On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices*

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Abstract

We study how environmental, social and governance (ESG) investing reshapes information aggregation by prices. We develop a rational expectations equilibrium model in which traditional and green investors are informed about financial and ESG risks but have different preferences over them. Because of the preference heterogeneity, traditional and green investors trade in the opposite directions based on the same information. We show that the equilibrium price may not be uniquely determined. An increase in the fraction of green investors and an improvement in the ESG information quality can reduce price informativeness about the financial payoff and raise the cost of capital.

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1 Introduction

One of the fastest growing phenomena in the financial industry in recent years has been the focus on environmental, social and governance (ESG) issues. An important part of this phenomenon is that many institutional investors are now explicitly integrating ESG principles into their investment strategies. In the United States alone, about \$17.1 trillion of investment funds' assets were invested with ESG considerations in 2020, a 2.5-fold increase from the \$6.6 trillion in 2014 (GSIA, 2021).

This trend might fundamentally change the equilibrium in financial markets. A key function of financial markets is to produce and aggregate information about the fundamentals of the traded assets (Hayek, 1945). This price informativeness in turn is important for welfare as it affects resource allocation in the real economy, through its effect on the cost of capital or through active learning by decision makers in the real economy. Traditionally, investors in financial markets were thought to be uniform, at least in their general objectives, as they were all focused on firms' cash flows (and the risks involved with these cash flows). Hence, this is what information in financial markets was expected to be about. However, now that a new class of investors, namely ESG investors, emerged and is becoming increasingly prominent, this classic paradigm should be challenged and the implications should be explored.

Several questions come up. First, it is less clear what prices will be informative about and to what extent.[1] Second, these changes in price informativeness may affect the firms' cost of capital and hence resource allocation and aggregate welfare. The overall effect is a priori unclear. Third, given the general lack of transparency about ESG performance, some may hope that the market can help in generating such information,[2] but it is not clear whether this might conflict with the role of generating information about cash flows. In this paper, we provide a model to address these questions.

Our model is in the tradition of the noisy rational expectations equilibrium (REE) models a la Hellwig (1980), where rational investors observe and trade on heterogeneous signals about firms' payoffs. They also learn from the price and incorporate the information

[1] Multiple empirical studies find that asset prices react to ESG news (e.g. Flammer, 2013; Krüger, 2015; Capelle-Blancard and Petit, 2019) and are informative about firms' ESG performances (Ng and Rezaee, 2020).

[2] For example, Berg, Kölbel, and Rigobon (2020) document rating divergence of six major ESG rating agencies. Practitioners (BlackRock, 2021) and academics (Huij, Laurs, Stork, and Zwinkels, 2021) suggest using asset prices to infer firm-specific ESG risks.

they learn in their trading. Finally, the market also has traders who trade for exogenous reasons and bring noise to the price. We amend this model in a couple of ways. First, we assume that the "payoff" consists of two risky components: a financial cash flow and an ESG component. Second, we assume that the financial market is populated with two groups of risk-averse rational investors who receive heterogeneous informative signals about both payoff components but have distinct preferences about them. Specifically, traditional investors value only the financial payoff while ESG ("green") investors value both financial and ESG payoffs.

Mapping this setup to the real-world environment, a few points are important to make. First, green investors' preferences can be interpreted as non-pecuniary warm glow utility from investing in assets with high ESG performance, or alternatively, these investors can be the managers of ESG funds, whose compensation and reputation hinge on identifying good ESG assets. Second, investing on ESG also involves risk because of the large uncertainty surrounding which assets are truly good ESG assets. This can be a major risk for ESG investors and, perhaps even more, for ESG asset managers. Third, private signals about ESG performance are a result of proprietary research on the topic, motivated to a large extent by the uncertainty ESG presents.[3] Fourth, while only green investors care about the ESG component directly, traditional investors care about it indirectly and want to learn about it to better interpret the price and enhance profitability of their trading in equilibrium. Therefore, it is reasonable to assume that both types of investors have information about the ESG component (as well as about the traditional component).[4]

The driving force of our model is the strategic interaction between traditional and green investors through learning and trading a given security. Because of their heterogeneous preferences, the two investor groups seek to learn different information from the price. They also end up trading differently on similar signals. In particular, when receiving positive signals about the firm's ESG payoff, green investors increase their demand for the stock, while traditional investors reduce their demand. The latter is an equilibrium response, since a positive signal on ESG, combined with their learning from the price, lead them to infer a worse realization of the financial payoff. As a result, trades by one investor group make the price more informative about what investors from this group

[3]For example, according to the BlackRock's ESG Integration Statement (BlackRock, 2021), the firm develops proprietary measurement tools to provide its portfolio managers with assessments of material ESG performance indicators.

[4]Although we do not explicitly model information acquisition, both types of investors have incentive to acquire ESG information if the cost of doing so is not prohibitively large.

care about and less informative about what investors from the other group care about. In this way, trades by traditional and green investors contaminate price informativeness to each other. Key to this is the idea that investors with heterogeneous preferences trade the same security differently because they care about different payoff components of the same security.

Based on these forces, we identify a feedback loop between investors' trading intensities and the amount of information contained in the price about the two different components. This leads to multiple equilibria in the trading game, such that the price can end up being dominated by one factor in one equilibrium or the other factor in another equilibrium. Specifically, if investors of one group trade more intensively on their private signals than investors of the other group, the preferences of the dominating group are reflected by the price more. As a result, the price becomes more informative to them and so they face less uncertainty when holding the stock. This justifies their more intensive trading. The feedback loop implies that two equilibria can coexist. In one equilibrium, the stock is predominately traded by traditional investors, the equilibrium price primarily loads on the financial component and is not particularly informative to green investors. In the other equilibrium, green investors dominate the trading, and the equilibrium price is more aligned with their preferences.

We characterize how the emergence of multiple equilibria depends on four parameters of the model. First, multiplicity arises when noise traders' demand is not too volatile. Otherwise, the price will be a poor signal about both payoff components and thus uninformative to all rational investors, preventing the above feedback loop from developing. Second, multiplicity requires that the preference heterogeneity is sufficiently strong. If this is not the case, traditional and green investors seek to learn similar information from the price. Third, it is also important that traditional and green investors receive informative signals about both payoff components. Otherwise, they will not be able to trade against the signal about the component they do not value and they will not prevent the price from being informative about it. Finally, for multiplicity to arise, the masses of traditional and green investors should not be too different from each other. If the investor base is very unbalanced and strongly tilted towards one investor type, investors of this type always dominate the trading.

This last property highlights one of the most important implications of our model for how the current transition in financial markets could impact the equilibrium. The increased presence of ESG investors in the market implies that we are shifting from a world where the only equilibrium is one where the cash flow component dominates the market to a world where both equilibria are possible. Hence, there could be a sudden shift for some stocks from a cash-flow-dominated price to an ESG-dominated price, and so a sudden shift in what kind of information investors can glean from the price. Aside from jumps across equilibria, we also study the impact of the increase in the share of green investors in the market for a given equilibrium. We show that, as the green investor share increases, the price becomes less informative to traditional investors, that is, less informative about the financial component, and more informative to green investors in any stable equilibrium.

This result has important implications for the firm's cost of capital. As is standard in REE models (e.g. Easley and O'Hara, 2004), the cost of capital reflects the average information risk faced by rational investors. We find that the cost of capital is non-monotone in the share of green investors and increases when the masses of traditional and green investors are similar. When the investor base is balanced, the price is not particularly informative to any investor group, and so both require high compensation for bearing the informational risk, which drives up the cost of capital. On the other hand, with an unbalanced investor base, the dominant group finds the price informative and drives down the cost of capital. Again, as we think about the current transition in financial markets, this implies that the information channel leads to an increase in the cost of capital when the market moves to be less dominated by traditional investors. This result is helpful to reconcile two seemingly contradictory empirical observations. On the one hand, green investors are willing to sacrifice financial payoff for non-pecuniary benefits (e.g. Martin and Moser, 2016; Riedl and Smeets, 2017; Barber, Morse, and Yasuda, 2021; Li, Ruan, Titman, and Xiang, 2022), which implies a lower cost of capital for green firms with a larger fraction of green investors. On the other hand, direct empirical evidence on the cost of capital for green firms is rather mixed.[5] Our results suggest that, although green firms may attract green investors demanding a lower financial return, entry of such investors can amplify the informational risks faced by existing traditional investors and lead them to demand a higher financial return.

Another important development in ESG investing in recent years is the regulatory push worldwide for improving the quality of ESG information (see, for example, the report on non-financial and sustainability reporting by van der Lugt, van de Wijs, and Petrovics, 2020). Our final exercise extends the model to examine the implications of such regulatory

[5] In Section 4.2, we allow the firm's ESG performance to be positive on average. In this extension, an increase in the green investor share has an additional negative impact on the cost of capital. We discuss empirical implications of this model and existing empirical evidence in Section 4.3.

changes. Holding the pricing function fixed, better ESG information benefits traditional investors as it helps them to interpret the price more accurately and learn more about the financial component from it. At the same time, green investors, who value the ESG payoff directly, benefit from better ESG information more. In particular, they respond by substantially increasing their trading intensities. Changes in trading intensities affect the equilibrium pricing function. We show that, if the preference heterogeneity across traditional and green investors is sufficiently strong, the price becomes less associated with the financial component and less informative to traditional investors. Furthermore, we show that the decrease in the price informativeness to traditional investors can dominate the increase in the price informativeness to green investors, leading to an increase in the cost of capital. This is an unintended consequence of improved quality of ESG information that should be considered in the policy circles.

The essential feature of the model is that the stock payoff consists of two distinct components. Throughout most of the paper, we assume that the financial and ESG components are uncorrelated. In reality, they are potentially correlated, and a priori, it is not clear whether the correlation is positive or negative. On the one hand, a firm with a good ESG performance can attract socially-concerned customers and is likely to be resilient against regulations such as carbon taxes. On the other hand, improving the ESG performance may be financially costly. To explore the correlation between the two payoff components, we explore an extension and show that a positive/negative correlation between them is equivalent to investors' preferences being more/less aligned. Therefore, our key message holds if the two components are correlated. Moreover, our results on the role of preference heterogeneity also shed light on the role of correlation structure in shaping prices in the market.

Overall, our model shows the novel implications of having different groups of traders with different preferences but comparable sets of information interacting in the financial market for a given security. The multiplicity of equilibria, where prices can be dominated by different factors, is unique to this setting. To the best of our knowledge, other implications, such as impacts of the investor base composition and the quality of ESG information on price informativeness and cost of capital, are also novel. While we relate this setup to the emerging ESG phenomenon, as we think it features its key properties, there are other applications that our model could be suitable for. For example, funds pursuing different strategies might care about different components of stock payoffs to fulfill different investment needs. As another example, investors with different investment horizons assign different weights to short-term payouts and long-term values (Bushee, 2001) which might be driven by distinct shocks. Similarly, investors might have heterogeneous preferences about dividends and capital gains (Graham and Kumar, 2006; Harris, Hartzmark, and Solomon, 2015). Even in the universe of ESG investors, preference heterogeneity might matter as some investors focus more on the environmental aspects while others might focus more on the social or governance aspects.

Literature There is a recent theoretical literature that investigates the impact of ESG investing on asset prices. Papers in this literature include Fama and French (2007), Luo and Balvers (2017), Baker, Bergstresser, Serafeim, and Wurgler (2018), Baker, Hollifield, and Osambela (2020), Zerbib (2020), Pastor, Stambaugh, and Taylor (2021b), and Pedersen, Fitzgibbons, and Pomorski (2021). Like us, they start from the premise that some investors derive utility from investing in assets with high ESG performance. Unlike these papers, we investigate how information about this performance gets into asset prices through investors who trade on private information, and how price informativeness is shaped between the ESG component and the traditional cash-flow component. Uncertainty about the ESG payoff is also featured in Avramov, Cheng, Lioui, and Tarelli (2021) and in Friedman and Heinle (2016), but there is no investigation of trading based on private information and the resulting price informativeness in these papers.

Another important question in the emerging ESG literature is about the impact of ESG investors on firms' production decisions. A natural way to achieve impact is through engagement by activist green investors, as in papers by Gollier and Pouget (2014), Chowdhry, Davies, and Waters (2019), Landier and Lovo (2020), Oehmke and Opp (2020), Green and Roth (2021), and Gupta, Kopytov, and Starmans (2021). In papers like ours, where investors are atomistic, such effects are not present. Instead, investors' decisions in financial markets affect firms' cost of capital, which may indirectly affect their production. Heinkel, Kraus, and Zechner (2001) show that firms excluded by green investors suffer a reduction in risk sharing in their investor base and thus have a higher cost of capital. The cost of capital channel is also at work in asset pricing models that are discussed in the previous paragraph (e.g. Pastor et al., 2021b). Hart and Zingales (2017) and Broccardo, Hart, and Zingales (2020) study engagement and exclusion in a unified model. Our model reveals a novel effect of ESG investing on the cost of capital through the information channel. In particular, we show that the presence of green investors can lead to an *increase* in the firm's cost of capital.[6] We further discuss empirical implications of our theoretical results and review related empirical literature in Sections 4.3 and 5.4.

On the methodology, our model contributes to the noisy REE literature, pioneered by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). This literature has grown a lot over the years and features many different settings. Hence, it cannot be fully reviewed here, but we will describe the most related papers below. Overall, to the best of our knowledge, our model is the first to combine the following two features. First, the market is populated by investors with heterogeneous preferences over multiple fundamentals. Second, investors are not restricted to be informed only about the fundamental they value. In our view, this combination is particularly relevant to describe financial markets as they start to transition to a new ESG reality. As we describe above, there are other possible applications for this setting.

A few papers analyze models with multiple fundamentals under homogeneous investor preferences. Goldstein and Yang (2015) build a model in which asset payoff is affected by two fundamentals while investors receive heterogeneous information about the fundamentals. Cespa and Foucault (2014) construct a two-asset economy to study cross-asset learning and liquidity spillovers. Ganguli and Yang (2009) and Manzano and Vives (2011) consider settings in which investors possess information about asset payoff and aggregate supply shock (see also Amador and Weill, 2010 and Davila and Parlatore, 2021); in Brunnermeier, Sockin, and Xiong (2021), investors can choose to learn about the fundamental or government action. Unlike these papers, our paper features preference heterogeneity, and this generates a new implication: investors with different preferences use the same information to trade in opposite directions, thus making the price noisier to each other.[7] This is the key force behind our results about different information regimes in the market and the comparative statics for price informativeness and the cost of capital.

A few other papers introduce heterogeneous valuations in the REE framework. Vives (2011 and 2014), Vanwalleghem (2017), Rahi and Zigrand (2018) and Rahi (2021) study models in which agents have private valuations but, different from our paper, receive only information about their private valuations. In our model, the financial and ESG components are firm-specific and thus all investors can learn about them. The fact that

[6] In Pedersen et al. (2021), the presence of ESG-unaware investors can boost expected returns of green stocks. In our model, all rational investors are aware of ESG and financial risks and the cost of capital increase is due to the information channel, which is specific to our paper.

[7] From this perspective, our paper is related to Goldstein, Li, and Yang (2014), where investors' objectives might be different due to different investment opportunities.

investors receive signals about both factors in our model is critical for the key force that they end up trading in opposite directions on similar information, and this leads to our main results.

The paper proceeds as follows. Section 2 presents a simplified model to highlight the key mechanisms. Section 3 lays out the main model and characterizes equilibria. Sections 4 and 5 study the growth of green investors and the improvement in ESG information quality, respectively. Section 6 concludes. Appendix contains all proofs missing from the main text.

2 A simplified model

To highlight the key mechanisms, we start by presenting a simplified version of our model in which we are able to get closed-form solutions.

2.1 Setup

Two assets are traded in the financial market: a risk-free bond and a risky stock of a firm. The bond is in unlimited supply. It pays off one and its price is normalized to one. The stock is a claim on the firm's output which consists of two risky components: a financial component $\tilde{z}$ and an ESG component $\tilde{\delta}$. The financial part can be interpreted as a cash flow generated by the firm. The ESG part can be interpreted as the firm's contribution to social good, for example, the amount of environmentally harmful carbon emissions taken with a negative sign. The two payoff components are uncorrelated normal random variables, $\tilde{z}, \tilde{\delta} \sim N\left(0, \tau^{-1}\right)$. The stock is in unit supply, and its price $\tilde{p}$ is determined endogenously by market clearing in equilibrium.

There are two groups of rational investors with a combined mass of $m > 0$. Half of them are traditional investors who only value the firm's financial output $\tilde{z}$. The other half are green investors who only value the ESG output $\tilde{\delta}$. Green investors derive a warm-glow utility from holding stocks with high ESG performance. That is, green investors care about greenness of their portfolios but, being atomistic, do not consider how their investments contribute to public good. This is a standard way to model preferences of atomistic green investors (e.g. Fama and French, 2007; Pastor et al., 2021b).[8]

[8]In contrast, if investors have size (as in, for example, Oehmke and Opp, 2020), they might internalize

Green investors in our model can be viewed not only as investors who derive non-pecuniary utility from good ESG performance of their investments. They can be also seen as managers of ESG-focused funds whose flows depend on managers' success in identifying assets with good ESG performances. Compensations of these managers are thus tied to their ability to pick such assets.[9] Under this interpretation, green investors want to invest in assets with good ESG performances because to them it translates into high monetary payoffs.

Both traditional and green investors have constant absolute risk aversion (CARA) utilities with the same risk aversion parameter γ. Specifically, if an investor of type $j \in \{t, g\}$ has an initial wealth W_0 and chooses to hold q shares, then her expected utility is

$$\mathbb{E}\left\{-\exp\left(-\gamma\left[W_0 + q\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} - \tilde{p}\right)\right]\right)\right\}, \tag{1}$$

where $\beta_z^t = 1$, $\beta_\delta^t = 0$, $\beta_z^g = 0$ and $\beta_\delta^g = 1$.[10] In addition to rational traders, there are noise traders whose stock demand is $\tilde{n} \sim N\left(0, \tau_n^{-1}\right)$.

Utility function (1) implies that green investors are averse to risk in the stock's ESG performance. This is consistent with the fact that many real-life ESG investors are concerned about uncertainty about firms' ESG performances. For example, Avramov et al. (2021) document that demand of ESG-sensitive institutional investors for stocks diminish in their ESG rating uncertainty. Notably, aversion to risk in the firm's ESG output might stem from a conventional risk aversion about monetary payoffs. As discussed above, green investors in our model can be viewed as ESG fund managers whose compensations are linked to their ability to select stocks with good ESG performances. For such managers, investing in a stock with uncertain ESG output is risky as it might lead to an outflow and reduction in fees if this stock turns out to have weak ESG performance.

Rational investors trade based on information contained in the stock price and their private signals. Traditional and green investors receive signals about both financial and ESG fundamentals, namely, an investor i observes $\tilde{s}_z^i \sim N\left(\tilde{z}, \tau_s^{-1}\right)$ and $\tilde{s}_\delta^i \sim N\left(\tilde{\delta}, \tau_s^{-1}\right)$.

their impacts on aggregate outcomes and public good.

[9] Hartzmark and Sussman (2019) document that mutual funds that are ranked high in sustainability by Morningstar attract additional flows.

[10] That is, type-j investors assign weights β_z^j and β_δ^j to per-dollar financial and ESG stock returns, respectively. Alternatively, one can consider the specification in which an investor assigns the same weight β_z^j to all monetary payoffs. The characterization of the equilibrium pricing function remains very similar. In particular, as we will discuss later, the key equilibrium objects—normalized price coefficients ξ_z and ξ_δ in the pricing function (4)—remain the same.

This assumption on the information structure differentiates our paper from existing works on rational expectation models featuring agents with heterogeneous private valuations of a risky asset (e.g. Vives, 2014; Rahi and Zigrand, 2018). In those works, investors receive informative signals only about their private asset valuations. In our model, $\tilde{z}$ and $\tilde{\delta}$ are firm-specific payoff components that all investors can learn about. For example, investors are likely to learn about both payoff components by reading analyst and investor reports that describe firm's performance and risks comprehensively.

In our main model presented in Section 3, we relax several assumptions that we make in the simplified model. In particular, we allow masses of traditional and green investors to differ. We also allow green investors to value both financial and ESG payoff components. We discuss how nonzero correlation between the two components affects our results. In Appendix E, we consider a general information structure featuring different information precisions for different types of investors and different payoff components. Such a model is much less tractable. Nevertheless, we show that our key results hold as long as traditional and green investors receive informative signals about both payoff components, not necessarily of equal precisions.

2.2 Market clearing and equilibrium

As is standard in a CARA-normal setup, the demand for the stock from an investor i of type $j \in \{t, g\}$ is

$$d^{ij}\left(\mathcal{F}^i\right) = \frac{\mathbb{E}\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} | \mathcal{F}^i\right) - \tilde{p}}{\gamma \mathbb{V}\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} | \mathcal{F}^i\right)}, \tag{2}$$

where the information set $\mathcal{F}^i = \{\tilde{s}_z^i, \tilde{s}_\delta^i, \tilde{p}\}$ includes investor i's private signals and publicly observable stock price. Aggregating individual demands of rational investors and adding the demand from noise traders, we obtain the following market clearing condition:

$$D^t\left(\tilde{z}, \tilde{\delta}, \tilde{p}\right) + D^g\left(\tilde{z}, \tilde{\delta}, \tilde{p}\right) + \tilde{n} = 1, \tag{3}$$

where $D^j\left(\tilde{z}, \tilde{\delta}, \tilde{p}\right) = \int_{i \in \mathcal{T}_j} d^{ij}\left(\mathcal{F}^i\right) di$ is the total demand for the stock from investors of type j; $\mathcal{T}_j$ denotes the set of investors of type $j \in \{t, g\}$.

Throughout the paper, we focus on rational expectation equilibria (REE) with linear prices,

$$\tilde{p} = p_0 + p_z \tilde{z} + p_\delta \tilde{\delta} + p_n \tilde{n} = p_0 + p_n \left(\xi_z \tilde{z} + \xi_\delta \tilde{\delta} + \tilde{n} \right), \tag{4}$$

where we define normalized price coefficients $\xi_z = \frac{p_z}{p_n}$ and $\xi_\delta = \frac{p_\delta}{p_n}$.

2.3 Equilibrium characterization

2.3.1 Trading intensities and feedback loop

Equilibrium price coefficients ξ_z and ξ_δ are shaped by trades of rational investors based on their private signals about $\tilde{z}$ and $\tilde{\delta}$. A main ingredient of our model is heterogeneity in preferences of traditional and green investors. It has important implications on how investors use their information to trade. Consider a traditional investor. Denote her trading intensities with respect to her private signals $\tilde{s}_z$ and $\tilde{s}_\delta$ as i_z^t and i_δ^t, respectively, where trading intensities are defined as[11]

$$i_z^t \equiv \frac{\partial d^t \left(\tilde{s}_z, \tilde{s}_\delta, \tilde{p} \right)}{\partial \tilde{s}_z} = \frac{\tau_s}{\gamma}, \tag{5}$$

$$i_\delta^t \equiv \frac{\partial d^t \left(\tilde{s}_z, \tilde{s}_\delta, \tilde{p} \right)}{\partial \tilde{s}_\delta} = -\frac{\tau_s}{\gamma} \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}}. \tag{6}$$

To understand what drives the traditional investor's trading intensities, it is useful to look at how she infers information about $\tilde{z}$, the payoff component that she values, from the price and her signals. Specifically, she expects to receive the following payoff from holding one share:

$$\mathbb{E}(\tilde{z} | \tilde{s}_z, \tilde{s}_\delta, \tilde{p}) = \underbrace{\tilde{s}_z \frac{\tau_s}{\tau_s + \tau}}_{\text{Signal inference}} + \underbrace{\frac{p_z \frac{1}{\tau + \tau_s} \left[\tilde{p} - \left(p_0 + p_z \tilde{s}_z \frac{\tau_s}{\tau_s + \tau} + p_\delta \tilde{s}_\delta \frac{\tau_s}{\tau_s + \tau} \right) \right]}{p_z^2 \frac{1}{\tau + \tau_s} + p_\delta^2 \frac{1}{\tau + \tau_s} + p_n^2 \frac{1}{\tau_n}}}_{\text{Price inference}}. \tag{7}$$

Upon receiving a higher $\tilde{s}_z$, a given traditional investor directly infers from her signal that $\tilde{z}$ is higher ("Signal inference" term in (7)). At the same time, for a given price $\tilde{p}$, a higher $\tilde{s}_z$ implies that other investors receive lower signals about $\tilde{z}$ and the information about $\tilde{z}$ contained in the price is worse ("Price inference" term in (7)).

Posterior uncertainty about $\tilde{z}$ for a traditional investor equals to uncertainty about $\tilde{z}$

[11] To lighten the notation, we use the fact that investors within each type have identical preferences and omit investor-specific indices where possible.

after observing a private signal net of a reduction in uncertainty due to learning from the price:

$$\mathbb{V}\left(\tilde{z}|\tilde{s}_z, \tilde{s}_\delta, \tilde{p}\right) = \frac{1}{\tau+\tau_s} - \frac{\left(p_z \frac{1}{\tau+\tau_s}\right)^2}{p_z^2 \frac{1}{\tau+\tau_s} + p_\delta^2 \frac{1}{\tau+\tau_s} + p_n^2 \frac{1}{\tau_n}}. \tag{8}$$

In particular, if the price is strongly associated with the ESG component (p_δ is high) or with the noise traders' demand (p_n is high), then traditional investors cannot learn much about $\tilde{z}$ from the price, and the uncertainty reduction term is small.

Plugging (7) and (8) in the demand function (2), it is easy to derive that trading intensity i_z^t of a traditional investor with respect to $\tilde{s}_z$ is positive and constant, as shown in (5). This is a standard result (Hellwig, 1980).

More interestingly, the traditional investor's trading intensity with respect to $\tilde{s}_\delta$ is negative and depends on the equilibrium price coefficients. Because a traditional investor does not value the ESG payoff component, a better realization of $\tilde{s}_\delta$ does not directly affect the expected stock payoff for such an investor. However, she uses her signal on $\tilde{\delta}$ to infer $\tilde{z}$ from the price. In particular, for a given price, she infers that a higher $\tilde{s}_\delta$ implies worse aggregate information about $\tilde{z}$. Therefore, she reduces her demand in response to a higher $\tilde{s}_\delta$.

The magnitude of i_δ^t is high if traditional investors are able to infer a lot about $\tilde{z}$ from the price based on their $\tilde{\delta}$-signals. This price inference effect is strong if the equilibrium price responds strongly to changes in $\tilde{\delta}$ (that is, ξ_δ is high) and, at the same time, informative about $\tilde{z}$ (ξ_z is high). The price inference effect is captured by the numerator of the expression (6). At the same time, if the price is a noisy signal about $\tilde{z}$, either due to its strong association with $\tilde{\delta}$ or due to noise traders, traditional investors do not trade the stock actively. This reduces the magnitude of i_δ^t. The price noisiness effect is captured by the denominator of (6).

Analogously, the trading intensities of a green investor are

$$i_z^g \equiv \frac{\partial d^g\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{p}\right)}{\partial \tilde{s}_z} = -\frac{\tau_s}{\gamma} \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau+\tau_s}{\tau_n}}, \tag{9}$$

$$i_\delta^g \equiv \frac{\partial d^g\left(\tilde{s}_z, \tilde{s}_\delta, \tilde{p}\right)}{\partial \tilde{s}_\delta} = \frac{\tau_s}{\gamma}. \tag{10}$$

Because traditional and green investors value different fundamentals, they trade in the opposite directions based on the same signals. Both investor types trade with equal and constant intensities on signals about the payoff components they value: $i_z^t = i_\delta^g = \frac{\tau_s}{\gamma}$. At the same time, their trading intensities on signals about the fundamentals they do not value, i_δ^t and i_z^g, depend on the equilibrium price coefficients and, in particular, on the riskiness of the stock payoff. Recall that an investor of type j trades more intensively on signals about the fundamental she does not value when facing smaller residual uncertainty or, equivalently, when the equilibrium price is more informative to her. Defining the price informativeness to a type-j investor as

$$PI_j \equiv \mathbb{V}\left(\beta_z^j \tilde{z} + \beta_\delta^j \tilde{\delta} | \mathcal{F}^i\right)^{-1},$$

it is easy to see that

$$\frac{i_\delta^t}{i_z^g} = \frac{\xi_z^2 + \frac{\tau + \tau_s}{\tau_n}}{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}} = \frac{PI_t}{PI_g} \equiv v, \tag{11}$$

where v is the relative price informativeness. If $v > 1$, the price is more informative to traditional investors, and their trading against their $\tilde{\delta}$-signals is more intense than trading of green investors against their $\tilde{z}$-signals. The opposite is true if $v < 1$.

The trading intensities of traditional and green investors determine information content of the price, that is, the equilibrium price coefficients. The market clearing condition (3) implies

$$\xi_z = \frac{m}{2}\left(i_z^t + i_z^g\right), \tag{12}$$

$$\xi_\delta = \frac{m}{2}\left(i_\delta^t + i_\delta^g\right). \tag{13}$$

Expression (11) and the system (12)-(13) indicate that there exists a feedback loop between the trading intensities i_δ^t, i_z^g and the price coefficients. On the one hand, if traditional investors trade more aggressively against their $\tilde{\delta}$-signals than green investors against their $\tilde{z}$-signals, the price incorporates less information about the ESG component, so that $\xi_\delta < \xi_z$. On the other hand, if the price reflects less ESG information, it is more informative to traditional investors. They face less residual uncertainty about the stock payoff, which justifies why they trade more aggressively than green investors in the first place. An analogous feedback loop exists if green investors dominate the trading.

2.3.2 Equilibrium multiplicity

The feedback loop described above has profound impacts on the equilibrium outcomes. In particular, it might lead to multiple equilibria in the trading stage and, thus, multiple equilibrium pricing functions. Using the expressions for the trading intensities (5)-(6) and (9)-(10), the system of equations (12)-(13) can be rewritten as

$$\xi_z = \frac{\tau_s}{\gamma}\frac{m}{2}\left[1 - \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau+\tau_s}{\tau_n}}\right], \tag{14}$$

$$\xi_\delta = \frac{\tau_s}{\gamma}\frac{m}{2}\left[1 - \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n}}\right]. \tag{15}$$

Expressing ξ_z as a function of ξ_δ from (15) and plugging it in (14), we get the following equation for ξ_δ:

$$\left(\xi_\delta^3 + \frac{\tau+\tau_s}{\tau_n}\xi_\delta - \frac{\tau_s}{\gamma}\frac{m}{2}\frac{\tau+\tau_s}{\tau_n}\right)\left(\xi_\delta^2 - \frac{\tau_s}{\gamma}\frac{m}{2}\xi_\delta + \frac{\tau+\tau_s}{\tau_n}\right) = 0. \tag{16}$$

Due to the symmetry of the system (14)-(15), it is natural to split the analysis in two cases.

Case 1: $\xi_z = \xi_\delta$.

Plugging $\xi_z = \xi_\delta$ in (15), we obtain

$$\xi_\delta^3 + \frac{\tau+\tau_s}{\tau_n}\xi_\delta - \frac{\tau_s}{\gamma}\frac{m}{2}\frac{\tau+\tau_s}{\tau_n} = 0.$$

This is the first term in the left-hand side of (16). Clearly, this equation always has a unique and positive real root. This solution corresponds to a symmetric equilibrium in which traditional and green investors trade equally actively, $i_\delta^t = i_z^g$. This results in the price being equally informative to the two investor groups, $v = 1$.

Case 2: $\xi_z \neq \xi_\delta$.

Recall that the system for ξ_z and ξ_δ (14)-(15) can be simplified to one equation in ξ_δ (16). The first term in the left-hand side of (16) corresponds to Case 1 in which $\xi_z = \xi_\delta$.

Therefore, if $\xi_z \neq \xi_\delta$, it must be that

$$\xi_\delta^2 - \frac{\tau_s}{\gamma}\frac{m}{2}\xi_\delta + \frac{\tau + \tau_s}{\tau_n} = 0.$$

This equation has two real roots if $\tau_n > \tau_n^* \equiv 4\left(\tau + \tau_s\right)\left(\frac{\tau_s}{\gamma}\frac{m}{2}\right)^{-2}$, that is, if the demand from noise traders is not too volatile:[12]

$$\xi_\delta = \frac{1}{2}\left[\frac{\tau_s}{\gamma}\frac{m}{2} \pm \sqrt{\left(\frac{\tau_s}{\gamma}\frac{m}{2}\right)^2 - 4\frac{\tau + \tau_s}{\tau_n}}\right] \quad \text{and} \quad \xi_z = \frac{\tau_s}{\gamma}\frac{m}{2} - \xi_\delta.$$

In the equilibrium with $\xi_\delta > \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2} > \xi_z$, the price is mostly driven by the ESG component and is more informative to green investors, $v < 1$. We refer to this equilibrium as a *G-equilibrium*. The other one is referred to as a *T-equilibrium*: there $\xi_\delta < \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2} < \xi_z$ and the price is more informative to traditional investors, $v > 1$.

The G- and T-equilibria coexist if

$$\tau_n > \tau_n^* \equiv 4\left(\tau + \tau_s\right)\left(\frac{\tau_s}{\gamma}\frac{m}{2}\right)^{-2}.$$

That is, the multiplicity emerges if the exogenous noise is small. More specifically, it emerges if the volatility of noise traders' demand τ_n^{-1} is small, signals are precise relative to priors (high τ_s and low τ), and the mass of informed investors m is large.

If the exogenous noise is small, the feedback loop described in the previous section is more pronounced. In particular, if τ_n is large, the relative price informativeness (11) is very sensitive to the price coefficients. As a result, multiple equilibria marked by different relative price informativeness arise. If, on the contrary, τ_n is small, the price is mostly driven by the noise traders' demand, the relative price informativeness is always close to one, and the feedback loop is weak. In this case, the only possible equilibrium is the one described in Case 1.

To sum up, equilibrium is unique when the exogenous noise is large. Otherwise, there exist three equilibria. In the G- and T-equilibria, trading is dominated by a particular group of investors and the price is more informative to investors of the dominant group. In the third equilibrium, neither of the two groups is dominating, and the price is equally

[12] If $\tau_n = \tau_n^*$, the root in Case 2 is unique and coincides with that in Case 1: $\xi_\delta = \xi_z = \frac{1}{2}\frac{\tau_s}{\gamma}\frac{m}{2}$.

informative to all investors. In what follows, we refer to this equilibrium as an *M-equilibrium*.

2.3.3 Discussion of key model features

The key mechanism behind the feedback loop and equilibria multiplicity is that investors trade in the opposite directions when receiving the same signals. This mechanism requires that the two investor groups have, first, the incentives to trade against each other and, second, the means of doing so.

The incentives arise due to the preference heterogeneity. Because investors value different payoff components, they use the same information differently. By trading against signals about the fundamental they do not value, investors of one group make the price noisier to the other group. Facing riskier stock payoff, investors of the other group choose to trade less actively. The feedback loop between the trading intensities and the price informativeness gives rise to multiple equilibria. In the absence of the preference heterogeneity, all investors trade in the same way, and the price is always equally informative to everyone. In that case, our model reduces to a fairly standard REE setting with a unique equilibrium.[13]

The ability of investors to trade in the opposite directions relies on availability of information about the payoff components that they value and not. In the context of responsible investing, traditional investors might put less value to firms' ESG performances but still receive related information from news articles or comprehensive disclosure statements. Receiving such information makes it possible for traditional investors to trade against green investors.[14] If investors receive information only about the component they value, the feedback loop disappears. In Appendix E, we show that in the setting with heterogeneous preferences, multiple equilibria emerge unless investors receive information *only* about the payoff components they value (as in, for example, Vives, 2014; Rahi and Zigrand, 2018).

[13] Goldstein and Yang (2015) study a model in which investors with homogeneous preferences trade the stock whose payoff is affected by two components. They show equilibrium uniqueness under assumption that there are two investor groups and investors within each group are informed about one fundamental. Appendix F verifies this result under a more general information structure.

[14] Importantly, investors value information about payoff components that they do not directly care about because it helps them to interpret the price. It would be interesting to formally study the information acquisition problem of heterogeneous investors, although we believe that analytic tractability of such a model is substantially limited.

3 The main model

3.1 Setup

This section presents our main model which extends the simplified version of the previous section along two dimensions. All proofs and derivations are in the Appendix.

First, we relax the equal-mass assumption such that the green investor share is $\alpha \in (0,1)$. The market is thus populated with a mass $m_t = (1-\alpha)m$ of traditional investors and a mass $m_g = \alpha m$ of green investors.

Second, we allow for partially aligned preferences of traditional and green investors. In particular, traditional investors still only value the financial payoff component, i.e. $\beta_z^t = 1, \beta_\delta^t = 0$. However, green investors might value both the financial and ESG components. The stock payoff to them is $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$, where $\beta_z \geq 0$ and $\beta_\delta > 0$ are utility weights. Therefore, β_δ captures the degree of preference heterogeneity across traditional and green investors. We normalize $\beta_z^2 + \beta_\delta^2 = 1$, so that the ex-ante variances of the stock payoff is the same for traditional and green investors. Otherwise, a change in the investor composition α would reshape the overall risk attitude and thus affect the asset price in a mechanical way.

In our main model, the financial and ESG payoff components are uncorrelated, while they can be correlated negatively or positively in reality. On the one hand, implementing socially responsible policies can be monetarily costly. On the other hand, a firm using a more responsible way to produce can attract a higher demand from socially concerned customers, thus enhancing its financial performance. We do not take a stand on the sign of the correlation in this paper. Instead, we explore an extension with correlated payoff components in Appendix D and show that incorporating correlation between the two payoff components is equivalent to considering the model with partially aligned preferences.[15] In particular, the model in which the two payoff components have a correlation of ρ and green investors have preference weights (β_z, β_δ) is equivalent to the model in which the payoff components are uncorrelated but green investors' preference weights on the "purely" financial and ESG components are $(\beta_z + \rho\beta_\delta, \beta_\delta\sqrt{1-\rho^2})$. Therefore, the payoff components being more/less correlated in the extension is equivalent to the

[15]More specifically, Appendix D considers the case $\mathrm{Corr}(\tilde{z}, \tilde{\delta}) = \mathrm{Corr}(\tilde{s}_z^i, \tilde{s}_\delta^i) = \rho$. That is, the two payoff components and investors' signals about them share the same correlation coefficient. A richer correlation structure substantially complicates analytical characterization.

preferences of traditional and green investors being more/less aligned in the main model. Our results on the role of the preference heterogeneity presented in the following sections can be also interpreted as the results about the role of the correlation between the payoff components.

3.2 Equilibrium characterization

The analyses and intuitions of the simplified model can be extended to the main model. From a green investor's perspective, the stock payoff is $\tilde{y} = \beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. Correspondingly, $\tilde{x} = \beta_\delta \tilde{z} - \beta_z \tilde{\delta}$ is orthogonal to $\tilde{y}$ and thus represents the payoff component that green investors do not value. In this setting, differential usage of information by the two investor groups becomes less stark. Specifically, a green investor receiving a better signal about the financial component still infers a worse realization of the ESG component from the price. However, as long as she directly values firm's financial performance, i.e. $\beta_z > 0$, she has a weaker incentive to trade against high $\tilde{z}$-signals. As a result, her trading intensity on her financial signal becomes

$$i_z^g = \frac{\tau_s}{\gamma} \frac{\beta_z \left(\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n}\right) - \xi_z \xi_\delta \beta_\delta}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \frac{\tau+\tau_s}{\tau_n}}. \tag{17}$$

Similarly, a green investor has a weaker incentive to increase her demand for the stock following a better ESG signal,

$$i_\delta^g = \frac{\tau_s}{\gamma} \frac{\beta_\delta \left(\xi_z^2 + \frac{\tau+\tau_s}{\tau_n}\right) - \xi_z \xi_\delta \beta_z}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \frac{\tau+\tau_s}{\tau_n}}. \tag{18}$$

Although the preferences of traditional and green investors are partially aligned, the feedback loop described in Section 2 still arises as long as their preferences are not entirely homogeneous. In particular, the relative price informativeness (11) becomes

$$v \equiv \frac{PI_t}{PI_g} = \frac{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2 + \frac{\tau+\tau_s}{\tau_n}}. \tag{19}$$

If traditional investors dominate the trading, the price is mostly aligned with their preferences and less noisy to them, i.e. $\xi_\delta < \xi_x \equiv \xi_z \beta_\delta - \xi_\delta \beta_z$, where ξ_x is the price coefficient associated with payoff component $\tilde{x}$ that green investors do not value. Then green investors do not trade the stock actively, as can be seen from expression for trading intensities (17)-(18). The opposite is true if green investors dominate the trading.

In the main model, analytic characterization becomes substantially more cumbersome than in the simplified model, so we delegate derivations and proofs to Appendix A. In particular, we show there that the equilibrium characterization boils down to a fixed point problem for ξ_δ,

$$\xi_\delta = J\left(\xi_\delta\right). \tag{20}$$

It can be simplified to the following quintic equation in ξ_δ:

$$\xi_\delta^5 - \frac{\tau_s}{\gamma}\alpha m\beta_\delta\xi_\delta^4 + 2\frac{\tau+\tau_s}{\tau_n}\xi_\delta^3 - 2\frac{\tau_s}{\gamma}\alpha m\beta_\delta\frac{\tau+\tau_s}{\tau_n}\xi_\delta^2 + \left[\left(\frac{\tau+\tau_s}{\tau_n}\right)^2 + \left(\frac{\tau_s}{\gamma}\left(1-\alpha\right)m\beta_\delta\right)^2\frac{\tau+\tau_s}{\tau_n}\right]\xi_\delta - \frac{\tau_s}{\gamma}\alpha m\beta_\delta\left(\frac{\tau+\tau_s}{\tau_n}\right)^2 = 0. \tag{21}$$

Note that (21) simplifies to (16) if $\alpha = \frac{1}{2}$ and $\beta_\delta = 1$.

Proposition 1. *There exists a multiplicity threshold $\tau_n^*\left(\alpha, \beta_\delta\right) > 0$ such that*[16]

(i) if $\tau_n \in (0, \tau_n^*)$, there is a unique equilibrium;

(ii) if $\tau_n = \tau_n^*$, there are two equilibria if $\alpha \neq \frac{1}{2}$ and one equilibrium if $\alpha = \frac{1}{2}$;

(iii) if $\tau_n > \tau_n^*$, there are three equilibria.

In any equilibrium, $p_0 < 0$, $p_z > 0$, $p_\delta > 0$ and $p_n > 0$.

Proposition 1 confirms that, same as in the simplified model, multiple equilibria can arise when the exogenous noise is sufficiently small, i.e. $\tau_n > \tau_n^*$. As illustrated earlier, in that case, the price informativeness and trading intensities are sensitive to the equilibrium price coefficients, which strengthens the feedback loop. When τ_n is small, the feedback loop is weak, resulting in a unique equilibrium.

Proposition 2. *The multiplicity threshold $\tau_n^*\left(\alpha, \beta_\delta\right)$ behaves such that (i) $\frac{d\tau_n^*(\alpha,\beta_\delta)}{d\beta_\delta} < 0$; (ii) $\frac{d\tau_n^*(\alpha,\beta_\delta)}{d\alpha} \lesseqgtr 0$ if $\alpha \lesseqgtr \frac{1}{2}$.*

[16] We write the threshold τ_n^* as a function of α and β_δ. In general, τ_n^* depends on other model parameters. However, we do not mention them explicitly because we do not focus on them in our analyses.

Proposition 2 characterizes how the multiplicity threshold τ_n^* varies with the degree of preference heterogeneity β_δ and the green investor share α. The equilibrium multiplicity is more likely to arise when the preference heterogeneity is large in the entire investor base, that is, when the ESG utility weight of green investors β_δ is large and the masses of the two groups are similar (α is close to $\frac{1}{2}$). If the investor base consists mainly of investors of one type, or if traditional and green investors' preferences are closely aligned, the aggregate preference heterogeneity is small. For example, if there are only a few green investors ($\alpha \to 0$), or green investors mostly value the financial payoff ($\beta_\delta \to 0$), the investor base is nearly homogeneous, and the model reduces to a standard REE model with a unique pricing function.

When multiple equilibria are possible, they can be ranked by the relative price informativeness. Formally, price informativeness to traditional and green investors are

$$PI_t \equiv \mathbb{V}\left(\tilde{z}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2+\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}}, \tag{22}$$

$$PI_g \equiv \mathbb{V}\left(\beta_z\tilde{z}+\beta_\delta\tilde{\delta}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2+\xi_\delta^2+\frac{\tau+\tau_s}{\tau_n}}{\left(\xi_\delta\beta_z-\xi_z\beta_\delta\right)^2+\frac{\tau+\tau_s}{\tau_n}}, \tag{23}$$

and so the relative price informativeness $v = \frac{PI_t}{PI_g}$ is given by (19). Using the same terminology as in the simplified model, if there are three equilibria, we call the one with the smallest v the G-equilibrium, the one with the largest v the T-equilibrium, and the one with a medium v the M-equilibrium.[17] Formally, we have

Proposition 3. *When there are three equilibria, they can be ranked according to the relative price informativeness to traditional investors v. In the T-equilibrium $v^T > 1$; in the G-equilibrium $v^G < 1$; in the M-equilibrium $v^M \in \left(v^G, v^T\right)$.*

Possibility of equilibrium multiplicity naturally raises the question about equilibrium selection. A common selection approach suggests that stable equilibria are more likely to be played. Recall that from (20) ξ_δ is a fixed point of $J(\xi_\delta)$. We call an equilibrium stable if the dynamics around the equilibrium ξ_δ^* are locally stable, i.e. $\frac{\partial[J(\xi_\delta)-\xi_\delta]}{\partial\xi_\delta}\Big|_{\xi_\delta=\xi_\delta^*} < 0$.[18]

[17] By Proposition 1, two equilibria exist when $\tau_n = \tau_n^*$ and $\alpha \neq \frac{1}{2}$. In what follows, we do not analyze this knife-edge case to save space. Results are available upon request.

[18] Under this criterion, a fixed point of the nonlinear differential equation $\frac{d\xi_{\delta,t}}{dt} = J\left(\xi_{\delta,t}\right) - \xi_{\delta,t}$ is locally stable. A formal evaluation of stability requires a dynamic extension of our model, which is beyond the scope of this paper. However, this criterion is similar to the one derived in the literature introducing recursive-least-squares learning in settings a la Grossman and Stiglitz (1980) (Bray, 1982; Marcet and Sargent, 1989; Heinemann, 2009).

Under this criterion, if the system is pushed to an off-equilibrium point $\xi_\delta^* + \epsilon$, it tends to move back to the equilibrium point ξ_δ^* if $|\epsilon|$ is sufficiently small.

Proposition 4. *If equilibrium is unique, it is stable. If there are three equilibria, the T- and G-equilibria are stable and the M-equilibrium is unstable.*

Proposition 4 suggests that investors are unlikely to coordinate on the M-equilibrium when the G- and T-equilibria exist. The M-equilibrium also has counter-intuitive properties. For example, in the M-equilibrium, when the mass of one investor group increases, the price becomes *less* informative to investors of this group (this is formally established in Proposition 5 below). In other words, investors should coordinate to trade *less* actively when there are more investors with the same preferences. In what follows, we characterize all equilibria but put less focus on the M-equilibrium when the multiplicity is possible.

4 Growth of green investors

In this section, we examine impacts of the recent trend of growing investors' awareness about firms' ESG performances. Using the model of Section 3, we characterize how the price informativeness and the firm's cost of capital respond to an increase in the green investor share α in Sections 4.1 and 4.2. We then discuss empirical implications of our results in Section 4.3. Proofs and derivations for this section are in Appendix B.

4.1 Price informativeness

Proposition 5 characterizes how absolute and relative price informativeness, PI_t, PI_g, and v, change with the green investor share α.

Proposition 5. *If $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there is a unique equilibrium in which $\frac{dPI_t}{d\alpha} < 0$, $\frac{dPI_g}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$. If $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there exists $\underline{\alpha} \in \left(0, \frac{1}{2}\right)$ and $\bar{\alpha} = 1 - \underline{\alpha}$ such that*[19]

(i) if $\alpha < \underline{\alpha}$, there is a unique T-equilibrium in which $v^T > 1$;

(ii) if $\alpha > \bar{\alpha}$, there is a unique G-equilibrium in which $v^G < 1$;

[19] Although there is a unique equilibrium if $\alpha < \underline{\alpha}$ and $\alpha > \bar{\alpha}$, we refer to it as either a T- or G-equilibrium, respectively, because the equilibrium outcomes, such as the price coefficients and price informativeness, are continuous at $\alpha = \underline{\alpha}$ and $\alpha = \bar{\alpha}$ as shown in panel (B) of Figure 1.

(iii) if $\alpha \in (\underline{\alpha}, \bar{\alpha})$, there are three equilibria and $v^T > v^M > v^G$.

Moreover, in the T- and G-equilibria, $\frac{dPI_t}{d\alpha} < 0$, $\frac{dPI_g}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$; in the M-equilibrium, $\frac{dPI_t}{d\alpha} > 0$, $\frac{dPI_g}{d\alpha} < 0$, and $\frac{dv}{d\alpha} > 0$.

Suppose first that the exogenous noise is large, i.e. $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$. By Proposition 2, the multiple equilibria region is largest when the investor base consists of equal masses of green and traditional investors, $\alpha = \frac{1}{2}$. Therefore, if equilibrium is unique for $\alpha = \frac{1}{2}$, it is unique for all $\alpha \in (0, 1)$.

As α increases, the equilibrium price coefficients change such that the price becomes more informative to green investors and less informative to traditional investors. First, for given individual trading intensities, a larger α means that the price becomes more aligned with the preferences of green investors and thus more informative to them because they are responsible for a larger share of trades in the market. Second, individual trading intensities adjust. Green investors, facing a lower residual risk, trade more actively, whereas traditional investors reduce their trading activity. Panel (A) in Figure 1 illustrates how the relative price informativeness varies with the green investor share.

If the exogenous noise is small, i.e. $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, equilibrium multiplicity is possible when masses of traditional and green investors are similar, that is, α is close to $\frac{1}{2}$. Start from an economy with few green investors ($\alpha < \underline{\alpha}$). Here, traditional investors significantly outweigh green investors. There exists a unique T-equilibrium in which the price is informative mostly about the financial component, resulting in $v > 1$. As α increases and crosses $\underline{\alpha}$, the feedback look becomes sufficiently strong to support the G-equilibrium in which the price is more informative to green investors, $v < 1$. Interestingly, the G-equilibrium is sustainable even if green investors constitute a minority in the investor base, i.e. $\alpha < \frac{1}{2}$. Eventually, when the share of green investors becomes sufficiently large, $\alpha > \bar{\alpha}$, there exists a unique G-equilibrium.

Panel (B) in Figure 1 shows relative price informativeness v in this case. Similar to the case of large exogenous noise, as α increases, the price becomes more informative to green investors and less informative to traditional investors in the stable T- and G-equilibria. Different from the case of large exogenous noise, however, there can be discontinuous jumps in the price informativeness due to switches across equilibria.

(A) Unique equilibrium, $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$ (B) Multiplicity is possible, $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$

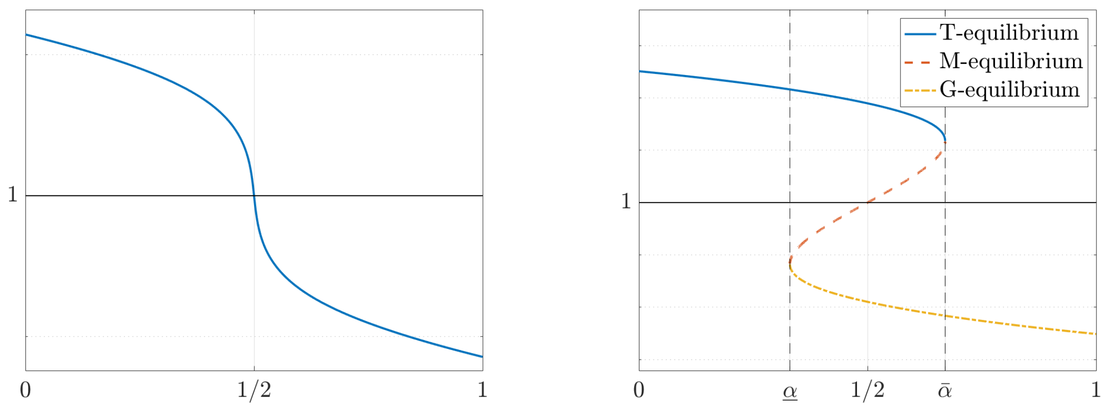

Figure 1: Relative price informativeness to traditional investors v as a function of the green investor share α. Y-axes are in the log scale.

4.2 Cost of capital

In our model, the financial return on the risky asset is $\tilde{z} - \tilde{p}$. Therefore, the expected financial return is

$$\mathbb{E}(\tilde{z} - \tilde{p}) = -p_0 = \frac{\gamma}{m_t PI_t + m_g PI_g}. \tag{24}$$

In what follows, we refer to $\mathbb{E}(\tilde{z} - \tilde{p})$ as the firm's cost of capital and denote it by CoC. CoC, therefore, is the expected financial return which captures the firm's cost of capital from the perspective of the manager who only values firm's financial performance. Such a metric is also commonly considered as a cost of capital measure in the literature because it captures the difference between expected cash flows and market value (e.g. Pastor, Stambaugh, and Taylor, 2021a).[20] $\mathbb{E}(\tilde{z} - \tilde{p})$ is also closely related to the earnings-to-price ratio that is frequently analyzed in empirical research and by practitioners.

As is standard in the REE settings (e.g. Easley and O'Hara, 2004), the cost of capital defined by (24) reflects the compensation required by risk-averse investors for their investment risks. In our environment, it is determined by the weighted average of price informativeness to traditional and green investors. Proposition 6 characterizes how CoC changes with the share of green investors α.

[20]In our setting, the expected financial return does not necessarily capture the expected return for all firm investors because green investors also value firm's ESG performance. However, if $\tilde{z}$ and $\tilde{\delta}$ have zero means, the expected return for green investors is the same as (24): $\mathbb{E}(\beta_z \tilde{z} + \beta_\delta \tilde{\delta} - \tilde{p}) = -p_0$. At the end of this section, we discuss the cost of capital measure if $\tilde{z}$ and $\tilde{\delta}$ have non-zero means.

Proposition 6. *If $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, there is a unique equilibrium in which $\frac{dCoC}{d\alpha} \gtreqless 0$ if $\alpha \lesseqgtr \frac{1}{2}$. If $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, in the T-equilibrium, $\frac{dCoC}{d\alpha} > 0$; in the G-equilibrium, $\frac{dCoC}{d\alpha} < 0$; in the M-equilibrium, $\frac{dCoC}{d\alpha} \gtreqless 0$ if $\alpha \lesseqgtr \frac{1}{2}$.*

Consider first the case of large exogenous noise, i.e. $\tau_n \leq \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$, such that there always exist a unique equilibrium. This case is illustrated by panel (A) of Figure 2. Suppose that $\alpha < \frac{1}{2}$, that is, the mass of traditional investors is larger than the mass of green investors. A marginal effect of α on CoC can be decomposed in two components,

$$\frac{dCoC}{d\alpha} = -\frac{\gamma}{\left((1-\alpha)PI_t + \alpha PI_g\right)^2} \frac{1}{m} \left(\underbrace{PI_g - PI_t}_{\text{Direct effect}} + \underbrace{(1-\alpha)\frac{dPI_t}{d\alpha} + \alpha \frac{dPI_g}{d\alpha}}_{\text{Indirect effect}} \right),$$

The direct effect reflects the change in the cost of capital due to to the change in the investor base composition holding price informativeness PI_t and PI_g fixed. If $\alpha < \frac{1}{2}$, $PI_g < PI_t$ by Proposition 5, that is, green investors face higher residual risk when investing in the stock. As a result, the direct effect drives the cost of capital up.

The indirect effect captures the change in the cost of capital due to adjustments in the equilibrium price coefficients and, hence, price informativeness. By Proposition 5, the price informativeness to traditional and green investors move in the opposite directions as the composition of investor base changes: $\frac{dPI_t}{d\alpha} < 0$ and $\frac{dPI_g}{d\alpha} > 0$. Nevertheless, the indirect effect also pushes the cost of capital up if $\alpha < \frac{1}{2}$. The key force behind this result is as follows. As the share of green investors α grows, the price becomes more associated with the ESG component, i.e. ξ_δ goes up. However, an increase in ξ_δ also allows traditional investors to use their ESG signals more actively to trade against green investors along the $\tilde{\delta}$-dimension. Trades by traditional investors thus prevent ξ_δ and PI_g from sharp increases. This effect is particularly strong if the magnitude of traditional investors' trading intensity is high, namely, if traditional investors face low investment risk ($\alpha < \frac{1}{2}$ and $PI_t > PI_g$).

In sum, when the investor base consists mostly of traditional investors, an increase in the green investor share leads to an increase in the overall information risk and, therefore, in the cost of capital. In contrast, when the majority of investors have green preferences ($\alpha > \frac{1}{2}$), the signs of both direct and indirect effects flip, and the cost of capital declines in α. The cost of capital reaches its maximum when the masses of the two groups are equal, that is, when investor heterogeneity is high and trades by green and traditional investors introduce substantial amounts of noise to each other.

Suppose now that the exogenous noise is small, i.e. $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$. Then multiple equilibria are possible. The comparative statics of CoC with respect to α for this case is shown in Panel (B) of Figure 2. In the T-equilibrium, traditional investors dominate the trading and $PI_t > PI_g$. Similar to the unique equilibrium case, an increase in α leads to a larger CoC through both direct and indirect channels. The opposite is true in the G-equilibrium in which the stock is primarily traded by green investors.

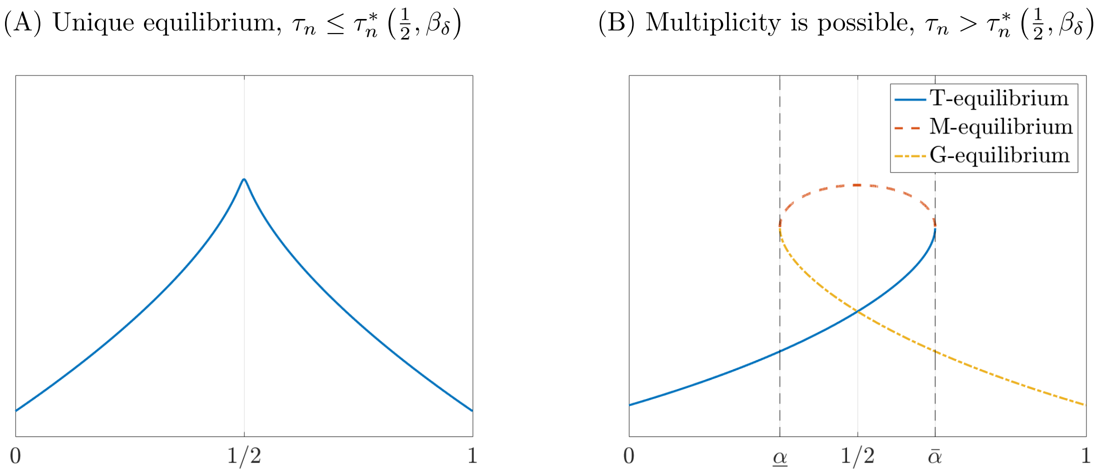

Figure 2: Cost of capital CoC as a function of the green investor share α. Y-axes are in the log scale.

So far we have analyzed the cost of capital for a firm with zero average financial and ESG payoffs, that is, when both $\tilde{z}$ and $\tilde{\delta}$ have zero means. We now characterize how the cost of capital changes with the green investor share for a firm with non-zero expected payoffs.

Corollary 1. *Suppose that* $\tilde{z} \sim N\left(\mu_z, \tau^{-1}\right)$ *and* $\tilde{\delta} \sim N\left(\mu_\delta, \tau^{-1}\right)$*. Then*

$$CoC = \mathbb{E}(\tilde{z} - \tilde{p}) = \frac{\gamma}{m_t PI_t + m_g PI_g} + c_z \mu_z + c_\delta \mu_\delta,$$

where $c_z = \frac{(1-\beta_z)\xi_\delta}{\beta_\delta \xi_z + (1-\beta_z)\xi_\delta} > 0$ *and* $c_\delta = -\frac{\beta_\delta \xi_\delta}{\beta_\delta \xi_z + (1-\beta_z)\xi_\delta} < 0$*. Moreover,* $\frac{dc_z}{d\alpha} > 0$ *and* $\frac{dc_\delta}{d\alpha} < 0$ *except for the M-equilibrium. In the M-equilibrium,* $\frac{dc_z}{d\alpha} < 0$ *and* $\frac{dc_\delta}{d\alpha} > 0$*.*

Corollary 1 delivers two main results. First, the firm's cost of capital increases in its expected financial output μ_z and decreases in its expected ESG output μ_δ. Recall that we define CoC as the expected financial return, that is, as the cost of capital from the perspective of the firm's manager who values only the financial output $\tilde{z}$. From the manager's perspective, an increase in μ_z is not fully reflected in the stock price in the presence of green investors because they do not value financial performance as much as the manager. In contrast, a higher μ_δ implies a higher demand from green investors, which drives the stock price up. However, the expected financial output is unchanged.

Second, as the green investor share increases, the cost of capital becomes more sensitive to both μ_z and μ_δ, that is, the absolute values of c_z and c_δ increase in α. With more investors valuing ESG performance, the average preference of investors deviates more from that of the firm manager. As a result, from the manager's perspective, the firm is more under-compensated for an increase in μ_z and more over-compensated for an increase in μ_δ.

Finally, it is worth commenting on a proper measure of the cost of capital in our model. The expected financial return (24) measures the firm's cost of capital from the perspective of the firm manager who only cares about firm's financial performance. In reality, firm managers are likely to have such preferences because their compensations are usually tied to stock prices and firms' earnings rather than ESG metrics such as carbon emissions. Within our framework, one may also consider a manager who values both payoff components such that the cost of capital is $\mathbb{E}\left(\beta_z^F \tilde{z} + \beta_\delta^F \tilde{\delta} - \tilde{p}\right)$. If $\tilde{z}$ and $\tilde{\delta}$ have zero means, the comparative statics of CoC described by Proposition 6 remain the same. If the means are nonzero, the results of Corollary 1 are preserved if the preferences of the manager and traditional investors are sufficiently close.[21]

4.3 Empirical implications

In this section, we discuss some empirical implications of our theoretical results. One can test directly the mechanism of our theory using observed trading behavior of investors or indirectly the empirical implications of our theory using asset prices.

Investor trading behavior One general implication of our model is that traditional and ESG investors react to financial and ESG information differently. With the access to data on trading accounts, one can test if ESG investors, such as ESG mutual funds, respond to news of different types, i.e. ESG versus earnings news, in a different way

[21] An interesting question in this respect is how the manager's preferences are related to those of heterogeneous investors, some of whom have non-pecuniary considerations (Hart and Zingales, 2017). We leave this for future exploration.

compared to traditional investors. When doing so, one should take into account that ESG news may also be informative about a firm's cash flows. As we have highlighted in the beginning of Section 3 and shown in Appendix D, if this is the case, trading behaviors of traditional and green investors may be more or less diverse depending on the correlation between the ESG and financial performance components. Another prediction of the model is that assets that are actively traded by ESG investors are risky to traditional investors. One can investigate how an inflow of ESG investors in certain assets affects behavior of traditional investors.

Price informativeness Proposition 5 states that as the green investor share increases, asset price becomes more informative about the ESG payoff and less informative about the financial payoff. To test this result, the first step is to estimate price informativeness about financial and ESG fundamentals. The approach developed by Davila and Parlatore (2018) can be useful here. Using their methodology, one can recover price informativeness about various payoff components via simple regressions of (changes in) individual stock prices on (changes in) earnings and some measures of ESG output (e.g., ESG ratings or firm carbon emissions). Another useful approach is the one by Bai, Philippon, and Savov (2016), which in our setting would imply estimating the predictive power of asset prices for future variation in earnings and ESG output. The second step is to test the relationship between these price informativeness measures and the investor composition (e.g., the ratio of asset holdings by ESG funds to that by non-ESG funds). Since the investor composition of a firm is likely to be correlated with the characteristics of its production technology (i.e., green investors hold on average greener stocks), one can in addition control for firm emissions or ESG ratings.

Cost of capital There is ample empirical evidence suggesting that investors care about ESG aspects of firms' operations and are willing to sacrifice financial returns (Martin and Moser, 2016; Riedl and Smeets, 2017; Barber et al., 2021; Li et al., 2022). However, when comparing the asset returns of green and traditional firms, existing empirical literature documents mixed results. For stocks, some papers find a negative relationship between ESG performance and returns (e.g. Hong and Kacperczyk, 2009; El Ghoul, Guedhami, Kwok, and Mishra, 2011; Chava, 2014; Bolton and Kacperczyk, 2021), whereas others find a positive relationship (e.g. Derwall, Guenster, Bauer, and Koedijk, 2005 and Pastor et al., 2021a). For bonds, some papers find lower yields for green bonds (e.g. Baker et al., 2018; Zerbib, 2019), while some others find no significant difference (e.g. Larcker and Watts, 2020; Tang and Zhang, 2020; Flammer, 2021).

Our results can be useful to interpret the aforementioned mixed empirical evidence.[22] In particular, Proposition 6 and Corollary 1 describe the impacts of average ESG performance and exposure to ESG risk on firm's cost of capital. On the one hand, Corollary 1 implies that firms with a high expected ESG output (e.g. firms operating in inherently greener industries) enjoy a lower cost of capital because green investors are willing to pay a premium for their greenness. On the other hand, green investors tend to divest traditional firms and invest in green firms. As a result, green firms are likely to have a more diverse investor base than traditional firms. According to Proposition 6, this implies a higher cost of capital for green firms due to a higher aggregate information risk. Note that the two channels can be tested separately. For example, to tease out the effect of the investor base diversity, one can compare costs of capital of firms with similar ESG ratings but different investor bases.

It should be mentioned that some papers (Ardia, Bluteau, Boudt, and Inghelbrecht, 2020; Pastor et al., 2021a) attribute high returns of green stocks in recent years to an unexpected increase in environmental concerns among investors rather than to risk premium. We do not take a stand on the interpretation of recent high returns of green stocks. Rather, we view the information channel as a novel force that can be complementary to the role of unexpected changes in the investor base composition in shaping financial returns. In fact, an unexpected inflow of green investors in our model also increases the realized price of a green firm with a high expected ESG output μ_δ.

Price volatility and trading volume Another theoretical prediction of our model is that there might be multiple equilibria in the financial markets. When multiplicity is possible, asset price might experience large fluctuations due to equilibrium switches. Such switches are also likely to be associated with large trading volumes because different equilibria are marked by different trading intensities by traditional and green investors. Our results suggest that multiple equilibria are more likely to arise when the masses of green and traditional investors are similar and exogenous noise is small. One can test if stocks with these properties indeed are more likely to experience price jumps and large flows across traditional and green investors.

[22] Our model features one risky asset, so our results cannot be directly used to make predictions about cross-sectional returns. A formal analysis of a multi-asset economy is beyond the scope of this paper.

5 Improvements in ESG information

Despite the growing interest toward ESG investing, there is a lack of clarity and consistency in the definition and measurement of firms' ESG performances. For example, the average correlation of ESG ratings provided by six large raters is only 0.54 (Berg et al., 2020). To address this problem, policy makers around the world have made a series of efforts to improve the quality of information about firms' ESG performances available to investors. For instance, in May of 2020 the SEC Investor Advisory Committee recommended updating public company reporting requirements to include ESG factors (SEC, 2020), while the EU regulator has already put in place a disclosure regulation that requires market participants and financial advisers to provide ESG-related information about certain financial products (Regulation EU 2019/2088). According to Carrots & Sticks, there are more than 600 ESG reporting requirements across over 80 countries, including the world's 60 largest economies (van der Lugt et al., 2020). In addition, firms also increasingly disclose ESG-related information voluntarily. Governance & Accountability Institute finds that in 2019 90% of companies included in S&P500 published ESG reports, a marked increase from 20% in 2011 (GAI, 2020).

In this section, we consider an improvement in the precision of ESG information. Sections 5.1-5.3 lay out the model and describe our results. Section 5.4 discusses empirical implications. Proofs and derivations for this section are in Appendix C.

5.1 Extended setup

To study how improvements in ESG information affect the outcomes, we generalize the information structure of our main model. First, we assume that the prior precisions of the two fundamentals are no longer identical, $\tilde{z} \sim N\left(0, \tau^{-1}\right)$ and $\tilde{\delta} \sim N\left(0, (\lambda\tau)^{-1}\right)$, where $\lambda > 0$. Second, the precisions of private signals that investors receive also differ by a factor of λ, i.e. $\tilde{s}_z^i \sim N\left(\tilde{z}, \tau_s^{-1}\right)$ and $\tilde{s}_\delta^i \sim N\left(\tilde{\delta}, (\lambda\tau_s)^{-1}\right)$ for any investor i. The extended setup reduces to our main model when $\lambda = 1$.[23] Comparative statics with respect to λ reveal the impacts of changes in the quality of ESG information.

Equilibrium characterization of the extended setup is similar to that of the main model.

[23] The model becomes much less tractable if the factor λ for priors is different from that for signals. We have verified the generality of the main results of Section 5.2 with numerical examples in this more general case (not reported to save space).

In particular, we show that the system of equilibrium conditions takes the same form as that in the main model after a proper change of variables. As a result, main results of Section 3 hold in this case. Specifically, there are up to three equilibria with two of them being stable that differ in their relative price informativeness. To save space, we delegate these analyses to Appendix C and below analyze comparative statics of interest.

Analytically characterizing how the key equilibrium outcomes change with respect to λ for all possible values of this parameter is challenging in this quite general setup. In Section 5.2, we focus on the case where λ is small, that is, ESG information is noisy in comparison with financial information. This assumption makes analytical characterization feasible. At the same time, we believe that it also reflects the current state of things for many companies.

5.2 Price informativeness and cost of capital

Price informativeness Price informativeness to traditional and green investors in the extended setup are given by

$$PI_t \equiv \mathbb{V}\left(\tilde{z}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2\lambda+\xi_\delta^2+\lambda\frac{\tau+\tau_s}{\tau_n}}{\xi_\delta^2+\lambda\frac{\tau+\tau_s}{\tau_n}}, \tag{25}$$

$$PI_g \equiv \mathbb{V}\left(\beta_z\tilde{z}+\beta_\delta\tilde{\delta}|\mathcal{F}^{ij}\right)^{-1} = (\tau+\tau_s)\frac{\xi_z^2\lambda+\xi_\delta^2+\lambda\frac{\tau+\tau_s}{\tau_n}}{\left(\xi_\delta\beta_z-\xi_z\beta_\delta\right)^2+\left(\beta_z^2\lambda+\beta_\delta^2\right)\frac{\tau+\tau_s}{\tau_n}}. \tag{26}$$

An improvement in the quality of ESG information affects price informativeness through two channels. First, a higher λ directly helps investors make better inferences, resulting in higher in PI_t and PI_g. Specifically, holding price coefficients ξ_z and ξ_δ fixed, it is easy to verify that $\frac{\partial PI_t}{\partial\lambda}>0$ and $\frac{\partial PI_g}{\partial\lambda}>0$. Notably, although traditional investors do not value the ESG payoff, more precise information about it allows them to make a better inference about the financial payoff from the price. Second, there is an indirect effect of an increase in λ. Specifically, an increase in λ changes investors' trading behaviors and thus the equilibrium price coefficients. Proposition 7 describes the comparative statics results of the price coefficients and the price informativeness with respect to λ.

Proposition 7. *There exists a $\bar{\lambda}>0$ such that if $\lambda\in(0,\bar{\lambda})$, equilibrium is unique, and*

(i) $\frac{d\xi_\delta}{d\lambda}>0$; $\frac{d\xi_z}{d\lambda}\lesseqgtr 0$ *if* $\beta_z\lesseqgtr\frac{\left(\frac{\tau_s}{\gamma}m_t\right)\left(\frac{\tau_s}{\gamma}m_g\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}}$;

(ii) $\frac{dPI_g}{d\lambda} > 0$; $\frac{dPI_t}{d\lambda} \lessgtr 0$ *if* $\beta_z \lessgtr \frac{3}{2}\frac{\left(\frac{\tau_s}{\gamma}m_t\right)\left(\frac{\tau_s}{\gamma}m_g\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}}$.

Proposition 7 shows that better ESG information always leads to an increase in the price coefficient ξ_δ and makes the price more informative to green investors. Since green investors value the ESG payoff, they increase their trading intensity on their $\tilde{\delta}$-signals in response to an increase in λ. As a result, more $\tilde{\delta}$-information gets incorporated in the price.

At the same time, the impacts of better ESG information on ξ_z and the price informativeness to traditional investors are more convoluted. Specifically, if the preference heterogeneity across traditional and green investors is large, green investors not only increase their trading intensity along the $\tilde{\delta}$-dimension but also trade substantially more aggressively against their $\tilde{z}$-signals in response to an increase in λ. As a result, less financial information gets incorporated in the price: ξ_z decreases. Furthermore, if the preference heterogeneity is sufficiently large, the indirect channel dominates the direct channel, and the price informativeness to traditional investors declines.

As shown by the cutoffs in Proposition 7, the responses of ξ_z and PI_t to changes in λ depend on other model parameters, in particular, on the mass of green investors m_g. If m_g is high, green investors' aggregate trading against their $\tilde{z}$-information is strong, strengthening the negative indirect channel. Hence, the price informativeness to traditional investors declines even if the preference heterogeneity is not that large.

Cost of capital That price informativeness PI_t and PI_g can respond to changes in λ in opposite directions suggests that the impact of better ESG information on the cost of capital may be positive or negative. The expression for the cost of capital in (24) preserves in this extended setup. Differentiating it with respect to λ, we get

$$\frac{dCoC}{d\lambda} = -\gamma\frac{m_t\frac{dPI_t}{d\lambda} + m_g\frac{dPI_g}{d\lambda}}{(m_tPI_t + m_gPI_g)^2}.$$

The sign of this derivative depends on the weighted average of the changes in the price informativeness across the two investor groups.

Proposition 8. *There exists a* $\bar{\lambda} > 0$ *such that if* $\lambda \in (0, \bar{\lambda})$, $\frac{dCoC}{d\lambda} \gtrless 0$ *if* $\beta_z \lessgtr \frac{3}{2}\frac{\left(\frac{\tau_s}{\gamma}m_t\right)\left(\frac{\tau_s}{\gamma}m_g\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}} - \frac{1}{2}\frac{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\frac{\tau+\tau_s}{\tau_n}}{\left(\frac{\tau_s}{\gamma}m_t\right)^2}$.

Proposition 7 shows that, if the preference heterogeneity is large, price informativeness PI_g and PI_t move in the opposite directions in response to an increase in λ. Proposition 8 establishes a related result for the cost of capital: For a sufficiently large preference heterogeneity, the reduction in PI_t dominates the improvement in PI_g, and the cost of capital increases in λ. Note, however, that the cost of capital always declines in λ if the cutoff for β_z in Proposition 8 is negative. This happens, for example, if the mass of green investors is small. In this case, green investors' elevated trading activity after an increase in λ do not diminish price informativeness to traditional investors too much.

5.3 Precise ESG information

In this section, we demonstrate via numerical example that the results of Propositions 7 and 8 tend to hold for a wide range of λ's. We pick parameters so that $\frac{dPI_t}{d\lambda} < 0$ and $\frac{dCoC}{d\lambda} > 0$ for sufficiently imprecise ESG information. We compute PI_t, PI_g and CoC as functions of λ and plot them in Figure 3. We find that multiple equilibria are possible when λ is close to one, that is, when financial and ESG information have similar precisions. When λ is small, the only possible equilibrium is the T-equilibrium, in which trading is dominated by traditional investors. Green investors choose not to trade actively because the ESG payoff is very uncertain. Naturally, this equilibrium exists as long as λ is sufficiently small. Importantly, we find that the comparative statics results established in Propositions 7 and 8 hold for all values of λ if the T-equilibrium is played. This finding is reassuring because it confirms that our predictions continue to hold even if λ is not small.

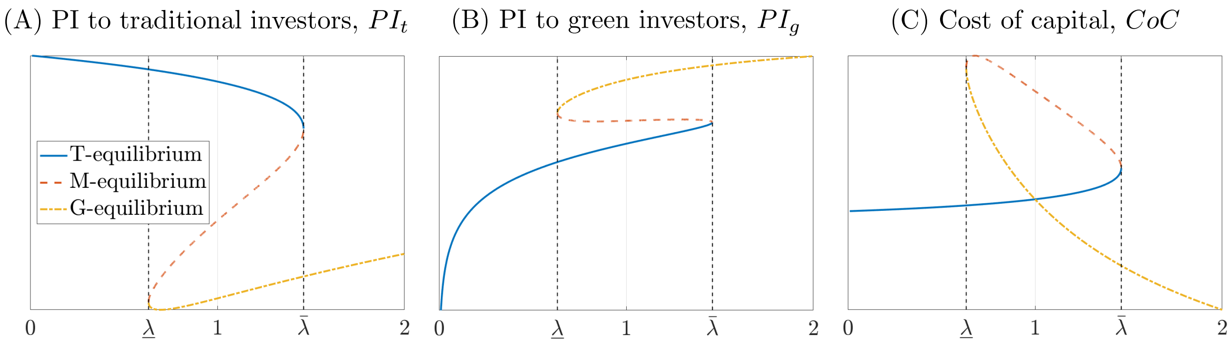

Figure 3: Price informativeness to traditional (panel A) and green (panel B) investors and cost of capital (panel C) as functions of the relative precision of financial information λ. Y-axes are in the log scale. Parameters used: $m_t = m_g = 1$, $\beta_\delta = \beta_z = \frac{1}{\sqrt{2}}$, $\gamma = 1$, $\tau_s = 5$, $\tau = 1$, $\tau_n = 4$.

5.4 Empirical implications

Proposition 7 studies implications of improving quality of ESG-related information on price informativeness. Importantly, it emphasizes that the conventional wisdom—more precise information always helps investors make more informed decisions—does not necessarily hold if investors have heterogeneous preferences. In particular, we show that better ESG information may encourage green investors to trade more actively against their cash flow signals. As a result, better ESG information can reduce the price informativeness about firm's cash flows which adversely affects traditional investors' learning from the price and increases their investment risk.[24]

Proposition 8 shows that improving the precision of ESG information might actually increase the cost of capital for a firm, which may discourage it from voluntarily disclosing ESG information. This potentially can explain why, despite regulators' efforts, the quality of ESG-related information is still unsatisfactory to market participants (Eccles, Kastrapeli, and Potter, 2017; Berg et al., 2020; Ilhan, Krueger, Sautner, and Starks, 2020). Even though firms are mandated to publish more ESG reports, they may benefit from limiting the informativeness of these reports.

Existing empirical literature provides mixed evidence on the relationship between the quality of ESG information and firms' cost of capital (see the review by Christensen, Hail, and Leuz, 2019). Focusing on voluntary disclosure, Richardson and Welker (2001) document a significantly positive relation between ESG disclosure quality and cost of equity capital, while Plumlee, Brown, Hayes, and Marshall (2015) present evidence for a negative association. Clarkson, Fang, Li, and Richardson (2013) find no significant association and Dhaliwal, Li, Tsang, and Yang (2011) find a significantly negative association only for firms with high ESG performance and no significant association overall. It should be emphasized that firms may voluntarily disclose ESG information to signal about good ESG performance, which is not captured by our model. Therefore, predictions of our model might be more applicable for understanding the impacts of mandatory disclosure (e.g. Chen, Hung, and Wang, 2018; Krueger, Sautner, Tang, and Zhong, 2021).

Importantly, when examining the role of ESG information, one should take into account that it can also be informative about cash flows if the financial and ESG payoff components are correlated. In particular, in our model a positive correlation between the two

[24]It is important to mention that our paper does not make any welfare statements. Our goal is rather to point out that there might be unintended adverse consequences of policies aimed at improving the quality of ESG information, such as mandatory ESG disclosure.

components is equivalent to preferences of traditional and green investors being more aligned (see discussions in Section 3.1). In this case, an improvement in the quality of ESG information does not intensify trading in the opposite directions by traditional and ESG investors as much as when correlation is zero or negative. As a result, better ESG information is more likely to be beneficial for all investors and reduce overall information risk.

6 Conclusion

Investors' growing ESG appetite has ignited significant changes in the asset management industry and has led to the establishment of numerous ESG funds. The ESG trend challenges the traditional view that firms' financial fundamentals are the major drivers of asset prices, and that market price is informative about financial fundamentals. In the presence of ESG investors, it is crucial to reconsider the price formation process and the information content of market price.

Our paper analyzes the interactions between traditional and ESG investors and highlights a tension between financial and ESG information contained in asset prices. As one asset price reflects both financial and ESG performances, trading for one naturally dilutes price informativeness about the other. Due to preference heterogeneity, the two groups of investors trade in different directions based on the same information, thus making the price noisier to each other. Such interactions give rise to a number of novel results. First, multiple equilibria with different pricing function may emerge. Second, an increase in the number of green investors or an improvement in ESG information quality can reduce the price informativeness about firm's financial performance, which may increase its cost of capital.

Going forward, our model can be extended along several dimensions. One extension would be to investigate how asset prices are formed in a setting with multiple firms that differ in terms of their financial and ESG performances. Another extension is to build a dynamic model to analyze price evolution in the presence of heterogeneous investors. One can also explore real implications of our results, that is, a feedback from the financial market to corporate decisions. Finally, it is interesting to analyze information acquisition in our setting. As discussed in Section 2.3.3, the key mechanism of our model relies on the fact that heterogeneous investors observe informative signals about both payoff components. Importantly, investors have incentives to acquire information about payoff components they do not directly care about because it helps them to interpret the price. Therefore, we believe that our main results will hold even if information acquisition is endogenized. We leave formal analyses of these extensions to future work.

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Appendix

Appendix A: A.1 derives equation (21); A.2 proves Proposition 1; A.3 proves Proposition 2; A.4 proves Proposition 4. Appendix B contains proofs for Section 4. It also proves Proposition 3. Appendix C contains proofs for Section 5. Appendix D considers the case in which the two payoff components are correlated. Appendix E analyzes the model with a general information structure and discusses conditions required for multiplicity of equilibria in the trading stage. Appendix F shows that trading stage features unique equilibrium when investors have homogeneous preferences but heterogeneous information.

Proofs frequently involve some tedious yet straightforward algebraic manipulations, which we perform via Matlab Symbolic Math Toolbox. Therefore, we often omit intermediate steps and present only final results. These omitted derivations are available upon request.

A Equilibrium characterization

A.1 Preliminary derivations

We start by deriving the quintic equation (21) for ξ_δ. From (2), it follows that the aggregate demand for stock from investors of group $j \in \{t, g\}$ is given by

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{p}) = m^j \frac{1}{\gamma} \frac{\tilde{z}\beta_z^j \frac{\tau_s}{\tau+\tau_s} + \tilde{\delta}\beta_\delta^j \frac{\tau_s}{\tau+\tau_s} + \left(p_z\beta_z^j \frac{1}{\tau+\tau_s} + p_\delta\beta_\delta^j \frac{1}{\tau+\tau_s}\right) \frac{\tilde{p}-p_0-\tilde{z}p_z\frac{\tau_s}{\tau_s+\tau}-\tilde{\delta}p_\delta\frac{\tau_s}{\tau_s+\tau}}{p_z^2\frac{1}{\tau+\tau_s}+p_\delta^2\frac{1}{\tau+\tau_s}+p_n^2\frac{1}{\tau_n}} - \tilde{p}}{\frac{1}{\tau+\tau_s} - \frac{\left(p_z\beta_z^j\frac{1}{\tau+\tau_s}+p_\delta\beta_\delta^j\frac{1}{\tau+\tau_s}\right)^2}{p_z^2\frac{1}{\tau+\tau_s}+p_\delta^2\frac{1}{\tau+\tau_s}+p_n^2\frac{1}{\tau_n}}}. \tag{27}$$

Plugging (27) in the market clearing condition (3) and equalizing coefficients in front of $\tilde{z}$, $\tilde{\delta}$ and $\tilde{n}$, we get

$$\xi_z = \frac{\tau_s}{\gamma}\left[m_t + m_g \frac{\beta_z\left(\xi_\delta^2+\kappa\right) - \xi_\delta\xi_z\beta_\delta}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + \kappa}\right], \tag{28}$$

$$\xi_\delta = \frac{\tau_s}{\gamma}\left[-m_t \frac{\xi_\delta\xi_z}{\xi_\delta^2+\kappa} + m_g \frac{\beta_\delta\left(\xi_z^2+\kappa\right) - \xi_\delta\xi_z\beta_z}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + \kappa}\right], \tag{29}$$

where we define $\xi_z = \frac{p_z}{p_n}$ and $\xi_\delta = \frac{p_\delta}{p_n}$ as the normalized price coefficients and denote $\kappa = \frac{\tau+\tau_s}{\tau_n}$ to simplify notations.

Consider a linear combination of the two equations:

$$\xi_z \beta_z + \xi_\delta \beta_\delta = \frac{\tau_s}{\gamma}\left(m_g + \beta_z m_t - \beta_\delta m_t \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \kappa}\right),$$

from which ξ_z can be expressed as a function of ξ_δ as

$$\xi_z = \frac{\left(\frac{\tau_s}{\gamma} m_g + \frac{\tau_s}{\gamma}\beta_z m_t - \xi_\delta \beta_\delta\right)\left(\xi_\delta^2 + \kappa\right)}{\beta_z\left(\xi_\delta^2 + \kappa\right) + \frac{\tau_s}{\gamma}\beta_\delta m_t \xi_\delta}. \tag{30}$$

Substituting in this expression for ξ_z, we can reduce the system (28)-(29) into equation (21) of one unknown ξ_δ. For brevity, we re-scale the masses of investors and define $\hat{m}_g = \frac{\tau_s}{\gamma}\beta_\delta m_g$ and $\hat{m}_t = \frac{\tau_s}{\gamma}\beta_\delta m_t$. Then equation (21) can be written as

$$\xi_\delta^5 - \hat{m}_g \xi_\delta^4 + 2\kappa\xi_\delta^3 - 2\hat{m}_g\kappa\xi_\delta^2 + \left(\kappa^2 + \hat{m}_t^2\kappa\right)\xi_\delta - \hat{m}_g\kappa^2 = 0. \tag{31}$$

A.2 Number of equilibria and noise precision

This section proves Proposition 1. The proof consists of three parts. A.2.1 establishes that equation (31) has at least one and at most three real roots. A.2.2 proves the existence of the threshold τ_n^*. A.2.3 shows that non-normalized price coefficients have conventional signs, i.e. $p_0 < 0$, $p_z, p_\delta, p_n > 0$.

A.2.1 Number and signs of roots

Claim 1. *Equation (31) has at least one and at most three real roots. All real roots are positive and are below $\hat{m}_g$.*

Proof of Claim 1.

All real roots of equation (31) are positive because coefficients of odd powers of ξ_δ are positive and coefficients of even powers of ξ_δ are negative. It is also easy to see that all roots are below $\hat{m}_g$ because the left-hand side of equation (31) is clearly positive for all $\xi_\delta \geq \hat{m}_g$.

In principle, equation (31) can have from one to five real roots. Below we show that it can have at most three real roots. Denote the left-hand side of (31) by

$$f(\xi_\delta) = \xi_\delta^5 - \hat{m}_g \xi_\delta^4 + 2\kappa \xi_\delta^3 - 2\kappa \hat{m}_g \xi_\delta^2 + \left(\kappa^2 + \hat{m}_t^2 \kappa\right) \xi_\delta - \hat{m}_g \kappa^2. \tag{32}$$

Taking first and second derivatives of $f(\xi_\delta)$, we get

$$\begin{aligned}
\frac{\partial f}{\partial \xi_\delta} &= 5\xi_\delta^4 - 4\hat{m}_g \xi_\delta^3 + 6\kappa \xi_\delta^2 - 4\hat{m}_g \kappa \xi_\delta + \kappa^2 + \hat{m}_t^2 \kappa, \\
\frac{\partial^2 f}{\partial \xi_\delta^2} &= 20\xi_\delta^3 - 12\hat{m}_g \xi_\delta^2 + 12\kappa \xi_\delta - 4\hat{m}_g \kappa.
\end{aligned}$$

The equation

$$\frac{\partial^2 f}{\partial \xi_\delta^2} = 0 \tag{33}$$

has a unique real root because its discriminant is negative: $\Delta \propto -\kappa \left(\left(\hat{m}_g^2 - \kappa \right)^2 + 4\kappa^2 \right) < 0$, where $\propto$ denotes proportionality up to a positive constant. The root of (33) is positive because coefficients of odd powers of ξ_δ are positive, and coefficients of even powers of ξ_δ are negative. Moreover, it is below $\hat{m}_g$ because $\frac{\partial^2 f}{\partial \xi_\delta^2}\big|_{\xi_\delta \geq \hat{m}_g} > 0$. Hence, $f(\xi_\delta)$ has a unique inflection point $\xi_\delta^{infl} \in (0, \hat{m}_g)$ such that $f(\xi_\delta)$ is concave if $\xi_\delta < \xi_\delta^{infl}$ and convex if $\xi_\delta > \xi_\delta^{infl}$. Given also that $f(\xi_\delta)$ is a continuous function, it follows that it can have at most three intersections with the zero line. $\square$

A.2.2 Number of roots and precision of noise trading

Rewrite (31) as

$$\frac{1}{\kappa} \left(\xi_\delta^3 + \kappa \xi_\delta - \alpha \hat{m} \kappa \right) \left(\xi_\delta^2 - \alpha \hat{m} \xi_\delta + \kappa \right) = - (1 - 2\alpha)\, \hat{m}^2 \xi_\delta, \tag{34}$$

where $\hat{m} = \hat{m}_g + \hat{m}_t$ and $\alpha = \frac{m_g}{m}$.

Denote the left-hand side of the expression above by

$$g(\xi_\delta) = \frac{1}{\kappa} \left(\xi_\delta^3 + \kappa \xi_\delta - \alpha \hat{m} \kappa \right) \left(\xi_\delta^2 - \alpha \hat{m} \xi_\delta + \kappa \right). \tag{35}$$

We start by establishing several useful properties of $g(\xi_\delta)$ in Lemma 1.

Lemma 1. *Define $g(\xi_\delta)$ as in (35). Define $\xi_\delta^* = \xi_\delta^*(\kappa, \alpha\hat{m})$ implicitly as*

$$(\xi_\delta^*)^3 + \kappa\xi_\delta^* - \alpha\hat{m}\kappa = 0. \tag{36}$$

1. *$g(\xi_\delta)$ has a unique inflection point ξ_δ^{infl} such that $g(\xi_\delta)$ is concave on $(-\infty, \xi_\delta^{infl})$ and convex on $(\xi_\delta^{infl}, \infty)$.*

2. *If $\kappa \geq \frac{1}{4}\alpha^2\hat{m}^2$ then $\frac{\partial g}{\partial \xi_\delta} > 0$; equation $g(\xi_\delta) = 0$ has a unique solution $\xi_\delta^* \in (0, \alpha\hat{m})$; $g(\xi_\delta)$ is convex on $(\xi_\delta^*, \alpha\hat{m})$.*

3. *If $\kappa < \frac{1}{4}\alpha^2\hat{m}^2$ then equation $g(\xi_\delta) = 0$ has three solutions, $\xi_\delta^{g,1}$, $\xi_\delta^{g,2}$ and ξ_δ^*, such that $0 < \xi_\delta^{g,1} < \sqrt{\kappa} < \xi_\delta^* < \xi_\delta^{g,2} < \alpha\hat{m}$.*

Proof of Lemma 1.

The first statement of this lemma directly follows from the proof of Claim 1 because $\frac{\partial^2 g}{\partial \xi_\delta^2} = \frac{1}{\kappa}\frac{\partial^2 f}{\partial \xi_\delta^2}$. In what follows, we prove the second and third statements of the lemma.

Case 1: $\kappa \geq \frac{1}{4}\alpha^2\hat{m}^2$ (Statement 2 of Lemma 1).

Take the first derivative of $g(\xi_\delta)$:

$$\kappa\frac{\partial g}{\partial \xi_\delta} = 5\xi_\delta^4 - 4\alpha\hat{m}\xi_\delta^3 + 6\kappa\xi_\delta^2 - 4\alpha\hat{m}\kappa\xi_\delta + \kappa^2 + \alpha^2\hat{m}^2\kappa =$$
$$\frac{1}{16}\left(2\xi_\delta - \alpha\hat{m}\right)^2\left(20\xi_\delta^2 + 4\alpha\hat{m}\xi_\delta + 5\alpha^2\hat{m}^2\right) + \left(\kappa - \frac{1}{4}\alpha^2\hat{m}^2\right)\left(6\xi_\delta^2 - 4\alpha\hat{m}\xi_\delta + \kappa + \frac{5}{4}\alpha^2\hat{m}^2\right).$$

Because $\kappa \geq \frac{1}{4}\alpha^2\hat{m}^2$, $\frac{\partial g}{\partial \xi_\delta} > 0$ and $g(\xi_\delta)$ is an increasing function. Furthermore, $g(0) < 0$ and $g(\alpha\hat{m}) > 0$, so $g(\xi_\delta) = 0$ has a unique solution $\xi_\delta^* \in (0, \alpha\hat{m})$. Note that ξ_δ^* satisfies (36). Indeed, consider (35) that defines $g(\xi_\delta)$. If $\kappa > \frac{1}{4}\alpha^2\hat{m}^2$ then $\xi_\delta^2 - \alpha\hat{m}\xi_\delta + \kappa$ is always positive. If $\kappa = \frac{1}{4}\alpha^2\hat{m}^2$ then the solution to $\xi_\delta^2 - \alpha\hat{m}\xi_\delta + \kappa = 0$ coincides with ξ_δ^*, defined by (36).

Finally, we show that $g(\xi_\delta)$ is convex on $(\xi_\delta^*, \alpha\hat{m})$. Take second derivative of $g(\xi_\delta)$:

$$\frac{\kappa}{4}\frac{\partial^2 g}{\partial \xi_\delta^2} = 5\xi_\delta^3 - 3\alpha\hat{m}\xi_\delta^2 + 3\kappa\xi_\delta - \alpha\hat{m}\kappa. \tag{37}$$

Plugging (36) to (37), we get

$$\frac{\kappa}{4}\frac{\partial^2 g}{\partial \xi_\delta^2} = 5\xi_\delta^3 - 3\alpha\hat{m}\xi_\delta^2 + 3\kappa\xi_\delta - (\xi_\delta^*)^3 - \kappa\xi_\delta^* \overset{\xi_\delta \ge \xi_\delta^*}{\ge} \left(4\xi_\delta^2 - 3\alpha\hat{m}\xi_\delta + 2\kappa\right)\xi_\delta.$$

The largest real root of the term in parentheses (if exists) is given by

$$\frac{3\alpha\hat{m} + \sqrt{9\alpha^2\hat{m}^2 - 32\kappa}}{8} \overset{\kappa \ge \frac{1}{4}\hat{\alpha}^2 m^2}{\le} \frac{1}{2}\alpha\hat{m}.$$

Note that $\xi_\delta^* \ge \frac{1}{2}\alpha\hat{m}$ if $\kappa \ge \frac{1}{4}\alpha^2\hat{m}^2$. Indeed, if $\kappa = \frac{1}{4}\alpha^2\hat{m}^2$, (36) implies $\xi_\delta^* = \frac{1}{2}\alpha\hat{m}$. Furthermore, applying the implicit function theorem to (36), we can see that $\frac{d\xi_\delta^*}{d\kappa} > 0$ for $\xi_\delta^* \in (0, \alpha\hat{m})$. Therefore, $\frac{\partial^2 g}{\partial \xi_\delta^2} > 0$ if $\xi_\delta \in (\xi_\delta^*, \alpha\hat{m})$.

Case 2: $\kappa < \frac{1}{4}\alpha^2\hat{m}^2$ (Statement 3 of Lemma 1).

Consider equation $g(\xi_\delta) = 0$, where $g(\xi_\delta)$ is given by (35). Define $\xi_\delta^{g,1}$ and $\xi_\delta^{g,2}$ as roots of

$$\xi_\delta^2 - \alpha\hat{m}\xi_\delta + \kappa = 0.$$

Then

$$\xi_\delta^{g,1} = \frac{\alpha\hat{m} - \sqrt{\alpha^2\hat{m}^2 - 4\kappa}}{2} \quad \text{and} \quad \xi_\delta^{g,1} = \frac{\alpha\hat{m} + \sqrt{\alpha^2\hat{m}^2 - 4\kappa}}{2}.$$

Clearly, $0 < \xi^{g,1} < \sqrt{\kappa} < \xi_\delta^{g,2} < \alpha\hat{m}$.

The third root of $g(\xi_\delta)$ is given by ξ_δ^* that solves (36). Since $\kappa < \frac{1}{4}\alpha^2\hat{m}^2$, it is easy to verify from (36) that $\xi_\delta^* > \sqrt{\kappa}$. Furthermore, evaluate the left-hand side of (36) at $\xi_\delta^{g,2}$:

$$\left(\frac{\alpha\hat{m} + \sqrt{\alpha^2\hat{m}^2 - 4\kappa}}{2}\right)^3 + \kappa\frac{-\alpha\hat{m} + \sqrt{\alpha^2\hat{m}^2 - 4\kappa}}{2} \overset{\kappa < \frac{1}{4}\alpha^2\hat{m}^2}{>} \left(\frac{\alpha\hat{m} + \sqrt{\alpha^2\hat{m}^2 - 4\kappa}}{2}\right)^3 - \frac{\alpha^3\hat{m}^3}{8} > 0.$$

Therefore, $\xi_\delta^* < \xi_\delta^{g,2}$.

□

We now proceed to proving the main result of Section A.2.2.

Claim 2. *For any $\alpha = \frac{m_g}{m_t + m_g} \in (0, 1)$ and $\hat{m} = \frac{\tau_s}{\gamma}\beta_\delta(m_t + m_g)$, $\exists \tau_n^*(\alpha, \hat{m}) > 0$ such that*

$\forall \tau_n \in (0, \tau_n^*)$ equation (31) has a unique solution; for $\tau_n = \tau_n^*$ it has two solutions when $\alpha \neq \frac{1}{2}$ and a unique solution when $\alpha = \frac{1}{2}$; $\forall \tau_n > \tau_n^*$ it has three solutions.

Proof of Claim 2.

First, note that the statement of the claim for $\alpha = \frac{1}{2}$ follows from Lemma 1 because equation (31) can be simplified to $g(\xi_\delta) = 0$, where $g(\xi_\delta)$ is given by (35). In what follows, we focus on the case with $\alpha \neq \frac{1}{2}$.

The proof proceeds in several steps. Recall that equations (31) and (34) are equivalent. In Lemmas 2 and 3, we show that there exist $\underline{\kappa}$ and $\bar{\kappa}$ such that equation (34) has one solution when $\kappa > \bar{\kappa}$ and three solutions when $\kappa < \underline{\kappa}$. In Lemma 4, we show that if for a given κ equation (34) has one (three) solutions, then it has one (three) solutions for any $\hat{\kappa}$ above (below) the given κ, respectively. Finally, we show that there exists κ^* such that equation (34) has two solutions, and any increase or decrease in κ implies that (34) has one or three solutions, respectively. Since $\kappa = \frac{\tau + \tau_s}{\tau_n}$, there is a one-to-one mapping between κ and τ_n for any given τ and τ_s. The conditions on κ then can be translated into conditions on τ_n.

Lemma 2. *$\forall \kappa \geq \bar{\kappa} = \frac{1}{4}\alpha^2 \hat{m}^2$ equation (34) has a unique solution.*

Proof of Lemma 2.

Suppose that $\alpha < \frac{1}{2}$. Equation (34) has a unique solution because the left-hand side increases in ξ_δ by Lemma 1, $g(0) < 0$ and $g(\alpha \hat{m}) > 0$, while the right-hand side decreases in ξ_δ and its value at $\xi_\delta = 0$ is zero. This case is illustrated by the intersecting solid blue line and dashed red line in Figure 4.

Suppose now that $\alpha > \frac{1}{2}$. In this case, both the left-hand side and the right-hand side of equation (34) increase in ξ_δ. They still have only one intersection because, by Lemma 1, the left-hand side of (34) is an increasing convex function $\forall \xi_\delta \in (\xi_\delta^*, \alpha \hat{m})$, where ξ_δ^* is the unique real root of equation $g(\xi_\delta) = 0$. This case is illustrated by the intersecting solid blue line and dot-dashed yellow line in Figure 4.

□

Lemma 3. *$\exists \underline{\kappa} \in (0, \bar{\kappa})$ such that $\forall \kappa \in (0, \underline{\kappa})$ equation (34) has three solutions.*

Proof of Lemma 3.

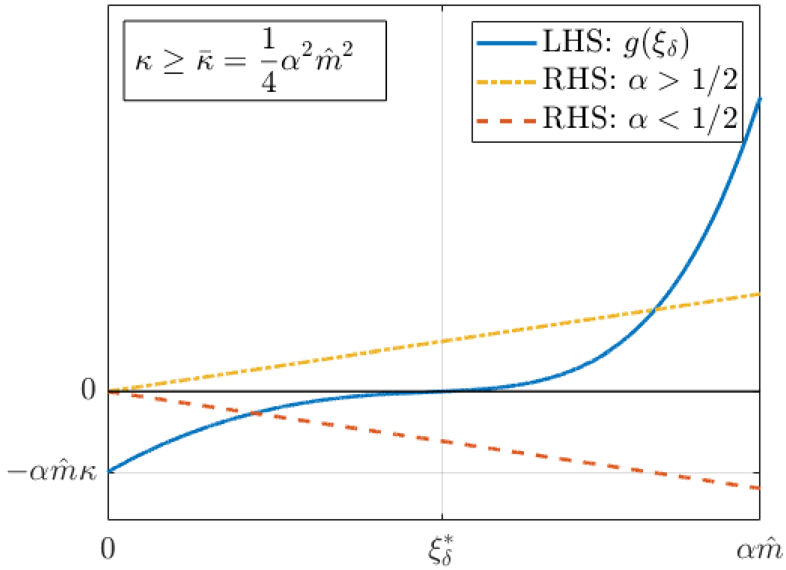

Figure 4: Unique solution to equation (34).

Write (34) in its original form as in (31),

$$\begin{aligned} f(\xi_\delta) = & \xi_\delta^5 - \alpha\hat{m}\xi_\delta^4 + 2\kappa\xi_\delta^3 - 2\alpha\hat{m}\kappa\xi_\delta^2 + \left(\kappa^2 + (1-\alpha)^2\hat{m}^2\kappa\right)\xi_\delta - \alpha\hat{m}\kappa^2 = \\ & (\xi_\delta - \alpha\hat{m})\left(\xi_\delta^4 + 2\kappa\xi_\delta^2 + \kappa^2\right) + (1-\alpha)^2\hat{m}^2\kappa\xi_\delta = 0. \end{aligned}$$

Notice that $f(\alpha\hat{m}) > 0$. At the same time, we can always pick a sufficiently small $\kappa_1 > 0$ such that $\forall\kappa \in (0, \kappa_1)$,

$$\begin{aligned} & f\left(\alpha\hat{m} - \sqrt{\kappa}\right) = \\ & -\sqrt{\kappa}\left(\left(\alpha\hat{m} - \sqrt{\kappa}\right)^4 + 2\kappa\left(\alpha\hat{m} - \sqrt{\kappa}\right)^2 + \kappa^2\right) + (1-\alpha)^2\hat{m}^2\kappa\left(\alpha\hat{m} - \sqrt{\kappa}\right) < 0. \end{aligned}$$

Notice also that $f(0) < 0$. Evaluate $f(\cdot)$ at $\frac{\alpha}{(1-\alpha)^2\hat{m}}\kappa + \kappa$,

$$\begin{aligned} & f\left(\frac{\alpha}{(1-\alpha)^2\hat{m}}\kappa + \kappa\right) = \kappa^2\Bigg[\kappa^3\left(\frac{\alpha}{(1-\alpha)^2\hat{m}} + 1\right)^5 - \alpha\hat{m}\kappa^2\left(\frac{\alpha}{(1-\alpha)^2\hat{m}} + 1\right)^4 + \\ & 2\kappa^2\left(\frac{\alpha}{(1-\alpha)^2\hat{m}} + 1\right)^3 - 2\alpha\hat{m}\kappa\left(\frac{\alpha}{(1-\alpha)^2\hat{m}} + 1\right)^2 + \kappa\left(\frac{\alpha}{(1-\alpha)^2\hat{m}} + 1\right) + \\ & (1-\alpha)^2\hat{m}^2\Bigg]. \end{aligned}$$

$\exists \kappa_2 > 0$ such that $\forall \kappa \in (0, \kappa_2)$, $f\left(\frac{\alpha}{(1-\alpha)^2 \hat{m}}\kappa + \kappa\right) > 0$ because the last term in the expression in brackets, $(1-\alpha)^2 \hat{m}^2$, does not depend on κ, while the other terms are proportional to κ^b, $b = 1, 2, 3$.

Finally, define κ_3 such that $\frac{\alpha}{(1-\alpha)^2 \hat{m}}\kappa_3 + \kappa_3 = \alpha\hat{m} - \sqrt{\kappa_3}$. Therefore, $\forall \kappa \in (0, \kappa_3)$, $\frac{\alpha}{(1-\alpha)^2 \hat{m}}\kappa + \kappa < \alpha\hat{m} - \sqrt{\kappa}$. Define $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$. Then $\forall \kappa \in (0, \underline{\kappa})$ a continuous function $f(\xi_\delta)$ changes its sign from negative to positive (at least) twice. Hence equation (34) has (at least) three solutions. Since it cannot have more than three solutions by Claim 1, it must be that it has exactly three solutions. $\square$

Lemma 4. *For any $\kappa > 0$, if equation (34) has three solutions at κ, then it has three solutions $\forall \hat{\kappa} \in (0, \kappa)$; if equation (34) has one solution at κ, then it has one solution $\forall \hat{\kappa} > \kappa$.*

Proof of Lemma 4.

Since the result trivially holds when $\kappa \in (0, \underline{\kappa}]$ and $\kappa \geq \bar{\kappa}$, where $\bar{\kappa}$ and $\underline{\kappa}$ are defined in Lemmas 2 and 3, we focus on the case when $\kappa \in (\underline{\kappa}, \bar{\kappa})$.

Consider equation $g(\xi_\delta) = 0$, where $g(\xi_\delta)$ is defined by (35). For $\kappa < \bar{\kappa} = \frac{1}{4}\alpha^2 \hat{m}^2$, this equation has three solutions by Lemma 1. Differentiate $g(\xi_\delta)$ with respect to κ:

$$\frac{\partial g}{\partial \kappa} = -\frac{1}{\kappa^2}\left(\xi_\delta^2 + \kappa\right)\left(\xi_\delta + \sqrt{\kappa}\right)\left(\xi_\delta - \sqrt{\kappa}\right)\left(\xi_\delta - \alpha\hat{m}\right).$$

Then $\frac{\partial g}{\partial \kappa} < 0$ if $\xi_\delta \in (0, \sqrt{\kappa})$ and $\frac{\partial g}{\partial \kappa} > 0$ if $\xi_\delta \in (\sqrt{\kappa}, \alpha\hat{m})$. In particular, notice that $\frac{\partial g}{\partial \kappa}\Big|_{\xi_\delta = \xi_\delta^*} > 0$ where ξ_δ^* solves (36). This is because $\xi_\delta^* \in (\sqrt{\kappa}, \alpha\hat{m})$ by Lemma 1.

In what follows, we evaluate the number of roots of equation (34). We split our analyses in two cases.

Case 1: $\alpha < \frac{1}{2}$

Suppose $\exists \kappa \in (\underline{\kappa}, \bar{\kappa})$ such that equation (34) has three solutions. This case is illustrated in Figure 5. From the graph it is evident that the smallest root ξ_δ^i of (34) is smaller than $\xi_\delta^{g,1}$—the smallest root of $g(\xi_\delta) = 0$ defined in Lemma 1. By Lemma 1, $\xi_\delta^{g,1} < \sqrt{\kappa}$, therefore, $\xi_\delta^i < \xi_\delta^{g,1} < \sqrt{\kappa}$. Furthermore, this solution $\xi_\delta^i < \sqrt{\kappa}$ exists for any κ.

The other two roots are above ξ_δ^* and $\sqrt{\kappa}$: $\xi_\delta^{iii} > \xi_\delta^{ii} > \xi_\delta^* > \sqrt{\kappa}$. In the region $\xi_\delta \in (\sqrt{\kappa}, \alpha\hat{m})$, $\frac{\partial g}{\partial \kappa} > 0$ and a marginal decrease in κ shifts $g(\xi_\delta)$ (blue solid line) downwards. At the same time, the right-hand side of equation (34) (red dashed line) does not depend

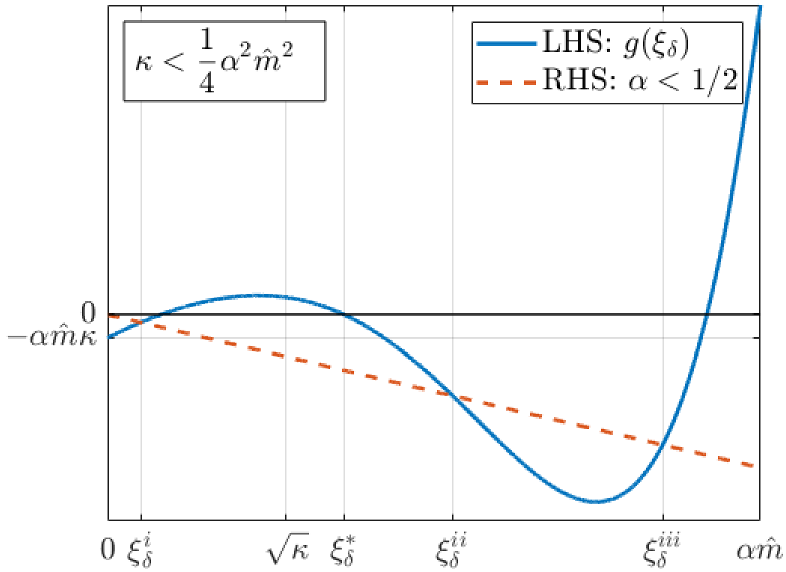

Figure 5: Three solutions to equation (34) when $\alpha < \frac{1}{2}$.

on κ and thus does not move. Therefore, for a marginally smaller κ equation (34) still has three solutions.

An analogous argument holds if for a given κ there is a unique solution to (34). In particular, the unique solution is $\xi_\delta^i < \sqrt{\kappa}$, and there is no intersection between the left-hand side, $g(\xi_\delta)$, and the right-hand side of (34) on $\xi_\delta \in (\sqrt{\kappa}, \alpha\hat{m})$. A marginal increase in κ shifts $g(\xi_\delta)$ up, while the right-hand side line does not move, which implies that equation (34) still has a unique solution.

Case 2: $\alpha > \frac{1}{2}$

Suppose $\exists\kappa \in (\underline{\kappa}, \bar{\kappa})$ such that equation (34) has three solutions. This case is illustrated in Figure 6. In this graph, two black lines (marked with crosses and circles) are tangent to the convex and the concave parts of $g(\xi_\delta)$, respectively. Recall from Lemma 1 that $g(\xi_\delta)$ has a unique inflection point ξ_δ^{infl}, and it is concave on $\xi_\delta \in (0, \xi_\delta^{infl})$ and convex on $(\xi_\delta^{infl}, \alpha\hat{m})$. Two tangent points, $\xi_\delta^{tang,1} < \xi_\delta^{infl} < \xi_\delta^{tang,2}$, solve

$$h(\xi_\delta) = \frac{\partial g(\xi_\delta)}{\partial \xi_\delta}\xi_\delta - g(\xi_\delta) = \frac{1}{\kappa}\left(4\xi_\delta^5 - 3\alpha\hat{m}\xi_\delta^4 + 4\kappa\xi_\delta^3 - 2\alpha\hat{m}\kappa\xi_\delta^2 + \alpha\hat{m}\kappa^2\right) = 0.$$

Notice that $\frac{\partial h}{\partial \xi_\delta} = \xi_\delta \frac{\partial^2 g}{\partial \xi_\delta^2}$. Therefore, $h(\xi_\delta)$ is decreasing on $\xi_\delta \in (0, \xi_\delta^{infl})$ and increasing on $\xi_\delta \in (\xi_\delta^{infl}, \alpha\hat{m})$.

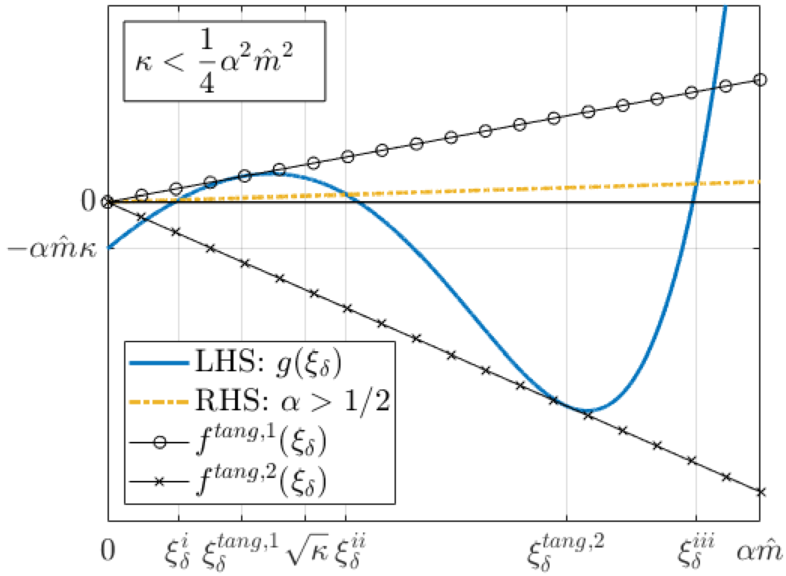

Figure 6: Three solutions to equation (34): $\alpha > \frac{1}{2}$.

Evaluate $h(\xi_\delta)$ at $\sqrt{\kappa}$:

$$h\left(\sqrt{\kappa}\right) = 4\kappa\left(2\sqrt{\kappa} - \alpha\hat{m}\right) \overset{\kappa<\bar{\kappa}=\frac{1}{4}\alpha^2\hat{m}^2}{<} 0.$$

Because $h(0) > 0$, $h(\alpha\hat{m}) > 0$ and $h\left(\sqrt{\kappa}\right) < 0$, $h(\xi_\delta) = 0$ has two solutions $\xi_\delta^{tang,1} < \sqrt{\kappa} < \xi_\delta^{tang,2}$.[25] The two tangent lines shown in Figure 6 go through zero and are thus described by equations $f^{tang,k}(\xi_\delta) = \frac{g\left(\xi_\delta^{tang,k}\right)}{\xi_\delta^{tang,k}}\xi_\delta$, $k = 1, 2$.

The right-hand side of equation (34), i.e. $-(1-2\alpha)\,\hat{m}^2\xi_\delta$, intersects $g(\xi_\delta)$ three times when its slope is smaller than the slope of the tangent line $f^{tang,1}(\xi_\delta)$, so the smallest root of (34) is $\xi_\delta^i < \xi_\delta^{tang,1} < \sqrt{\kappa}$. In the region $\xi_\delta \in (0, \sqrt{\kappa})$, $\frac{\partial g}{\partial \kappa} < 0$ and a marginal decrease in κ shifts $g(\xi_\delta)$ (blue solid line) upwards. At the same time, the right-hand side of equation (34) (yellow dot-dashed line) does not depend on κ and does not move in response to the change in κ. Therefore, for a marginally smaller κ equation (34) still has three solutions.

An analogous argument holds if for a given κ there is a unique solution to (34), i.e. when the slope of $f^{tang,1}(\xi_\delta)$ is below the slope of the right-hand side of (34). In the region

[25] Notice also that $\xi_\delta^{tang,1} < \frac{\alpha\hat{m}}{2} < \xi_\delta^{tang,2}$ because $h\left(\frac{\alpha\hat{m}}{2}\right) = \alpha\hat{m}\frac{1}{\kappa}\left(-\frac{\alpha^2\hat{m}^2}{4} + \kappa\right)\left(\frac{\alpha^2\hat{m}^2}{4} + \kappa\right) \overset{\kappa<\frac{1}{4}\alpha^2\hat{m}^2}{<} 0$. This will be used in the proof of Proposition 2.

$\xi_\delta \in (0, \sqrt{\kappa})$, there is no intersection between the left-hand side, $g(\xi_\delta)$, and the right-hand side of (34). A marginal increase in κ shifts $g(\xi_\delta)$ down, while the right-hand side line does not move, which implies that equation (34) still has one solution. $\square$

Having proved Lemmas 2, 3, 4, we are now ready to complete the proof of Claim 2. These lemmas imply that there exist $\bar{\kappa}^* \geq \underline{\kappa}^* > 0$ such that equation (34) has three solutions if $\kappa \in (0, \underline{\kappa}^*)$, two solutions if $\kappa \in [\underline{\kappa}^*, \bar{\kappa}^*]$, and one solution if $\kappa > \bar{\kappa}^*$. In addition, it must be the case that $\underline{\kappa}^* = \bar{\kappa}^* = \kappa^*$. To see this, focus on the case $\alpha > \frac{1}{2}$ without loss of generality. Equation (34) then has two solutions if and only if the right-hand side of (34) coincides with the tangent line $f^{tang,1}(\xi_\delta)$ (see Figure 6). However, from the proof of Lemma 4 it follows that any marginal increase or decrease in κ leaves equation (34) with one or three solutions, respectively.

Finally, recall that by definition $\kappa = \frac{\tau+\tau_s}{\tau_n}$. Given τ and τ_s, define $\tau_n^* = \frac{\tau+\tau_s}{\kappa^*}$. Then equation (34) has two solutions if $\tau_n = \tau_n^*$, one solution if $\tau_n \in (0, \tau_n^*)$ and three solutions if $\tau_n > \tau_n^*$. $\square$

A.2.3 Signs of the price coefficients

Claim 3. *$p_0 < 0$, $p_z > 0$, $p_\delta > 0$, $p_n > 0$.*

Proof of Claim 3.

By Claim 1, all solutions to (31) are positive and below $\hat{m}_g = \frac{\tau_s}{\gamma}\beta_\delta m_g$. Equation (30) implies

$$\xi_z = \frac{\left(\frac{\tau_s}{\gamma}m_g + \frac{\tau_s}{\gamma}\beta_z m_t - \xi_\delta\beta_\delta\right)\left(\xi_\delta^2+\kappa\right)}{\beta_z\left(\xi_\delta^2+\kappa\right) + \frac{\tau_s}{\gamma}\beta_\delta m_t \xi_\delta} \overset{\xi_\delta < \hat{m}_g}{>} \frac{\left(\frac{\tau_s}{\gamma}m_g + \frac{\tau_s}{\gamma}\beta_z m_t - \frac{\tau_s}{\gamma}\beta_\delta^2 m_g\right)\left(\xi_\delta^2+\kappa\right)}{\beta_z\left(\xi_\delta^2+\kappa\right) + \frac{\tau_s}{\gamma}\beta_\delta m_t \xi_\delta} \overset{\beta_\delta \leq 1}{>} 0.$$

Recall that $\xi_\delta = \frac{p_\delta}{p_n}$ and $\xi_z = \frac{p_z}{p_n}$. Therefore, p_z, p_δ and p_n have the same sign.

Matching coefficients in the market clearing condition (3) implies

$$\frac{1}{\tau+\tau_s} = \frac{1}{\gamma}p_n\left[m_t\frac{p_z^2+p_\delta^2+p_n^2\kappa - p_z}{p_\delta^2+p_n^2\kappa} + m_g\frac{p_z^2+p_\delta^2+p_n^2\kappa - (p_z\beta_z + p_\delta\beta_\delta)}{\left(p_z\beta_\delta - p_\delta\beta_z\right)^2 + p_n^2\kappa}\right], \tag{38}$$

where, as above, $\kappa = \frac{\tau+\tau_s}{\tau_n}$. Clearly, if p_z, p_δ and p_n are all negative, then the right-hand side is negative. Therefore, p_z, p_δ and p_n are all positive.

We are left to show that $p_0 < 0$. Again, by matching coefficients in the market clearing condition, we have

$$p_0 = -\frac{\gamma}{\tau + \tau_s}\left[m_t \frac{\xi_z^2 + \xi_\delta^2 + \kappa}{\xi_\delta^2 + \kappa} + m_g \frac{\xi_z^2 + \xi_\delta^2 + \kappa}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \kappa}\right]^{-1}.$$

Therefore, $p_0 < 0$. □

Proposition 1 follows from Claims 1, 2 and 3. Note that Claim 2 expresses the multiplicity threshold τ_n^* as a function of α and $\hat{m}$: $\tau_n^* = \tau_n^*(\alpha, \hat{m})$. Because $\hat{m} = \frac{\tau_s}{\gamma}\beta_\delta m$ and β_δ does not show up elsewhere in equation (34), we can alternatively express the multiplicity threshold τ_n^* as a function of α and β_δ, $\tau_n^* = \tau_n^*(\alpha, \beta_\delta)$.

A.3 Comparative statics of τ_n^*

This section establishes comparative statics properties of τ_n^* stated in Proposition 2.

Proof of Proposition 2.

Claim 2 implies that the multiplicity threshold τ_n^* can be written as a function of $\hat{m}$ and α, $\tau_n^* = \tau_n^*(\alpha, \hat{m})$. Below, we explore comparative statics with respect to $\hat{m}$ and α. Because $\hat{m} = \frac{\tau_s}{\gamma}\beta_\delta m$ and β_δ does not show up elsewhere in equation (34), the comparative statics with respect to β_δ is equivalent to the one with respect to $\hat{m}$.

Comparative statics with respect to $\hat{m}$

Divide (34) by $\hat{m}^2$ to get

$$\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa) = \frac{1}{\kappa}\left[\frac{1}{\hat{m}^2}(\xi_\delta - \alpha\hat{m})\left(\xi_\delta^2 + \kappa\right)^2 + \alpha^2 \xi_\delta\right] = -(1 - 2\alpha)\,\xi_\delta. \tag{39}$$

Then

$$\frac{\partial\left[\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa)\right]}{\partial \hat{m}} = \frac{\alpha\hat{m} - 2\xi_\delta}{\kappa \hat{m}^3}\left(\xi_\delta^2 + \kappa\right)^2,$$

so $\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa)$ increases in $\hat{m}$ when $\xi_\delta \in \left(0, \frac{\alpha\hat{m}}{2}\right)$ and decreases in $\hat{m}$ when $\xi_\delta \in \left(\frac{\alpha\hat{m}}{2}, \alpha\hat{m}\right)$.

Suppose that $\alpha < \frac{1}{2}$. Fix $\hat{m}_1 > 0$. By definition, at $\tau_n = \tau_n^*(\alpha, \hat{m}_1)$ equation (39) has two solutions. This is illustrated by the solid blue and the red dashed lines in Figure 7,

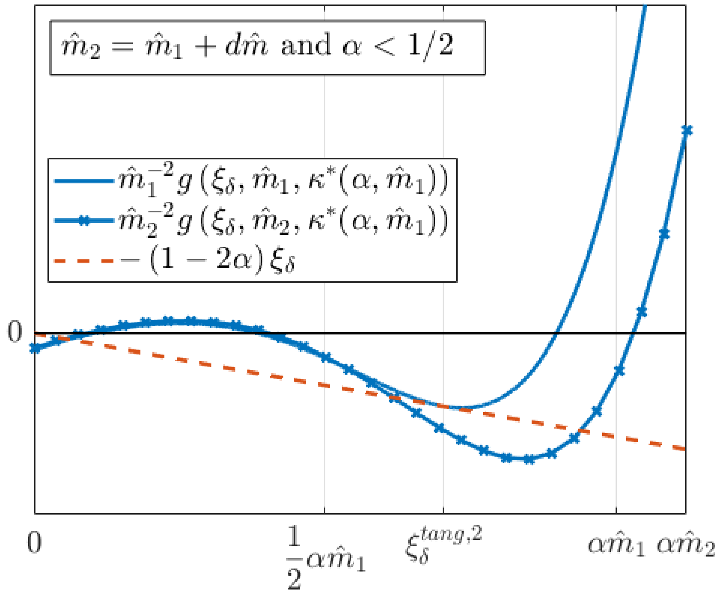

Figure 7: Comparative statics of $\tau_n^* (\alpha, \hat{m})$ with respect to $\hat{m}$.

which intersect twice (in particular, the largest intersection $\xi_\delta^{tang,2}$ is a tangent point). Recall that $\xi_\delta^{tang,2} > \frac{\alpha\hat{m}_1}{2}$ (see footnote 25). Therefore, a marginal increase in $\hat{m}$ from $\hat{m}_1$ to $\hat{m}_1 + d\hat{m}$ shifts the curve $\hat{m}_1^{-2} g(\xi_\delta, \hat{m}_1, \kappa^*(\alpha, \hat{m}_1))$ down $\forall \xi_\delta \in \left(\frac{\alpha\hat{m}_1}{2}, \alpha\hat{m}_1\right)$, as shown in Figure 7 (crossed blue solid line). The right-hand side of (39) does not depend on $\hat{m}$ and thus does not move. Therefore, there exist three solutions to (39) if $\hat{m} = \hat{m}_1 + d\hat{m}$. From Lemma 4 it then follows that $\frac{\partial \tau_n^*}{\partial \hat{m}} < 0$.

Analogous arguments can be made to show that $\frac{\partial \tau_n^*}{\partial \hat{m}} < 0$ if $\alpha > \frac{1}{2}$.

If $\alpha = \frac{1}{2}$, we can get the analytical solution for $\tau_n^* (\alpha, \hat{m})$ (see Lemma 1),

$$\tau_n^* \left(\frac{1}{2}, \hat{m}\right) = \frac{16(\tau + \tau_s)}{\hat{m}^2}.$$

Therefore, $\frac{\partial \tau_n^*}{\partial \hat{m}} < 0$ if $\alpha = \frac{1}{2}$.

Comparative statics with respect to α

Equation (31) can be rewritten as

$$\frac{1}{\kappa} \left(\xi_\delta^5 - \alpha\hat{m}\xi_\delta^4 + 2\kappa\xi_\delta^3 - 2\alpha\hat{m}\kappa\xi_\delta^2 + \kappa^2\xi_\delta - \alpha\hat{m}\kappa^2\right) = -(1-\alpha)^2 \hat{m}^2\xi_\delta. \tag{40}$$

Denote the left-hand side of (40) by $\tilde{g}(\xi_\delta)$. Note that $\tilde{g}(\xi_\delta) = g(\xi_\delta) - \alpha^2\hat{m}^2\xi_\delta$, where

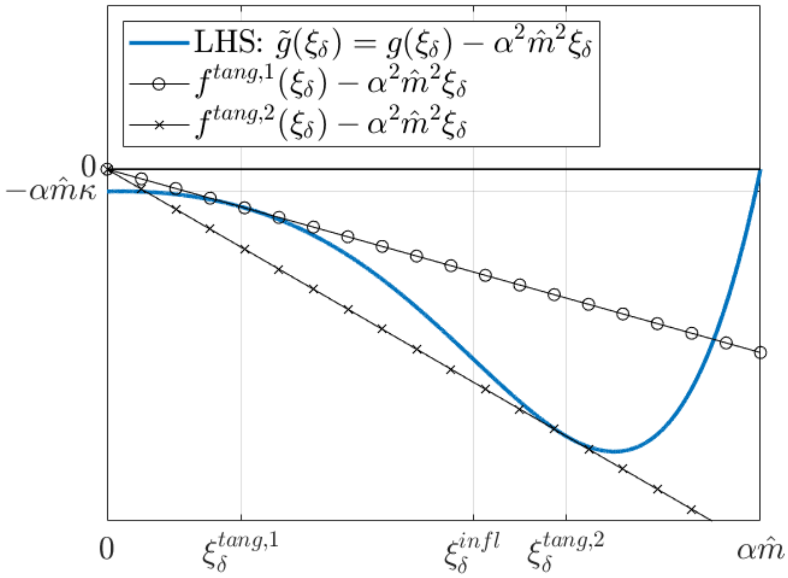

Figure 8: Comparative statics of $\tau_n^*(\alpha, \hat{m})$ with respect to α.

$g(\xi_\delta)$ is given by (35). Since $\tilde{g}(\xi_\delta)$ and $g(\xi_\delta)$ differ only by a linear term in ξ_δ, they are both concave if $\xi_\delta < \xi_\delta^{infl}$ and convex if $\xi_\delta > \xi_\delta^{infl}$. In addition, they have the same two tangent points $\xi_\delta^{tang,1} < \xi_\delta^{infl} < \xi_\delta^{tang,2}$ defined in the proof of Lemma 4.

By definition, if $\tau_n = \tau_n^*(\alpha, \hat{m})$ and $\alpha \neq \frac{1}{2}$, equation (40) has two solutions. (40) has two solutions if and only if its right-hand side is tangent to $\tilde{g}(\xi_\delta)$. In particular, if $\alpha > \frac{1}{2}$, (40) has two solutions if its right-hand side is $f^{tang,1}(\xi_\delta) - \alpha^2\hat{m}^2\xi_\delta$, where $f^{tang,1}(\xi_\delta)$ is defined in the proof of Lemma 4. Similarly, if $\alpha < \frac{1}{2}$, (40) has two solutions if its right-hand side is $f^{tang,2}(\xi_\delta) - \alpha^2\hat{m}^2\xi_\delta$. Figure 8 illustrates both cases.

Suppose that $\alpha < \frac{1}{2}$ and $\tau_n = \tau_n^*(\alpha, \hat{m})$. Following a marginal increase in α, the left-hand side of (40), shown by the blue solid line in Figure 8, moves downwards. At the same time, the right-hand side, shown by the black crossed line in Figure 8, rotates counterclockwise around the zero point. Therefore, (40) has three solutions. From Lemma 4 it then follows that $\frac{\partial \tau_n^*}{\partial \alpha} < 0$.

Analogous arguments can be made to show that $\frac{\partial \tau_n^*}{\partial \alpha} > 0$ when $\alpha > \frac{1}{2}$.

$\square$

A.4 Stability of equilibria

Plugging expression (30) for $\xi_z(\xi_\delta)$ in the right-hand side of (29), we can write $\xi_\delta = J(\xi_\delta)$. Moreover, $J(\xi_\delta) - \xi_\delta = -k(\xi_\delta) \times f(\xi_\delta)$, where $k(\xi_\delta) > 0$ $\forall \xi_\delta$ and $f(\xi_\delta)$ is given by (32). Then at any solution ξ_δ^{root} such that $f(\xi_\delta^{root}) = 0$, we have

$$\left.\frac{\partial\left[J(\xi_\delta) - \xi_\delta\right]}{\partial \xi_\delta}\right|_{\xi_\delta = \xi_\delta^{root}} = -k(\xi_\delta^{root}) f'(\xi_\delta^{root}).$$

By Claim 1, $f(\xi_\delta) = 0$ has at least one root and at most three roots, and all roots are positive. Furthermore, $f(0) < 0$. Therefore, if there exists a unique root to $f(\xi_\delta) = 0$, $f'(\xi_\delta^{root}) > 0$. When there are three roots $\xi_\delta^i < \xi_\delta^{ii} < \xi_\delta^{iii}$, $f'(\xi_\delta^{root}) > 0$ for $\xi_\delta^{root} = \xi_\delta^i, \xi_\delta^{iii}$ and $f'(\xi_\delta^{root}) < 0$ for $\xi_\delta^{root} = \xi_\delta^{ii}$. Since ξ_δ^i, ξ_δ^{ii} and ξ_δ^{iii} correspond to T-, M- and G-equilibria, respectively, Proposition 4 follows.

B Growth of green investors

B.1 Price informativeness

In this section, we analyze how price informativeness changes as the fraction of green investors α increases and prove Proposition 5. We also argue at the end of this section that Proposition 3 follows from the proof of Proposition 5.

Proof of Proposition 5.

Denote $\tilde{x} = \beta_\delta \tilde{z} - \beta_z \tilde{\delta}$ and $\xi_x = \beta_\delta \xi_z - \beta_z \xi_\delta$. Using (31) and $\xi_z = \xi_z(\xi_\delta)$ from (30), we can rewrite the system of equations (28)-(29) as

$$\xi_\delta = \frac{\alpha \hat{m} \kappa}{\xi_x^2 + \kappa}, \tag{41}$$

$$\xi_x = \frac{(1-\alpha)\hat{m}\kappa}{\xi_\delta^2 + \kappa}, \tag{42}$$

where $\hat{m} = \frac{\tau_s}{\gamma} \beta_\delta m$. Clearly, both ξ_x and ξ_δ are positive. Taking derivatives of (41)-(42)

with respect to α, we obtain

$$
\begin{aligned}
&2\xi_x\xi_\delta\xi_x' + (\xi_x^2 + \kappa)\xi_\delta' - \kappa\hat{m} = 0, \\
&(\xi_\delta^2 + \kappa)\xi_x' + 2\xi_x\xi_\delta\xi_\delta' + \kappa\hat{m} = 0.
\end{aligned}
$$

Here we use the prime symbol to denote derivatives with respect to α. Simplifying these equations, we get

$$
\xi_x' = -\frac{\xi_x^2 + 2\xi_x\xi_\delta + \kappa}{\xi_\delta^2 + 2\xi_x\xi_\delta + \kappa}\xi_\delta'. \tag{43}
$$

Rewriting definitions (22) and (23), we get

$$
\begin{aligned}
PI_t &= \frac{\tau + \tau_s}{\beta_\delta^2}\frac{2\beta_z\xi_x\xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta_\delta^2\kappa}{\xi_\delta^2 + \kappa}, \\
PI_g &= \frac{\tau + \tau_s}{\beta_\delta^2}\frac{2\beta_z\xi_x\xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta_\delta^2\kappa}{\xi_x^2 + \kappa}, \\
v &= \frac{PI_t}{PI_g} = \frac{\xi_x^2 + \kappa}{\xi_\delta^2 + \kappa}.
\end{aligned} \tag{44}
$$

Below we analyze the comparative statics of PI_t, PI_g and v with respect to α.

Comparative statics of PI_t and PI_g with respect to α

$$
\frac{dPI_t}{d\alpha} = \frac{2\left(\tau + \tau_s\right)}{\beta_\delta^2}\frac{\left(\beta_z\xi_x'\xi_\delta + \beta_z\xi_x\xi_\delta' + \xi_\delta\xi_\delta' + \xi_x\xi_x'\right)\left(\xi_\delta^2 + \kappa\right) - \left(2\beta_z\xi_x\xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta_\delta^2\kappa\right)\xi_\delta\xi_\delta'}{\left(\xi_\delta^2 + \kappa\right)^2}.
$$

Substituting in ξ_x' from (43), we can rewrite the above expression as

$$
\frac{dPI_t}{d\alpha} = -\xi_\delta' \times A_1\left(\xi_\delta, \xi_x\right),
$$

where $A_1\left(\xi_\delta, \xi_x\right)$ is a function that takes positive values for $\xi_\delta > 0$ and $\xi_x > 0$. Hence, the sign of $\frac{dPI_t}{d\alpha}$ is the same as the sign of $-\xi_\delta'$.

Using the same approach as for PI_t, we find that

$$
\frac{dPI_g}{d\alpha} = \xi_\delta' \times A_2\left(\xi_\delta, \xi_x\right),
$$

where $A_2(\xi_\delta, \xi_x)$ is a function that takes positive values for $\xi_\delta > 0$ and $\xi_x > 0$. Hence, the sign of $\frac{dPI_g}{d\alpha}$ is the same as the sign of ξ'_δ.

Comparative statics of v with respect to α

$$\frac{dv}{d\alpha} = \frac{\frac{dPI_t}{d\alpha}PI_g - \frac{dPI_g}{d\alpha}PI_t}{PI_g^2} = \frac{-\xi'_\delta\left(A_1(\xi_\delta, \xi_x)PI_g + A_2(\xi_\delta, \xi_x)PI_t\right)}{PI_g^2}.$$

Hence, the sign of $\frac{dv}{d\alpha}$ is the same as the sign of $-\xi'_\delta$.

Comparative statics of ξ_δ with respect to α

ξ_δ is implicitly defined by equation (31), which we also show below.

$$f(\xi_\delta, \alpha) = \xi_\delta^5 - \alpha\hat{m}\xi_\delta^4 + 2\kappa\xi_\delta^3 - 2\alpha\hat{m}\kappa\xi_\delta^2 + \left[\kappa^2 + (1-\alpha)^2\hat{m}^2\kappa\right]\xi_\delta - \alpha\hat{m}\kappa^2 = 0,$$

where we again denote $\hat{m} = \frac{\tau_s}{\gamma}\beta_\delta m$. Using the implicit function theorem, we get

$$\xi'_\delta = \frac{\hat{m}\xi_\delta^4 + 2\hat{m}\kappa\xi_\delta^2 + 2(1-\alpha)\hat{m}^2\kappa\xi_\delta + \hat{m}\kappa^2}{\frac{\partial f}{\partial \xi_\delta}}.$$

Therefore, the sign of ξ'_δ is the same as the sign of $\frac{\partial f}{\partial \xi_\delta}$.

By Claim 1, $f(\xi_\delta) = 0$ has at least one root and at most three roots, and all roots are positive. Furthermore, $f(0) < 0$. Therefore, if there exists a unique root to $f(\xi_\delta) = 0$, $f'(\xi_\delta^{root}) > 0$. When there are three roots $\xi_\delta^i < \xi_\delta^{ii} < \xi_\delta^{iii}$, $f'(\xi_\delta^{root}) > 0$ for $\xi_\delta^{root} = \xi_\delta^i, \xi_\delta^{iii}$ and $f'(\xi_\delta^{root}) < 0$ for $\xi_\delta^{root} = \xi_\delta^{ii}$. Since ξ_δ^i, ξ_δ^{ii} and ξ_δ^{iii} correspond to T-, M- and G-equilibria, respectively, comparative statics results of Proposition 5 follow.

Relative price informativeness across equilibria

Suppose that multiple equilibria are possible, that is, $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$. The existence of $\bar{\alpha}$ and $\underline{\alpha}$, defined in Proposition 5, follows from Proposition 2. Notice that at $\alpha = \underline{\alpha}$ and $\alpha = \bar{\alpha}$, there are two equilibria such that $\xi_\delta^T(\underline{\alpha}) < \xi_\delta^M(\underline{\alpha}) = \xi_\delta^G(\underline{\alpha})$ and $\xi_\delta^T(\bar{\alpha}) = \xi_\delta^M(\bar{\alpha}) < \xi_\delta^G(\bar{\alpha})$.

Figure 9 shows ξ_δ as a function of α, where the monotonicity properties of ξ_δ with respect to α have been established above. In particular, $\xi_\delta(\alpha)$ is an increasing function in the T- and G-equilibria and is a decreasing function in the M-equilibrium.

Equations (41)-(42) imply $\xi_\delta(\alpha)$ and $\xi_x(\alpha)$ are symmetric around $\alpha = \frac{1}{2}$ such that that $\xi_\delta(\alpha) = \xi_x(1-\alpha)$. This is illustrated in Figure 9 for the T-equilibrium. This symmetry implies that $1 - \underline{\alpha} = \bar{\alpha}$. Further, it implies that in the T-equilibrium $\xi_\delta(\alpha) < \xi_x(\alpha)$. Using the definition of the relative price informativeness υ (44), we conclude that in the T-equilibrium $\upsilon^T > 1$. Analogously, in the G-equilibrium $\upsilon^G < 1$. Finally, $\upsilon^T > \upsilon^M > \upsilon^G$. □

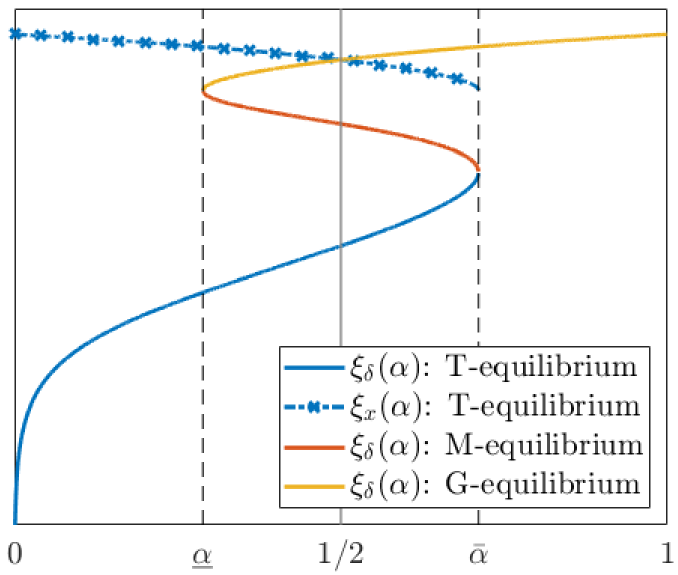

Figure 9: ξ_δ and $\xi_x = \beta_\delta \xi_z - \beta_z \xi_\delta$ as functions of the fraction of green investors α. Y-axes are in the log scale. Parameters are selected such that the equilibrium multiplicity is possible, that is, $\tau_n > \tau_n^*\left(\frac{1}{2}, \beta_\delta\right)$.

Note that the last part of the above proof also proves Proposition 3. This proof also implies the following corollary that we will use below.

Corollary 2. *If equilibrium is unique,* $\xi'_\delta > 0$ *and* $\xi_\delta \gtreqless \xi_x$ *if* $\alpha \gtreqless \frac{1}{2}$. *If there are multiple equilibria,* $\xi'_\delta > 0$ *and* $\xi_\delta < \xi_x$ *in the T-equilibrium,* $\xi'_\delta > 0$ *and* $\xi_\delta > \xi_x$ *in the G-equilibrium, and* $\xi'_\delta < 0$ *and* $\xi_\delta \gtreqless \xi_x$ *if* $\alpha \lesseqgtr \frac{1}{2}$ *in the M-equilibrium.*

B.2 Cost of capital

In this section, we prove Proposition 6 and Corollary 1.

First, we express the cost of capital in its general form when the firm's expected output is non-zero, i.e.

$$CoC = \mathbb{E}[\tilde{z} - \tilde{p}] = \mu_z - p_0 - p_z\mu_z - p_\delta\mu_\delta. \tag{45}$$

With non-zero expected $\tilde{z}$ and $\tilde{\delta}$, the aggregate demand by type-j investors is given by

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{p}) =$$

$$\frac{m^j}{\gamma} \frac{\beta_z^j \frac{\tau_s \tilde{z} + \tau\mu_z}{\tau_s + \tau} + \beta_\delta^j \frac{\tau_s \tilde{\delta} + \tau\mu_\delta}{\tau_s + \tau} + \left(p_z \beta_z^j \frac{1}{\tau + \tau_s} + p_\delta \beta_\delta^j \frac{1}{\tau + \tau_s}\right) \frac{\tilde{p} - p_0 - p_z \frac{\tau_s \tilde{z} + \tau\mu_z}{\tau_s + \tau} - p_\delta \frac{\tau_s \tilde{\delta} + \tau\mu_\delta}{\tau_s + \tau}}{p_z^2 \frac{1}{\tau + \tau_s} + p_\delta^2 \frac{1}{\tau + \tau_s} + p_n^2 \frac{1}{\tau_n}} - \tilde{p}}{\frac{1}{\tau + \tau_s} - \frac{\left(p_z \beta_z^j \frac{1}{\tau + \tau_s} + p_\delta \beta_\delta^j \frac{1}{\tau + \tau_s}\right)^2}{p_z^2 \frac{1}{\tau + \tau_s} + p_\delta^2 \frac{1}{\tau + \tau_s} + p_n^2 \frac{1}{\tau_n}}}.$$

This expression is analogous to (27) in the zero-mean case. Plugging the above expression in the market clearing condition (3) and equalizing coefficients in front of $\tilde{z}$, $\tilde{\delta}$, and $\tilde{n}$, we can verify that the equilibrium price coefficients p_z, p_δ and p_n remain the same as in the zero-mean case. However, p_0 is different,

$$p_0 = \frac{\gamma\left(\frac{\tau}{\tau_s}(\mu_z\xi_z + \mu_\delta\xi_\delta) - 1\right)}{m_t PI_t + m_g PI_g},$$

where PI_t and PI_g are the price informativeness to traditional and green investors, given by (22) and (23), respectively. We express the non-normalized price coefficients p_z and p_δ in terms of the normalized price coefficients ξ_z and ξ_δ. Rewriting (38), we obtain

$$p_n = (\tau + \tau_s) \frac{\frac{\gamma}{\tau + \tau_s} + m_t \frac{\xi_z}{\xi_\delta^2 + \kappa} + m_g \frac{\xi_z\beta_z + \xi_\delta\beta_\delta}{(\xi_z\beta_\delta - \xi_\delta\beta_z)^2 + \kappa}}{m_t PI_t + m_g PI_g}.$$

Using the system (41)-(42) and the expressions for p_0, p_n, $p_z = \xi_z \times p_n$, $p_\delta = \xi_\delta \times p_n$, we can rewrite (45) as

$$CoC = c_z\mu_z + c_\delta\mu_\delta + \frac{\gamma}{m_t PI_t + m_g PI_g}, \tag{46}$$

where $c_z = \frac{(1-\beta_z)\xi_\delta}{\beta_\delta\xi_z + (1-\beta_z)\xi_\delta}$ and $c_\delta = -\frac{\beta_\delta\xi_\delta}{\beta_\delta\xi_z + (1-\beta_z)\xi_\delta}$. Note that if $\mu_z = \mu_\delta = 0$, this expression reduces to (24).

Proof of Proposition 6.

If $\mu_z = \mu_\delta = 0$,

$$CoC = \frac{\gamma}{m_t PI_t + m_g PI_g}.$$

Differentiating CoC with respect to α and substituting in ξ_x' from (43), we get

$$\frac{dCoC}{d\alpha} = -\left(\xi_\delta - \xi_x\right)\xi_\delta' \times A_3(\xi_\delta, \xi_x),$$

where $A_3(\xi_\delta, x)$ is a function that takes positive values for any $\xi_\delta > 0$ and $\xi_x > 0$. Then the comparative statics of CoC with respect to α follow from Corollary 2. □

Proof of Corollary 1.

Equation (46) shows that CoC is linear in μ_z and μ_δ. Below we analyze the comparative statics of c_z and c_δ with respect to α. Note that $\frac{dc_\delta}{d\alpha}$ and $\frac{dc_z}{d\alpha}$ always have opposite signs because $c_z = -\frac{1-\beta_z}{\beta_\delta}c_\delta$,. Therefore, in what follows, we focus on the sign of $\frac{dc_\delta}{d\alpha}$.

Recall that $\xi_x = \beta_\delta \xi_z - \beta_z \xi_\delta$. Hence,

$$c_\delta = -\frac{\beta_\delta \xi_\delta}{\xi_x + \xi_\delta} \Rightarrow c_\delta' = \frac{dc_\delta}{d\alpha} = -\beta_\delta \frac{\xi_\delta' \xi_x - \xi_\delta \xi_x'}{(\xi_x + \xi_\delta)^2}.$$

Substitute ξ_x' from (43) to obtain $c_\delta' = -\xi_\delta' \times A_4(\xi_\delta, x)$, where $A_4(\xi_\delta, \xi_x)$ is a function that takes positive values for $\xi_\delta > 0$ and $\xi_x > 0$. Then the comparative statics of c_δ with respect to α follow from Corollary 2. □

C Improvements in ESG information

In this section, we consider the setting discussed in Section 5. When $\lambda > 0$, demand for the stock from investors of type j is

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{p}) = \frac{m^j}{\gamma} \frac{\tilde{z}\beta_z^j \frac{\tau_s}{\tau+\tau_s} + \tilde{\delta}\beta_\delta^j \frac{\lambda\tau_s}{\lambda\tau+\lambda\tau_s} + \left(p_z \beta_z^j \frac{1}{\tau+\tau_s} + p_\delta \beta_\delta^j \frac{1}{\lambda\tau+\lambda\tau_s}\right) \frac{\tilde{p} - p_0 - \tilde{z}p_z \frac{\tau_s}{\tau_s+\tau} - \tilde{\delta}p_\delta \frac{\lambda\tau_s}{\lambda\tau_s+\lambda\tau}}{p_z^2 \frac{1}{\tau+\tau_s} + p_\delta^2 \frac{1}{\lambda\tau+\lambda\tau_s} + p_n^2 \frac{1}{\tau_n}} - \tilde{p}}{\left(\beta_z^j\right)^2 \frac{1}{\tau+\tau_s} + \left(\beta_\delta^j\right)^2 \frac{1}{\lambda\tau+\lambda\tau_s} - \frac{\left(p_z \beta_z^j \frac{1}{\tau+\tau_s} + p_\delta \beta_\delta^j \frac{1}{\lambda\tau+\lambda\tau_s}\right)^2}{p_z^2 \frac{1}{\tau+\tau_s} + p_\delta^2 \frac{1}{\lambda\tau+\lambda\tau_s} + p_n^2 \frac{1}{\tau_n}}}.$$

Imposing the market clearing condition (3), we obtain the system

$$\xi_z = \frac{\tau_s}{\gamma}\left[m_t + m_g \frac{\beta_z\left(\xi_\delta^2 + \lambda\kappa\right) - \xi_\delta \xi_z \beta_\delta}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + \left(\beta_z^2\lambda + \beta_\delta^2\right)\kappa}\right], \tag{47}$$

$$\xi_\delta = \frac{\lambda\tau_s}{\gamma}\left[-m_t\frac{\xi_\delta\xi_z}{\xi_\delta^2 + \lambda\kappa} + m_g \frac{\beta_\delta\left(\xi_z^2 + \kappa\right) - \xi_\delta \xi_z \beta_z}{\left(\xi_z\beta_\delta - \xi_\delta\beta_z\right)^2 + \left(\beta_z^2\lambda + \beta_\delta^2\right)\kappa}\right]. \tag{48}$$

Denote $\check{\kappa} = \lambda\kappa = \lambda\frac{\tau+\tau_s}{\tau_n}$, $\check{\beta}_\delta = \frac{\beta_\delta}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\check{\beta}_z = \sqrt{1-\check{\beta}_\delta^2} = \frac{\sqrt{\lambda}\beta_z}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\check{m}_g = m_g\lambda\frac{1}{\sqrt{\lambda\beta_z^2+\beta_\delta^2}}$, $\check{m}_t = m_t\sqrt{\lambda}$ and $\check{\xi}_z = \xi_z\sqrt{\lambda}$. Then the system becomes

$$\check{\xi}_z = \frac{\tau_s}{\gamma}\left[\check{m}_t + \check{m}_g \frac{\check{\beta}_z\left(\xi_\delta^2 + \check{\kappa}\right) - \xi_\delta \check{\xi}_z \check{\beta}_\delta}{\left(\check{\xi}_z\check{\beta}_\delta - \xi_\delta\check{\beta}_z\right)^2 + \check{\kappa}}\right],$$

$$\xi_\delta = \frac{\tau_s}{\gamma}\left[-\check{m}_t\frac{\xi_\delta\check{\xi}_z}{\xi_\delta^2 + \check{\kappa}} + \check{m}_g \frac{\check{\beta}_\delta\left(\check{\xi}_z^2 + \check{\kappa}\right) - \xi_\delta \check{\xi}_z \check{\beta}_z}{\left(\check{\xi}_z\check{\beta}_\delta - \xi_\delta\check{\beta}_z\right)^2 + \check{\kappa}}\right].$$

Note that it has the same structure as (28)-(29). Therefore, adjusted versions of Propositions 1 and 2 hold, where m, α and β_δ are substituted by, respectively, $\check{m} = \check{m}_t + \check{m}_g$, $\check{\alpha} = \frac{\check{m}_g}{\check{m}}$ and $\check{\beta}_\delta$.

Analytically characterizing comparative statics of the endogenous objects such as price coefficients, price informativeness and cost of capital, with respect to λ is nontrivial. In what follows, we investigate the model under assumption that λ is small. We linearize price coefficients ξ_z and ξ_δ around $\lambda = 0$ and investigate the comparative statics of the linearized solution. To do so, we proceed in three steps. First, we solve the model for the case with $\lambda = 0$. Second, we use the system of equations (47)-(48), derived under assumption $\lambda > 0$, to get equation (49) that implicitly defines ξ_δ. We then show that if λ is sufficiently small, there exists a unique solution to this equation that is smooth in λ around 0. Moreover, the solution to this equation coincides with the solution derived in Step 1 when $\lambda = 0$. In the third step, we linearize the solution of equation (49) around $\lambda = 0$ and prove Propositions 7 and 8.

Step 1: Solving the model when $\lambda = 0$.

When $\lambda = 0$, prior and signals about the ESG component $\tilde{\delta}$ are infinitely imprecise. Therefore, the price cannot be informative about $\tilde{\delta}$ in any equilibrium so that $p_\delta = 0$ and $\tilde{p} = p_0 + p_z\tilde{z} + p_n\tilde{n} = p_0 + p_n\left(\xi_z\tilde{z} + \tilde{n}\right)$. Green investors do not trade the stock because its payoff is infinitely risky to them. As a result, the equilibrium price coefficients are shaped by trading activities of traditional and noise investors only. In particular, demand for the stock from traditional investors is

$$D^t(\tilde{z}, \tilde{p}) = m_t \frac{1}{\gamma} \frac{\tilde{z}\frac{\tau_s}{\tau+\tau_s} + p_z \frac{1}{\tau+\tau_s} \frac{\tilde{p} - p_0 - \tilde{z} p_z \frac{\tau_s}{\tau_s+\tau}}{p_z^2 \frac{1}{\tau+\tau_s} + p_n^2 \frac{1}{\tau_n}} - \tilde{p}}{\frac{1}{\tau+\tau_s} - \frac{\left(p_z \frac{1}{\tau+\tau_s}\right)^2}{p_z^2 \frac{1}{\tau+\tau_s} + p_n^2 \frac{1}{\tau_n}}}.$$

The market clearing condition is $D^t(\tilde{z}, \tilde{p}) + \tilde{n} = 1$. By matching the price coefficients, it is straightforward to show that $p_n > 0$, $p_z > 0$, $p_0 < 0$, $\xi_z = \frac{\tau_s}{\gamma} m_t$. As mentioned earlier, the price cannot be informative about $\tilde{\delta}$, therefore we have $p_\delta = 0$, $\xi_\delta = 0$.

Step 2: Equation for ξ_δ when $\lambda > 0$.

When $\lambda > 0$, we can use system (47)-(48) to get a quintic equation of ξ_δ, analogous to equation (21) in the main text:

$$\begin{aligned} f(\xi_\delta) = & \xi_\delta^5 - \left(\frac{\tau_s}{\gamma} m_g\right) \lambda \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2} \xi_\delta^4 + 2\lambda\kappa \xi_\delta^3 - 2\left(\frac{\tau_s}{\gamma} m_g\right) \lambda^2 \kappa \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2} \xi_\delta^2 + \\ & \left[\lambda^2\kappa^2 + \left(\frac{\tau_s}{\gamma} m_t\right)^2 \lambda^2 \kappa \frac{\beta_\delta^2}{\lambda \beta_z^2 + \beta_\delta^2}\right] \xi_\delta - \left(\frac{\tau_s}{\gamma} m_g\right) \lambda^3 \kappa^2 \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2} = 0, \end{aligned} \tag{49}$$

which can be rewritten as

$$f(\xi_\delta) = \left(\xi_\delta - \left(\frac{\tau_s}{\gamma} m_g\right) \lambda \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2}\right) \left(\xi_\delta^2 + \lambda\kappa\right)^2 + \left(\frac{\tau_s}{\gamma} m_t\right)^2 \lambda^2 \kappa \frac{\beta_\delta^2}{\lambda \beta_z^2 + \beta_\delta^2} \xi_\delta = 0.$$

If $\lambda = 0$, this equation has a unique solution $\xi_\delta = 0$, which coincides with the one derived in Step 1. If $\lambda > 0$, there always exists a positive solution because $f(0) < 0$ and $f(\infty) > 0$. Moreover, all solutions are below $\frac{\tau_s}{\gamma} m_g \lambda \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2}$. When λ is sufficiently small, the solution is unique. Indeed, differentiate (49) and observe that for $\xi_\delta \in \left(0, \frac{\tau_s}{\gamma} m_g \lambda \frac{\beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2}\right)$,

$$\frac{\partial f}{\partial \xi_\delta} > -4\left(\frac{\tau_s}{\gamma} m_g \frac{\lambda \beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2}\right)^4 - 4\left(\frac{\tau_s}{\gamma} m_g \frac{\lambda \beta_\delta}{\lambda \beta_z^2 + \beta_\delta^2}\right)^2 \lambda\kappa + \lambda^2\kappa^2 + \left(\frac{\tau_s}{\gamma} m_t\right)^2 \frac{\lambda^2 \kappa \beta_\delta^2}{\lambda \beta_z^2 + \beta_\delta^2}.$$

If λ is sufficiently small, the last positive term is larger in absolute terms than the first two negative terms combined. Therefore, $f(\xi_\delta)$ is strictly increasing on the relevant interval, which guarantees that there exists a unique solution $\xi_\delta(\lambda)$. Moreover, the function $\xi_\delta(\lambda)$ is smooth in the neighborhood of zero because $f(\xi_\delta, \lambda)$ is smooth in the neighborhood of $(0, 0)$.

Step 3: Linearization.

Because $\xi_\delta(\lambda)$ is smooth around $\lambda = 0$, we can use its Taylor series to approximate it around this point. Write $\xi_\delta = \xi_{\delta,1}\lambda + o(\lambda)$, where $\xi_{\delta,1}$ does not depend on λ, and plug it in (49). Omitting higher order terms, we obtain

$$\xi_{\delta,1} = \frac{\left(\frac{\tau_s}{\gamma}m_g\right)\kappa}{\beta_\delta\left(\left(\frac{\tau_s}{\gamma}m_t\right)^2 + \kappa\right)} > 0.$$

Similarly, we have $\xi_z = \xi_{z,0} + \xi_{z,1}\lambda + o(\lambda)$. Using equation (47), we get

$$\xi_{z,0} = \frac{\tau_s}{\gamma}m_t \quad \text{and} \quad \xi_{z,1} = \xi_{\delta,1}\frac{1}{\beta_\delta}\left[\beta_z - \frac{\left(\frac{\tau_s}{\gamma}m_t\right)\left(\frac{\tau_s}{\gamma}m_g\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2 + \kappa}\right].$$

The linear term of ξ_z is negative if and only if $\beta_z < \frac{\left(\frac{\tau_s}{\gamma}m_t\right)\left(\frac{\tau_s}{\gamma}m_g\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\kappa}$, which proves part (i) of Proposition 7.

Next, we linearize price informativeness, given by (25)-(26). We repeat them below.

$$PI_t = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda\kappa}{\xi_\delta^2 + \lambda\kappa} \quad \text{and} \quad PI_g = (\tau + \tau_s)\frac{\xi_z^2\lambda + \xi_\delta^2 + \lambda\kappa}{(\xi_\delta\beta_z - \xi_z\beta_\delta)^2 + (\beta_z^2\lambda + \beta_\delta^2)\kappa}.$$

For green investors, we have

$$PI_{g,0} + PI_{g,1}\lambda = (\tau + \tau_s)\frac{\xi_{z,0}^2\lambda + \lambda\kappa}{\xi_{z,0}^2\beta_\delta^2 + \beta_\delta^2\kappa} \Rightarrow PI_{g,0} = 0, \quad PI_{g,1} = (\tau + \tau_s)\frac{1}{\beta_\delta^2} > 0.$$

For traditional investors, we have

$$PI_{t,0} + PI_{t,1}\lambda = (\tau + \tau_s)\frac{\xi_{z,0}^2\lambda + 2\xi_{z,1}\xi_{z,0}\lambda^2 + \xi_{\delta,1}^2\lambda^2 + \lambda\kappa}{\xi_{\delta,1}^2\lambda^2 + \lambda\kappa} \Rightarrow$$

$$PI_{t,0} = (\tau + \tau_s)\frac{\left(\frac{\tau_s}{\gamma}m_t\right)^2 + \kappa}{\kappa}, PI_{t,1} = (\tau + \tau_s)\frac{2\xi_{z,0}\xi_{\delta,1}}{\kappa\beta_\delta}\left(\beta_z - \frac{3}{2}\frac{\left(\frac{\tau_s}{\gamma}m_g\right)\left(\frac{\tau_s}{\gamma}m_t\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2 + \kappa}\right).$$

Clearly, $PI_{t,1} < 0$ if and only if $\beta_z - \frac{3}{2}\frac{\left(\frac{\tau_s}{\gamma}m_g\right)\left(\frac{\tau_s}{\gamma}m_t\right)}{\left(\frac{\tau_s}{\gamma}m_t\right)^2+\kappa} < 0$, which proves part (ii) of Proposition 7.

Finally, recall that $CoC = \frac{\gamma}{m_t PI_t + m_g PI_g}$ and so

$$\frac{dCoC}{d\lambda} \propto -\left(m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda}\right),$$

where $\propto$ denotes proportionality up to a positive term. Using $PI_{g,1}$ and $PI_{t,1}$ derived above, we have

$$m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda} = m_t PI_{t,1} + m_g PI_{g,1} + o\left(1\right) =$$
$$\frac{\tau+\tau_s}{\beta_\delta^2} \frac{2m_g \left(\frac{\tau_s}{\gamma} m_t\right)^2}{\left(\frac{\tau_s}{\gamma} m_t\right)^2 + \kappa} \left[\beta_z - \frac{3}{2} \frac{\left(\frac{\tau_s}{\gamma} m_t\right)\left(\frac{\tau_s}{\gamma} m_g\right)}{\left(\frac{\tau_s}{\gamma} m_t\right)^2 + \kappa} + \frac{1}{2} \frac{\left(\frac{\tau_s}{\gamma} m_t\right)^2 + \kappa}{\left(\frac{\tau_s}{\gamma} m_t\right)^2}\right] + o(1).$$

If the expression in the brackets is negative, CoC increases in λ for a sufficiently small λ. This proves Proposition 8.

D Correlated payoff components

This section considers the model in which the financial payoff component $\tilde{z}$ and the ESG payoff component $\tilde{\delta}$ are correlated with a correlation coefficient $\rho \in (-1, 1)$. As in the main model, the stock payoff to traditional investors is $\tilde{z}$ and the stock payoff to green investors is $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. We normalize $\beta_z^2 + 2\beta_z\beta_\delta\rho + \beta_\delta^2 = 1$ such that traditional and green investors are exposed to the same ex-ante variance when holding the stock.

Define orthogonalized payoff components

$$\tilde{x} = \tilde{z}, \tag{50}$$

$$\tilde{y} = \frac{\tilde{\delta} - \rho\tilde{z}}{\sqrt{1-\rho^2}}. \tag{51}$$

By construction, $\tilde{x}$ and $\tilde{y}$ have the same variance τ^{-1} as $\tilde{z}$ and $\tilde{\delta}$. Furthermore, they are uncorrelated. Intuitively, $\tilde{y}$ represents "pure" ESG output that is completely unrelated to cash flows.

We can write investors' preferences over the orthogonalized payoff components (50)-(51) in the following way. Traditional investors still value only one component $\tilde{x}$. For green investors, the stock payoff is $\beta_x\tilde{x} + \beta_y\tilde{y}$, where

$$\beta_x = \beta_z + \beta_\delta\rho,$$
$$\beta_y = \beta_\delta\sqrt{1-\rho^2}.$$

Note that $\beta_x^2 + \beta_y^2 = \beta_z^2 + 2\beta_z\beta_\delta\rho + \beta_\delta^2 = 1$.

We assume that each investor i irrespective of her type observes two uncorrelated private signals $\tilde{s}_x^i \sim N\left(\tilde{x}, \tau_s^{-1}\right)$ and $\tilde{s}_y^i \sim N\left(\tilde{y}, \tau_s^{-1}\right)$. Signals about $\tilde{y}$ represent information about "pure" ESG output that is unrelated to cash flows; signals about $\tilde{x}$ represent information about cash flows, including cash flow-relevant information that investors extract from a firm's ESG performance. For example, $\tilde{s}_x^i$ might include information about how eco-friendly a firm's products are and thus how strong the demand from eco-conscious consumers is going to be. Note that this information environment is equivalent to the one in which investors receive correlated signals about the non-orthogonalized payoff components $\tilde{s}_z^i \sim N\left(\tilde{z}, \tau_s^{-1}\right)$ and $\tilde{s}_\delta^i \sim N\left(\tilde{\delta}, \tau_s^{-1}\right)$, with the same correlation coefficient ρ as between $\tilde{z}$ and $\tilde{\delta}$.[26]

Therefore, by orthogonalizing the payoff components and defining investors' preferences over these components, we get back to the main model of Section 3, in which the payoff components are uncorrelated. The following proposition summarizes the equivalence result.

Proposition 9. *The following two models are equivalent:*

1. *A model in which payoff components are correlated, $Corr(\tilde{z}, \tilde{\delta}) = \rho$; signals are correlated $Corr(\tilde{s}_z^i, \tilde{s}_\delta^i) = \rho$ for any investor i; stock payoff to traditional investors is $\tilde{z}$; stock payoff to green investors is $\beta_z\tilde{z} + \beta_\delta\tilde{\delta}$.*

2. *A model in which payoff components $\tilde{x} = \tilde{z}$ and $\tilde{y} = \frac{\tilde{\delta}-\rho\tilde{z}}{\sqrt{1-\rho^2}}$ and investor signals about them are uncorrelated; stock payoff to traditional investors is $\tilde{x}$; stock payoff to green investors is $(\beta_z + \beta_\delta\rho)\tilde{x} + (\beta_\delta\sqrt{1-\rho^2})\tilde{y}$.*

Proposition 9 is intuitive. In particular, it states that a positive correlation between the payoff components effectively makes traditional and green investors' preferences more aligned. That is, a high realization of the ESG payoff component benefits not only green

[26]Equal correlation $\mathrm{Corr}(\tilde{z}, \tilde{\delta}) = \mathrm{Corr}(\tilde{s}_z^i, \tilde{s}_\delta^i) = \rho$ is crucial to keep the model analytically tractable.

investors, who directly value the ESG output, but also traditional investors because it tends to be associated with a higher realization of the financial payoff component.

E General information structure

In the main text, we consider an analytically tractable case when traditional and green investors have access to information of the same quality. In this section, we explore the role of information structure for our main results. In particular, we establish that our results about the existence of multiple equilibria in the trading game and the nature of these equilibria are robust to general assumptions about information available to investors.

First, we allow $\tilde{z}$ and $\tilde{\delta}$ to have different ex ante variances, τ_z^{-1} and τ_δ^{-1}, respectively. Second, traditional and green investors receive informative signals about $\tilde{z}$ and $\tilde{\delta}$ of potentially different precisions. In particular, investor i of type $j \in \{t, g\}$ receives two private signals, $\tilde{s}_z^{ij} \sim N\left(\tilde{z}, \left(\tau_{s_z}^j\right)^{-1}\right)$ and $\tilde{s}_\delta^{ij} \sim N\left(\tilde{\delta}, \left(\tau_{s_\delta}^j\right)^{-1}\right)$. Given their preferences, we assume that traditional (green) investors receive some useful information about $\tilde{z}$ ($\tilde{\delta}$), namely, $\tau_{s_z}^t > 0$ and $\tau_{s_\delta}^g > 0$. Other signals can be in principle uninformative, $\tau_{s_\delta}^t \geq 0$ and $\tau_{s_z}^g \geq 0$. Finally, we maintain our baseline assumptions: masses of traditional and green investors are positive, $m_t > 0$ and $m_g > 0$; investors' risk aversion parameter is $\gamma > 0$; traditional investors care only about $\tilde{z}$ and green investors care about $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$, where $\beta_z \geq 0$ and $\beta_\delta > 0$; noise traders' demand is $\tilde{n} \sim N\left(0, \tau_n^{-1}\right)$.

Under general information structure, the system of equations (28)-(29) for ξ_z and ξ_δ becomes

$$\xi_z = m_t \tau_{s_z}^t + m_g \tau_{s_z}^g \frac{\beta_z \left(\xi_\delta^2 + \frac{\tau_\delta + \tau_{s_\delta}^g}{\tau_n}\right) - \xi_\delta \xi_z \beta_\delta}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \left(\tau_\delta + \tau_{s_\delta}^g\right) + \beta_\delta^2 \left(\tau_z + \tau_{s_z}^g\right)\right) \frac{1}{\tau_n}}, \tag{52}$$

$$\xi_\delta = -m_t \tau_{s_\delta}^t \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau_\delta + \tau_{s_\delta}^t}{\tau_n}} + m_g \tau_{s_\delta}^g \frac{\beta_\delta \left(\xi_z^2 + \frac{\tau_z + \tau_{s_z}^g}{\tau_n}\right) - \xi_\delta \xi_z \beta_z}{\left(\xi_z \beta_\delta - \xi_\delta \beta_z\right)^2 + \left(\beta_z^2 \left(\tau_\delta + \tau_{s_\delta}^g\right) + \beta_\delta^2 \left(\tau_z + \tau_{s_z}^g\right)\right) \frac{1}{\tau_n}}. \tag{53}$$

In (52)-(53), we set $\gamma = 1$. This is without loss of generality because it is equivalent to redefining the masses of traditional and green investors.

Proposition 10. *Fix $m_t > 0$, $m_g > 0$, $\gamma > 0$, $\beta_z \geq 0$, $\beta_\delta > 0$, $\tau_{s_z}^t > 0$, $\tau_{s_\delta}^t \geq 0$, $\tau_{s_z}^g \geq 0$, $\tau_{s_\delta}^g > 0$. For any $\tau_n > 0$, an equilibrium with a linear price $\tilde{p} = p_0 + p_z \tilde{z} + p_\delta \tilde{\delta} + p_n \tilde{n}$ exists. Moreover, for a sufficiently large τ_n multiple equilibria exist if one of the following* *conditions is satisfied:*

(i) $\tau^t_{s_\delta} > 0$ *and* $\tau^g_{s_z} > 0$*;*

(ii) $\tau^t_{s_\delta} > 0,\ \tau^g_{s_z} = 0,$ *and either* $\dfrac{4\beta_\delta^2 m_t^2 \tau^t_{s_z}\tau^t_{s_\delta}}{m_g^2\left(\tau^g_{s_\delta}\right)^2} < 1$ *or* $\beta_z > 0$*;*

(iii) $\tau^t_{s_\delta} = 0,\ \tau^g_{s_z} > 0,$ *and* $\dfrac{4m_g\tau^g_{s_\delta}\left(\tau^g_{s_z}m_g + \beta_z m_t \tau^t_{s_z}\right)}{\beta_\delta^2 m_t^2\left(\tau^t_{s_z}\right)^2} < 1$*;*

(iv) $\tau^t_{s_\delta} = 0,\ \tau^g_{s_z} = 0,\ \beta_z > 0$ *and* $\dfrac{4\beta_z m_g \tau^g_{s_\delta}}{\beta_\delta^2 m_t \tau^t_{s_z}} < 1.$

Below, we first discuss Proposition 10 and then formally prove it at the end of this section.

Discussion

Proposition 10 emphasizes the importance of the information structure for the existence of multiple equilibria in the trading stage. In particular, they arise when investors have access to information about fundamentals that they value differently. To see it clearly, it is instructive to consider a special case when green investors care only about the $\tilde{\delta}$-component, i.e. $\beta_z = 0$. For a sufficiently small exogenous noise (large τ_n), multiple equilibria always arise as long as traditional and green investors receive *some* informative signals about $\tilde{\delta}$ and $\tilde{z}$, respectively. In an equilibrium that resembles the T-equilibrium, the price is closely associated with $\tilde{z}$ and is thus very informative to traditional investors. This incentivizes them to trade the stock intensively. In particular, they actively trade against their $\tilde{\delta}$-signals, virtually offsetting the impact of green investors who trade in the opposite direction. The price is, therefore, weakly associated with $\tilde{\delta}$. Analogously, there is an equilibrium that resembles the G-equilibrium, where the price is closely associated with $\tilde{\delta}$.

Notice that the multiplicity is possible even if only one investor group receives signals about the factor they do not value, e.g. $\tau^t_{s_\delta} > 0$ and $\tau^g_{s_z} = 0$ (the case with $\tau^t_{s_\delta} = 0$ and $\tau^g_{s_z} > 0$ is analogous). In the absence of relevant signals about $\tilde{z}$, green investors are not able to offset traditional investors' trading along the $\tilde{z}$-dimension. The price is always informative to traditional investors because the price coefficient ξ_z is shaped solely by their trading activities. The multiplicity is still possible due to trading in the opposite directions along the $\tilde{\delta}$-dimension. It requires, however, that the mass of traditional investors is small and their private signals are not precise relative to those of green investors, i.e. $\frac{4\beta_\delta^2 m_t^2 \tau_{s_z}^t \tau_{s_\delta}^t}{m_g^2\left(\tau_{s_\delta}^g\right)^2} < 1$. If this is not the case, traditional investors dominate the trading along the $\tilde{\delta}$-dimension and the price is uniquely determined. Note that if green investors care about the $\tilde{z}$-component, multiple equilibria are always possible for a sufficiently small noise. If $\beta_z > 0$, preferences of green and traditional investors are partially aligned. Green investors benefit to some extent from traditional investors' trading as they can learn about $\tilde{z}$ from the price. The price is less noisy to them, and they trade more aggressively based on their $\tilde{\delta}$-signals.

Finally, the equilibrium is always unique if investors are informed only about the factors they care about, i.e. $\tau_{s_\delta}^t = \tau_{s_z}^g = 0$. In this case, there is no trading in the opposite directions because the investors' information sets are orthogonal. This case is studied in Rahi and Zigrand (2018) and Rahi (2021). As in the previous case, multiple equilibria might arise if $\beta_z > 0$. In this case, signals received by green investors are not perfectly aligned with what they value and, therefore, they benefit from the information about $\tilde{z}$ contained in the price.

Overall, Proposition 10 shows that, under fairly general assumptions on the information structure, the price might not be uniquely pinned down if the stock is traded by investors with heterogeneous valuations. Equilibria differ in terms of which investor group most actively trades the stock and which factors the price is mostly informative about. There are two key requirements for the multiplicity to emerge. First, investors of one group need to possess *some* information about the fundamental that investors of the other group value. That allows investors with heterogeneous preferences to trade against each other based on the same information. Second, the amount of exogenous noise should be small; otherwise, the price is always an imprecise signal to all rational investors.

Proof of Proposition 10.

As in other proofs, this one involves many tedious yet straightforward algebraic manipulations, which we frequently perform via Matlab Symbolic Math Toolbox and do not show.

The first part of the proof involves the reduction of (52)-(53) to a polynomial equation either for ξ_δ or ξ_z. Depending on the values of signal precisions, this equation is either cubic or have a higher odd order. For cubic equations, we investigate the number of roots using the sign of the discriminant. For higher order equations, the analysis is conceptually similar to the proof of Lemma 3. In particular, we prove that for a sufficiently large τ_n, there are at least three distinct real roots by showing that the polynomial changes its sign at least three times.

Getting a polynomial equation for either ξ_z or ξ_δ from the system (52)-(53) involves different steps when $\beta_z = 0$ and $\beta_z > 0$, so we analyze these two cases separately. Each case is further split into four subcases that jointly cover all possible values of signal precisions. In some of those subcases, we introduce new notation. Since subcases are independent from one another, the additional notation is case-specific, that is, we might use the same notation in different subcases to denote different objects.

Case 1: $\beta_z = 0$.

Note that we do not impose the restriction that $\beta_z^2 + \beta_\delta^2 = 1$ as in our main model, which makes it possible for the two groups of investors to be exposed to different risk levels. The system (52)-(53) simplifies to

$$\begin{aligned} \xi_z &= m_t \tau_{s_z}^t - \hat{m}_g \tau_{s_z}^g \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau_z + \tau_{s_z}^g}{\tau_n}}, \\ \xi_\delta &= \hat{m}_g \tau_{s_\delta}^g - m_t \tau_{s_\delta}^t \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau_\delta + \tau_{s_\delta}^t}{\tau_n}}, \end{aligned} \tag{54}$$

where we denote $\hat{m}_g = \frac{1}{\beta_\delta} m_g$.

Case 1.1: $\tau_{s_\delta}^t = \tau_{s_z}^g = 0$.

If investors receive signals only about fundamentals they care about, the equilibrium in the trading stage is trivially unique: $\xi_z = m_t \tau_{s_z}^t$ and $\xi_\delta = \hat{m}_g \tau_{s_\delta}^g$.

Case 1.2: $\tau_{s_\delta}^t = 0$ **and** $\tau_{s_z}^g > 0$**.**

If only green investors receive informative signals about $\tilde{\delta}$, their trading activity solely determines the corresponding price coefficient, $\xi_\delta = \hat{m}_g \tau_{s_\delta}^g$. ξ_z solves the following equation:

$$\xi_z^3 - \xi_z^2 \left[m_t \tau_{s_z}^t \right] + \xi_z \left[\frac{\tau_z + \tau_{s_z}^g}{\tau_n} + \hat{m}_g^2 \tau_{s_z}^g \tau_{s_\delta}^g \right] - m^t \tau_{s_z}^t \frac{\tau_z + \tau_{s_z}^g}{\tau_n} = 0. \tag{55}$$

This equation has at least one real root because it is cubic, and the real root(s) must be positive since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i \left(\frac{1}{\tau_n}\right)^i,$$

where $d_0 = \hat{m}_g^4 m_t^2 \left(\tau_{s_z}^g \tau_{s_\delta}^g \tau_{s_z}^t\right)^2 - 4\hat{m}_g^6 \left(\tau_{s_z}^g \tau_{s_\delta}^g\right)^3$. For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (55) has three distinct real roots if

$$\frac{4\hat{m}_g^2 \tau_{s_\delta}^g \tau_{s_z}^g}{m_t^2 \left(\tau_{s_z}^t\right)^2} = \frac{4 m_g^2 \tau_{s_\delta}^g \tau_{s_z}^g}{\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2} < 1.$$

Case 1.3: $\tau_{s_\delta}^t > 0$ and $\tau_{s_z}^g = 0$.

This case is symmetric to **Case 1.2**. There are three solutions to (54) if τ_n is sufficiently large and

$$\frac{4 m_t^2 \tau_{s_z}^t \tau_{s_\delta}^t}{\hat{m}_g^2 \left(\tau_{s_\delta}^g\right)^2} = \frac{4\beta_\delta^2 m_t^2 \tau_{s_z}^t \tau_{s_\delta}^t}{m_g^2 \left(\tau_{s_\delta}^g\right)^2} < 1.$$

Case 1.4: $\tau_{s_\delta}^t, \tau_{s_z}^g > 0$.

Since the first equation of (54) is linear in ξ_δ, we can straightforwardly write $\xi_\delta = \xi_\delta(\xi_z)$. Plugging it in the second equation of the system, we obtain the following equation for ξ_z:

$$f(\xi_z) = \sum_{i=0}^{9} a_i \xi_z^i = 0. \tag{56}$$

Moreover, $a_9 = 1$ and $a_0 = a_{0,3} \left(\frac{1}{\tau_n}\right)^3$, where $a_{0,3} < 0$ does not depend on τ_n. Then there exists at least one positive real root. Let's now show that there exists at least three positive real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.

We can write

$$a_0 = a_{0,3}\left(\frac{1}{\tau_n}\right)^3,$$
$$a_1 = a_{1,2}\left(\frac{1}{\tau_n}\right)^2 + a_{1,3}\left(\frac{1}{\tau_n}\right)^3,$$
$$a_2 = a_{2,2}\left(\frac{1}{\tau_n}\right)^2 + a_{2,3}\left(\frac{1}{\tau_n}\right)^3,$$
$$a_3 = a_{3,1}\frac{1}{\tau_n} + a_{3,2}\left(\frac{1}{\tau_n}\right)^2 + a_{3,3}\left(\frac{1}{\tau_n}\right)^3,$$

where $a_{i,j} \neq 0$ are coefficients that do not depend on τ_n. Moreover, $a_{0,3} < 0$ and $a_{1,2} > 0$. Then, evaluating $f(\cdot)$ at $-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f\left(-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) = a_{1,2}c_1\left(\frac{1}{\tau_n}\right)^3 + o\left(\left(\frac{1}{\tau_n}\right)^3\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) > 0$.

Next, we can write (56) also as a polynomial of $\frac{1}{\tau_n}$:

$$f\left(\xi_z\right) = \sum_{i=0}^{3} b_i(\xi_z)\left(\frac{1}{\tau_n}\right)^i,$$

where

$$b_0(\xi_z) = \xi_z^4\left(\xi_z - m_t\tau_{s_z}^t\right)^2\left(\xi_z^2\left(\xi_z - m_t\tau_{s_z}^t\right) + \hat{m}_g^2\tau_{s_z}^g\tau_{s_\delta}^g\xi_z\right) + \hat{m}_g^2 m_t\left(\tau_{s_z}^g\right)^2\tau_{s_\delta}^t\xi_z^5\left(\xi_z - m_t\tau_{s_z}^t\right).$$

Then, evaluating $f\left(\cdot\right)$ at $m_t\tau_{s_z}^t - \left(\frac{1}{\tau_n}\right)^{1/2}$, we obtain

$$f\left(m_t\tau_{s_z}^t - \left(\frac{1}{\tau_n}\right)^{1/2}\right) = -\hat{m}_g^2 m_t\left(\tau_{s_z}^g\right)^2\tau_{s_\delta}^t\left(m_t\tau_{s_z}^t\right)^5\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

Therefore, for a sufficiently large τ_n, $f\left(m_t\tau_{s_z}^t - \left(\frac{1}{\tau_n}\right)^{1/2}\right) < 0$ and $m_t\tau_{s_z}^t - \left(\frac{1}{\tau_n}\right)^{1/2} > -\frac{a_{0,3}}{a_{1,2}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$. Furthermore, because $a_9 > 0$, for any $\tau_n > 0$ $f\left(\xi_z\right) > 0$ if ξ_z is sufficiently large. Hence, we have shown that (56) has at least three (positive real) solutions for ξ_z if τ_n is sufficiently large.

Case 2: $\beta_z > 0$.

We now work with the system (52)-(53).

Case 2.1: $\tau^t_{s_\delta} = \tau^g_{s_z} = 0$.

The price coefficient ξ_z is $m_t\tau^t_{s_z}$. ξ_δ solves

$$\xi_\delta^3[\beta_z^2] - \xi_\delta^2\left[2\beta_z\beta_\delta m_t\tau^t_{s_z}\right] + \xi_\delta\left[\left(\beta_z^2\left(\tau_\delta + \tau^g_{s_\delta}\right) + \beta_\delta^2\tau_z\right)\frac{1}{\tau_n} + m_g m_t\tau^g_{s_\delta}\tau^t_{s_z}\beta_z + \left(\beta_\delta m_t\tau^t_{s_z}\right)^2\right] - m_g\tau^g_{s_\delta}\beta_\delta\left(\left(m_t\tau^t_{s_z}\right)^2 + \frac{\tau_z}{\tau_n}\right) = 0. \tag{57}$$

This equation has at least one real root because it is cubic, and the real root(s) must be positive since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i\left(\frac{1}{\tau_n}\right)^i,$$

where $d_0 = \beta_z^4 m_g^2 m_t^3\left(\tau^t_{s_z}\right)^3\left(\tau^g_{s_\delta}\right)^2\left(m_t\tau^t_{s_z}\beta_\delta^2 - 4\beta_z m_g\tau^g_{s_\delta}\right)$. For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (57) has three distinct real roots if

$$\frac{4\beta_z m_g\tau^g_{s_\delta}}{\beta_\delta^2 m_t\tau^t_{s_z}} < 1.$$

Case 2.2: $\tau^t_{s_\delta} = 0$ **and** $\tau^g_{s_z} > 0$.

Notice that $\xi_\delta\beta_\delta\tau^g_{s_z} + \xi_z\beta_z\tau^g_{s_\delta}$ is constant, so $\xi_z(\xi_\delta)$ is a linear function. Plugging it back to (53), we obtain the following equation for ξ_δ:

$$f(\xi_\delta) = \sum_{i=0}^{3} a_i\xi_\delta^i = 0, \tag{58}$$

where $a_1, a_3 > 0$ and $a_0, a_2 < 0$. This equation has a real root because it is cubic. It has only positive real roots since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$D = \sum_{i=0}^{3} d_i \left(\frac{1}{\tau_n}\right)^i,$$

where

$$d_0 = \frac{m_g^2}{\beta_z^2}\left(m_g \tau_{s_z}^g + \beta_z m_t \tau_{s_z}^t\right)^2 \left(\tau_{s_\delta}^g \beta_z^2 + \tau_{s_z}^g \beta_\delta^2\right)^2 \left(\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2 - 4\tau_{s_z}^g \tau_{s_\delta}^g m_g^2 - 4\beta_z m_g m_t \tau_{s_\delta}^g \tau_{s_z}^t\right).$$

For a sufficiently large τ_n, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large τ_n, (58) has three distinct real roots if

$$\frac{4\tau_{s_z}^g \tau_{s_\delta}^g m_g^2 + 4\beta_z m_g m_t \tau_{s_\delta}^g \tau_{s_z}^t}{\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2} = \frac{4 m_g \tau_{s_\delta}^g \left(\tau_{s_z}^g m_g + \beta_z m_t \tau_{s_z}^t\right)}{\beta_\delta^2 m_t^2 \left(\tau_{s_z}^t\right)^2} < 1.$$

Case 2.3: $\tau_{s_\delta}^t > 0$ and $\tau_{s_z}^g = 0$.

The price coefficient ξ_z is $m_t \tau_{s_z}^t$. ξ_δ solves

$$f(\xi_\delta) = \sum_{i=1}^{5} a_i \xi_\delta^i = 0. \tag{59}$$

Moreover, $a_5 = \beta_z^2$ and $a_0 = a_{0,1}\frac{1}{\tau_n} + a_{0,2}\left(\frac{1}{\tau_n}\right)^2$, where $a_{0,1}, a_{0,2} < 0$ do not depend on τ_n. Then there exists at least one positive real root. Let's now show that there exists at least three positive real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.

We can write

$$a_0 = a_{0,1}\frac{1}{\tau_n} + a_{0,2}\left(\frac{1}{\tau_n}\right)^2,$$
$$a_1 = a_{1,0} + a_{1,1}\frac{1}{\tau_n} + a_{1,2}\left(\frac{1}{\tau_n}\right)^2,$$

where $a_{i,j} \neq 0$ are coefficients that do not depend on τ_n. Moreover, $a_{0,1} < 0$ and $a_{1,0} > 0$.

Then, evaluating $f(\cdot)$ at $-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n}+c_1\frac{1}{\tau_n}>0$ for some $c_1>0$, we obtain

$$f\left(-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n}+c_1\frac{1}{\tau_n}\right)=a_{1,0}c_1\frac{1}{\tau_n}+o\left(\frac{1}{\tau_n}\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n}+c_1\frac{1}{\tau_n}\right)>0$.

Next, we can write (59) also as a polynomial of $\frac{1}{\tau_n}$:

$$f\left(\xi_\delta\right)=\sum_{i=0}^{2}b_i(\xi_\delta)\left(\frac{1}{\tau_n}\right)^i,$$

where

$$b_0(\xi_\delta)=\xi_z\left(\xi_z^2+m_t^2\tau_{s_z}^t\tau_{s_\delta}^t\right)\left(\beta_z\xi_\delta-\beta_\delta m_t\tau_{s_z}^t\right)^2+m_tm_g\tau_{s_\delta}^g\tau_{s_z}^t\xi_\delta^2\left(\beta_z\xi_\delta-\beta_\delta m_t\tau_{s_z}^t\right).$$

Then, evaluating $f\left(\cdot\right)$ at $\frac{1}{\beta_z}\left(\beta_\delta m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}\right)$, we obtain

$$f\left(\frac{1}{\beta_z}\left(\beta_\delta m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}\right)\right)=-m_tm_g\tau_{s_\delta}^g\tau_{s_z}^t\left(\frac{1}{\beta_z}\beta_\delta m_t\tau_{s_z}^t\right)^2\left(\frac{1}{\tau_n}\right)^{1/2}+o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

Therefore, for a sufficiently large τ_n, $f\left(\frac{1}{\beta_z}\left(\beta_\delta m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}\right)\right)<0$ and, at the same time, $\frac{1}{\beta_z}\left(\beta_\delta m_t\tau_{s_z}^t-\left(\frac{1}{\tau_n}\right)^{1/2}\right)>-\frac{a_{0,1}}{a_{1,0}}\frac{1}{\tau_n}+c_1\frac{1}{\tau_n}>0$. Furthermore, because $a_5>0$, for any $\tau_n>0$ $f\left(\xi_\delta\right)>0$ if ξ_δ is sufficiently large. Hence, we have shown that (59) has at least three (positive real) solutions for ξ_δ if τ_n is sufficiently large.

Case 2.4: $\tau_{s_\delta}^t,\tau_{s_z}^g>0$.

Notice that $\xi_z\beta_z\tau_{s_\delta}^g+\xi_\delta\beta_\delta\tau_{s_z}^g$ is linear in ξ_z, so we can straightforwardly write $\xi_z=\xi_z(\xi_\delta)$. Plugging it back to (53), we obtain the following equation for ξ_δ:

$$f(\xi_\delta)=\sum_{i=1}^{9}a_i\xi_\delta^i=0. \tag{60}$$

Moreover, $a_9>0$ and $a_0<0$. Then there exists at least one positive real root. Let's now show that there exists at least three real roots for a sufficiently large τ_n. Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.

We can write

$$a_0 = a_{0,4}\left(\frac{1}{\tau_n}\right)^3 + a_{0,5}\left(\frac{1}{\tau_n}\right)^4,$$
$$a_1 = a_{1,3}\left(\frac{1}{\tau_n}\right)^2 + a_{1,4}\left(\frac{1}{\tau_n}\right)^3 + a_{1,5}\left(\frac{1}{\tau_n}\right)^4,$$
$$a_2 = a_{2,3}\left(\frac{1}{\tau_n}\right)^2 + a_{2,4}\left(\frac{1}{\tau_n}\right)^3,$$
$$a_3 = a_{3,2}\left(\frac{1}{\tau_n}\right) + a_{3,3}\left(\frac{1}{\tau_n}\right)^2 + a_{3,4}\left(\frac{1}{\tau_n}\right)^3,$$
$$a_4 = a_{4,2}\left(\frac{1}{\tau_n}\right) + a_{4,3}\left(\frac{1}{\tau_n}\right)^2,$$

where $a_{i,j} \neq 0$ are coefficients that do not depend on τ_n. Moreover, $a_{0,4} < 0$ and $a_{1,3} > 0$. Then, evaluating $f(\cdot)$ at $-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f\left(-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) = a_{1,3}c_1\left(\frac{1}{\tau_n}\right)^3 + o\left(\left(\frac{1}{\tau_n}\right)^3\right).$$

For a sufficiently large τ_n, $f\left(-\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}\right) > 0$.

Next, we can write (60) also as a polynomial of $\frac{1}{\tau_n}$:

$$f(\xi_\delta) = \sum_{i=0}^{4} b_i(\xi_\delta)\left(\frac{1}{\tau_n}\right)^i,$$

where $b_0(\xi_\delta)$ has a root at $\xi_\delta = \check{\xi}_\delta = \frac{\beta_\delta\left[m_g\tau^g_{s_z}\tau^g_{s_\delta} + \beta_z m_t\left(\tau^t_{s_z}\tau^g_{s_\delta} - \tau^g_{s_z}\tau^t_{s_\delta}\right)\right]}{\left(\beta_z^2\tau^g_{s_\delta} + \beta_\delta^2\tau^g_{s_z}\right)}$. Note that under our benchmark assumptions, $\tau^t_{s_z}\tau^g_{s_\delta} - \tau^g_{s_z}\tau^t_{s_\delta} = 0$ and $\check{\xi}_\delta > 0$. Moreover, $\check{\xi}_\delta > 0$ as long as traditional (green) investors are relatively better informed about $\tilde{z}$-component ($\tilde{\delta}$-component). Therefore, we consider $\check{\xi}_\delta \geq 0$ as a more empirically relevant case. However, for the sake of completeness, we also study the case $\check{\xi}_\delta < 0$ separately.

Case 2.4.1: $\check{\xi}_\delta > 0$. Evaluate $b_0(\cdot)$ at $\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}$ to obtain

$$b_0\left(\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}\right) = -c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_2 is a positive coefficient which does not depend on τ_n. Then, evaluating $f(\cdot)$ at the same point, we obtain

$$f\left(\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}\right) = -c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is negative and, at the same time, $\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2} > -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n} > 0$. Furthermore, because $a_9 > 0$, for any $\tau_n > 0$ $f(\xi_\delta) > 0$ if ξ_δ is sufficiently large. Hence, we have shown that (60) has at least three (positive real) solutions for ξ_δ if τ_n is sufficiently large.

Case 2.4.2: $\check{\xi}_\delta < 0$. Evaluate $b_0(\cdot)$ at $\check{\xi}_\delta - \left(\frac{1}{\tau_n}\right)^{1/2}$ to obtain

$$b_0\left(\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_2 is the same positive coefficient as in Case 2.4.1. Then, evaluating $f(\cdot)$ at the same point, we obtain

$$f\left(\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_2\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is positive and, at the same time, $\check{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2} < 0 < -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}$. Furthermore, because $a_9 > 0$, for any $\tau_n > 0$ $f(\xi_\delta) < 0$ if ξ_δ is sufficiently large in absolute terms and negative. Hence, we have shown that (60) has at least three real solutions for ξ_δ if τ_n is sufficiently large (recall that $f(0) = a_0 < 0$).

Case 2.4.3: $\check{\xi}_\delta = 0$.

In this case, $b_0(\cdot)$ can be written as

$$b_0(\xi_\delta) \overset{\check{\xi}_\delta = 0}{=} A\xi_\delta^6 \sum_{i=0}^{3} b_{0,i}\xi_\delta^i,$$

where $A > 0, b_{0,3} > 0, b_{0,2} < 0, b_{0,1} > 0, b_{0,0} > 0$. Then there exists $\hat{\xi}_\delta < 0$ that solves

$b_0(\xi_\delta) = 0$ such that

$$b_0\left(\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_3\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right),$$

where c_3 is a positive constant. Moreover, at this point

$$f\left(\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_3\left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large τ_n, the above expression is positive and, at the same time, $\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2} < 0 < -\frac{a_{0,4}}{a_{1,3}}\frac{1}{\tau_n} + c_1\frac{1}{\tau_n}$. Furthermore, because $a_9 > 0$, for any $\tau_n > 0$ $f(\xi_\delta) < 0$ if ξ_δ is sufficiently large in absolute terms and negative. Hence, we have shown that (60) has at least three real solutions for ξ_δ if τ_n is sufficiently large (recall that $f(0) = a_0 < 0$).

□

F Investors with homogeneous preferences

The key assumption we make throughout the paper is that there are two groups of investors with heterogeneous stock valuations. Because of the preference heterogeneity, they use information about the same fundamentals differently and trade in the opposite directions, which might give rise to multiple equilibria that differ in the relative price informativeness about the two fundamentals. We show the robustness of this result to general assumptions on the information structure in Appendix E.

The goal of this appendix is to show that the preference heterogeneity is an essential ingredient for equilibrium multiplicity. In particular, we explore a model that features two groups of investors that have homogeneous preferences but might have different information about the two fundamentals. The key difference between our setting and Goldstein and Yang (2015) is that we allow investors of both groups to receive informative signals about both fundamentals. As we discuss in Appendix E, this is crucial to support multiple equilibria in the trading stage when investors' preferences are heterogeneous. Our key result here is that equilibrium in the trading stage is unique when preferences are homogeneous.

We consider the same framework as described in Section 3 with several differences. First, we assume that both groups of investors have the same stock valuation, $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. For consistency, we keep denoting the two groups using t and g subscripts. The masses of the two groups are m_t and m_g. Without loss of generality, we set the utility weights $\beta_z = \beta_\delta = 1$ and the risk aversion parameter $\gamma = 1$. Further, we assume that t-investors (g-investors) specialize in particular types of information and, thus, receive signals about $\tilde{z}$ and $\tilde{\delta}$ with precisions of τ_s ($\lambda\tau_s$) and $\lambda\tau_s$ (τ_s), respectively. Without loss of generality, we assume $\lambda \in [0, 1]$. The priors for $\tilde{z}$ and $\tilde{\delta}$ are assumed to be the same, $\tau_z = \tau_\delta = \tau$.[27]

Market clearing implies the following system of equations for ξ_z and ξ_δ:

$$\xi_z = \tau_s \left[m_t \frac{\xi_\delta^2 + \frac{\tau + \lambda\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\lambda)) \frac{1}{\tau_n}} + m_g \lambda \frac{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\lambda)) \frac{1}{\tau_n}} \right],$$

$$\xi_\delta = \tau_s \left[m_t \lambda \frac{\xi_z^2 + \frac{\tau + \tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\lambda)) \frac{1}{\tau_n}} + m_g \frac{\xi_z^2 + \frac{\tau + \lambda\tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1+\lambda)) \frac{1}{\tau_n}} \right].$$

Denote $x \equiv \xi_\delta - \xi_z$. It is easy to see that ξ_z and ξ_δ are uniquely pinned down for a given x. Furthermore, the system can be simplified to the following quintic equation for x:

$$f(x)g(x) = (1-\lambda)^2 (1+\lambda) \tau_n \tau_s^3 m_g m_t x, \tag{61}$$

where

$$f(x) = x \left(x^2 \tau_n + 2\tau + \tau_s (1+\lambda) \right) + x \tau_n \tau_s^2 \left(\lambda \left(m_g^2 + m_t^2 \right) + m_g m_t \left(1 + \lambda^2 \right) \right) - \tau \tau_s (m_g - m_t)(1-\lambda),$$

$$g(x) = x^2 \tau_n + 2\tau + \tau_s (1+\lambda).$$

Clearly, (61) has a unique solution $x = 0$ when $\lambda = 1$. Suppose now that $\lambda < 1$ and $m_g > m_t$ (case of $m_g \leq m_t$ can be considered analogously). Our goal is to show that (61) has a unique solution.

We first show that there exists a unique solution to (61) on $x \geq 0$. Since $f(0) < 0$ and $f(x)$ is an increasing and convex function, there exists a unique $\underline{x} > 0$ such that $f(\underline{x}) = 0$ and that $f(x) > 0$ if and only if $x > \underline{x}$. Moreover, $f(x)g(x)$ is an increasing convex

[27] These assumptions on the information structure can be further relaxed (at the expense of tractability but without changing the final result) by allowing for different prior precisions and more general signal precisions. The analyses are available upon request.

function on $x \geq \underline{x}$. Therefore, there exists exactly one solution to (61) on $x \geq 0$.

We then verify that there is no solution on $x < 0$. First, $f(0)g(0) < 0$. Second, $f(x)g(x)$ is increasing and concave on $x < 0$. Finally, the derivative of $f(x)g(x)$ at 0 is $f'(0)g(0) + f(0)g'(0) > (1+\lambda^2)(1+\lambda)\tau_n\tau_s^3 m_g m_t > (1-\lambda)^2(1+\lambda)\tau_n\tau_s^3 m_g m_t$. So the right-hand side of (61) is always above the left-hand side on $x < 0$. We, therefore, have established the following proposition.

Proposition 11. *If investors have homogeneous preferences, there exists a unique equilibrium with a linear price.*

We conclude that the equilibrium multiplicity in the trading game requires investors to have heterogeneous stock valuations. Otherwise, trading behaviors of investors are aligned and the price is always simultaneously informative to both investor groups.