

PRINCETON UNIVERSITY  
Economics Department  
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**FINANCIAL CRISES**  
**ECO 575 (Part II)**

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Section 5: Bubbles and Crises

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(All materials available on my website:  
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## 1. Introduction

So far the realizations of returns that have driven the crises have been taken as exogenous, i.e. they are simply bad outcomes.

In practice it seems that there are many cases where the crisis is brought on by an asset price bubble. It is the bursting of the bubble that causes the crisis.

In his famous book *Manias, Panics and Crashes* Kindleberger recognizes that often such bubbles are driven by an expansion of money and credit.

Bubbles and ensuing crises have a very long history:

Tulipmania

South Sea Bubble

Mississippi Land Company

Wall Street Crash of 1929

They have very much been a problem of the 1980's and 1990's:

Emerging countries:

Mexico

Developed countries:

Norway

Finland

Sweden

Japan

The Internet Bubble

*Typical sequence of events* (See, e.g., Kaminsky and Reinhart (1996, 1999) and Higgins and Osler (1997))

- There is financial liberalization and this leads to credit expansion with significant bank lending.
- Some of this lending finances new investment but much of it is used to buy assets in fixed supply such as real estate and stocks.
- The prices of real estate and stocks rise above their "fundamentals" (i.e. what would occur if everybody was investing their own money).
- This process continues until there is some real event and returns are low or the central bank is forced to restrict credit because of fears such as "overheating" and inflation.
- The prices of real estate and stocks collapse.
- A banking crisis results because people had borrowed using assets valued at "bubble" prices as collateral.
- There may be a foreign exchange crisis as investors try to avoid the banking crisis.
- The crisis spills over to the real economy and there is a recession.

- Are these bubbles and crises consistent with rational behavior by investors? We argue they are.
- Standard models of asset pricing assume people invest with their own money. We identify the price of an asset in this benchmark case as the "fundamental". A *bubble* is said to occur when the price of an asset rises above this fundamental.
- Allen and Gale (2000) develop a simple model where the people who make investment decisions do so with borrowed money. Lenders cannot observe the riskiness of the project invested in so there is an agency problem.
- There are two assets:
  - A safe asset in variable supply  
(manufacturing)
  - A risky asset in fixed supply  
(real estate, stocks)
- Marginal returns on the two assets are equated.
- If they borrow money then because of default they are only interested in the upper part of the distribution of returns of the risky asset. As a result there is a *risk shifting problem* and the price of the risky asset is bid up above its fundamental so there is a bubble.

## A Simple Example to Illustrate the Basic Idea

There are two assets:

	<u>t = 1</u>	<u>t = 2</u>
Safe asset:	For each 1 invested	-----> 1.5
Risky asset:	Fixed supply 1	-----> 6 with prob. 0.25 1 " " 0.75
	Price P	Expected payoff = 2.25

All investors are risk neutral

When investors borrow money then because of default they are only interested in the upper part of the distribution of returns of the risky asset. As a result there is a *risk shifting problem* and the price of the risky asset is bid up above the benchmark so there is a bubble.

## The Fundamental

This is the price that somebody would be willing to pay if investing their own money

Suppose all investors have wealth 1

Given risk neutrality the marginal returns on the two assets would be equated:

$$\frac{(0.25 \times 6 + 0.75 \times 1)}{P_F} = \frac{1.5}{1}$$

$$P_F = \frac{2.25}{1.5} = 1.5$$

The benchmark value of the asset is 1.5 and any price above this is termed a bubble.

## Intermediated case

Investors have no wealth of their own.

They can borrow money to buy assets at a rate of 33.33 percent. The most they can borrow is 1. If they borrow 1 they repay 1.33.

<u>t = 1</u>	<u>t = 2</u>
Borrow	Pay back
1	1.33
	if able

Lenders can't observe how loans are invested.

Can  $P = 1.5$  be the equilibrium price?

Suppose an investor borrows 1 and invests in the safe asset.

$$\begin{aligned}\text{Marginal return safe asset} &= 1.5 - 1.33 \\ &= 0.17\end{aligned}$$

Suppose an investor borrows 1 and invests in the risky asset. She purchases  $1/1.5$  units of the asset. When the payoff is 6 she repays the loan and keeps what remains and when it is 1 she defaults.

$$\begin{aligned}\text{Marginal return risky asset} &= 0.25 \left( \frac{1}{1.5} \times 6 - 1.33 \right) + 0.75 \times 0 \\ &= 0.67\end{aligned}$$

The risky asset is clearly preferred. This is the *risk shifting problem*. The risky asset is more attractive because part of the risk is shifted to the lender since the loan is repaid only part of the time.

This is similar to the standard result in corporate finance that managers of debt financed firms will be willing to accept negative net present value investments.

What is the equilibrium price of the risky asset?

In an equilibrium where the safe asset is produced, the price of the risky asset,  $P$ , will be bid up since it is in fixed supply, until the profit of borrowers is the same for both the risky and the safe asset:

$$0.25\left(\frac{1}{P}\times 6 - 1.33\right) + 0.75\times 0 = 1.5 - 1.33$$

$$P = 3.$$

There is a *bubble* with the price of the risky asset above the benchmark of 1.5

The more risk there is the greater is the risk shifting and the larger the bubble

Note that default and the potential for a financial crisis occurs in this model when the return on the risky asset is low, i.e. 1

Risk shifting can occur in equilibrium provided there is a spread between the rate depositors receive and the rate the bank lends at

The bank's depositors bear the loss and this requires markets are segmented to be possible in equilibrium

## Closing the model

- Banks take deposits and lend out funds. They cannot distinguish between safe and risky assets. Banks pay depositors  $r_D$  and charge borrowers 1.33.
- The banking sector is perfectly competitive so banks earn zero profits.
- The fixed supply of the risky asset is 1.
- The amount of funds depositors have is 10 and the number of borrowers is 10.
- In the equilibrium where  $P = 3$ , in order for the fixed supply of 1 unit to be invested 3 of the borrowers invest in the risky asset and 7 in the safe.
- The zero profit condition for banks is

$$0.7 \times 1.33 + 0.3(0.75 \times (1/3) \times 1 + 0.25 \times 1.33) = r_D$$

This implies

$$r_D = 1.11$$

- Example illustrates importance of *segmented markets* to the theory. Depositors receive 11% but investments in the safe asset receive 50%. Justification is that many institutional investors do make a high rate.

The effect of an increase in risk

Now suppose the payoffs to the risky asset are:

	<u>t = 1</u>	<u>t = 2</u>
Safe asset:	For each 1 invested	-----> 1.5
Risky asset:	Fixed supply 1	-----> 9 with prob. 0.25 0 " " 0.75
	Price P	Expected payoff = 2.25

The price of the risky asset is given by

$$0.25 \left( \frac{1}{P} \times 9 - 1.33 \right) + 0.75 \times 0 = 1.5 - 1.33$$

$$P = 4.5.$$

There is more incentive to shift risk so the price is bid up to a higher level.

## Credit and Interest Rate Determination

The amount of credit and the interest rate have so far been taken as exogenous. Now suppose the central bank sets the amount of credit  $B$  and the interest rate  $r$  is determined by the marginal product of capital in the economy.

The interest rate  $r$  depends on the amount of consumption good  $x$  invested at date 1 in the productive technology to produce  $f(x)$  units of output at date 2

Since there is 1 unit of the risky asset

$$x = B - P$$

and it is assumed

$$f(x) = 3(B - P)^{0.5}$$

The interest rate is then given by

$$r = f'(B-P) = 1.5(B - P)^{-0.5}$$

The pricing equation is now

$$0.25\left(\frac{1}{P} \times 6 - r\right) + 0.75 \times 0 = 0$$

Substituting for  $r$  and solving

$$P = 8(-1 + \sqrt{1 + 0.25B})$$

The fundamental is given by

$$P_F = \frac{2.25}{r}$$

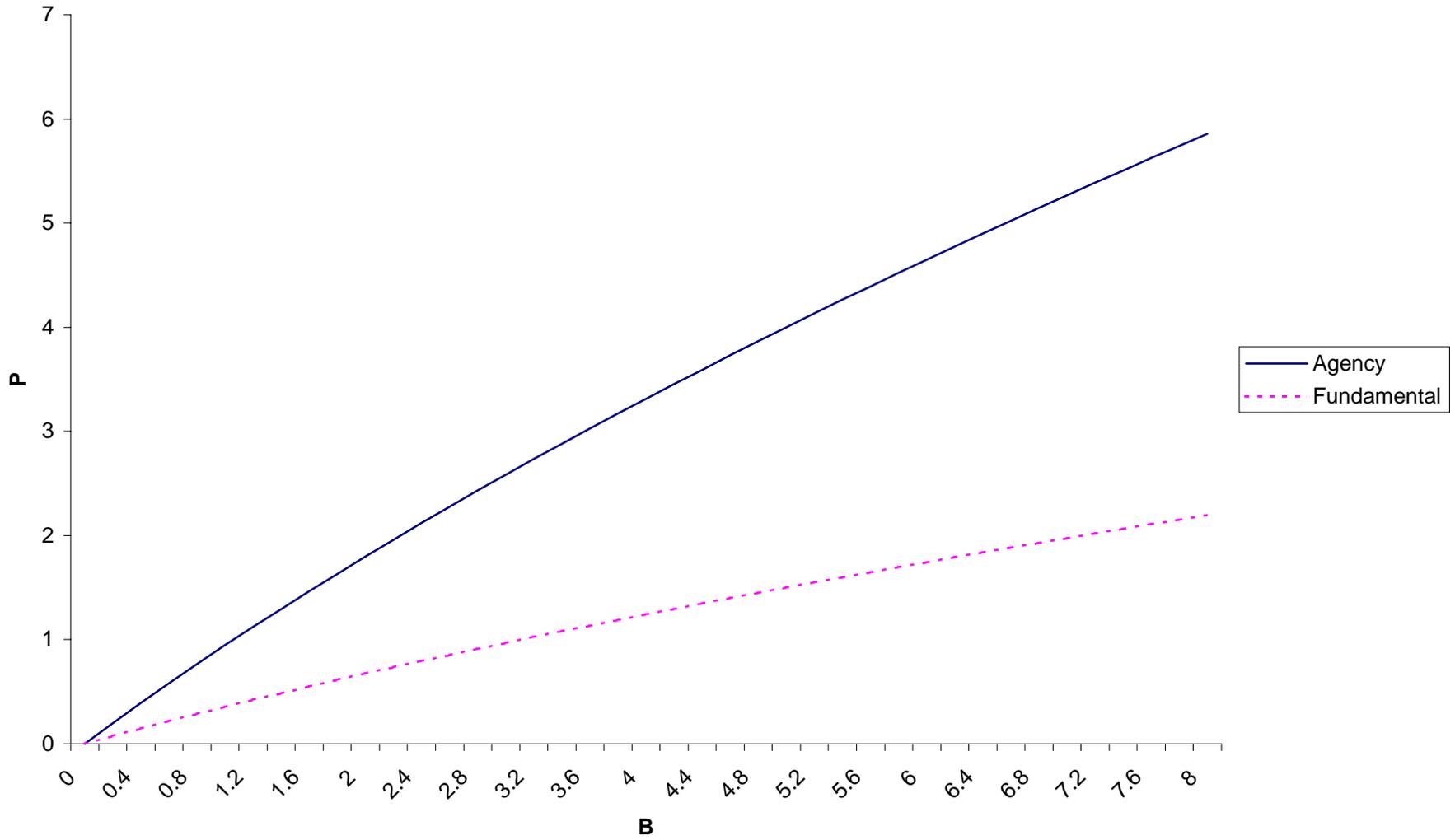
The relationship between P and B is shown for both cases in Figure 1.

Solid line: The case with an agency problem

Dotted line: The fundamental

Changes in aggregate credit cause a relatively large changes in asset prices when there is an agency problem.

**Figure 1**  
Credit and Asset Prices



## **Financial Risk**

A version of the model can be developed to consider the dynamic relationship between the amount of credit available to investors and asset prices.

What is meant by financial risk? The amount of credit and hence interest rates are taken as random variables by investors and this uncertainty leads to uncertainty in asset prices.

In making decisions initially investors must anticipate the amount of credit at the subsequent date since this determines asset prices at the subsequent date. If credit turns out to be too low asset prices will not be high enough for loans to be repaid and default will occur. Risk results from uncertainty about credit and so is financial.

The risk shifting effect operates for financial risk in the same way as it does for real risk. There can again be a bubble. The possibility of credit expansion over a period of years may create a great deal of uncertainty about how high the bubble may go and when it may collapse.

Suppose that the risk in B has the following form. The corresponding prices and interest rates are found from the relationship in Figure 1.

Probability	B	P	r
0.5	5	4	1.5
0.5	7	5.27	1.14

In this case at date 0 the borrower borrows 1 and buy  $1/P_0$  units of the asset. If the borrower invested in the safe asset they would receive 0 since there is no surplus. Hence the pricing equation is:

$$0.5 \left( \frac{1}{P_0} \times 5.27 - r_0 \right) + 0.5 \times 0 = 0$$

Substituting for  $r_0$ ,

$$P_0 = \frac{5.27}{1.5} (B_0 - P_0)^{0.5}$$

Solving for

$$r_0 = 1.27$$

$$P_0 = 4.61$$

Hence there is a bubble since the expected payoff on the asset at date 2 is 2.25.

As with real uncertainty an increase in risk leads to a bigger bubble

Probability	B	P	r
0.5	4	3.14	1.81
0.5	8	5.86	1.03

In this case it can be shown

$$r_0 = 1.27$$

$$P_0 = 4.61$$

so the bubble is increased.

## Discussion

- This version illustrates the basic idea. Allen and Gale (2000) develop a more formal version of the model and characterize the equilibrium.
- The formal version is more plausible in that each entrepreneur undertakes both activities and there are decreasing returns and various other assumptions to ensure interior equilibrium.
- Note that the price of the risky asset depends not only on the payoffs of the assets but also on the amount of credit investors have available.
- The central bank controls the amount of credit investors have available through reserve requirements and the amount of assets available to be used as reserves.
- The higher the amount of credit available to investors the higher the price of the asset in fixed supply.

## 2. Uncertainty Generated by the Real Sector

### **The Model**

The model is simple to make the analysis transparent

	<u>t = 1</u>		<u>t = 2</u>
<b>Safe asset:</b>	For each x invested	----->	rx

r is determined by marginal product of capital:

Aggregate production function:

<u>t = 1</u>	<u>t = 2</u>
x	f(x)

where  $f'(x) > 0$ ,  $f''(x) < 0$ ,  $f'(0) = \infty$ ,  $f'(\infty) = 0$ .

$$r = f'(x)$$

<b>Risky asset:</b>	Fixed supply 1		----->	R with prob. h(R)
	Price P			Expected payoff = $\bar{R}$

Non-pecuniary cost of collecting returns from the risky asset is  $c(x)$  where  $c(0) = c'(0) = 0$ ,  $c'(x) > 0$ ,  $c''(x) > 0$ .

- Continuum of small, risk neutral *investors* (firms or individuals)
- Continuum of small, risk neutral *banks*, each having  $B > 0$  units to lend. Banks can't distinguish between good and bad assets and so must lend to investors
- Contracts between banks and investors cannot be made contingent on the (unobservable) payoffs so simple debt contracts with interest rate  $r$  are used
- Banks cannot observe what investors do with the funds

$X_S$  is the investor's investment in the safe asset

$X_R$  " " " " " " risky "

Investor's total borrowing is  $X_S + PX_R$  and net monetary payoff is

$$rX_S + RX_R - r(X_S + PX_R) = RX_R - rPX_R$$

### Investor's decision problem:

$$\max_{X_R} \int_{R^*}^{R_{MAX}} [RX_R - rPX_R] h(R) dR - c(X_R) \quad (1)$$

where  $R^* = rP$  is the return at which default occurs

### Equilibrium conditions:

Market clearing requires:

$$X_R = 1 \quad (2)$$

$$X_S + P = B \quad (3)$$

$$r = f'(X_S) \quad (4)$$

Using the first-order condition for (1) and substituting (2) gives,

$$\int_{R^*}^{R_{MAX}} [R - rP] h(R) dR = c'(1) \quad (5)$$

From (3) and (4)

$$r = f'(B - P) \quad (6)$$

Together (5) and (6) determine the equilibrium values.

## Benchmark Case

Consider the decision when the representative agent is investing her own money:

$$\max \int_0^{R_{\max}} [rX_S + RX_R] h(R) dR - c(X_R) \quad (7)$$

$$\text{subject to } X_S + PX_R = B$$

Substituting in the budget constraint and choosing  $X_R$  to maximize (7) gives

$$\int_0^{R_{\max}} Rh(R) dR - rP = c'(X_R) \quad (8)$$

## Comparing prices:

*Benchmark case:*

Setting  $X_R = 1$  in (8) gives

$$\bar{P} = \frac{1}{r} (\bar{R} - c'(1)) \quad (9)$$

*Intermediated case:*

From (5)

$$P = \frac{1}{r} \left( \frac{\int_{R^*}^{R_{\text{MAX}}} R h(R) dR - c'(1)}{\Pr[R \geq R^*]} \right) \quad (10)$$

Comparing these gives:

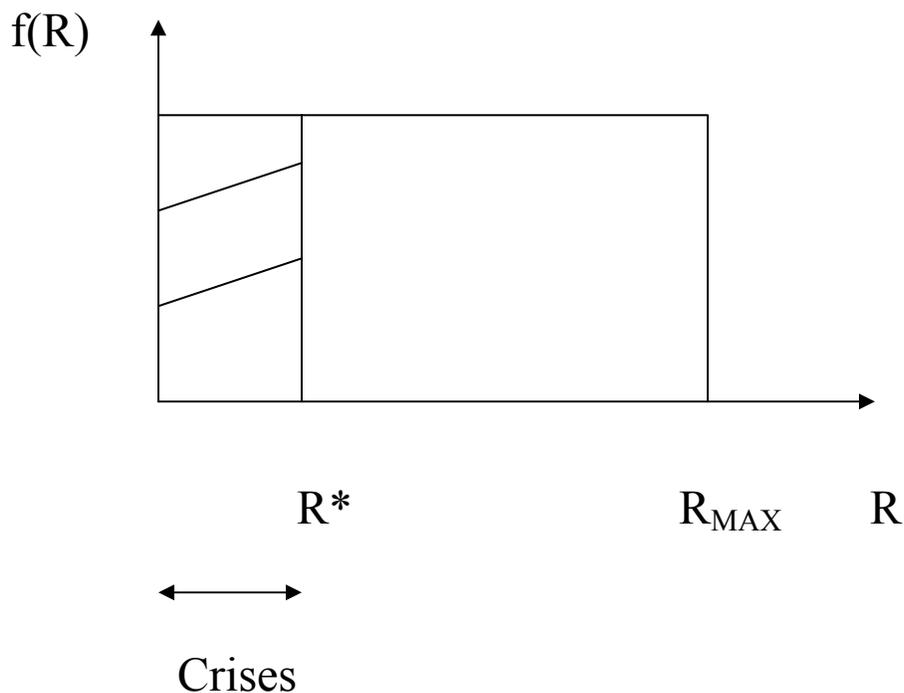
**Proposition 1:** There is a bubble in the intermediated equilibrium provided there is a positive probability of bankruptcy. In other words,  $P > \bar{P}$  provided  $\Pr[R \geq R^*] < 1$ .

The bubble occurs because of the risk shifting problem. In the intermediated equilibrium the only part of the distribution that is relevant for asset pricing is  $R \geq R^*$  whereas in the benchmark case the whole distribution is.

An increase in risk amplifies the bubble

**Proposition 2:** A mean preserving spread in  $R$  leads the bubble to be larger.

In addition to the risk shifting problem leading to a bubble, the other important thing to notice is that a financial crisis arises here when  $R < R^*$



The cause of a financial crisis in this version of the model is thus a *real event*

An example is the crisis that occurred in Norway in the mid 1980's when oil prices collapsed



There are a number reasons  $B_1$  can be random

- Central bank may only be able to control it imperfectly

But there may also be changes of

- policy preference
- government
- external environment

The asset substitution effect operates for financial risk in the same way it does for real risk. There is again a bubble.

- Dates 1 and 2 are similar to before except risky asset payoff at date 2 is  $\bar{R}$  with certainty so

$$P_1 = \frac{1}{f'(B_1 - P_1)} (\bar{R} - c'(1))$$

Hence  $P_1(B_1)$  is random and risk arises because of the uncertainty about  $B_1$

- Otherwise model is similar to before with overlapping generations of investors
- Similarly to (1) the safe asset drops out and the date 0 investor's decision problem is to choose  $X_{0R}$ :

$$\max_{X_{0R}} \int_{B_1^*}^{\infty} [P_1(B_1)X_{0R} - r_0 P_0 X_{0R}] k(B_1) dB_1 - c(X_{0R}) \quad (14)$$

where  $B_1^*$  is the level of credit at which default occurs:

$$P_1(B_1^*) = r_0 P_0 \quad (15)$$

**Equilibrium conditions:**

Market clearing requires:

$$X_{0R} = 1 \quad (16)$$

$$X_{0S} + P_0 = B_0 \quad (17)$$

$$r_0 = f'(X_{0S}) \quad (18)$$

**Equilibrium conditions:**

The first-order condition for (14) and (16)-(18) give

$$\int_{B_1^*}^{B_{1\text{MAX}}} [P_1(B_1) - r_0 P_0] k(B_1) dB_1 = c'(1) \quad (19)$$

$$r_0 = f'(B_1 - P_0)$$

$$P_1(B_1^*) = r_0 P_0$$

- The risk shifting effect operates for financial risk in the same way as it does for real risk and there is again a bubble.
- Similarly to before it can be shown that in the benchmark case

$$\bar{P}_0 = \frac{1}{r_0} (E[P_1(B_1)] - c'(1)) \quad (21)$$

The equivalent expression from (14) for the intermediated case is:

$$P_0 = \frac{1}{r_0} \left( \frac{\int_{B_1^*}^{B_{1\text{MAX}}} P_1(B_1) k(B_1) dB_1 - c'(1)}{\Pr[B_1 \geq B_1^*]} \right)$$

Comparing these gives:

**Proposition 3:** There is a bubble in the intermediated equilibrium provided there is a positive probability of bankruptcy. In other words,  $P_0 > \bar{P}_0$  provided  $\Pr[B_1 \geq B_1^*] < 1$ .

- The possibility of credit expansion over a period of years may create a great deal of uncertainty about how high the bubble may go and when it may collapse so the bubble may be large. This is particularly true when there is financial liberalization.
- A financial crisis with widespread default occurs whenever  $B_1 < B_1^*$ .
- What kind of dynamic credit policies, i.e. the sequence  $B_0, B_1$ , will lead to a financial crisis?
- Under what circumstances is this likely to occur?
- Is it simply necessary for the central bank to restrict credit to cause a financial crisis?
- This is the issue of *financial robustness and fragility* that we turn to next.

#### 4. Robustness and Fragility

- To illustrate circumstances in which a financial crisis is almost inevitable consider (19) again:

$$\int_{B_1^*}^{B_{1MAX}} [P_1(B_1) - r_0 P_0] k(B_1) dB_1 = c'(1) \quad (19)$$

**Proposition 4:** As  $c'(1) \rightarrow 0$  the default level  $B_1^* \rightarrow B_{1MAX}$  so credit expansion must not only be positive but close to the upper bound of the support of  $B_1$  to ensure a crisis is avoided.

- In general two regimes can be identified:

*Robust regime:* If the amount of credit stays the same or is increased asset prices are such that there is no financial crisis

*Fragile regime:* Unless the amount of credit grows at a sufficient rate people are unable to repay loans and a financial crisis occurs

**Example:**

$B_1$  is uniform on  $[0,2]$

$$B_0 = 1; f(X_S) = 4X_S^{0.5}; \bar{R} - c'(1) = 4$$

Now

$$P_1(B_1) = 2[(1 + B_1)^{0.5} - 1]$$

Varying  $c'(1)$  gives a number of cases of interest:

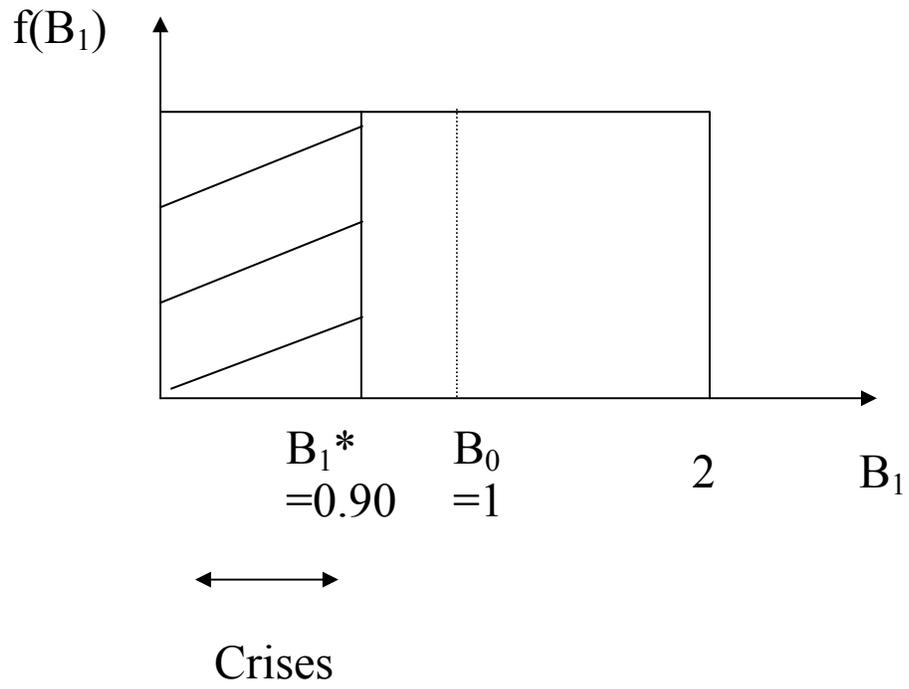
**Table 1**

<b><math>c'(1)</math></b>	<b><math>B_1^*</math></b>	<b>Prob. crisis</b>	<b>Inter. <math>P_0</math></b>	<b>Fund. <math>\bar{P}_0</math></b>	<b>Bubble <math>P_0 - \bar{P}_0</math></b>
0.2	0.90	0.45	0.31	0.25	0.06
0.1	1.21	0.61	0.38	0.27	0.11
0.01	1.74	0.87	0.47	0.29	0.18

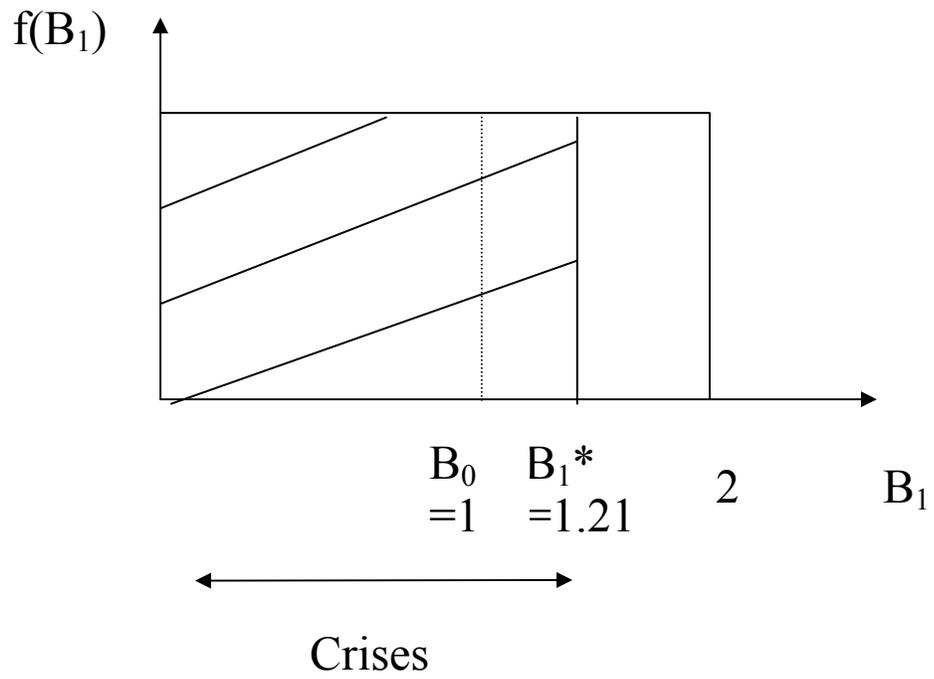
The first line illustrates a robust regime.

The second and third lines illustrate fragile regimes.

Robust case:



Fragile case:



## 5. Policy Issues

Allen and Gale (1999) discusses the policy issues

- Policies to prevent bubbles

The theory emphasizes the importance of the level and volatility of credit for asset price determination

Governments and central banks should try to avoid unnecessary expansion in the level of credit and uncertainty about the path of credit expansion

Financial liberalization is a particularly risky process

- Policies to minimize after-effects

Financial problems of the banking sector spill over into the real economy because of *debt overhang* and *inefficient liquidation*

Effective solution in

Norway

Ineffective solution in

Japan

## 6. Concluding remarks

- Bubbles and ensuing financial crises have a long history.
- We argue they can occur because investments are debt financed. This creates an asset substitution problem and leads to high prices for risky assets in fixed supply such as real estate and stocks.
- They can also occur because of agency problems between investors and intermediaries such as mutual funds (see Allen and Gorton (1994)).
- Central banks control the credit available to investors and hence the level of asset prices.
- In a fragile regime, the central bank must keep increasing the amount of credit or asset prices will not be high enough for loans to be repaid. There are limits to the amount credit can be increased. Eventually a crisis may occur and this can spill over into the real economy.
- Bubbles are likely to occur when the variance of returns is high and there are significant agency problems because of intermediation.
- Is the US currently in a bubble?

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