Moral Hazard

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Principal-Agent Problem

- Basic problem in corporate finance: separation of ownership and control:
  - The owners of the firm are typically not those who manage it on a daily basis.
  - Owners (principal) delegate tasks to managers (agent).
  - Yet, managers have their own objective function. They may not exert much effort, for example, because it is costly for them.

Compensation Contracts

- The way to solve the problem would be to write a contract that compensates the manager on the basis of his effort.
    - Yet, the effort is typically unobservable (**Hidden Action**).
- Hence, we write contracts that compensate the manager based on performance, which is a noisy signal of the manager's effort.
- This might be costly when the manager is risk averse, since extra compensation is needed for the risk taken.

Basic Setup (Mas-Colell, Whinston, Green, Ch. 14.B)

Technology and Effort

- π denotes the observable profit, e denotes the manager's effort.
- For simplicity, the manager has two possible efforts: $e_H > e_L$.
- The distribution of profits $f(\pi|e)$ depends on the level of effort:
  - The distribution conditional on e_H first-order stochastically dominates the one conditional on e_L: $F(\pi|e_H) \leq F(\pi|e_L)$ at all $\pi \in [\underline{\pi}, \overline{\pi}]$ with strict inequality on some open set.
  - As a result: $\int \pi f(\pi|e_H) d\pi > \int \pi f(\pi|e_L) d\pi$.

Preferences

- Manager maximizes utility function $u(w, e)$ over wage and effort.
  - $u_w(w, e) > 0$; $u_{ww}(w, e) \leq 0$; $u(w, e_H) < u(w, e_L)$.
- Concentrate on: $u(w, e) = v(w) - g(e)$.
  - $v'(w) > 0$; $v''(w) \leq 0$; $g(e_H) > g(e_L)$.
- The owner receives the profit minus the wage. We assume here that he is risk neutral, and thus tries to maximize his expected payoff.
- Assumption that manager is risk averse and owner is risk neutral can be justified by patterns of diversification.

Optimal Contract with Observable Effort

- A contract specifies effort level (e_H or e_L) and wage function $w(\pi)$.

- Owner solves following problem:

$$\max_{e \in \{e_L, e_H\}, w(\pi)} \int \big(\pi - w(\pi)\big) f(\pi|e) d\pi,$$

subject to **participation constraint** (always binding):

$$\text{s.t.} \int v\big(w(\pi)\big) f(\pi|e) d\pi - g(e) \geq \overline{u}$$

- Find $w(\pi)$ for a given e, and then find optimal e.

- Solution often referred to as **first-best**.

- Given e, owner's problem is equivalent to:

$$\min_{w(\pi)} \int w(\pi) f(\pi|e) d\pi,$$

$$\text{s.t.} \int v\big(w(\pi)\big) f(\pi|e) d\pi - g(e) \geq \bar{u}.$$

- Denoting the multiplier on the constraint as γ:

$$f(\pi|e) - \gamma v'\big(w(\pi)\big) f(\pi|e) = 0, \text{ and so: } \frac{1}{v'\big(w(\pi)\big)} = \gamma.$$

- Hence, when the manager is strictly risk averse, the owner offers a fixed compensation: **Risk sharing**.

- For effort level e, the owner offers w_e^*, such that $v(w_e^*) - g(e) = \overline{u}$.
- Then, the owner chooses e that maximizes:

$$\int \pi f(\pi|e)d\pi - v^{-1}\big(\overline{u} + g(e)\big).$$

- If $\int \pi f(\pi|e_H)d\pi - v^{-1}\big(\overline{u} + g(e_H)\big) > \int \pi f(\pi|e_L)d\pi - v^{-1}\big(\overline{u} + g(e_L)\big)$,
  - then $e = e_H$ and $w = v^{-1}\big(\overline{u} + g(e_H)\big)$,
  - otherwise, $e = e_L$ and $w = v^{-1}\big(\overline{u} + g(e_L)\big)$.
- With risk neutrality, same spirit, but fixed wage is not necessary.

Optimal Contract with Unobservable Effort: Risk Neutral Manager

- Suppose that $v(w) = w$.

- With observable effort, owner solves:

$$\max_{e \in \{e_L, e_H\}} \int \pi f(\pi|e) d\pi - \overline{u} - g(e).$$

- A basic result is that the owner can achieve the same value with a compensation contract when effort is unobservable.

- This contract then must be optimal because the owner cannot do better under unobservable effort than under observable effort.

- Consider a compensation schedule of the form: $w(\pi) = \pi - \alpha$.
    - This is effectively like selling the project to the manager for α.
- The manager then chooses e to maximize:

$$\int \pi f(\pi|e)d\pi - \alpha - g(e).$$

And thus chooses the same e^* as in the first-best solution.

- Setting $\alpha = \int \pi f(\pi|e^*)d\pi - \overline{u} - g(e^*)$, will then give the owner the same value as in the first-best solution.
- When risk is not a problem, it is easy to incentivize the manager.

Optimal Contract with Unobservable Effort: Risk Averse Manager

- As before, find $w(\pi)$ for the level e that we choose to implement, and then find optimal e.

- The owner solves:

$$\min_{w(\pi)} \int w(\pi) f(\pi|e) d\pi,$$

$$\text{s.t. (1)} \int v\big(w(\pi)\big) f(\pi|e) d\pi - g(e) \geq \bar{u},$$

$$\text{(2) } e \text{ solves } \max_{\tilde{e}} \int v\big(w(\pi)\big) f(\pi|\tilde{e}) d\pi - g(\tilde{e}),$$

where (2) is the **incentive compatibility constraint**, ensuring that the manager chooses the right level of effort.

- Suppose that the first-best level of effort (achieved under observable effort) is e_L:

$$\int \pi f(\pi|e_H)d\pi - v^{-1}\big(\overline{u} + g(e_H)\big) < \int \pi f(\pi|e_L)d\pi - v^{-1}\big(\overline{u} + g(e_L)\big).$$

  - The owner can implement e_L in exactly the same way as he did when effort was observable, that is, by paying the manager a fixed wage: $w = v^{-1}\big(\overline{u} + g(e_L)\big)$.

  - Since the manager's wage does not depend on his performance, he always chooses the low effort, and e_L is implemented.

  - So, when e_L is first-best, non-observability of effort is costless.

- Suppose that the first-best level of effort is e_H:

$$\int \pi f(\pi|e_H)d\pi - v^{-1}\big(\overline{u} + g(e_H)\big) > \int \pi f(\pi|e_L)d\pi - v^{-1}\big(\overline{u} + g(e_L)\big).$$

  - Implementing e_H implies incentive compatibility constraint:

$$\int v\big(w(\pi)\big)f(\pi|e_H)d\pi - g(e_H) \geq \int v\big(w(\pi)\big)f(\pi|e_L)d\pi - g(e_L).$$

  - Denoting the multipliers on the participation and incentive constraints as γ and μ, respectively:

$$f(\pi|e_H) - \gamma v'\big(w(\pi)\big)f(\pi|e_H)$$

$$-\mu[f(\pi|e_H) - f(\pi|e_L)]v'\big(w(\pi)\big) = 0.$$

- Hence, the condition becomes:

$$\frac{1}{v'\big(w(\pi)\big)} = \gamma + \mu\left[1 - \frac{f(\pi|e_L)}{f(\pi|e_H)}\right].$$

- It is straightforward to show that γ and μ must be strictly positive in any solution, and thus both the participation constraint and the incentive constraint are binding.
  - If $\gamma = 0$, condition is violated for π where $\frac{f(\pi|e_L)}{f(\pi|e_H)} > 1$.
  - If $\mu = 0$, wage must be constant, but then there is no way to implement e_H.

Implementing High Effort with Risk Aversion: Insights and Results

- Consider fixed $\hat{w}$: $\frac{1}{v'(\hat{w})} = \gamma$ (optimal without incentive constraints).
  - $w(\pi) > \hat{w}$ if $\frac{f(\pi|e_L)}{f(\pi|e_H)} < 1$.
  - $w(\pi) < \hat{w}$ if $\frac{f(\pi|e_L)}{f(\pi|e_H)} > 1$.
  - The optimal contract compensates the manager more at profit realizations that are statistically more likely with high effort.
  - The gap between $f(\pi|e_L)$ and $f(\pi|e_H)$ determines the extent of deviation from fixed wage.

- For the compensation contract to be monotonically increasing in π, $\frac{f(\pi|e_L)}{f(\pi|e_H)}$ has to be decreasing in π.
  - This condition is called the **monotone likelihood ratio property** (MLRP). It implies that high profits are relatively more likely with high effort than low profits.
  - It is not guaranteed by first-order stochastic dominance.
  - Hence, non-monotone compensation contracts are possible in this model. Compensation contracts here are complicated.

- Given the variability in wages, the expected wage here is higher than under observable effort (where wage is fixed).
  - Formally,
    - Under observable effort, the wage is $v^{-1}\left(\bar{u} + g(e_H)\right)$.
    - Here, $E\left[v\left(w(\pi)\right)\right] = \bar{u} + g(e_H)$.
    - By Jensen's inequality, because $v'' < 0$,

$$v(E[w(\pi)]) > E\left[v\left(w(\pi)\right)\right] = \bar{u} + g(e_H).$$

$$\Rightarrow E[w(\pi)] > v^{-1}\left(\bar{u} + g(e_H)\right).$$

- The main conclusion is that providing incentives to a risk averse manager to choose high effort when effort is unobservable is costly.
  - If the owner chooses to implement the high effort when effort is unobservable, he pays more than when effort is observable.
  - Given that the manager always gets his reservation utility, the solution under unobservable effort is always inferior to that under observable effort.
    - The owner gets a lower utility, and the manager gets the same utility.

- In some cases, moving to unobservable effort will be so costly that it will lead to a shift from high to low effort.
    - The owner picks the level of effort to implement by comparing the difference in expected profits between high effort and low effort with the difference in the associated compensation cost.
    - Relative to the case of observable effort, nothing is changed except that the cost of wage to implement high effort increases.
    - Hence, it is possible that due to the non-observability of effort, there will be a shift from high to low effort.

- Important:
    - Non-observability of effort is a problem even though in equilibrium the principal knows exactly what effort the agent is choosing.
        - He designs a contract that ensures that the agent is choosing a particular level of effort.
    - Yet, the level of effort is not observable and cannot be contracted upon.
    - The contract has to ensure it will be desirable and this is costly.

Additional Information

- The analysis demonstrates that non-observability of effort is costly.
- Since effort is generally believed to be impossible to observe in most settings, the analysis goes on to consider other signals and their ability to improve the allocation of profits and risks.
- Suppose that in addition to π, both parties observe a signal y. The condition becomes:

$$\frac{1}{v'\big(w(\pi,y)\big)} = \gamma + \mu\left[1 - \frac{f(\pi,y|e_L)}{f(\pi,y|e_H)}\right].$$

- $\frac{f(\pi,y|e_L)}{f(\pi,y|e_H)}$ may now change with *y*. Hence, for the same level of profit π, the agent may receive different wages for different levels of *y*.
  - The observation of *y* provides more information about whether the agent chose the desired action, and thus conditioning the wage on *y* helps provide incentive without harming risk sharing.
  - For example, *y* can represent average profits in the industry, which generate changes in π that are beyond the control of the manager, and thus changes in π that are associated with changes in *y* should not affect wages much.

- When exactly will wages depend on y in addition to π?
  - When π is not a **sufficient statistic** for y with respect to e.
  - Formally, we can write:

$$f(\pi, y|e) = f_1(\pi|e) f_2(y|\pi, e).$$

  - When $f_2(y|\pi, e)$ does not depend on e (π is a sufficient statistic) it will cancel out, and the wage will not depend on y.
  - When y doesn't add information on the effort (e.g., it is pure noise), there is no reason to condition w on it and add variation.

Holmstrom (1979)

- The classic paper formalizing these ideas and others is by Holmstrom (1979).
  - His work extends or builds on earlier work by Mirrlees (1976), Harris and Raviv (1979), and others.
  - His formulation of the problem is more general than that considered above, allowing for risk aversion on the principal side and for continuous choice of effort.
  - Let us review his basic setup.

Technology and Preferences

- Agent can take an action $a \in A$, which together with a random state of nature θ determines profit $x(a, \theta)$.

- The principal's utility function $G(w)$ is defined over wealth, and the agent's utility function is defined over wealth and action: $H(w, a) = U(w) - V(a)$.

- Assumptions about the functions:

  - $U' > 0, G' > 0, U'' < 0, G'' \leq 0, V' > 0, x_a > 0$.

Solving for Contract

- Let $s(x)$ denote the share of x that goes to the agent, and $r(x) = x - s(x)$ the share that goes to the principal.

- Then, $s(x)$ and a are the result of the following constrained optimization problem:

$$\max_{s(x),a} E\big[G\big(x - s(x)\big)\big],$$

$$\text{s.t. } E[H(s(x), a)] \geq \overline{H},$$

$$a \in \operatorname{argmax}_{a' \in A} E[H(s(x), a')].$$

- The condition for optimal contract that emerges from the system is:

$$\frac{G'\big(x - s(x)\big)}{U'\big(s(x)\big)} = \gamma + \mu \frac{f_a(x,a)}{f(x,a)}$$

- This condition is equivalent to what we had before, except:
    - $\frac{G'(x-s(x))}{U'(s(x))}$ stands for the ratio of marginal utilities that without incentive constraints (in first-best world) should be fixed.
    - $\frac{f_a(x,a)}{f(x,a)}$ is the continuous version of the incentive term. When it is high, $s(x)$ is high to provide more incentive. The MLRP implies that the ratio is increasing in *x*.

Moral Hazard in Teams: Holmstrom (1982)

- Another key problem in corporate finance and firm organization stems from the fact that output is produced by a group of agents, and only the joint output (not individual efforts) is observed.
    - Even if agents are being compensated on the basis of the observed output (which, as we learned before, is a crucial feature for incentive provision), they will **free ride** on others' efforts.
    - Put simply, when exerting effort, agents bear its full cost, but only share the resulting output, and thus tend to put too little.

- Unlike in the principal-agent problem with one agent, moral hazard arises here even if output is certain (based on effort).
    - Even when output perfectly reveals the group's aggregate effort, agents still want to free ride on each other.
- Interestingly, introducing a principal in the team setting will be part of the solution.
    - It will serve to break the **budget-balancing** constraint and enable deviation from sharing the total output among the team members.

Model Setup

- There are n agents. Each agent i takes a non-observable action a_i, which generates a private cost $v_i(a_i)$: $v_i' > 0$, $v_i'' > 0$, $v_i(0) = 0$.

- Denote $a = (a_1, \dots, a_n)$, and $a_{-i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots a_n)$.

- Agents' actions determine the joint monetary output $x(a)$, which is strictly increasing and concave, and where $x(0) = 0$.

- Agent i receives share $s_i(x)$ of the output, and the utility function over compensation and action is $u_i(m_i, a_i) = m_i - v_i(a_i)$.

Pareto Optimality

- Suppose that a social planner can decide on the efforts of all agents in the team.

- He will set $a = a^*$ to maximize total surplus:

$$a^* = \underset{a}{\operatorname{argmax}} \left[x(a) - \sum_{i=1}^{n} v_i(a_i) \right]$$

- This implies that for every i:

$$x_i' - v_i' = 0.$$

  - The marginal benefit of every effort equals its marginal cost.

Non-cooperative Game

- However, in reality agents choose their own levels of effort, and they do it without knowing what choices other agents make.
- To analyze the result of this interaction, we need to apply the tools from **game theory**. In particular, we will use the **Nash equilibrium concept**:
    - Every agent chooses his action to maximize his utility, given his belief about other agents' actions.
    - All beliefs about other agents' actions are correct in equilibrium.

- Formally, agent *i* solves:

$$\max_{a_i} \left[s_i \left(x\big((a_i, a_{-i})\big) \right) - v_i(a_i) \right],$$

  - and a_{-i} is the equilibrium strategy of other agents.

- Assuming a differentiable sharing rule (for simplicity, you can think about each agent getting a fixed share of *x*), this implies:

$$s_i' x_i' - v_i' = 0.$$

  - The marginal benefit that the effort yields for the agent *himself* (considering his share of the profit) equals the marginal cost.

Balanced Budget

- The choices of efforts in equilibrium depend on the sharing rule.
- A natural benchmark to consider is that of a balanced budget, where the profit x is fully allocated among the agents:

$$\sum_{i=1}^{n} s_i(x) = x, \text{ for all } x.$$

  - This would be the case in a partnership.
- A striking result is that under a balanced budget, Pareto optimal production cannot be achieved in equilibrium.

- For Pareto optimality to hold in equilibrium, we require that

$$s_i' = 1 \text{ for all } i.$$

  - But under balanced budget:

$$\sum_{i=1}^{n} s_i' = 1.$$

  - Under balanced budget, when the agent shirks, he saves the full cost, and loses only a share in the profit. For efficiency, we need to penalize all agents for the full consequence of their decision, but this is impossible when they always fully share the output.

- The proof also extends to non-differentiable sharing rules.

Non-Balanced Budget

- Consider the following sharing rule:

$$s_i(x) = \begin{cases} b_i & if \quad x \geq x(a^*) \\ 0 & if \quad x < x(a^*) \end{cases},$$

where $\sum_i b_i = x(a^*)$, and $b_i > v_i(a_i^*)$.

  - Here, there is no balanced budget because when $x < x(a^*)$, the output does not go to the team members.

- Then, there is a Nash equilibrium, where all agents choose a_i^*, and the Pareto optimal outcome is obtained.

- Under the belief that $a_{-i} = a^*_{-i}$, agent i knows that:
  - If he chooses a_i^* he gets $b_i - v_i(a_i^*)$.
  - If he chooses $a_i > a_i^*$, he gets $b_i - v_i(a_i) < b_i - v_i(a_i^*)$.
  - If he chooses $a_i < a_i^*$, he gets $0 < b_i - v_i(a_i^*)$.
  - Hence, it is optimal for the agent to choose a_i^*.
- The deviation from a balanced budget enabled us to make agents internalize the consequence of deviating from a_i^*. The aggregate penalty is severe enough to deter agents from free riding.

- Of course, key for this to work is the ability to commit to throw away output when $x < x(a^*)$.
  - This is where the role of the principal comes up. He stands ready to penalize for output decrease (which will not happen).
- Note that this is not the only Nash equilibrium with the above sharing rule. There is an equilibrium where all agents put 0 effort.
  - Expecting 0 effort from others, agents know that with reasonable levels of effort, they will not get more than 0 compensation.
  - This equilibrium is worse than under a balanced budget.

Introducing Uncertainty

- Assume that x depends also on the realization of some state of nature, so that the density function $f(x,a)$ summarizes the distribution of x given agents' actions a.

- Denote $E_i(a)$, as the effect of agent i's effort on the expected output: $\partial Ex(a)/\partial a_i$. Then, a^* being Pareto optimal implies that for every i:

$$E_i(a^*) - v_i'(a^*) = 0.$$

- We can again implement this with a sharing rule that has a group penalty when x falls below a certain threshold.

- Consider the following sharing rule:

$$s_i(x) = \begin{cases} s_i x & if \quad x \geq \overline{x} \\ s_i x - k_i & if \quad x < \overline{x} \end{cases},$$

where $\sum_i s_i = 1$, and $k_i > 0$.

- To ensure a^* as an equilibrium solution, we need that for every *i*:

$$s_i E_i(a^*) - k_i F_i(\overline{x}, a^*) - v_i'(a^*) = 0,$$

where $F_i(\overline{x}, a^*)$ is the effect of agent *i*'s effort on the probability that output falls below the threshold $\overline{x}$.

- This is achieved by setting $k_i = \left(s_i E_i(a^*) - v_i'(a^*)\right)/F_i(\overline{x}, a^*)$.
- This is not perfect, however, because even with a^*, there is a chance that output will fall below $\overline{x}$, generating loss of value.
    - The amount of lost value is:

$$\sum_i k_i F(\overline{x}, a^*) = \sum_i \left(s_i E_i(a^*) - v_i'(a^*)\right) \frac{F(\overline{x}, a^*)}{F_i(\overline{x}, a^*)}.$$

- Assuming that $\frac{F(\overline{x}, a^*)}{F_i(\overline{x}, a^*)}$ approaches 0 as x approaches its lower bound, we can guarantee that the payoff scheme replicates the Pareto efficient allocation almost perfectly.

- Of course, the problem here is that we might need agents to pay large penalties k_i when output falls below the threshold.
    - This might not be feasible when they have limited endowment.
- There is a parallel solution that involves the principal paying bonuses when output goes above a certain threshold.
    - If the principal has deep enough pockets, there is no problem of feasibility.
    - Of course, with parallel condition, we can guarantee that the expected amount he has to pay is low.

Multitasking: Holmstrom and Milgrom (1991)

- The basic theory of moral hazard suggests that compensation should depend strongly on performance in order to create incentives.
- In the real world, things are more complicated:
    - Agents are often required to perform multiple tasks, or make effort in multiple dimensions.
    - Tasks are related, being complements or substitutes.
    - Success in some tasks can be easily measured and in others not.

- For example:
  - Assistant professors are expected to produce a large volume of research that will have a long lasting impact.
  - Volume is easily measured, but long-term impact is not.
  - Thinking in terms of the one-dimensional model, we would think it is a good idea to compensate assistant professors based on their volume of research.
  - But, this might divert their effort from making long-term impact. Hence, performance-based compensation might be a mistake.

Basic Setup (Salanie (1998), Ch. 5)

- There is one agent, who decides on a vector a of efforts. For simplicity, assume two effort decisions: a_1 and a_2.

- Effort generates a private cost $C(a_1, a_2)$, which is strictly convex.

  - C can be negative for a non-financial benefit (if this is different than the benefit to the principal, an agency problem exists).

- There is an output vector x that goes to the risk-neutral principal:

$$\begin{cases} x_1 = a_1 + \epsilon_1 \\ x_2 = a_2 + \epsilon_2 \end{cases}.$$

- The noise in the production function is distributed as follows:

$$\begin{pmatrix}\epsilon_1 \\ \epsilon_2\end{pmatrix} \sim N\left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \Sigma = \begin{bmatrix}\sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2\end{bmatrix}\right).$$

- The principal pays wage $w(x_1, x_2)$ to the agent. Based on results from Holmstrom and Milgrom (1987), we focus on linear form:

$$w(x_1, x_2) = \alpha' x + \beta = \alpha_1 x_1 + \alpha_2 x_2 + \beta.$$

- The agent has **CARA utility** over wage and effort:

$$-exp\Big(-r\big(w(x_1, x_2) - C(a_1, a_2)\big)\Big).$$

Solving for the Contract

- Under this contract, the principal's expected profit is:

$$a_1 + a_2 - \alpha_1 a_1 - \alpha_2 a_2 - \beta.$$

- The agent's **certainty equivalent** is:

$$\alpha_1 a_1 + \alpha_2 a_2 + \beta - C(a_1, a_2) - \frac{r}{2}\alpha\prime\Sigma\alpha.$$

- The principal's problem is to maximize the sum of the certainty equivalents – the output from effort minus the costs of effort minus the cost of risk to the agent – subject to the incentive constraint.

- Formally,

$$\max_{a_1,a_2} a_1 + a_2 - C(a_1, a_2) - \tfrac{r}{2}\alpha\prime\Sigma\alpha,$$

$$\text{s.t. } (a_1, a_2) \in \operatorname{argmax}_{a_1',a_2'} \alpha_1 a_1' + \alpha_2 a_2' - C(a_1, a_2).$$

- From the incentive constraint, we get that for $i = 1{,}2$:

$$\alpha_i = C_i(a_1, a_2)$$

- By differentiating the objective function with respect to a_i, we get:

$$1 - C_i(a_1, a_2) - r\alpha'\Sigma\tfrac{\partial\alpha}{\partial a_i} = 0.$$

- Plugging the incentive constraint inside the first-order condition:

$$1 - \alpha_i - r\alpha'\Sigma\frac{\partial \alpha}{\partial a_i} = 0.$$

- From the incentive constraint, we can express $\frac{\partial \alpha}{\partial a}$ as:

$$\frac{\partial \alpha}{\partial a} = C'' \equiv \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix}.$$

- Then, with a little algebraic manipulation, we get:

$$\alpha = (I + rC''\Sigma)^{-1} \binom{1}{1}.$$

- This tells us the sensitivity of pay to performance in different tasks.

Optimal Contract: Results and Insights

- As a benchmark, consider the case where the two tasks are completely independent.
  - o This will happen when $C'' = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$.
  - o Plugging this in the expression for α, we get the traditional results from a model of one-dimensional principal-agent model:

$$\text{For each } i,\ \alpha_i = \frac{1}{1 + rC_{ii}\sigma_i^2}.$$

- The sensitivity to performance on a task depends only on the parameters of this task.
  - Sensitivity decreases in risk aversion.
  - Sensitivity decreases in noisiness of the link between output and effort: with a noisy process, sensitivity adds too much ris to the agent.
  - Sensitivity decreases in the how fast costs rise with effort: Fast rising costs will cause the agent to respond less to incentives, and hence it is optimal to reduce them.

- Now, let us consider the example that motivated us:
  - The two tasks are related to each other in their cost structure.
  - One task can be easily measured and the other one not.
  - Formally, $C'' = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \infty \end{bmatrix}$.
  - Plugging this in the expression for α, and after some algebra (remember l'Hopital's rule…) we get:

$$\alpha_1 = \frac{1 - C_{12}/C_{22}}{1 + r\sigma_1^2 (C_{11} - (C_{12})^2/C_{22})},\ \alpha_2 = 0.$$

- We can now see very clearly how the interaction between the two tasks affects the compensation structure.
  - When the tasks are substitutes, putting effort in one task increases the marginal cost of the other, and thus, $C_{12} > 0$.
  - Then, we want to give the agent a lower incentive to put an effort in one task even when it is easily measurable. This is because we do not want to deter him from putting effort in the other task, which cannot be measured and thus directly motivated.

- Interestingly, the incentive to perform the task can even be negative.

- But, when the tasks are complements, $C_{12} < 0$, the opposite happens. Then, the agent will be motivated very strongly to perform a task in hope that this will lead to better outcomes in an immeasurable task.

o It should be noted that the results would weaken if the other task can be measured as well, since then incentives for the other task can be settled directly.

- This theory can shed light on some empirical phenomena that cannot be understood in the one-dimensional model.
    - It provides an explanation for why many employees do not receive performance-based pay, or are provided rather weak incentives.
        - Compensating teachers based on students’ test scores will deter them from promoting higher-level thinking.
    - It can open some discussion on the boundaries of the firm, which are taken as given in traditional agency models.

- If a firm hires the services of a non-employee, the market might give him too strong incentives to perform one task that will decrease the incentive to perform another. By hiring him and putting restrictions on working outside the firm, the firm can make sure that incentives for the other activity are weak.

- It sheds light on why some important dimensions are often left outside a contract.
- Providing explicit incentive to a contractor to finish the work quickly will divert effort from providing quality.