\ w_\lambda}}\exp[-a(R\bar{M}_i + (E\theta - \frac{Ex}{D})\bar{X}_i + \frac{1}{2a}(\frac{(Ex)^2}{D^2\mathrm{Var}(u|w_\lambda)} - \frac{\zeta^2}{\xi}))] . \tag{A6}$$

Simplifying this expression using

$$a\left[\frac{\lambda(1-\eta)}{a\sigma_{\varepsilon}^{2}}\ \frac{\text{Var } w_{\lambda}}{D}\left(\frac{\lambda}{a\sigma_{\varepsilon}^{2}}+\frac{(1-\lambda)}{a\text{Var}(u|w_{\lambda})}\ \eta\right)-D\text{Var}(u|w_{\lambda})\text{Var } w_{\lambda}\ \xi\right]=-1 \qquad \text{(A7)}$$

it can be shown that (A6) simplifies to (43).

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