

# Stock price manipulation, market microstructure and asymmetric information

Franklin Allen and Gary Gorton\*

*University of Pennsylvania, Philadelphia, PA 19104, USA*

## 1. Introduction

Is it possible for an uninformed trader to buy a stock, drive the price up and then sell the stock at this higher price, thereby earning a profit? In this paper, we show that this type of trade-based manipulation is possible under very natural conditions and develop an example where there is an equilibrium amount of manipulation.

The possibility of this type of price manipulation has historically been a subject of great concern to market participants and governments [see Allen and Gale (1990) for a discussion] and more recently to academic researchers [see, e.g., Kyle (1984), Vila (1987, 1989), Jarrow (1989), Allen and Gale (1990), Bagnoli and Lipman (1990), Benabou and Laroque (1990), Kumar and Seppi (1990), and Black (1991)]. In contrast to these papers, which develop special models to demonstrate the possibility of manipulation, we use an existing model and introduce an asymmetry. We argue that this asymmetry is a more natural assumption than the original one which was primarily made for analytical convenience. In this modified model profitable manipulation is possible.

In recent years there has been a large literature concerned with the pricing of stocks by a specialist who trades with investors who are better informed than him. Two models that have received wide attention are those of Glosten and Milgrom (1985) and Kyle (1985). One of the assumptions of both these models is that, in addition to the informed investors, there are 'liquidity traders' who must trade the stock for exogenous reasons. The usual justification for the existence of these liquidity traders is that there are people

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with pressing needs for cash who are forced to *sell* securities. However, Glosten and Milgrom (1985), Kyle (1985) and the large subsequent literature treat liquidity traders as asymmetric, that is, equally likely to be buyers as sellers. It is difficult to understand what the motivation is for traders who have pressing needs to *buy* securities. Traders wishing to buy securities can more freely choose the time to buy and thus minimize losses to insiders. For example, liquidity buyers could trade immediately following earnings or other important announcements when there is a lower probability of informed insiders profiting at their expense. This flexibility is in sharp contrast to traders who need to sell because of an immediate need for cash.

The natural asymmetry between liquidity purchases and liquidity sales leads to an asymmetry in price responses. If liquidity sales are more likely than liquidity purchases, there is less information in a sale than in a purchase because it is less likely the trader is informed. The bid price then moves less in response to a sale than does the ask price in response to a purchase. This asymmetry of price elasticities can create an opportunity for profitable price manipulation. A manipulator can repeatedly buy stock, causing a relatively large effect on prices, and then sell with relatively little effect.

Another important symmetry assumption in Glosten and Milgrom (1985) and Kyle (1985) is that sellers and buyers are equally likely to be informed. This assumption is again questionable because of factors such as short sale constraints which make it easier to exploit good news than bad news. If there is a different probability of a buyer being informed than a seller, the effects of purchases and sales on prices will again be asymmetric (even if liquidity trading is symmetric). This can again produce opportunities for profitable manipulation.

These possibilities are illustrated below with a series of examples based on the model of Glosten and Milgrom (1985). This is briefly described in section 2. It is then demonstrated in section 3 that when the asymmetries discussed above are introduced, manipulators can sometimes enter breaking the standard equilibrium. Section 4 shows that the model can be adapted to allow for an equilibrium level of manipulation. Finally, section 5 contains concluding remarks.

## **2. The Glosten and Milgrom model**

Glosten and Milgrom model a perfectly competitive specialist system in which a specialist posts a bid price  $B$ , and an ask price,  $A$ , at which he agrees to buy one unit of stock and sell one unit respectively. Only one unit can be traded at each point in time and all orders are market orders. After posting initial bid and ask prices, the sequence of events is as follows. A trader arrives and submits an order to either buy or sell one unit of stock at the

posted bid and ask prices. After the transaction, the specialist may revise the posted prices. Then another trader arrives and so on.

The value of the share at some specified future date is uncertain; it may be high,  $V_H$ , or low,  $V_L$ . Initially, the specialist's prior probability the high value occurs is  $\pi_0$ . There are two types of trader other than the specialist: privately informed traders and liquidity traders. Privately informed traders are informed about whether the value of the share at the future date will be  $V_H$  or  $V_L$ . The probability an arriving trader is an insider is  $\alpha$ . Liquidity traders have no private information; they must buy or sell at the current price for exogenous reasons. The probability that a liquidity trader is a seller is  $\delta$ . All agents are risk neutral.

The specialist is aware of the structure of the model. Potential entry by other specialists ensures that he sets prices so as to break even. His posted prices are equal to the stock's expected value conditional on whether the trade will be a buy or a sell. This ensures that he does not come to regret the prices when he actually finds out whether the next trade is a buy or a sell. For example, suppose the specialist's current prior the stock's value is  $V_H$  is  $\pi_0$ . The specialist anticipates that if the next trader is a purchaser, then the share should be sold at a price which takes into account the fact that the trader may be informed and know that the stock has value  $V_H$ . In setting the ask price, the specialist therefore uses Bayes rule to update his prior to  $\pi_1^A = \pi_0[\alpha + (1-\alpha)(1-\delta)] / \{\pi_0[\alpha + (1-\alpha)(1-\delta)] + (1-\pi_0)(1-\alpha)(1-\delta)\}$  to allow for this possibility and his posted ask price is  $A_1 = \pi_1^A V_H + (1-\pi_1^A) V_L$ . Similarly, in setting his posted bid price  $B_1$ , he takes into account the seller may know the stock's value is  $V_L$  so  $\pi_1^B = \pi_0(1-\alpha)\delta / \{\pi_0(1-\alpha)\delta + (1-\pi_0)[\alpha + (1-\alpha)\delta]\}$  and  $B_1 = \pi_1^B V_H + (1-\pi_1^B) V_L$ . Following the trade, the specialist changes his beliefs from  $\pi_0$  to  $\pi_1^A$  or  $\pi_1^B$  depending on whether the trader was a purchaser or buyer. He then uses Bayes rule to calculate  $\pi_2^A$  and  $\pi_2^B$  and uses these to set  $A_2$  and  $B_2$  in anticipation of the next trade. Similarly for subsequent dates.

### 3. Possibilities for manipulation

Manipulation is not possible if the model is symmetric as Glosten and Milgrom assume. To see this consider an example where  $\pi_0=0.5$ ;  $\alpha=0.1$ ;  $V_H=10$ ;  $V_L=0$  and  $\delta=0.5$  so purchases and sales by liquidity traders are equally likely. Table 1 shows the ask and bid prices set by the specialist for a particular sequence of trades.

Suppose a manipulator attempted to just buy and sell the stock. He would buy at the initial ask of 5.50, the specialist would change the ask to 5.99 and the bid to 5.00 and the manipulator could sell at 5.00. The manipulator would therefore make a net loss of  $-5.50 + 5.00 = -0.50$ . Similarly, if the manipulator tried a strategy of {buy, buy, sell, sell} the transactions would be

Table 1  
The symmetric case.

Trading date	Trade	$\pi_t^A$	$\pi_t^B$	Ask	Bid	Cumulative profits
1	Buy	0.550	0.450	<u>5.50</u>	4.50	-5.50
2	Buy	0.599	0.500	<u>5.99</u>	5.00	-11.49
3	Sell	0.646	0.550	6.46	<u>5.50</u>	-5.99
4	Sell	0.599	0.500	5.99	<u>5.00</u>	-0.99

Table 2  
Manipulation with asymmetric liquidity traders.

Trading date	Trade	$\pi_t^A$	$\pi_t^B$	Ask	Bid	Cumulative profits
1	Buy	0.679	0.471	6.79	4.71	-6.79
2	Buy	0.817	0.653	<u>8.17</u>	6.53	-14.96
3	Sell	0.904	0.799	9.04	<u>7.99</u>	-6.97
4	Sell	0.893	0.779	8.93	<u>7.79</u>	+0.82

at the ask and bid prices underlined in table 1 and his cumulative profit at the end would be  $-0.99$  as shown in the last column. Similarly, any manipulative strategy will lead to a loss since the specialist is setting prices to protect himself from an informed trader.

Next, suppose that there is an asymmetry where liquidity selling is more likely than liquidity buying as discussed in the introduction. Small degrees of asymmetry do not permit profitable manipulation. However, for  $\delta > 0.770$  profitable manipulation is possible. Table 2 is the counterpart of table 1 with  $\delta = 0.9$ .

Once again it can be seen from the table that a strategy of buying and selling is not profitable for a manipulator since it gives a net profit of  $-6.79 + 6.53 = -0.26$ . In fact, a strategy of {buy, sell} can never be profitable since a date  $t+1$  bid can never be above a date  $t$  ask. This is because a sale will never be made by an informed trader who knows the stock is worth  $V_H$  so  $\pi_t^A \geq \pi_{t+1}^B$ . A strategy of {buy, buy, sell, sell} is profitable, however. Buying a share at dates 1 and 2 drives both the ask and the bid prices up by a significant amount because there are few liquidity purchases. This position can then be unwound without moving the bid price down by very much because there are many liquidity sales. Overall, it can be seen from the table that this strategy yields a cumulative profit of 0.82.

Finally, suppose that the asymmetry in the model comes from a different source. Consider the case where there are informed buyers but because there are short sale constraints and informed traders start with zero endowment of the stock, there are no informed sellers. In this case,  $\pi_1^A$  is given by the same

Table 3  
 Manipulation with asymmetric informed trading.

Trading date	Trade	$\pi_1^A$	$\pi_1^B$	Ask	Bid	Cumulative profits
1	Buy	0.550	0.500	5.50	5.00	-5.50
2	Buy	0.599	0.550	5.99	5.50	-11.49
3	Sell	0.646	0.599	6.46	5.99	-5.50
4	Sell	0.646	0.599	6.46	5.99	+0.49

expression as before but  $\pi_1^B = \pi_0$  since a sale contains no information. The profitability of a {buy, buy, sell, sell} strategy when  $\delta=0.5$  and the other parameters are as before is shown in table 3.

It can be seen that the figures in table 3 are the same as those in table 1, except that prices are not changed by sales since there are no informed sellers. As a result the {buy, buy, sell, sell} strategy is again profitable.

#### 4. An equilibrium level of manipulation

Tables 2 and 3 demonstrate that the notion of equilibrium used by Glosten and Milgrom needs to be altered if the model is extended to allow for the possibility of asymmetries in liquidity trading or informed investors' trades. In both the cases above where a manipulation strategy was profitable, the specialist only took account of the fact that there were informed traders and liquidity traders. In equilibrium, it is necessary for the specialist to take account of the presence of manipulators in setting prices and the level of manipulation will be such that manipulators just break even so there is no incentive for further manipulators to enter.

In general, allowing for manipulative strategies will be complex because of the many sequences of trades a manipulator could undertake. In order to illustrate the properties of an equilibrium where manipulation occurs, consider the case where there are four dates. Since {buy, sell} can never be a profitable manipulation the only possible sequence of manipulative trades is {buy, buy, sell, sell}. Suppose that the probability that this sequence is made by a manipulator is  $\phi$ . Apart from this the example is similar to that for table 2. The probability a buyer or seller is informed is  $\alpha$  and the probability a liquidity trader is a seller is  $\delta$ .

When the specialist sets his date 1 prices he takes into account the fact that there might be manipulation. In calculating the ask price  $A_1$ , the probability share value is high is  $\pi_1^A = \pi_0[\phi + \alpha + l] / \{\pi_0[\phi + \alpha + l] + (1 - \pi_0)[\phi + l]\}$ , since the probability the purchase is made by a manipulator is  $\phi$ , the probability it is made by an informed trader is  $\alpha$  and the probability it is made by a liquidity trader is  $l = (1 - \alpha - \phi)(1 - \delta)$ . In calculating the bid

Table 4  
Equilibrium manipulation with asymmetric liquidity traders.

Trading date	Trade	$\pi_t^A$	$\pi_t^B$	Ask	Bid	Cumulative profits
1	Buy	0.671	0.471	6.71	4.71	-6.71
2	Buy	0.745	0.653	<u>7.45</u>	6.53	-14.16
3	Sell	0.904	0.721	9.04	<u>7.21</u>	-6.95
4	Sell	0.893	0.695	8.93	<u>6.95</u>	+0.00

price  $B_1$ ,  $\pi_1^B = \pi_0(1-\alpha)\delta / \{\pi_0(1-\alpha)\delta + (1-\pi_0)[\alpha + (1-\alpha)\delta]\}$  as before. This is because the specialist knows that if the first trade is a sale there cannot be any manipulation since the only manipulative strategy involves a purchase at date 1. In fact, if at any time the trading sequence deviates from {buy, buy, sell, sell} the specialist knows that there is no manipulation and updating is the same as in the example for table 2. Hence the only path, that needs to be considered is {buy, buy, sell, sell}.

Consider what happens at date 2 given there has been a buy at date 1. In setting  $A_2$ , the specialist considers how a {buy, buy} sequence might arise. One possibility is that he is trading with a manipulator. The other possibilities are that there could be a sequence of two informed traders, two liquidity traders or one liquidity trader and one informed trader in either order. Given these possibilities it is possible to use Bayes rule to calculate  $\pi_2^A$  and use this to set  $A_2$ . The setting of the bid prices at dates 3 and 4 along the {buy, buy, sell, sell} sequence can be similarly analyzed.

The equilibrium level of manipulation such that manipulators just break even when the parameter values are the same as for table 2 is  $\phi = 0.00654$ . The prices that occur in this case are shown in table 4.

It can be seen by comparing tables 2 and 4 that the possibility of equilibrium manipulation means that the specialist adjusts prices sluggishly along the manipulation path. Apart from that prices are the same as before.

## 5. Concluding remarks

The examples given above are clearly special. Their purpose is to illustrate that the symmetry assumptions of standard market microstructure models are not as innocuous as they seem at first sight. Allowing for asymmetries introduces the possibility of profitable manipulation. This means that it is necessary to change the notion of equilibrium used so that the specialist takes account of the possibility that he is trading with a manipulator and in equilibrium manipulators just break even. This was demonstrated in the context of the Glosten and Milgrom (1985) model but similar results seem likely to hold in Kyle's (1985) model.

The first task to be undertaken is to investigate the notion of equilibrium

with manipulation in more general models than the simple examples considered above. An important element of such models will be to endogenize the timing of liquidity purchases. In the long run the number of sales and purchases must usually balance. However, in the short run they will not balance since buyers who can control the timing of their trades will cluster to minimize losses to insiders. This clustering of buyers will be similar to that in Admati and Pfleiderer's (1988) theory of intra-day trading where both uninformed buyers and sellers trade at certain times of the day to avoid losses to insiders.

Once tractable general models with manipulation have been constructed, the empirical implications of manipulation for the pattern of stock prices can be considered. In the examples above there is positive serial correlation of prices due to the activities of manipulators; it will be interesting to see whether this property holds in more general models and what other time series properties hold. Once these properties have been established an important task will be to test for the presence of manipulation in actual series of stock prices.

In addition to these positive issues there are also important welfare questions. The elimination of manipulation has long been a policy goal in a number of countries. The question of whether or not it is desirable to eliminate the type of manipulation identified above and if so how this could be accomplished need to be addressed. These are all topics for future research.

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