

OPTIMAL DEPOSIT INSURANCE*

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Abstract

This paper studies the optimal determination of deposit insurance when bank runs are possible. We show that the welfare impact of changes in the level of deposit insurance coverage can be generally expressed in terms of a small number of sufficient statistics, which include the level of losses in specific scenarios and the probability of bank failure. We characterize the wedges that determine the optimal ex-ante regulation, which map to asset- and liability-side regulation. We demonstrate how to employ our framework in an application to the most recent change in coverage in the United States, which took place in 2008.

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1 Introduction

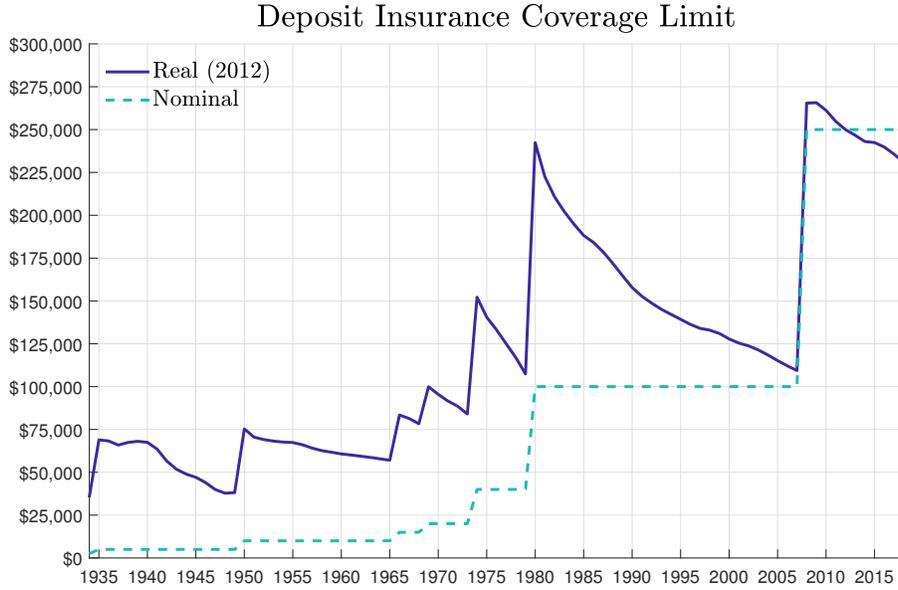
Bank failures have been a recurrent phenomenon in the United States and many other countries throughout modern history. A sharp change in the United States banking system occurred with the introduction of federal deposit insurance in 1934, which dramatically reduced the number of bank failures. For reference, more than 13,000 banks failed between 1921 and 1933, of which 4,000 banks failed in 1933 alone. In contrast, a total of 4,057 banks failed in the United States between 1934 and 2014.¹ In many other countries, the design of deposit insurance schemes is still in progress and is the subject of ongoing debates; see, e.g., Demirgüç-Kunt, Kane and Laeven (2014) for a recent account of deposit insurance systems around the world. As of today, deposit insurance is a crucial pillar of financial regulation in most economies and represents the most salient explicit government guarantee to the financial sector.

Despite its success in reducing bank failures, deposit insurance entails costs when it has to be paid and affects the ex-ante behavior of market participants — these responses to the policy are often referred to as moral hazard. Hence, in practice, deposit insurance only guarantees a fixed level of deposits. As shown in Figure 1, this level of coverage has changed over time in the US. Starting from the original \$2,500 in 1934, the nominal insured limit per account in the US has been \$250,000 since October 2008. A natural question to ask is how the level of this guarantee should be determined to maximize social welfare. In particular, what is the optimal level of deposit insurance coverage? Are \$250,000, the current value in the US, and €100,000, the current value in most European countries, the optimal levels of deposit insurance coverage for these economies? How should emerging economies set their insured limits? Which variables ought to be measured to optimally determine the level of deposit insurance coverage in a given economy?

This paper provides an analytical characterization, written as a function of observable or potentially recoverable variables, which directly addresses these questions. Although existing research has been effective at understanding several of the theoretical tradeoffs associated with deposit insurance, a general framework that incorporates the most relevant tradeoffs and that can be used to provide explicit guidance to policymakers when facing these questions has been missing. With this paper, we provide a first step in that direction.

We initially derive the main results of the paper in a version of the canonical model of bank runs of Diamond and Dybvig (1983), augmented to consider depositors who hold different levels of deposits. In our framework, banks offer a predetermined interest rate on a deposit contract to share risks between early and late depositors in an environment with aggregate uncertainty about the profitability of banks' investments. Due to the demandable nature of the deposit contract, depending on the aggregate state, both fundamental-based and panic-based bank failures are possible. Mimicking actual deposit insurance arrangements, we assume that deposits are guaranteed by the government up to a deposit insurance limit of δ dollars and then focus on the implications for social welfare of varying the level of coverage δ . We also assume that any funding shortfall associated with deposit insurance payments entails a distortionary

¹These values come from the FDIC Historical Statistics on Banking. Weighting bank failures by the level of banks' assets or correcting by the total number of banks still generates a significant discontinuity on the level of bank failures after the introduction of deposit insurance.



Note: Figure 1 shows the evolution of the level of deposit insurance coverage between 1934 and 2018 in nominal and real terms. Nominal values are from the FDIC. Real values are reported in 2012 dollars using a consumption expenditure deflator.

Figure 1: Evolution of Deposit Insurance Coverage Limit

fiscal cost.

After characterizing how changes in the level of coverage δ affect equilibrium outcomes, in particular depositors' withdrawal choices and bank failure probabilities, we focus on the welfare implications of varying δ . We initially characterize the marginal welfare change of varying the level of deposit insurance coverage, which provides an exact test for whether it is desirable to increase or decrease the level of coverage. In order to implement this test in practice, a policymaker would need detailed information on individual deposit balances and consumption across different scenarios. While gathering this information is conceivable, the informational requirements on the policymaker would be large. To make our results more applicable, we provide an approximate characterization of the marginal welfare change that can be expressed in terms of a few sufficient statistics that can be constructed relying exclusively on bank-level aggregates. The approximation of the marginal welfare change of varying the level of deposit insurance coverage takes the form

$$\begin{aligned} \frac{dW}{d\delta} = & -\text{Sensitivity of bank failure probability to an increase in } \delta \times \text{Consumption gain of preventing marginal failure} \\ & -\text{Probability of bank failure} \times \text{Expected marginal social cost of intervention.} \end{aligned} \tag{1}$$

Equation (1) embeds the fundamental tradeoffs regarding the optimal determination of deposit insurance. On the one hand, when a marginal change in δ substantially reduces the likelihood of bank failure at the same time that there are significant gains from avoiding a marginal bank failure, it is optimal to increase the level of coverage. On the other hand, when bank failures are frequent and when the social cost of ex-post intervention associated with them — for instance, when it is very costly to raise resources through distortionary taxation — is substantial, it is optimal to decrease the level of coverage.

Our formulation in terms of sufficient statistics is appealing for three reasons. First, conceptually, we

show that the same characterization of the marginal welfare impact of a change in the level of coverage is valid for a large set of primitives. In that sense, the high-level variables that we identify are not specific to a particular set of modeling assumptions. Second, in practice, it is possible to directly infer or recover the different elements that determine $\frac{dW}{d\delta}$ using aggregate information at the bank level. By directly measuring the variables in Equation (1), our framework provides direct guidance to policymakers regarding which variables ought to be measured to determine the optimal level of deposit insurance. Once the relevant variables are known, the policymaker does not need any other information to consider changes in the level of coverage, at least locally. Third, within a structural model, the sufficient statistics that we identify can be used as calibration targets or as intermediate outcomes that shed light on the connection between primitives and welfare assessments.

Our characterization can also be used to derive several analytical insights. In particular, we show that in an environment in which banks never fail and government intervention is never required in equilibrium, it is optimal to guarantee deposits fully. This result, which revisits the classic finding by Diamond and Dybvig (1983), follows from Equation (1) when the probability of bank failure tends towards zero. We also describe the conditions under which a zero, non-zero, or a maximal level of coverage are optimal.

Moreover, we discuss how to use our framework to understand the role played by alternative arrangements of mutual insurance either across banks or between banks and other agents/institutions in a *laissez-faire* setup. We consider two benchmarks. First, we study the case in which bank failures are idiosyncratic. In this case, we show that it may be possible to set up ex-post transfers across banks that eliminate funding shortfalls, by transferring funds from surviving to failed banks. In terms of the sufficient statistics that we identify, we show that this impacts the expected marginal cost of intervention, defined above. Second, we study the case in which bank failures are system-wide, which is effectively the case considered throughout the paper. In this case, we explore the impact of having a deposit insurance fund or having access to a third party that acts as insurer, and provide conditions under which our characterization of welfare effects in terms of sufficient statistics remains valid.

Although we initially derive our results when banks' deposit rates are predetermined, we also study the scenarios in which banks face no ex-ante regulation or perfect ex-ante regulation. First, we show that the changes in the behavior of unregulated competitive banks in response to the policy (often referred to as moral hazard) only modify the optimal policy formula directly through a fiscal externality caused by banks.² Next, we use our framework to explore the optimal ex-ante regulation, which in practice corresponds to optimally setting deposit insurance premia or deposit rate regulations. In particular, we show that the optimal ex-ante regulation, which requires jointly restricting banks' asset and liability choices, is designed so that banks internalize the fiscal externalities of their actions. We characterize the wedges that banks must face when the optimal ex-ante regulation is implemented, sharply distinguishing

²We use the term fiscal externality to refer to the social resource cost associated with the need to raise funds through distortionary taxation, as in the public finance literature. This result does not contradict common wisdom, which emphasizes the role of moral hazard as the primary welfare loss created by having a deposit insurance system. Our results simply show that the changes in banks' behavior associated with changes in the level of coverage are subsumed into the sufficient statistics that we identify. In other words, even though high levels of coverage can induce unregulated banks to make decisions that will increase the likelihood and severity of bank failures, only their effects through the fiscal externality that we identify have a first-order impact on welfare.

between the corrective and revenue-raising roles of ex-ante regulations. In practice, our results imply that deposit insurance premia, even if optimally determined, are not sufficient when banks can adjust their asset allocation, so regulating banks' asset allocations is necessary. Our results also imply that fairly-priced deposit insurance is neither necessary nor sufficient for the optimal regulation. Note that our results with optimal regulation can be interpreted as implementing a laissez-faire co-insurance outcome between banks and a set of outside agents, rather than relying on markets to do so.

Next, we demonstrate how to employ our framework in an application to the most recent change in deposit insurance coverage in the US, which took place in 2008. We describe how a policymaker, armed with our framework, would have set the optimal level of coverage in early 2008, sometime before the moment in which the change in coverage took place. Our quantitative application features two complementary approaches.

First, we provide direct measures of the sufficient statistics that we identify and implement the test that determines whether it is optimal to increase or decrease coverage. This approach has the advantage of sidestepping the need to specify model parameters and functional forms. Using the best empirical counterparts of the sufficient statistics that we can construct, we explain why our test finds that an increase in the level of coverage was desirable and discuss the associated welfare gains.

Second, using the sufficient statistics that we identify — along with additional information — as calibration targets for our structural model, we explore the quantitative results that the model generates. We draw four main conclusions. First, we find that the welfare gains from increasing the level of coverage when starting from low levels of coverage are very large. This result implies that having some form of deposit insurance is highly valuable. This should not be surprising, given that arguably no other financial regulation has had a more significant impact than the introduction of deposit insurance. Second, given our assumptions, we find that the optimal level of coverage in the scenario that we consider would have been \$381,000. This magnitude is larger than the \$250,000 that was chosen, but is perhaps more aligned with the extended guarantees that were implemented soon after. Third, we explain why a drop in confidence (modeled as an increase in the probability of a sunspot) is associated with a higher optimal level of coverage. We also explain why an increase in the riskiness of bank investments is associated with a lower optimal level of coverage. Lastly, we find that increasing the level of coverage increases the welfare of most depositors most of the time but not always.³ In particular, there are situations in which large depositors may be worse off when the level of coverage increases.

Finally, we explain how our framework accommodates additional features relevant for the determination of deposit insurance. We formally show how the sufficient statistics of the baseline model continue to be valid exactly or suitably modified once we allow for i) depositors with a consumption-savings decision and portfolio decisions, ii) banks that face an arbitrary set of investment opportunities with different liquidity and return properties, iii) alternative equilibrium selection mechanisms (e.g., global games), and iv) spillovers among banks. Lastly, we discuss how to integrate additional channels within our framework.

³Note that we model depositors and taxpayers as separate groups. We also show that the welfare losses for taxpayers are non-monotonic in the level of coverage.

Related Literature This paper is directly related to the well-developed literature on financial fragility, banking, and bank runs that follows Diamond and Dybvig (1983), which includes contributions by Allen and Gale (1998), Rochet and Vives (2004), Goldstein and Pauzner (2005), Uhlig (2010), Keister (2016), and Fernandez-Villaverde et al. (2021), among others. As originally pointed out by Diamond and Dybvig (1983), bank runs can be prevented by either modifying the trading structure, in particular by suspending convertibility, or by introducing deposit insurance. Both ideas have been further developed ever since. A sizable literature on mechanism design, including Peck and Shell (2003), Green and Lin (2003), and Ennis and Keister (2009), among others, has focused on the optimal design of contracts to prevent runs. Schilling (2018) has recently studied the optimal delay of bank resolution. Instead, taking the contracts used as a primitive, we focus on the optimal determination of the deposit insurance limit, a policy measure implemented in most modern economies.

Purely from a theoretical perspective, our paper expands on previous work by developing a new tractable framework with a rich cross-section of depositors. Allowing for depositors with different deposit balances turns out to be a key element for studying the optimal level of deposit insurance coverage, since changes in the level of coverage vary the composition of the set of fully insured depositors at the margin. Along this dimension, the recent work of Mitkov (2020) is the most closely related — see also Cooper and Kempf (2016). Building on the framework of Keister (2016), Mitkov (2020) studies the optimal ex-post government response (bailouts) to banking failures, relating inequality to financial fragility. While his focus is different (ex-post bailouts), his work also features a non-trivial distribution of deposit sizes and a cost of public funds that determines the size of the intervention. We connect our results to his at different points in the paper.

The papers by Merton (1977), Kareken and Wallace (1978), Pennacchi (1987, 2006), Chan, Greenbaum and Thakor (1992), Dreyfus, Saunders and Allen (1994), Matutes and Vives (1996), Hazlett (1997), Freixas and Rochet (1998), Freixas and Gabilon (1999), Cooper and Ross (2002), Duffie et al. (2003), Manz (2009), and Acharya, Santos and Yorulmazer (2010) have explored different dimensions of the deposit insurance institution. In particular, they study the role of moral hazard and the determination of appropriately priced deposit insurance for an imperfectly informed policymaker. More recently, Allen et al. (2018) show that government guarantees, including deposit insurance, are welfare improving within a global games framework, while Kashyap, Tsomocos and Vardoulakis (2019) study optimal asset and liability regulations with credit and run risk, but abstract from modeling deposit insurance. In this paper, we depart from the existing literature, which has exclusively provided theoretical results, by developing a general but tractable framework that provides direct guidance to policymakers regarding the set of variables that must be measured to set the level of deposit insurance optimally. Our approach crucially relies on characterizing optimal policy prescriptions as a function of potentially observable variables.

Our emphasis on measurement is related to a growing quantitative literature on the implications of bank runs and deposit insurance. Demirgüç-Kunt and Detragiache (2002), Ioannidou and Penas (2010), Iyer and Puri (2012), and Martin, Puri and Ufier (2017) are examples of recent empirical studies that shed light on how deposit insurance affects the behavior of banks and depositors in practice. Lucas

with $D_0(i)$ dollars, which are deposited in a bank. Hence, the aggregate initial mass of deposits is given by $\overline{D}_0 = \int_{i \in I} D_0(i) dG(i)$. We denote the smallest and largest deposit balance by \underline{D} and \overline{D} , respectively.

Depositors, whose preferences are identical ex-ante, are uncertain about their preferences over future consumption. Some will be early depositors (e), who only consume at date 1, and some will be late depositors (ℓ), who only consume at date 2. We index a generic early or late depositor by $x \in \{e, \ell\}$. At date 0, depositors know the probabilities of being an early or a late depositor, respectively λ and $1 - \lambda$, which are constant across depositor types. At date 1, depositors privately learn whether they are of the early or the late type. Under a law of large numbers, λ and $1 - \lambda$ are the exact shares of early and late depositors, respectively, for every depositor type i .⁴

Formally, the ex-ante utility of a type i depositor, $V(i)$, is given by

$$V(i) = \lambda \mathbb{E}_s [U(C_1(i, e, s))] + (1 - \lambda) \mathbb{E}_s [U(C_2(i, \ell, s))], \quad (2)$$

where $C_t(i, x, s)$ denotes the consumption at date t of a type i early depositor (if $x = e$) or late depositor (if $x = \ell$) for a given realization of the state s . Depositors' flow utility $U(\cdot)$ satisfies standard regularity conditions: $U'(\cdot) > 0$, $U''(\cdot) < 0$, and $\lim_{C \rightarrow 0} U'(C) = \infty$.⁵

Early depositors receive a stochastic endowment $Y_1(i, e, s) > 0$ at date 1 while late depositors receive a stochastic endowment $Y_2(i, \ell, s) > 0$ at date 2. Late depositors also have access to a storage technology between dates 1 and 2. At date 1, after learning their early/late status $x \in \{e, \ell\}$ and observing the state s , depositors can change their deposit balance by choosing a new deposit level $D_1(i, x, s)$, which is the single choice variable for depositors. Anticipating the possibility of multiple equilibria, we assume that there is an i.i.d. sunspot at date 1 for every realization of the state s .

Banks' Technology and Deposit Contract At date 0, banks have access to a production technology with the following properties. Every unit of consumption good invested at date 0 is transformed into $\rho_1(s) \geq 0$ units of consumption good at date 1. Every unit of consumption good held by banks at the end of date 1 is transformed into $\rho_2(s) \geq 0$ units of consumption good at date 2.⁶ For simplicity, we assume that banks do not have access to an additional storage technology at date 1 with returns that differ from $\rho_2(s)$.

We assume that both $\rho_1(s)$ and $\rho_2(s)$ are continuous and increasing in the realization of the state s , so high (low) realizations of s correspond to states in which banks are more (less) profitable. We further assume i) that $\rho_2(\underline{s}) \leq 1$, which guarantees the existence of fundamental bank failures, ii) that $\rho_1(s) \leq 1$ whenever $\rho_2(s) \leq 1$, which simplifies the exposition by limiting the cases to consider, and

⁴In previous versions of this paper, as in Wallace (1988, 1990) and Chari (1989), among others, we allowed for the share of early depositors to vary with the state s , by making λ a function of s , as in $\lambda(s)$. This introduces an additional source of aggregate risk but does not affect the main insights of the paper. Similarly, the shares of early/late depositors could depend on the level of deposits, by assuming that λ is also a function of i , as in $\lambda(i)$. Our framework can also accommodate this possibility.

⁵Our framework can accommodate preferences $U(\cdot)$ that vary with a depositor's type i and the state s . Because depositors have external resources, our model remains well-behaved even when depositors' utility satisfies an Inada condition.

⁶Many models in the Diamond and Dybvig (1983) tradition often set $\rho_1(s) = 1, \forall s$. Allowing $\rho_1(s)$ to take values different from 1 is necessary to guarantee that there are regions in which banks cannot fail even when all depositors withdraw their funds. Goldstein and Pauzner (2005) make an equivalent assumption to generate an upper-dominance region. By flexibly modeling $\rho_1(s)$ and $\rho_2(s)$ our framework accommodates illiquidity and insolvency scenarios.

iii) that $\rho_2(s)$ is strictly increasing, which guarantees that the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ (defined below) are uniquely defined.

The only contract available to depositors is a deposit contract, so that a depositor who deposits $D_0(i)$ at date 0 is entitled to withdraw on demand up to $D_0(i)R_1$ dollars at either date 1 or date 2. At date 1, depositors can withdraw funds or leave them in the bank, but cannot deposit new funds, so $D_1(i, x, s) \in [0, D_0(i)R_1]$. When $D_1(i, x, s) < D_0(i)R_1$, a depositor withdraws a strictly positive amount of deposits at date 1. When $D_1(i, x, s) = D_0(i)R_1$, a depositor leaves his deposit balance unchanged. We denote aggregate net withdrawals in state s by $\Omega(s)$, given by

$$\Omega(s) = \bar{D}_0 R_1 - \bar{D}_1(s), \quad (\text{Aggregate net withdrawals})$$

where \bar{D}_0 and $\bar{D}_1(s)$, which denote the aggregate mass of bank deposits at date 0 and at date 1 in state s , respectively, are given by

$$\begin{aligned} \bar{D}_0 &= \int_{i \in I} D_0(i) dG(i) \\ \bar{D}_1(s) &= \lambda \int_{i \in I} D_1(i, e, s) dG(i) + (1 - \lambda) \int_{i \in I} D_1(i, \ell, s) dG(i). \end{aligned}$$

Depositors make withdrawal decisions at date 1 simultaneously. Similarly to [Allen and Gale \(1998\)](#), funds are allocated proportionally in case of failure among all depositors. That is, if, given withdrawal decisions, banks anticipate being unable to satisfy all promised claims at date 1 or 2, they enter into a liquidation process in which funds are distributed on a proportional basis among claimants after the deposit insurance guarantee has been satisfied.⁷

Hence, the actual payoff received by a given depositor at either date 1 or date 2 depends on the realization of the state, the promised deposit rate, the behavior of all depositors, and the level of deposit insurance — as described in Equations (4) and (5) below. If a bank does not fail at date 1, similarly to [Diamond and Dybvig \(1983\)](#), all remaining proceeds of banks' investments at date 2 are distributed to depositors in the form of a return on deposits higher than the promised R_1 .

Deposit Rate Determination Throughout the paper, we consider three alternative assumptions regarding the determination of the deposit rate. First, we assume that the deposit rate R_1 is predetermined and invariant to the level of deposit insurance coverage δ . That is, we take R_1 as a primitive of the model. This assumption simplifies the characterization of the equilibrium and allows for a transparent derivation of the optimal policy formulas.

Subsequently, in [Section 3](#), we re-derive our results in two scenarios in which the deposit rate is endogenously determined, allowing for changes in banks' choices induced by varying the level of coverage — this behavior is often referred to as moral hazard. We first study the scenario in which R_1 is chosen

⁷In previous versions of this paper, we adopted a sequential service constraint, without affecting our conclusions. The current formulation, which is substantially more tractable, eliminates the need to keep track of which specific depositors are first in line when banks cannot pay back all depositors in full. See [Ennis and Keister \(2009, 2010\)](#) for a detailed exploration of the dynamics of deposit withdrawals during runs.

by a benevolent planner and then the case in which R_1 is chosen by competitive banks. Comparing both solutions allows us to study the optimal ex-ante deposit rate regulation.

Deposit Insurance and Taxpayers The level of deposit insurance δ , expressed in dollars (units of the consumption good), is the single instrument available to the planner. It is modeled to mimic actual deposit insurance policies: in any event, depositors are guaranteed the promised return on their deposits up to a predetermined amount δ . The level of the deposit insurance guarantee, $\delta \geq 0$, is chosen under commitment at date 0 through a planning problem.

In case of bank failure, the deposit insurance authority recovers a fraction $\chi(s) \in [0, 1]$ of any resources held by the banks to cover the deposit insurance guarantee. The remaining fraction $1 - \chi(s)$ captures deadweight losses associated with bank failure. We allow for the recovery rate $\chi(s)$ to vary with the realization of the state s and, to preserve the differentiability of the planner's problem, we assume that $\chi(s)$ is continuous and that $\chi(s) = 0$.

Whenever the resources recovered from a failed bank are sufficient to cover all insured deposits, the remaining funds are distributed proportionally among the partially insured depositors, as described in Equations (4) and (5) below. Whenever the resources recovered from a failed bank are not sufficient to cover the deposit insurance guarantee, the recovery rate on uninsured deposits is zero. In the latter scenario, the funding shortfall in state s , denoted by $T(s)$, must be covered through taxation. Any dollar raised through taxation is associated with a resource loss of $\kappa(T(s)) \geq 0$ dollars, which represents the cost of public funds. We assume that $\kappa(\cdot)$ is a weakly increasing and convex function that satisfies $\kappa(0) = 0$ and $\lim_{T \rightarrow \infty} \kappa(T) = \infty$.⁸

Finally, we assume that the taxes necessary to cover the funding shortfall and the associated deadweight losses are borne by taxpayers (equivalently, a representative taxpayer), who have the same flow utility $U(\cdot)$ as depositors. For simplicity, taxpayers only consume at date 1. We assume that the endowment of taxpayers $Y(\tau, s)$ is sufficiently large to cover the funding shortfall $T(s)$ in any state. Modeling depositors and taxpayers as distinct groups of agents highlights the fiscal implications of the deposit insurance policy.

Equilibrium Definition An equilibrium, for a given level of deposit insurance δ and a given deposit rate R_1 , is defined as consumption allocations $C_1(i, e, s)$ and $C_2(i, \ell, s)$ and deposit choices $D_1(i, x, s)$, for $x \in \{e, \ell\}$, such that depositors maximize their utility, given that other depositors behave optimally, and taxpayers cover the funding shortfall.

Remarks on the Environment We conclude the description of the environment with five remarks.

First, following most of the literature on bank runs, we take the noncontingent nature of deposits and their demandability as primitives. With this, we depart from the approach that regards deposit contracts as the outcome of a mechanism. The upside of our approach is that we can map banks' choices and outcomes to observables, like deposit rates and failure probabilities, as opposed to focusing on more abstract assignment procedures.

⁸It is trivial to make the fiscal distortion endogenous by endowing taxpayers with a labor supply choice and assuming that raising public funds distorts their consumption-leisure decision. The model can also accommodate a cost of public funds that varies with the state by making s an additional argument of $\kappa(\cdot)$.

Second, we restrict our attention to the choice of a single policy instrument under commitment: a maximum amount of deposit insurance coverage. Consequently, we study a second-best problem in the Ramsey tradition. The form of the policy that we consider (deposits are insured 100% up to a maximum, and 0% insured above that amount) matches well the policies implemented in many deposit insurance systems. The arguments in [Mitkov \(2020\)](#) can be adapted to show that this form of policy can be credible ex-post (for a given s) in our model under plausible circumstances, as follows. It is evident that deposit insurance policies can only be effective if they provide full coverage of deposits for at least some depositors.⁹ In principle, there is some indeterminacy regarding which depositors should be fully insured. However, whenever a planner has a preference for protecting the consumption of smaller depositors, perhaps because they are poorer, a deposit insurance policy of the form we consider in this paper is ex-post optimal. That said, policies that are explicitly or implicitly state-contingent, for instance, lender-of-last-resort policies, can bring social welfare closer to the first-best. Even when those policies are available, independently of whether they are chosen optimally, our main characterization and the insights associated with it remain valid as long as these additional policies do not restore the first-best, as we discuss in [Section 5](#). In the quantitative analysis in [Section 4](#), we show how alternative calibrations of our model can be used to explore how the optimal level of coverage varies with financial/business cycle conditions. Note also that the assumption of full commitment may require credible fiscal backing in practice, as highlighted by [Ennis and Keister \(2009\)](#) — see [Bonfim and Santos \(2020\)](#) for recent evidence consistent with this view.

Third, note that the level of coverage in our paper is not depositor-specific. One could envision a case in which banks offer different deposit contracts to depositors and in which deposit contracts feature different levels of deposit insurance coverage for different depositors. This is a different dimension in which we study a second-best problem, leaving the door open to further considering richer alternative welfare-enhancing policies.

Fourth, our paper departs from [Diamond and Dybvig \(1983\)](#) in three significant ways. First, we allow for a non-degenerate distribution of deposit balances, which is crucial to capture the extensive margin effects of deposit insurance. Second, the profitability of banks' investments at dates 1 and 2 is subject to aggregate risk, which is necessary to observe bank failures in equilibrium under the optimal deposit insurance policy, as in [Goldstein and Pauzner \(2005\)](#). Finally, instead of a sequential service constraint, we adopt a proportional sharing rule for the distribution of funds in the case of bank failure. This formulation, similar to [Allen and Gale \(1998\)](#), allows us to eliminate the ex-post consumption heterogeneity among depositors of the same type that emerges under sequential service and to simplify the model solution, but it is otherwise inessential.

Finally, our baseline model should be interpreted as describing a single representative bank within a banking sector. Therefore, deposit withdrawals in our model should be interpreted as transfers to cash. In [Section 4](#), we explain how our framework can be used to build system-wide welfare assessments. In [Section 5](#), we discuss the role of general equilibrium spillovers among banks, identifying interactions

⁹See [Goldsmith-Pinkham and Yorulmazer \(2010\)](#) for an account of the Northern Rock failure episode in 2007 in the UK, which illustrates why partial and delayed coverage of deposits fails to stop runs.

between banks absent in our baseline framework, for instance through the relocation of deposits via interbank markets.

2.2 Equilibrium Characterization

We first characterize depositors' equilibrium choices at date 1. Subsequently, we study the planning problem that determines δ^* .

Depositors' Optimal Choices The amount of aggregate deposit withdrawals determines the funds available to banks to satisfy their promises to depositors. Two scenarios may arise, depending on the aggregate level of deposits at date 1, $\bar{D}_1(s)$. In the no bank failure scenario, banks have sufficient funds to satisfy their commitments. In the bank failure scenario, banks do not have sufficient funds to satisfy their commitments to depositors either at date 1 or at date 2. In that case, banks fail and depositors resort to the deposit insurance guarantee. Formally, bank failure is determined by

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \bar{D}_0 - \Omega(s) \right) < \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \bar{D}_0 - \Omega(s) \right) \geq \bar{D}_1(s), \end{aligned} \quad (3)$$

where the left-hand side of the inequalities in Equation (3) represents the total resources available to banks to satisfy deposits at date 2.

We must separately consider the behavior of i) early depositors, ii) fully insured late depositors, and iii) partially insured late depositors, in both the failure and no-failure scenarios. Under our assumptions, regardless of the actions of other depositors, it is optimal for early depositors to withdraw all their deposits at date 1, setting $D_1^*(i, e, s) = 0, \forall s$. Hence, the equilibrium consumption of early depositors is given by

$$C_1(i, e, s) = \begin{cases} \min \{D_0(i) R_1, \delta\} + \alpha_F(s) \max \{D_0(i) R_1 - \delta, 0\} + Y_1(i, e, s), & \text{Bank Failure} \\ D_0(i) R_1 + Y_1(i, e, s), & \text{No Bank Failure,} \end{cases} \quad (4)$$

where $\alpha_F(s) \geq 0$ corresponds to the equilibrium recovery rate on uninsured deposits, characterized in Equation (15) below.

Fully insured late depositors are those whose deposit balances are weakly less than the level of deposit insurance coverage, that is, $D_0(i) R_1 \leq \delta$. Regardless of the actions of other depositors, fully insured late depositors are indifferent between withdrawing or leaving all their funds inside the banks in case of failure, as long as they have access to a perfect storage technology. They also weakly prefer to leave all deposits inside the banks if there is no bank failure. We restrict our attention to equilibria in which fully insured late depositors leave all their funds in banks at date 1, so $D_1^*(i, \ell, s) = D_0(i) R_1$ if $D_0(i) R_1 \leq \delta$. This equilibrium behavior is consistent with a small fixed cost of withdrawing funds or an imperfect storage technology.

Partially insured late depositors are those whose deposit balances are larger than the level of deposit insurance coverage, that is, $D_0(i) R_1 > \delta$. If banks do not fail, it is weakly optimal for these depositors

to set $D_1^*(i, \ell, s) = D_0(i) R_1$, since they will receive a positive net return on their deposits between dates 1 and 2, as shown below. In the case of bank failure, we restrict our attention to equilibria in which these depositors leave up to the level of coverage inside the banks, setting $D_1^*(i, \ell, s) = \delta$. In net terms, this behavior is consistent with the recent evidence uncovered by Martin, Puri and Ufier (2017), which shows that depositors rarely exceed the level of deposit insurance coverage when a bank failure is likely.¹⁰

Formally, the equilibrium consumption of both fully insured and partially insured late depositors can be expressed as

$$C_2(i, \ell, s) = \begin{cases} \min \{D_0(i) R_1, \delta\} + \alpha_F(s) \max \{D_0(i) R_1 - \delta, 0\} + Y_2(i, \ell, s), & \text{Bank Failure} \\ \alpha_N(s) D_0(i) R_1 + Y_2(i, \ell, s), & \text{No Bank Failure,} \end{cases} \quad (5)$$

where $\alpha_N(s) \geq 1$ corresponds to the additional gross return earned by those deposits that stay within the bank until date 2. When a bank does not fail, late depositors receive a higher return relative to early depositors, modulated by $\alpha_N(s)$, which is fully characterized in Equation (15) below. Note that the consumption of early and late depositors with the same deposit balance is identical in the case of bank failure.

Equilibria at Date 1 After characterizing the optimal individual behavior of depositors for a given level of aggregate withdrawals, we now show that two different types of equilibria may emerge at date 1, depending on the realization of s . We refer to the first type of equilibrium as a *no-failure equilibrium*. In that equilibrium, partially insured depositors keep their deposits in banks, allowing banks to honor their promises at dates 1 and 2. We refer to the second type of equilibrium as a *failure equilibrium*. In that equilibrium, partially insured depositors withdraw all deposits in excess of the level of coverage, making banks unable to honor their promises either at date 1 or date 2. As explained above, in both types of equilibria early depositors find it optimal to withdraw all their funds, and fully insured late depositors find it optimal not to withdraw any of their funds.

Note that we can reformulate Equation (3), which determines the type of equilibrium that arises, as follows:

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \tilde{D}_1(s) > \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \tilde{D}_1(s) \leq \bar{D}_1(s), \end{aligned} \quad (6)$$

where the deposit failure threshold $\tilde{D}_1(s)$ is given by

$$\tilde{D}_1(s) = \begin{cases} \frac{(R_1 - \rho_1(s)) \bar{D}_0}{1 - \frac{1}{\rho_2(s)}}, & \text{if } \rho_2(s) > 1 \\ \infty, & \text{if } \rho_2(s) \leq 1, \end{cases} \quad (7)$$

¹⁰Martin, Puri and Ufier (2017) provide the most detailed available evidence on the behavior of depositors in the case of a representative bank failure in the US. They show that a fraction of existing depositors abandon the bank in question when it is close to failure. They also show that these depositors are replaced by new depositors who hold exactly up to the level of coverage. In net terms, which is the relevant dimension for the problem we study, our model is consistent with their evidence. Our model can also accommodate a type of failure equilibrium in which partially insured late depositors optimally set $D_1^*(i, \ell, s) = 0$, yielding similar conclusions.

and where $\bar{D}_1(s)$ corresponds to the aggregate level of deposits in state s , which can potentially take two values, depending on the behavior of partially insured depositors.¹¹ If partially insured late depositors decide to withdraw all their uninsured deposits, the aggregate level of deposits $\bar{D}_1(s)$ is given by the total amount of insured deposits among late depositors, that is,

$$\bar{D}_1(s) = D_1^-(\delta, R_1) \equiv (1 - \lambda) \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i). \quad (8)$$

Alternatively, if partially insured late depositors decide not to withdraw their deposits, the aggregate level of deposits $\bar{D}_1(s)$ corresponds to

$$\bar{D}_1(s) = D_1^+(R_1) \equiv (1 - \lambda) \bar{D}_0 R_1. \quad (9)$$

Figure 3 illustrates how Equation (6) determines whether there is a unique equilibrium or multiple equilibria. There are three possibilities. First, for sufficiently low realizations of s , both $D_1^+(R_1)$ and $D_1^-(\delta, R_1)$ are less than the deposit failure threshold $\tilde{D}_1(s)$. Within this region, even if there are no withdrawals by late depositors, bank profitability is so low that early depositors' withdrawals make bank failure unavoidable. In this case, a unique failure equilibrium exists. We refer to bank failures in this region as fundamental failures.¹² Second, for intermediate realizations of s , if the level of aggregate deposits corresponds to $D_1^+(R_1)$, banks are able to honor their promises, and a no-failure equilibrium exists. However, if the level of aggregate deposits corresponds to $D_1^-(\delta, R_1)$, banks are unable to honor their promises, and a failure equilibrium exists. Within this region, there are multiple equilibria. We refer to bank failures in this region as panic failures. Finally, for sufficiently high realizations of s , both $D_1^+(R_1)$ and $D_1^-(\delta, R_1)$ are higher than the deposit failure threshold $\tilde{D}_1(s)$. Within this region, even if partially insured late depositors decide to withdraw all their uninsured funds, bank profitability is high enough to be able to honor all promises, so a unique no-failure equilibrium exists.

Figure 3 also illustrates the mechanism through which deposit insurance affects the set of equilibria. Since the value of $D_1^-(\delta, R_1)$ is increasing in δ , a higher level of deposit insurance coverage reduces the multiplicity region. Note that $\lim_{\delta \rightarrow \bar{D}_1 R_1} D_1^-(\delta, R_1) = D_1^+(R_1)$, so bank failure is possible even when all deposits are covered. In this case, when the realization of s is sufficiently low, the withdrawals of early depositors are sufficient to make banks fail. Note also that if $\delta \rightarrow 0$, the equilibrium still features three regions. For very low realizations of the state s , there is a unique fundamental failure equilibrium, while for very high realizations of s , there is a unique no-failure equilibrium. In an intermediate region of s there are multiple equilibria. Therefore, high enough levels of deposit insurance eliminate the failure equilibrium as long as banks are not completely insolvent. Interestingly, the expression for the deposit failure threshold $\tilde{D}_1(s)$ features a “multiplier” $\frac{1}{1 - \frac{1}{\rho_2(s)}} > 1$. Intuitively, every dollar left inside the banks not only reduces the net loss on investments that must be liquidated, but also earns the extra marginal

¹¹If $R_1 < \rho_1(s)$, the deposit failure threshold $\tilde{D}_1(s)$ can be negative and only the no-failure equilibrium trivially exists.

¹²There exists a long tradition that distinguishes between fundamental failures (business cycle view) and panic failures (sunspot view). Our model purposefully accommodates both. See the earlier work by Chari and Jagannathan (1988), Gorton (1988), and Jacklin and Bhattacharya (1988), among others, as well as the more recent discussions by Allen and Gale (1998) and Goldstein (2012).

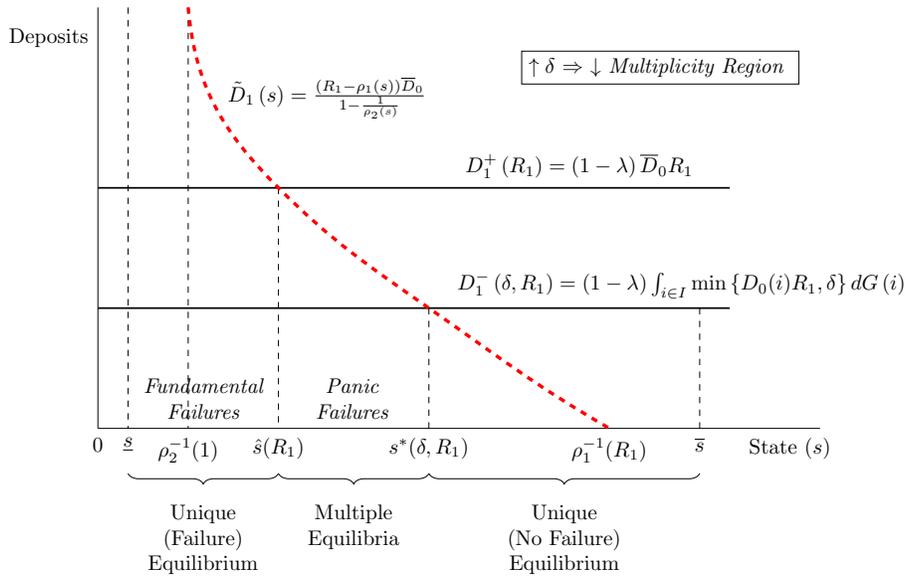


Figure 3: Equilibrium Regions

Note: Figure 3 illustrates, for a given level of deposit insurance coverage δ and for a given deposit rate R_1 , whether there exists a unique equilibrium or multiple equilibria for different realizations of the state s . The red dashed line is defined in Equation (7). The black solid lines are defined in Equations (8) and (9). The intersections between the red dashed line and the black solid lines define the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$, characterized in Equations (10) and (11) and represented in Figure 4 as a function of the level of coverage δ .

net return on banks' investments. This mechanism amplifies the impact of deposit insurance.

To characterize ex-ante behavior and welfare, it is useful to formally define the regions of s that determine the different type of equilibria that may arise at date 1. Formally,

$$\begin{aligned}
 \text{Unique (Failure) equilibrium,} & \quad \text{if } \underline{s} \leq s < \hat{s}(R_1) \\
 \text{Multiple equilibria,} & \quad \text{if } \hat{s}(R_1) \leq s < s^*(\delta, R_1) \\
 \text{Unique (No-Failure) equilibrium,} & \quad \text{if } s^*(\delta, R_1) \leq s \leq \bar{s},
 \end{aligned}$$

where the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ are defined as follows:

$$\hat{s}(R_1) = \left\{ s \mid D_1^+(R_1) = \tilde{D}_1(s) \right\} \quad (10)$$

$$s^*(\delta, R_1) = \left\{ s \mid D_1^-(\delta, R_1) = \tilde{D}_1(s) \right\}, \quad (11)$$

where $s^*(\delta, R_1) = \bar{s}$ whenever the Equation $D_1^-(\delta, R_1) = \tilde{D}_1(s)$ cannot be satisfied for any value of s . Figure 4 illustrates the three regions graphically. In Section C of the Online Appendix, we explicitly establish the relevant properties of the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$. We show that

$$\frac{\partial s^*}{\partial \delta} \leq 0, \quad \frac{\partial s^*}{\partial R_1} \geq 0, \quad \text{and} \quad \frac{\partial \hat{s}}{\partial R_1} \geq 0.$$

That is, the region of multiplicity decreases with the level of deposit insurance while the region with a unique failure equilibrium increases in the deposit rate offered by banks. The region of multiplicity can increase or decrease with the deposit rate offered by banks.

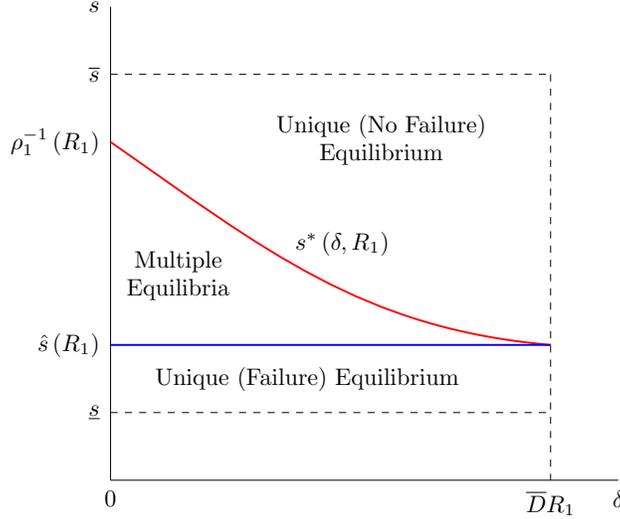


Figure 4: Regions Defined by $s^*(\delta, R_1)$ and $\hat{s}(R_1)$

Note: For a given deposit rate R_1 , Figure 4 illustrates which realizations of the state s are associated with a unique equilibrium and its type, or with multiple equilibria, for different levels of deposit insurance coverage δ .

Probability of Bank Failure In order to compute ex-ante welfare whenever there are multiple equilibria at date 1, we must take a stance on which equilibrium materializes for every realization of s . For now, a sunspot coordinates depositors' behavior: for a given realization of s , the failure equilibrium occurs with probability $\pi \in [0, 1]$ and the no-failure equilibrium occurs with probability $1 - \pi$.¹³ Alternatively, we could have introduced imperfect common knowledge of fundamentals, as in Goldstein and Pauzner (2005), which would allow us to endogenize the probability of bank failure. We explain in Section 5 how the main insights of the paper extend to that case.

Therefore we can write the unconditional probability of bank failure in this economy, which we denote by $q^F(\delta, R_1)$, as

$$q^F(\delta, R_1) = F(\hat{s}(R_1)) + \pi [F(s^*(\delta, R_1)) - F(\hat{s}(R_1))]. \quad (\text{Failure Probability}) \quad (12)$$

The unconditional probability of bank failure $q^F(\cdot)$ inherits the properties of $s^*(\cdot)$ and $\hat{s}(\cdot)$. Formally, we express the sensitivity of the probability of failure to a change in the level of coverage holding the deposit rate constant, $\frac{\partial q^F}{\partial \delta}$, which is a key input for the optimal determination of δ , and the sensitivity of the probability of failure to a change in R_1 , $\frac{\partial q^F}{\partial R_1}$, as follows:

$$\frac{\partial q^F}{\partial \delta} = \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0 \quad (13)$$

$$\frac{\partial q^F}{\partial R_1} = (1 - \pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} \geq 0, \quad (14)$$

where $f(s)$ is the probability density associated with $F(s)$. Intuitively, holding the deposit rate constant, a higher level of deposit insurance coverage decreases the likelihood of bank failures in equilibrium by

¹³Our model also accommodates the case in which the sunspot probability π varies with the state s , as in $\pi(s)$.

reducing the region in which there are multiple equilibria. Figure 4 illustrates why $\frac{\partial q^F}{\partial \delta}$ is weakly negative. Similarly, holding the level of deposit insurance constant, a higher deposit rate increases the likelihood of bank failure both by reducing the region with a unique no-failure equilibrium, $\frac{\partial s^*}{\partial R_1} \geq 0$, and by enlarging the region with a unique failure equilibrium, $\frac{\partial \hat{s}}{\partial R_1} \geq 0$. Note that deposit insurance is more effective in reducing bank failures whenever depositors are more likely to coordinate in the failure equilibrium, that is, when $\pi \rightarrow 1$.

Depositors' Equilibrium Consumption To determine depositors' consumption in equilibrium, it is necessary to characterize the equilibrium objects $\alpha_N(s)$ and $\alpha_F(s)$. As shown in Section C of the Online Appendix, the recovery rate on uninsured claims in case of failure $\alpha_F(s)$ and the additional gross return in case of no-failure $\alpha_N(s)$ are respectively given by

$$\alpha_F(s) = \frac{\max\{\chi(s)\rho_1(s)\bar{D}_0 - \int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i), 0\}}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)} \quad \text{and} \quad \alpha_N(s) = \rho_2(s) \frac{\rho_1(s) - \lambda R_1}{(1 - \lambda)R_1}. \quad (15)$$

Figure 5 illustrates how both $\alpha_F(s)$ and $\alpha_N(s)$ vary with the state s . Intuitively, the recovery rate on uninsured claims in case of failure is given by the ratio of total funds available after insurance payments to uninsured claims. The funds available after liquidation correspond to the difference between the total amount of bank resources $\chi(s)\rho_1(s)\bar{D}_0$ and the level of insured payments, $\int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i)$. The level of uninsured claims corresponds to $\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)$. Note that for sufficiently low values of bank profitability at date 1 or their recovery rate on assets $\chi(s)$, $\alpha_F(s)$ can be zero in some states, implying that the recovery rate on uninsured deposits is zero. The funding shortfall will be positive in those scenarios. The value of $\alpha_F(s) \in [0, 1)$ is decreasing in the deposit rate R_1 and in the level of coverage δ , and it is increasing in the realization of the state s .

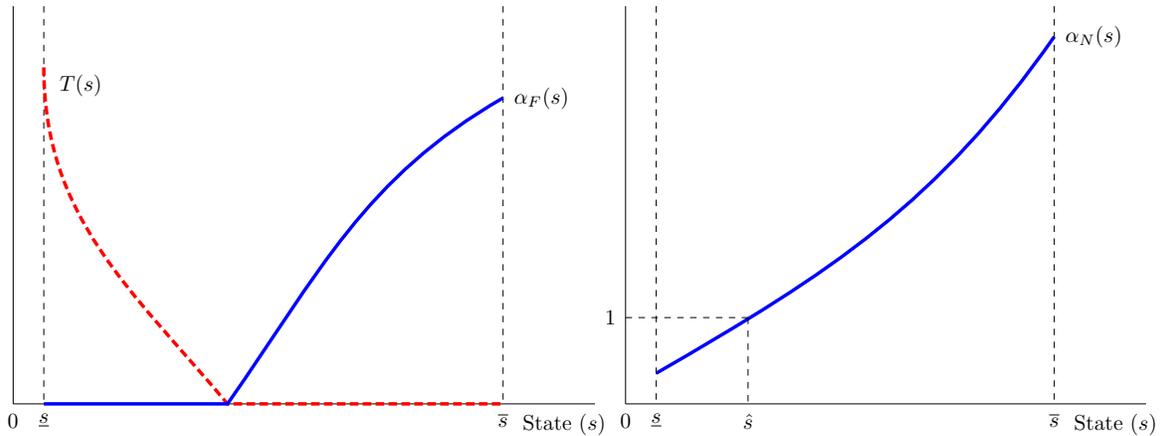


Figure 5: Depositors' Equilibrium Consumption Determinants and Funding Shortfall

Note: For a given level of deposit insurance coverage δ and a given deposit rate R_1 , the left panel in Figure 5 shows the recovery rate on uninsured deposits in case of failure, $\alpha_F(s)$, as well as the funding shortfall, $T(s)$, for different values of the realizations of the state s . For the same levels of δ and R_1 , the right panel in Figure 5 shows the additional gross return earned by the deposits that stay within the bank until date 2, $\alpha_N(s)$. Note that \hat{s} can also be defined as the value of s such that $\alpha_N(\hat{s}) = 1$.

The additional gross return in case of no-failure, $\alpha_N(s)$, corresponds to the ratio of available funds at date 2, given by $\rho_2(s)(\rho_1(s) - \lambda R_1)\bar{D}_0$, to the level of date 1 deposits, given by $(1 - \lambda)\bar{D}_0 R_1$. The

value of $\alpha_N(s)$ is increasing in the realization of the state s and decreasing in λ and R_1 .

As we show below, a key determinant of the optimal test for whether to increase or decrease the optimal level of deposit insurance is the consumption gap between failure and no-failure equilibria. Formally, for a given realization of s , the consumption gaps for early and late depositors are respectively given by

$$C_1^N(i, e, s) - C_1^F(i, e, s) = \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Unrecovered Uninsured Deposits}} \quad (\text{Early Depositors}) \quad (16)$$

$$C_2^N(i, \ell, s) - C_2^F(i, \ell, s) = \underbrace{(\alpha_N(s) - 1) D_0(i) R_1}_{\text{Additional Net Return}} + \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Unrecovered Uninsured Deposits}}. \quad (\text{Late Depositors}) \quad (17)$$

Note that the consumption gap between failure and no-failure equilibria is zero for early depositors who are fully insured. The consumption gap for partially insured early depositors corresponds to the funds that are not recovered in the case of bank failure. The consumption gap for late depositors contains an additional term relative to early depositors that captures the forgone additional net return on deposits between dates 1 and 2.

Funding Shortfall and Taxpayers' Equilibrium Consumption Finally, we characterize the funding shortfall in state s , $T(s)$, given by

$$T(s) = \max \left\{ \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0, 0 \right\}. \quad (\text{Funding Shortfall}) \quad (18)$$

The funding shortfall is positive when the total amount of deposit insurance claims exceeds the funds available after liquidation. In this case, the recovery rate on uninsured deposits is zero, that is, $\alpha_F(s) = 0$. The funding shortfall is zero when the funds available after liquidation are sufficient to cover all insured deposits. Figure 5 illustrates how $T(s)$ varies with the state s and how $T(s)$ and $\alpha_F(s)$ are related.

Given that the deadweight loss of taxation $\kappa(T(s))$ is borne by taxpayers, we can express taxpayers' equilibrium consumption $C(\tau, s) = \{C^N(\tau, s), C^F(\tau, s)\}$ in failure and no-failure scenarios as

$$C^F(\tau, s) = Y(\tau, s) - T(s) - \kappa(T(s)) \quad \text{and} \quad C^N(\tau, s) = Y(\tau, s), \quad (19)$$

where $T(s)$ is defined in Equation (18). The consumption gap between failure and no-failure equilibria for taxpayers is simply given by the funding shortfall augmented by the deadweight loss of taxation, that is,

$$C^N(\tau, s) - C^F(\tau, s) = T(s) + \kappa(T(s)). \quad (20)$$

2.3 Normative Analysis

After characterizing the equilibrium of this economy for a given level of deposit insurance coverage δ , we now study how changes in the level of coverage affect social welfare. We initially consider a scenario in which the deposit rate offered by banks is predetermined and invariant to the level of coverage δ . This case provides a tractable benchmark from which we study multiple departures in the next section.

We denote by $V(i, \delta, R_1)$ and $V(\tau, \delta, R_1)$ the ex-ante indirect utilities of type i depositors and taxpayers, respectively, for given levels of deposit insurance and the deposit rate, which are given by

$$\begin{aligned} V(i, \delta, R_1) &= \lambda \overbrace{\mathbb{E}_s [U(C_1(i, e, s))]}^{\equiv V(i, e, \delta, R_1)} + (1 - \lambda) \overbrace{\mathbb{E}_s [U(C_2(i, \ell, s))]}^{\equiv V(i, \ell, \delta, R_1)} & (\text{Depositors}) \\ V(\tau, \delta, R_1) &= \mathbb{E}_s [U(C(\tau, s))], & (\text{Taxpayers}) \end{aligned}$$

where $V(i, e, \delta, R_1)$ and $V(i, \ell, \delta, R_1)$ denote the indirect utility of type i depositors conditional on being of the early or late type, respectively. In the Appendix, we provide explicit characterizations of $\mathbb{E}_s [U(C_1(i, e, s))]$, $\mathbb{E}_s [U(C_2(i, \ell, s))]$, and $\mathbb{E}_s [U(C(\tau, s))]$, which account for the possibility of multiple equilibria. Going forward, to simplify the exposition, we use the index j to jointly refer to early and late depositors of type i , as well as taxpayers. For instance, we use $V(j, \delta, R_1)$, to refer to $V(i, e, \delta, R_1)$, $V(i, \ell, \delta, R_1)$, and $V(\tau, \delta, R_1)$. When integrating over j , we define a new measure $H(j)$ that accounts for the mass of agents in each group.

Because our model features a rich cross-section of depositors, we must set a criterion to aggregate welfare. Instead of directly maximizing a weighted sum of the utilities of depositors and taxpayers, we assess the aggregate welfare gains/losses of a marginal change in the level of coverage by aggregating the money-metric utility change (in dollars of the marginal failure state) across all agents. This approach can be interpreted as selecting a set of “generalized social marginal welfare weights” — see Saez and Stantcheva (2016) — for all agents. As we show in Section E of the Online Appendix, there is a one-to-one mapping between using generalized welfare weights and selecting a particular set of traditional social welfare weights. There are two main advantages to using our approach. First, this approach allows us to quantify aggregate marginal welfare changes in dollars. Second, it facilitates aggregation by making (approximate) welfare assessments exclusively a function of bank-level aggregates, as we formally show in Proposition 2 below.

Formally, we express the change in social welfare induced by a marginal change in the level of deposit insurance coverage δ , $\frac{dW}{d\delta}$, as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \int \omega(j) \frac{dV_m(j, \delta, R_1)}{d\delta} dH(j) \\ &= \lambda \underbrace{\int_{i \in I} \omega(i, e) \frac{dV_m(i, e, \delta, R_1)}{d\delta} dG(i)}_{\text{Early Depositors}} + (1 - \lambda) \underbrace{\int_{i \in I} \omega(i, \ell) \frac{dV_m(i, \ell, \delta, R_1)}{d\delta} dG(i)}_{\text{Late Depositors}} + \underbrace{\omega(\tau) \frac{dV_m(\tau, \delta, R_1)}{d\delta}}_{\text{Taxpayers}}, \quad (21) \end{aligned}$$

where $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{dV(j, \delta, R_1)}{U'(C^F(j, s^*))}$ denotes the money-metric change in indirect utility, using the marginal failure state, s^* , as reference.¹⁴ The subindex m indicates that $\frac{dV_m}{d\delta}$ is a “money-metric” welfare representation. The weights $\omega(j) = \{\omega(i, e), \omega(i, \ell), \omega(\tau)\}$ are generalized social marginal welfare weights. We derive Proposition 1 for general weights, although we specialize to the case of uniform weights — $\omega(j) = 1, \forall j$ — in Proposition 2.

Given the definition of $\frac{dW}{d\delta}$, Proposition 1, which presents a central result of this paper, provides an

¹⁴In principle, any state could be chosen as reference for the money-metric normalization. By choosing s^* , we slightly simplify the characterization of Proposition 1. In Section E of the Online Appendix, we re-derive Equation (22) for any reference state and show that Proposition 2 remains valid in that case after suitably redefining $m(j, s)$.

exact test that determines whether it is optimal to increase or decrease the level of deposit insurance coverage.

Proposition 1. (Exact directional test) *The change in social welfare induced by a marginal change in the level of deposit insurance coverage δ , $\frac{dW}{d\delta}$, is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left(\underbrace{-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right)}_{\text{Marginal Benefit}} + \underbrace{q^F \mathbb{E}_s^F \left[m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right]}_{\text{Marginal Cost}} \right) dH(j), \quad (22)$$

where $\omega(j)$ denotes the generalized social welfare weight for agent j , $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))}$ denotes the stochastic discount factor of agent j in state s relative to the marginal failure state s^* , $\mathbb{E}_s^F[\cdot]$ denotes the conditional expectation over bank failure states, and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

Proposition 1 characterizes the effect on social welfare of a marginal change in the level of deposit insurance, and formalizes the tradeoffs that determine the optimal deposit insurance limit. The first element of the weighted sum in Equation (22) can be interpreted as the marginal benefit of increasing the level of deposit insurance by a dollar. A marginal increase in the deposit insurance limit decreases the likelihood of bank failure by $\frac{\partial q^F}{\partial \delta}$.¹⁵ The marginal utility gain associated with such a reduction in the probability of bank failure is captured by the differences in utilities between the failure and no-failure equilibria evaluated at the marginal failure state s^* , $U(C^N(j, s^*)) - U(C^F(j, s^*))$. For each agent, this difference in utilities is determined by the difference in consumption, characterized in Equations (16), (17), and (20).

To better understand the aggregate marginal benefit of increasing coverage, in Lemma 1 we formally characterize the aggregate consumption difference between failure and no-failure equilibria at the marginal failure state.

Lemma 1. (Aggregate consumption difference between failure and no-failure equilibria) *The aggregate consumption change induced by a bank failure in the marginal failure state s^* is given by*

$$\int (C^N(j, s^*) - C^F(j, s^*)) dH(j) = \underbrace{(\rho_2(s^*) - 1)(\rho_1(s^*) - \lambda R_1) \bar{D}_0}_{\text{Net Return Loss}} + \underbrace{(1 - \chi(s^*)) \rho_1(s^*) \bar{D}_0}_{\text{Bank Failure Deadweight Loss}} + \underbrace{\kappa(T(s^*))}_{\text{Net Cost of Public Funds}}. \quad (23)$$

As we show below, Equation (23) corresponds to the approximate social gain from avoiding the marginal bank failure. Its first term corresponds to the marginal net return loss caused by bank failure. Intuitively, at date 1, a bank failure forfeits the net return $\rho_2(s^*) - 1$ per unit of available funds $(\rho_1(s^*) - \lambda R_1) \bar{D}_0$. The second term corresponds to the deadweight loss on banks' assets associated

¹⁵As we show in Section 3, the total impact of a change in coverage on the probability of failure when deposit rates react to the level of δ can be decomposed as $\frac{dq^F}{d\delta} = \frac{\partial q^F}{\partial \delta} + \frac{\partial q^F}{\partial R_1} \frac{dR_1}{d\delta}$. In this section, note that we adopt the partial derivative notation, even though $\frac{dR_1}{d\delta} = 0$. It will become clear in Sections 3 and 5 that the partial derivative is the relevant object of interest more generally.

with bank failure. The final term is the total cost of public funds, which is non-zero at the margin whenever banks do not have enough resources after liquidation to pay for all insurance claims at the marginal failure state s^* . Part of the marginal benefit of preventing a bank failure comes from avoiding fiscal distortions at the marginal state.

The second element of the weighted sum in Equation (22) can be interpreted as the marginal cost of increasing the level of deposit insurance by a dollar. A marginal increase in the level of deposit insurance changes the consumption of depositors and taxpayers in the case of bank failure by $\frac{\partial C^F(j,s)}{\partial \delta}$ over the set of failure states, which take place with probability q^F . Each agent values consumption changes in state s according to his stochastic discount factor relative to the marginal failure state: $m(j,s) = \frac{U'(C^F(j,s))}{U'(C^F(j,s^*))}$. In Section C of the Online Appendix, we provide explicit characterizations of $\frac{\partial C^F(j,s)}{\partial \delta}$ for both depositors and taxpayers. There we show that $\frac{\partial C^F(j,s)}{\partial \delta}$ is zero for fully insured depositors and can be positive (for depositors whose deposits are right above the coverage limit) or negative (for depositors with large uninsured balances) for partially insured depositors. We also show that the aggregate effect among depositors, $\lambda \int \frac{\partial C^F(i,e,s)}{\partial \delta} dG(i) + (1-\lambda) \int \frac{\partial C^F(i,\ell,s)}{\partial \delta} dG(i)$, is weakly positive and is zero when the funding shortfall is zero, i.e., $T(s) = 0$. We also show that $\frac{\partial C^F(\tau,s)}{\partial \delta}$ is weakly negative for taxpayers.

To better understand the aggregate marginal cost of increasing coverage, in Lemma 2 we formally characterize the aggregate change in consumption in case of bank failure in state s induced by an increase in the level of coverage δ .

Lemma 2. (*Aggregate consumption change induced by a change in coverage in failure states*) *The aggregate consumption change in case of bank failure in state s induced by a marginal change in the level of deposit insurance coverage δ is given by*

$$\int \frac{\partial C^F(j,s)}{\partial \delta} dH(j) = \begin{cases} - \overbrace{\kappa'(T(s))}^{\text{Marginal Cost of Public Funds}} \overbrace{\int_{i \in \mathcal{PI}} dG(i)}^{\text{Mass of Partially Insured Depositors}}, & \text{if } T(s) > 0 \\ 0, & \text{if } T(s) = 0, \end{cases} \quad (24)$$

where $\mathcal{PI} = \{i \mid D_0(i) R_1 > \delta\}$ denotes the set of partially insured depositors.

Lemma 2 shows that the marginal cost of increasing δ is increasing in the marginal cost of public funds $\kappa'(\cdot)$ and the mass of partially insured depositors $\int_{i \in \mathcal{PI}} dG(i)$. The value of $\int \frac{\partial C^F(j,s)}{\partial \delta} dH(j)$ is strictly negative whenever the transfer of resources among different agents associated with the deposit insurance system is distortionary, in this case due to the deadweight losses of taxation. Intuitively, the net social cost of a marginal increase in δ is given by the deadweight loss associated with transferring a dollar from taxpayers to the partially insured depositors. Equation (24) highlights that only partially insured depositors are marginal as the coverage limit changes. In other words, a marginal change in the coverage limit has no marginal cost impact on already fully insured depositors. Taking an expectation

over the failure states, we can express the marginal cost of increasing the level of coverage as

$$q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right] = -q^F \cdot q^{T+|F} \cdot \mathbb{E}_s^F [\kappa'(T(s)) | T(s) > 0] \cdot \int_{i \in \mathcal{PI}} dG(i). \quad (25)$$

Intuitively, the cost increasing the level of coverage by a dollar is given by the marginal cost of public funds, $\kappa'(T(s))$, which has to be paid to partially insured depositors, $\int_{i \in \mathcal{PI}} dG(i)$, whenever banks fail (which occurs with probability q^F) and the funding shortfall is positive (which occurs with probability $q^{T+|F}$ conditional on bank failure).

Even though the test characterized in Proposition 1 is exact, it is challenging to operationalize in practice by directly measuring its constituents. A policymaker would need detailed information on individual deposit balances and consumption across different scenarios. While gathering this information is conceivable, the informational requirements on the policymaker would be large. Instead, in Proposition 2, we introduce an approximate directional test that determines whether it is optimal to increase or decrease the level of coverage relying exclusively on *aggregate outcomes* at the bank level. As we explain in our remarks below, the elements of Equation (26) are sufficient statistics to determine whether to increase or decrease the level of coverage.

Proposition 2. (Approximate directional test based on bank-level aggregates) *When the planner i) sets uniform generalized marginal social welfare weights, i.e., $\omega(j) = 1, \forall j$, ii) approximates $U(C^N(j, s^*))$ linearly around $C^F(j, s^*)$, and iii) values consumption equally across agents and states, i.e., computes welfare as if $m(j, s) = 1, \forall j, \forall s$, the change in social welfare induced by a marginal change in the level of deposit insurance coverage δ , $\frac{dW}{d\delta}$, is given by*

$$\frac{dW}{d\delta} \approx -\frac{\partial q^F}{\partial \delta} \int (C^N(j, s^*) - C^F(j, s^*)) dH(j) + q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right], \quad (26)$$

where $\mathbb{E}_s^F[\cdot]$ denotes the conditional expectation over bank failure states, q^F denotes the unconditional probability of bank failure, and where $\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)$ and $\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j)$ are characterized in Lemmas 1 and 2, respectively. If $\frac{dW}{d\delta} > (<) 0$, it is approximately optimal to locally increase (decrease) the level of coverage.

The approximate test characterized in Proposition 2 is based on three premises. First, it uses uniform generalized marginal social welfare weights. This choice of weights eliminates distributional motives by valuing resources equally among all agents, using the state s^* as reference. Second, it approximates the difference in utilities at the marginal failure state s^* as $U'(C^F(j, s^*)) (C^N(j, s^*) - C^F(j, s^*))$, which is necessary to express individual valuations in terms of marginal utilities. Note that this approximation also allows us to express $\frac{dW}{d\delta}$ in terms of consumption differences, $C^N(j, s^*) - C^F(j, s^*)$, and not in consumption levels, $C^N(j, s^*)$ and $C^F(j, s^*)$, which makes the result substantially more applicable. Finally, it imposes that all agents value resources equally in all (failure) states, which further eliminates any desire to redistribute across agents with different valuations. As a whole, these three requirements allow the planner to give equal weight to dollar transfers across different agents and different states,

abstracting away from distributional issues.¹⁶ Since the distributional implications of policies may be an important practical concern for policymakers, any conclusion obtained from applying Equation (26) should be understood as a reference point.

In practice, Equation (26) allows anyone interested in making approximate welfare assessments to rely only on information on failure probabilities, q^F and $\frac{\partial q^F}{\partial \delta}$, and aggregate consumption, $\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)$ and $\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j)$. In Section 4, we show how to combine Proposition 2 with Lemmas 1 and 2 to find specific estimates of Equation (26) in a particular scenario, illustrating how to implement our approximate test in practice.

We conclude our normative analysis with five remarks. In these remarks, we focus on the implications of Proposition 2, because it is more widely applicable, although similar insights emerge when we consider the test in Proposition 1.

Remark 1. Sufficient statistics. Proposition 2 provides a simple test for whether to increase or decrease the level of coverage that exclusively relies on a few potentially observable sufficient statistics. These sufficient statistics are: i) the probability of bank failure, ii) its sensitivity to changes in the level of coverage, iii) the aggregate consumption losses associated with a marginal bank failure, and iv) the marginal impact on aggregate consumption in failure states induced by changing the level of coverage. These sufficient statistics can a) be potentially recovered from measured data, or b) be used to shed light on the results of a calibrated structural model. In Section 4, we make use of both approaches within a particular application. Even though we characterize $\frac{dW}{d\delta}$ locally, the welfare change caused by a non-local change in the level of coverage can be recovered by integrating over the values of $\frac{dW}{d\delta}$. Formally, for a non-local policy change from δ to δ' , we can write the welfare change as follows: $W(\delta') - W(\delta) = \int_{\delta}^{\delta'} \frac{dW}{d\delta}(\tilde{\delta}) d\tilde{\delta}$, where $\frac{dW}{d\delta}(\cdot)$ is determined in Proposition 1. Therefore, direct measurement of these variables for different levels of δ is sufficient to assess the welfare impact of any change in the level of coverage.

Remark 2. Diamond and Dybvig (1983) revisited. In the baseline version of their model, which features no aggregate risk, Diamond and Dybvig (1983) show that it is optimal to provide unlimited deposit insurance coverage, eliminating bank failure equilibria altogether. The prescription of optimal unlimited coverage also extends to the setting in which the share of early consumers is stochastic, hence featuring aggregate risk, but in which deposit insurance can be made contingent on such share. Importantly, in either version of their model, deposit insurance never has to be paid in equilibrium. In our model, due to the fact that δ is not contingent on the aggregate state, there are scenarios in which deposit insurance must be paid even if the level of coverage is unlimited, which makes unlimited coverage suboptimal. Equation (26) allows us to heuristically recover the Diamond and Dybvig (1983) result by setting $q^F = 0$ and assuming that $\frac{\partial q^F}{\partial \delta} < 0$. In this case, banks never fail, so there is no cost of intervention, but increasing the level of coverage reduces the probability of failure, making unlimited deposit insurance

¹⁶In Section F.8 of the Online Appendix, we describe the quantitative impact of the three conditions needed to derive Equation (26). There we show that removing redistributive concerns calls for higher levels of coverage. Moreover, in Section E.4 of the Online Appendix, we describe an alternative approach that leads to the exact same characterization of Equation (26). This derivation, which does not involve approximations, relies on a welfare assessment based on dynamic stochastic generalized social marginal social welfare weights, introduced in Dávila and Schaab (2021).

optimal. This logic extends more broadly to other models of multiple equilibria, in which policies that costlessly eliminate bad equilibria are optimal.

Remark 3. Convexity and limiting results. Our assumptions guarantee that the planner's problem is continuous and differentiable in δ . When numerically solving the model, we find that the planner's problem is well-behaved for standard functional forms and distributional assumptions, although the convexity of the planner's problem is not guaranteed in general, as in most normative problems. Note that Equation (26) can be used to conclude whether a non-zero or a maximal level of coverage is desirable. For instance, if the marginal cost of a small increase in the level of coverage is zero, because q^F , $q^{T+|F}$, or $\kappa'(\cdot)$ are zero when $\delta = 0$, but a small increase in the level of coverage is effective at reducing the probability of failure, $\frac{\partial q^F}{\partial \delta} \Big|_{\delta=0} < 0$, then Equation (26) implies that $\frac{dW}{d\delta} \Big|_{\delta=0} > 0$, so a strictly positive level of coverage is optimal. Note that as long as banks fail in equilibrium when coverage is unlimited, $\delta = \overline{\overline{DR}}_1$, and fiscal costs are positive, $\kappa'(\cdot) > 0$, a maximal level of coverage is not optimal, since $\frac{\partial q^F}{\partial \delta} \Big|_{\delta=\overline{\overline{DR}}_1} = 0$, which implies that $\frac{dW}{d\delta} \Big|_{\delta=\overline{\overline{DR}}_1} < 0$.

Remark 4. Optimal level of coverage δ^ .* At an interior optimum, the optimal level of deposit insurance δ^* satisfies $\frac{dW}{d\delta}(\delta^*) = 0$, which implies the following relations exactly and approximately:

$$\delta^* = \varepsilon_\delta^q \frac{\int \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} dH(j)}{\mathbb{E}_s^F \left[\int m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right]} \approx \varepsilon_\delta^q \frac{\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)}{\mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right]}, \quad (27)$$

where $\varepsilon_\delta^q = -\frac{\partial \log q^F}{\partial \log(\delta)}$ denotes the elasticity of the probability of bank failure to a change in the level of coverage. Intuitively, a high (low) value for δ^* is optimal when $|\varepsilon_\delta^q|$ and $|C^N(j, s^*) - C^F(j, s^*)|$ are large (small), or when $\left| \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right] \right|$ is large (small), all else equal. As it is common in optimal policy exercises, δ^* cannot be written as a function of primitives, since all right-hand side variables in Equation (27) are endogenous to the level of δ .¹⁷ Since solving for the fixed point that determines δ^* in Equation (27) would require finding measures of all right-hand side variables for every level of δ , we focus on characterizing $\frac{dW}{d\delta}$, which can be computed for a given level of δ .

Remark 5. Role of the approximation/welfare weights. It is not obvious whether the approximate test in Proposition 2 delivers results that are similar to those obtained using Proposition 1. In Section F of the Online Appendix, we address this issue in detail within the calibrated quantitative model from Section 4.2. First, as expected, we conclude that the choice of welfare weights is important. Using generalized welfare weights that are not uniform will deliver different conclusions regarding the desirability of changing the level of coverage. For instance, we find that a classic utilitarian planner who values resources in the hands of smaller depositors more would prefer lower levels of coverage. Second, we find that the actual approximations, i.e., approximating $U(C^N(j, s^*))$ linearly around $C^F(j, s^*)$ and setting $m(j, s) = 1, \forall j, \forall s$, have a small quantitative impact, at least for our calibration. As we explain in the Online Appendix, a planner who uses the approximate results tends to overestimate the welfare gains from increasing the level of coverage, mostly by *underweighting* the marginal cost of providing

¹⁷This logic is similar to conventional characterizations of optimal taxes. For instance, optimal Ramsey commodity taxes are a function of demand elasticities, which are endogenous to the level of taxes.

public funds for taxpayers. Moreover, it is worth highlighting that there are welfare weights under which no deposit insurance at all is optimal in our framework. Formally, since taxpayers are always worse off when there is deposit insurance, by putting an increasingly large welfare weight on taxpayers (equivalently, a vanishingly small weight on depositors) a planner may find zero deposit insurance to be optimal.

2.4 A Laissez-Faire Interpretation

Up to now, we have assumed that the funds needed to pay for deposit insurance are raised directly from taxpayers. We do so because, in the presence of large aggregate shocks, governments typically act as ultimate sources of funding. However, our framework can be used to understand the role played by alternative arrangements of mutual insurance either across banks or between banks and other agents/institutions.

There are two benchmark scenarios to consider. First, there is the case in which bank failures are idiosyncratic — we study this case in Section E.1 of the Online Appendix, summarizing here our conclusions. There, we consider an environment in which there is a continuum of ex-ante identical banks of the form studied so far. Instead of assuming that all banks fail in the multiple equilibria region according to an aggregate sunspot, we assume that, whenever the realization of s lies in the multiple equilibria region, a fraction of banks fails, which makes the risk of failure idiosyncratic in those states. In this case, we show that it may be possible to set up ex-post transfers across banks that eliminate funding shortfalls, by transferring funds from surviving to failed banks. In terms of the sufficient statistics that we identify, the probability of facing a funding shortfall conditional on failure, q^{T+F} , subsumes the impact of introducing idiosyncratic risk and ex-post transfers across banks.

Second, there is the case in which bank failures are system-wide, which is effectively the case considered throughout the paper. In this case, by construction, deposit insurance funds must come from outside of the banking sector. In this case, there are two natural possibilities. The first one, which we study in Section E.2 of the Online Appendix, is one in which a deposit insurance fund funded by contributions of insured banks is responsible for paying insured deposits in case of failure. Our analysis in the Appendix shows that our main characterization remains valid in that case, provided that the returns on the resources held in the fund are commensurate with the returns obtained by banks. The second possibility involves banks obtaining insurance against system-wide failures from a third party. That case can be mapped to our results with optimal regulation — which we describe next — in which the level of coverage and the optimal regulation fully internalize the welfare of depositors and the outside sector. In fact, our results with optimal regulation can be interpreted as implementing a laissez-faire co-insurance outcome between banks and a set of outside agents, rather than relying on markets to do so.

3 Endogenous Deposit Rate and Optimal Regulation

So far, we have considered the case in which the deposit rate R_1 offered by banks is predetermined. We now analyze two environments in which R_1 is endogenously determined. First, we consider an environment in which a regulator can directly determine the deposit rate offered by banks. Next, we consider a different environment in which competitive banks choose the deposit rate offered to depositors. Finally, by comparing the solution to both problems, we characterize the optimal deposit rate regulation.

We draw three major conclusions from this analysis. First, we show that the equation that characterizes $\frac{dW}{d\delta}$ when deposit rates are fixed is *identical* to the equation that characterizes $\frac{dW}{d\delta}$ under the optimal deposit rate regulation. Therefore, in both scenarios, the same set of sufficient statistics is needed to determine the optimal policy. Second, we show that this equation only has to be augmented by the fiscal externality induced by banks' behavior when deposit rates can vary freely. Finally, we show that the optimal deposit rate regulation should be designed to counteract the fiscal externality caused by banks, regardless of whether deposit insurance is "fairly-priced".

3.1 Regulated Deposit Rate

We now allow the policymaker to jointly determine the welfare maximizing deposit rate along with the optimal level of deposit insurance. Letting the planner choose the deposit rate directly is analogous to allowing for a rich set of ex-ante policies that modify banks' behavior at date 0. Deposit rate regulation has been commonly used in practice, in particular before the financial deregulation wave at the end of the last century. We first characterize the set of constrained efficient policies and then discuss possible decentralizations, including, for instance, imposing deposit rate ceilings or requiring a deposit insurance premium.

Formally, we let the planner jointly choose the level of δ and the deposit rate offered to households. Going forward, we assume that the planner uses uniform generalized marginal social welfare weights, i.e., $\omega(j) = 1$. In this case, the optimal choice of $R_1 \in [1, \bar{R}_1]$ is characterized by the solution to $\frac{\partial W}{\partial R_1} = 0$, where social welfare is now a function of both δ and R_1 . Importantly, the planner internalizes the effect of changing R_1 on the funding shortfall $T(s)$. In Section B of the Online Appendix, we formally characterize the expression that determines the optimal rate. Here, we directly characterize the directional test for how social welfare varies with the level of coverage.

Proposition 3. (Directional test for δ under perfect ex-ante regulation) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ when R_1 is optimally determined by the planner is given by*

$$\frac{dW}{d\delta} = \int \left(-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[\int m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dH(j), \quad (28)$$

where $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))}$ denotes the stochastic discount factor of agent j in state s relative to the marginal failure state s^* , $\mathbb{E}_s^F[\cdot]$ denotes the conditional expectation over bank failure states, and q^F

denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

By comparing Equations (22) and (28), we observe that the marginal change in welfare caused by a change in the level of coverage can be expressed in identical form when R_1 is predetermined and when R_1 is optimally chosen by the planner. Once again, information about depositors' and taxpayers' consumption and failure probabilities is sufficient to determine the welfare effect of changes in the level of coverage. Intuitively, any impact on welfare induced by the change in deposit rates generated by a change in δ must be 0 when R_1 is optimally chosen by perfectly regulated banks.

If one were to solve for the optimal value of δ by setting $\frac{dW}{d\delta} = 0$, the solutions when R_1 is predetermined and optimally chosen would differ, because the endogenous elements (consumption and failure probabilities) vary with the level of R_1 . However, from the perspective of understanding the welfare impact of changes in the level of coverage, the set of relevant sufficient statistics is the same. This reasoning motivates the use of Equation (28) or, equivalently, Equation (22) for the purpose of direct measurement exercises, as we do in Section 4.1.

3.2 Unregulated Deposit Rate

We now allow banks to freely choose the deposit rate that they offer to depositors. In environments with a representative depositor, including Diamond and Dybvig (1983), the assumption of perfect competition among banks translates into an objective function for banks that simply maximizes depositors' welfare. However, specifying the objective function of banks in an environment with heterogeneous depositors is far from trivial.¹⁸

Here we proceed as follows. For a given level of coverage δ , we assume that banks set the single deposit rate $R_1 \in [1, \bar{R}_1]$ competitively at date 0 to maximize a money-metric sum of depositors' utilities. Formally, we let R_1 be pinned down by the solution to

$$\frac{\partial V}{\partial R_1} = \lambda \int \frac{\partial V_m(i, e, \delta, R_1)}{\partial R_1} dG(i) + (1 - \lambda) \int \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial R_1} dG(i) = 0, \quad (29)$$

where $\frac{\partial V_m(i, x, \delta, R_1)}{\partial R_1} = \frac{\frac{\partial V(i, x, \delta, R_1)}{\partial R_1}}{U'(C^F(j, s^*))}$ denotes the money-metric change in indirect utility for depositors with types i and x induced by an increase in the deposit rate.¹⁹ Our definition of equilibrium needs to be augmented to incorporate that R_1 is optimally chosen by banks at date 0, for a given level of deposit insurance δ .

As we show in the Online Appendix, the choice of R_1 determines the optimal degree of risk sharing between early and late types and across depositors, accounting for the level of aggregate uncertainty and

¹⁸Formally modeling how banks compete for depositors and how depositors end up grouped in different banks is outside of the scope of the paper. This is an important question that has not received much attention. On the theoretical side, Mitkov (2020) addresses this problem by assuming that each bank serves only depositors of the same wealth level, so the objective of the bank is clearly defined. He then provides conditions under which the equilibrium outcome is unchanged if depositors with different wealth levels are grouped together in the same bank. Quantitatively, Egan, Hortaçsu and Matvos (2017) have structurally estimated a quantitative model for demand deposits.

¹⁹Our model can be augmented to allow banks to set different deposit rates $R_1(i)$ for different types of depositors — see Jacewitz and Pogach (2018) for evidence consistent with this possibility.

incorporating the costs associated with bank failure. Overall, banks internalize that varying R_1 not only changes the consumption of depositors in both failure and no-failure states (intensive margin terms) but also the likelihood of experiencing a bank failure (extensive margin terms). Importantly, banks do not take into account how their choice of R_1 affects the need to raise resources through taxation to pay for deposit insurance.

In principle, the equilibrium deposit rate R_1 can increase or decrease with the level of coverage δ , due to conflicting income effects and direct effects on the size of the failure/no-failure regions. However, in most cases, it is reasonable to expect R_1 to increase with δ , that is, $\frac{dR_1^*}{d\delta} > 0$.²⁰ Intuitively, we expect competitive banks to offer higher deposit rates when the level of coverage is higher, since they know that the existence of deposit insurance partially shields depositors' consumption. This result is a form of increased moral hazard by banks. We can now characterize the directional test for how social welfare varies with the level of coverage.

Proposition 4. (Directional test for δ without ex-ante regulation) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ when R_1 is determined by competitive banks as described in Equation (A2) is given by*

$$\begin{aligned} \frac{dW}{d\delta} = & -\frac{\partial q^F}{\partial \delta} \int \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) dH(j) + q^F \mathbb{E}_s^F \left[\int m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right] \\ & + \underbrace{\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \frac{dR_1}{d\delta}}_{\text{Fiscal Externality}}, \end{aligned} \quad (30)$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states, q^F denotes the unconditional probability of bank failure, and $\frac{\partial V_m(\tau)}{\partial R_1}$ can be expressed, in terms of a risk-neutral approximation, as $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s) + \kappa(T(s))]}{\partial R_1}$. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

It is clear that when banks choose their deposit rate freely, a new set of effects must be accounted for to understand the welfare impact of changes in the level of coverage. The derivation of Equation (30) repeatedly exploits the fact that banks choose the value of R_1 to provide insurance across types optimally, while taking into account how that may change the likelihood of bank failure. The third term of Equation (30) corresponds to the impact of the distortions on banks' behavior induced by the change in the level of deposit insurance. As shown in the Appendix, the fiscal externality dimension features both an intensive and extensive margin. At the intensive margin, an increase in R_1 increases the level of claims that must be satisfied in failure states. At the extensive margin, an increase in R_1 increases the set of states in which bank failures occur and fiscal costs must be incurred. Under a risk-neutral approximation similar to the one used in Proposition 2, $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s) + \kappa(T(s))]}{\partial R_1}$, which corresponds to the direct impact on tax revenue and deadweight losses induced by a change in the deposit rate.

We also show in the Appendix that the fiscal externality term is negative, so the third term in

²⁰In a global games framework, Allen et al. (2018) explicitly find this result in a special case of our model.

Equation (30) increases the marginal cost of increasing the deposit insurance limit. Because it directly affects the funds that need to be raised by the government, we refer to it as a fiscal externality. It is worth emphasizing how “moral hazard” considerations affect our results in the following remark.

Remark. Banks’ changes in behavior (often referred to as moral hazard) only affect social welfare directly through the fiscal externality term. We indeed expect banks to quote higher deposit rates when the level of deposit insurance is higher, since they know the presence of deposit insurance partially shields depositors’ consumption. However, because banks are competitive and maximize depositors’ welfare, only the fiscal consequences of their change in behavior, which materialize when the fiscal authority actually has to pay for deposit insurance, matter. This result remains valid even when banks make endogenous liquidity and investment choices — see Section 5. Therefore, accounting for banks’ moral hazard simply augments the directional test for δ by including a fiscal externality component. Indirectly, changes in bank behavior affect i) the level of gains from reducing bank failures (numerator of Equation (27)), ii) the region in which deposit insurance is paid (denominator of Equation (27)), and iii) the value attached to a dollar in the different states (captured by depositors’ and taxpayers’ marginal utilities), but these effects are subsumed into the identified sufficient statistics.

3.3 Optimal Ex-Ante Regulation

By comparing the optimal deposit rate chosen by the regulator and by competitive banks, we can provide insights into the form of the optimal ex-ante regulation of deposit rates.²¹

Proposition 5. (Optimal ex-ante deposit rate regulation) *The optimal corrective policy modifies banks’ optimal choice of deposit rates by introducing a wedge in their deposit rate decision given by*

$$\tau_{R_1} = -\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \approx \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1},$$

which is set to counteract the fiscal externality term defined in Proposition 4.

Proposition 5 shows how to correct banks’ deposit rates so that they internalize the fiscal externality that their choices generate. Importantly, the existing literature has not previously identified this fiscal externality as the relevant object of interest that defines the optimal ex-ante regulation of banks. Consistent with Equation (30), an increase in the deposit rate offered by banks varies overall welfare according to $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1}$. Proposition 5 shows that this object can be expressed as the marginal change in the expected funding shortfall, augmented to include the cost of public funds. We show in the Appendix that this derivative accounts for the increased resource loss faced by taxpayers in the case of bank failure and the induced change in the unconditional probability of bank failure. Note that, even if there are no fiscal costs, implying that $\kappa(T(s)) = 0$, there is a role for corrective regulation emerging from the fact

²¹Note that we consider two extreme scenarios. In one, there is no ex-ante regulation, so banks freely choose their deposit rate. In the other one, regulation is perfectly targeted. In practice, the set of policy instruments available to policymakers may be constrained. In that case, our results in this paper are key inputs for the optimal second-best policy (Dávila and Walther, 2020).

that banks do not internalize that taxpayers must pay for funding shortfalls.²²

In general, the implementation of the optimal ex-ante corrective policy is not unique, although in this particular case a single instrument affecting the choice of deposit rate is sufficient. Because the funds used to pay for deposit insurance are raised through distortionary taxation, any Pigovian corrective policy in which the deposit insurance authority raises revenue may generate a “double-dividend” (Goulder, 1995). That is, a policy that corrects the ex-ante behavior of banks and at the same time reduces the need for raising revenue when required improves welfare along two different margins. The double-dividend logic supports an implementation of the optimal corrective policy through a deposit insurance fund financed with deposit insurance premia paid by participating banks. However, if the return of the deposit insurance fund is less than the return earned by the banks themselves, it may be preferred to set a different type of ex-ante corrective policy, like a deposit rate ceiling. We highlight the distinction between the corrective role of ex-ante policies (optimal corrective deposit insurance premium) versus their revenue-raising role (fairly-priced deposit insurance premium) in the following remark.

Remark. Optimal corrective regulation vs. fairly-priced deposit insurance. The existing literature has emphasized the study of deposit insurance schemes that are fairly-priced or actuarially fair. A deposit insurance fund is said to be actuarially fair if deposit insurance premia are such that the deposit insurance fund breaks even in expectation. Our formulation shifts the emphasis from setting deposit insurance premia that cover the expected fiscal cost to implementing regulations that distort banks’ choices at the margin. This distinction is often blurred in existing discussions of deposit insurance premia. In Section D of the Online Appendix, we show how to account for risk choices in a more general framework, allowing for a form of risk-based premia.

4 Quantitative Application: Revisiting the 2008 Change in Coverage

On October 3, 2008, the level of coverage in the US changed from \$100,000 to \$250,000. In this section, we apply our framework to that particular scenario. That is, we describe how a policymaker, armed with our framework, would have set the optimal level of coverage in early 2008, sometime before the moment in which the change in coverage took place. We study this specific scenario because it is the one for which we can obtain the most credible measures of the relevant sufficient statistics.²³

Initially, in Section 4.1, we describe how to measure the empirical counterparts of the sufficient statistics identified in Proposition 2 and implement the appropriate directional test for whether to increase or decrease the level of coverage.²⁴ We explain why our test finds that an increase in the level of coverage was desirable and discuss the associated welfare gains.

²²The exact expression for $\frac{\partial V_m(\tau)}{\partial R_1}$, given in Equation (A2) in the Appendix, shows that the optimal corrective policy must in general account for aggregate and systematic risk. In the context of optimally setting deposit insurance premia, a similar argument has been emphasized by Pennacchi (2006), Acharya, Santos and Yorulmazer (2010), and Lucas (2019), among others.

²³All measures of the sufficient statistics are in principle state- and time-dependent. The advantage of focusing on a specific scenario is that we can construct credible measures for that particular situation.

²⁴Note that we abstract from changes in bank behavior associated with changes in the level of coverage. Given our results in Section 3, we should interpret our results as the marginal welfare change associated with a change in coverage which is implemented along with the optimal ex-ante regulation.

Next, in Section 4.2, using these sufficient statistics — along with additional information — as calibration targets for our structural model, we explore the quantitative results that the model generates. We first provide a welfare decomposition in terms of the marginal benefits and costs identified in Section 2. Then, we conduct sensitivity analysis on the model parameters and describe how the optimal level of coverage varies for alternative scenarios/sets of parameters. Finally, we describe the distributional consequences of changing the level of coverage.

We draw four main conclusions. First, we find that the welfare gains from increasing the level of coverage when starting from low levels of coverage are very large. This result implies that having some form of deposit insurance is highly valuable. Second, given our assumptions, we find that the optimal level of coverage in the scenario that we consider would have been \$381,000. This magnitude is larger than the \$250,000 that was chosen, but is perhaps more aligned with the extended guarantees that were implemented soon after. Third, we explain why a drop in confidence (modeled as an increase in the probability of a sunspot) is associated with a higher optimal level of coverage. We also explain why an increase in the riskiness of bank investments is associated with a lower optimal level of coverage. Finally, we find that increasing the level of coverage increases the welfare of most depositors most of the time but not always. In particular, there are situations in which large depositors may be worse off when the level of coverage increases.

4.1 Direct Measurement of the Sufficient Statistics

Throughout the whole Section 4, we focus on measuring social welfare for a hypothetical representative bank.²⁵ In order to avoid relying on account- or depositor-level information, we make use of the test characterized in Proposition 2, which expresses the change in social welfare induced by a change in the level of coverage in terms of variables aggregated at the bank level. Moreover, to better map the model to observables, we focus on the marginal welfare change (expressed in dollars) per deposit account, given by $\frac{dW}{d\delta}/\bar{G}$, where $\bar{G} = \int dG(i)$ denotes the mass of deposit accounts in our representative bank.

Formally, starting from Proposition 2, we can express $\frac{dW}{d\delta}/\bar{G}$ as follows:

$$\frac{dW}{d\delta}/\bar{G} \approx q^F \left(-\frac{\partial \log q^F}{\partial \delta} \frac{\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)}{\bar{G}} - q^{T^+|F} \mathbb{E}_s^F [\kappa'(\cdot) | T > 0] \frac{\int_{i \in \mathcal{PI}} dG(i)}{\bar{G}} \right), \quad (31)$$

where q^F denotes the probability of bank failure, $\frac{\partial \log q^F}{\partial \delta} \equiv \frac{\partial q^F/q^F}{\partial \delta}$ denotes the semi-elasticity of bank failure with respect to a change in the level of coverage, $\frac{\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)}{\bar{G}}$ corresponds to the resource losses per account in case of failure, $q^{T^+|F}$ corresponds to the probability of facing a funding shortfall conditional on bank failure, $\mathbb{E}_s^F [\kappa'(\cdot) | T > 0]$ denotes the average marginal cost of public funds whenever these have to be paid, and $\frac{\int_{i \in \mathcal{PI}} dG(i)}{\bar{G}}$ is the share of partially insured deposit accounts. Consequently, once $\frac{dW}{d\delta}/\bar{G}$ is measured, we can scale up or down the size of the welfare gains/losses

²⁵Ideally, if more disaggregated data were available, one would first measure the relevant sufficient statistics for each bank and then aggregate these measures to conduct system-wide assessments. Differences in bank-specific sufficient statistics would account for differences in banks' funding, e.g., wholesale vs. retail, and the composition of banks' investments, among other characteristics.

Table 1: Direct Measurement: Sufficient Statistics

	Description	Variable	Value
	Probability of bank failure	q^F	2.5%
Mg. Benefit	Sensitivity of log-failure probability to DI limit	$\frac{\partial \log q^F}{\partial \delta}$	$-\frac{0.3}{150,000}$
	Resource losses per account after failure	$\int (C^N(j, s^*) - C^F(j, s^*)) dH(j) / \bar{G}$	\$13,810
Mg. Cost	Conditional probability of funding shortfall	$q^{T+ F}$	1
	Expected net marginal cost of public funds	$\mathbb{E}_s^F [\kappa'(\cdot) T > 0]$	0.15
	Share of partially insured deposit accounts	$\int_{i \in \mathcal{P}\mathcal{I}} dG(i) / \bar{G}$	0.064

Note: Table 1 includes the baseline measures of the relevant sufficient statistics. The probability of bank failure as well as the sensitivity of the probability of bank failure to a change in the coverage limit are based on CDS data, as described in the text. The measure of resource losses per account after failure combines information from Martin, Puri and Ufier (2017) with estimates from Granja, Matvos and Seru (2017) and Bennett and Unal (2015). The choice of the conditional probability of funding shortfall is based on the behavior of the Deposit Insurance Fund, as explained in the text. The marginal cost of public funds is consistent with Kleven and Kreiner (2006) and Dahlby (2008). The share of partially insured depositors comes from Martin, Puri and Ufier (2017).

associated with a change in the level of coverage according to the number of deposit accounts in a given bank.

We interpret the horizon of the model as a one-year period in the data. We summarize our preferred measures of the sufficient statistics required to compute Equation (31) in Table 1. Next, we describe the data sources that support those choices. Note that we factor out the probability of failure q^F in Equation (31), which allows us to express the marginal benefit of increasing coverage in terms the semi-elasticity $\frac{\partial \log q^F}{\partial \delta}$, instead of $\frac{\partial q^F}{\partial \delta}$. Hence, for a given value of the semi-elasticity $\frac{\partial \log q^F}{\partial \delta}$, the probability of failure q^F does not affect the sign of the $\frac{dW}{d\delta} / \bar{G}$, only its magnitude.

Probability of bank failure Measures of bank failure probabilities can be based on historical occurrences of bank failures or extracted from the expectations of market participants who trade CDS (Credit Default Swaps) on banks. A direct estimate of historical bank failure probabilities, using the FDIC Historical Statistics on Banking between 1934 and 2017, yields estimates of yearly failure probabilities of roughly 0.42%. This historical estimate is implausibly low to describe the actual probability of failure in early 2008.

We also use CDS data (from Markit) to compute yearly implied default probabilities for the sample of banks for which this instruments is traded — see Section F of the Online Appendix for a detailed explanation of our calculations with CDS data. We find an average implied default probability across banks and trading dates between January and June of 2008 of 1.23%.²⁶ However, the average implied default probability across banks on the date of the policy change, October 3, 2008, was 6.67%. Given these estimates, we select 2.5% as our baseline measure for q^F .

Marginal benefit Here we describe the measures of the sufficient statistics that determine the marginal benefit of changing the level of coverage. First, by using the change in the implied probability of failure around the change in the level of coverage from \$100,000 to \$250,000 we can provide a sense of

²⁶For reference, the average implied default probability during the post-crisis period 2012-2014 was 1.58%, while the average between 2004 and 2014 is 1.1%.

how failure probabilities react to changes in the level of coverage. In the Online Appendix, we document that the average proportional reduction in the implied probability of failure among the banks for whom failure probabilities went down was roughly 13%, and that average failure probabilities moved from 6.67% to 6.11%. Unfortunately, this approach is fraught with difficulties since the change in the level of coverage is not random and is only one of the measures in the Emergency Economic Stabilization Act passed on that date. We also document average proportional reductions in failure probabilities on October 14, 2008 of roughly 53%, in which the FDIC guaranteed in full noninterest-bearing transaction accounts. Given these estimates, we suppose that a change in the level of coverage of \$150,000 is associated with a proportional reduction of 30% in the probability of bank failure, that is, we set

$$\frac{\partial \log q^F}{\partial \delta} = -\frac{0.3}{150,000} = -2 \times 10^{-6}. \quad (32)$$

Next, we must compute the consumption difference at the bank level between failure and no-failure equilibria. To do so, we leverage Lemma 1. First, to simplify the computation, we set $\frac{\kappa(T(s^*))}{\rho_1(s^*)\bar{D}_0} \approx 0$ and $\lambda R_1 \approx 0$, which barely affects the final calculation. In that case, we can express the term $\frac{\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)}{\bar{G}}$ as a function of three terms, as in Equation (33) below. First, we need the net return on assets, $\rho_2(s^*) - 1$, which the FDIC reports to be roughly 1%. Second, we need the deadweight losses of default, $1 - \chi(s^*)$, which we set to 0.28, consistent with the recovery rate on bank assets after failure of 72% estimated in Granja, Matvos and Seru (2017) — see also Bennett and Unal (2015). Finally, we need the ratio of bank assets to deposit accounts, $\frac{\rho_1(s^*)\bar{D}_0}{\bar{G}}$, which we take from Martin, Puri and Ufier (2017). They report that the bank they study has roughly 42,000 accounts and 2 billion in assets, which implies an assets-to-accounts ratio of $\frac{\$2B}{42,000} = \$47,619$. Therefore, our best measure of the resource losses per account after failure is

$$\frac{\int (C^N(j, s^*) - C^F(j, s^*)) dH(j)}{\bar{G}} = \left(\underbrace{\rho_2(s^*) - 1}_{0.01} + \underbrace{1 - \chi(s^*)}_{0.28} \right) \underbrace{\frac{\rho_1(s^*)\bar{D}_0}{\bar{G}}}_{\frac{\$2B}{42,000}} = \$13,810. \quad (33)$$

Marginal cost We now turn to the marginal cost estimates. First, we approach the measurement of $q^{T^+|F}$ and $\mathbb{E}_s^F[\kappa'(\cdot) | T > 0]$ as a joint task. By setting $q^{T^+|F}$ to 1 and $\mathbb{E}_s^F[\kappa'(\cdot) | T > 0] = 0.15$, our choices imply that every marginal dollar promised to partially insured depositors is associated with an average deadweight loss of 15%. We set our measure for the net marginal cost of public funds to be somewhat higher than the 13% estimate for the US from Kleven and Kreiner (2006).²⁷ Our choice of $q^{T^+|F} = 1$ is based on the evidence — included in the Online Appendix — that the Deposit Insurance Fund faced a negative balance in 2009 and 2010, which we interpret as widespread funding shortfalls. Since managing a deposit insurance fund in general may be costly, by setting $q^{T^+|F} = 1$ we can map the cost of transferring resources to partially insured depositors in case of failure to the choice of $\mathbb{E}_s^F[\kappa'(\cdot) | T > 0]$.²⁸ Finally,

²⁷The estimate of 13% is within the lower end of estimates. Through alternative methods, Ballard, Shoven and Whalley (1985) find a range of estimates between 0.17 and 0.56. See Dahlby (2008) for a comprehensive review of the literature.

²⁸As we show in Section E.2 of the Online Appendix, the marginal cost of public funds ought to be linked to the deadweight losses associated with keeping resources in a deposit insurance fund (commonly invested in low-maturity treasuries and other

we use 6.4% as the percentage of partially insured deposit accounts, based on the description of the bank studied in [Martin, Puri and Ufier \(2017\)](#), which is, somewhat surprisingly, the only source to our knowledge that reports this information.

Test implementation/Welfare gains Combining the measures of the sufficient statistics that we have just introduced, we can use Equation (31) to compute the marginal welfare gain of changing the level of coverage. First, we find that the marginal welfare gain per deposit account associated with a one-dollar increase in the level of coverage, measured using our framework as of early 2008, is given by

$$\frac{dW}{d\delta} = 0.025 \left(\frac{0.3}{150,000} \times 13,810 - 0.15 \times 0.064 \right) = \$4.5 \times 10^{-4}, \quad (34)$$

where each of the elements in Equation (34) come either directly from Table 1 or indirectly through Equations (32) and (33). Since (34) has a positive sign, our approach implies that an increase in the level of coverage would have been welfare improving.

To gauge the magnitude of the gains, it is natural to normalize the marginal welfare gains by the level of assets of a bank instead of the number of accounts. Relying again on the information in [Martin, Puri and Ufier \(2017\)](#), we can express $\frac{dW}{d\delta}$ for a representative bank as follows:

$$\frac{dW}{d\delta} \Big|_{\text{assets}} = \underbrace{\frac{dW}{d\delta}}_{\$4.5 \times 10^{-4}} \times \underbrace{\frac{\bar{G}}{\text{assets}}}_{\frac{42,000}{\$2B}} = \$9.46 \times 10^{-9}, \quad (35)$$

which implies that the marginal welfare gain per dollar of banks assets associated with a one-dollar increase in the level of coverage is $\$9.46 \times 10^{-9}$. Therefore, Equation (35) implies that an increase in coverage of \$100,000 is associated with a welfare gain of 0.000946 (9.46bps) per dollar of bank assets.

Finally, we can use our estimate of the marginal welfare gain per level of assets in Equation (35) to find an estimate for the whole banking sector. Given that the level of assets for the whole banking sector in 2008 was of roughly \$14 trillion, we can compute the welfare gain of a dollar increase in the level of coverage for the whole banking sector as follows:

$$\frac{dW}{d\delta} \Big|_{\text{all banks}} = \underbrace{\frac{dW}{d\delta}}_{\$9.46 \times 10^{-9}} \times \underbrace{\text{total bank assets}}_{\$14T} = \$1.32 \times 10^5. \quad (36)$$

Therefore, Equation (36) implies that an increase in coverage of \$100,000 is associated with a welfare gain of \$13.2 billion for the whole banking sector. We should note that the \$13.2 billion estimate measures yearly flow welfare gains, which makes it a non-negligible magnitude. However, we should also note our measurement exercise is local, so extrapolating far away from the pre-existing level of coverage may overestimate the potential gains from increasing coverage. For that reason, it may be useful to rely on a fully specified model, as we do next.

low-yield securities) and transferring them to banks, relative to keeping these funds inside the banks. At the margin, one would expect that the costs of raising funds through bank contributions and other forms of taxation are roughly equal, which justifies our choices.

Since one of our goals is to guide future measurement efforts, we conclude with the following remark.

Remark. Implications for future measurement. There are three objects that warrant additional measurement efforts. First, the main challenge of the direct measurement approach is to find appropriate values for the semi-elasticity $\frac{\partial \log q^F}{\partial \delta}$. Changes in the level of coverage are often a response to banks' distress, which obviously biases naive estimates of this semi-elasticity. Our approach suggests that finding quasi-experimental variation in δ , perhaps exploiting a change in the level of coverage unrelated to bank profitability and failure probabilities, can be highly informative for policymakers. Second, better measures of bank assets relative to the number of deposit accounts can be highly informative. This ratio is important since the marginal benefit of increasing the level of coverage is linked to the level of bank assets while the marginal cost is linked to the number of (partially insured) accounts. Finally, having more detailed information on the composition of fully insured and partially insured accounts is important. While it is common to report measures of uninsured and insured deposits as a whole, our results show that what is relevant at the margin is whether an account is partially insured or not, not as much the amount of total insured and uninsured deposits. We hope that our results spur further effort to measure, report, and monitor the relevant sufficient statistics that we have identified.

4.2 Model-Based Quantification

While the direct measurement approach has the advantage of sidestepping the need to fully specify model primitives, it cannot be used, for instance, to think about the optimal level of coverage, at least given the current sets of available measures.²⁹ We now describe how our results can be used in the context of a fully specified quantitative model. We first explain the calibration of the model, followed by a decomposition of the welfare impact of policy changes. In both cases, we rely on our theoretical characterization of sufficient statistics. By explicitly computing the sufficient statistics in a fully specified model, we provide an intermediate step between primitives and welfare assessments.³⁰ Finally, we conduct sensitivity analysis on the model parameters and describe the distributional consequences of changing the level of coverage.

Calibration Here we describe our choice of functional forms and parameter values, which we report in Table 2. A period in the model coincides with a year. For the purpose of reporting the model parameters, we choose \$100,000 as the unit of account. That is, for instance, $\delta = 1$ corresponds to a level of coverage of \$100,000.

We combine a mix of externally chosen parameters and internally calibrated targets. As in the direct measurement exercise, we choose targets consistent with the early-2008 period, so the model is calibrated for a level of coverage of $\delta = 1$. Importantly, as shown in Table OA-1 in the Online Appendix, our calibration is designed to match the measures of sufficient statistics reported in Table 1 that we used

²⁹The direct measurement approach could be used to find the optimal level of coverage if one were able to construct measures of the sufficient statistics for different levels of coverage.

³⁰The results from this approach should be of interest to the growing quantitative structural literature on banking, since our characterization allows us to provide further insights into how to interpret the normative implications of calibrated structural models.

Table 2: Parameter Values — Calibration

		Parameter	Value
Depositors	Distribution of Deposits	μ_D	-3.8
		σ_D	2.2
	Endowment Early Depositors	$y_1(i, s)$	3
	Endowment Late Depositors	$y_2(i, s)$	3.13
	Early Depositor Share	λ	0.05
	Sunspot Probability	π	0.3
	Utility Curvature	γ	1.5
Banks	Return on Assets	μ_s	0.08
		σ_s	0.033
		φ	0.25
	Interest Rate	R_1	0.02
	Default Recovery Rate	χ_1	1.051
		χ_2	0.147
		χ_3	1
Taxpayers	Deadweight Loss	κ_1	0.13
		κ_2	5.5
	Endowment Taxpayers	$Y(\tau, s)$	5.02

Note: The bounds on the distribution of deposits are $[\underline{D}, \overline{D}] = [0.01, 15]$. The bounds on the distribution of returns are $[\underline{s}, \overline{s}] = [1, 1.35]$.

for the direct measurement exercise.

First, we describe the parameters that relate to depositors. We assume that the distribution of deposit accounts/balances is log-normally distributed, with parameters (μ_D, σ_D) and a truncated support $[\underline{D}, \overline{D}]$. By following this approach, we are effectively normalizing $\overline{G} = \int_{i \in I} dG(i) = 1$, so our welfare calculations can be interpreted on a per-account basis.³¹ We choose $\mu_D = -3.8$, $\sigma_D = 2.2$, and $[\underline{D}, \overline{D}] = [0.01, 1.5]$ to jointly match i) a share of partially insured accounts of 6.4% reported in [Martin, Puri and Ufier \(2017\)](#), ii) a share of insured deposits of 62% reported by the FDIC, and iii) a median and mean deposit balance of \$6,000 and \$30,000, on the larger end of values from the Survey of Consumer Finances, which only includes households.

We further assume that depositors' outside sources of wealth scale proportionally with the level of their deposits, that is, $Y_1(i, s) = y_1(s) D_0(i)$ and $Y_2(i, s) = y_2(s) D_0(i)$. We set $y_1(s) = 3$ and $y_2(s) = 3.075$, implying that deposits account for roughly 25% of a depositor's endowment and that the first-best equilibrium rate set by a utilitarian bank is 1.6%. Without direct evidence on withdrawals, we choose a small share of early depositors, $\lambda = 0.05$, letting the other parameters modulate the likelihood of bank failure.

When needed, we assume that depositors have isoelastic utility with an elasticity of intertemporal substitution $\frac{1}{\gamma}$, that is, $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In our baseline parametrization, we set $\gamma = -c \frac{U''(c)}{U'(c)} = 1.5$, a conventional choice. As discussed in Section F of the Online Appendix, the choice of γ does not affect

³¹The only source of heterogeneity among depositors in the quantitative model is the level of deposits, so there is a one-to-one mapping between i and $D_0(i)$.

most of the conclusions in this section, since we measure welfare changes as described in Proposition 2.

Next, we describe how we jointly select the probability of a sunspot and the parameters that relate to banks. We initially normalize the date 2 return to be $\rho_2(s) = s$ and assume that the date 1 return takes the form

$$\rho_1(s) = 1 + \varphi(s - 1),$$

which is consistent with the assumptions on $\rho_1(s)$ made in Section 2. We assume that the state s is log-normally distributed with a truncated support $[\underline{s}, \bar{s}] = [1, 1.35]$. While the choice of \bar{s} barely affects the results, the choice of \underline{s} does matter to pin down the likelihood of fundamental failures. We jointly choose $\mu_s = 0.08$, $\sigma_s = 0.033$, $\varphi = 0.25$, $\underline{s} = 1$, and $\pi = 0.3$ to target the following five moments: i) a probability of failure without deposit insurance ($\delta = 0$) of 15%, consistent with pre-FDIC failure rates; ii) a probability of failure at the preexisting level of coverage ($\delta = 1$) of 2.5%, as in our direct measurement approach; iii) a probability of fundamental failure of 2%, consistent with the observed failure rates in 2009/2010; iv) a sensitivity of the log-failure probability to the deposit insurance limit of -0.2 , targeting our measure in Section 4.1; and v) a level of resource losses per account after failure of 0.138, also targeting our measure in Section 4.1.

We parametrize the recovery rate/deadweight losses of bank failure according to

$$\chi(s) = \chi_1(s - \chi_3)^{\chi_2},$$

where we set χ_1 , χ_2 , and χ_3 so that $\chi(\underline{s}) = 0$, $\chi(\bar{s}) = 1$, and so that the average deadweight loss is equal to 28%, as measured by Granja, Matvos and Seru (2017). We set $R_1 = 1.02$, consistent with rates on savings accounts in early 2008.³²

Finally, we consider a marginal cost of public funds $\kappa(T)$ of the exponential-affine form:

$$\kappa(T) = \frac{\kappa_1}{\kappa_2} \left(e^{\kappa_2 T} - 1 \right),$$

for which the parameter $\kappa_1 = \kappa'(0)$ represents the marginal cost of public funds for a small intervention and the parameter $\kappa_2 = \frac{\kappa''(T)}{\kappa'(T)}$ modulates how quickly the cost of public funds increase with T . We set $\kappa_1 = 0.13$, consistent with the estimate in Kleven and Kreiner (2006), and $\kappa_2 = 5.5$, to match an average marginal cost of public funds of 0.15. When needed, we assume that the endowment of taxpayers $Y(\tau, s)$ is equivalent to the total endowment of early depositors in the 97.5th percentile of the distribution, consistent with the fact that most taxes are paid by individuals in the top 5% of the income distribution. As in the case of γ , the choice of $Y(\tau, s)$ does not impact the results of this section.

Optimal Level of Coverage/Welfare Decomposition Given our calibration, Figure 6 illustrates how the marginal welfare change, $\frac{dW}{d\delta}$, and its determinants vary with the level of coverage δ . In the body of the paper, we report and describe at all times money-metric welfare gains/losses, as described in Equation (21). There are three findings worth highlighting.

³²While we focus on the case in which the deposit rate is predetermined, there is scope to further explore the quantitative implications of the model under perfect and imperfect regulation.

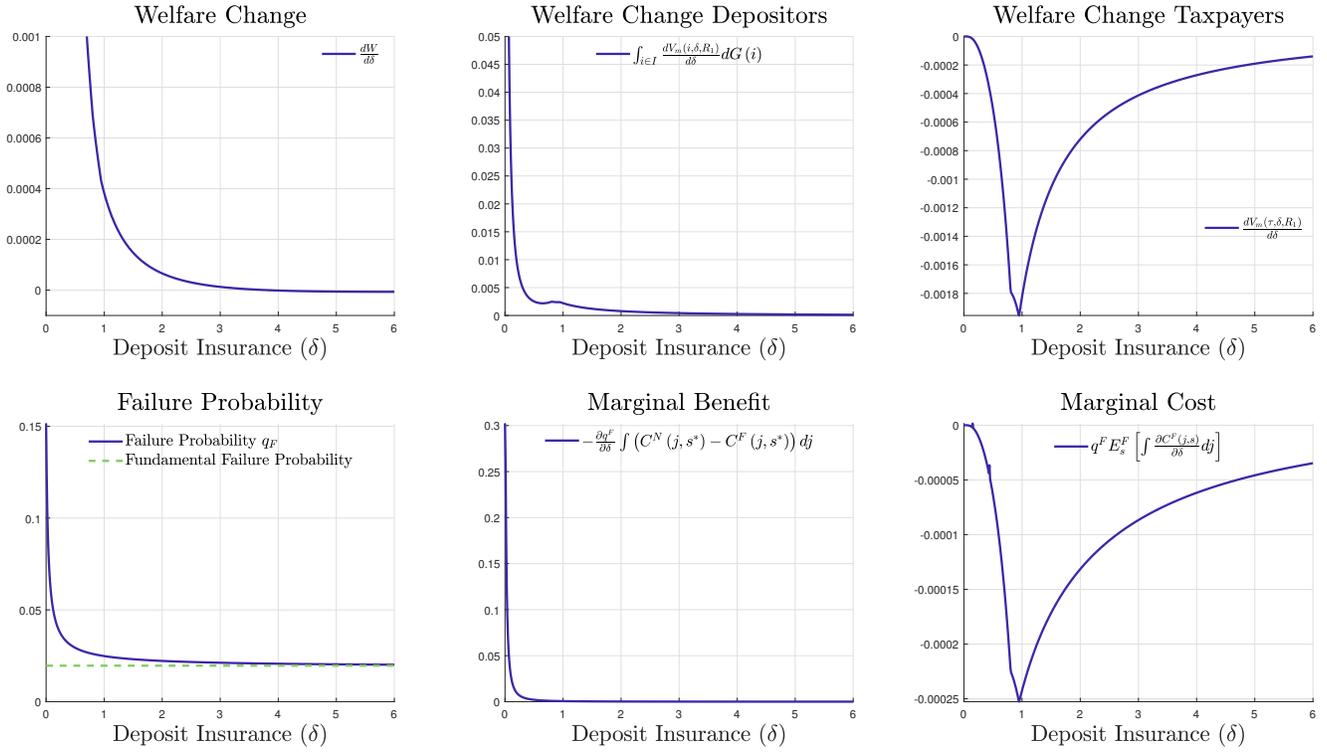


Figure 6: Social Welfare Decomposition

Note: The top left panel in Figure 6 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage, $\frac{dW}{d\delta}$, as described in Equation (26). The top middle and right panels respectively show the welfare change for depositors, $\lambda \frac{dV_m(i, \delta, R_1)}{d\delta} + (1 - \lambda) \frac{dV_m(i, \ell, \delta, R_1)}{d\delta}$, and taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$. The bottom left panel shows the probability of failure, $q^F(\delta, R_1)$, and the probability of fundamental failure $F(\hat{s}(R_1))$. The bottom middle and right panels show the marginal benefit and marginal cost of increasing the level of coverage, given by $-\frac{\partial q^F}{\partial \delta} \int (C^N(j, s^*) - C^F(j, s^*)) dH(j)$ and $q^F E_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right]$, respectively.

First, the marginal welfare gains from increasing the level of coverage are remarkably high for very low levels of coverage, which implies that the welfare gains from having a deposit insurance system at all are very large. Intuitively, when δ is low, the marginal impact of δ on reducing the probability of failure $\frac{\partial q^F}{\partial \delta}$ is large, since the behavior of many small depositors is affected at the margin, which directly increases the marginal benefit of increasing the level of coverage. Also, the funding shortfall is small when the level of coverage is low, since banks' resources are often enough to cover the claims of insured depositors, which contributes to a low marginal cost of increasing the level of coverage. Put together, both channels make the welfare gains from increasing δ when $\delta \approx 0$ very large. This should not be surprising, given that arguably no other financial regulation has had a more significant impact than the introduction of deposit insurance.

Second, the marginal welfare gains from decreasing the level of coverage are small for levels of coverage higher than the optimum. This result implies that, in case of doubt, the losses from overshooting on the level of coverage are smaller than the losses from setting a level that is too low.

Third, we find that the optimal level of coverage given our calibration is $\delta^* = \$381,000$, which is a level of coverage larger than the actual level of coverage chosen by policymakers on October 2008,

although it is roughly of the same order of magnitude. There are several observations that may explain the discrepancy between our solution and the policy chosen. First of all, there is no reason to believe that policymakers followed our framework or, even if they inadvertently considered the same tradeoffs, reached the same conclusions we did on the measurement of the relevant inputs. Second, it is the case that other guarantees were implemented around that time, which may explain why we find a higher value for the optimal level of coverage when it is the single policy instrument. Third, our model abstracts from joint accounts, for which coverage limits are higher. With adequate information on joint accounts, it would be possible to re-calibrate our model and find optimal levels of coverage for single and joint accounts.

In the remaining of this section, we explore the sensitivity of our quantitative results to changes in the sunspot probability and the riskiness of bank returns. By exploring these scenarios, we can explain how the optimal policy varies with financial conditions and business cycle conditions.³³ As shown in Figure OA-10 in the Online Appendix, the optimal level of coverage can vary significantly when parameters change. Finally, we describe the distributional implications of changing the level of coverage for depositors with different deposit balances.

Sensitivity Analysis: Sunspot Probability and Banks' Riskiness Changes in the level of confidence in the economy, captured by the sunspot probability π (a high value of π has the interpretation of low confidence), do affect the desirability of changing the level of coverage. Given our calibration, changes in π have a very strong impact on $\frac{dW}{d\delta}$ and, ultimately, on the optimal level of coverage. When π is high, the likelihood of a run in the multiple equilibria region is large, which makes increasing the level of coverage a very powerful tool, increasing the marginal benefit of higher coverage. While the marginal cost of increasing δ also grows, because — all else equal — failure is more likely, the increase in the marginal benefit is substantially larger, which implies that the optimal level of coverage is increasing in π .

By studying how the predictions of our framework change with the riskiness of banks' investments σ_s , we aim to capture different business cycle conditions, in the form of a risk shock to banks' investment. A higher value of σ_s unambiguously reduces the welfare of taxpayers, since negative realizations of s , in which bank failures are more prevalent and costly, are more likely to occur. However, a higher value of σ_s has an ambiguous impact on depositors' welfare, depending on the level of δ . When the level of coverage is low, the increased volatility generates worse and more frequent failures, lowering depositors' welfare. When the level of coverage is high, depositors benefit from the increase in volatility, since they receive all the upside when bank returns are high, but are shielded from bank failure by the generous level of coverage. Given our calibration, the net welfare effects on taxpayers' and depositors' imply that high riskiness of banks' investments is associated with lower levels of the optimal coverage limit.

Distributional Considerations Even though we have purposefully focused on reaching conclusions based on aggregate outcomes at the bank level since those are appealing from a practical perspective,

³³See Figures OA-8 and OA-7 in Section F of the Online Appendix. There we also explore the sensitivity of our results to the cost of public funds, which can be interpreted as fiscal shock. Note that we do not re-calibrate the model when conducting the sensitivity analyses, so our results can be interpreted as comparative statics exercises.

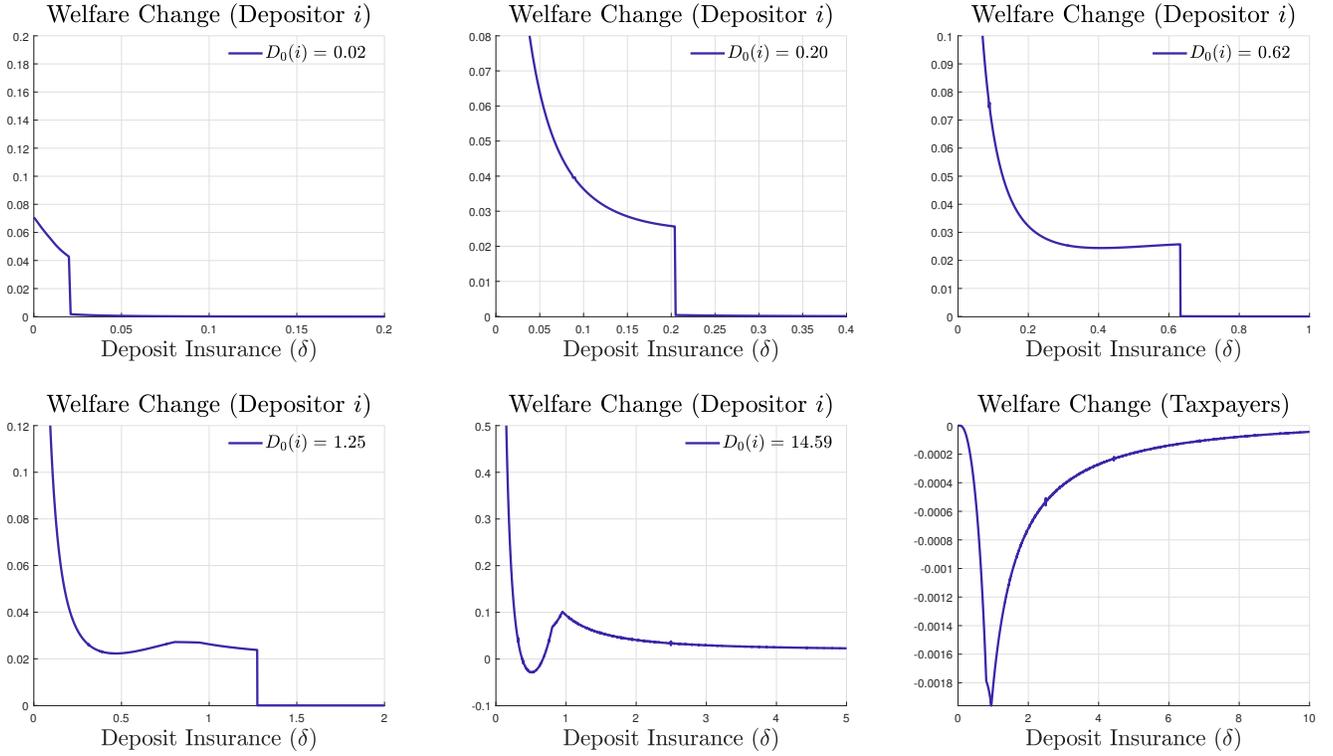


Figure 7: Distributional Considerations

Note: The top panels and the left and middle bottom panels in Figure 7 show the change in welfare induced by a marginal change in the level of deposit insurance coverage, $\lambda \frac{dV_m(i, \epsilon, \delta, R_1)}{d\delta} + (1 - \lambda) \frac{dV_m(i, \ell, \delta, R_1)}{d\delta}$, for different depositors with deposit balances $\{0.02, 0.2, 0.62, 1.25, 14.59\}$, which correspond to percentiles $\{0.25, 0.75, 0.9, 0.95, 0.99\}$ in the distribution of deposits. The bottom right panel shows the change in welfare induced by a marginal change in the level of deposit insurance coverage for taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$.

varying the level of coverage has different distributional implications for different individuals. Before concluding, we would like to illustrate some of the distributional considerations of our policy.

Figure 7 illustrates the money-metric marginal welfare change for depositors with different balances, $\lambda \frac{dV_m(i, \epsilon, \delta, R_1)}{d\delta} + (1 - \lambda) \frac{dV_m(i, \ell, \delta, R_1)}{d\delta}$, and taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$. Several insights are worth highlighting. First, the money-metric welfare change for depositors is strictly positive for most depositors most of the time, but not always. While depositors as a whole are better off by increasing the level of coverage, since they receive a net transfer from taxpayers, it is conceivable that some large depositors can be made worse off by increasing the level of coverage at times. This may occur to the largest depositors in situations in which an increase in δ substantially reduces the recovery rate on partially insured deposits $\alpha_F(s)$. For instance, Figure 7 shows that an increase in the level of coverage when δ is roughly 0.6 makes the depositor in the middle bottom plot locally worse off.

Second, since $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$ is weakly negative for taxpayers, a Pareto improvement could only be potentially reached in a scenario in which for sufficiently low levels of coverage the funding shortfall is zero across all states. In our model, taxpayers are worse off for any level of deposit insurance since the funding shortfall is non-zero for some states when $\delta = 0$, but quantitatively the funding shortfall is very small for low levels of coverage, which implies that there should always be support for having a positive level of coverage when welfare is reasonably aggregated.

Finally, Figure 7 illustrates how depositors that become fully insured turn out to be effectively inframarginal in determining $\frac{dW}{d\delta}$ and δ^* .³⁴ This is an important takeaway from modeling a rich cross-section of depositors. In other words, changes in the level of coverage mostly affect at the margin those deposit accounts that are partially insured, so information about these depositors is critical when considering changes in the level of coverage.

5 Extensions

Before concluding, we explain how our framework accommodates additional features relevant for the determination of deposit insurance.

5.1 Formal Extensions

First, we describe the conclusions from the three formal extensions executed in Section D of the Online Appendix. There we show that Proposition 1 continues to be valid exactly or suitably modified once we relax several of the model assumptions.

Banks’ Moral Hazard: General Portfolio and Investment Decisions In our baseline formulation, neither depositors nor banks make portfolio decisions. Including both sets of decisions is important to allow banks or depositors to adjust their risk-taking behavior in response to changes in the level of coverage — these effects are also often referred to as moral hazard. We show that introducing a consumption-savings decision and portfolio choices for depositors does not modify the set of sufficient statistics already identified under perfect regulation. However, allowing unregulated banks to make investment choices requires accounting for a new set of fiscal externality terms. The new set of fiscal externalities, which capture the direct effects of banks’ changes in behavior on taxpayers’ welfare should be targeted with asset-side and liability-side regulation.

Alternative Equilibrium Selection Mechanisms In the baseline model, we assume that depositors coordinate following an exogenous sunspot. However, it is well-known that incorporating dispersed information among depositors would yield a unique equilibrium — see Goldstein and Pauzner (2005). Even though studying a global game model, as in Goldstein and Pauzner (2005) or Allen et al. (2018), is appealing because the probability of failure is endogenously determined by fundamentals, we show that the particular information structure considered and the equilibrium selection procedure only enter in the expression of $\frac{dW}{d\delta}$ through the sufficient statistics identified in this paper.

General Equilibrium Spillovers/Macroprudential Considerations In our baseline formulation, bank decisions do not affect equilibrium prices or other aggregate variables. However, when the decisions made by banks affect aggregate variables, for instance, asset prices, further exacerbating the possibility of a bank failure, the optimal deposit insurance formula should incorporate a macroprudential correction.

³⁴We say effectively because the marginal benefit of an increase in coverage is strictly positive even for fully insured depositors. This occurs because an increase in coverage reduces the probability of failure and fully insured depositors are better off when banks do not fail, as implied by Equation (17). This effect is quantitatively very small, as Figure 7 shows.

In this extension, we model liquidation spillovers as a tractable way to capture equilibrium interactions in interbank markets. Importantly, we show that ex-ante regulation can directly target the wedges caused by aggregate spillovers, so the characterization of $\frac{dW}{d\delta}$ remains valid under perfect ex-ante regulation.

5.2 Additional Channels

Here we discuss how incorporating additional features relevant for the determination of the optimal level of coverage into our framework may affect our main characterization. Even though extending our model along several of these dimensions may require additional information to account for the welfare impact of deposit insurance policies, the channels identified in this paper do not vanish.

Lender of Last Resort/Bailout In our baseline formulation, we exclusively consider the level of deposit insurance coverage as a single policy instrument. In practice, in addition to the level of coverage, banks often receive alternative forms of government support through lender of last resort policies or bailouts. Within our framework, we can interpret this form of intervention as a state-contingent policy that increases the resources available to banks in certain states. Formally, we can consider the counterpart to Equation (3):

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \bar{D}_0 - \Omega(s) \right) + \Lambda(s) < \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \bar{D}_0 - \Omega(s) \right) + \Lambda(s) \geq \bar{D}_1(s), \end{aligned}$$

where $\Lambda(s)$ captures the size of the ex-post intervention in state s . Propositions 1 and 3, and the associated sufficient statistics, remain valid in this case when $\Lambda(s)$ is predetermined or when it can be optimally designed. However, for our results to be meaningful, it must be the case that the lender of last resort policy is imperfect and unable to fully eliminate the existence of coordination failures.

Multiple Deposit Accounts Our baseline model does not explicitly allow a given depositor to have multiple accounts in different banks, although, in practice, deposit limits are defined at the account level in most countries. However, as long as there is a cost of switching/opening deposit accounts, making deposits partially inelastic, which is consistent with the evidence in Egan, Hortaçsu and Matvos (2017), Proposition 1 remains valid once suitably reinterpreted. In this case, as we discuss in Section 2, the relevant marginal cost of varying δ needs to account for the insured/partially insured status of a given account, not the status of an individual depositor. See Shy, Stenbacka and Yankov (2016) for a model in which depositors can explicitly open multiple deposit accounts.

Equityholders/Debtholders/Liquidity Benefits Since we build on the Diamond and Dybvig (1983) framework, our baseline formulation does not incorporate a role for equityholders and debtholders, and does not allow for demand deposits to have non-pecuniary benefits. Richer funding structures call for extending Propositions 1 through 4 to include all stakeholders. Beyond that, on aggregate, the sufficient statistics that we identify already capture differences in capital structure choices across banks. For instance, one would expect banks with more fragile capital structures — perhaps more likely to face debt rollover concerns — to be more likely to fail and potentially more sensitive to interventions.

Departures from Bank Value Maximization: Imperfect Competition and Agency Frictions

Allowing for imperfect competition and agency frictions that depart from value maximization introduces additional terms when extending Propositions 1 through 4, with an a priori indeterminate impact on the optimal level of coverage. For instance, increasing the level of coverage when banks have market power can encourage banks to make safer investments to preserve their franchise value but also to make less careful investment and funding choices. Similarly, if non-competitive banks fund projects with negative net present values, ex-ante regulation would be needed. In general, if there are specific regulatory tools designed to ex-ante correct for the impact of imperfect competition or managerial distortions, it would be optimal to make use of them, allowing us to rely again on our baseline characterization.

Unregulated Sector Throughout the paper, every bank is subject to deposit insurance and ex-ante regulation. Our framework implies that all sectors subject to coordination failures benefit from deposit insurance guarantees. In general, the optimal level of coverage must account for leakage from the regulated deposit sector into unregulated sectors, and vice versa. In models with imperfect policy instruments — see Plantin (2014) and Ordoñez (2018) in the context of shadow banking, or Dávila and Walther (2020) more generally — the optimal policy absent an unregulated sector that we characterize in this paper is a key input to the second-best optimal policy when some agents or activities cannot be perfectly regulated.

6 Conclusion

We have developed a framework to study the tradeoffs associated with the optimal determination of deposit insurance coverage. Our analysis identifies the set of variables that have a first-order effect on welfare and become sufficient statistics for assessing changes in the level of deposit insurance coverage. Our results provide a step forward towards building a microfounded theory of measurement for financial regulation that can be applied to a wide variety of environments.

There are several avenues for further research that build on our results. From a theoretical perspective, exploring alternative forms of asset- or liability-side competition among banks or introducing dynamic considerations are non-trivial extensions worth exploring. However, the most promising implications of this paper for future research come from the measurement perspective. Recovering robust and credible estimates of the sufficient statistics that we have uncovered in this paper, in particular, the sensitivity of bank failures to changes in the level of coverage and the relevant fiscal externalities associated with such a policy change, has the potential to directly discipline future regulatory decisions.

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ONLINE APPENDIX

FOR ONLINE PUBLICATION ONLY

Sections **A** and **B** of this Online Appendix include proofs and derivations of the results in Sections 2 and 3 in the text. Section **C** of this Online Appendix includes extended analytical derivations. Section **D** describes the three formal extensions discussed in Section 5 in the body of the paper, along with the associated proofs. Section **E** presents additional results, including the introduction of a deposit insurance fund, a comparison between traditional and generalized social welfare weights, and a more general directional test. Finally, Section **F** includes additional material supporting the quantitative application in Section 4 of the paper.

A Proofs and Derivations: Section 2

Proof of Proposition 1. (Exact directional test) Exploiting the envelope theorem and the fact that $\frac{\partial C^N(j,s)}{\partial \delta} = 0$, we can express the the marginal impact of varying δ on early depositors, late depositors, or taxpayers' welfare as follows:

$$\begin{aligned} \frac{dV(j, \delta, R_1)}{d\delta} &= \overbrace{\int_{\underline{s}}^{\hat{s}} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} dF(s)}^{=q^F \mathbb{E}_s^F \left[U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} \right]} \\ &\quad + \left[U(C^F(j, s^*)) - U(C^N(j, s^*)) \right] \frac{\partial q^F}{\partial \delta}, \end{aligned}$$

where q^F is defined in Equation (12) and $\frac{\partial q^F}{\partial \delta} = \pi f(s^*) \frac{\partial s^*}{\partial \delta}$. Therefore, we can express $\frac{dV_m(j, \delta, R_1)}{d\delta}$ as follows:

$$\frac{dV_m(j, \delta, R_1)}{d\delta} = -\frac{\partial q^F}{\partial \delta} \left[\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right] + q^F \mathbb{E}_s^F \left[\frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right].$$

Given these, Equation (22) in Proposition 1 follows immediately.

Proof of Lemma 1. (Aggregate consumption difference between failure and no-failure equilibria) Equation (23) follows from Equations (16) and (17), as well as from the definitions of $\alpha_F(s)$ and $\alpha_N(s)$. See Section C for a step-by-step derivation.

Proof of Lemma 2. (Aggregate consumption change induced by a change in coverage in failure states) Note that

$$\lambda \int_{i \in I} \frac{\partial C_1^F(i, e, s)}{\partial \delta} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial C_2^F(i, \ell, s)}{\partial \delta} dG(i) = \int_{i \in PI} dG(i),$$

whenever $T(s) > 0$; it is 0 otherwise. Note also that $\frac{\partial C^F(\tau, s)}{\partial \delta} = -(1 + \kappa'(T(s))) \int_{i \in \mathcal{PI}} dG(i)$ whenever $T(s) > 0$; it is 0 otherwise. Equation (24) follows immediately from these two observations. See Section C for a step-by-step derivation. Note also that Equation (25) follows from Equation (24) after defining

$$q^{T^+|F} = \frac{\int_{\underline{\hat{s}}}^{\hat{s}} \mathbb{I}[T(s) > 0] dF(s) + \pi \int_{\hat{s}}^{s^*} \mathbb{I}[T(s) > 0] dF(s)}{\int_{\underline{\hat{s}}}^{\hat{s}} dF(s) + \pi \int_{\hat{s}}^{s^*} dF(s)},$$

where $\mathbb{I}[\cdot]$ denotes the indicator function.

Proof of Proposition 2. (Approximate directional test for a change in the level of coverage)

Note that we can linearly approximate $U(C^N(j, s^*))$ around $C^F(j, s^*)$ as follows:

$$U(C^N(j, s^*)) - U(C^F(j, s^*)) \approx U'(C^F(j, s^*)) (C^N(j, s^*) - C^F(j, s^*)). \quad (\text{A1})$$

Hence, Equation (26) follows immediately from Equation (22) after substituting Equation (A1), setting $\omega(j) = 1, \forall j$, and $m(j, s) = 1, \forall j, \forall s$.

B Proofs and Derivations: Section 3

Proof of Proposition 3. (Directional test for δ under perfect ex-ante regulation)

For a planner that aggregates welfare using uniform generalized social welfare weights, $\frac{dW}{d\delta} = \int \frac{dV_m(j, \delta, R_1)}{d\delta} dH(j)$, where $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{dV(j, \delta, R_1)}{U'(C^F(j, s^*))}$ and where $\frac{dV(j, \delta, R_1)}{d\delta}$ is given by

$$\begin{aligned} \frac{dV(j, \delta, R_1)}{d\delta} &= \int_{\underline{s}}^{\hat{s}} U'(C^F(j, s)) \frac{dC^F(j, s)}{d\delta} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(j, s)) \frac{dC^F(j, s)}{d\delta} dF(s) \\ &+ (1 - \pi) \int_{\hat{s}}^{s^*} U'(C^N(j, s)) \frac{dC^N(j, s)}{d\delta} dF(s) + \int_{s^*}^{\bar{s}} U'(C^N(j, s)) \frac{dC^N(j, s)}{d\delta} dF(s) \\ &+ [U(C^F(j, \hat{s})) - U(C^N(j, \hat{s}))] (1 - \pi) f(\hat{s}) \frac{d\hat{s}}{d\delta} + [U(C^F(j, s^*)) - U(C^N(j, s^*))] \pi f(s^*) \frac{ds^*}{d\delta}, \end{aligned}$$

and the impact on consumption can be decomposed as $\frac{dC^F(j, s)}{d\delta} = \frac{\partial C^F(j, s)}{\partial \delta} + \frac{\partial C^F(j, s)}{\partial R_1} \frac{dR_1}{d\delta}$ and $\frac{dC^N(j, s)}{d\delta} = \frac{\partial C^N(j, s)}{\partial R_1} \frac{dR_1}{d\delta}$, while the impact on the thresholds \hat{s} and s^* satisfies $\frac{d\hat{s}}{d\delta} = \frac{\partial \hat{s}}{\partial R_1} \frac{dR_1}{d\delta}$, and $\frac{ds^*}{d\delta} = \frac{\partial s^*}{\partial \delta} + \frac{\partial s^*}{\partial R_1} \frac{dR_1}{d\delta}$.

In the particular case of taxpayers, note that we can express $\frac{dV(\tau, \delta, R_1)}{d\delta}$ as follows:

$$\begin{aligned} \frac{dV(\tau, \delta, R_1)}{d\delta} &= \int_{\underline{s}}^{\hat{s}} U'(C^F(\tau, s)) \frac{dC^F(\tau, s)}{d\delta} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(\tau, s)) \frac{dC^F(\tau, s)}{d\delta} dF(s) \\ &+ [U(C^F(\tau, \hat{s})) - U(C^N(\tau, \hat{s}))] (1 - \pi) f(\hat{s}) \frac{d\hat{s}}{d\delta} + [U(C^F(\tau, s^*)) - U(C^N(\tau, s^*))] \pi f(s^*) \frac{ds^*}{d\delta}, \end{aligned}$$

where it is the case that $\frac{dC^F(\tau, s)}{d\delta} = \frac{\partial C^F(\tau, s)}{\partial \delta} + \frac{\partial C^F(\tau, s)}{\partial R_1} \frac{dR_1}{d\delta}$. Note that we rely on the fact that $\frac{dC^N(\tau, s)}{d\delta} = 0$.

Under the optimal regulation, R_1 must satisfy $\frac{\partial W}{\partial R_1} = \int \frac{\partial V_m(j, \delta, R_1)}{\partial R_1} dH(j)$, where $\frac{\partial V_m(j, \delta, R_1)}{\partial R_1} = \frac{\frac{\partial V(j, \delta, R_1)}{\partial R_1}}{U'(C^F(j, s^*))}$ and $\frac{\partial V(j, \delta, R_1)}{\partial R_1} = 0$, and where

$$\begin{aligned} \frac{\partial V(j, \delta, R_1)}{\partial R_1} &= \int_{\underline{s}}^{\hat{s}} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial R_1} dF(s) \\ &+ (1 - \pi) \int_{\hat{s}}^{s^*} U'(C^N(j, s)) \frac{\partial C^N(j, s)}{\partial R_1} dF(s) + \int_{s^*}^{\bar{s}} U'(C^N(j, s)) \frac{\partial C^N(j, s)}{\partial R_1} dF(s) \\ &+ [U(C^F(j, s^*)) - U(C^N(j, s^*))] \pi f(s^*) \frac{\partial s^*}{\partial R_1} + [U(C^F(j, \hat{s})) - U(C^N(j, \hat{s}))] (1 - \pi) \frac{\partial \hat{s}}{\partial R_1} f(\hat{s}). \end{aligned}$$

In the particular case of taxpayers:

$$\begin{aligned} \frac{\partial V(\tau, \delta, R_1)}{\partial R_1} &= \int_{\underline{s}}^{\hat{s}} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s) \\ &+ [U(C^F(\tau, s^*)) - U(C^N(\tau, s^*))] \pi f(s^*) \frac{\partial s^*}{\partial R_1} + [U(C^F(\tau, \hat{s})) - U(C^N(\tau, \hat{s}))] (1 - \pi) \frac{\partial \hat{s}}{\partial R_1} f(\hat{s}), \end{aligned}$$

where we use the fact that $\frac{\partial C^N(\tau, s)}{\partial R_1} = 0$. Therefore, given the optimal ex-ante regulation, we can express

$\frac{dW}{d\delta}$ as follows:

$$\begin{aligned} \frac{dW}{d\delta} = & \int \left(\underbrace{\int_{\underline{s}}^{\hat{s}} \frac{U'(C^F(j,s))}{U'(C^F(j,s^*))} \frac{\partial C^F(j,s)}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} \frac{U'(C^F(j,s))}{U'(C^F(j,s^*))} \frac{\partial C^F(j,s)}{\partial \delta} dF(s)}_{=q^F \mathbb{E}_s^F \left[\frac{U'(C^F(j,s))}{U'(C^F(j,s^*))} \frac{\partial C^F(j,s)}{\partial \delta} \right]} \right) dH(j) \\ & + \pi f(s^*) \frac{\partial s^*}{\partial \delta} \int \left[\frac{U(C^F(j,s^*)) - U(C^N(j,s^*))}{U'(C^F(j,s^*))} \right] dH(j), \end{aligned}$$

which corresponds exactly to Equation (28) in the text.

Optimal Deposit Rate Determination Formally, note that $\frac{\partial V(i,x,\delta,R_1)}{\partial R_1}$, which is the key input of Equation (29) is given by

$$\begin{aligned} \frac{\partial V(i,x,\delta,R_1)}{\partial R_1} = & q^F \mathbb{E}_s^F \left[U'(C_t^F(i,x,s)) \frac{\partial C_t^F(i,x,s)}{\partial R_1} \right] + (1-q^F) \mathbb{E}_s^N \left[U'(C_t^N(i,x,s)) \frac{\partial C_t^N(i,x,s)}{\partial R_1} \right] \\ & + (1-\pi) \left(U(C_t^F(i,x,\hat{s})) - U(C_t^N(i,x,\hat{s})) \right) \frac{\partial \hat{s}}{\partial R_1} f(\hat{s}) \\ & + \pi \left(U(C_t^F(i,x,s^*)) - U(C_t^N(i,x,s^*)) \right) \frac{\partial s^*}{\partial R_1} f(s^*), \end{aligned}$$

where $\mathbb{E}_s^F[\cdot]$ and $\mathbb{E}_s^N[\cdot]$ respectively denote conditional expectations over failure and no-failure states. The intensive margin effects are captured by $\frac{\partial C_t^N(i,x,s)}{\partial R_1}$ and $\frac{\partial C_t^F(i,x,s)}{\partial R_1}$. We show in Section C of the Online Appendix that the term $\frac{\partial C_t^N(i,x,s)}{\partial R_1}$ takes on positive values for early depositors and negative values for late depositors. These effects capture the ex-ante risk sharing gains between early and late types generated by a higher deposit rate.³⁵ $\frac{\partial C_t^F(i,x,s)}{\partial R_1}$ is positive for fully insured depositors, but can turn negative for partially insured depositors. We show that $\int \frac{\partial C_t^F(i,x,s)}{\partial R_1} dG(i)$ is weakly positive, which can be interpreted as a form of moral hazard, at least on aggregate. Intuitively, banks internalize that an increase in the deposit rate increases the consumption of insured depositors in failure states, at the expense of taxpayers. On the extensive margin, banks take into account that offering a high deposit rate makes bank failures more likely. This is captured by the positive sign of $\frac{\partial \hat{s}}{\partial R_1}$ and $\frac{\partial s^*}{\partial R_1}$, which, combined with the sign of $U(C_t^F(i,x,s)) - U(C_t^N(i,x,s))$, which we show to be negative, makes increasing R_1 less desirable.

Proof of Proposition 4. (Directional test for δ without ex-ante regulation) Without ex-ante regulation, R_1 is given by the solution to

$$\lambda \int_{i \in I} \frac{\partial V_m(i,e,\delta,R_1)}{\partial R_1} dG(i) + (1-\lambda) \int_{i \in I} \frac{\partial V_m(i,\ell,\delta,R_1)}{\partial R_1} dG(i) = 0,$$

where $\frac{\partial V_m(i,e,\delta,R_1)}{\partial R_1}$ and $\frac{\partial V_m(i,\ell,\delta,R_1)}{\partial R_1}$ follow from the characterization of $\frac{\partial V_m(i,x,\delta,R_1)}{\partial R_1}$ above.

³⁵When $s^* \rightarrow \underline{s}$ and $\hat{s} \rightarrow \underline{s}$, there are no bank failures in equilibrium, and Equation (??) defines the optimal arrangement that equalizes marginal rates of substitution across types with the expected marginal rate of transformation, determined by $\rho_2(s)$. In that case, banks set R_1 exclusively to provide insurance between early and late types across deposit levels.

Therefore, we can express $\frac{dW}{d\delta}$ as follows:

$$\begin{aligned}
\frac{dW}{d\delta} &= \int \frac{dV_m(j, \delta, R_1)}{d\delta} dH(j) \\
&= \lambda \int_{i \in I} \frac{\partial V_m(i, e, \delta, R_1)}{\partial \delta} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial \delta} dG(i) + \frac{\partial V_m(\tau, \delta, R_1)}{\partial \delta} \\
&\quad + \left(\lambda \int_{i \in I} \frac{\partial V_m(i, e, \delta, R_1)}{\partial R_1} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial R_1} dG(i) + \frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \right) \frac{dR_1}{d\delta} \\
&= \lambda \int_{i \in I} \frac{\partial V_m(i, e, \delta, R_1)}{\partial \delta} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial \delta} dG(i) + \frac{\partial V_m(\tau, \delta, R_1)}{\partial \delta} + \frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \frac{dR_1}{d\delta},
\end{aligned}$$

where $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1}$ is given by

$$\begin{aligned}
\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} &= q^F \mathbb{E}_s^F \left[\frac{U'(C^F(\tau, s))}{U'(C^F(\tau, s^*))} \frac{\partial C^F(\tau, s)}{\partial R_1} \right] + \left[\frac{U(C^F(\tau, s^*)) - U(C^N(\tau, s^*))}{U'(C^F(\tau, s^*))} \right] \pi f(s^*) \frac{\partial s^*}{\partial R_1} \\
&\quad + \left[\frac{U(C^F(\tau, \hat{s})) - U(C^N(\tau, \hat{s}))}{U'(C^F(\tau, s^*))} \right] (1 - \pi) \frac{\partial \hat{s}}{\partial R_1} f(\hat{s}).
\end{aligned} \tag{A2}$$

Under the same conditions used in the approximation in Proposition 2, we can express $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1}$ as

$$\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \approx - \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1}.$$

Note that Equation (A9) guarantees that this fiscal externality term is negative, as described in the body of the paper.

Proof of Proposition 5. (Optimal ex-ante deposit rate regulation) The choice of R_1 under perfect ex-ante regulation is given by

$$\lambda \int_{i \in I} \frac{\partial V_m(i, e, \delta, R_1)}{\partial R_1} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial R_1} dG(i) + \frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} = 0.$$

The choice of R_1 subject to a cost τ_{R_1} per unit of R_1 offered ($-\tau_{R_1} R_1$) corresponds to

$$\lambda \int_{i \in I} \frac{\partial V_m(i, e, \delta, R_1)}{\partial R_1} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\partial V_m(i, \ell, \delta, R_1)}{\partial R_1} dG(i) - \tau_{R_1} = 0.$$

Therefore, the optimal regulation is associated with a wedge $\tau_{R_1} = -\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1}$, which, as shown above, can be approximated as $\frac{\partial V_m(\tau, \delta, R_1)}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1}$.

C Extended Analytical Results

In this section, to facilitate the understanding of the results, we provide detailed analytical characterizations of several outcomes of the model.

C.1 Thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$

The threshold $\hat{s}(R_1)$ is given by the minimum among the value of s that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1 - \lambda) R_1, \quad (\text{A3})$$

and \bar{s} . Note that this threshold is not a function of δ . Similarly, the value of $s^*(\delta, R_1)$ is given by the minimum among the value of s that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1 - \lambda) R_1 \zeta(\delta, R_1), \quad (\text{A4})$$

and \bar{s} , where $\zeta(\delta, R_1) \equiv \frac{\int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i)}{D_0 R_1}$ denotes the share of insured deposits.³⁶ Note that $\zeta(\delta, R_1) \in [0, 1]$, $\frac{\partial \zeta}{\partial \delta} \geq 0$, and $\frac{\partial \zeta}{\partial R_1} \leq 0$.

The left-hand side of both equations, $z(s, R_1) \equiv \frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}}$, is a decreasing function of s , since both $\rho_1(s)$ and $\rho_2(s)$ are monotonically increasing in s and $\rho_2(s)$ strictly so. Since we have assumed that $\rho_2(\underline{s}) < 1$, it is always guaranteed that $\hat{s}(R_1) > \underline{s}$. Note that

$$\lim_{\rho_2(s) \rightarrow 1^+} z(s, R_1) = \infty \quad \text{and} \quad \lim_{\rho_1(s), \rho_2(s) \rightarrow \infty} z(s, R_1) < 0,$$

which is sufficient to establish that both Equation (A3) and Equation (A4) have a unique solution strictly higher than $\hat{s}(R_1)$. Since $\zeta(\delta, R_1) \in [0, 1]$, we can also conclude that $\hat{s}(R_1) \leq s^*(\delta, R_1)$, with equality only when all deposits are insured, $\delta \rightarrow \overline{\overline{D}}R_1$, since $\lim_{\delta \rightarrow \overline{\overline{D}}R_1} \zeta(\delta, R_1) = 1$.

In order for $s^*(\delta, R_1) < \bar{s}$, as in Figure 3 in the text, it must be that $\rho_1(\bar{s}) > R_1$. In that case, there are three regions (unique failure equilibrium, multiple equilibria, and unique no-failure equilibrium) for any value of δ , including $\delta = 0$. If $\rho_1(\bar{s}) < R_1$, then there are only two regions (unique failure equilibrium and multiple equilibria) for small values of δ .

The relevant comparative statics for $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ are the following. First, it follows directly from Equation (A4) that

$$\frac{\partial s^*}{\partial \delta} \leq 0,$$

since its right-hand side is increasing in δ . The effect of δ on $s^*(\delta, R_1)$ is modulated by the behavior of

³⁶When $\rho_1(s) = 1$ and $\rho_2(s) = s$, the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ can be explicitly computed as

$$\hat{s}(R_1) = \frac{(1 - \lambda) R_1}{1 - \lambda R_1} \quad \text{and} \quad s^*(\delta, R_1) = \min \left\{ \rho_2^{-1} \left(\frac{(1 - \lambda) R_1}{(1 - \lambda) R_1 - \frac{R_1 - 1}{\zeta}} \right), \bar{s} \right\}.$$

$\zeta(\delta, R_1)$. Second, similar arguments imply that

$$\frac{\partial \hat{s}}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^*}{\partial R_1} \geq 0. \quad (\text{A5})$$

Finally, it also follows immediately from Equations (A3) and (A4) that

$$\frac{\partial \hat{s}}{\partial \lambda} \geq 0 \quad \text{and} \quad \frac{\partial s^*}{\partial \lambda} \geq 0,$$

since the right-hand side of both equations is decreasing in λ . Intuitively, all else constant, an increase in the mass of early depositors, who withdraw their deposits inelastically, increases the probability of failure.

C.1.1 Parametric assumptions

Under the following parametric assumptions: $\rho_2(s) = s$ and $\rho_1(s) = 1 + \varphi(s - 1)$, we can express the thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ implied by Equations (A3) and (A4) as follows:

$$\begin{aligned} \hat{s}(R_1) &= \left\{ s \mid 0 = \varphi s^2 - (\lambda R_1 + \varphi - 1)s - (1 - \lambda)R_1 \right\} \\ s^*(\delta, R_1) &= \left\{ s \mid 0 = \varphi s^2 - ((1 - (1 - \lambda)\zeta)R_1 + \varphi - 1)s - (1 - \lambda)R_1\zeta \right\}, \end{aligned}$$

where

$$\hat{s}(R_1) = \frac{\lambda R_1 + \varphi - 1 \pm \sqrt{(\lambda R_1 + \varphi - 1)^2 + 4\varphi(1 - \lambda)R_1}}{2\varphi} \quad (\text{A6})$$

$$s^*(\delta, R_1) = \frac{(1 - (1 - \lambda)\zeta)R_1 + \varphi - 1 \pm \sqrt{((1 - (1 - \lambda)\zeta)R_1 + \varphi - 1)^2 + 4\varphi(1 - \lambda)R_1\zeta}}{2\varphi}. \quad (\text{A7})$$

Both quadratic equations have a unique positive root. Note that by setting $\zeta = 1$, Equation (A7) collapses to Equation (A6). Note also that $\hat{s}(R_1)$ is exclusively a function of φ , λ , and R_1 . The threshold $s^*(\delta, R_1)$ is a function of φ , λ , R_1 , and ζ , which in turn depends on R_1 , δ , and the distribution of deposits $G(i)$. Explicitly characterizing $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ simplifies the numerical solution of the model.

C.2 Probability of bank failure

Starting from Equation (12), we can express $\frac{\partial q^F}{\partial \delta}$ and $\frac{\partial q^F}{\partial R_1}$ as follows:

$$\begin{aligned} \frac{\partial q^F}{\partial \delta} &= \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0 \\ \frac{\partial q^F}{\partial R_1} &= (1 - \pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} \geq 0, \end{aligned}$$

where the sign results follow from Equation (A5). As explained in the text, higher levels of coverage reduce the probability of failure, holding R_1 constant, while higher deposit rates increase the probability of failure, holding δ constant.

C.3 Insured/uninsured deposits

A fraction of a depositor's claims at date 1 is insured, while the remaining is potentially uninsured. Formally,

$$D_0(i) R_1 = \underbrace{\min \{D_0(i) R_1, \delta\}}_{\text{Insured}} + \underbrace{\max \{D_0(i) R_1 - \delta, 0\}}_{\text{Uninsured}}. \quad (\text{A8})$$

We can express aggregate insured and uninsured deposits at date 1 as follows:

$$\begin{aligned} \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) &= \int_{i \in \mathcal{FI}} D_0(i) R_1 dG(i) + \delta \int_{i \in \mathcal{PI}} dG(i) \quad (\text{Insured Deposits}) \\ \int_{i \in I} \max \{D_0(i) R_1 - \delta, 0\} dG(i) &= \int_{i \in \mathcal{PI}} (D_0(i) R_1 - \delta) dG(i), \quad (\text{Uninsured Deposits}), \end{aligned}$$

where we formally define the sets of fully insured (\mathcal{FI}) and partially insured depositors (\mathcal{PI}) as

$$\begin{aligned} \mathcal{FI} &= \{i \mid D_0(i) R_1 \leq \delta\} \\ \mathcal{PI} &= \{i \mid D_0(i) R_1 > \delta\}. \end{aligned}$$

The counterpart of Equation (A8) in the aggregate is given by

$$\bar{D}_0 R_1 = \underbrace{\int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i)}_{\text{Insured Deposits}} + \underbrace{\int_{i \in I} \max \{D_0(i) R_1 - \delta, 0\} dG(i)}_{\text{Uninsured Deposits}}.$$

We repeatedly use the fact that

$$\begin{aligned} \frac{d \left(\int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) \right)}{dR_1} &= \int_{i \in \mathcal{FI}} D_0(i) dG(i) \\ \frac{d \left(\int_{i \in I} \max \{D_0(i) R_1 - \delta, 0\} dG(i) \right)}{dR_1} &= \int_{i \in \mathcal{PI}} D_0(i) dG(i), \end{aligned}$$

as well as

$$\begin{aligned} \frac{d \left(\int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) \right)}{d\delta} &= \int_{i \in \mathcal{PI}} dG(i) \\ \frac{d \left(\int_{i \in I} \max \{D_0(i) R_1 - \delta, 0\} dG(i) \right)}{d\delta} &= - \int_{i \in \mathcal{PI}} dG(i). \end{aligned}$$

Note that while $\hat{s}(R_1)$ only depends on R_1 and λ , $s^*(\delta, R_1)$ also depends on those two objects in addition to the whole distribution of deposits, through its impact on the share of insured deposits.

C.4 Properties of depositors' and taxpayers' consumption

C.4.1 Individual consumption levels

For reference, we reproduce here the expressions for depositors' equilibrium consumption in the cases of failure and no-failure:

$$\begin{aligned} C_t^F(i, x, \delta, R_1) &= \min\{D_0(i) R_1, \delta\} + \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} + Y_t(i, s) \\ C_1^N(i, e, R_1) &= D_0(i) R_1 + Y_1(i, s) \\ C_2^N(i, \ell, R_1) &= \alpha_N(s) D_0(i) R_1 + Y_2(i, s). \end{aligned}$$

The equilibrium objects $\alpha_F(s)$ and $\alpha_N(s)$ are given by

$$\begin{aligned} \alpha_F(s) &= \frac{\max\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\}}{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)} = \max\left\{1 - \frac{(R_1 - \chi(s) \rho_1(s)) \bar{D}_0}{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)}, 0\right\} \\ \alpha_N(s) &= \rho_2(s) \frac{\rho_1(s) - \lambda R_1}{(1 - \lambda) R_1}. \end{aligned}$$

The rate $\alpha_N(s)$ captures the additional gross return obtained by late depositors at date 2 when there is no bank failure.³⁷ The rate $\alpha_F(s)$ corresponds to the individual recovery rate on uninsured deposits in the case of bank failure. Note that $\alpha_F(s)$ is a function of s only through $\chi(s) \rho_1(s)$ and that $\alpha_N(s)$ is a function of s through $\rho_2(s)$ and $\rho_1(s)$. Note that $\alpha_N(s) D_0(i) R_1 = \rho_2(s) \frac{\rho_1(s) - \lambda R_1}{1 - \lambda} D_0(i)$. The fact $\rho_1(s) > R_1$ is incompatible with the existence of a failure equilibrium, which implies that $\alpha_F(s) < 1$. Given our assumptions on $\chi(s)$, $\rho_1(s)$, and $\rho_2(s)$, it follows that $\frac{\partial \alpha_F(s)}{\partial s} \geq 0$ and $\frac{\partial \alpha_N(s)}{\partial s} > 0$. It also follows that $\frac{\partial T(s)}{\partial s} \leq 0$.

Note that we can also express $\alpha_F(s)$ as

$$\alpha_F(s) = \frac{\max\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\}}{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)},$$

which implies that when $\alpha_F(s) > 0$, $T(s) = 0$, and when $\alpha_F(s) = 0$, $T(s) > 0$ (the funding shortfall is introduced in Equation (18)). Note also that, whenever $\alpha_F(s) > 0$, we can express $1 - \alpha_F(s)$ as

$$1 - \alpha_F(s) = \frac{(R_1 - \chi(s) \rho_1(s)) \bar{D}_0}{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)}.$$

Finally, note that we can express consumption ratios of the form $\frac{C^F(i, x, s)}{C^F(i, x, s^*)}$ as

$$\frac{C^F(i, x, s)}{C^F(i, x, s^*)} = \begin{cases} \frac{D_0(i) R_1 + Y_t(i, x, s)}{D_0(i) R_1 + Y_t(i, x, s^*)}, & \text{if } D_0(i) R_1 < \delta \\ \frac{\delta + \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} + Y_t(i, x, s)}{\delta + \alpha_F(s^*) \max\{D_0(i) R_1 - \delta, 0\} + Y_t(i, x, s^*)}, & \text{if } D_0(i) R_1 \geq \delta \end{cases}$$

³⁷Note that \hat{s} can also be defined as the value of s such that $\alpha_N(\hat{s}) = 1$.

for depositors and

$$\frac{C^F(\tau, s)}{C^F(\tau, s^*)} = \frac{Y(\tau, s) - T(s) - \kappa(T(s))}{Y(\tau, s^*) - T(s^*) - \kappa(T(s^*))}$$

for taxpayers. In order to understand whether $\frac{C^F(j, s)}{C^F(j, s^*)}$ takes values above or below 1 across failure states, it is necessary to understand how $\alpha_F(s)$, $T(s)$, $Y_t(i, s)$, and $Y(\tau, s)$ vary with s . Under the assumption that $Y_t(i, s)$ and $Y(\tau, s)$ are increasing in s , and, given that $\alpha_F(s)$ is increasing in s and $T(s)$ is decreasing in s , it follows that $\frac{C^F(j, s)}{C^F(j, s^*)} \leq 1$ for both depositors and taxpayers. Note that if $Y_t(i, s)$ is independent of s , $\frac{C^F(i, s)}{C^F(i, s^*)} = 1$ for fully insured depositors.

C.4.2 Comparative statics

We can show that $\alpha_F(s)$ is decreasing in both R_1 and δ , as follows:

$$\frac{\partial \alpha_F(s)}{\partial R_1} = \begin{cases} -\frac{\int_{i \in \mathcal{F}\mathcal{I}} D_0(i) dG(i) (\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i)) + (\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in \mathcal{I}} \min\{D_0(i) R_1, \delta\} dG(i)) \int_{i \in \mathcal{P}\mathcal{I}} D_0(i) dG(i)}{(\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i))^2} \leq 0, & \text{if } T(s) = 0 \\ 0, & \text{if } T(s) > 0 \end{cases}$$

$$\frac{\partial \alpha_F(s)}{\partial \delta} = \begin{cases} -\frac{(R_1 - \chi(s) \rho_1(s)) \bar{D}_0 \int_{i \in \mathcal{P}\mathcal{I}} dG(i)}{(\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i))^2} = -(1 - \alpha_F(s)) \frac{\int_{i \in \mathcal{P}\mathcal{I}} dG(i)}{\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i)} \leq 0, & \text{if } T(s) = 0 \\ 0, & \text{if } T(s) > 0, \end{cases}$$

since $R_1 - \chi(s) \rho_1(s) \geq 0$ in any failure equilibrium. Note that $\frac{\int_{i \in \mathcal{P}\mathcal{I}} dG(i)}{\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i)}$ is the ratio of partially insured accounts to uninsured deposits.

In no-failure states, depositors' consumption levels vary with R_1 as follows:

$$\frac{\partial C_1^N(i, e, s)}{\partial R_1} = D_0(i) \geq 0$$

$$\frac{\partial C_2^N(i, \ell, s)}{\partial R_1} = -\rho_2(s) \frac{\lambda}{1 - \lambda} D_0(i) \leq 0.$$

In no-failure states, depositors' consumption is not directly affected by δ , so $\frac{\partial C_t^N(i, s)}{\partial \delta} = 0$.

In failure states, we can derive the following comparative statics, which are relevant inputs for the characterization of the optimal deposit insurance policy:

$$\frac{\partial C_t^F(i, x, s)}{\partial R_1} = \begin{cases} D_0(i) \geq 0, & \text{if } D_0(i) R_1 < \delta \\ \alpha_F(s) D_0(i) + \frac{\partial \alpha_F(s)}{\partial R_1} (D_0(i) R_1 - \delta) \geq 0, & \text{if } D_0(i) R_1 \geq \delta \end{cases}$$

$$\frac{\partial C_t^F(i, x, s)}{\partial \delta} = \begin{cases} 0, & \text{if } D_0(i) R_1 < \delta \\ = (1 - \alpha_F(s)) \left(1 - \frac{\int_{i \in \mathcal{P}\mathcal{I}} dG(i)}{\int_{i \in \mathcal{I}} \max\{D_0(i) R_1 - \delta, 0\} dG(i)} (D_0(i) R_1 - \delta) \right) \geq 0, & \text{if } D_0(i) R_1 \geq \delta. \end{cases}$$

Hence, when $T(s) > 0$, $\alpha_F(s) = 0$ and $\frac{\partial \alpha_F(s)}{\partial \delta} = 0$, so $\frac{\partial C_t^F(i, x, s)}{\partial \delta} = 1$ for all uninsured depositors —

those for which $D_0(i) R_1 \geq \delta$. Note that when aggregated among depositors

$$\int_{i \in I} \frac{\partial C_t^F(i, x, s)}{\partial \delta} dG(i) = \begin{cases} 0, & \text{if } T(s) = 0 \\ \int_{i \in \mathcal{PT}} dG(i), & \text{if } T(s) > 0. \end{cases}$$

Therefore, we can express $\int \int_{i \in I} \frac{\partial C_t^F(i, x, s)}{\partial \delta} dG(i) dF(s) = \int_{i \in \mathcal{PT}} dG(i) \int \mathbb{I}[T(s) > 0] dF(s)$, where $\mathbb{I}[\cdot]$ denotes the indicator function. Although for some individual depositors $\frac{\partial C_t^F(i, x, s)}{\partial R_1}$ and $\frac{\partial C_t^F(i, x, s)}{\partial \delta}$ can take negative values (this is more likely to occur to depositors with large uninsured balances) since, as shown above, $\frac{\partial \alpha_F(s)}{\partial \delta} \leq 0$, we show below that the aggregate consumption response among depositors to δ and R_1 is positive.

We can derive similar comparative statics for taxpayers' consumption as follows:

$$\begin{aligned} \frac{\partial C^F(\tau, s)}{\partial R_1} = - (1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial R_1} &= \begin{cases} 0, & \text{if } T(s) = 0 \\ - (1 + \kappa'(\cdot)) \int_{i \in \mathcal{FT}} D_0(i) dG(i) \leq 0, & \text{if } T(s) > 0 \end{cases} \quad (\text{A9}) \\ \frac{\partial C^F(\tau, s)}{\partial \delta} = - (1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial \delta} &= \begin{cases} 0, & \text{if } T(s) = 0 \\ - (1 + \kappa'(\cdot)) \int_{i \in \mathcal{PT}} dG(i) \leq 0, & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

C.4.3 Individual consumption differences

We can express $C_1^N(i, e, s) - C_1^F(i, e, s)$ for early depositors as

$$\begin{aligned} C_1^N(i, e, s) - C_1^F(i, e, s) &= D_0(i) R_1 - \min\{D_0(i) R_1, \delta\} - \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} \\ &= \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}}, \end{aligned}$$

and similarly $C_2^N(i, \ell, s) - C_2^F(i, \ell, s)$ for late depositors as

$$\begin{aligned} C_2^N(i, \ell, s) - C_2^F(i, \ell, s) &= \alpha_N(s) D_0(i) R_1 - \min\{D_0(i) R_1, \delta\} - \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} \\ &= \underbrace{(\alpha_N(s) - 1) D_0(i) R_1}_{\text{Net Return}} + \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}}. \end{aligned}$$

C.5 Properties of aggregate consumption

C.5.1 Aggregate consumption differences

The aggregate change in consumption among early depositors is given by

$$\begin{aligned} \int_{i \in I} (C_1^N(i, e, s) - C_1^F(i, e, s)) dG(i) &= (1 - \alpha_F(s)) \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\ &= \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) - \max\left\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\right\} \\ &= \begin{cases} (R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i), & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

The aggregate change in consumption among late depositors is given by

$$\begin{aligned}
\int_{i \in I} (C_2^N(i, \ell, s) - C_2^F(i, \ell, s)) dG(i) &= (\alpha_N(s) - 1) \bar{D}_0 R_1 + (1 - \alpha_F(s)) \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\
&= (\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\
&\quad - \max\left\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\right\} \\
&= \begin{cases} (\alpha_N(s) R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ (\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i), & \text{if } T(s) > 0. \end{cases}
\end{aligned}$$

The aggregate change in consumption among depositors and taxpayers is given by

$$\begin{aligned}
\int (C^N(j, s) - C^F(j, s)) dH(j) &= \lambda \int_{i \in I} (C_1^N(i, s) - C_1^F(i, s)) dG(i) + (1 - \lambda) \int_{i \in I} (C_2^N(i, s) - C_2^F(i, s)) dG(i) + C^N(\tau, s) - C^F(\tau, s) \\
&= \begin{cases} \lambda(R_1 - \chi(s) \rho_1(s)) \bar{D}_0 + (1 - \lambda)(\alpha_N(s) R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ (1 - \lambda)(\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) + T(s) + \kappa(T(s)), & \text{if } T(s) > 0 \end{cases} \\
&= [(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1 - \chi(s)) \rho_1(s)] \bar{D}_0 + \kappa(T(s)).
\end{aligned}$$

Therefore, we can express $\int (C^N(j, s) - C^F(j, s)) dH(j)$ in Equation (23) as follows:

$$\int (C^N(j, s) - C^F(j, s)) dH(j) = [(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1 - \chi(s)) \rho_1(s)] \bar{D}_0 + \kappa(T(s)),$$

where $T(s) = \max\left\{\int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0, 0\right\}$.

C.5.2 Aggregate consumption levels

Aggregate consumption among depositors in the case of bank failure is given by

$$C_{\text{dep}} = \lambda \int_{i \in I} C_t^F(i, e, \delta, R_1) dG(i) + (1 - \lambda) \int_{i \in I} C_t^F(i, \ell, \delta, R_1) dG(i),$$

where

$$\begin{aligned}
C_{\text{dep}} &= \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) + \max\left\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\right\} + \bar{Y}(s) \\
&= \begin{cases} \chi(s) \rho_1(s) \bar{D}_0 + \bar{Y}(s), & \text{if } T(s) = 0 \\ \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) + \bar{Y}(s), & \text{if } T(s) > 0 \end{cases} \\
&= \max\left\{\chi(s) \rho_1(s) \bar{D}_0, \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i)\right\} + \bar{Y}(s),
\end{aligned}$$

where we define $\bar{Y}(s) = \lambda \int_{i \in I} Y_t(i, e, s) dG(i) + (1 - \lambda) \int_{i \in I} Y_t(i, \ell, s) dG(i)$.

Aggregate consumption among depositors and taxpayers in the case of bank failure is given by

$$\begin{aligned} C_{\text{dep}} + C^F(\tau) &= \max \left\{ \chi(s) \rho_1(s) \bar{D}_0, \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) \right\} \\ &\quad - \max \left\{ \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0, 0 \right\} - \kappa(T(s)) + \bar{Y}_j(s) \\ &= \chi(s) \rho_1(s) \bar{D}_0 - \kappa(T(s)) + \bar{Y}_j(s), \end{aligned}$$

where we define $\bar{Y}_j(s) = \lambda \int_{i \in I} Y_t(i, e, s) dG(i) + (1 - \lambda) \int_{i \in I} Y_t(i, e, s) dG(i) + Y(\tau, s)$.

Therefore we can easily calculate $\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j)$, which is a relevant input to set the optimal level of coverage, as well as $\int \frac{\partial C^F(j, s)}{\partial R_1} dH(j)$, as follows:

$$\begin{aligned} \int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) &= \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(T(s)) \int_{i \in \mathcal{P}\mathcal{I}} dG(i), & \text{if } T(s) > 0 \end{cases} \\ \int \frac{\partial C^F(j, s)}{\partial R_1} dH(j) &= \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(T(s)) \int_{i \in \mathcal{F}\mathcal{I}} D_0(i) dG(i), & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

Finally, aggregate consumption among depositors if there is no bank failure is given by

$$\begin{aligned} \lambda \int_{i \in I} C_t^N(i, e, \delta, R_1) dG(i) + (1 - \lambda) \int_{i \in I} C_t^N(i, \ell, \delta, R_1) dG(i) &= \lambda \bar{D}_0 R_1 + (1 - \lambda) \alpha_N(s) \bar{D}_0 R_1 + \bar{Y}(s) \\ &= \lambda \bar{D}_0 R_1 + \rho_2(s) (\rho_1(s) - \lambda R_1) \bar{D}_0 + \bar{Y}(s). \end{aligned}$$

C.6 Regularity conditions and limits

Continuity and differentiability of the problems faced by banks and regulators are guaranteed whenever distributions and parameters that vary with the realization of the state s are sufficiently smooth. The one potential source of non-differentiability that emerges in the model is related to the form of the fiscal costs. To guarantee that social welfare is differentiable, it must be that $\frac{dW}{d\delta}$ is continuous. For this to be the case, it must be that either $\min_s \{\chi(s) \rho_1(s)\} = 0$ or $\lim_{T \rightarrow 0} \kappa'(T) = 0$. Otherwise, for sufficiently low values of δ it is the case that there is no need to raise fiscal resources for any realization of s , so the second term in Equation (22) changes from 0 to a positive value at a point, making the planner's objective non-differentiable.

As usual in normative exercises, it is hard to guarantee the convexity of the planning problem (quasi-concavity of the planner's objective) in general: there are no simple conditions on primitives that guarantee that the planner's problem is well-behaved. In practice, for natural parametrizations of the model, $W(\delta)$ is well-behaved and features a single interior optimum. Similarly, it is not easy to establish the convexity of the problem solved by competitive banks (quasi-concavity of the bank's objective) to choose R_1 , although the problem solved by banks is also well-behaved in practice for standard parametrizations, utility, and distributional choices.

In Remark 3 on page 23, we make a statement about the behavior of $\frac{dW}{d\delta}$ in the limit when $\delta \rightarrow 0$.

Formally, we can write $\lim_{\delta \rightarrow 0^+} \frac{dW}{d\delta}$ as

$$\lim_{\delta \rightarrow 0^+} -\frac{\partial q^F}{\partial \delta} \int \left[U \left(C^N(j, s^*) \right) - U \left(C^F(j, s^*) \right) \right] dH(j) + \lim_{\delta \rightarrow 0^+} q^F \mathbb{E}_s^F \left[\int U' \left(C^F(j, s) \right) \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right].$$

Since $U \left(C^N(j, s^*) \right) - U \left(C^F(j, s^*) \right)$ is non-negative for both depositors and taxpayers, the sign of the first element is given by $\lim_{\delta \rightarrow 0^+} \left(-\frac{\partial q^F}{\partial \delta} \right)$. Similarly, since q^F is strictly positive for any δ , including $\delta = 0$, the sign of the second term depends on whether the average marginal cost of funds across states is zero or positive when $\delta \rightarrow 0$. In a previous version of this paper, we explored a scenario in which $\lim_{\delta \rightarrow 0^+} \left(-\frac{\partial q^F}{\partial \delta} \right) = 0$ and $q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right] \Big|_{\delta=0} < 0$. In that case, as long as there is a fiscal cost of paying for deposit insurance, increasing coverage around $\delta = 0$ may decrease welfare locally, since very small coverage levels are costly in equilibrium but are not enough to reduce the probability of bank failure. This is likely to be a very local result.

D Extensions

For simplicity, we study every extension separately, and focus on the characterization of marginal changes in the level of deposit insurance under perfect regulation of the deposit rate, although the analysis can be extended to other scenarios along the lines of Section 3. When appropriate, we discuss the implications for the optimal design of ex-ante regulation.³⁸

D.1 Banks' Moral Hazard: General Portfolio and Investment Decisions

In our baseline formulation, neither depositors nor banks make portfolio decisions.³⁹ Allowing for both sets of decisions is important to allow banks or depositors to adjust their risk-taking behavior in response to changes in the level of coverage — these effects are also often referred to as moral hazard. Depositors now have a consumption-savings decision at date 0 and a portfolio decision among K securities. In particular, depositors have access to $k = 1, 2, \dots, K$ assets, with returns $\theta_{1k}(s)$ at date 1 in state s for early depositors and returns $\theta_{2k}(s)$ at date 2 in state s for late depositors. Hence, the resources of early and late depositors are respectively given by $Y_1(i, e, s) = \sum_k \theta_{1k}(s) y_k(i)$ and $Y_2(i, \ell, s) = \sum_k \theta_{2k}(s) y_k(i)$. We preserve the structure of the distribution of deposits. Therefore, the budget constraint of depositors at date 0 is given by

$$\sum_k y_k(i) + D_0(i) + C_0(i) = Y_0(i), \quad (\text{A10})$$

where $Y_0(i)$, which denotes the initial endowment of depositor i , and $D_0(i)$ are primitives of the model. Subject to Equation (A10), the ex-ante utility of depositors now corresponds to

$$U(C_0(i)) + \lambda \mathbb{E}_s [U(C_1(i, e, s))] + (1 - \lambda) \mathbb{E}_s [U(C_2(i, \ell, s))], \quad (\text{A11})$$

where $C_1(i, e, s)$ and $C_2(i, \ell, s)$ respectively denote the consumption of early and late depositors with initial deposits $D_0(i)$ for a given realization of the state s . Depositors optimally choose their holdings of the different assets $y_k(i)$ to maximize their expected utility.

Additionally, banks have access to $h = 1, 2, \dots, H$ investment opportunities, which offer a gross return $\rho_{1h}(s)$ at date 1 and a return $\rho_{2h}(s)$ between dates 1 and 2 in state s . Hence, at date 0, banks must choose shares ψ_h for every investment opportunity such that $\sum_h \psi_h = 1$. We assume that banks liquidate an equal share of every type of investment at date 1. This is a particularly tractable formulation to introduce multiple investment opportunities. Our results could be extended to the case in which different investments have different liquidation rates at date 1 and banks have the choice of liquidating different investments in different proportions.

Given our assumptions, we can show that the counterpart to the failure threshold $\tilde{D}_1(s)$ in Equation

³⁸In this section, for simplicity, we do not make explicit some of the additional arguments of $V(i)$ and $V(\tau)$.

³⁹The endowments at dates 1 and 2 in the baseline model can be interpreted as the net payoffs on the rest of a depositor's portfolio.

(7) is given by

$$\tilde{D}_1(s) = \frac{(R_1 - \sum_h \rho_{1h}(s) \psi_h) \bar{D}_0}{1 - \frac{1}{\sum_h \rho_{2h}(s) \rho_{1h}(s) \psi_h}}, \quad (\text{A12})$$

allowing us to characterize the equilibrium thresholds \hat{s} and s^* as in the baseline model. It is equally straightforward to generalize the values taken by $\alpha_F(s)$, $\alpha_N(s)$, and $T(s)$. We characterize below the optimal choices of $y_k(i)$ and ψ_h by depositors and banks and focus again on the directional test for how welfare varies with the level of coverage.

Proposition 6. (Directional test for δ under general investment opportunities) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left(-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[\int \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dH(j),$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

Proposition 6 extends the results of the baseline model by showing that introducing a consumption-savings and portfolio choices for depositors does not modify the set of sufficient statistics already identified under perfect regulation. However, allowing unregulated banks to make investment choices requires accounting for a new set of fiscal externality terms. The new set of fiscal externalities, which capture the direct effects of banks' changes in behavior on taxpayers welfare, is now given by

$$\underbrace{\frac{\partial V_m(\tau)}{\partial R_1} \frac{dR_1}{d\delta}}_{\text{Liability-side regulation}} + \underbrace{\sum_h \frac{\partial V_m(\tau)}{\partial \psi_h} \frac{d\psi_h}{d\delta}}_{\text{Asset-side regulation}}. \quad (\text{A13})$$

As in Section 3.2, we expect more generous levels of coverage to increase the deposit rate, so $\frac{\partial V_m(\tau)}{\partial R_1} \frac{dR_1}{d\delta} < 0$, making it socially costlier to increase δ , since banks do not internalize the fiscal consequences of offering higher deposit rates. In principle, it is impossible to individually sign each of the H terms $\frac{\partial V_m(\tau)}{\partial \psi_h} \frac{d\psi_h}{d\delta}$ that determine the regulation of banks' asset allocations. However, in most cases, it is reasonable to expect that the sum of all these terms takes negative values, since competitive banks have incentives to increase their risk-taking when the level of coverage is higher. Previous research has nonetheless shown that the risk-taking behavior of banks is sensitive to the details of the market environment; see, for instance, Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010). In imperfectly competitive environments, it should not be surprising for the asset-side regulation term in Equation (A13) to take positive values.

However, regardless of their sign, our results robustly point out that both liability-side regulations, controlling the deposit rate offered by banks, and asset-side regulations, controlling the investment portfolio of banks, are in general needed to maximize social welfare when ex-ante policies are feasible.⁴⁰

⁴⁰In practice, capital requirements and net stable funding ratios are forms of liability-side regulations, while liquidity

The optimal corrective policy introduces wedges on banks' choices that can be approximated as follows:

$$\tau_{R_1} = -\frac{\partial V_m(\tau)}{\partial R_1} \approx \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1} \quad \text{and} \quad \tau_{\psi_h} = -\frac{\partial V_m(\tau)}{\partial \psi_h} \approx \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial \psi_h}. \quad (\text{A14})$$

As discussed above, restrictions on the set of ex-ante instruments available to the planner deliver intermediate outcomes between the two extremes analyzed here. Equation (A14) provides direct guidance on how to set ex-ante policies to correct the ex-ante distortions on banks' behavior caused by deposit insurance.

D.2 Alternative Equilibrium Selection Mechanisms

In the baseline model, depositors coordinate following an exogenous sunspot. We now show that varying the information structure and the equilibrium selection procedure does not change the sufficient statistics we identify. We consider a global game structure in which late depositors observe at date 1 an arbitrarily precise private signal about the date 2 return on banks' investments before deciding $D_1(i, s)$. With that information structure, Goldstein and Pauzner (2005) show, in a model which can be mapped to our baseline model with no deposit insurance, that there exists a unique equilibrium in threshold strategies in which depositors withdraw their deposits when they receive a sufficiently low signal but leave their deposits in the bank otherwise.

Since our goal in this paper is to show the robustness of our optimal policy characterization and to directly use the set of sufficient statistics that we identify, we take the outcome of a global game as a primitive. In particular, we take as a prediction of the global game that there exists a threshold $s^G(\delta, R_1)$ such that when $s \leq s^G(\delta, R_1)$ there is a bank failure with certainty but when $s > s^G(\delta, R_1)$ no-failure occurs, with the following properties:

$$\frac{\partial s^G}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^G}{\partial \delta} \leq 0.$$

Goldstein and Pauzner (2005) formally show that $\frac{\partial s^G}{\partial R_1} \geq 0$, while Allen et al. (2018) formally show that $\frac{\partial s^G}{\partial \delta} \leq 0$ in a special case of our framework. In fact, any model of behavior which generates a threshold with these properties, not necessarily a global game, is consistent with our results.

Therefore, given the behavior of depositors at date 1, the ex-ante welfare of depositors is now given by

$$\lambda \int_{i \in I} V(i, e, \delta, R_1) dG(i) + (1 - \lambda) \int_{i \in I} V(i, \ell, \delta, R_1) dG(i),$$

where

$$V(i, x, \delta, R_1) = \int_{\underline{s}}^{s^G(\delta, R_1)} U(C_t^F(i, x, s)) dF(s) + \int_{s^G(\delta, R_1)}^{\bar{s}} U(C_t^N(i, x, s)) dF(s), \quad (\text{A15})$$

coverage ratios are an example of asset-side regulations. See Diamond and Kashyap (2016) for a recent assessment of these policy measures in a model of runs and Hachem and Song (2017) for a study of its ex-ante consequences in an environment with strategic banks.

and where early and late depositors' consumption is defined by Equations (4) and (5). We can then show that the characterization of $\frac{dW}{d\delta}$ remains valid in this context.

Proposition 7. (Directional test for δ under an alternative equilibrium selection) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left(-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[\int \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dH(j), \quad (\text{A16})$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

The particular information structure considered and the equilibrium selection procedure only enter in the expression of $\frac{dW}{d\delta}$ through the sufficient statistics identified in this paper. In particular, this is true even though the sensitivity of the probability of bank failure to changes in the level of coverage $\frac{\partial q^F}{\partial \delta}$ will depend on the assumptions on the informational structure of the economy. Studying a global game model, as in Allen et al. (2018), is appealing because it makes it possible to understand how the probability of failure is endogenously determined. However, Proposition 7 shows that it is enough to measure the sufficient statistics identified in this paper.

D.3 General Equilibrium Spillovers/Macroprudential Considerations

In our baseline formulation, as in Diamond and Dybvig (1983), bank decisions do not affect aggregate variables, so our analysis so far can be defined as microprudential. When the decisions made by banks affect aggregate variables, for instance, asset prices, further exacerbating the possibility of a bank failure, the optimal deposit insurance formula may incorporate a macroprudential correction. These general equilibrium effects arise in models in which economy-wide outcomes determined by decentralized choices directly interact with coordination failures. Our extension captures in a simple form the macro implications of banks' choices, which may operate through pecuniary externalities or aggregate demand externalities (Dávila and Korinek, 2018; Farhi and Werning, 2016).

Formally, we now assume that, given a level of aggregate withdrawals $\bar{\Omega}(s) = \bar{D}_0 R_1 - \bar{D}_1(s)$, banks must liquidate $\theta(\bar{\Omega}(s))$ of their investments, where $\theta(\cdot) \geq 1$ is a well-behaved increasing function. By assuming that banks have to liquidate more than one-for-one their investments at a rate that increases with the aggregate level of liquidations, we capture the possibility of illiquidity in financial markets when many banks unwind existing investments. This is a parsimonious way of incorporating aggregate linkages, but there is scope for richer modeling of interbank markets as in, for instance, Freixas, Martin and Skeie (2011). Under this assumption, the level of resources available to banks with withdrawals $\Omega(s)$, when the level of total withdrawals is $\bar{\Omega}(s)$, is given by

$$\rho_2(s) \left(\rho_1(s) \bar{D}_0 - \theta(\bar{\Omega}(s)) \Omega(s) \right). \quad (\text{A17})$$

Equation (A17) generalizes the left-hand side of Equation (3). When $\theta(\cdot) > 1$, it captures that the unit price of liquidating investments is increasing in the aggregate level of withdrawals. Following the same logic used to solve the baseline model, we can define thresholds \hat{s} and s^* , which now have $\bar{\Omega}(s)$ as a new argument. When the regulator sets δ optimally, she takes into account the effects of individual banks' choices on the aggregate level of withdrawals $\bar{\Omega}(s)$. Under these assumptions, we show that $\frac{dW}{d\delta}$ satisfies the same equation as in our baseline model when ex-ante regulation is available, although it must incorporate a macroprudential correction when ex-ante regulation is not available.

Proposition 8. (Directional test for δ incorporating aggregate spillovers) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left(-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^G)) - U(C^F(j, s^G))}{U'(C^F(j, s^G))} \right) + q^F \mathbb{E}_s^F \left[\frac{U'(C^F(j, s))}{U'(C^F(j, s^G))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dH(j), \quad (\text{A18})$$

where $\mathbb{E}_s^F[\cdot]$ stands for a conditional expectation over bank failure states and q^F denotes the unconditional probability of bank failure. If $\frac{dW}{d\delta} > (<) 0$, it is optimal to locally increase (decrease) the level of coverage.

In this case, ex-ante regulation can directly target the wedges caused by aggregate spillovers. In this case, the ex-ante regulation faced by banks partly addresses both the fiscal externality that emerges from the presence of deposit insurance and the externality induced by the aggregate spillovers caused by competitive deposit rate setting. Similar formulas would apply when banks have general portfolio decisions, as in our analysis earlier in this section.

As in the case of moral hazard, it is possible to correct the welfare impact of aggregate spillovers with ex-ante regulation. The optimal corrective policy can be expressed in this case as

$$\tau_{R_1} = \underbrace{-\frac{\partial V_m^P(\tau)}{\partial R_1}}_{\text{Fiscal. Ext.}} - \underbrace{\left(\lambda \int_{i \in I} \left(\frac{\partial V_m^P(i, e)}{\partial R_1} - \frac{\partial V_m(i, e)}{\partial R_1} \right) dG(i) + (1 - \lambda) \int_{i \in I} \left(\frac{\partial V_m^P(i, \ell)}{\partial R_1} - \frac{\partial V_m(i, \ell)}{\partial R_1} \right) dG(i) \right)}_{\text{Spillovers}},$$

where the superscript P corresponds to the welfare assessment from the planner's perspective, as described below. The first term accounts for banks' fiscal externalities, as studied above. The second term, which accounts for the general equilibrium spillovers of banks decisions, is a function of the terms $\frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1}$ and $\frac{\partial \hat{s}_P}{\partial R_1} - \frac{\partial \hat{s}}{\partial R_1}$, which account for the fact that the planner acknowledges that when banks offer higher rates, withdrawals are higher and bank failures more likely.

D.4 Proofs and Derivations

Proof of Proposition 6. (Directional test for δ under general investment opportunities)

First, we establish the new failure threshold, which corresponds to Equation (A12) in the text. For a

given common liquidation rate φ , the resources at date 2 for a bank are now given by

$$\begin{aligned} \sum_h \rho_{2h}(s) \left(\rho_{1h}(s) \psi_h \bar{D}_0 - \varphi \rho_{1h}(s) \psi_h \bar{D}_0 \right) &= \sum_h \rho_{2h}(s) \left(\rho_{1h}(s) \psi_h \bar{D}_0 - \frac{\rho_{1h}(s) \psi_h \bar{D}_0}{\sum_h \rho_{1h}(s) \psi_h \bar{D}_0} \Omega(s) \right) \\ &= \sum_h \rho_{2h}(s) \left(\rho_{1h}(s) \psi_h \bar{D}_0 - \frac{\rho_{1h}(s) \psi_h}{\sum_h \rho_{1h}(s) \psi_h} \left(\bar{D}_0 R_1 - \bar{D}_1(s) \right) \right), \end{aligned}$$

where we use the fact that the level of withdrawals $\Omega(s)$ pins down the liquidation rate $\varphi = \frac{\Omega(s)}{\sum_h \rho_{1h}(s) \psi_h \bar{D}_0}$. It is therefore easy to show that the threshold for the level of deposits that delimits the probability of failure is

$$\tilde{D}_1(s) = \frac{(R_1 - \sum_h \rho_{1h}(s) \psi_h) \bar{D}_0}{1 - \frac{1}{\sum_h \frac{\rho_{2h}(s) \rho_{1h}(s) \psi_h}{\rho_{1h}(s) \psi_h}}},$$

which depends on R_1 and ψ_h . It is straightforward to compute consumption for early and late depositors, as in Equations (4) and (5). In this case

$$T(s) = \max \left\{ \int_{i \in I} \min \{ D_0(i) R_1, \delta \} dG(i) - \sum_h \chi_h(s) \rho_{1h}(s) \psi_h \bar{D}_0, 0 \right\}.$$

We can express $\frac{dW}{d\delta}$ as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial \delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial \delta}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \delta}}{U'(C^F(\tau,s^*))} \\ &+ \left(\lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial R_1}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial R_1}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial R_1}}{U'(C^F(\tau,s^*))} \right) \frac{dR_1}{d\delta} \\ &+ \sum_h \left(\lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial \psi_h}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial \psi_h}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \psi_h}}{U'(C^F(\tau,s^*))} \right) \frac{d\psi_h}{d\delta} \\ &+ \left(\lambda \int_{i \in I} \sum_k \frac{\frac{\partial V(i,e)}{\partial y_k(i)}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \sum_k \frac{\frac{\partial V(i,\ell)}{\partial y_k(i)}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial y_k}}{U'(C^F(\tau,s^*))} \right) \frac{dy_k}{d\delta}, \end{aligned}$$

where

$$\begin{aligned} &\lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial \delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial \delta}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \delta}}{U'(C^F(\tau,s^*))} = \\ &- \int \frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j,s^*)) - U(C^F(j,s^*))}{U'(C^F(i,s^*))} \right) dH(j) + \int q^F \mathbb{E}_s^F \left[\frac{U'(C^F(j,s))}{U'(C^F(\tau,s^*))} \frac{\partial C^F(j,s)}{\partial \delta} \right] dH(j), \end{aligned}$$

which corresponds to $\frac{dW}{d\delta}$ under perfect regulation. The definition of $V(i)$ now corresponds to the updated utility specification (A11), and it is subject to Equations (A10) and $\sum_h \psi_h = 1$.

Unregulated banks and depositors optimally set

$$\begin{aligned} \lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial R_1}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial R_1}}{U'(C^F(i,\ell,s^*))} dG(i) &= 0 \\ \lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial \psi_h}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial \psi_h}}{U'(C^F(i,\ell,s^*))} dG(i) &= 0, \forall h \\ \lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial y_k(i)}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial y_k(i)}}{U'(C^F(i,\ell,s^*))} dG(i) &= 0, \forall i, \forall k. \end{aligned}$$

Since $T(s)$ is independent of $y_k(i)$, it is always the case that $\frac{\partial V_m(\tau)}{\partial y_k(i)} = 0$. Therefore, in that case, $\frac{dW}{d\delta}$ corresponds to

$$\begin{aligned} \frac{dW}{d\delta} &= \lambda \int_{i \in I} \frac{\frac{\partial V(i,e)}{\partial \delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell)}{\partial \delta}}{U'(C^F(i,\ell,s^*))} dG(i) \\ &\quad + \frac{\frac{\partial V(\tau)}{\partial R_1}}{U'(C^F(\tau,s^*))} \frac{dR_1}{d\delta} + \sum_h \frac{\frac{\partial V(\tau)}{\partial \psi_h}}{U'(C^F(\tau,s^*))} \frac{d\psi_h}{d\delta}. \end{aligned}$$

We can express $\frac{\partial V(\tau)}{\partial R_1}$ exactly as in Equation (A2), and $\frac{\partial V(\tau)}{\partial \psi_h}$ as follows:

$$\begin{aligned} \frac{\partial V(\tau)}{\partial \psi_h} &= q^F \mathbb{E}_s^F \left[U'(C^F(\tau,s)) \frac{\partial C^F(\tau,s)}{\partial \psi_h} \right] + [U(C^F(\tau,s^*)) - U(C^N(\tau,s^*))] \pi f(s^*) \frac{\partial s^*}{\partial \psi_h} \\ &\quad + (U(C^F(\tau,\hat{s})) - U(C^N(\tau,\hat{s}))) (1-\pi) \frac{\partial \hat{s}}{\partial \psi_h} f(\hat{s}), \end{aligned}$$

where

$$q^F \mathbb{E}_s^F \left[U'(C^F(\tau,s)) \frac{\partial C^F(\tau,s)}{\partial \psi_h} \right] = \int_{\underline{s}}^{\hat{s}} U'(C^F(\tau,s)) \frac{\partial C^F(\tau,s)}{\partial \psi_h} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(\tau,s)) \frac{\partial C^F(\tau,s)}{\partial \psi_h} dF(s).$$

Using an approximation as in Proposition 2, we can write

$$\frac{\partial V_m(\tau)}{\partial R_1} \approx - \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1} \quad \text{and} \quad \frac{\partial V_m(\tau)}{\partial \psi_h} \approx - \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial \psi_h}.$$

Proof of Proposition 7. (Directional test for δ under an alternative equilibrium selection)

Under the new equilibrium selection assumption, $V(i,x,\delta,R_1)$ is defined in Equation (A15) and $V(\tau,\delta,R_1)$ is defined as

$$V(\tau,\delta,R_1) = \int_{\underline{s}}^{s^G(\delta,R_1)} U(C^F(\tau,s)) dF(s) + \int_{s^G(\delta,R_1)}^{\bar{s}} U(C^N(\tau,s)) dF(s).$$

Therefore, under perfect regulation, we can express $\frac{dW}{d\delta}$ as follows:

$$\frac{dW}{d\delta} = \lambda \int_{i \in I} \frac{\frac{\partial V(i,e,\delta,R_1)}{\partial \delta}}{U'(C^F(i,e,s^G))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V(i,\ell,\delta,R_1)}{\partial \delta}}{U'(C^F(i,\ell,s^G))} dG(i) + \frac{\frac{\partial V(\tau,\delta,R_1)}{\partial \delta}}{U'(C^F(\tau,s^G))},$$

where

$$\begin{aligned} \frac{\partial V(i, x, \delta, R_1)}{\partial \delta} &= \left(U \left(C_t^F(i, x, s^G) \right) - U \left(C_t^N(i, x, s^G) \right) \right) \underbrace{f(s^G) \frac{\partial s^G}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} \\ &+ \underbrace{\int_{\underline{s}}^{s^G} U' \left(C_t^F(i, x, s) \right) \frac{\partial C_t^F(i, x, s)}{\partial \delta} dF(s)}_{= q^F \mathbb{E}_s^F \left[U' \left(C_t^F(i, x, s) \right) \frac{\partial C_t^F(i, x, s)}{\partial \delta} \right]}, \end{aligned}$$

since $\frac{\partial C_t^N(i, x, s)}{\partial \delta} = 0$, and where we can write

$$\begin{aligned} \frac{\partial V(\tau, \delta, R_1)}{\partial \delta} &= \left(U \left(C^F(\tau, s^G) \right) - U \left(C^N(\tau, s^G) \right) \right) \underbrace{f(s^G) \frac{\partial s^G}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} + \underbrace{\int_{\underline{s}}^{s^G} U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial \delta} dF(s)}_{= q^F \mathbb{E}_s^F \left[U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial \delta} \right]}. \end{aligned}$$

Equation (A16) follows immediately.

Proof of Proposition 8. (Directional test for δ incorporating aggregate spillovers) First, we establish the new failure threshold, which corresponds to Equation (A17) in the text. That is, the total resources available to a given bank at date 2, given aggregate withdrawals $\bar{\Omega}(s)$, corresponds to $\rho_2(s) \left(\rho_1(s) \bar{D}_0 - \theta \left(\bar{\Omega}(s) \right) \Omega(s) \right)$, which can be expressed as

$$\rho_2(s) \left(\theta \left(\bar{\Omega}(s) \right) \bar{D}_1(s) + \left(\rho_1(s) - \theta \left(\bar{\Omega}(s) \right) R_1 \right) \bar{D}_0 \right).$$

As in the baseline model, we can implicitly define a threshold level of deposits, denoted by $\tilde{D}_1(s)$ and given by

$$\tilde{D}_1(s) = \frac{\theta \left(\bar{\Omega}(s) \right) R_1 - \rho_1(s) \bar{D}_0}{\theta \left(\bar{\Omega}(s) \right) - \frac{1}{\rho_2(s)}},$$

which delimits the failure regions. When banks choose R_1 unregulated, they do not internalize that deposit rates affect $\theta(\cdot)$. In that case, we can define two types of thresholds. We denote the thresholds used by banks ex-ante to choose R_1 by $\hat{s}(R_1)$ and $s^*(\delta, R_1)$. Those perceived by the deposit insurance authority, incorporating the effects on aggregate withdrawals $\bar{\Omega}(s) = \bar{D}_0 R_1 - \bar{D}_1(s)$, are denoted by $\hat{s}_P(R_1)$ and $s_P^*(\delta, R_1)$. In equilibrium, $\hat{s}(R_1) = \hat{s}_P(R_1)$ and $s^*(\delta, R_1) = s_P^*(\delta, R_1)$, even though, crucially, the partial derivatives of each set of thresholds with respect to R_1 are different.

As above, we can express $\frac{dW}{d\delta}$ as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \lambda \int_{i \in I} \frac{\frac{dV^P(i,e)}{d\delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{dV^P(i,\ell)}{d\delta}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{dV^P(\tau)}{d\delta}}{U'(C^F(\tau,s^*))} \\ &= \lambda \int_{i \in I} \frac{\frac{\partial V^P(i,e)}{\partial \delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V^P(i,\ell)}{\partial \delta}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau,s^*))} \\ &\quad + \left(\lambda \int_{i \in I} \frac{\frac{\partial V^P(i,e)}{\partial R_1}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V^P(i,\ell)}{\partial R_1}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial R_1}}{U'(C^F(\tau,s^*))} \right) \frac{dR_1}{d\delta}, \end{aligned}$$

where we use the P notation to emphasize that $V^P(i,x)$ and $V^P(\tau)$ are calculated from the perspective of a planner who uses thresholds $\hat{s}_P(R_1)$ and $s_P^*(\delta, R_1)$, which account for equilibrium spillovers. Note that the first two terms are given by

$$\begin{aligned} &\lambda \int_{i \in I} \frac{\frac{\partial V^P(i,e)}{\partial \delta}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V^P(i,\ell)}{\partial \delta}}{U'(C^F(i,\ell,s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau,s^*))} = \\ &\int q^F \mathbb{E}_s^F \left[\frac{U'(C^F(j,s))}{U'(C^F(j,s^*))} \frac{\partial C^F(j,s)}{\partial \delta} \right] dH(j) + \underbrace{\pi f(s^*) \frac{\partial s^*}{\partial \delta}}_{= \frac{\partial q^F}{\partial \delta}} \int \left(\frac{U(C^F(j,s^*)) - U(C^N(j,s^*))}{U'(C^F(j,s^*))} \right) dH(j). \end{aligned}$$

We can express

$$\begin{aligned} &\lambda \int_{i \in I} \frac{\frac{\partial V^P(i,e)}{\partial R_1}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V^P(i,\ell)}{\partial R_1}}{U'(C^F(i,\ell,s^*))} dG(i) = \\ &\mathbb{E}_\lambda \left[\int_{i \in I} \left(\frac{\frac{\partial V^P(i,x)}{\partial R_1}}{U'(C^F(i,x,s^*))} - \frac{\frac{\partial V(i,x)}{\partial R_1}}{U'(C^F(i,x,s^*))} \right) dG(i) \right] \end{aligned}$$

as follows:⁴¹

$$\begin{aligned} &\lambda \int_{i \in I} \frac{\frac{\partial V^P(i,e)}{\partial R_1}}{U'(C^F(i,e,s^*))} dG(i) + (1-\lambda) \int_{i \in I} \frac{\frac{\partial V^P(i,\ell)}{\partial R_1}}{U'(C^F(i,\ell,s^*))} dG(i) = \\ &\mathbb{E}_\lambda \left[\int_{i \in I} \left(\frac{U(C_t^F(i,x,s^*)) - U(C_t^N(i,x,s^*))}{U'(C^F(i,x,s^*))} \right) \pi f(s^*) \left(\frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1} \right) dG(i) \right] \\ &+ \int_{i \in I} \left(\frac{U(C_t^F(i,x,\hat{s})) - U(C_t^N(i,x,\hat{s}))}{U'(C^F(i,x,s^*))} \right) (1-\pi) f(\hat{s}_P) \left(\frac{\partial \hat{s}_P}{\partial R_1} - \frac{\partial \hat{s}}{\partial R_1} \right) dG(i) \end{aligned}$$

where we use the fact that for depositors $\mathbb{E}_\lambda \left[\int_{i \in I} \frac{\frac{\partial \mathbb{E}_s[U(C_t(i,s))]}{\partial R_1}}{U'(C^F(i,s^*))} dG(i) \right] = 0$. Similarly, we can express

⁴¹Here we slightly abuse notation using $\mathbb{E}_\lambda[\cdot] = \lambda \int \dots dG(i) + (1-\lambda) \int \dots dG(i)$.

$\frac{\partial V^P(\tau)}{\partial R_1}$ as follows:

$$\begin{aligned} \frac{\partial V^P(\tau)}{\partial R_1} &= q^F \mathbb{E}_s^F \left[U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial R_1} \right] + \left(U \left(C^F(\tau, s^*) \right) - U \left(C^N(\tau, s^*) \right) \right) \pi f(s^*) \frac{\partial s_P^*}{\partial R_1} \\ &\quad + \left[U \left(C^F(\tau, \hat{s}) \right) - U \left(C^N(\tau, \hat{s}) \right) \right] (1 - \pi) \frac{\partial \hat{s}_P}{\partial R_1} f(\hat{s}), \end{aligned}$$

where

$$q^F \mathbb{E}_s^F \left[U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial R_1} \right] = \int_{\underline{s}}^{\hat{s}} U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U' \left(C^F(\tau, s) \right) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s).$$

Using an approximation as in Proposition 2, we can write $\frac{\partial V_m(\tau, s)}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s) + \kappa(T(s))]}{\partial R_1}$.

Therefore, under perfect regulation, $\frac{dW}{d\delta} = \int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))}$, which corresponds to Equation (A18). The optimal regulation of banks, set so that

$$\lambda \int_{i \in I} \frac{\frac{\partial V^P(i, e)}{\partial R_1}}{U'(C^F(i, e, s^*))} dG(i) + (1 - \lambda) \int_{i \in I} \frac{\frac{\partial V^P(i, \ell)}{\partial R_1}}{U'(C^F(i, \ell, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial R_1}}{U'(C^F(\tau, s^*))} = 0,$$

now incorporates a correction that accounts for aggregate spillovers. The optimal regulation is set so that banks internalize their fiscal externality and their aggregate spillovers.

E Additional Results

E.1 Idiosyncratic Risk: Mutual Insurance

In this subsection, we describe one tractable way to introduce idiosyncratic risk in our model. We begin by considering an environment in which there is a continuum of banks of the form described in Section 2 of the paper. We index an individual bank by n . The state s continues to denote aggregate uncertainty, as in the baseline model. We assume that these banks are ex-identical, but we assume that at date 1 a share π of banks will fail whenever the state s is in multiple equilibria region, $[\hat{s}, s^*]$.

Hence, in this extension of the model, whenever $s \in [\underline{s}, \hat{s}]$, all banks fail, and when $s \in [\hat{s}, s^*]$, a share π of banks fail. Formally, we can express the change in social welfare for a given depositor/taxpayer associated with bank n induced by a marginal change in coverage for bank, $\frac{dV(j, \delta, R_1; n)}{d\delta}$, as follows:

$$\begin{aligned} \frac{dV(j, \delta, R_1; n)}{d\delta} &= \overbrace{\int_{\underline{s}}^{\hat{s}} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} dF(s) + \mathbb{I}_F(n) \int_{\hat{s}}^{s^*} U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} dF(s)}^{=q^F \mathbb{E}_s^F \left[U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} \right]} \\ &\quad + \left[U(C^F(j, s^*)) - U(C^N(j, s^*)) \right] \mathbb{I}_F(n) f(s^*) \frac{\partial s^*}{\partial \delta}, \end{aligned}$$

where $\mathbb{I}_F(n)$ corresponds to a failure indicator for bank n . When aggregating across all banks, we can express the change in social welfare induced by a marginal change in the level of deposit insurance coverage by

$$\frac{dW}{d\delta} = \int \frac{dW}{d\delta}(n) dM(n),$$

where $\frac{dW}{d\delta}(n) = \int \omega(j) \frac{dV_m(j, \delta, R_1; n)}{d\delta} dH(j)$, which takes the exact same form as in Proposition 1, after defining q^F as

$$q^F = \underbrace{\int_{\underline{s}}^{\hat{s}} dF(s)}_{F(\hat{s})} + \underbrace{\int_{\hat{s}}^{s^*} dF(s)}_{F(s^*) - F(\hat{s})} \pi$$

where now $\pi = \int \mathbb{I}_F(n) dM(n)$. Given our assumptions, the thresholds \hat{s} and s^* are identical for all banks, while q^F still corresponds to the probability of bank failure. In the augmented model, the probability of failure has a systemic component, captured by $F(\hat{s})$, and one that purely arises from fragility, π , in the region of multiple equilibria, $F(s^*) - F(\hat{s})$.

The key difference between this extension with idiosyncratic run risk and the baseline model emerges when characterizing the funding shortfall, since now there is the possibility of raising funds from non-failed banks to cover such shortfall. We define a new variable $\vartheta(s)$, which corresponds to the share of failed banks for a given realization of s , given by

$$\vartheta(s) = \begin{cases} 1, & \text{if } \underline{s} \leq s < \hat{s}(R_1) \\ \pi, & \text{if } \hat{s}(R_1) \leq s < s^*(R_1). \end{cases}$$

In this case, we can then express the funding shortfall as

$$T(s) = \max \left\{ \vartheta(s) \left(\int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0 \right) - (1 - \vartheta(s)) \tau^e, 0 \right\},$$

where τ^e corresponds to the funds levied from non-failed banks at date 1, which we take for now as a primitive, but whose determination we discuss below.

In this new environment, it will be the case that the failure thresholds \hat{s} and s^* are functions of τ^e . Formally, now the failure regions are

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \left(1 - \tau^f \right) \bar{D}_0 - \Omega(s) - \tau^e \right) < \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \left(1 - \tau^f \right) \bar{D}_0 - \Omega(s) - \tau^e \right) \geq \bar{D}_1(s). \end{aligned}$$

Under the assumption that τ^e is uniform across, it follows immediately that the thresholds \hat{s} and s^* are identical for all banks. For instance, we could solve for the value of τ^e that eliminates the funding shortfall, given by

$$\tau^e = \frac{\vartheta(s)}{1 - \vartheta(s)} \left(\max \left\{ \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0 \right\}, 0 \right). \quad (\text{A19})$$

Equation (A19) highlights that funding shortfalls are easier to cover when the share of failed banks is smaller. In particular, when $\vartheta(s) \rightarrow \frac{1}{2}$, half the banks fail, so τ^e simply equals the funding shortfall for an individual banks.⁴² When $\vartheta(s) \rightarrow 0$, a small τ^e is enough to eliminate the funding shortfall, while when $\vartheta(s) \rightarrow 1$, τ^e becomes increasingly large. At this point, one could go back and solve for \hat{s} and s^* as a function of τ^e . Importantly, for a given τ^e , our characterization of $\frac{dW}{d\delta}$ remains unchanged, and the only effect of allowing for mutual insurance across banks ex-post is captured by the funding shortfall, $T(s)$, in particular via $q^{T+|F}$.

E.2 Aggregate Risk: Deposit Insurance Fund

In this subsection, we describe a tractable way to explicitly introduce a deposit insurance fund in our environment. We assume that banks must set aside a fraction τ^f of their initial deposits to be invested in a fund. Therefore, the date 0 investment of a bank becomes $(1 - \tau^f) \bar{D}_0$. Consequently, bank failure is determined in this case by

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \left(1 - \tau^f \right) \bar{D}_0 - \Omega(s) \right) < \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \rho_2(s) \left(\rho_1(s) \left(1 - \tau^f \right) \bar{D}_0 - \Omega(s) \right) \geq \bar{D}_1(s), \end{aligned}$$

⁴²See [Fernandez-Villaverde et al. \(2021\)](#) for a related argument in which mutual insurance is sufficient to eliminate runs when risk is idiosyncratic.

which is the counterpart of Equation (3) in the text. Following the same steps as in the baseline model, the new deposit failure threshold is now given by

$$\tilde{D}_1(s) = \begin{cases} \frac{(R_1 - \rho_1(s)(1 - \tau^f))\bar{D}_0}{1 - \frac{1}{\rho_2(s)}}, & \text{if } \rho_2(s) > 1 \\ \infty, & \text{if } \rho_2(s) \leq 1, \end{cases}$$

which is the counterpart of Equation (7) in the text. It follows immediately that the thresholds \hat{s} and s^* are now increasing functions of τ^f . That is, the higher the amount contributed to the deposit fund, the higher the probability of failure.

We assume that the return on the resources held in the fund is $\rho_1^f(s)$. In this case, the funding shortfall $T(s)$ takes the form:

$$T(s) = \max \left\{ \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) (1 - \tau^f) \bar{D}_0 - \rho_1^f(s) \tau^f \bar{D}_0, 0 \right\}. \quad (\text{A20})$$

Compared to its counterpart in the text in Equation (18), now the funds available after liquidation are only $\chi(s) \rho_1(s) (1 - \tau^f) \bar{D}_0$, but there are $\rho_1^f(s) \tau^f \bar{D}_0$ funds available through the fund.⁴³ A similar adjustment applies to $\alpha_F(s)$ and $\alpha_N(s)$.

It is evident from Equation (A20) that whenever

$$\chi(s) \rho_1(s) = \rho_1^f(s),$$

the funding shortfall would be equivalent with or without a deposit insurance fund. In that case, for a given τ^f , our characterization of $\frac{dW}{d\delta}$ remains unchanged, after accounting for the allegedly small impact of τ^f on \hat{s} and s^* .

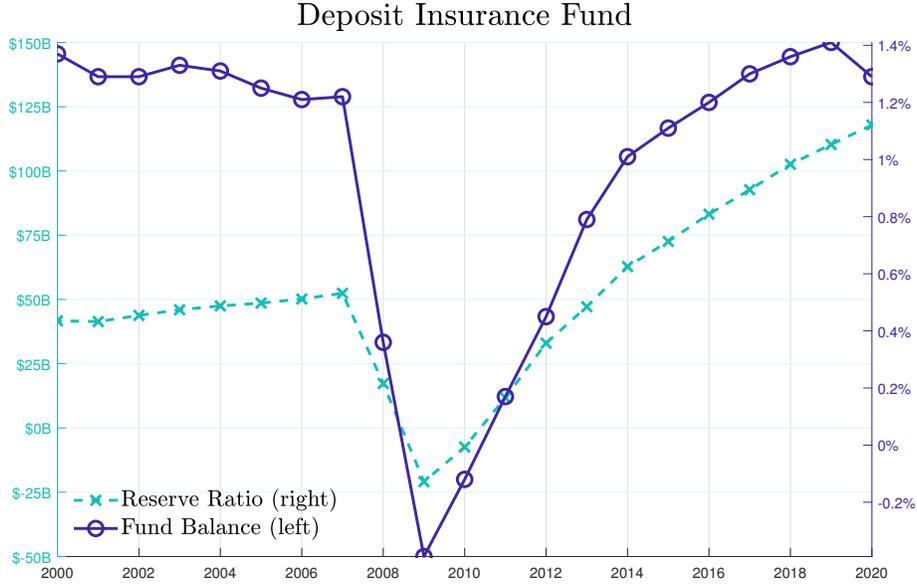
If the return on the deposit insurance fund is too low ($\chi(s) \rho_1(s) > \rho_1^f(s)$), and supposing that an increase in level of coverage is associated with a higher τ^f , there would be an additional cost to increasing the level of coverage. In any case, there is scope to explore further any issues related to the financing of deposit insurance obligations.

For reference, Figure OA-1 shows the evolution between 2000 and 2020 of i) the balance of the deposit insurance fund (in nominal terms) in the US and ii) the reserve ratio, which corresponds to the ratio between the balance of the deposit insurance fund and the total estimated insured deposits of the industry. In 2009 and 2010, the fund had a negative balance, although part of this fact was due to provisions to cover expected future losses. However, note that the ultimate backstop of the deposit insurance fund is a line of credit from the Treasury.⁴⁴ The Dodd-Frank Act of 2010 sets a minimum reserve ratio of 1.35%. Whenever the reserve ratio falls (or is expected to fall) below this threshold, the FDIC must adopt a restoration plan to return to this level within 8 years.⁴⁵

⁴³This formulation is valid for a representative bank. With multiple banks, it is necessary to model carefully the correlation of shocks across banks and understand whether bank failures are isolated events or systemic.

⁴⁴See <https://www.nytimes.com/2009/11/25/business/economy/25fdic.html> for a discussion of the 2009 episode.

⁴⁵See <https://www.fdic.gov/deposit/insurance/fund.html> for more information on the management of the deposit insurance fund.



Note: Figure OA-1 shows the evolution between 2000 and 2020 of i) the balance of the deposit insurance fund (in nominal terms) in the US and ii) the reserve ratio, which corresponds to the ratio between the balance of the deposit insurance fund and the total estimated insured deposits of the industry. Data are from the FDIC.

Figure OA-1: Evolution of Deposit Insurance Fund

E.3 Equivalence of Traditional and Generalized Social Welfare Weights

In this subsection, we show that there is a one-to-one mapping between traditional welfare weights and generalized social welfare weights. First, for completeness, we reproduce here Equation (21) in the text after substituting $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{\frac{dV(j, \delta, R_1)}{d\delta}}{U'(C^F(j, s^*))}$:

$$\frac{dW}{d\delta} = \int \omega(j) \frac{\frac{dV(j, \delta, R_1)}{d\delta}}{U'(C^F(j, s^*))} dH(j). \quad (\text{A21})$$

Alternatively, we can identify social welfare with a weighted sum of depositors' and taxpayers' ex-ante expected utility. We denote traditional welfare weights by $\theta(j) = \{\theta(i), \theta(\tau)\}$. In this case, social welfare $W(\delta)$ formally corresponds to

$$\begin{aligned} W(\delta) &= \int \theta(j) V(j, \delta, R_1) dH(j) \\ &= \underbrace{\lambda \int_{i \in I} \theta(i) V(i, e, \delta, R_1) dG(i) + (1 - \lambda) \int_{i \in I} \theta(i) V(i, \ell, \delta, R_1) dG(i)}_{\text{Depositors}} + \underbrace{\theta(\tau) V(\tau, \delta, R_1)}_{\text{Taxpayers}}. \end{aligned}$$

In this case, we can express $\frac{dW}{d\delta}$ as follows:

$$\frac{dW}{d\delta} = \int \theta(j) \frac{dV(j, \delta, R_1)}{d\delta} dH(j). \quad (\text{A22})$$

Figure OA-12 on page OA-43 illustrates the magnitudes taken by the traditional welfare weights in the context of our quantitative model.

Remark. (Equivalence of traditional and generalized social welfare weights) A direct comparison of Equations (A21) and (A22) concludes that the value of $\frac{dW}{d\delta}$ under both approaches is identical when

$$\theta(j) = \frac{\omega(j)}{U'(C^F(j, s^*))}. \quad (\text{A23})$$

Equation (A23) implies that using traditional utilitarian weights ($\theta(j) = 1$) is equivalent to choosing generalized welfare weights $\omega(j) = U'(C^F(j, s^*))$. Similarly, it also implies that using uniform generalized weights ($\omega(j) = 1$) is equivalent to using traditional weights of the form $\theta(j) = \frac{1}{U'(C^F(j, s^*))}$. In that sense, using generalized welfare weights is equivalent to working with a set of endogenous traditional welfare weights that possibly depend on the early/late status of a depositor. Note that once we select a set of generalized weights and find the optimal level of coverage and the equilibrium allocation associated with those weights, it is possible to use Equation (A23) to find the set of $\theta(j)$ that would deliver the same allocation.

Finally, note that we could have chosen a state other than s^* to compute the money-metric normalization. In that case, denoting the arbitrary state by \check{s} , the normalized welfare change is given by $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{dV(j, \delta, R_1)}{U'(C^F(j, \check{s}))}$, and the counterpart of Equation (22) becomes

$$\frac{dW}{d\delta} = \int \omega(j) \left(\underbrace{-\frac{\partial q^F}{\partial \delta} m(j, s^*) \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right)}_{\text{Marginal Benefit}} + \underbrace{q^F \mathbb{E}_s \left[m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right]}_{\text{Marginal Cost}} \right) dH(j),$$

where $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, \check{s}))}$. In this case, we simply have to normalize $m(j, s)$ by the reference state. It is evident that Proposition 2 remains valid in this case.

E.4 Dynamic Stochastic Generalized Welfare Weights

In this subsection, we provide an alternative approach that leads to the exact same characterization of Equation (26) in Proposition 2. This derivation relies on a welfare assessment based on dynamic stochastic generalized social marginal social welfare weights, introduced in Dávila and Schaab (2021).

Proposition 9. *(Alternative directional test based on bank-level aggregates) a) Using dynamic stochastic generalized social marginal social welfare weights, it is possible to decompose the change in social welfare induced by a marginal change in the level of deposit insurance coverage δ , $\frac{dW}{d\delta}$, as follows:*

$$\frac{dW}{d\delta} = \Xi_{AE} + \Xi_{RS} + \Xi_{RD},$$

where the terms Ξ_{AE} , Ξ_{RS} , and Ξ_{RD} are respectively defined in Equations (A26), (A27), and (A25) below, capture aggregate efficiency, risk-sharing, and redistribution.

b) When i) the individual component of the dynamic-stochastic weights is uniform across agents, i.e., $\tilde{\omega}(j) = 1$, ii) the stochastic components of the dynamic-stochastic weights is invariant across agents and

set to 1, i.e., $\tilde{\omega}_t^N(j, s^*) = \tilde{\omega}_t^F(j, s^*) = \tilde{\omega}_t(j, s) = 1$, then

$$\frac{dW}{d\delta} = -\frac{\partial q^F}{\partial \delta} \int \left(C^N(j, s^*) - C^F(j, s^*) \right) dH(j) + q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right], \quad (\text{A24})$$

which is the exact counterpart of Equation (26) in Proposition 2.

The main difference between Proposition 2 and Proposition 9 is that Proposition 2 describes an approximation for a planner with generalized welfare weights — see Saez and Stantcheva (2016) — while the characterization of $\frac{dW}{d\delta}$ in Proposition 9 is exact for a particular set of dynamic stochastic generalized welfare weights — see Dávila and Schaab (2021). Hence, the results in Section 4.1 can be equally interpreted through both approaches.

Proof of Proposition 9. (Alternative directional test based on bank-level aggregates) a)

We can start from an instantaneous social welfare functions, so $ISWF = \int V(j, \delta, R_1) dH(j)$, where

$$\begin{aligned} V(\tau, \delta, R_1) &= \int_{\underline{s}}^{\hat{s}(R_1)} \zeta(j, s) U\left(C^F(\tau, s)\right) dF(s) \\ &\quad + \int_{\hat{s}(R_1)}^{s^*(\delta, R_1)} \left(\pi \zeta(j, s) U\left(C^F(\tau, s)\right) + (1 - \pi) \zeta(j, s) U\left(C^N(\tau, s)\right) \right) dF(s) \\ &\quad + \int_{s^*(\delta, R_1)}^{\bar{s}} \zeta(j, s) U\left(C^N(\tau, s)\right) dF(s), \end{aligned}$$

where $\zeta(j, s)$ denotes instantaneous Pareto weights. Given this ISWF, we can express $\frac{dV(j, \delta, R_1)}{d\delta} = \frac{d\mathbb{E}_s[\zeta(j, s)U(C_t(j, s))]}{d\delta}$ as

$$\begin{aligned} \frac{dV(j, \delta, R_1)}{d\delta} &= \underbrace{\int_{\underline{s}}^{\hat{s}} \zeta(j, s) U'\left(C_t^F(j, s)\right) \frac{\partial C_t^F(j, s)}{\partial \delta} dF(s) + \pi \int_{\hat{s}}^{s^*} \zeta(j, s) U'\left(C_t^F(j, s)\right) \frac{\partial C_t^F(j, s)}{\partial \delta} dF(s)}_{=q^F \mathbb{E}_s^F \left[\zeta(j, s) U'\left(C_t^F(j, s)\right) \frac{\partial C_t^F(j, s)}{\partial \delta} \right]} \\ &\quad + \left[\zeta^-(j, s^*) U\left(C_t^F(j, s^*)\right) - \zeta^+(j, s^*) U\left(C_t^N(j, s^*)\right) \right] \pi f(s^*) \frac{\partial s^*}{\partial \delta}, \end{aligned}$$

Now, transforming instantaneous Pareto weights (defined over utilities) into dynamic-stochastic weights (defined over consumption) we can express $\frac{dV(j, \delta, R_1)}{d\delta}$ as

$$\frac{d\tilde{V}(j, \delta, R_1)}{d\delta} = q^F \mathbb{E}_s^F \left[\omega_t(j, s) \frac{\partial C_t^F(j, s)}{\partial \delta} \right] + \left[\omega_t^F(j, s^*) C_t^F(j, s^*) - \omega_t^N(j, s^*) C_t^N(j, s^*) \right] \frac{\partial q^F}{\partial \delta},$$

where we define $\frac{\partial q^F}{\partial \delta} = \pi f(s^*) \frac{\partial s^*}{\partial \delta}$, $\omega_t(j, s) = \zeta(j, s) U'\left(C_t^F(j, s)\right)$, $\omega_t^F(j, s^*) = \zeta^-(j, s^*) \frac{U\left(C_t^F(j, s^*)\right)}{C_t^F(j, s^*)}$, and $\omega_t^N(j, s^*) = \zeta^+(j, s^*) \frac{U\left(C_t^N(j, s^*)\right)}{C_t^N(j, s^*)}$. Without loss of generality — see Lemma 1 of Dávila and Schaab (2021) — we can decompose the dynamic stochastic weights into an individual component and a stochastic

component, as follows:

$$\omega_t^N(j, s^*) = \tilde{\omega}(j) \tilde{\omega}_t^N(j, s^*), \quad \omega_t^F(j, s^*) = \tilde{\omega}(j) \tilde{\omega}_t^F(j, s^*), \quad \omega_t(j, s) = \tilde{\omega}(j) \tilde{\omega}_t(j, s).$$

Hence, under the assumption that $\int dH(j) = 1$, which may require a normalization, we can now express a welfare assessment $\frac{dW}{d\delta}$ as follows:

$$\frac{dW}{d\delta} = \int \tilde{\omega}(j) \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} dH(j) = \mathbb{E}_j \left[\tilde{\omega}(j) \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} \right] = \int \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} dH(j) + \Xi_{RD},$$

where we use the fact that $\int dH(j) = 1$ and where

$$\Xi_{RD} = \text{Cov}_j \left[\tilde{\omega}(j), \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} \right]. \quad (\text{A25})$$

Note that we can write $\int \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} dH(j) = \mathbb{E}_j \left[\frac{d\tilde{V}(j, \delta, R_1)}{d\delta} \right]$ as

$$\begin{aligned} \int \frac{d\tilde{V}(j, \delta, R_1)}{d\delta} dH(j) &= -\frac{\partial q^F}{\partial \delta} \left(\mathbb{E}_j \left[\tilde{\omega}_t^N(j, s^*) C_t^N(j, s^*) \right] - \mathbb{E}_j \left[\tilde{\omega}_t^F(j, s^*) C_t^F(j, s^*) \right] \right) \\ &\quad + q^F \mathbb{E}_j \left[\mathbb{E}_s^F \left[\tilde{\omega}_t(j, s) \frac{\partial C_t^F(j, s)}{\partial \delta} \right] \right] \\ &= \Xi_{AE} + \Xi_{RS}, \end{aligned}$$

where

$$\begin{aligned} \Xi_{AE} &= -\frac{\partial q^F}{\partial \delta} \left(\mathbb{E}_j \left[\tilde{\omega}_t^N(j, s^*) \right] \mathbb{E}_j \left[C_t^N(j, s^*) \right] - \mathbb{E}_j \left[\tilde{\omega}_t^F(j, s^*) \right] \mathbb{E}_j \left[C_t^F(j, s^*) \right] \right) \\ &\quad + q^F \mathbb{E}_s^F \left[\mathbb{E}_j \left[\tilde{\omega}_t(j, s) \right] \mathbb{E}_j \left[\frac{\partial C_t^F(j, s)}{\partial \delta} \right] \right] \end{aligned} \quad (\text{A26})$$

$$\begin{aligned} \Xi_{RS} &= -\frac{\partial q^F}{\partial \delta} \left(\text{Cov}_j \left[\tilde{\omega}_t^N(j, s^*), C_t^N(j, s^*) \right] - \text{Cov} \left[\tilde{\omega}_t^F(j, s^*), C_t^F(j, s^*) \right] \right) \\ &\quad + q^F \mathbb{E}_s^F \left[\text{Cov}_j \left[\tilde{\omega}_t(j, s), \frac{\partial C_t^F(j, s)}{\partial \delta} \right] \right]. \end{aligned} \quad (\text{A27})$$

b) The results follow immediately using the definitions of Ξ_{AE} , Ξ_{RS} , and Ξ_{RD} . When $\tilde{\omega}(j) = 1$, $\Xi_{RD} = 0$. And when $\tilde{\omega}_t^N(j, s^*) = \tilde{\omega}_t^F(j, s^*)$ and $\tilde{\omega}_t(j, s) = 1$, $\Xi_{RS} = 0$ and Ξ_{AE} is exactly given by Equation (A24).

E.5 Directional Test: General Case

In this subsection, we provide a directional test for the level of coverage under minimal assumptions. While using the Diamond and Dybvig (1983) framework allows us to completely characterize a fully specified model, here we show that our insights extend more generally. As in the text, we focus on characterizing the welfare impact of a marginal change in the level of coverage under perfect regulation or when banks do not respond to the level of coverage. When banks are unregulated, our characterization

needs to be augmented by the fiscal externality component(s).

Consider an economy in which $V(j)$ denotes the utility of (early or late) depositors or taxpayers. We can then write

$$V(j) = \mathbb{E}_s [U_j(C(j, \delta, s); s)] = \int_{\mathcal{F}} U_j(C^F(j, \delta, s); s) dF(s) + \int_{\mathcal{N}} U_j(C^N(j, \delta, s); s) dF(s),$$

where \mathcal{F} and \mathcal{N} denote the set of failure and no-failure states, respectively. The value of $C(j, \delta, s)$ incorporates the final consumption by agent j in state s . By making utility state-dependent, we can implicitly account for early and late types.

Once we define the probability of bank failure as $q(\delta) = \int_{\mathcal{F}} dF(s)$, we can express the change in individual welfare as

$$\begin{aligned} \frac{dV(j)}{d\delta} &= -\frac{\partial q^F}{\partial \delta} \left(U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*) \right) + q^F \mathbb{E}_{\mathcal{F}} \left[U_j'(C^F(j, \delta, s); s) \frac{\partial C^F(j, \delta, s)}{\partial \delta} \right] \\ &+ (1 - q^F) \mathbb{E}_{\mathcal{N}} \left[U_j'(C^N(j, \delta, s); s) \frac{\partial C^N(j, \delta, s)}{\partial \delta} \right]. \end{aligned}$$

In money-metric form, $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{\frac{dV(j)}{d\delta}}{U_j'(C^F(j, \delta, s^*); s^*)}$ is given by

$$\begin{aligned} \frac{dV_m(j, \delta, R_1)}{d\delta} &= -\frac{\partial q^F}{\partial \delta} \left(\frac{U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*)}{U_j'(C^F(j, \delta, s^*); s^*)} \right) \\ &+ q^F \mathbb{E}_{\mathcal{F}} \left[\frac{U_j'(C^F(j, \delta, s); s)}{U_j'(C^F(j, \delta, s^*); s^*)} \frac{\partial C^F(j, \delta, s)}{\partial \delta} \right] \\ &+ (1 - q^F) \mathbb{E}_{\mathcal{N}} \left[\frac{U_j'(C^N(j, \delta, s); s)}{U_j'(C^F(j, \delta, s^*); s^*)} \frac{\partial C^N(j, \delta, s)}{\partial \delta} \right]. \end{aligned}$$

Since $C^N(\cdot)$ does not depend on δ directly, we can express $\frac{dW}{d\delta}$ as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \int -\frac{\partial q^F}{\partial \delta} \left(\frac{U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*)}{U_j'(C^F(j, \delta, s^*); s^*)} \right) dH(j) \\ &+ \int q^F \mathbb{E}_{\mathcal{F}} \left[\frac{U_j'(C^F(j, \delta, s); s)}{U_j'(C^F(j, \delta, s^*); s^*)} \frac{\partial C^F(j, \delta, s)}{\partial \delta} \right] dH(j). \end{aligned} \tag{A28}$$

Equation (A28) is a direct generalization of Equation (22) in the text.

F Quantitative Application: Additional Material

F.1 Measurement with CDS data

Data Description For our calculations, we use Markit CDS data, as distributed by Wharton Research Data Services (WRDS). Our full sample includes daily data from January 2006 until December 2014. We focus on five-year CDS spreads (these are the most liquid) on the following banks (ticker in parentheses): Bank of America Corp (BACORP), Bank of NY Mellon (BK), Citigroup Inc (C), Goldman Sachs (GS), JP Morgan Chase (JPM), Merrill Lynch & Co Inc (MER), Morgan Stanley (MWD), State Street Corp (STT), Wachovia Corp (WB), and Wells Fargo & Co (WFC). We exclusively consider CDS contracts with CR (Complete Restructuring) as a restructuring clause, so any restructuring event counts as a bank failure for our purposes. Similar results arise when using restructuring clauses MR (Modified Restructuring), MM (Modified Modified Restructuring), or XR (No Restructuring). We only consider CDS contracts on Senior Unsecured Debt and use the recovery rate provided by Markit.

Measurement The implied probability of failure can be read from spreads and recovery rates provided one is willing to make some assumptions. We use a simple constant hazard rate model (Hull, 2013), to calculate implied yearly default probabilities as follows:

$$\text{Implied Default Probability} = \frac{\text{5 Year Spread}}{1 - \text{Recovery Rate}}.$$

On October 3, 2008, President George W. Bush signed the Emergency Economic Stabilization Act of 2008, raising the limit on federal deposit insurance coverage from \$100,000 to \$250,000 per depositor. Initially, this change was temporary through the end of 2010, but it was made permanent by the Dodd-Frank Act in July 2010. Our discussion in the text is based on the jumps in the default probability caused by changes in the level of deposit insurance on October 3, 2008. We also gauge our measures by the impact of the Temporary Liquidity Guarantee Program (TLGP), announced on October 14, 2008, in which the FDIC guaranteed in full noninterest-bearing transaction accounts.⁴⁶

F.2 Model-Based Quantification: Matching Sufficient Statistics

Table OA-1 compares the sufficient statistics used in the direct measurement approach in Section 4.1 with the sufficient statistics implied by our calibration of the structural model in Section 4.2. The value of $\frac{dW}{d\delta}$ implied by Table 1 using the direct measurement approach is $\frac{dW}{d\delta} = \$4.5 \times 10^{-4}$. The value of $\frac{dW}{d\delta}$ implied by Table OA-1 using the model-based quantification is $\frac{dW}{d\delta} = \$3.9 \times 10^{-4}$. We have made sure that the calibration errs on the side of displaying somewhat lower marginal benefits than in the direct measurement approach to be able to robustly argue that the model implies a large level of coverage.

⁴⁶See <https://www.fdic.gov/regulations/resources/TLGP/index.html> for a description of the TLGP.

Table OA-1: Sufficient Statistics as Calibration Targets

	Variable	Direct Measurement	Calibrated Model
	q^F	0.025	0.0249
Mg. Benefit	$\frac{\partial \log q^F}{\partial \delta}$	-2×10^{-6}	-1.96×10^{-6}
	$\int (C^N(j, s^*) - C^F(j, s^*)) dH(j) / \bar{G}$	\$13,809	\$12,700
Mg. Cost	$q^{T^+ F}$	1	1
	$\mathbb{E}_s^F [\kappa'(\cdot) T > 0]$	0.15	0.15
	$\int_{i \in \mathcal{PI}} dG(i) / \bar{G}$	0.064	0.064

Note: To compare with Table 1, note that $-\frac{0.3}{150,000} = -2 \times 10^{-6}$. The value of $\frac{dW}{dG}$ implied by Table 1 using the direct measurement approach is $\frac{dW}{dG} = \$4.5 \times 10^{-4}$. The value of $\frac{dW}{dG}$ implied by Table OA-1 using the model-based quantification is $\frac{dW}{dG} = \$3.9 \times 10^{-4}$.

F.3 Model-Based Quantification: Additional Material

The left panel in Figure OA-2 shows the share of insured and uninsured deposits, respectively given by

$$\frac{\int \min \{D_0(i) R_1, \delta\} dG(i)}{\bar{D}_0 R_1} \quad \text{and} \quad \frac{\int \max \{D_0(i) R_1 - \delta, 0\} dG(i)}{\bar{D}_0 R_1}.$$

The right panel in Figure OA-2 shows the share of fully insured accounts/depositors and partially insured accounts/depositors, respectively given by

$$\int_{i \in \mathcal{FI}} dG(i) \quad \text{and} \quad \int_{i \in \mathcal{PI}} dG(i),$$

where $\mathcal{FI} = \{i | D_0(i) R_1 > \delta\}$ and $\mathcal{PI} = \{i | D_0(i) R_1 > \delta\}$. The fact that the distribution of depositors is heavily right-skewed is necessary to be able to match at the same time (when $\delta = 1$) a share of partially insured accounts of 6.4% and share of insured deposits of 62%.

Figures OA-3 and OA-4 are the counterparts in the calibrated model of Figures 4 and 5 in the text. Figure OA-3 shows how $s^*(\delta, R_1)$ and $\hat{s}(R_1)$ vary as a function of the level of deposit insurance coverage δ . The left panel in Figure OA-4 shows the recovery rate on uninsured deposits in case of failure, $\alpha_F(s)$, as well as the funding shortfall, $T(s)$, for different values of the realizations of the state s when $\delta = 1$. The right panel in Figure OA-4 shows the additional gross return earned by the deposits that stay within the bank until date 2, $\alpha_N(s)$.

The left panel in Figure OA-5 shows the average marginal cost of public funds in failure states, $\mathbb{E}_s^F [\kappa'(T(s))]$, and in failure states in which the funding shortfall is positive, $\mathbb{E}_s^F [\kappa'(T(s)) | T(s) > 0]$, as a function of the level of deposit insurance coverage δ . The right panel in Figure OA-5 shows the unconditional probability of a positive funding shortfall, q^{T^+} , and the probability of a positive funding shortfall condition on a bank failure taking place, as a function of the level of deposit insurance coverage

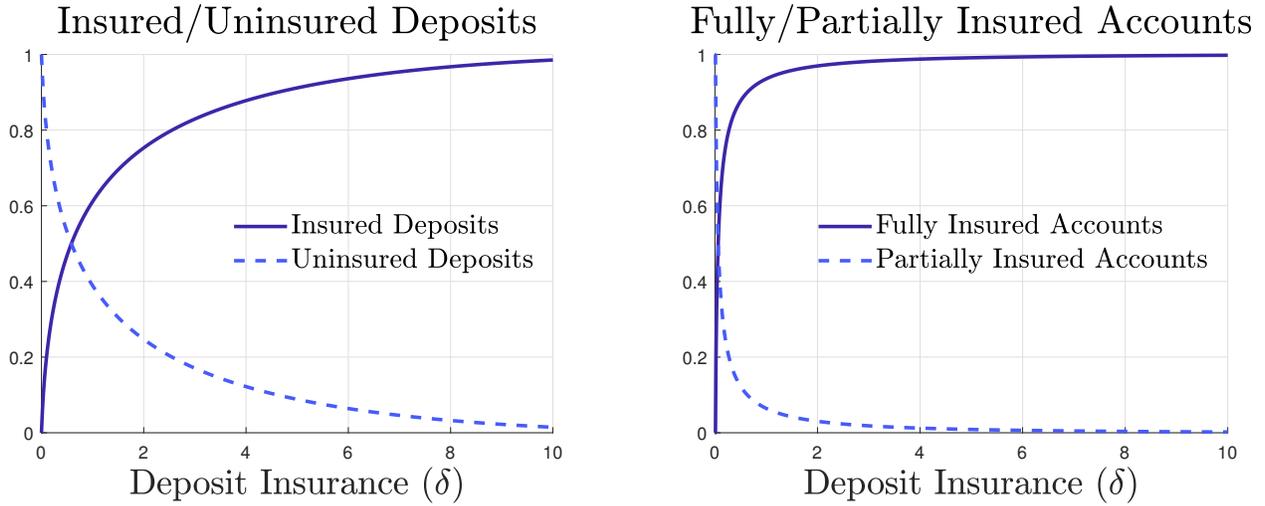


Figure OA-2: Share of Insured/Uninsured Deposits and Fully/Partially Insured Accounts

Note: The left panel in Figure OA-2 shows the share of insured deposits and its complement, the share of uninsured deposits, as a function of the level of deposit insurance coverage δ . The right panel in Figure OA-2 shows the share of fully insured accounts/depositors and its complement, the share of partially insured accounts/depositors, as a function of the level of deposit insurance coverage δ .

δ . Formally,

$$q^{T+} = \int_{\underline{s}}^{\hat{s}} \mathbb{I}[T(s) > 0] dF(s) + \pi \int_{\hat{s}}^{s^*} \mathbb{I}[T(s) > 0] dF(s)$$

$$q^{T+|F} = \frac{\int_{\underline{s}}^{\hat{s}} \mathbb{I}[T(s) > 0] dF(s) + \pi \int_{\hat{s}}^{s^*} \mathbb{I}[T(s) > 0] dF(s)}{\int_{\underline{s}}^{\hat{s}} dF(s) + \pi \int_{\hat{s}}^{s^*} dF(s)},$$

where $\mathbb{I}[\cdot]$ denotes the indicator function. Figure OA-6 integrates over $\frac{dW}{d\delta}$, as defined in Equation (26), to build a measure of social welfare. Figure OA-6 uses the normalization $W(0) = 0$.

F.4 Sensitivity Analysis: Sunspot Probability

Figure OA-8 illustrates how changes in the level of confidence in the economy, captured by the sunspot probability π (a high value of π has the interpretation of low confidence), affect the desirability of changing the level of coverage. Given our calibration, changes in π have a very strong impact on $\frac{dW}{d\delta}$ and, ultimately, on the optimal level of coverage. When π is high, the likelihood of a run in the multiple equilibria region is large, which makes increasing the level of coverage a very powerful tool, increasing the marginal benefit of higher coverage. While the marginal cost of increasing δ also grows, because — all else equal — failure is more likely, the increase in the marginal benefit is substantially larger, which implies that the optimal level of coverage is increasing in π .

F.5 Sensitivity Analysis: Banks' Riskiness

By studying how the predictions of our framework change with the riskiness of banks' investments we aim to capture different business cycle conditions, in the form of a risk shock to banks' investment. Figure

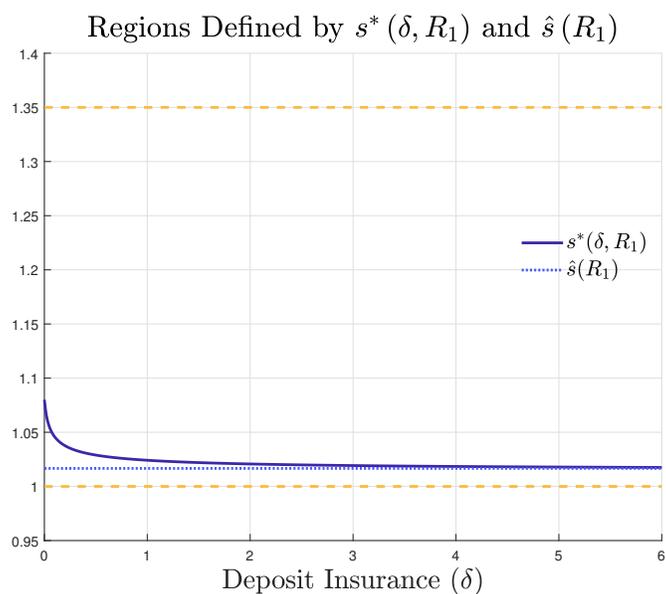


Figure OA-3: Regions Defined by $s^*(\delta, R_1)$ and $\hat{s}(R_1)$

Note: Figure OA-3 shows the thresholds $s^*(\delta, R_1)$ and $\hat{s}(R_1)$ as a function of the level of deposit insurance coverage δ .

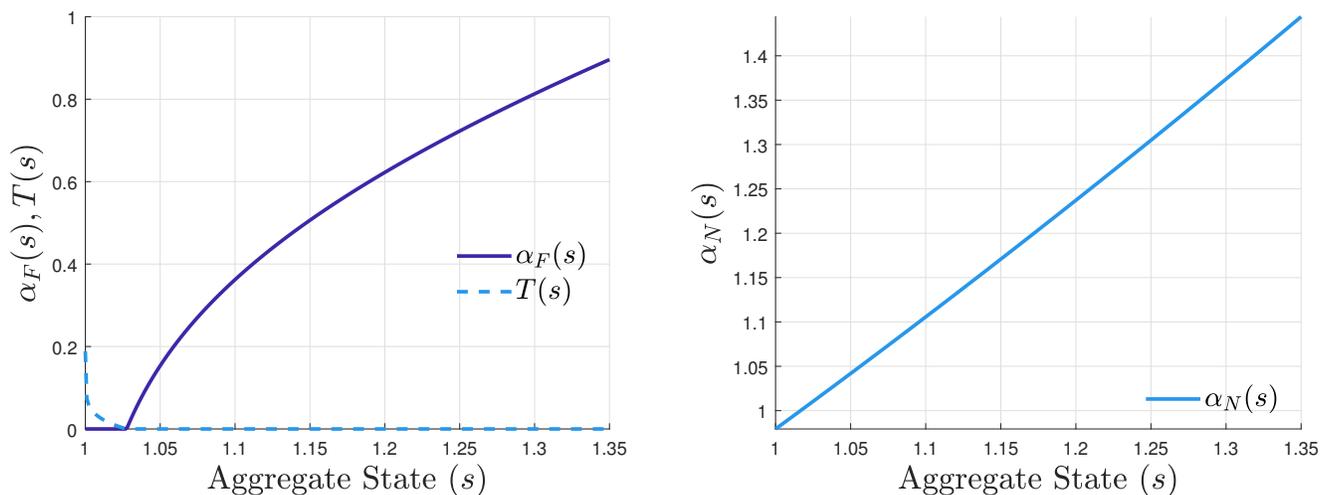


Figure OA-4: Depositors' Equilibrium Consumption Determinants and Funding Shortfall

Note: The left panel in Figure OA-4 shows the recovery rate on uninsured deposits in case of failure, $\alpha_F(s)$, as well as the funding shortfall, $T(s)$, as a function of the realizations of the state s when $\delta = 1$. The right panel in Figure OA-4 shows the additional gross return earned by the deposits that stay within the bank until date 2, $\alpha_N(s)$, as a function of the realizations of the state s when $\delta = 1$.

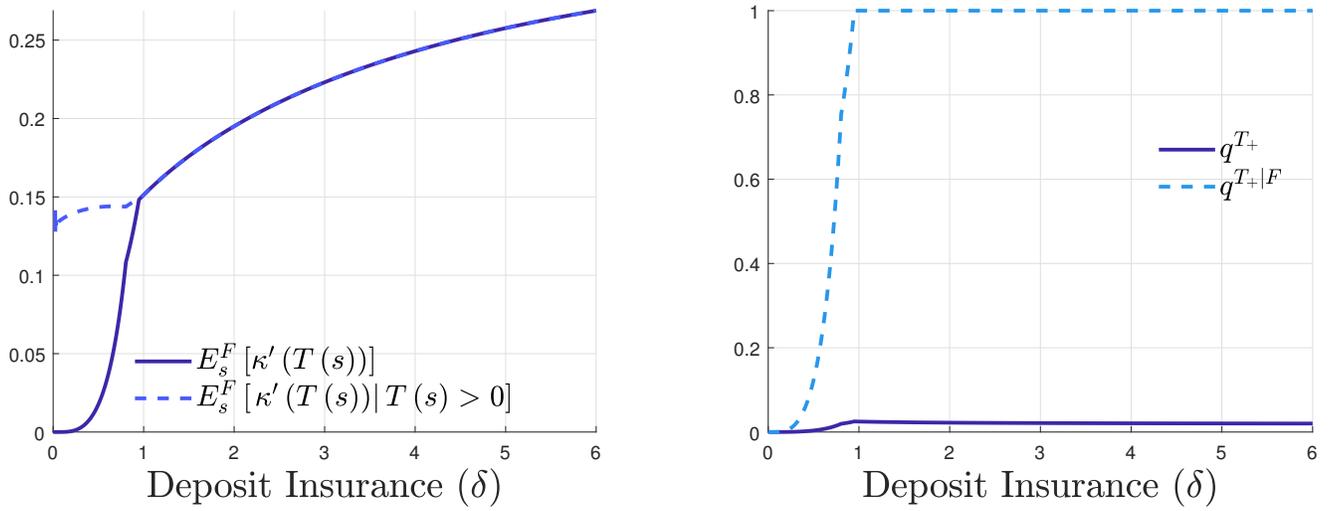


Figure OA-5: Marginal Cost of Public Funds/Probability of Funding Shortfall

Note: The left panel in Figure OA-5 shows the average marginal cost of public funds in failure states, $\mathbb{E}_s^F[\kappa'(T(s))]$, and in failure states in which the funding shortfall is positive, $\mathbb{E}_s^F[\kappa'(T(s)) | T(s) > 0]$, as a function of the level of deposit insurance coverage δ . The right panel in Figure OA-5 shows the unconditional probability of a positive funding shortfall, q^{T+} , and the probability of a positive funding shortfall condition on a bank failure taking place, as a function of the level of deposit insurance coverage δ .

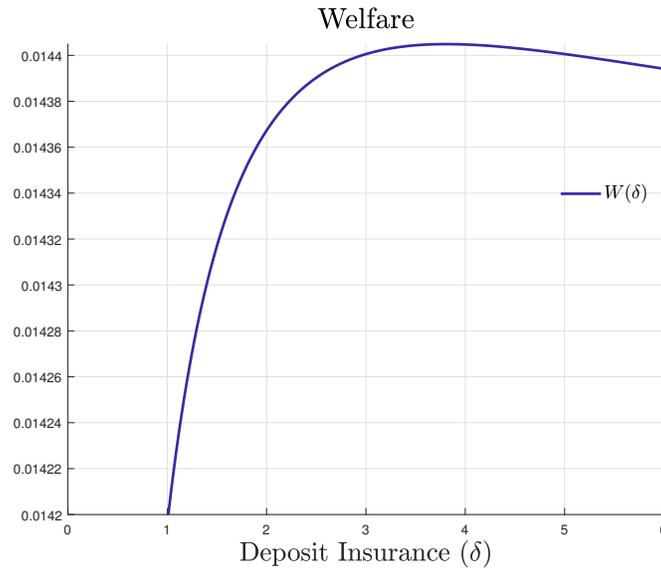


Figure OA-6: Social Welfare $W(\delta)$

Note: Figure OA-6 integrates over $\frac{dW}{d\delta}$, as defined in Equation (26), to build a measure of social welfare. This figure uses the normalization: $W(0) = 0$.

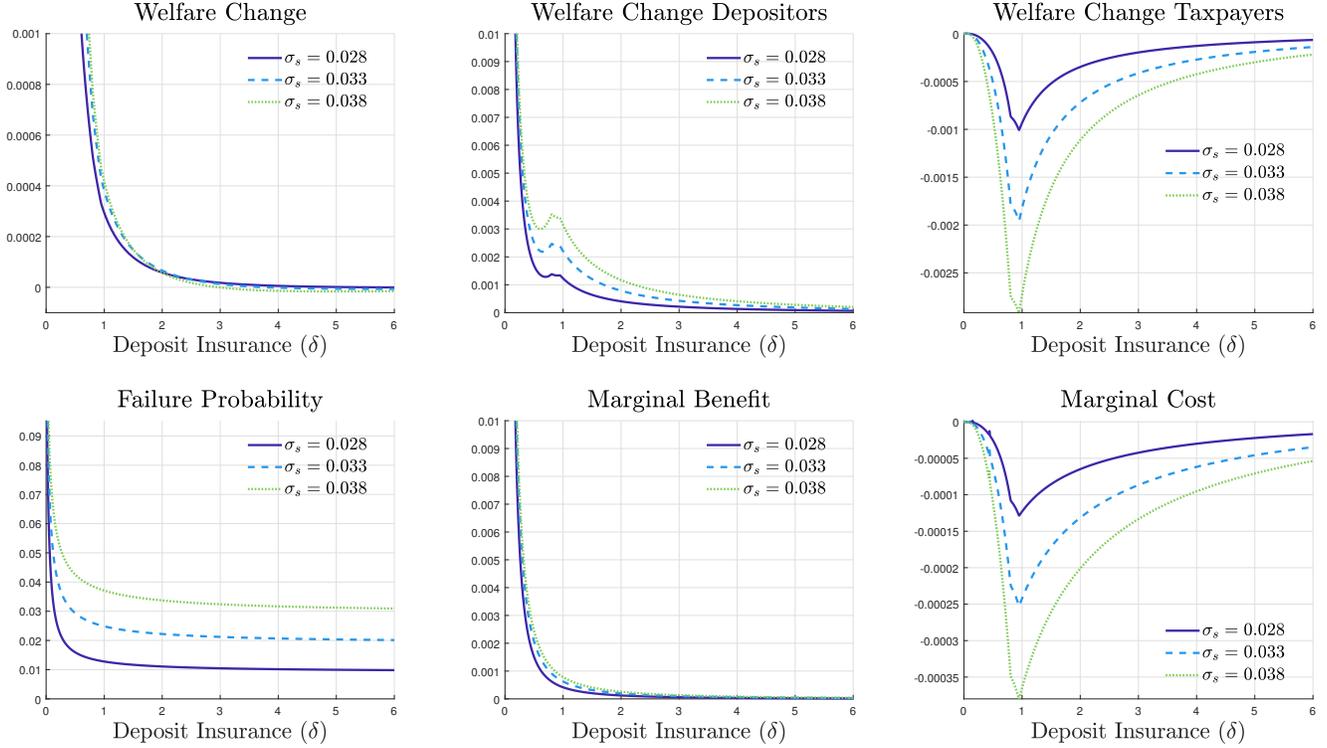


Figure OA-7: Sensitivity Analysis: Banks' Riskiness (σ_s)

Note: The top left panel in Figure OA-9 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage, $\frac{dW}{d\delta}$, as described in Equation (26), for $\sigma_s \in \{0.028, 0.033, 0.038\}$. The top middle and right panels respectively show the welfare change for depositors, $\lambda \int \frac{dV_m(i, \epsilon, \delta, R_1)}{d\delta} dG(i) + (1 - \lambda) \int \frac{dV_m(i, \ell, \delta, R_1)}{d\delta} dG(i)$, and taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$. The bottom left panel shows the probability of failure, $q^F(\delta, R_1)$, and the probability of fundamental failure $F(\hat{s}(R_1))$. The bottom middle and right panels show the marginal benefit and marginal cost of increasing the level of coverage, given by $-\frac{\partial q^F}{\partial \delta} \int (C^N(j, s^*) - C^F(j, s^*)) dH(j)$ and $q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right]$, respectively. The optimal levels of coverage are $\delta^* = 5.62$, $\delta^* = 3.81$, and $\delta^* = 2.97$ for $\sigma_s = 0.028$, $\sigma_s = 0.033$, and $\sigma_s = 0.038$, respectively.

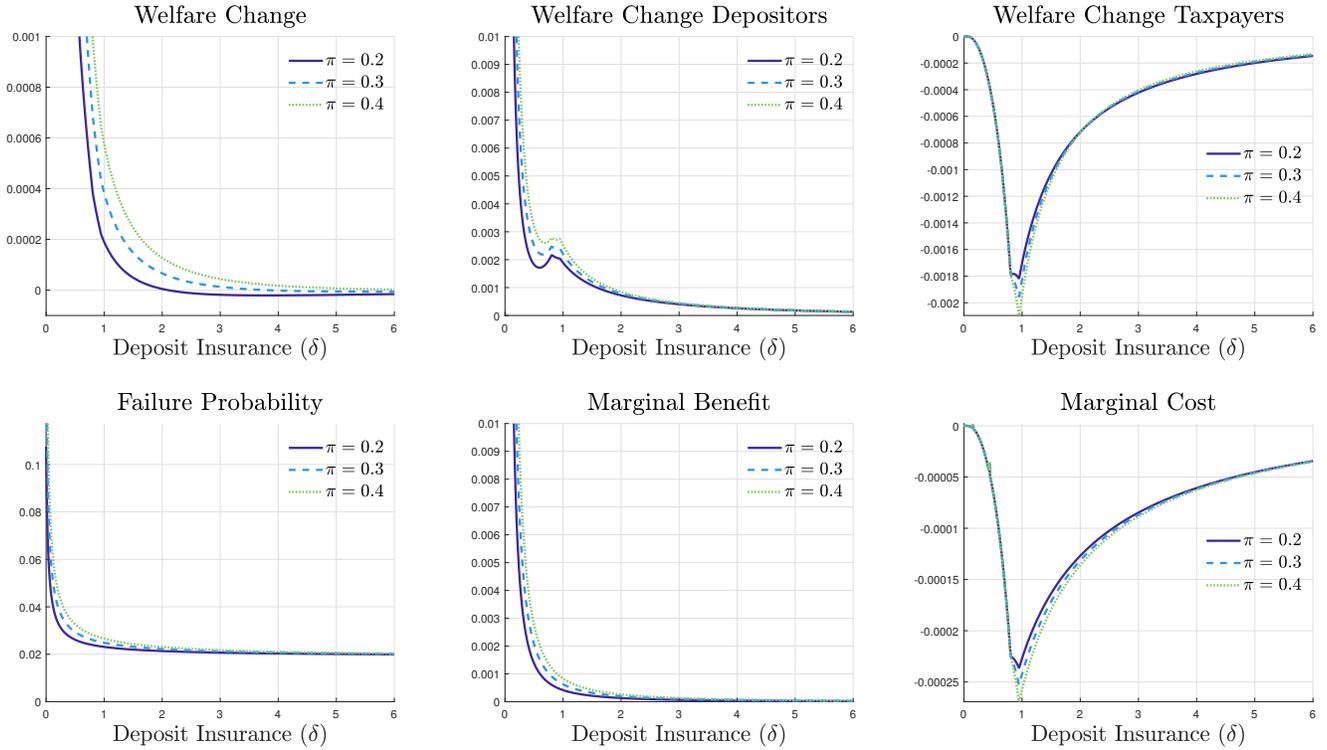


Figure OA-8: Sensitivity Analysis: Sunspot Probability (π)

Note: The top left panel in Figure OA-8 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage, $\frac{dW}{d\delta}$, as described in Equation (26), for $\pi \in \{0.2, 0.3, 0.4\}$. The top middle and right panels respectively show the welfare change for depositors, $\lambda \int \frac{dV_m(i, \epsilon, \delta, R_1)}{d\delta} dG(i) + (1 - \lambda) \int \frac{dV_m(i, \epsilon, \delta, R_1)}{d\delta} dG(i)$, and taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$. The bottom left panel shows the probability of failure, $q^F(\delta, R_1)$, and the probability of fundamental failure $F(\hat{s}(R_1))$. The bottom middle and right panels show the marginal benefit and marginal cost of increasing the level of coverage, given by $-\frac{\partial q^F}{\partial \delta} \int (C^N(j, s^*) - C^F(j, s^*)) dH(j)$ and $q^F \mathbb{E}_s^F \left[\int \frac{\partial C^F(j, s)}{\partial \delta} dH(j) \right]$, respectively. The optimal levels of coverage are $\delta^* = 2.1$, $\delta^* = 3.81$, and $\delta^* = 7.24$ for $\pi = 0.2$, $\pi = 0.3$, and $\pi = 0.4$, respectively.

OA-7 illustrates how changes in the level of σ_s affect the desirability of changing the level of coverage. A higher value of σ_s unambiguously reduces the welfare of taxpayers, since negative realizations of s , in which bank failures are more prevalent and costly, are more likely to occur. However, a higher value of σ_s has an ambiguous impact on depositors' welfare, depending on the level of δ . When the level of coverage is low, the increased volatility generates worse and more frequent failures, lowering depositors' welfare (for scaling reasons, this region is not visible in Figure OA-7). When the level of coverage is high, depositors benefit from the increase in volatility, since they receive all the upside when bank returns are high, but are shielded from bank failure by the generous level of coverage. Given our calibration, the net welfare effects on taxpayers' and depositors' imply that high riskiness of banks' investments is associated with lower levels of the optimal coverage limit.

F.6 Sensitivity Analysis: Cost of Public Funds

Figure OA-9 illustrates how changes in the fiscal capacity of the economy, captured by the marginal cost of public funds κ_1 , affect the desirability of changing the level of coverage. Changes in the marginal cost of public funds can be interpreted as a shock to the fiscal condition of the economy. Consistent with

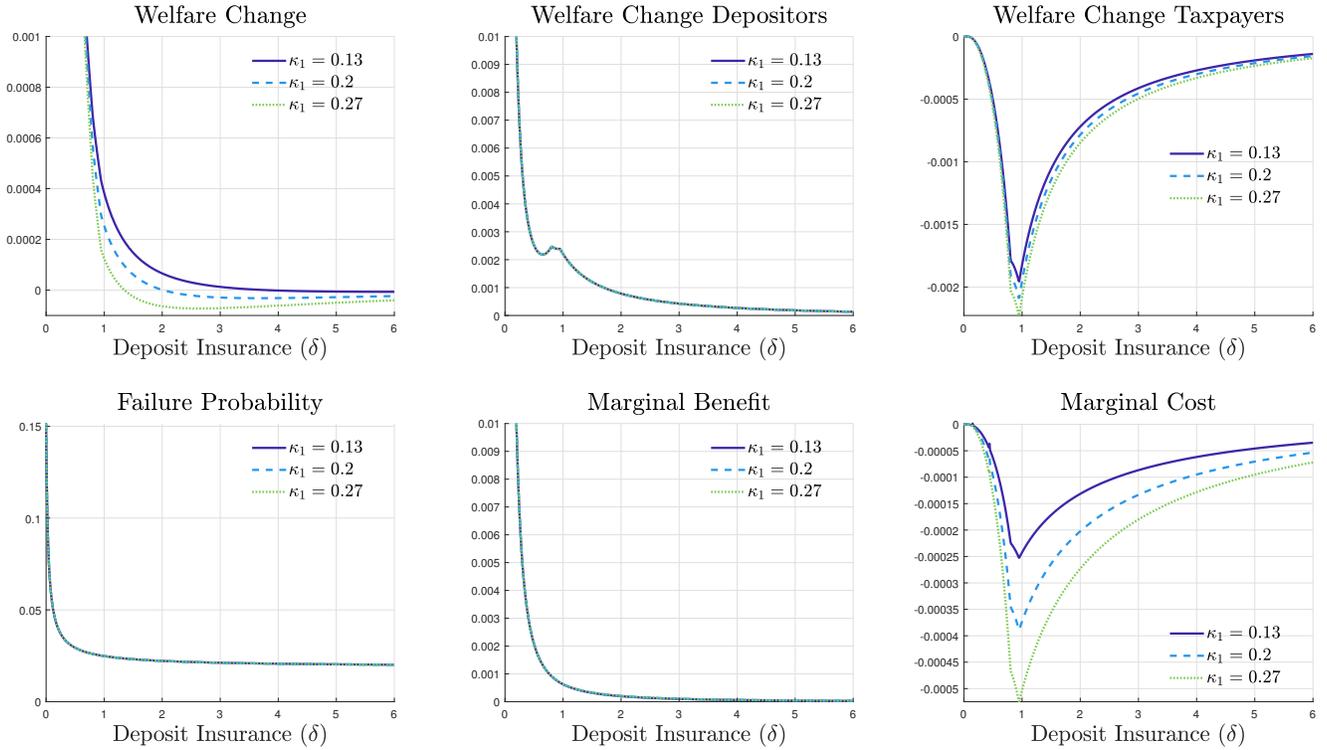


Figure OA-9: Sensitivity Analysis: Cost of Public Funds (κ_1)

Note: The top left panel in Figure OA-9 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage, $\frac{dW}{d\delta}$, as described in Equation (26), for $\kappa_1 \in \{0.13, 0.2, 0.27\}$. The top middle and right panels respectively show the welfare change for depositors, $\lambda \int \frac{dV_m(i, e, \delta, R_1)}{d\delta} dG(i) + (1 - \lambda) \int \frac{dV_m(i, e, \delta, R_1)}{d\delta} dG(i)$, and taxpayers, $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$. The bottom left panel shows the probability of failure, $q^F(\delta, R_1)$, and the probability of fundamental failure $F(\hat{s}(R_1))$. The bottom middle and right panels show the marginal benefit and marginal of increasing the level of coverage, as defined in Equation (26). The optimal levels of coverage are $\delta^* = 3.81$, $\delta^* = 2.02$, and $\delta^* = 1.35$ for $\kappa_1 = 0.13$, $\kappa_1 = 0.2$, and $\kappa_1 = 0.27$, respectively.

our analytical results, changes in the level of κ_1 exclusively affect taxpayers' welfare, leaving unchanged depositors' welfare. The main effect of an increase in the cost of public funds is that the marginal cost of paying for coverage becomes higher. Given our calibration, as one would expect, a higher κ_1 is associated with a lower optimal level of coverage δ^* , since the deadweight loss associated with covering funding shortfalls becomes higher.

F.7 Model-Based Quantification: Optimal Coverage Isoquants

Figure OA-10 describes the set of parameters that yield the same optimal level of coverage δ^* for combinations of π and σ_s and combinations π and κ_1 , respectively. The left panel in Figure OA-10 shows that higher values of the sunspot probability π and lower values of the riskiness of bank's investments are associated with a higher optimal level of coverage. The right panel in Figure OA-10 shows that higher values of the sunspot probability π and lower values of the marginal cost of public funds κ_1 are associated with a higher optimal level of coverage.

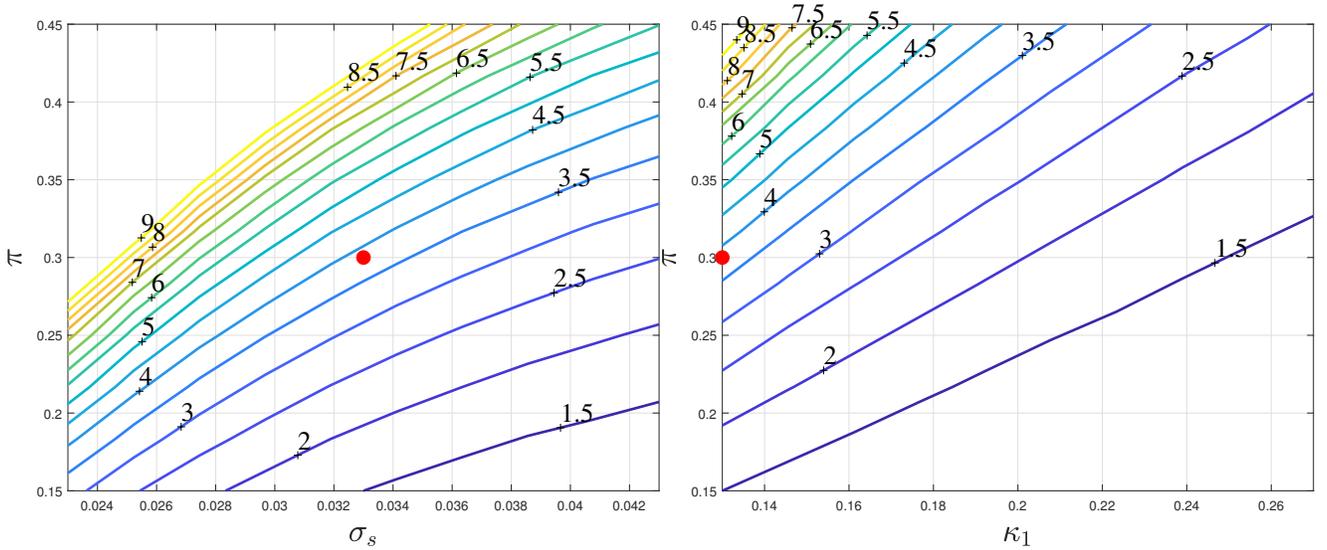


Figure OA-10: Optimal Deposit Insurance Coverage Isoquants

Note: The left panel in Figure OA-10 shows combinations of π and σ_s that yield the same optimal level of coverage δ^* . The right panel in Figure OA-10 shows combinations of π and κ_1 that yield the same optimal level of coverage δ^* . The red solid bullet point in each of the figures represents the baseline combination used in Section 4.2: $(\pi, \sigma_s) = (0.3, 0.033)$ and $(\pi, \kappa_1) = (0.3, 0.13)$, respectively.

F.8 Robustness of Approximation

In our model-based quantitative analysis in Section 4.2 we have used the characterization of the marginal welfare change from Proposition 2. The main advantage of working with the result in Proposition 2 is that we can characterize $\frac{dW}{d\delta}$ exclusively as a function of bank-level aggregates.

Here we explore the robustness of our results to the conditions necessary for Proposition 2 to hold. To simplify the exposition, we reproduce here Equation (22) in the text:

$$\frac{dW}{d\delta} = \int \omega(j) \left(-\frac{\partial q^F}{\partial \delta} \left(\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dH(j).$$

In Figure OA-11, we compute $\frac{dW}{d\delta}$ under three different sets of assumptions. First, we use uniform generalized social welfare weights, $\omega(j) = 1$, and use the approximation assumptions required for Proposition 2 to hold, that is, $\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \approx C^N(j, s^*) - C^F(j, s^*)$, and $m(j, s) = 1$. This is the approach we have followed in the body of the paper. Second, we use uniform generalized social welfare weights, $\omega(j) = 1$, but do not make any approximations. By comparing these two approaches, we can obtain a sense of the validity of the approximation. Finally, we compute $\frac{dW}{d\delta}$ using utilitarian traditional social welfare weights. As shown in Section E.3 of this Online Appendix, this corresponds to setting $\omega(j) = U'(C^F(j, s^*))$. By comparing the second and third approaches, we can understand the differences between using generalized and traditional welfare weights.

There are two main takeaways from this exploration. First, Figure OA-11 clearly shows that the choice of uniform generalized welfare weights versus utilitarian traditional welfare weights does matter for the welfare calculations. A planner using utilitarian traditional welfare weights finds a lower optimal

level of coverage because the gains from insuring the consumption depositors with medium to high levels of deposits have barely any weight in the planner's computations. This is illustrated in Figure OA-12 below, in which we compare the implied welfare weights $\omega(j)$ for depositors with different deposit levels and taxpayers under different assumptions. The solid line shows the implied generalized social welfare weights of depositors when using a traditional utilitarian objective. The dashed line shows the implied generalize social welfare weight for taxpayers also when using a traditional utilitarian objective. The dotted line shows for reference uniform generalized social weights.

Second, we find that the planner who uses the approximate results tends to overestimate the welfare gains from increasing the level of coverage, but the approximate are results reasonably accurate for our baseline calibration. The optimal level of coverage changes from $\delta^* = 3.81$ when using the approximation to $\delta^* = 3.42$ in the exact case. Whether the approximation results are closer to the exact ones does depend on the value of γ . Intuitively, the quality of the approximate results is a function of the curvature of the utility functions of the agents in the economy, since the two approximations are exact when agents are risk-neutral. With isoelastic utility, $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} = \left(\frac{C^F(j, s)}{C^F(j, s^*)}\right)^{-\gamma}$, and we show in Section C of this Online Appendix that $\frac{C^F(j, s)}{C^F(j, s^*)} \leq 1$ for both depositors and taxpayers. Therefore, increasing γ makes the value of $m(j, s)$ move away from 1, worsening the approximation. When using the approximation, the welfare losses suffered by taxpayers in states with very low s carry a lower weight than when using the exact solution (since $m(\tau, s) = 1$), which underestimates the marginal cost of increasing δ , making a higher level of coverage more desirable. Finally, note that these effects are also modulated by the value of $Y(\tau, s)$, since the marginal utility of taxpayers becomes more sensitive to all these effects when $Y(\tau, s)$ is lower.

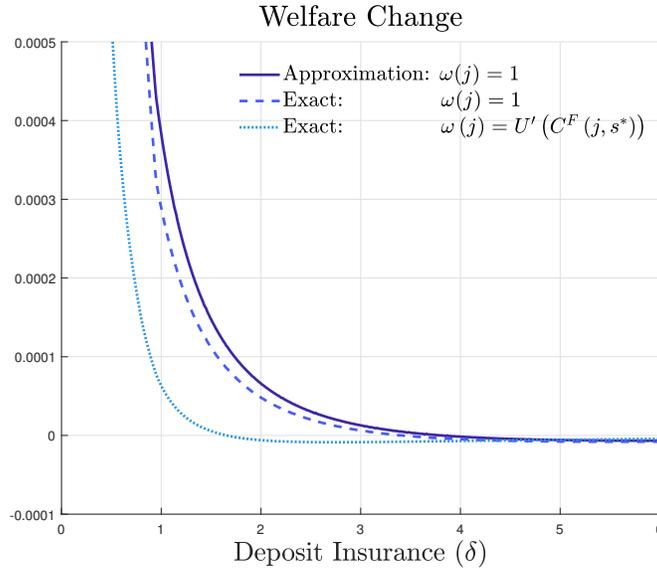


Figure OA-11: Robustness of Approximation

Note: Figure OA-11 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage, $\frac{dW}{d\delta}$, under different assumptions on how to aggregate and compute social welfare. First, the solid line computes $\frac{dW}{d\delta}$ as in Equation (26), that is, using uniform generalized social welfare weights and under the approximation required for Proposition 2 to hold. Second, the dashed line computes $\frac{dW}{d\delta}$ as in Equation (22) when setting $\omega(j) = 1$. Third, the dotted line computes $\frac{dW}{d\delta}$ as in Equation (22) when using utilitarian traditional social welfare weights, so $\omega(j) = U'(C^F(j, s^*))$. The optimal levels of coverage in each case are $\delta^* = 1.64$, $\delta^* = 3.41$, and $\delta^* = 3.81$, respectively.

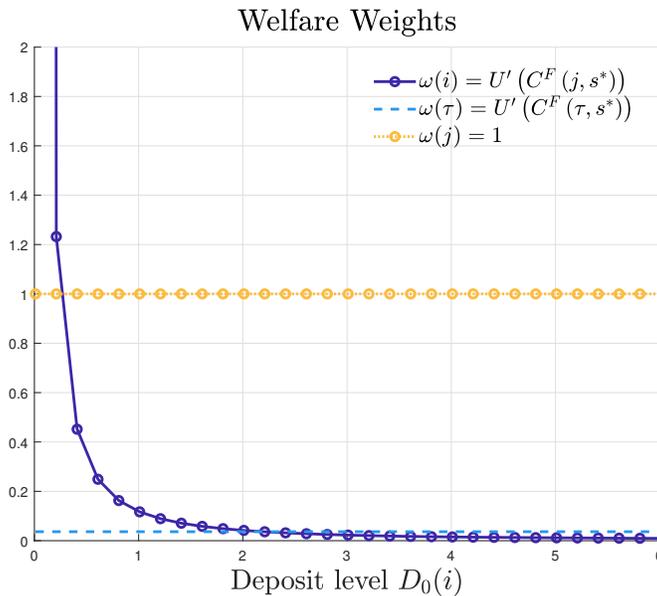


Figure OA-12: Generalized vs. Traditional Welfare Weighs

Note: Figure OA-12 shows the implied welfare weights $\omega(j)$ for depositors with different deposit levels and taxpayers under different assumptions. The solid dark blue line shows the implied generalized social welfare weights of depositors when using a traditional utilitarian objective. The dashed light blue line shows the implied generalize social welfare weight for taxpayers also when using a traditional utilitarian objective. The dotted orange line shows for reference uniform generalized social weights.