On ESG Investing: Heterogeneous Preferences, Information, and Asset Prices

Itay Goldstein†, Alexandr Kopytov‡, Lin Shen§, and Haotian Xiang¶

May 1, 2024

Abstract

We study how ESG investing reshapes information aggregation by prices. We document that the information content of asset prices changes with ESG investing. We then develop a rational expectations equilibrium model in which traditional and green investors are informed about financial and ESG performances of a firm but have different preferences about them. Two investor groups trade in opposite directions based on the same information, resulting in a potential multiplicity of equilibrium price. The growth of green investors and an improvement in ESG information quality can reduce price informativeness about a firm’s financial performance and raise its cost of capital.

JEL: G12, G14


†University of Pennsylvania and NBER: itayg@wharton.upenn.edu
‡University of Rochester: akopytov@simon.rochester.edu
§INSEAD: lin.shen@insead.edu
¶Peking University: xiang@gsm.pku.edu.cn
1 Introduction

One of the most prominent phenomena in the financial industry in recent years has been investors’ growing interest in environmental, social and governance (ESG) issues. In the United States alone, about $8.4 trillion of investment funds’ assets were invested with ESG considerations in 2022, an order of magnitude increase from a decade ago (GSIA, 2023). As many investors now integrate ESG principles into their investment strategies, they evaluate their portfolios in a different way from traditional investors. The rise of ESG investing, therefore, introduces substantial investor heterogeneity, which may fundamentally change the workings of financial markets.

A key function of financial markets is to produce and aggregate information about the fundamentals of the traded assets (Hayek, 1945). This price informativeness, in turn, is important for welfare as it affects resource allocation in the real economy through its effect on the cost of capital or through active learning by decision makers. Traditionally, investors in financial markets were thought to be uniform, at least in their general objectives, as they were all focused on firms’ cash flows (and the risks involved with these cash flows). Hence, this is what information in financial markets was expected to be about. However, now that a new class of investors, namely ESG investors, emerged and is becoming increasingly important across a wide range of asset classes and geographies, this classic paradigm faces unprecedented challenges.

Several questions come up. First, in the presence of ESG investors, it is less clear what prices will be informative about and to what extent. Second, the changes in price informativeness may affect the firms’ cost of capital and, hence, resource allocation and aggregate welfare. Third, given the general lack of transparency about firms’ ESG performances, some may hope that the market can help in generating such information, but it is not clear whether this might conflict with the role of generating information about cash flows. In this paper, we provide a model to address these questions.

Our model is in the tradition of the noisy rational expectations equilibrium (REE) models a la Hellwig (1980), where rational investors observe and trade on heterogeneous signals about firm output. They also learn from the asset price and incorporate the information they learn in their trading. Finally, the market also has traders who trade for exogenous reasons and bring noise to the price. We amend this model in two key ways. First, we assume that firm output consists of two risky components: a financial cash flow and an ESG component. Second, we assume that the financial market is populated with two groups of risk-averse rational investors who receive heterogeneous informative
signals about both components but have different preferences about them. Specifically, traditional investors value only the financial component, and ESG (or “green”) investors value both financial and ESG components. In this environment, we analyze how the price is formed and what it is informative about.

Green investors in our model can be directly interpreted as investors who derive a non-pecuniary warm-glow utility from holding assets with high ESG performances. Investors’ preferences for ESG can also stem from conventional pecuniary reasons. In particular, green investors can be viewed as managers of ESG funds who do not have non-pecuniary motives but whose compensation and reputation hinge on identifying good ESG assets. Alternatively, traditional and green investors can both value only the monetary payoff but disagree about the relevance of ESG for cash flows. Specifically, green investors believe that a firm’s good ESG performance will translate into a good financial performance. As such, they view ESG and financial outputs as two components of the firm’s cash flows. Traditional investors disagree and view ESG output as unrelated to the firm’s cash flows. Under each interpretation, traditional and green investors have different trading motives, which is key to our results. The exact interpretation is inconsequential, and throughout the paper, we use the first one when describing our model and results.

Before analyzing the model, we empirically explore whether the information content of asset prices changes with ESG investing, which is a key feature of our model. Specifically, we focus on the environmental component of ESG and proxy for it using firms’ carbon emissions. Using the methodology of Bai, Philippon, and Savov (2016), we find that, controlling for firms’ current and future financial performances, high stock prices today are associated with lower future growth in emissions for firms held more by active ESG mutual funds. This result suggests that ESG investing can indeed change the information content of asset prices.

Let us now discuss our model’s mechanisms and results. The driving force of the model is the strategic interaction between traditional and green investors through learning and trading. Because of their heterogeneous preferences, traditional and green investors seek to learn different information from the price. They also end up trading differently on similar signals. In particular, when receiving positive signals about the firm’s ESG output, green investors increase their demand for the stock, while traditional investors reduce their demand. The latter is an equilibrium response, since a positive signal on ESG, for a given

\footnote{According to recent surveys (Capital Group, 2022; Haber, Kepler, Larcker, Seru, and Tayan, 2022), the E component is the most important out of the E, S and G components for investors across various demographics and geographies. Furthermore, carbon emissions can be precisely measured and are first-order for typical E metrics (e.g., MSCI, 2024).}
price, leads them to infer a worse realization of the financial output. As a result, trades by one investor group make the price more informative about what investors from this group value and less informative about what investors from the other group value. In this way, trades by traditional and green investors contaminate price informativeness to each other. Key to this is the idea that investors with heterogeneous preferences trade the same security differently because they value different payoff components of the same security.

Based on these forces, we identify a feedback loop between investors’ trading intensities and the amount of information contained in the price about the two output components. This leads to multiple equilibria in the trading game, such that the price can end up being dominated by one component in one equilibrium or the other component in another equilibrium. Specifically, if investors of one group trade more intensively on their private signals than investors of the other group, the preferences of the dominating group are reflected by the price more. As a result, the price becomes more informative to them and so they face less uncertainty when holding the stock. This justifies their more intensive trading. The feedback loop implies that two equilibria can coexist. In one equilibrium, the stock is predominately traded by traditional investors, the equilibrium price primarily loads on the financial output and is not particularly informative to green investors. In the other equilibrium, green investors dominate the trading, and the equilibrium price is more aligned with their preferences.

We characterize how the emergence of multiple equilibria depends on key parameters of the model. First, multiplicity arises when noise traders’ demand is not too volatile. Otherwise, the price will be a poor signal about both financial and ESG outputs and thus uninformative to all rational investors, preventing the above feedback loop from developing. Second, multiplicity requires that preference heterogeneity is sufficiently strong. If this is not the case, traditional and green investors seek to learn similar information from the price. Third, it is important that traditional and green investors receive informative signals about both output components. Otherwise, they will not be able to trade against the signal about the component they do not value, and, thus, they will not prevent the price from being informative about it. Finally, for multiplicity to arise, the masses of traditional and green investors should not be too different from each other. If the investor base is strongly tilted toward one investor type, investors of this type always dominate the trading.

This last property highlights an important implication of our model for how the current transition in financial markets could impact the equilibrium. The increased presence
of ESG investors in the market implies that we are shifting from a world where the only equilibrium is the one where the cash flow component dominates the market to a world where both equilibria are possible. Hence, there could be a sudden shift for some stocks from a cash-flow-dominated price to an ESG-dominated price, and so a sudden shift in what kind of information investors can glean from the price. Aside from jumps across equilibria, we also study the impact of the increase in the share of green investors in the market for a given equilibrium. We show that, as the green investor share increases, the price becomes less informative to traditional investors, that is, less informative about the financial component, and more informative to green investors in any stable equilibrium.

This result has important implications for the firm’s cost of capital. As is standard in REE models (e.g., Easley and O’Hara, 2004), the cost of capital reflects the average information risk faced by rational investors. We find that the cost of capital is non-monotone in the share of green investors and is high when the masses of traditional and green investors are similar. When the investor base is balanced, the price is not particularly informative to any investor group, and so both require a high compensation for bearing the information risk, which drives up the cost of capital. With an unbalanced investor base, the dominant group finds the price informative and this drives down the cost of capital. As we think about the current transition in financial markets, this implies that the information channel can lead to an increase in the cost of capital when the market becomes less dominated by traditional investors.

Another important development in ESG investing in recent years is the regulatory push worldwide for improving the quality of ESG information (see, e.g., van der Lugt, van de Wijs, and Petrovics (2020) for a review of worldwide sustainability reporting policies). We extend our model to examine the implications of such regulatory changes. Holding the pricing function fixed, better ESG information benefits traditional investors as it helps them to interpret the price more accurately and learn more about the financial component from it. At the same time, green investors who value the ESG output directly benefit from better ESG information more. In particular, they respond by substantially increasing their trading intensities. Changes in their trading intensities affect the equilibrium pricing function. We show that if the preference heterogeneity across traditional and green investors is sufficiently strong, the price becomes less associated with the financial output and, thus, less informative to traditional investors. Furthermore, we show

\footnote{Implications of ESG disclosure in settings different from ours have been studied by Friedman, Heinle, and Luneva (2021), Aghamolla and An (2023), Chen and Schneemeier (2023), Gupta and Starmans (2023), Smith (2023), and Xue (2023).}
that the decrease in the price informativeness to traditional investors can dominate the
decrease in the price informativeness to green investors, leading to an increase in the cost of capital. This is an unintended consequence of the improved quality of ESG information that should be considered by policymakers.

The essential feature of the model is that the stock payoff to investors consists of
two distinct components. In the baseline analysis, we assume that the financial and ESG components are uncorrelated. We consider this as a natural benchmark as in reality there exist forces that push the correlation in either positive or negative directions. On the one hand, a firm with a good ESG performance can attract socially concerned customers and is likely to be resilient against regulations such as carbon taxes. On the other hand, improving the ESG performance may be financially costly. We consider an extension in which the two payoff components can be correlated. We show that introducing a positive/negative correlation is equivalent to making investors’ preferences more/less aligned. In particular, as long as the financial and ESG outputs are not perfectly correlated, our results are preserved.

To summarize, our model considers interactions of two groups of investors with heterogeneous preferences but similar information sets in the financial market. We uncover a set of novel implications in this framework. We relate our framework to ESG investing. Due to its massive growth in recent years, it gives rise to unprecedented investor heterogeneity across multiple asset classes and geographies. There are, however, other applications that our model could be suitable for. For example, funds pursuing different strategies might care about different components of stock payoffs to fulfill different investment needs. As another example, investors with different investment horizons assign different weights to short-term payouts and long-term values (Bushee, 2001) which might be driven by distinct shocks. Similarly, investors might have heterogeneous preferences about dividends and capital gains (Graham and Kumar, 2006; Harris, Hartzmark, and Solomon, 2015). Even in the universe of ESG investors, preference heterogeneity might matter as some investors focus more on the environmental aspects while others focus more on the social or governance aspects.3

Literature There is recent theoretical literature that investigates the impact of ESG investing on asset prices, including Heinkel, Kraus, and Zechner (2001), Fama and French (2007), Luo and Balvers (2017), Baker, Bergstresser, Serafeim, and Wurgler (2018), Pastor, Stambaugh, and Taylor (2021), Pedersen, Fitzgibbons, and Pomorski (2021), Baker, Hollifield, and Osambela (2022), and Zerbib (2022). Like us, they start from the premise

3Appendix G explores an extension in which green investors have heterogeneous preferences.
that some investors derive utility from investing in assets with good ESG performances. Unlike these papers, we investigate how the information about this performance gets into the price through investors’ trading, and how informative the price ends up being about the ESG and financial output components. Uncertainty about investors’ ESG payoff is also featured in Friedman and Heinle (2016) and Avramov, Cheng, Lioui, and Tarelli (2022), but there is no investigation of trading based on private information and the resulting price informativeness in these papers.

Another important question in the emerging ESG literature is about the impact of ESG investors on firms’ production decisions. A natural way to achieve impact is through engagement by activist green investors, as in papers by Gollier and Pouget (2014), Chowdhry, Davies, and Waters (2019), Landier and Lovo (2020), Oehmke and Opp (2020), Green and Roth (2021), and Gupta, Kopytov, and Starmans (2021). In papers like ours, where investors are atomistic, such effects are not present. Instead, investors’ decisions in financial markets affect firms’ cost of capital, which may indirectly affect their production. Heinkel et al. (2001) show that firms excluded by green investors suffer a reduction in risk sharing in their investor base and thus have a higher cost of capital. The cost of capital channel is also at work in asset pricing models that are discussed in the previous paragraph. Hart and Zingales (2017), Broccardo, Hart, and Zingales (2022) and Huang and Kopytov (2023) study engagement and exclusion in a unified model. Our model reveals a novel effect of ESG investing on the cost of capital through the information channel. In particular, we show that the growth of green investors can lead to an increase in the firm’s cost of capital.4

Methodologically, our model contributes to the noisy REE literature, pioneered by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). To the best of our knowledge, our model is the first to combine the following two features. First, the market is populated by investors with heterogeneous preferences over multiple fundamentals. Second, investors are not restricted to being informed only about the fundamentals they value. In our view, this combination is particularly relevant to describe financial markets as they start to transition to a new ESG reality. As we describe above, there are other possible applications for this setting.

A few papers analyze models with multiple fundamentals under homogeneous investor preferences. Goldstein and Yang (2015) build a model in which asset payoff is affected

4In Pedersen et al. (2021), the presence of ESG-unaware investors can boost expected returns of green stocks. In our model, all rational investors are aware of ESG and financial risks, and the cost of capital increase is due to the information channel, which is specific to our paper.
by two fundamentals while investors receive heterogeneous information about the fundamentals. Cespa and Foucault (2014) construct a two-asset economy to study cross-asset learning and liquidity spillovers. Ganguli and Yang (2009) and Manzano and Vives (2011) consider settings in which investors possess information about asset payoff and aggregate supply shock (see also Amador and Weill, 2010 and Davila and Parlatore, 2021). In the paper by Brunnermeier, Sockin, and Xiong (2021), investors can choose to learn about the fundamental or government action. Unlike these papers, our paper features preference heterogeneity, and this generates a new implication: investors with different preferences use the same information to trade in opposite directions, thus making the price noisier to each other. This is the key force behind our results about different information regimes in the market and the comparative statics for price informativeness and the cost of capital.

Several papers introduce heterogeneous valuations in the REE framework. Vives (2011 and 2014), Vanwalleghem (2017), Rahi and Zigrand (2018), Rahi (2021), and Glebkin and Kuong (2023) study models in which agents have private valuations but, different from our paper, receive only information about their private valuations. In our model, the financial and ESG components are firm-specific, and, thus, all investors can learn about them. The fact that investors receive signals about both factors in our model is critical for the key force that they end up trading in opposite directions on similar information, which gives rise to multiple equilibria with distinct pricing functions. While the paper by Glebkin and Kuong (2023) also features equilibrium multiplicity, it results from the heterogeneity in market power among investors. In the other referenced papers, equilibrium is unique in the trading stage.

As mentioned above, one interpretation of the investors’ heterogeneous preferences is their disagreement on whether ESG matters for firms’ cash flows. This interpretation connects our paper to the literature on differences of opinion, where investors agree to disagree on the valuation of an asset and trade based on their subjective beliefs. The existing literature has focused mainly on investors’ disagreement on the distribution of the same asset fundamental and the resulting effect on asset prices (Harrison and Kreps, 1978, Scheinkman and Xiong, 2003) and trading volume (Harris and Raviv, 1993, Kandel and Pearson, 1995). Different from these papers, in our model, traditional and green investors disagree on the importance (weights) of two fundamental factors, financial and ESG.

Footnote: From this perspective, our paper is related to Goldstein, Li, and Yang (2014), where investors’ objectives might be different due to different investment opportunities, and Cespa and Vives (2023), where investors’ objectives might be different due to different endowments of a non-tradable asset.
ESG, while they agree on the distribution of prior and private signals. This novel feature leads to new insights into how disagreement affects the information content of asset prices.

The paper proceeds as follows. Section 2 presents motivating evidence for our theory. Section 3 lays out the model and Section 4 characterizes equilibria. Section 5 studies the growth of green investors. Section 6 analyzes two extensions of the baseline model. Section 7 concludes and discusses the implications of our paper for future research. Appendix contains all proofs missing from the main text.

2 Motivating evidence

To motivate our theory, in this section, we briefly review existing evidence and provide new evidence linking the information content of asset prices to ESG investing.

Existing empirical evidence confirms that asset prices are affected by ESG news (e.g., Flammer, 2013; Krüger, 2015; Capelle-Blancard and Petit, 2019). At the same time, asset prices respond less to changes in firms’ financial performance if held by more ESG investors. In particular, Cao, Titman, Zhan, and Zhang (2022) document that prices of stocks held more by responsible institutions respond less to mispricing signals. Glebkin and Kuong (2023) find that stocks with higher ESG ratings that are held predominantly by ESG-concerned investors have smaller pre-earnings drifts and turnovers, consistent with ESG investors trading less on their information about future earnings announcements.

Adding to existing evidence, the remainder of this section shows that current prices can be a useful source of information about firms’ future ESG performances. This evidence directly relates to the central mechanism of our theory. Specifically, we focus on the environmental component of ESG. Arguably, among the E, S, and G components, the E component can be measured most reliably. Therefore, investors are more likely to trade actively on their information on it, thereby making the price more informative about it. Furthermore, recent surveys (Capital Group, 2022; Haber et al., 2022) find the environmental component is the most important for investors across various demographics and geographies. We proxy for firms’ environmental performances using their carbon emissions which have a first-order effect on typical E metrics (e.g., MSCI, 2024). We find that higher stock prices today imply lower future growth in emissions for firms held more by active ESG mutual funds.

Specifically, in line with Bai et al. (2016) and Kacperczyk, Sundaresan, and Wang
(2021), we run the following forecasting panel regression:

\[
\frac{C_{i,t+1}}{A_{i,t}} = \beta_0 + \beta_1 H_{i,t} \times \log \frac{M_{i,t}}{A_{i,t}} + \beta_2 H_{i,t} + \beta_3 \log \frac{M_{i,t}}{A_{i,t}} + \beta_4 \frac{C_{i,t}}{A_{i,t}} + Controls_{i,t} + Industry_i + Year_t + \epsilon_{i,t},
\]

where \( C_{i,t}, M_{i,t} \) and \( A_{i,t} \) are, respectively, firm \( i \)'s carbon emissions (Scope 1+2), market capitalization, and assets in year \( t \). Our sample spans between 2010 and 2021, and to improve the reliability of emission information, we drop firms for which more than 25% of emission data are missing.\(^6\) We forward market capitalization \( M_{i,t} \) by one quarter and adjust nominal values for inflation. \( H_{i,t} \) is a measure of active ESG fund holdings from Li, Ruan, Titman, and Xiang (2022), who identify active equity mutual funds that disclose the adoption of ESG investment principles in their SEC filings.\(^7\) We use the following two measures: the fraction of firm \( i \)'s outstanding shares held by ESG funds (\( \text{ESG}_{\text{ownership}} \)) and log of 1 plus the number of ESG funds holding stock \( i \) (\( \text{ESG}_{\#\text{funds}} \)). Control variables include ratio of EBITDA to assets, log market capitalization, market leverage, CAPM \( \beta \), logarithm of property, plant and equipment, ratio of capital expenditures to assets, growth rate of sales, volatility of monthly stock returns over the past 12 months, and the Amihud’s (2002) ratio. We follow Bolton and Kacperczyk (2021) to control for industry and year fixed effects.

Table 1 reports our results. We find that stock prices are positively associated with future emissions, i.e., \( \beta_3 \) in specification (1) is positive. This is not surprising because financial performance and emissions are positively correlated in the data, that is, firms that sell more emit more. More importantly, the coefficient of interest, \( \beta_1 \), is significantly negative. This implies that for firms held more by active ESG funds, an increase in current asset prices implies lower future growth in emissions. In Appendix D.2.3, we further show that \( \beta_1 \) remains significantly negative after controlling for future financial performances in specification (1). This implies that prices can provide useful information about future emissions that is unrelated to future financial performance.

Our results are complemented by the findings of a recent paper by Yang, Zhan, Zhang, and Zhang (2023). They test our theory with a different measure of price informativeness, that is, the future earnings response coefficient (Lundholm and Myers, 2002). Consis-

---

\(^6\)Details about our data sources and variable construction are in Appendix D.1. Appendix D.2.1 shows that our results are robust to using log market-to-book ratio instead of log market-to-asset ratio. Appendix D.2.2 presents a robustness analysis regarding sample selection.

\(^7\)In Appendix D.4, we show that ESG funds in our sample are averse to holding firms with high emissions.
Table 1: ESG ownership and information content of asset prices. The table reports the estimates of specification (1). Columns (1) and (2) use ESGownership as independent variable; Columns (3) and (4) use ESGfunds as independent variable. Controls include ratio of EBITDA to assets, log market capitalization, market leverage, CAPM β, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.

<table>
<thead>
<tr>
<th></th>
<th>Independent variable: $C_{i,t+1}/A_{i,t}$</th>
<th>$H=\text{ESG}_{ownership}$</th>
<th>$H=\text{ESG}_{#funds}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(1)$</td>
<td>$(2)$</td>
<td>$(3)$</td>
</tr>
<tr>
<td>$H_{i,t} \times \log(M_{i,t}/A_{i,t})$</td>
<td>-0.052**</td>
<td>-0.054**</td>
<td>-0.056***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.027)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$H_{i,t}$</td>
<td>-0.019</td>
<td>-0.012</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$\log(M_{i,t}/A_{i,t})$</td>
<td>0.082**</td>
<td>0.152**</td>
<td>0.161***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.067)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>$C_{i,t}/A_{i,t}$</td>
<td>0.975***</td>
<td>0.974***</td>
<td>0.975***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Controls</th>
<th>N</th>
<th>Y</th>
<th>N</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry/Year FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.947</td>
<td>0.948</td>
<td>0.947</td>
<td>0.948</td>
</tr>
<tr>
<td>Obs</td>
<td>7,182</td>
<td>7,033</td>
<td>7,182</td>
<td>7,033</td>
</tr>
</tbody>
</table>

In line 8, it is important to emphasize that we do not aim to establish a causal effect of ESG fund holdings. It can be that funds adjust their holdings in response to information about firms’ future emissions, thus making prices more informative. At the same time, it is also in the funds’ interest to utilize ESG information embedded in prices to make better investment decisions; thus, improved price informativeness may encourage their investment.\(^8\) Both features are present in our model in which trading decisions and the information content of asset prices are jointly determined.

\(^8\)Huij, Laurs, Stork, and Zwinkels (2021) argue that asset prices can be useful for assessing firms’ carbon risks.
3 Model

3.1 Setup

Two assets are traded in the financial market: a risk-free bond and a risky stock of a firm. The bond is in unlimited supply. It pays off one and its price is normalized to one. The stock is a claim on the firm’s output which consists of two risky components: a financial component $\hat{z}$ and an ESG component $\hat{\delta}$. The financial component can be interpreted as a cash flow generated by the firm. The ESG component can be interpreted as the firm’s contribution to social good, for example, a reduction in carbon emissions. The two payoff components are normal random variables, $\hat{z}, \hat{\delta} \sim N(0, \tau^{-1})$, which are uncorrelated in the baseline model. In Section 6.2, we consider an extension in which the two components can be correlated. The stock is in unit supply, and its price $\hat{p}$ is determined endogenously by market clearing.

There are two groups of rational investors with a combined mass of $m > 0$. Fraction $1 - \alpha \in [0, 1]$ of them are traditional investors who only value the firm’s financial output $\hat{z}$. Fraction $\alpha$ are investors who might value both the financial and ESG outputs, such that the stock payoff to them is $\beta_z \hat{z} + \beta_\delta \hat{\delta}$, where $\beta_z \geq 0$ and $\beta_\delta > 0$ are utility weights. We refer to these investors as green. We normalize $\beta_z^2 + \beta_\delta^2 = 1$ so that the ex-ante variances of the stock payoff are the same for traditional and green investors. Otherwise, a change in the investor composition $\alpha$ would reshape the overall risk attitude of the investor population and thus change the asset price in a mechanical way.

There are several interpretations of green investors’ ESG preferences. First, ESG preferences can stem from a non-monetary warm-glow utility from holding assets with good ESG performance. Investors’ preferences for ESG can also be driven by conventional pecuniary reasons. In particular, green investors can be viewed as managers of ESG funds whose compensation and reputation hinge on identifying good ESG assets. Alternatively, traditional and green investors might both value only monetary payoffs but disagree about the relevance of ESG for firms’ cash flows. Specifically, green investors believe that a firm’s good ESG performance will translate into a good financial performance. As such, they view ESG and financial outputs, $\hat{\delta}$ and $\hat{z}$, as two components of the firm’s cash flows. Traditional investors disagree and view ESG output as unrelated to the firm’s cash flows. Section 3.3 discusses existing empirical evidence on ESG preferences.

Both traditional and green investors have constant absolute risk aversion (CARA) utilities with the same risk aversion parameter $\gamma$. Specifically, if an investor of type
$j \in \{t, g\}$ has an initial wealth $W_0$ and chooses to hold $q$ shares, then her expected utility is

$$E\left\{ -\exp\left( -\gamma \left[ W_0 + q \left( \beta^t_z \tilde{z} + \beta^t_\delta \tilde{\delta} - \tilde{p} \right) \right] \right) \right\},$$

(2)

where $\beta^t_z = 1$, $\beta^t_\delta = 0$, $\beta^g_z = \beta_z$ and $\beta^g_\delta = \beta_\delta$.\(^9\) In addition to rational traders, there are noise traders whose stock demand is $\tilde{n} \sim N(0, \tau_n^{-1})$.

Utility function (2) implies that green investors are averse to risk in the stock’s ESG output. As discussed in Section 3.3, this is consistent with the fact that many real-life ESG investors are concerned about the uncertainty about firms’ ESG performances.

Rational investors trade based on information contained in the stock price and their private signals. Traditional and green investors receive signals about both financial and ESG fundamentals, namely, an investor $i$ observes $\tilde{s}_z^i = \tilde{z} + \tilde{\varepsilon}_z^i$ and $\tilde{s}_\delta^i = \tilde{\delta} + \tilde{\varepsilon}_\delta^i$, where signal noises are uncorrelated normal variables, $\tilde{\varepsilon}_z^i, \tilde{\varepsilon}_\delta^i \sim N(0, \tau_s^{-1})$. This assumption on the information structure differentiates our paper from existing works on rational expectation models featuring agents with heterogeneous private valuations of a risky asset (e.g., Vives, 2014; Rahi and Zigrand, 2018). In those works, investors receive informative signals only about their private asset valuations. In our model, $\tilde{z}$ and $\tilde{\delta}$ are firm-specific payoff components that all investors can learn about. For example, investors are likely to learn about both payoff components by reading analyst and investor reports that describe the firm’s performance and risks comprehensively.

In Appendix E, we consider a general information structure featuring different information precisions for different types of investors and different payoff components. Such a model is much less tractable. Nevertheless, we show that our key results hold as long as traditional and green investors receive informative signals about both payoff components, not necessarily of equal precisions.

\(^9\)That is, type-$j$ investors assign weights $\beta^t_z$ and $\beta^t_\delta$ to per-dollar financial and ESG stock returns, respectively. Alternatively, one can consider the specification in which an investor assigns the same weight $\beta^t_z$ to all financial payoffs. The characterization of the equilibrium pricing function remains very similar. In particular, the key equilibrium objects—normalized price coefficients $\xi_z$ and $\xi_\delta$ in the pricing function (5)—remain the same.
3.2 Market clearing

As is standard in a CARA-normal setup, the demand for the stock from an investor $i$ of type $j \in \{t, g\}$ is

$$d_{ij}(F_i) = \frac{E(\beta^j_z \bar{z} + \beta^j_\delta \bar{\delta}|F_i) - \bar{p}}{\gamma V(\beta^j_z \bar{z} + \beta^j_\delta \bar{\delta}|F_i)}, \quad (3)$$

where the information set $F_i = \{\bar{s}_i^z, \bar{s}_i^\delta, \bar{p}\}$ includes investor $i$’s private signals and publicly observable stock price. Aggregating the individual demands of rational investors and adding the demand from noise traders, we obtain the following market clearing condition:

$$D^t(\bar{z}, \bar{\delta}, \bar{p}) + D^g(\bar{z}, \bar{\delta}, \bar{p}) + \bar{n} = 1, \quad (4)$$

where $D^j(\bar{z}, \bar{\delta}, \bar{p}) = \int_{i \in T_j} d_{ij}(F_i) \, di$ is the total demand for the stock from investors of type $j$, and where $T_j$ denotes the set of investors of type $j \in \{t, g\}$.

We focus on rational expectation equilibria (REE) with linear prices, i.e.,

$$\bar{p} = p_0 + p_z \bar{z} + p_\delta \bar{\delta} + p_n \bar{n} = p_0 + p_n \left(\xi_z \bar{z} + \xi_\delta \bar{\delta} + \bar{n}\right), \quad (5)$$

where $\xi_z = \frac{p_z}{p_n}$ and $\xi_\delta = \frac{p_\delta}{p_n}$ are normalized price coefficients.

3.3 Empirical evidence on ESG preferences

Before analyzing the model, in this section, we briefly discuss existing evidence on the preferences and trading behaviors of ESG investors that is consistent with our modeling choices.

It is well documented that ESG investors hold more stocks with good ESG performances relative to their peers (e.g., Riedl and Smeets, 2017; Bolton and Kacperczyk, 2021; Avramov et al., 2022; Li et al., 2022).\footnote{The preference for ESG is also reflected by how investors adjust their portfolios in response to news (e.g., Bolton and Kacperczyk, 2021; Cao et al., 2022; Starks, Venkat, and Zhu, 2017).} Existing empirical evidence suggests that there may be several drivers of such behavior (see Starks (2023) for a discussion of various economic drivers of preferences for ESG). First, some sustainable investors exhibit warm-glow preferences and adjust their portfolios consistent with their ethical values (e.g., Bonnefon, Landier, Sastry, and Thesmar, 2022; Heeb, Kölbel, Paetzold, and Zeisberger, 2023). Second, ESG and traditional investors might disagree on whether a good ESG performance improves firms’ future financial performance (e.g., Starks et al., 2017;
Krueger, Sautner, and Starks, 2020). Giglio, Maggiori, Stroebel, Tan, Utkus, and Xu (2023) find evidence for both types of motivation for ESG investing among Vanguard’s clients. Furthermore, mutual fund managers’ compensation is tied to fund flows, and for ESG-concerned funds, flows depend on managers’ success in identifying assets with good ESG performances (e.g., Hartzmark and Sussman, 2019; Humphrey and Li, 2021). Therefore, even if ESG fund managers have purely pecuniary preferences, they have an incentive to invest in high ESG assets to generate a higher financial payoff.

Importantly, ESG investors are averse to ESG risks. Gibson, Krueger, and Schmidt (2021) find that stocks with high ESG rating disagreement earn excess returns. Avramov et al. (2022) document that conditional on the average ESG rating, ESG-concerned investors hold significantly fewer stocks with high ESG rating uncertainty. Experimental studies on charitable giving document that individuals donate less if the donation outcome is risky (Brock, Lange, and Ozbay, 2013; Exley, 2016). Naturally, ESG risks are also important for investors who value ESG for financial reasons due to a conventional aversion to risks in monetary payoffs. For example, for ESG fund managers, investing in a stock with an uncertain ESG output is risky as it might lead to an outflow and reduction in fees if this stock turns out to have a weak ESG performance.

4 Equilibrium characterization

In this section, we characterize equilibria of the model. In Section 4.1, we consider a special case in which we can provide closed-form solutions. This special case is useful for clearly seeing the economic forces at work. In Section 4.2, we consider our baseline model.

4.1 Special case

In the special case, we impose two simplifying assumptions. First, green investors value only the ESG output, i.e., $\beta_z = 0$ and $\beta_\delta = 1$. Second, the masses of green and traditional investors are equal, i.e., $\alpha = \frac{1}{2}$.

4.1.1 Trading intensities and feedback loop

Equilibrium price coefficients $\xi_z$ and $\xi_\delta$ are shaped by trades of rational investors based on their private signals about $\tilde{z}$ and $\tilde{\delta}$. Heterogeneity in preferences has important implications for how investors use their information to trade. Consider a traditional
investor. Denote her trading intensities with respect to her private signals $\tilde{s}_z$ and $\tilde{s}_\delta$ as $i^t_z$ and $i^t_\delta$, respectively, where trading intensities are defined as

$$i^t_z \equiv \frac{\partial d^t(\tilde{s}_z, \tilde{s}_\delta, \tilde{p})}{\partial \tilde{s}_z} = \frac{\tau_s}{\gamma}, \quad (6)$$

$$i^t_\delta \equiv \frac{\partial d^t(\tilde{s}_z, \tilde{s}_\delta, \tilde{p})}{\partial \tilde{s}_\delta} = -\frac{\tau_s \xi_\delta \xi_z}{\gamma \xi_\delta^2 + \gamma + \tau_s}. \quad (7)$$

To understand what drives the traditional investor’s trading intensities, it is useful to look at how she infers information about $\tilde{z}$, the payoff component that she values, from the price and her signals. Specifically, she expects to receive the following payoff from holding one share:

$$\mathbb{E}(\tilde{z} | \tilde{s}_z, \tilde{s}_\delta, \tilde{p}) = \tilde{s}_z \frac{\tau_s}{\gamma + \tau_s} + \frac{p_z}{\tau_s + \tau_s} \left[ \tilde{p} - \left( p_0 + p_z \tilde{s}_z \frac{\tau_s}{\gamma + \tau_s} + p_\delta \tilde{s}_\delta \frac{\tau_s}{\gamma + \tau_s} \right) \right]. \quad (8)$$

Upon receiving a higher $\tilde{s}_z$, a traditional investor directly infers from her signal that $\tilde{z}$ is higher (“Signal inference” term). At the same time, for a given price $\tilde{p}$, a higher $\tilde{s}_z$ implies that other investors have received lower signals about $\tilde{z}$, and this implies that the information about $\tilde{z}$ contained in the price is worse (“Price inference” term).

Posterior uncertainty about $\tilde{z}$ for a traditional investor equals uncertainty about $\tilde{z}$ after observing a private signal net of a reduction in uncertainty due to learning from the price:

$$\nabla(\tilde{z} | \tilde{s}_z, \tilde{s}_\delta, \tilde{p}) = \frac{1}{\tau + \tau_s} - \frac{\left( p_z \frac{1}{\tau + \tau_s} \right)^2}{p_z^2 \tau_s + p_\delta^2 \tau_s + p_n^2 \tau_n}. \quad (9)$$

In particular, if the price is strongly associated with the ESG output ($p_\delta$ is high) or with the noise traders’ demand ($p_n$ is high), then traditional investors cannot learn much about $\tilde{z}$ from the price, and the uncertainty reduction term is small.

Plugging (8) and (9) into the demand function (3), it is easy to derive that trading intensity $i^t_z$ of a traditional investor with respect to $\tilde{s}_z$ is positive and constant, as shown in (6). This is a standard result (Hellwig, 1980).

More interestingly, the traditional investor’s trading intensity with respect to $\tilde{s}_\delta$ is

\[\text{To lighten the notation, we use the fact that investors within each type have identical preferences and omit investor-specific indices where possible.}\]
negative and depends on the equilibrium price coefficients. Because a traditional investor does not value the ESG output, a better realization of $\tilde{s}_\delta$ does not directly affect the expected stock payoff to her. However, she uses her signal on $\tilde{\delta}$ to infer $\tilde{z}$ from the price. In particular, for a given price, she infers that a higher $\tilde{s}_\delta$ implies worse aggregate information about $\tilde{z}$. Therefore, she reduces her demand in response to a higher $\tilde{s}_\delta$.

The magnitude of $i^t_\delta$ is large if traditional investors are able to infer a lot about $\tilde{z}$ from the price based on their $\tilde{\delta}$-signals. This price inference effect is strong if the equilibrium price responds strongly to changes in $\tilde{\delta}$ ($\xi_\delta$ is high) and is informative about $\tilde{z}$ ($\xi_z$ is high). The price inference effect is captured by the numerator of the expression (7). At the same time, if the price is a noisy signal about $\tilde{z}$, either due to its strong association with $\tilde{\delta}$ or due to noise traders, traditional investors do not trade the stock actively. This reduces the magnitude of $i^t_\delta$. The price noisiness effect is captured by the denominator of (7).

Analogously, the trading intensities of a green investor are

$$i^g_\delta \equiv \frac{\partial d^g(\tilde{s}_z, \tilde{s}_\delta, \tilde{p})}{\partial \tilde{s}_\delta} = \frac{\tau_s \gamma}{\gamma \xi_z^2 + \tau_n^2 \tau_n},$$

(10)

$$i^g_z \equiv \frac{\partial d^g(\tilde{s}_z, \tilde{s}_\delta, \tilde{p})}{\partial \tilde{s}_z} = \frac{\tau_s}{\gamma}.$$

(11)

Because traditional and green investors value different outputs, they trade in opposite directions based on the same signals. Both investor groups trade with equal and constant intensities on signals about the output components they value: $i^t_z = i^g_\delta = \frac{\xi_z^2}{\gamma}$. At the same time, their trading intensities on signals about the output components they do not value, $i^t_\delta$ and $i^g_z$, depend on the equilibrium price coefficients and on the riskiness of the stock payoff. Recall that an investor of type $j$ trades more intensively on signals about the fundamental she does not value when facing a smaller residual uncertainty or, equivalently, when the equilibrium price is more informative to her. Defining the price informativeness to a type-$j$ investor as $PI_j \equiv \mathbb{V} \left( \beta^j_z \tilde{z} + \beta^j_\delta \tilde{\delta} | \mathcal{F}^t \right)^{-1}$, it is easy to see that

$$i^t_\delta \equiv \frac{\xi^2_\delta + \tau_n^2 \tau_n}{\xi^2_\delta + \tau_n^2} \equiv \frac{PI_t}{PI_g} \equiv v,$$

(12)

where $v$ is the relative price informativeness. If $v > 1$, the price is more informative to traditional investors, and they trade against their $\tilde{\delta}$-signals more intensively than green investors trade against their $\tilde{z}$-signals. The opposite is true if $v < 1$.

The trading intensities of traditional and green investors determine the information
content of the price, that is, the equilibrium price coefficients. The market clearing condition (4) implies

\[
\xi_z = \frac{m}{2} (i_z^t + i_z^g),
\]

(13)

\[
\xi_\delta = \frac{m}{2} (i_\delta^t + i_\delta^g).
\]

(14)

Expression (12) and the system (13)-(14) indicate that there exists a feedback loop between the trading intensities \(i_\delta^t, i_\delta^g\) and the price coefficients \(\xi_\delta, \xi_z\). On the one hand, suppose that traditional investors trade more intensively against their \(\tilde{\delta}\)-signals than green investors against their \(\tilde{z}\)-signals, that is, \(i_\delta^t < i_\delta^g < 0\). Then the price incorporates less information about the ESG output, so that \(\xi_\delta < \xi_z\). On the other hand, if the price contains less ESG information, i.e., \(\xi_\delta < \xi_z\), it is more informative to traditional investors, i.e., \(v > 1\). Traditional investors then face a smaller residual uncertainty about the stock payoff, which justifies why they trade more intensively than green investors in the first place. This feedback loop is illustrated in Figure 1. An analogous feedback loop exists if green investors dominate the trading.

![Feedback loop between the trading intensities and the price coefficients](image)

Figure 1: Feedback loop between the trading intensities \(i_\delta^t, i_\delta^g\) and the price coefficients \(\xi_\delta, \xi_z\) when traditional investors dominate the trading.

### 4.1.2 Equilibrium multiplicity

The feedback loop described above has profound impacts on the equilibrium outcomes. In particular, it can lead to multiple equilibria in the trading stage, that is, multiple equilibrium pricing functions. Using the expressions for the trading intensities (6)-(7) and (10)-(11), the system of equations (13)-(14) that pins down equilibrium price coefficients
can be rewritten as

\[
\xi_z = \frac{\tau_s m}{\gamma/2} \left[ 1 - \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau + \tau_s}{\tau_n}} \right], \tag{15}
\]

\[
\xi_\delta = \frac{\tau_s m}{\gamma/2} \left[ 1 - \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}} \right]. \tag{16}
\]

The system (15)-(16) can have two types of solutions.

**Symmetric case:** \( \xi_z = \xi_\delta \) Imposing \( \xi_z = \xi_\delta \) makes (15) and (16) identical. Each of these equations can be rewritten as

\[
\xi_\delta^3 + \frac{\tau + \tau_s}{\tau_n} \xi_\delta - \frac{\tau_s m}{\gamma/2} \frac{\tau + \tau_s}{\tau_n} = 0. \tag{17}
\]

Clearly, this equation always has a unique and positive real root. This solution corresponds to a symmetric equilibrium in which traditional and green investors trade equally actively, \( i^*_g = i^*_z \). This results in the price being equally informative to the two investor groups, \( v = 1 \).

**Asymmetric case:** \( \xi_z \neq \xi_\delta \) As shown in Appendix A.1, if the system (15)-(16) has solutions in which the price coefficients \( \xi_z \) and \( \xi_\delta \) are not equal, they must be symmetric around \( \frac{\tau_s m}{\gamma/2} \), i.e., \( \xi_z + \xi_\delta = \frac{\tau_s m}{\gamma/2} \). Imposing this restriction in (16), we can simplify it to

\[
\xi_\delta^2 - \frac{\tau_s m}{\gamma/2} \xi_\delta + \frac{\tau + \tau_s}{\tau_n} = 0.
\]

This equation has two distinct real roots if and only if

\[
\tau_n > \tau_n^* \equiv 4 (\tau + \tau_s) \left( \frac{\tau_s m}{\gamma/2} \right)^{-2}, \tag{18}
\]

that is, if the demand from noise traders is not too volatile.\(^{12}\) These roots are given by

\[
\xi_\delta = \frac{1}{2} \left[ \frac{\tau_s m}{\gamma/2} \pm \sqrt{\left( \frac{\tau_s m}{\gamma/2} \right)^2 - 4 \frac{\tau + \tau_s}{\tau_n}} \right] \quad \text{and} \quad \xi_z = \frac{\tau_s m}{\gamma/2} - \xi_\delta.
\]

In the equilibrium with \( \xi_\delta > \frac{\tau_s m}{\gamma/2} > \xi_z \), the price is mostly driven by ESG output and is more informative to green investors, \( v < 1 \). We refer to this equilibrium as a

\(^{12}\)If \( \tau_n = \tau_n^* \), the root in the asymmetric case is unique and coincides with that in the symmetric case: \( \xi_\delta = \xi_z = \frac{1}{2} \frac{\tau_s m}{\gamma/2} \).
The other one is referred to as a *T-equilibrium*; there $\xi_4 < \frac{1}{2} \frac{m}{\tau} < \xi_5$ and the price is more informative to traditional investors, $v > 1$. Notably, the symmetric equilibrium with $\xi_4 = \xi_5$ solving (17) always exists. In this equilibrium, which we refer to as *M-equilibrium*, the price is equally informative to the two investor groups, $v = 1$.

The G- and T-equilibria coexist if (18) is satisfied, that is, if noise trading is dominated by informed trading. Specifically, (18) is satisfied if the exogenous noise is small, namely, the variance of noise traders’ demand $\tau^{-1}$ is small, signals are precise relative to priors (high $\tau_s$ and low $\tau$), and the mass of informed investors $m$ is large. Under these conditions, the feedback loop described in the previous section is strong, and the relative price informativeness (12) is sensitive to the price coefficients. As a result, multiple equilibria arise. In the T-equilibrium (G-equilibrium), trading is dominated by traditional (green) investors, and the price is mostly informative about the financial (ESG) output, which makes it less noisy to the dominating investor group. If, on the contrary, the exogenous noise is large, the price is mostly driven by the noise traders’ demand, and the relative price informativeness is always close to one. The feedback loop becomes weak. In this case, the only possible equilibrium is the one described in the symmetric case.

### 4.1.3 Mechanism behind the feedback loop and equilibrium multiplicity

The key economic mechanism behind the feedback loop and equilibria multiplicity is that investors trade in opposite directions when receiving the same signals. This mechanism requires that the two investor groups have, first, the incentives to trade against each other and, second, the means to do so.

The incentives arise due to preference heterogeneity. Because investors value different output components, they use the same information differently. By trading against signals about the output they do not value, investors of one group make the price noisier to the other group. Facing a higher residual risk, investors of the other group choose to trade less actively. The feedback loop between the trading intensities and the price informativeness gives rise to multiple equilibria. In the absence of preference heterogeneity, all investors trade in the same way, and the price is always equally informative to everyone. In that case, our model reduces to a fairly standard REE setting with a unique equilibrium.$^{13}$

The ability of investors to trade in opposite directions relies on the availability of

---

$^{13}$Goldstein and Yang (2015) study a model in which investors with homogeneous preferences trade the stock whose payoff is affected by two components. They show equilibrium uniqueness under the assumption that there are two investor groups and investors within each group are informed about one fundamental. Appendix F verifies this result under a more general information structure.
information about both output components. In the context of ESG investing, even if traditional investors do not value firms’ ESG performances, they still may receive related information from news articles or firm disclosures. Receiving such information makes it possible for traditional investors to trade against green investors. If investors receive information only about the component they value, the feedback loop disappears. In Appendix E, we show that in the setting with heterogeneous preferences, multiple equilibria can never emerge only if investors receive information exclusively about the output components they value.

4.2 Baseline model

In the baseline model, we allow green investors to value both output components and the two investor groups to have different masses. Derivations and proofs are in Appendix A.

The analyses and intuitions of the special case of Section 4.1 can be extended to the baseline model. From a green investor’s perspective, the stock payoff is \( \tilde{y} = \beta_z \tilde{z} + \beta_\delta \tilde{\delta} \). Correspondingly, \( \tilde{x} = \beta_\delta \tilde{z} - \beta_z \tilde{\delta} \) is orthogonal to \( \tilde{y} \) and thus represents the output component that green investors do not value. In this setting, differential usage of information by the two investor groups becomes less stark. Specifically, a green investor receiving a better signal about the financial output still infers a worse realization of the ESG output from the price. However, as long as she values the firm’s financial output, i.e., \( \beta_z > 0 \), she has a weaker incentive to trade against her \( \tilde{z} \)-signals. As a result, her trading intensity on her financial signal becomes

\[
\tilde{i}_z^g = \frac{\tau_s \beta_z \left( \xi_z^2 + \frac{\tau_s + \tau_n}{\tau_n} \right) - \xi_z \xi_\delta \beta_\delta}{\gamma \left( \xi_z \beta_\delta - \xi_\delta \beta_z \right)^2 + \frac{\tau_s + \tau_n}{\tau_n}}.
\]  

(19)

Similarly, a green investor has a weaker incentive to increase her demand for the stock following a better ESG signal,

\[
\tilde{i}_\delta^g = \frac{\tau_s \beta_\delta \left( \xi_\delta^2 + \frac{\tau_s + \tau_n}{\tau_n} \right) - \xi_z \xi_\delta \beta_z}{\gamma \left( \xi_z \beta_\delta - \xi_\delta \beta_z \right)^2 + \frac{\tau_s + \tau_n}{\tau_n}}.
\]  

(20)

Although the preferences of traditional and green investors are partially aligned, the feedback loop described in Section 4.1 still arises as long as their preferences are not
entirely homogeneous. In particular, the relative price informativeness (12) becomes

\[
v = \frac{PI_t}{PI_g} = \frac{(\xi_z \beta_\delta - \xi_\delta \beta_z)^2 + \frac{\tau + \tau_s}{\tau_n}}{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}}.
\] (21)

If traditional investors dominate the trading, the price is mostly aligned with their preferences and less noisy to them, i.e., \(\xi_\delta < \xi_z \equiv \xi_z \beta_\delta - \xi_\delta \beta_z\), where \(\xi_z\) is the price coefficient associated with output component \(\tilde{x}\) that green investors do not value. Then green investors do not trade the stock actively, as can be seen from the expression for trading intensities (19)-(20). The opposite is true if green investors dominate the trading.

Proposition 1 shows that, as in the special case, multiple equilibria arise when the exogenous noise is sufficiently small.

**Proposition 1.** There exists a multiplicity threshold \(\tau^*_n > 0\) such that (i) if \(\tau_n \in (0, \tau^*_n)\), there is a unique equilibrium; (ii) if \(\tau_n = \tau^*_n\), there are two equilibria if \(\alpha \neq \frac{1}{2}\) and one equilibrium if \(\alpha = \frac{1}{2}\); (iii) if \(\tau_n > \tau^*_n\), there are three equilibria. In any equilibrium, \(p_0 < 0\), \(p_z > 0\), \(p_\delta > 0\) and \(p_n > 0\).

Proposition 2 characterizes how the multiplicity threshold \(\tau^*_n\) varies with the degree of preference heterogeneity \(\beta_\delta\) and the green investor share \(\alpha\). The equilibrium multiplicity is more likely when the preference heterogeneity is large in the entire investor base, that is, when the ESG utility weight of green investors \(\beta_\delta\) is large and the masses of the two groups are similar (\(\alpha\) is close to \(\frac{1}{2}\)). If the investor base consists mainly of investors of one type or if traditional and green investors’ preferences are closely aligned, the aggregate preference heterogeneity is small. For example, if there are only a few green investors (\(\alpha \to 0\)) or green investors mostly value the financial output (\(\beta_\delta \to 0\)), the investor base is nearly homogeneous, and the model reduces to a standard REE model with a unique pricing function.

**Proposition 2.** The multiplicity is more likely if investor preferences are more heterogeneous and investor groups have similar masses: (i) \(\frac{d\tau^*_n(\alpha, \beta_\delta)}{d\beta_\delta} < 0\); (ii) \(\frac{d\tau^*_n(\alpha, \beta_\delta)}{d\alpha} \leq 0\) if \(\alpha \leq \frac{1}{2}\).

When multiple equilibria are possible, they can be ranked by the relative price infor-
mativeness. Formally, price informativeness to traditional and green investors are

\begin{align*}
PI_t & \equiv \mathbb{V}(\tilde{z}|\mathcal{F}^t)^{-1} = (\tau + \tau_s) \frac{\xi_z^2 + \xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}}{\xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}}, \\
PI_g & \equiv \mathbb{V}(\beta_z\tilde{z} + \beta_\delta\tilde{\delta}|\mathcal{F}^t)^{-1} = (\tau + \tau_s) \frac{\xi_z^2 + \xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n}}{(\xi_\delta \beta_z - \xi_z \beta_\delta)^2 + \frac{\tau + \tau_s}{\tau_n}},
\end{align*}

(22)

(23)

and so the relative price informativeness \( v = \frac{PI_t}{PI_g} \) is given by (21). Using the same terminology as in the simplified model, if there are three equilibria, we call the one with the smallest \( v \) the G-equilibrium, the one with the largest \( v \) the T-equilibrium, and the one with a medium \( v \) the M-equilibrium.\(^{14}\) Formally, we have

**Proposition 3.** When there exist three equilibria, they can be ranked according to the relative price informativeness to traditional investors \( v \). In the T-equilibrium, \( v^T > 1; \) in the G-equilibrium, \( v^G < 1; \) in the M-equilibrium, \( v^M \in (v^G, v^T) \).

The possibility of equilibrium multiplicity naturally raises the question about equilibrium selection. A common selection approach suggests that stable equilibria are more likely to be played. As is typical in the literature, we call an equilibrium stable if the dynamics around the equilibrium price coefficients are locally stable—in particular, if the system is pushed to an off-equilibrium point, it tends to move back to the same equilibrium point if the shock is sufficiently small.\(^{15}\)

**Proposition 4.** If equilibrium is unique, it is stable. If there are three equilibria, the T- and G-equilibria are stable and the M-equilibrium is unstable.

Proposition 4 suggests that investors are unlikely to coordinate on the M-equilibrium when the G- and T-equilibria exist. The M-equilibrium also has counter-intuitive properties. For example, in the M-equilibrium, when the mass of one investor group increases, the price becomes less informative to investors of this group (this is formally established in Proposition 5 below). In other words, investors with the same preferences coordinate to trade less actively when there are more of them. In what follows, we characterize all equilibria but put less focus on the M-equilibrium when the multiplicity is possible.

\(^{14}\)By Proposition 1, two equilibria exist when \( \tau_n = \tau_n^* \) and \( \alpha \neq \frac{1}{2} \). We do not analyze this knife-edge case to save space.

\(^{15}\)It is worth noting that a formal evaluation of stability requires a dynamic extension of our model, which is beyond the scope of this paper. However, the criterion we use is similar to the one derived in the literature introducing recursive-least-squares learning in settings a la Grossman and Stiglitz (1980) (Bray, 1982; Marcet and Sargent, 1989; Heinemann, 2009). See Appendix A.5 for details.
5 Growth of green investors

In this section, we examine the impacts of the growth in the share of green investors. Specifically, we characterize how the price informativeness and the firm’s cost of capital respond to an increase in the green investor share $\alpha$ in Sections 5.1 and 5.2, respectively. Proofs and derivations for this section are in Appendix B.

5.1 Price informativeness

Proposition 5 characterizes how absolute and relative price informativeness—$PI_t$, $PI_g$, and $v$—change with the green investor share $\alpha$.

**Proposition 5.** If $\tau_n \leq \tau_n^* \left(\frac{1}{2}, \beta_\delta\right)$, there is a unique equilibrium in which $\frac{dP_{I_t}}{d\alpha} < 0$, $\frac{dP_{I_g}}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$. If $\tau_n > \tau_n^* \left(\frac{1}{2}, \beta_\delta\right)$, there exists $\alpha \in \left(0, \frac{1}{2}\right)$ and $\bar{\alpha} = 1 - \alpha$ such that

(i) if $\alpha < \alpha$, there is a unique $T$-equilibrium in which $v^T > 1$;

(ii) if $\alpha > \bar{\alpha}$, there is a unique $G$-equilibrium in which $v^G < 1$;

(iii) if $\alpha \in (\alpha, \bar{\alpha})$, there are three equilibria and $v^T > v^M > v^G$.

Moreover, in the $T$- and $G$-equilibria, $\frac{dP_{I_t}}{d\alpha} < 0$, $\frac{dP_{I_g}}{d\alpha} > 0$, and $\frac{dv}{d\alpha} < 0$; in the $M$-equilibrium, $\frac{dP_{I_t}}{d\alpha} > 0$, $\frac{dP_{I_g}}{d\alpha} < 0$, and $\frac{dv}{d\alpha} > 0$.

Suppose first that the exogenous noise is large, i.e., $\tau_n \leq \tau_n^* \left(\frac{1}{2}, \beta_\delta\right)$. By Proposition 2, the multiple equilibria region is the largest when the investor base consists of equal masses of green and traditional investors, $\alpha = \frac{1}{2}$. Therefore, if equilibrium is unique for $\alpha = \frac{1}{2}$, it is unique for all $\alpha \in (0, 1)$.

As $\alpha$ increases, the equilibrium price coefficients change such that the price becomes more informative to green investors and less informative to traditional investors. First, for given individual trading intensities, a larger $\alpha$ means that the price becomes more aligned with the preferences of green investors and, thus, more informative to them because they are responsible for a larger share of trades in the market. Second, individual trading intensities adjust. Green investors, facing a lower residual risk, trade more actively, whereas traditional investors reduce their trading activity. Panel (A) in Figure 2 illustrates how the relative price informativeness varies with the green investor share.

$^{16}$Although there is a unique equilibrium if $\alpha < \alpha$ and $\alpha > \bar{\alpha}$, we refer to it as either a $T$- or $G$-equilibrium, respectively, because the equilibrium outcomes, such as the price coefficients and price informativeness, are continuous at $\alpha = \alpha$ and $\alpha = \bar{\alpha}$ as shown in panel (B) of Figure 2.
If the exogenous noise is small, i.e., $\tau_n > \tau^*_n \left( \frac{1}{2}, \beta_\delta \right)$, equilibrium multiplicity is possible when masses of traditional and green investors are similar, that is, $\alpha$ is close to $\frac{1}{2}$. Start from an economy with few green investors ($\alpha < \alpha_0$). Here, traditional investors significantly outweigh green investors. There exists a unique T-equilibrium in which the price is informative mostly about the financial output, resulting in $v > 1$. As $\alpha$ increases and crosses $\alpha_0$, the feedback loop becomes sufficiently strong to support the G-equilibrium in which the price is more informative to green investors, $v < 1$. Notably, the G-equilibrium is sustainable even if green investors constitute a minority in the investor base, i.e., $\alpha < \alpha < \frac{1}{2}$. Eventually, when the share of green investors becomes sufficiently large, $\alpha > \bar{\alpha}$, there exists a unique G-equilibrium.

Panel (B) in Figure 2 shows relative price informativeness $v$ in this case. Similar to the case of large exogenous noise, as $\alpha$ increases, the price becomes more informative to green investors and less informative to traditional investors in the stable T- and G-equilibria. Different from the case of large exogenous noise, however, there can be discontinuous jumps in the price informativeness due to switches across equilibria. Therefore, we can expect a sudden shift from a cash-flow-dominated price to an ESG-dominated price if the share of green investors in the market keeps increasing.

![Figure 2: Relative price informativeness to traditional investors $v$ as a function of the green investor share $\alpha$. Y-axes are in the log scale.](image-url)
5.2 Cost of capital

In our model, the financial return on the risky asset is \( \bar{z} - \bar{p} \). Therefore, the expected financial return is

\[
\mathbb{E}(\bar{z} - \bar{p}) = -p_0 = \frac{\gamma}{m_t P I_t + m_g P I_g}.
\]  

(24)

In what follows, we refer to \( \mathbb{E}(\bar{z} - \bar{p}) \) as the firm’s cost of capital and denote it by \( CoC \). \( CoC \), therefore, is the expected financial return, which captures the firm’s cost of capital from the perspective of a manager who only values the firm’s financial output.\(^{17} \) \( \mathbb{E}(\bar{z} - \bar{p}) \) is also closely related to the earnings-to-price ratio that is frequently analyzed in empirical research and by practitioners.

As is standard in the REE settings (e.g., Easley and O’Hara, 2004), the cost of capital, defined by (24), reflects the compensation required by risk-averse investors for their investment risks. In our environment, it is determined by the weighted average of price informativeness to traditional and green investors. Proposition 6 characterizes how \( CoC \) changes with the share of green investors \( \alpha \).

**Proposition 6.** If \( \tau_n \leq \tau_n^*(\frac{1}{2}, \beta_\delta) \), there is a unique equilibrium in which \( \frac{dCoC}{d\alpha} \gtrless 0 \) if \( \alpha \lesssim \frac{1}{2} \). If \( \tau_n > \tau_n^*(\frac{1}{2}, \beta_\delta) \), in the T-equilibrium, \( \frac{dCoC}{d\alpha} > 0 \); in the G-equilibrium, \( \frac{dCoC}{d\alpha} < 0 \); in the M-equilibrium, \( \frac{dCoC}{d\alpha} \gtrless 0 \) if \( \alpha \lesssim \frac{1}{2} \).

Consider first the case of large exogenous noise, i.e., \( \tau_n \leq \tau_n^*(\frac{1}{2}, \beta_\delta) \), such that there always exists a unique equilibrium. This case is illustrated by panel (A) of Figure 3. Suppose that \( \alpha < \frac{1}{2} \), that is, the mass of traditional investors is larger than the mass of green investors. A marginal effect of \( \alpha \) on \( CoC \) can be decomposed in two components,

\[
\frac{dCoC}{d\alpha} = -\frac{\gamma}{(1 - \alpha) P I_t + \alpha P I_g} \frac{1}{m} \left( P I_g - P I_t + (1 - \alpha) \frac{dP I_t}{d\alpha} + \alpha \frac{dP I_g}{d\alpha} \right).
\]

The direct effect reflects the change in the cost of capital due to the change in the investor composition holding price informativeness \( P I_t \) and \( P I_g \) fixed. If \( \alpha < \frac{1}{2} \), \( P I_g < P I_t \) by Proposition 5, that is, green investors face a higher residual risk when investing in the

\(^{17}\)In our setting, the expected financial return does not necessarily capture the expected return for all firm investors because green investors also value the firm’s ESG output. However, if \( \bar{z} \) and \( \bar{\delta} \) have zero means, the expected return for green investors is the same as (24): \( \mathbb{E}(\beta_\delta \bar{z} + \beta_\delta \bar{\delta} - \bar{p}) = -p_0 \). At the end of this section, we discuss the cost of capital measure if \( \bar{z} \) and \( \bar{\delta} \) have non-zero means.
As a result, the direct effect drives the cost of capital up.

The indirect effect captures the change in the cost of capital due to adjustments in the equilibrium price coefficients and, hence, price informativeness. By Proposition 5, the price informativeness to traditional and green investors move in the opposite directions as the investor composition changes: \( \frac{dPI_t}{d\alpha} < 0 \) and \( \frac{dPI_g}{d\alpha} > 0 \). Nevertheless, the indirect effect also pushes the cost of capital up if \( \alpha < \frac{1}{2} \). The key force behind this result is that, if \( \alpha < \frac{1}{2} \), an increase in \( PI_g \) in response to a higher \( \alpha \) is modest, i.e., \( \frac{dPI_g}{d\alpha} \) is not too large. As the share of green investors \( \alpha \) grows, the price becomes more associated with the ESG output, i.e., \( \xi_\delta \) goes up. However, an increase in \( \xi_\delta \) also allows traditional investors to use their ESG signals more efficiently to trade against green investors along the \( \tilde{\delta} \)-dimension. This effect is strong because, when the mass of traditional investors is large, i.e., \( \alpha < \frac{1}{2} \), they face a relatively low investment risk and thus trade intensively against their \( \tilde{\delta} \)-signals. Active trading by traditional investors, therefore, prevents \( \xi_\delta \) and \( PI_g \) from increasing sharply.

In sum, when the investor base consists mostly of traditional investors, an increase in the green investor share leads to an increase in the overall information risk and in the cost of capital. In contrast, when the majority of investors have green preferences (\( \alpha > \frac{1}{2} \)), the signs of both direct and indirect effects flip, and the cost of capital declines in \( \alpha \). The cost of capital reaches its maximum when the masses of the two groups are equal, that is, when investor heterogeneity is high, and trades by green and traditional investors introduce substantial amounts of noise to each other.

Suppose now that the exogenous noise is small, i.e., \( \tau_n > \tau_n^* (\frac{1}{2}, \beta_\delta) \). Then multiple equilibria are possible. The comparative statics of CoC with respect to \( \alpha \) for this case is shown in Panel (B) of Figure 3. In the T-equilibrium, traditional investors dominate the trading and \( PI_t > PI_g \). Similar to the unique equilibrium case with \( \alpha < \frac{1}{2} \), an increase in \( \alpha \) leads to a larger CoC through both direct and indirect effects. The opposite is true in the G-equilibrium in which the stock is primarily traded by green investors.

So far we have analyzed the cost of capital for a firm with zero average financial and ESG outputs, that is, when both \( \tilde{z} \) and \( \tilde{\delta} \) have zero means. We now characterize how the cost of capital changes with the green investor share for a firm with non-zero expected outputs.

**Corollary 1.** Suppose that \( \tilde{z} \sim N (\mu_z, \tau^{-1}) \) and \( \tilde{\delta} \sim N (\mu_\delta, \tau^{-1}) \). \( PI_t, PI_g, \xi_z \) and \( \xi_\delta \) do

---

\(^{18}\)Specifically, \( v(\alpha) = 1 \) if \( \alpha = \frac{1}{2} \) and, since \( v(\alpha) \) is a decreasing function by Proposition 5, it must be that \( v(\alpha) > 1 \Leftrightarrow PI_t(\alpha) > PI_g(\alpha) \) if \( \alpha < \frac{1}{2} \).
not depend on $\mu_z$ and $\mu_\delta$. The cost of capital is
\[
\text{CoC} \equiv \mathbb{E}(\tilde{z} - \tilde{p}) = \frac{\gamma}{m_t P_t + m_g P_g} + c_z \mu_z + c_\delta \mu_\delta,
\]
where $c_z = \frac{(1-\beta_z)\xi_z}{\beta_\delta \xi_x + (1-\beta_z)\xi_z} > 0$ and $c_\delta = -\frac{\beta_\delta \xi_\delta}{\beta_\delta \xi_x + (1-\beta_z)\xi_z} < 0$. Moreover, $\frac{dc_z}{d\alpha} > 0$ and $\frac{dc_\delta}{d\alpha} < 0$ except for the M-equilibrium. In the M-equilibrium, $\frac{dc_z}{d\alpha} < 0$ and $\frac{dc_\delta}{d\alpha} > 0$.

Corollary 1 delivers two main results. First, the firm’s cost of capital increases in its expected financial output $\mu_z$ because $c_z > 0$, and decreases in its expected ESG output $\mu_\delta$ because $c_\delta < 0$. Recall that we define CoC as the expected financial return, that is, as the cost of capital from the perspective of a manager who values only the financial output $\tilde{z}$. From this manager’s perspective, an increase in $\mu_z$ is not fully reflected in the stock price because of the presence of green investors who do not value financial output as much as the manager. Consequently, an increase in $\mu_z$ has a stronger positive effect on the expected financial output $\mathbb{E}\tilde{z}$ than on the expected price $\mathbb{E}\tilde{p}$. In contrast, a higher $\mu_\delta$ does not affect the expected financial output but leads to a higher expected price due to an increased demand from green investors. As a result, a higher $\mu_\delta$ drives CoC down. This result echoes existing theoretical literature that highlights the role of ESG investors in lowering the cost of capital for green firms because these investors are willing to sacrifice financial returns to increase the non-pecuniary benefits of their investments.

Second, as the green investor share increases, the cost of capital becomes more sensitive to both $\mu_z$ and $\mu_\delta$, that is, the absolute values of $c_z$ and $c_\delta$ increase in $\alpha$. With more investors valuing ESG output, the average preferences of investors deviates more from
that of the firm manager. From the manager’s perspective, the firm is thus more under-compensated for an increase in $\mu_z$ and more over-compensated for an increase in $\mu_\delta$.

Finally, it is worth commenting on a proper measure of the cost of capital in our model. The expected financial return (24) measures the firm’s cost of capital from the perspective of the firm manager who only values the firm’s financial performance. Within our framework, one may also consider a manager who values both payoff components such that the cost of capital is $E \left( \beta z \hat{z} + \beta \delta \hat{\delta} - \hat{p} \right)$. If $\hat{z}$ and $\hat{\delta}$ have zero means, the comparative statics of CoC described by Proposition 6 remain the same. If the means are non-zero, the results of Corollary 1 are preserved if the preferences of the manager and traditional investors are sufficiently close.\(^{19}\)

## 6 Extensions

In this section, we extend our baseline model along two dimensions. In Section 6.1, we consider a model in which the precision of financial and ESG information can be different and explore the effects of improvements in the quality of ESG information. In Section 6.2, we allow the financial and ESG outputs to be correlated.

### 6.1 Improvements in ESG information quality

Despite the growing interest in ESG investing, there is a lack of clarity and consistency in the measurement of firms’ ESG performances. For example, the average correlation of ESG ratings provided by six large raters is only 0.54 (Berg, Kölbl, and Rigobon, 2022).\(^{20}\) To address this problem, policymakers around the world have made a series of efforts to improve the quality of information about firms’ ESG performances available to investors. For instance, in May of 2020, the SEC Investor Advisory Committee recommended updating public company reporting requirements to include ESG factors (SEC, 2020), while the EU regulator has already put in place a disclosure regulation that requires market participants and financial advisers to provide ESG-related information about certain financial products (Regulation EU 2019/2088). There are currently more than 600 ESG reporting requirements across over 80 countries, including the world’s 60

\(^{19}\)An interesting question in this respect is how the manager’s preferences are related to those of heterogeneous investors, some of whom have non-pecuniary considerations (see, e.g., Hart and Zingales (2017) and Geelen, Hajda, and Starmans (2023) for related work). We leave this for future exploration.

\(^{20}\)Such a low correlation might also result from different weightings of E, S and G components in different ratings. At the same time, ESG investors may also have heterogeneous preferences about these three sub-components. We discuss implications of this heterogeneity in Section 7 and Appendix G.
largest economies (van der Lugt et al., 2020). In addition, firms also increasingly disclose ESG information voluntarily. Governance & Accountability Institute finds that in 2019, 90% of S&P500 companies published ESG reports, a marked increase from 20% in 2011 (GAI, 2020).

In this section, we extend our baseline model to investigate the effects of an improvement in the precision of ESG information. The novel insight coming out of our analysis is that improved ESG information quality can impair price informativeness about the financial output and lead to an increase in the cost of capital. Policymakers should, therefore, be cautious about such unintended consequences.

6.1.1 Setup

To study how an improvement in ESG information quality affects the equilibrium outcomes, we generalize the information structure of our baseline model as follows. First, we assume that the prior precisions of the two output components are no longer identical, \( \tilde{z} \sim N(0, \tau^{-1}) \) and \( \tilde{\delta} \sim N(0, (\lambda \tau)^{-1}) \), where \( \lambda > 0 \). Second, the precisions of private signals that investors receive also differ by a factor of \( \lambda \), i.e., \( \tilde{s}_z^i = \tilde{z} + \tilde{\varepsilon}_z^i \) and \( \tilde{s}_\delta^i = \tilde{\delta} + \tilde{\varepsilon}_\delta^i \), where \( \tilde{\varepsilon}_z^i \sim N(0, \tau_s^{-1}) \) and \( \tilde{\varepsilon}_\delta^i \sim N(0, (\lambda \tau_s)^{-1}) \) for any investor \( i \).

The parameter \( \lambda \) captures the quality of ESG information. The extended setup reduces to our baseline model when \( \lambda = 1 \).

Equilibrium characterization in this extension is similar to that of the baseline model. In particular, we show in Appendix C that the system of equations that determine normalized price coefficients \( \xi_z \) and \( \xi_\delta \) takes the same form as that in the baseline model after a proper change of variables. As a result, the main results in Section 4 carry through. Specifically, there are up to three equilibria, with two of them being stable, that differ in their relative price informativeness. To save space, we delegate these analyses to Appendix C and below focus on the comparative statics of interest.

Analytically characterizing how the key equilibrium outcomes change with respect to \( \lambda \) globally is challenging. In Section 6.1.2, we focus on the case where \( \lambda \) is small, that is, ESG information is much noisier than financial information. This assumption makes analytical characterization feasible. At the same time, we believe that it also reflects current circumstances for many companies. In Section 6.1.3, we consider a numerical example in which \( \lambda \) is not necessarily small.

---

21Using numerical analysis, we have verified that our results in Section 6.1 hold if the factor \( \lambda \) for priors is different from that for signals. These results are available upon request.
6.1.2 Imprecise ESG information

**Price informativeness** Price informativeness to traditional and green investors in the extended setup are given by

\[
\begin{align*}
PI_t &\equiv \mathbb{V}(\tilde{z}|\mathcal{F}^j)^{-1} = (\tau + \tau_s) \frac{\xi^2 \lambda + \xi^2 \delta + \lambda \tau + \tau_s}{\xi^2 \delta + \lambda \tau + \tau_s}, \quad (25) \\
PI_g &\equiv \mathbb{V}(\beta \tilde{z} + \beta \delta|\mathcal{F}^j)^{-1} = (\tau + \tau_s) \frac{\xi^2 \lambda + \xi^2 \delta + \lambda \tau + \tau_s}{(\xi \beta z - \xi \beta \delta)^2 + (\beta^2 \lambda + \beta^2 \delta) \tau + \tau_s}. \quad (26)
\end{align*}
\]

An improvement in the quality of ESG information affects price informativeness through two channels. First, a higher \( \lambda \) directly helps investors make better inferences, resulting in higher \( PI_t \) and \( PI_g \). Specifically, holding price coefficients \( \xi_z \) and \( \xi_\delta \) fixed, it is easy to verify that \( \frac{\partial PI_t}{\partial \lambda} > 0 \) and \( \frac{\partial PI_g}{\partial \lambda} > 0 \). Notably, although traditional investors do not value the ESG output, more precise information about it allows them to make a better inference about the financial output from the price. Second, there is an indirect effect of an increase in \( \lambda \): An increase in \( \lambda \) changes investors’ trading behaviors and, thus, the equilibrium price coefficients. Proposition 7 describes the comparative statics results of the price coefficients and the price informativeness with respect to \( \lambda \) when \( \lambda \) is small.

**Proposition 7.** There exists a \( \bar{\lambda} > 0 \) such that if \( \lambda \in (0, \bar{\lambda}) \), equilibrium is unique, and

\[
\begin{align*}
(i) \quad & \frac{d \xi_\delta}{d \lambda} > 0; \quad \frac{d \xi_z}{d \lambda} \leq 0 \text{ if } \beta \leq \frac{\left(\frac{3}{4} m_t \left(\frac{3}{4} m_g\right)\right)}{\xi_z \xi_\delta \left(\frac{1}{4} m_t \right) + \tau + \tau_s}; \\
(ii) \quad & \frac{d PI_g}{d \lambda} > 0; \quad \frac{d PI_t}{d \lambda} \leq 0 \text{ if } \beta \leq \frac{3}{2} \frac{\left(\frac{3}{4} m_t \left(\frac{3}{4} m_g\right)\right)}{\xi_z \xi_\delta \left(\frac{1}{4} m_t \right) + \tau + \tau_s}.
\end{align*}
\]

Under small \( \lambda \), ESG information is noisy, which implies that green investors do not trade the stock actively. Consequently, there exists a unique T-equilibrium. An improvement in the ESG information quality leads to an increase in the price coefficient \( \xi_\delta \) and makes the price more informative to green investors. Since green investors value the ESG output, they increase their trading intensity on their \( \tilde{\delta} \)-signals in response to an increase in \( \lambda \). As a result, more \( \tilde{\delta} \)-information gets incorporated into the price.

At the same time, the impacts of better ESG information quality on the price coefficient \( \xi_z \) and the price informativeness to traditional investors are more convoluted. Specifically, if preference heterogeneity across traditional and green investors is large, green investors not only increase their trading intensity along the \( \tilde{\delta} \)-dimension but also trade substantially more aggressively against their \( \tilde{z} \)-signals in response to an increase in
As a result, less financial information gets incorporated in the price: $\xi_z$ decreases. Furthermore, if preference heterogeneity is sufficiently large, the indirect channel dominates the direct channel, and the price informativeness to traditional investors declines.

As shown by the cutoffs for $\beta_z$ in Proposition 7, the responses of $\xi_z$ and $PI_t$ to changes in $\lambda$ depend on other model parameters, in particular, on the mass of green investors $m_g$. If $m_g$ is high, green investors’ aggregate trading against their $\tilde{z}$-information is strong, meaning that the indirect channel is significantly negative. Hence, the price informativeness to traditional investors declines even if preference heterogeneity is not that large (i.e., the cutoffs for $\beta_z$ in Proposition 7 increase in $m_g$).

**Cost of capital** The fact that price informativeness $PI_t$ and $PI_g$ can respond to changes in $\lambda$ in opposite directions suggests that the impact of better ESG information quality on the cost of capital can be positive or negative. The expression for the cost of capital in (24) preserves in this extended setup. Differentiating it with respect to $\lambda$, we get

$$\frac{dCoC}{d\lambda} = -\gamma \frac{m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda}}{(m_t PI_t + m_g PI_g)^2}.$$ 

The sign of this derivative depends on the weighted average of the changes in the price informativeness across the two investor groups.

**Proposition 8.** There exists a $\bar{\lambda} > 0$ such that if $\lambda \in (0, \bar{\lambda})$, $\frac{dCoC}{d\lambda} \geq 0$ if $\beta_z \leq \frac{3}{2} (\frac{\tau_s}{m_t})(\frac{\tau_s}{m_g}) (\frac{\tau_s}{m_t})^2 (\frac{\tau_s}{m_g})^2 - \frac{1}{2} \frac{(\tau_s m_t)^2 + \tau_s m_g}{(\tau_s m_t)^2 + \tau_s m_g}.$

Proposition 7 shows that if preference heterogeneity is large, price informativeness $PI_g$ and $PI_t$ move in opposite directions in response to an increase in $\lambda$. Proposition 8 establishes a related result for the cost of capital: For a sufficiently large preference heterogeneity, the reduction in $PI_t$ dominates the improvement in $PI_g$, and the cost of capital increases in $\lambda$. Note, however, that the cost of capital always declines in $\lambda$ if the cutoff for $\beta_z$ in Proposition 8 is negative. This happens, for example, if the mass of green investors is small. In this case, green investors’ elevated trading activity after an increase in $\lambda$ has only a small negative effect on price informativeness to traditional investors.

### 6.1.3 Precise ESG information

In this section, we demonstrate via a numerical example that the results of Propositions 7 and 8 tend to hold for a wide range of $\lambda$’s. We pick parameters so that $\frac{dPI_t}{d\lambda} < 0$
and \( \frac{d\text{CoC}}{d\lambda} > 0 \) for sufficiently imprecise ESG information. We compute \( PI_t \), \( PI_g \) and \( \text{CoC} \) as functions of \( \lambda \) and plot them in Figure 4. We find that multiple equilibria are possible when \( \lambda \) is close to one, that is, when financial and ESG information have similar precisions. When \( \lambda \) is small, the only possible equilibrium is the T-equilibrium, in which trading is dominated by traditional investors. Green investors do not trade actively because the ESG payoff is very uncertain. Naturally, this equilibrium exists as long as \( \lambda \) is sufficiently small. Importantly, we find that the comparative statics results established in Propositions 7 and 8 hold for all values of \( \lambda \) if the T-equilibrium is played. This finding is reassuring because it confirms that our predictions continue to hold even if \( \lambda \) is not small.

(A) PI to traditional investors, \( PI_t \)  
(B) PI to green investors, \( PI_g \)  
(C) Cost of capital, \( \text{CoC} \)

Figure 4: Price informativeness to traditional (panel A) and green (panel B) investors and cost of capital (panel C) as functions of the relative precision of ESG information \( \lambda \). Y-axes are in the log scale. Parametrization: \( m_t = m_g = 1, \beta_\delta = \beta_z = \frac{1}{\sqrt{2}}, \gamma = 1, \tau_s = 5, \tau = 1, \tau_n = 4. \)

### 6.2 Correlated financial and ESG outputs

In our baseline model, the financial and ESG outputs are uncorrelated. We think of the zero-correlation case as a natural benchmark. In reality, the sign and the strength of this correlation can be ambiguous. On the one hand, implementing ESG policies can be financially costly. On the other hand, a firm using a more responsible way to produce can attract a higher demand from ESG-concerned customers, thus enhancing its financial performance. It can also be better equipped to comply with environmental regulations.

In this section, we consider an extension in which the financial and ESG outputs can be correlated. Section 6.2.1 considers a special case that imposes a specific structure on the correlation of investors’ signals, making the model analytically tractable. Section 6.2.2 relaxes this assumption and shows via a numerical example that the results of the special case are robust.
6.2.1 Special case

Denote the correlation between the financial output \( \tilde{z} \) and the ESG output \( \tilde{\delta} \) by \( \rho \in (-1, 1) \). As in the baseline model, the stock payoff to traditional investors is \( \tilde{z} \) and the stock payoff to green investors is \( \beta_z \tilde{z} + \beta_\delta \tilde{\delta} \). We normalize \( \beta_z^2 + 2\beta_z\beta_\delta \rho + \beta_\delta^2 = 1 \) such that traditional and green investors are exposed to the same ex-ante variance when holding the stock.

Define orthogonalized output components \( \tilde{u} = \tilde{z} \) and \( \tilde{v} = \tilde{\delta} - \rho \tilde{z} \sqrt{1 - \rho^2} \). By construction, \( \tilde{u} \) and \( \tilde{v} \) have the same variance \( \tau^{-1} \) as \( \tilde{z} \) and \( \tilde{\delta} \). Furthermore, they are uncorrelated. Intuitively, \( \tilde{v} \) represents “pure” ESG output that is completely unrelated to cash flows. We can write investors’ preferences over the orthogonalized output components in the following way. Traditional investors still value only one component \( \tilde{u} \). For green investors, the stock payoff is \( \beta_u \tilde{u} + \beta_v \tilde{v} \), where \( \beta_u = \beta_z + \beta_\delta \rho \) and \( \beta_v = \beta_\delta \sqrt{1 - \rho^2} \).

We assume that each investor \( i \), irrespective of her type, observes two uncorrelated private signals, \( \tilde{s}^i_u = \tilde{u} + \tilde{\varepsilon}^i_u \) and \( \tilde{s}^i_v = \tilde{v} + \tilde{\varepsilon}^i_v \), where signal noises are uncorrelated normal variables, \( \tilde{\varepsilon}^i_u, \tilde{\varepsilon}^i_v \sim N(0, \tau^{-1}_i) \). Signals about \( \tilde{v} \) represent information about the “pure” ESG output that is unrelated to cash flows, and signals about \( \tilde{u} \) represent all information related to cash flows. In particular, \( \tilde{s}^i_u \) might include information about how the eco-friendliness of the firm’s products affects the demand. Note that this information environment is equivalent to the one in which investors receive correlated signals about the non-orthogonalized output components \( \tilde{s}^i_z \) and \( \tilde{s}^i_\delta \), with the same correlation coefficient \( \rho \) as between \( \tilde{z} \) and \( \tilde{\delta} \). Equal correlation \( \text{Corr}(\tilde{z}, \tilde{\delta}) = \text{Corr}(\tilde{s}^i_z, \tilde{s}^i_\delta) = \rho \) is crucial for analytic tractability.

From our discussion above, it follows that by orthogonalizing the output components and defining investors’ preferences over these components, we get back to the baseline model of Section 3, in which the output components are uncorrelated. The following proposition summarizes the equivalence result.

**Proposition 9.** The following two models are equivalent:

1. A model in which the output components are correlated, \( \text{Corr}(\tilde{z}, \tilde{\delta}) = \rho \); signals are correlated \( \text{Corr}(\tilde{s}^i_z, \tilde{s}^i_\delta) = \rho \) for any investor \( i \); stock payoff to traditional investors is \( \tilde{z} \); stock payoff to green investors is \( \beta_z \tilde{z} + \beta_\delta \tilde{\delta} \).

2. A model in which the output components \( \tilde{u} = \tilde{z} \) and \( \tilde{v} = \frac{\delta - \rho \tilde{z}}{\sqrt{1 - \rho^2}} \) and investor signals about them are uncorrelated; stock payoff to traditional investors is \( \tilde{u} \); stock payoff to green investors is \( (\beta_z + \beta_\delta \rho)\tilde{u} + (\beta_\delta \sqrt{1 - \rho^2})\tilde{v} \).
Proposition 9 shows that a positive correlation between two output components effectively makes traditional and green investors’ preferences more aligned compared to the zero-correlation benchmark. That is, a high ESG output benefits not only green investors, who directly value the ESG payoff, but also traditional investors because it tends to be associated with a higher financial payoff. Therefore, our results on the role of preference heterogeneity also shed light on the role of the correlation between outputs. In particular, if traditional and green investors’ preferences are partially aligned, i.e. $\beta_z > 0$, a negative correlation between the two output components implies a greater heterogeneity in investors’ trading behaviors, leading to a higher likelihood of multiple equilibria (Proposition 2). Importantly, as long as the financial and ESG outputs are not perfectly correlated, the results of the baseline model apply.

6.2.2 General case

In this section, we explore the robustness of our results of the previous section to the case in which investors’ signals about the two output components may have a different correlation from that between the output components themselves. Namely, $\text{Corr}(\hat{z}, \hat{\delta}) = \rho$ and $\text{Corr}(\hat{s}_i^z, \hat{s}_i^\delta) = \rho_s$, where $\rho_s$ and $\rho$ are potentially different. We were not able to analytically characterize the model with $\rho \neq \rho_s$, and so we rely on a numerical example to explore how our results of Section 6.2.1 change in the more general case.

Specifically, we consider the following parametrization: $m = 1, \gamma = 1, \beta_z = 0, \beta_\delta = 1, \tau = 1, \tau_s = \tau_n = 10, \rho = 0.5$. Figure 5 plots the normalized price coefficients on the financial (panel A) and ESG (panel B) outputs as the share of green investors changes from 0.35 to 0.65 in two cases. In the first case (solid lines), $\rho_s = \rho = 0.5$, such that the results of Section 6.2.1 apply. In the second case (dashed lines), $\rho_s = \rho = 0.45$, which corresponds to the case where signal noises are uncorrelated.\(^{22}\)

According to Figure 5, both models feature multiple equilibria under our parametrization. However, if $\rho = \rho_s$, the multiplicity region is smaller. By Proposition 9, the model with $\rho = \rho_s > 0$ is equivalent to the baseline model in which the output components are uncorrelated but investors’ preferences are more aligned. If investors’ preferences are more aligned, their trading patterns are also more alike, and hence equilibrium multiplicity is less likely (Proposition 2). If $\rho_s < \rho$, investor signals are less positively correlated than the output components. Intuitively, this implies that investors’ trades are less aligned

\(^{22}\)Specifically, we have $\hat{s}_i^z = \hat{z} + \tilde{\varepsilon}_i^z$ and $\hat{s}_i^\delta = \hat{\delta} + \tilde{\varepsilon}_i^\delta$, where $\tilde{\varepsilon}_i^z \sim N(0, \tau_s^{-1})$ and $\tilde{\varepsilon}_i^\delta \sim N(0, \tau_s^{-1})$. If $\tilde{\varepsilon}_i^z$ and $\tilde{\varepsilon}_i^\delta$ are uncorrelated, then $\text{Corr}(\hat{s}_i^z, \hat{s}_i^\delta) = \rho \frac{\tau_s}{\tau_s + \tau}$.\[34]
than in the case with $\rho_s = \rho > 0$. As a result, equilibrium multiplicity is more likely. Furthermore, the T- and G-equilibria become more extreme, in the sense that the dominating investor group (green investors in the G-equilibrium and traditional investors in the T-equilibrium) tilts the pricing function more strongly toward their preferences. For example, in the T-equilibrium, $\xi_z$ is higher and $\xi_\delta$ is lower if signals are less positively correlated, i.e., $\rho_s < \rho$.

7 Concluding remarks

The recent ESG trend challenges the traditional view that firms’ financial fundamentals are the only drivers of asset prices. In the presence of ESG investors, it is crucial to reconsider the price formation process and the information content of prices. In this paper, we analyze the interactions between traditional and ESG investors and highlight the tension between financial and ESG information contained in asset prices. As one asset price reflects both financial and ESG performances, trading for one dilutes price informativeness about the other. Due to preference heterogeneity, the two groups of investors trade in different directions based on the same information, thus making the price noisier to each other. Such interactions give rise to a number of novel results. First, multiple equilibria with different pricing functions may emerge. Second, an increase in the number of green investors or an improvement in ESG information quality can reduce the price informativeness about the firm’s financial performance and increase its cost of capital.
Going forward, our theoretical model can be extended along several dimensions. We discuss a few of them below.

**Heterogeneous ESG preferences**  Green investors in our paper have homogeneous preferences and uniformly value the total ESG output. In reality, individual investors may assign distinct utility weights to various components of ESG. In Appendix G, we provide a preliminary exploration of our model in which there are two independent ESG outputs and two types of green investors who put different weights on them. In line with our baseline model, we find that heterogeneity in preferences implies that green investors of different types trade differently based on their information. As a result, the impact of ESG investment can be diluted, in the sense that the price becomes noisier to all green investors but, at the same time, more informative about the firm’s financial output.

**Information acquisition**  We assume that rational investors receive informative signals but do not analyze their incentives to acquire information. As discussed in Section 4.1.3, the key mechanism of our model relies on the fact that heterogeneous investors observe informative signals about both financial and ESG outputs. Importantly, investors have incentives to acquire information about the output component that they do not value because such information helps them better interpret the price. Zhou and Kang (2023) confirm this conjecture: By considering a simplified version of our model, they find that both traditional and green investors have incentives to acquire ESG information, and the demand for ESG information increases with the share of green investors.

**Feedback from the financial market to corporate decisions**  Our results on how ESG investing affects price informativeness and the cost of capital can have implications for the decisions of corporate managers. On the one hand, an increase in the share of green investors can lead to an increase in the overall information risk, resulting in a higher cost of capital for the firm. Naturally, this can reduce real investment activity by the firm. On the other hand, an increase in the share of green investors makes the price more informative about the firm’s ESG performance, which can guide firm manager’s sustainability efforts and make their decisions along this dimension more informed (e.g., Goldstein, 2023). Furthermore, the firm’s investment activity may shift toward more sustainable projects. In particular, if a firm manager’s compensation is tied to the stock price, and the stock price becomes more sensitive to the firm’s ESG performance, the manager will be incentivized to select more sustainable projects.\(^{23}\)

\(^{23}\)A common approach to promote sustainable investment by companies is to tie executive compensation to explicit ESG targets (e.g., Cohen, Kadach, Ormazabal, and Reichelstein (2023) discuss international evidence on this practice). Our paper suggests that, in the presence of ESG investors whose trading affects the market price, tying executive compensation to the price can serve the same goal.
Multiple assets As is typical for noisy REE frameworks, our model considers a setting with a single risky asset. Novel implications of our theory are based on the presumption that this risky asset is traded by both traditional and green investors. A potential concern is that in a multi-asset framework, investors with heterogeneous preferences might form polarized portfolios such that high-ESG assets will be held by green investors and low-ESG assets will be held by traditional investors. However, even though heterogeneous investors may tilt their portfolios according to their preferences, they are still likely to trade a diverse set of assets for speculative and diversification reasons. Consistently, such an extreme investor polarization is not observed in practice. Therefore, we believe that our key assumption on investor heterogeneity is relevant for a large set of securities.

Furthermore, the presence of heterogeneous investors creates a tension between financial and ESG information contained in the price. The tension stems from the fact that one asset price cannot be a good source of information about two distinct output components simultaneously. Therefore, a firm manager may have incentives to issue multiple securities: For example, in addition to a common share, the manager might be willing to issue a security whose payoff is tied solely to the firm’s ESG output. Hypothetically, the price of this security could be a good source of ESG information, which then could be used to filter financial information from the price of the common share. In practice, however, creating well-functioning markets for such securities can be challenging, particularly if measuring the firm’s ESG output precisely is difficult. In the absence of liquid markets, prices of such securities are unlikely to be a good source of information.

Overall, we view our paper as the first step in understanding how heterogeneity in investors’ preferences affects information aggregation by asset prices. Going forward, one could study a richer model featuring multiple assets that differ in their ESG and financial profiles, in the spirit of Admati (1985).

References


MSCI (2024): “ESG Ratings Methodology,” MSCI ESG Research LLC.


VAN DER LUGT, C., P. P. VAN DE WIJS, AND D. PETROVICS (2020): Carrots & Sticks: Sustainability Reporting Policy: Global Trends in Disclosure as the ESG Agenda Goes Mainstream, Global Reporting Initiative (GRI) and the University of Stellenbosch Business School (USB).


Appendix

Appendix A: A.1 contains details about the special case considered in Section 4.1; A.2 derives the system of equations for the normalized price coefficients in the baseline model; A.3 proves Proposition 1; A.4 proves Proposition 2; A.5 proves Proposition 4. Appendix B contains proofs for Section 5. It also proves Proposition 3. Appendix C contains proofs for Section 6.1. Appendix D provides additional details about our empirical results in Section 2: D.1 describes our data sources; D.2 conducts robustness analysis; D.3 analyzes how current asset prices are associated with future financial performance; D.4 establishes the aversion of active ESG funds to carbon emission intensity. Appendix E analyzes the model with a general information structure and discusses conditions required for the multiplicity of equilibria in the trading stage. Appendix F shows that the trading stage features unique equilibrium when investors have homogeneous preferences but heterogeneous information. Appendix G considers an extension with heterogeneous ESG investors.

Proofs frequently involve tedious yet straightforward algebraic manipulations, which we perform via Matlab Symbolic Math Toolbox. Therefore, we often omit intermediate steps and present only final results. These omitted derivations are available upon request.

A Equilibrium characterization

A.1 Special case

The main text derives a system of equations (13)-(14) for \( \xi_z \) and \( \xi_\delta \). From (14),

\[
\xi_z (\xi_\delta) = \frac{\left( \frac{\tau_s m}{\gamma} - \xi_\delta \right) \left( \xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n} \right)}{\frac{\tau_s m}{\gamma} \xi_\delta}.
\]  

(27)

Plugging this expression in (13), we can derive the following quintic equation for \( \xi_\delta \):

\[
\left( \xi_\delta^3 + \frac{\tau + \tau_s}{\tau_n} \xi_\delta - \frac{\tau_s m}{\gamma} \frac{\tau + \tau_s}{\tau_n} \right) \left( \xi_\delta^2 - \frac{\tau_s m}{\gamma} \frac{\tau + \tau_s}{\tau_n} \right) = 0.
\]  

(28)

The first term on the left-hand side of (28) corresponds to the symmetric case in which \( \xi_z = \xi_\delta \). The second term on the left-hand side of (28) corresponds to the asymmetric case in which \( \xi_z \neq \xi_\delta \). In the asymmetric case we have \( \xi_\delta^2 + \frac{\tau + \tau_s}{\tau_n} = \frac{\tau_s m}{\gamma} \xi_\delta \). Plugging it in (27), we get \( \xi_z = \frac{\tau_s m}{\gamma} - \xi_\delta \).
A.2 Preliminary derivations

From (3), the aggregate demand for stock from investors of group $j \in \{t, g\}$ is

$$D_j(z, \delta, p) = m_j \left[ \frac{1}{\gamma} \frac{\gamma \beta_j^2 \frac{r_z}{\tau + r_s} + \gamma \beta_j^2 \frac{r_z}{\tau + r_s} + \left( p_z \beta_j^2 \frac{1}{\gamma} \frac{r_z}{\tau + r_s} + p_\delta \beta_j^2 \frac{1}{\gamma} \frac{r_z}{\tau + r_s} \right) \frac{\dot{p} - p_0 - \frac{r_z}{\tau + r_s} \frac{r_z}{\tau + r_s} - \delta p_0 \frac{r_z}{\tau + r_s}}}{p_1 \frac{r_z}{\tau + r_s} + p_2 \frac{r_z}{\tau + r_s} + p_3 \frac{r_z}{\tau + r_s} - \frac{r_z}{\tau + r_s}} \right].$$

(29)

Plugging (29) in (4) and equalizing coefficients in front of $\hat{z}, \hat{\delta}$, we get

$$\xi_z = \frac{\tau_s}{\gamma_s} \left[ m_t + m_g \frac{\beta_z (\xi_\delta^2 + \kappa) - \xi_\delta \xi_z \beta_\delta}{(\xi_\delta \beta_\delta - \xi_\delta \beta_z)^2 + \kappa} \right],$$

(30)

$$\xi_\delta = \frac{\tau_s}{\gamma_s} \left[ -m_t \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \kappa} + m_g \frac{\beta_\delta (\xi_z^2 + \kappa) - \xi_\delta \xi_z \beta_z}{(\xi_\delta \beta_\delta - \xi_\delta \beta_z)^2 + \kappa} \right],$$

(31)

where we define $\xi_z = \frac{p_z}{p_n}$ and $\xi_\delta = \frac{p_\delta}{p_n}$ and denote $\kappa = \frac{r_z}{\tau + r_s}$ to simplify notations. Combining the two equations, we get

$$\xi_z \beta_z + \xi_\delta \beta_\delta = \frac{\tau_s}{\gamma} \left( m_g + \beta_z m_t - \beta_\delta m_t \frac{\xi_z}{\xi_\delta} \right),$$

from which $\xi_z$ can be expressed as a function of $\xi_\delta$ as

$$\xi_z = \frac{\left( \frac{\tau_s}{\gamma_s} m_g + \frac{\tau_s}{\gamma_s} \beta_z m_t - \xi_\delta \beta_\delta \right) (\xi_\delta^2 + \kappa)}{\beta_z (\xi_\delta^2 + \kappa) + \frac{\tau_s}{\gamma_s} \beta_\delta m_t \xi_\delta}.$$  

(32)

Using this expression, we can reduce the system (30)-(31) into the equation with one unknown $\xi_\delta$:

$$\xi_\delta^5 - \hat{m}_g \xi_\delta^4 + 2\kappa \xi_\delta^3 - 2\hat{m}_g \kappa \xi_\delta^2 + \left( \kappa^2 + \hat{m}_t^2 \kappa \right) \xi_\delta - \hat{m}_g \kappa^2 = 0,$$

where, for brevity, we define $\hat{m}_g = \frac{\tau_s}{\gamma_s} \beta_\delta m_g$ and $\hat{m}_t = \frac{\tau_s}{\gamma_s} \beta_\delta m_t$.

A.3 Number of equilibria and noise precision

This section proves Proposition 1. The proof consists of three parts. A.3.1 establishes that equation (33) has at least one and at most three real roots. A.3.2 proves the existence of the threshold $\tau_n^*$. A.3.3 shows that $p_0 < 0, p_z, p_\delta, p_n > 0$. 

46
A.3.1 Number and signs of roots

Claim 1. Equation (33) has at least one and at most three real roots. All real roots are positive and are below \( \hat{m}_g \).

Proof of Claim 1.

All real roots of (33) are positive because coefficients of odd powers of \( \xi_\delta \) are positive and coefficients of even powers of \( \xi_\delta \) are negative. It is also easy to see that all roots are below \( \hat{m}_g \) because the left-hand side of (33) is clearly positive for all \( \xi_\delta \geq \hat{m}_g \).

In principle, (33) can have from one to five real roots. Below we show that it can have at most three real roots. Denote the left-hand side of (33) by

\[
f(\xi_\delta) = \xi_\delta^5 - \hat{m}_g \xi_\delta^4 + 2\kappa \xi_\delta^3 - 2\kappa \hat{m}_g \xi_\delta^2 + (\kappa^2 + \hat{m}_g^2 \kappa) \xi_\delta - \hat{m}_g \kappa^2.
\] (34)

Differentiating \( f(\xi_\delta) \), we get

\[
\frac{\partial^2 f}{\partial \xi_\delta^2} = 20\xi_\delta^3 - 12\hat{m}_g \xi_\delta^2 + 12\kappa \xi_\delta - 4\hat{m}_g \kappa.
\]

The equation \( \frac{\partial^2 f}{\partial \xi_\delta^2} = 0 \) has a unique real root because its discriminant is negative:

\[
\Delta \propto -\kappa \left( (\hat{m}_g^2 - \kappa)^2 + 4\kappa^2 \right) < 0,
\]

where \( \propto \) denotes proportionality up to a positive constant. This root is positive because coefficients of odd powers of \( \xi_\delta \) are positive, and coefficients of even powers of \( \xi_\delta \) are negative. Moreover, it is below \( \hat{m}_g \) because \( \frac{\partial^2 f}{\partial \xi_\delta^2} \bigg|_{\xi_\delta \geq \hat{m}_g} > 0 \). Hence, \( f(\xi_\delta) \) has a unique inflection point \( \xi_\delta^{\text{infl}} \in (0, \hat{m}_g) \) such that \( f(\xi_\delta) \) is concave if \( \xi_\delta < \xi_\delta^{\text{infl}} \) and convex if \( \xi_\delta > \xi_\delta^{\text{infl}} \). Given also that \( f(\xi_\delta) \) is a continuous function, it follows that it can have at most three intersections with the zero line. \( \square \)

A.3.2 Number of roots and precision of noise trading

Rewrite (33) as

\[
\frac{1}{\kappa} \left( \xi_\delta^3 + \kappa \xi_\delta - \alpha \hat{m}_g \kappa \right) \left( \xi_\delta^2 - \alpha \hat{m}_g \xi_\delta + \kappa \right) = -(1 - 2\alpha) \hat{m}_g^2 \xi_\delta,
\] (35)

where \( \hat{m} = \hat{m}_g + \hat{m}_t \) and \( \alpha = \frac{m_g}{m} \). Denote the left-hand side of the expression above by

\[
g(\xi_\delta) = \frac{1}{\kappa} \left( \xi_\delta^3 + \kappa \xi_\delta - \alpha \hat{m}_g \kappa \right) \left( \xi_\delta^2 - \alpha \hat{m}_g \xi_\delta + \kappa \right).
\] (36)

We start by establishing several useful properties of \( g(\xi_\delta) \) in Lemma 1.

Lemma 1. Define \( g(\xi_\delta) \) as in (36). Define \( \xi_\delta^* = \xi_\delta^*(\kappa, \alpha \hat{m}) \) implicitly as

\[
(\xi_\delta^*)^3 + \kappa \xi_\delta^* - \alpha \hat{m} \kappa = 0.
\] (37)
1. $g(\xi_\delta)$ has a unique inflection point $\xi_{\delta}^{infl}$ such that $g(\xi_\delta)$ is concave on $(-\infty, \xi_{\delta}^{infl})$ and convex on $(\xi_{\delta}^{infl}, \infty)$.

2. If $\kappa \geq \frac{1}{4} \alpha^2 \hat{m}^2$ then $\frac{\partial g}{\partial \xi_\delta} > 0$; equation $g(\xi_\delta) = 0$ has a unique solution $\xi_{\delta}^* \in (0, \alpha \hat{m})$; $g(\xi_\delta)$ is convex on $(\xi_{\delta}^*, \alpha \hat{m})$.

3. If $\kappa < \frac{1}{4} \alpha^2 \hat{m}^2$ then equation $g(\xi_\delta) = 0$ has three solutions, $\xi_{\delta}^{a,1}$, $\xi_{\delta}^{a,2}$ and $\xi_{\delta}^*$, such that $0 < \xi_{\delta}^{a,1} < \sqrt[3]{\kappa} < \xi_{\delta}^* < \xi_{\delta}^{a,2} < \alpha \hat{m}$.

Proof of Lemma 1.
Statement 1 directly follows from the proof of Claim 1 because $\frac{\partial^2 g}{\partial \xi_\delta^2} = \frac{1}{\kappa} \frac{\partial^2 f}{\partial \xi_\delta^2}$.

Case 1: $\kappa \geq \frac{1}{4} \alpha^2 \hat{m}^2$ (Statement 2 of Lemma 1).

Take the first derivative of $g(\xi_\delta)$:

$$\kappa \frac{\partial g}{\partial \xi_\delta} = 5\xi_{\delta}^3 - 4\alpha \hat{m} \xi_{\delta}^2 + 6\kappa \xi_{\delta}^2 - 4\alpha \hat{m} \kappa \xi_{\delta} + \kappa^2 + \alpha^2 \hat{m}^2 \kappa =$$

$$= \frac{1}{16} (2\xi_{\delta} - \alpha \hat{m})^2 (20\xi_{\delta}^2 + 4\alpha \hat{m} \xi_{\delta} + 5\alpha^2 \hat{m}^2) + \left( \kappa - \frac{1}{4} \alpha^2 \hat{m} \right) \left( 6\xi_{\delta}^2 - 4\alpha \hat{m} \xi_{\delta} + \kappa + \frac{5}{4} \alpha^2 \hat{m}^2 \right).$$

Because $\kappa \geq \frac{1}{4} \alpha^2 \hat{m}^2$, $\frac{\partial g}{\partial \xi_\delta} > 0$ and $g(\xi_\delta)$ is an increasing function. Furthermore, $g(0) < 0$ and $g(\alpha \hat{m}) > 0$, so $g(\xi_\delta) = 0$ has a unique solution $\xi_{\delta}^* \in (0, \alpha \hat{m})$. Note that $\xi_{\delta}^*$ satisfies (37). Indeed, consider (36) that defines $g(\xi_\delta)$. If $\kappa > \frac{1}{4} \alpha^2 \hat{m}^2$ then $\xi_{\delta}^2 - \alpha \hat{m} \xi_{\delta} + \kappa$ is positive. If $\kappa = \frac{1}{4} \alpha^2 \hat{m}^2$ then the solution to $\xi_{\delta}^2 - \alpha \hat{m} \xi_{\delta} + \kappa = 0$ coincides with $\xi_{\delta}^*$, defined by (37).

Finally, we show that $g(\xi_\delta)$ is convex on $(\xi_{\delta}^*, \alpha \hat{m})$. Take second derivative of $g(\xi_\delta)$:

$$\frac{\kappa}{4} \frac{\partial^2 g}{\partial \xi_\delta^2} = 5\xi_{\delta}^3 - 3\alpha \hat{m} \xi_{\delta}^2 + 3\kappa \xi_{\delta} - \alpha \hat{m} \kappa. \quad (38)$$

Plugging (37) to (38), we get

$$\frac{\kappa}{4} \frac{\partial^2 g}{\partial \xi_\delta^2} = 5\xi_{\delta}^3 - 3\alpha \hat{m} \xi_{\delta}^2 + 3\kappa \xi_{\delta} - (\xi_{\delta}^*)^3 - \kappa \xi_{\delta}^* \xi_{\delta}^* \geq (4\xi_{\delta}^2 - 3\alpha \hat{m} \xi_{\delta} + 2\kappa) \xi_{\delta}^*.$$  

The largest real root of the term in parentheses (if exists) is given by $\frac{3\alpha \hat{m} + \sqrt{9\alpha^2 \hat{m}^2 - 32\kappa}}{8} \leq \frac{1}{2} \alpha \hat{m}$, where the inequality holds because $\kappa \geq \frac{1}{4} \alpha^2 \hat{m}^2$. Note that $\xi_{\delta}^* \geq \frac{1}{2} \alpha \hat{m}$ if $\kappa \geq \frac{1}{4} \alpha^2 \hat{m}^2$. Indeed, if $\kappa = \frac{1}{4} \alpha^2 \hat{m}^2$, (37) implies $\xi_{\delta}^* = \frac{1}{2} \alpha \hat{m}$. Furthermore, applying the implicit function theorem to (37), we can see that $\frac{\partial \xi_{\delta}^*}{\partial \kappa} > 0$ for $\xi_{\delta}^* \in (0, \alpha \hat{m})$. Therefore, $\frac{\partial^2 g}{\partial \xi_\delta^2} > 0$ if $\xi_{\delta} \in (\xi_{\delta}^*, \alpha \hat{m})$.

Case 2: $\kappa < \frac{1}{4} \alpha^2 \hat{m}^2$ (Statement 3 of Lemma 1).
Consider equation \( g(\xi_\delta) = 0 \), where \( g(\xi_\delta) \) is given by (36). Define \( \xi^{g:1}_\delta \) and \( \xi^{g:2}_\delta \) as roots of \( \xi^2_\delta - \alpha \hat{m} \xi_\delta + \kappa = 0 \). Then \( \xi^{g:1}_\delta = \frac{\alpha \hat{m} - \sqrt{\alpha^2 \hat{m}^2 - 4 \kappa}}{2} \) and \( \xi^{g:2}_\delta = \frac{\alpha \hat{m} + \sqrt{\alpha^2 \hat{m}^2 - 4 \kappa}}{2} \). Clearly, \( 0 < \xi^{g:1}_\delta < \sqrt{\kappa} < \xi^{g:2}_\delta < \alpha \hat{m} \).

The third root of \( g(\xi_\delta) \) is given by \( \xi^{*}_\delta \) that solves (37). Since \( \kappa < \frac{1}{4} \alpha^2 \hat{m}^2 \), it is easy to verify from (37) that \( \xi^{*}_\delta > \sqrt{\kappa} \). Furthermore, evaluate the left-hand side of (37) at \( \xi^{g:2}_\delta \):

\[
\left( \frac{\alpha \hat{m} + \sqrt{\alpha^2 \hat{m}^2 - 4 \kappa}}{2} \right)^3 + \kappa \frac{\alpha \hat{m} + \sqrt{\alpha^2 \hat{m}^2 - 4 \kappa}}{2} \kappa < \frac{1}{4} \alpha^2 \hat{m}^2 \frac{\alpha^3 \hat{m}^3}{8} - \frac{\alpha^3 \hat{m}^3}{8} = 0.
\]

Therefore, \( \xi^{*}_\delta < \xi^{g:2}_\delta \).

We now proceed to proving the main result of Section A.3.2.

**Claim 2.** For any \( \alpha = \frac{m_g}{m_l + m_g} \in (0, 1) \) and \( \hat{m} = \frac{\gamma}{4} \beta_\delta (m_l + m_g) \), \( \exists \tau^*_n (\alpha, \hat{m}) > 0 \) such that \( \forall \tau_n \in (0, \tau^*_n) \) equation (33) has a unique solution; for \( \tau_n = \tau^*_n \) it has two solutions when \( \alpha \neq \frac{1}{2} \) and a unique solution when \( \alpha = \frac{1}{2} \); \( \forall \tau_n > \tau^*_n \) it has three solutions.

**Proof of Claim 2.**

First, note that the statement of the claim for \( \alpha = \frac{1}{2} \) follows from Lemma 1 because equation (33) can be simplified to \( g(\xi_\delta) = 0 \), where \( g(\xi_\delta) \) is given by (36). In what follows, we focus on the case with \( \alpha \neq \frac{1}{2} \).

The proof proceeds in several steps. Recall that equations (33) and (35) are equivalent. In Lemmas 2 and 3, we show that there exist \( \kappa \) and \( \bar{\kappa} \) such that equation (35) has one solution when \( \kappa > \bar{\kappa} \) and three solutions when \( \kappa < \bar{\kappa} \). In Lemma 4, we show that if for a given \( \kappa \) equation (35) has one (three) solutions, then it has one (three) solutions for any \( \bar{\kappa} \) above (below) the given \( \kappa \), respectively. Finally, we show that there exists \( \kappa^* \) such that equation (35) has two solutions, and any increase or decrease in \( \kappa \) implies that (35) has one or three solutions, respectively. Since \( \kappa = \frac{\tau + \tau_s}{\tau_n} \), there is a one-to-one mapping between \( \kappa \) and \( \tau_n \) for any given \( \tau \) and \( \tau_s \). The conditions on \( \kappa \) then can be translated into conditions on \( \tau_n \).

**Lemma 2.** \( \forall \kappa \geq \bar{\kappa} = \frac{1}{4} \alpha^2 \hat{m}^2 \) equation (35) has a unique solution.

**Proof of Lemma 2.**

Suppose that \( \alpha < \frac{1}{2} \). Equation (35) has a unique solution because the left-hand side increases in \( \xi_\delta \) by Lemma 1, \( g(0) < 0 \) and \( g(\alpha \hat{m}) > 0 \), while the right-hand side decreases in \( \xi_\delta \) and its value at \( \xi_\delta = 0 \) is zero. This case is illustrated by the intersecting solid blue line and dashed red line in Figure 6.
Suppose now that $\alpha > \frac{1}{2}$. In this case, both the left-hand side and the right-hand side of equation (35) increase in $\xi_\delta$. They still have only one intersection because, by Lemma 1, the left-hand side of (35) is an increasing convex function $\forall \xi_\delta \in (\xi_\delta^*, \alpha \hat{m})$, where $\xi_\delta^*$ is the unique real root of equation $g(\xi_\delta) = 0$. This case is illustrated by the intersecting solid blue line and dot-dashed yellow line in Figure 6.

**Lemma 3.** $\exists \kappa \in (0, \bar{\kappa})$ such that $\forall \kappa \in (0, \kappa)$ equation (35) has three solutions.

**Proof of Lemma 3.**

Write (35) in its original form as in (33),

$$f(\xi_\delta) = \xi_\delta^5 - \alpha \hat{m} \xi_\delta^4 + 2\kappa \xi_\delta^3 - 2\alpha \hat{m} \kappa \xi_\delta^2 + (\kappa^2 + (1 - \alpha)^2 \hat{m}^2 \kappa) \xi_\delta - \alpha \hat{m} \kappa^2 = (\xi_\delta - \alpha \hat{m}) \left( \xi_\delta^4 + 2\kappa \xi_\delta^2 + \kappa^2 \right) + (1 - \alpha)^2 \hat{m}^2 \kappa \xi_\delta = 0.$$ 

Notice that $f(\alpha \hat{m}) > 0$. At the same time, we can always pick a sufficiently small $\kappa_1 > 0$ such that $\forall \kappa \in (0, \kappa_1)$,

$$f(\alpha \hat{m} - \sqrt{\kappa}) = -\sqrt{\kappa} \left( (\alpha \hat{m} - \sqrt{\kappa})^4 + 2\kappa (\alpha \hat{m} - \sqrt{\kappa})^2 + \kappa^2 \right) + (1 - \alpha)^2 \hat{m}^2 \kappa (\alpha \hat{m} - \sqrt{\kappa}) < 0.$$
Notice also that $f(0) < 0$. Evaluate $f(.)$ at $\frac{\alpha}{(1-\alpha)^2 m} \kappa + \kappa$,

$$
f \left( \frac{\alpha}{(1-\alpha)^2 m} \kappa + \kappa \right) = \kappa^2 \left( \frac{\alpha}{(1-\alpha)^2 m} \right)^3 + \alpha \hat{m} \kappa \left( \frac{\alpha}{(1-\alpha)^2 m} \right)^4 + 2 \kappa^2 \left( \frac{\alpha}{(1-\alpha)^2 m} + 1 \right) - 2 \alpha \hat{m} \kappa \left( \frac{\alpha}{(1-\alpha)^2 m} + 1 \right)^2 + \kappa \left( \frac{\alpha}{(1-\alpha)^2 m} + 1 \right) + (1-\alpha)^2 m^2 \right].
$$

$\exists \kappa_2 > 0$ such that $\forall \kappa \in (0, \kappa_2)$, $f \left( \frac{\alpha}{(1-\alpha)^2 m} \kappa + \kappa \right) > 0$ because the last term in the expression in brackets, $(1-\alpha)^2 \hat{m}^2$, does not depend on $\kappa$, while the other terms are proportional to $\kappa^b$, $b = 1, 2, 3$.

Finally, define $\kappa_3$ such that $\frac{\alpha}{(1-\alpha)^2 m} \kappa_3 + \kappa_3 = \alpha \hat{m} - \sqrt{\kappa_3}$. Therefore, $\forall \kappa \in (0, \kappa_3)$, $\frac{\alpha}{(1-\alpha)^2 m} \kappa + \kappa < \alpha \hat{m} - \sqrt{\kappa}$. Define $\kappa = \min \{ \kappa_1, \kappa_2, \kappa_3 \}$. Then $\forall \kappa \in (0, \kappa)$ a continuous function $f(\xi_\delta)$ changes its sign from negative to positive (at least) twice. Hence equation (35) has (at least) three solutions. Since it cannot have more than three solutions by Claim 1, it must be that it has exactly three solutions.  

\[ \blacksquare \]

**Lemma 4.** For any $\kappa > 0$, if equation (35) has three solutions at $\kappa$, then it has three solutions $\forall \kappa \in (0, \kappa)$; if equation (35) has one solution at $\kappa$, then it has one solution $\forall \kappa > \kappa$.

**Proof of Lemma 4.**

Since the result trivially holds when $\kappa \in (0, \tilde{\kappa}]$ and $\kappa \geq \tilde{\kappa}$, where $\hat{\kappa}$ and $\kappa$ are defined in Lemmas 2 and 3, we focus on the case when $\kappa \in (\kappa, \tilde{\kappa})$.

Consider equation $g(\xi_\delta) = 0$, where $g(\xi_\delta)$ is defined by (36). For $\kappa < \tilde{\kappa} = \frac{1}{4} \alpha^2 \hat{m}^2$, this equation has three solutions by Lemma 1. Differentiate $g(\xi_\delta)$ with respect to $\kappa$:

$$
\frac{\partial g}{\partial \kappa} = -\frac{1}{\kappa^2} \left( \xi_\delta^2 + \kappa \right) \left( \xi_\delta + \sqrt{\kappa} \right) \left( \xi_\delta - \sqrt{\kappa} \right) \left( \xi_\delta - \alpha \hat{m} \right).
$$

Then $\frac{\partial g}{\partial \kappa} < 0$ if $\xi_\delta \in (0, \sqrt{\kappa})$ and $\frac{\partial g}{\partial \kappa} > 0$ if $\xi_\delta \in (\sqrt{\kappa}, \alpha \hat{m})$. In particular, notice that $\frac{\partial g}{\partial \kappa} |_{\xi_\delta = \xi_\delta^*} > 0$ where $\xi_\delta^*$ solves (37). This is because $\xi_\delta^* \in (\sqrt{\kappa}, \alpha \hat{m})$ by Lemma 1.

In what follows, we evaluate the number of roots of equation (35). We split our analyses in two cases.

**Case 1:** $\alpha < \frac{1}{2}$

Suppose $3 \tilde{\kappa} \in (\kappa, \tilde{\kappa})$ such that equation (35) has three solutions. This case is illustrated in panel (A) of Figure 7. From the graph it is evident that the smallest root $\xi_\delta^1$ of (35) is smaller than $\xi_\delta^{2,1}$—the smallest root of $g(\xi_\delta) = 0$ defined in Lemma 1. By Lemma
1. \( \xi_{\delta}^{g,1} < \sqrt{\kappa} \), therefore, \( \xi_{\delta}^{i} < \xi_{\delta}^{g,1} < \sqrt{\kappa} \). Furthermore, this solution \( \xi_{\delta}^{i} < \sqrt{\kappa} \) exists for any \( \kappa \).

The other two roots are above \( \xi_{\delta}^{i} \) and \( \sqrt{\kappa} \): \( \xi_{\delta}^{ii} > \xi_{\delta}^{i} > \xi_{\delta}^{i} > \sqrt{\kappa} \). In the region \( \xi_{\delta} \in (\sqrt{\kappa}, \alpha \hat{\kappa}) \), \( \frac{\partial g}{\partial \kappa} > 0 \) and a marginal decrease in \( \kappa \) shifts \( g(\xi_{\delta}) \) (blue solid line) downwards. At the same time, the right-hand side of equation (35) (red dashed line) does not depend on \( \kappa \) and thus does not move. Therefore, for a marginally smaller \( \kappa \) equation (35) still has three solutions.

An analogous argument holds if for a given \( \kappa \) there is a unique solution to (35). In particular, the unique solution is \( \xi_{\delta}^{i} < \sqrt{\kappa} \), and there is no intersection between the left-hand side, \( g(\xi_{\delta}) \), and the right-hand side of (35) on \( \xi_{\delta} \in (\sqrt{\kappa}, \alpha \hat{m}) \). A marginal increase in \( \kappa \) shifts \( g(\xi_{\delta}) \) up, while the right-hand side line does not move, which implies that equation (35) still has a unique solution.

\[
(A) \quad \alpha < \frac{1}{2} \quad \text{and} \quad (B) \quad \alpha > \frac{1}{2}
\]

Figure 7: Three solutions to equation (35) when \( \alpha < \frac{1}{2} \) (panel A) and \( \alpha > \frac{1}{2} \) (panel B).

**Case 2:** \( \alpha > \frac{1}{2} \)

Suppose \( \exists \kappa \in (\kappa, \bar{\kappa}) \) such that equation (35) has three solutions. This case is illustrated in panel (B) of Figure 7. In this graph, two black lines (marked with crosses and circles) are tangent to the convex and the concave parts of \( g(\xi_{\delta}) \), respectively. Recall from Lemma 1 that \( g(\xi_{\delta}) \) has a unique inflection point \( \xi^{\text{infl}}_{\delta} \), and it is concave on \( \xi_{\delta} \in (0, \xi^{\text{infl}}_{\delta}) \) and convex on \( (\xi^{\text{infl}}_{\delta}, \alpha \hat{\kappa}) \). Two tangent points, \( \xi^{\text{tang},1}_{\delta} < \xi^{\text{infl}}_{\delta} < \xi^{\text{tang},2}_{\delta} \), solve

\[
h(\xi_{\delta}) = \frac{\partial g(\xi_{\delta})}{\partial \xi_{\delta}} - g(\xi_{\delta}) = \frac{1}{\kappa} \left( 4 \xi_{\delta}^{5} - 3 \alpha \hat{m} \xi_{\delta}^{4} + 4 \kappa \xi_{\delta}^{3} - 2 \alpha \hat{m} \kappa \xi_{\delta}^{2} + \alpha \hat{m} \kappa \right) = 0.
\]

Notice that \( \frac{\partial h}{\partial \xi_{\delta}} = \xi_{\delta} \frac{\partial^2 g}{\partial \xi_{\delta}^2} \). Therefore, \( h(\xi_{\delta}) \) is decreasing on \( \xi_{\delta} \in (0, \xi^{\text{infl}}_{\delta}) \) and increasing
on $\xi_\delta \in (\xi^{in}_\delta, \alpha \hat{m})$.

Evaluate $h(\xi_\delta)$ at $\sqrt{\kappa}$: $h(\sqrt{\kappa}) = 4\kappa (2\sqrt{\kappa} - \alpha \hat{m}) < 0$ because $\kappa < \bar{\kappa} = \frac{1}{4} \alpha^2 \hat{m}^2$. Because $h(0) > 0$, $h(\alpha \hat{m}) > 0$ and $h(\sqrt{\kappa}) < 0$, $h(\xi_\delta) = 0$ has two solutions $\xi^{tang,1}_\delta < \sqrt{\kappa} < \xi^{tang,2}_\delta$. The two tangent lines shown in panel (B) of Figure 7 go through zero and are thus described by equations $f^{tang,k}(\xi_\delta) = g(\xi^{tang,k}_\delta) \xi_\delta$, $k = 1, 2$.

The right-hand side of equation (35), i.e., $-(1 - 2\alpha) \hat{m}^2 \xi_\delta$, intersects $g(\xi_\delta)$ three times when its slope is smaller than the slope of the tangent line $f^{tang,1}(\xi_\delta)$, so the smallest root of (35) is $\xi_\delta < \xi^{tang,1}_\delta < \sqrt{\kappa}$. In the region $\xi_\delta \in (0, \sqrt{\kappa})$, $\frac{\partial g}{\partial \kappa} < 0$ and a marginal decrease in $\kappa$ shifts $g(\xi_\delta)$ (blue solid line) upwards. At the same time, the right-hand side of equation (35) (yellow dot-dashed line) does not depend on $\kappa$ and does not move in response to the change in $\kappa$. Therefore, for a marginally smaller $\kappa$ equation (35) still has three solutions.

An analogous argument holds if for a given $\kappa$ there is a unique solution to (35), i.e., when the slope of $f^{tang,1}(\xi_\delta)$ is below the slope of the right-hand side of (35). In the region $\xi_\delta \in (0, \sqrt{\kappa})$, there is no intersection between the left-hand side, $g(\xi_\delta)$, and the right-hand side of (35). A marginal increase in $\kappa$ shifts $g(\xi_\delta)$ down, while the right-hand side line does not move, which implies that equation (35) still has one solution.

Having proved Lemmas 2, 3, 4, we are now ready to complete the proof of Claim 2. These lemmas imply that there exist $\bar{\kappa} > 0$ such that equation (35) has three solutions if $\kappa \in (0, \bar{\kappa})$, two solutions if $\kappa \in [\bar{\kappa}, \kappa^*]$, and one solution if $\kappa > \kappa^*$. In addition, it must be the case that $\kappa^* = \bar{\kappa} = \kappa^*$. To see this, focus on the case $\alpha > \frac{1}{2}$ without loss of generality. Equation (35) then has two solutions if and only if the right-hand side of (35) coincides with the tangent line $f^{tang,1}(\xi_\delta)$ (see panel (B) Figure 7). However, from the proof of Lemma 4 it follows that any marginal increase or decrease in $\kappa$ leaves equation (35) with one or three solutions, respectively.

Finally, recall that $\kappa = \frac{\tau_\delta - \tau_\tau}{\tau_s}$. Given $\tau$ and $\tau_s$, define $\tau^*_n = \frac{\tau_\delta + \tau_\tau}{\tau_s}$. Then (35) has two solutions if $\tau_n = \tau^*_n$, one solution if $\tau_n \in (0, \tau^*_n)$ and three solutions if $\tau_n > \tau^*_n$. \hfill \Box

### A.3.3 Signs of the price coefficients

**Claim 3.** $p_0 < 0$, $p_z > 0$, $p_\delta > 0$, $p_n > 0$.

**Proof of Claim 3.**

\[^{24}\text{Notice also that } \xi^{tang,1}_\delta < \frac{\alpha \hat{m}}{2} < \xi^{tang,2}_\delta \text{ because } h\left(\frac{\alpha \hat{m}}{2}\right) = \alpha \hat{m} \left(-\frac{\alpha^2 \hat{m}^2}{4} + \kappa\right) \left(-\frac{\alpha^2 \hat{m}^2}{4} + \kappa\right) < 0.\] This will be used in the proof of Proposition 2.
By Claim 1, all roots of (33) are positive and below \( \hat{m}_g = \frac{z_2}{\gamma} \beta_g m_g \). From (32), it then follows that \( \xi_z > 0 \). Recall that \( \xi_\delta = \frac{p_\delta}{p_n} \) and \( \xi_z = \frac{p_z}{p_n} \). Therefore, \( p_z \), \( p_\delta \) and \( p_n \) have the same sign.

Matching coefficients in the market clearing condition (4) implies

\[
\frac{1}{\tau + \tau_s} = \frac{1}{\gamma} p_n \left[ m_t \left( p_z^2 + p_\delta^2 + p_n^2 \kappa - p_z \right) \right] + m_g \left( p_z^2 + p_\delta^2 + p_n^2 \kappa - (p_z \beta_z + p_\delta \beta_\delta) \right),
\]

(39)

where, as above, \( \kappa = \frac{z_1 + z_2}{\tau_n} \). Clearly, if \( p_z \), \( p_\delta \) and \( p_n \) are all negative, then the right-hand side is negative. Therefore, \( p_z \), \( p_\delta \) and \( p_n \) are all positive.

We are left to show that \( p_0 < 0 \). Again, by matching coefficients in the market clearing condition, we have

\[
p_0 = -\frac{\gamma}{\tau + \tau_n} \left[ m_t \frac{\xi_\delta^2 + \xi_z^2 + \kappa}{\xi_z^2 + \kappa} + m_g \frac{\xi_\delta^2 + \xi_z^2 + \kappa}{(\xi_\delta - \xi_z \delta)^2 + \kappa} \right]^{-1} < 0.
\]

\( \Box \)

Proposition 1 follows from Claims 1, 2 and 3. Note that Claim 2 expresses the multiplicity threshold \( \tau_n^* \) as a function of \( \alpha \) and \( \hat{m} \): \( \tau_n^* = \tau_n^*(\alpha, \hat{m}) \). Because \( \hat{m} = \frac{z_1}{\gamma} \beta_\delta m \) and \( \beta_\delta \) do not show up elsewhere in equation (35), we can alternatively express the multiplicity threshold \( \tau_n^* \) as a function of \( \alpha \) and \( \beta_\delta \), \( \tau_n^* = \tau_n^*(\alpha, \beta_\delta) \).

### A.4 Comparative statics of \( \tau_n^* \)

This section establishes comparative statics properties of \( \tau_n^* \) stated in Proposition 2.

**Proof of Proposition 2.**

Claim 2 implies that the multiplicity threshold \( \tau_n^* \) can be written as a function of \( \hat{m} \) and \( \alpha \), \( \tau_n^* = \tau_n^*(\alpha, \hat{m}) \). Below, we explore comparative statics with respect to \( \hat{m} \) and \( \alpha \). Because \( \hat{m} = \frac{z_1}{\gamma} \beta_\delta m \) and \( \beta_\delta \) do not show up elsewhere in equation (35), the comparative statics with respect to \( \beta_\delta \) is equivalent to the one with respect to \( \hat{m} \).

**Comparative statics with respect to \( \hat{m} \)**

Divide (35) by \( \hat{m}^2 \) to get

\[
\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa) = \frac{1}{\kappa} \left[ \frac{1}{\hat{m}^2} (\xi_\delta - \alpha \hat{m}) (\xi_\delta^2 + \kappa)^2 + \alpha^2 \xi_\delta \right] = -(1 - 2\alpha) \xi_\delta.
\]

(40)

Then \( \frac{\partial}{\partial \hat{m}} \left( \frac{\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa)}{\hat{m}} \right) = \frac{\hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa)}{\hat{m}^2} (\xi_\delta^2 + \kappa)^2 \), so \( \hat{m}^{-2} g(\xi_\delta, \hat{m}, \kappa) \) increases in \( \hat{m} \) when \( \xi_\delta \in (0, \frac{a_n}{2}) \) and decreases in \( \hat{m} \) when \( \xi_\delta \in (\frac{a_n}{2}, a_m) \).

Suppose that \( \alpha < \frac{1}{2} \). Fix \( \hat{m}_1 > 0 \). By definition, at \( \tau_n = \tau_n^*(\alpha, \hat{m}_1) \) equation (40) has two solutions. This is illustrated by the solid blue and the red dashed lines in panel (A) of Figure 8, which intersect twice (in particular, the largest intersection \( \xi_\delta^{tang, 2} \) is a tangent
point). Recall that $\xi_{\text{tang}}^{2} > \frac{a\hat{m}_{1}}{2}$ (see footnote 24). Therefore, a marginal increase in $\hat{m}$ from $\tilde{m}_{1}$ to $\hat{m}_{1} + d\hat{m}$ shifts the curve $\hat{m}_{1}^{-2}g(\xi_{5}, \tilde{m}_{1}, \kappa^{*}(\alpha, \hat{m}_{1}))$ down $\forall \xi_{\delta} \in \left( \frac{a\hat{m}_{1}}{2}, a\hat{m}_{1} \right)$, as shown in panel (A) of Figure 8 (crossed blue solid line). The right-hand side of (40) does not depend on $\hat{m}$ and thus does not move. Therefore, there exist three solutions to (40) if $\hat{m} = \hat{m}_{1} + d\hat{m}$. From Lemma 4 it then follows that $\frac{\partial \tau^{*}_{n}}{\partial \hat{m}} < 0$.

(A) With respect to $\hat{m}$

\begin{equation}
\hat{m}_{2} = \hat{m}_{1} + d\hat{m} \text{ and } \alpha < 1/2
\end{equation}

(B) With respect to $\hat{\alpha}$

\begin{equation}
\tau_{n}^{*}(\alpha, \hat{m}) \text{ with respect to } \hat{m} \text{ (panel A)} \text{ and } \alpha \text{ (panel B)}.
\end{equation}

Figure 8: Comparative statics of $\tau_{n}^{*}(\alpha, \hat{m})$ with respect to $\hat{m}$ (panel A) and $\alpha$ (panel B).

Analogous arguments can be made to show that $\frac{\partial \tau^{*}_{n}}{\partial \hat{m}} < 0$ if $\alpha > \frac{1}{2}$.

If $\alpha = \frac{1}{2}$, we can get the analytical solution for $\tau_{n}^{*}(\alpha, \hat{m})$ (see Lemma 1), $\tau_{n}^{*}(\frac{1}{2}, \hat{m}) = \frac{16(\tau + \tau_{s})}{\hat{m}^{2}}\hat{m}$. Therefore, $\frac{\partial \tau^{*}_{n}}{\partial \hat{m}} < 0$ if $\alpha = \frac{1}{2}$.

**Comparative statics with respect to $\alpha$**

Equation (33) can be rewritten as

\begin{equation}
\frac{1}{\kappa} \left( \xi_{\delta}^{5} - a\hat{m}\xi_{\delta}^{4} + 2\kappa\xi_{\delta}^{3} - 2a\hat{m}\kappa\xi_{\delta}^{2} + \kappa^{2}\xi_{\delta} - \alpha\hat{m}\kappa^{2} \right) = -(1 - \alpha)^{2}\hat{m}^{2}\xi_{\delta}.
\end{equation}

Denote the left-hand side of (41) by $\tilde{g}(\xi_{\delta})$. Note that $\tilde{g}(\xi_{\delta}) = g(\xi_{\delta}) - \alpha^{2}\hat{m}^{2}\xi_{\delta}$, where $g(\xi_{\delta})$ is given by (36). Since $\tilde{g}(\xi_{\delta})$ and $g(\xi_{\delta})$ differ only by a linear term in $\xi_{\delta}$, they are both concave if $\xi_{\delta} < \xi_{\delta}^{\text{infl}}$ and convex if $\xi_{\delta} > \xi_{\delta}^{\text{infl}}$. In addition, they have the same two tangent points $\xi_{\delta}^{\text{tang},1} < \xi_{\delta}^{\text{infl}} < \xi_{\delta}^{\text{tang},2}$ defined in the proof of Lemma 4.

By definition, if $\tau_{n} = \tau_{n}^{*}(\alpha, \hat{m})$ and $\alpha \neq \frac{1}{2}$, equation (41) has two solutions. (41) has two solutions if and only if its right-hand side is tangent to $\tilde{g}(\xi_{\delta})$. In particular, if $\alpha > \frac{1}{2}$, (41) has two solutions if its right-hand side is $f^{\text{tang},1}(\xi_{\delta}) - \alpha^{2}\hat{m}^{2}\xi_{\delta}$, where $f^{\text{tang},1}(\xi_{\delta})$ is defined in the proof of Lemma 4. Similarly, if $\alpha < \frac{1}{2}$, (41) has two solutions if its right-hand side is $f^{\text{tang},2}(\xi_{\delta}) - \alpha^{2}\hat{m}^{2}\xi_{\delta}$. Panel (B) of Figure 8 illustrates both cases.

Suppose that $\alpha < \frac{1}{2}$ and $\tau_{n} = \tau_{n}^{*}(\alpha, \hat{m})$. Following a marginal increase in $\alpha$, the
left-hand side of (41), shown by the blue solid line in Figure Panel (B) of Figure 8, moves downwards. At the same time, the right-hand side, shown by the black crossed line in the same figure, rotates counterclockwise around the zero point. Therefore, (41) has three solutions. From Lemma 4 it then follows that \( \frac{\partial \tau_n^*}{\partial \alpha} < 0 \).

Analogous arguments can be made to show that \( \frac{\partial \tau_n^*}{\partial \alpha} > 0 \) when \( \alpha > \frac{1}{2} \).

A.5 Stability of equilibria

Plugging expression (32) for \( \xi_z(\xi_\delta) \) in the right-hand side of (31), we can write \( \xi_\delta = J(\xi_\delta) \). Moreover, \( J(\xi_\delta) - \xi_\delta = -k(\xi_\delta) \times f(\xi_\delta) \), where \( k(\xi_\delta) > 0 \) \( \forall \xi_\delta \) and \( f(\xi_\delta) \) is given by (34). We call an equilibrium stable if the dynamics around the equilibrium \( \xi_\delta^* \) are locally stable, i.e.,

\[
\frac{\partial [J(\xi_\delta) - \xi_\delta]}{\partial \xi_\delta} \bigg|_{\xi_\delta = \xi_\delta^*} = -k(\xi_\delta^*)f'(\xi_\delta^*) < 0.
\]

At any solution \( \xi_{\delta_0}^{root} \) such that \( f(\xi_{\delta_0}^{root}) = 0 \), we have

\[
\frac{\partial [J(\xi_\delta) - \xi_\delta]}{\partial \xi_\delta} \bigg|_{\xi_\delta = \xi_{\delta_0}^{root}} = -k(\xi_{\delta_0}^{root})f'(\xi_{\delta_0}^{root}).
\]

By Claim 1, \( f(\xi_\delta) = 0 \) has at least one root and at most three roots, and all roots are positive. Furthermore, \( f(0) < 0 \). Therefore, if there exists a unique root to \( f(\xi_\delta) = 0 \), \( f'(\xi_{\delta_0}^{root}) > 0 \). When there are three roots \( \xi_{i_1}^* < \xi_{i_2}^* < \xi_{i_3}^* \), \( f'(\xi_{\delta_0}^{root}) > 0 \) for \( \xi_{\delta_0}^{root} = \xi_{i_1}^* \), \( \xi_{i_2}^* \) and \( \xi_{i_3}^* \). Since \( \xi_{i_1}^* \), \( \xi_{i_2}^* \) and \( \xi_{i_3}^* \) correspond to T-, M- and G-equilibria, respectively, Proposition 4 follows.

B Growth of green investors

B.1 Price informativeness

In this section, we analyze how price informativeness changes as the fraction of green investors \( \alpha \) increases and prove Proposition 5. We also argue at the end of this section that Proposition 3 follows from the proof of Proposition 5.

Proof of Proposition 5.

Denote \( \tilde{x} = \beta_\delta \tilde{z} - \beta_\alpha \tilde{\delta} \) and \( \xi_x = \beta_\delta \xi_z - \beta_\alpha \xi_\delta \). Using (33) and \( \xi_z = \xi_z(\xi_\delta) \) from (32), we can rewrite the system of equations (30)-(31) as

\[
\xi_\delta = \frac{\alpha \hat{m} \kappa}{\xi_z^2 + \kappa} \quad \text{and} \quad \xi_x = \frac{(1 - \alpha) \hat{m} \kappa}{\xi_\delta^2 + \kappa},
\]

(42)

where \( \hat{m} = \frac{\tau_s^*}{\tau_s} \beta_\delta \hat{m}. \) Clearly, both \( \xi_x \) and \( \xi_\delta \) are positive. Taking derivatives of (42) with respect to \( \alpha \), we obtain \( 2 \xi_x \xi_\delta \xi_\delta' + (\xi_z^2 + \kappa) \xi_\delta' - \kappa \hat{m} = 0 \) and \( (\xi_\delta^2 + \kappa) \xi_x' + 2 \xi_x \xi_\delta \xi_\delta' + \kappa \hat{m} = 0. \)
Here we use the prime symbol to denote derivatives with respect to \( \alpha \). Simplifying these equations, we get

\[
\xi'_x = \frac{\xi_x^2 + 2 \xi_x \xi_\delta + \kappa \xi'_\delta}{\xi_\delta^2 + 2 \xi_x \xi_\delta + \kappa \xi'_\delta}. \tag{43}
\]

Rewriting definitions (22) and (23), we get

\[
PI_t = \frac{\tau + \tau_s}{\beta^2 \delta} \left( \frac{\beta \xi_x \xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta^2 \kappa}{\xi_\delta^2 + \kappa} \right) \quad \text{and} \quad PI_g = \frac{\tau + \tau_s}{\beta^2 \delta} \left( \frac{2 \beta \xi_x \xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta^2 \kappa}{\xi_\delta^2 + \kappa} \right).
\]

Below we analyze the comparative statics of \( PI_t \), \( PI_g \) and \( v \) with respect to \( \alpha \).

**Comparative statics of \( PI_t \) and \( PI_g \) with respect to \( \alpha \)**

\[
\frac{dPI_t}{d\alpha} = 2 \frac{(\tau + \tau_s)}{\beta^2} \left( \beta \xi_x \xi_\delta + \beta \xi_x \xi'_\delta + \xi_\delta \xi'_\delta + \xi_x \xi'_x \right) \left( \frac{\xi_\delta^2 + \kappa}{\xi_\delta^2 + \kappa} \right) - \left( 2 \beta \xi_x \xi_\delta + \xi_\delta^2 + \xi_x^2 + \beta^2 \kappa \right) \xi_x \xi'_\delta.
\]

Substituting in \( \xi'_x \) from (43), we can rewrite the above expression as \( \frac{dPI_t}{d\alpha} = -\xi'_\delta \times A_1(\xi_\delta, \xi_x) \), where \( A_1(\xi_\delta, \xi_x) \) is a function that takes positive values for \( \xi_\delta > 0 \) and \( \xi_x > 0 \). Hence, the sign of \( \frac{dPI_t}{d\alpha} \) is the same as the sign of \( -\xi'_\delta \).

Using the same approach as for \( PI_t \), we find that \( \frac{dPI_g}{d\alpha} = A_2(\xi_\delta, \xi_x) \), where \( A_2(\xi_\delta, \xi_x) \) is a function that takes positive values for \( \xi_\delta > 0 \) and \( \xi_x > 0 \). Hence, the sign of \( \frac{dPI_g}{d\alpha} \) is the same as the sign of \( \xi'_\delta \).

**Comparative statics of \( v \) with respect to \( \alpha \)**

\[
\frac{dv}{d\alpha} = \frac{dPI_g}{d\alpha} PI_g - \frac{dPI_g}{d\alpha} PI_t = -\xi'_\delta A_1(\xi_\delta, \xi_x) PI_g + A_2(\xi_\delta, \xi_x) PI_t.
\]

Hence, the sign of \( \frac{dv}{d\alpha} \) is the same as the sign of \( -\xi'_\delta \).

**Comparative statics of \( \xi_\delta \) with respect to \( \alpha \)**

\( \xi_\delta \) is implicitly defined by equation (33). Using the implicit function theorem, we get \( \xi'_\delta = \left( \frac{df}{\delta \xi_\delta} \right)^{-1} (\hat{m} \xi_\delta^4 + 2 \hat{m} \kappa \xi_\delta^2 + 2(1-\alpha)\hat{m} \kappa \xi_\delta + \hat{m} \kappa^2) \). Therefore, the sign of \( \xi'_\delta \) is the same as the sign of \( \frac{df}{\delta \xi_\delta} \).

By Claim 1, \( f(\xi_\delta) = 0 \) has at least one root and at most three roots, and all roots are positive. Furthermore, \( f(0) < 0 \). Therefore, if there exists a unique root to \( f(\xi_\delta) = 0 \), \( f'(\xi_\delta^{\text{root}}) > 0 \). When there are three roots \( \xi_\delta^i < \xi_\delta^ii < \xi_\delta^iii \), \( f'(\xi_\delta^{\text{root}}) > 0 \) for \( \xi_\delta^{\text{root}} = \xi_\delta^i, \xi_\delta^ii \) and
\( f'(\xi_{\text{root}}) < 0 \) for \( \xi_{\text{root}} = \xi_{\text{ii}} \).
Since \( \xi_{\text{ii}}, \xi_{\text{iii}} \) and \( \xi_{\text{iii}} \) correspond to T-, M- and G-equilibria, respectively, comparative statics results of Proposition 5 follow.

Relative price informativeness across equilibria

Suppose that multiple equilibria are possible, that is, \( \tau_n > \tau_n^* (\frac{1}{2}, \beta_{\delta}) \). The existence of \( \bar{\alpha} \) and \( \alpha \), defined in Proposition 5, follows from Proposition 2. Notice that at \( \alpha = \alpha \) and \( \alpha = \bar{\alpha} \), there are two equilibria such that \( \xi_{\text{ii}}^T(\alpha) = \xi_{\text{ii}}^M(\alpha) = \xi_{\text{ii}}^G(\alpha) \) and \( \xi_{\text{ii}}^T(\bar{\alpha}) = \xi_{\text{ii}}^M(\bar{\alpha}) < \xi_{\text{ii}}^G(\bar{\alpha}) \).

Figure 9 shows \( \xi_{\delta} \) as a function of \( \alpha \), where the monotonicity properties of \( \xi_{\delta} \) with respect to \( \alpha \) have been established above. In particular, \( \xi_{\delta}(\alpha) \) is an increasing function in the T- and G-equilibria and is a decreasing function in the M-equilibrium.

Equations (42) imply \( \xi_{\delta}(\alpha) \) and \( \xi_{\delta}(\alpha) \) are symmetric around \( \alpha = \frac{1}{2} \) such that that \( \xi_{\delta}(\alpha) = \xi_{\delta}(1 - \alpha) \). This is illustrated in Figure 9 for the T-equilibrium. This symmetry implies that \( 1 - \alpha = \bar{\alpha} \). It also implies that in the T-equilibrium \( \xi_{\delta}(\alpha) < \xi_{\delta}(\alpha) \). Using the definition of the relative price informativeness (44), we conclude that in the T-equilibrium \( v_T > 1 \). Analogously, in the G-equilibrium \( v_G < 1 \). Finally, \( v_T > v_M > v_G \).

Corollary 2. If equilibrium is unique, \( \xi_{\delta}' > 0 \) and \( \xi_{\delta} \geq \xi_{\delta} \) if \( \alpha \geq \frac{1}{2} \). If there are multiple equilibria, \( \xi_{\delta}' > 0 \) and \( \xi_{\delta} < \xi_{\delta} \) in the T-equilibrium, \( \xi_{\delta}' > 0 \) and \( \xi_{\delta} > \xi_{\delta} \) in the G-equilibrium, and \( \xi_{\delta}' < 0 \) and \( \xi_{\delta} \geq \xi_{\delta} \) if \( \alpha \leq \frac{1}{2} \) in the M-equilibrium.
B.2 Cost of capital

In this section, we prove Proposition 6 and Corollary 1. First, we express the cost of capital in its general form when the firm’s expected output is non-zero, i.e.,

$$CoC = E[\hat{z} - \hat{p}] = \mu_z - p_0 - p_z \mu_z - \rho_\delta \mu_\delta.$$  \hspace{1cm} (45)

With non-zero expected \( \hat{z} \) and \( \hat{\delta} \), the aggregate demand by type-\( j \) investors is given by

$$D^j(\hat{z}, \hat{\delta}, \hat{p}) = \frac{m^j \beta_0^{1 + \tau_s + \nu_{\alpha}} + \beta_\delta^{1 + \nu_{\alpha}} + \beta_z^{1 + \nu_{\alpha}}}{\gamma} \frac{p_0 \beta_0^{1 + \nu_{\alpha}}}{\gamma} \frac{p_0 \beta_\delta^{1 + \nu_{\alpha}}}{\gamma} \frac{p_0 \beta_z^{1 + \nu_{\alpha}}}{\gamma} - \hat{p}.$$

This expression is analogous to (29) in the zero-mean case. Plugging the above expression in (4) and equalizing coefficients in front of \( \hat{z}, \hat{\delta}, \) and \( \hat{n} \), we can verify that the equilibrium price coefficients \( p_z, p_\delta \) and \( p_n \) remain the same as in the zero-mean case. However, \( p_0 \) is different, \( p_0 = \frac{\gamma (\hat{z}_x (\mu_x + \mu_\xi) - 1)}{m_t PI_t + m_g PI_g} \), where \( PI_t \) and \( PI_g \) are the price informativeness to traditional and green investors, given by (22) and (23), respectively. We express the non-normalized price coefficients \( p_z \) and \( p_\delta \) in terms of the normalized price coefficients \( \xi_z \) and \( \xi_\delta \). Rewriting (39), we obtain

$$p_n = (\tau + \tau_s) \frac{m_t \xi_z + \rho \xi_\delta}{m_t PI_t + m_g PI_g}.$$

Using (42) and expressions for \( p_0, p_n, p_z = \xi_z \times p_n, p_\delta = \xi_\delta \times p_n \), we can write (45) as

$$CoC = c_z \mu_z + c_\delta \mu_\delta + \frac{\gamma}{m_t PI_t + m_g PI_g},$$

(46)

where \( c_z = \frac{\beta_z \xi_z}{\beta_z \xi_z + (1 - \beta_z) \xi_\delta} \) and \( c_\delta = -\frac{\beta_\delta \xi_\delta}{\beta_\delta \xi_\delta + (1 - \beta_\delta) \xi_z} \). Note that if \( \mu_z = \mu_\delta = 0 \), this expression reduces to (24).

**Proof of Proposition 6.**

If \( \mu_z = \mu_\delta = 0 \), \( CoC = \frac{\gamma}{m_t PI_t + m_g PI_g} \). Differentiating \( CoC \) with respect to \( \alpha \) and substituting in \( \xi_z \) from (43), we get \( \frac{dCoC}{d\alpha} = -(\xi_\delta - \xi_z) \xi_\delta \times A_3(\xi_\delta, \xi_z) \), where \( A_3(\xi_\delta, \xi_z) \) is a function that takes positive values for any \( \xi_\delta > 0 \) and \( \xi_z > 0 \). Then the comparative statics of \( CoC \) with respect to \( \alpha \) follow from Corollary 2.

**Proof of Corollary 1.**

59
As was established at the beginning of this appendix, $p_z$, $p_{\delta}$ and $p_n$ do not depend on $\mu_z$ and $\mu_\delta$. Therefore, $\xi_z$, $\xi_\delta$, $PI_t$ and $PI_\delta$ do not depend on $\mu_z$ and $\mu_\delta$ as well.

Equation (46) shows that CoC is linear in $\mu_z$ and $\mu_\delta$. Below we analyze the comparative statics of $c_z$ and $c_\delta$ with respect to $\alpha$. Note that $\frac{dc_z}{d\alpha}$ and $\frac{dc_\delta}{d\alpha}$ always have opposite signs because $c_z = \frac{1-\beta_z}{\beta_\delta} c_\delta$. Therefore, in what follows, we focus on the sign of $\frac{dc_\delta}{d\alpha}$.

Recall that $\xi_x = \lambda_\delta \xi_z - \lambda_\delta \xi_\delta$. Hence, $c_\delta = -\beta_\delta \xi_\delta$, and so $c'_\delta = \frac{dc_\delta}{d\alpha} = -\beta_\delta \frac{\xi_x' \xi_\delta - \xi_\delta' \xi_x}{(\xi_x + \xi_\delta)}$. Substitute $\xi_x'$ from (43) to obtain $c'_\delta = -\xi'_\delta \times A_4(\xi_\delta, x)$, where $A_4(\xi_\delta, x)$ is a function that takes positive values for $\xi_\delta > 0$ and $\xi_x > 0$. Then the comparative statics of $c_\delta$ with respect to $\alpha$ follow from Corollary 2.

C Improvements in ESG information

In this section, we consider the setting discussed in Section 6.1. When $\lambda > 0$, demand for the stock from investors of type $j$ is

$$D^j(\tilde{z}, \tilde{\delta}, \tilde{p}) = \frac{m_j \tilde{p}^j_{\beta} \lambda_{\tau_e + \lambda_{\tau_t}} + \tilde{p} \lambda_{\beta} \lambda_{\tau_e + \lambda_{\tau_t}} + \left(p_z \beta_z \lambda_{\tau_e + \lambda_{\tau_t}} + p_{\delta} \beta_\delta \lambda_{\tau_e + \lambda_{\tau_t}}\right) \tilde{p} - p_0 \tilde{z} \lambda_{\tau_e + \lambda_{\tau_t}} - \tilde{p}}{\left(\beta_z \lambda_{\tau_e + \lambda_{\tau_t}} + \beta_\delta \lambda_{\tau_e + \lambda_{\tau_t}}\right) \tilde{p} - p_0 \tilde{z} \lambda_{\tau_e + \lambda_{\tau_t}} - \tilde{p}}.$$  

Imposing the market clearing condition (4), we obtain the system

$$\xi_z = \frac{\tau_e}{\gamma} \left[m_t + m_g \frac{\beta_z (\xi^2_z + \lambda) - \xi_\delta \xi_z \beta_\delta}{(\xi_z \beta_\delta - \xi_\delta \beta_z)^2 + (\beta_z \lambda + \beta_\delta^2) \lambda} \right],$$  

$$\xi_\delta = \frac{\lambda \tau_e}{\gamma} \left[-m_t \xi^2_z + \lambda \lambda + m_g \frac{\beta_\delta (\xi^2_\delta + \lambda) - \lambda \xi_z \beta_\delta}{(\xi_z \beta_\delta - \xi_\delta \beta_z)^2 + (\beta_z \lambda + \beta_\delta^2) \lambda} \right].$$

Denote $\tilde{k} = \lambda = \lambda_{\tau_e + \tau_n} / \beta_\delta$, $\beta_\delta = \beta_\delta \sqrt{\lambda_{\tau_e + \lambda_{\tau_t}} + \beta_\delta^2}$, $\beta_z = \sqrt{1 - \beta_z^2} = \sqrt{\lambda_{\tau_e + \lambda_{\tau_t}}}$, $\tilde{m}_t = m_t \sqrt{\lambda}$ and $\xi_\delta = \xi_z \sqrt{\lambda}$. Then the system becomes

$$\tilde{\xi}_z = \frac{\tau_e}{\gamma} \left[\tilde{m}_t + \tilde{m}_g \frac{\beta_z (\xi^2_\delta + \tilde{k}) - \xi_\delta \xi_\delta \beta_\delta}{(\xi_\delta \beta_\delta - \xi_\delta \beta_\delta)^2 + \tilde{k}} \right],$$  

$$\xi_\delta = \frac{\tau_e}{\gamma} \left[-\tilde{m}_t \xi_\delta \xi_\delta + \tilde{m}_g \frac{\beta_\delta (\xi^2_\delta + \tilde{k}) - \lambda \xi_z \beta_\delta}{(\xi_\delta \beta_\delta - \xi_\delta \beta_\delta)^2 + \tilde{k}} \right].$$

Note that it has the same structure as (30)-(31). Therefore, adjusted versions of Propositions 1 and 2 hold, where $m, \alpha$ and $\beta_\delta$ are substituted by, respectively, $\tilde{m}_t = \tilde{m}_t + \tilde{m}_g$, 60
Analytically characterizing comparative statics of the endogenous objects such as price coefficients, price informativeness and cost of capital, with respect to $\lambda$ is nontrivial. In what follows, we investigate the model under assumption that $\lambda$ is small. We linearize price coefficients $\xi_{\delta}$ and $\xi_{\delta}$ around $\lambda = 0$ and investigate the comparative statics of the linearized solution. To do so, we proceed in three steps. First, we solve the model for the case with $\lambda = 0$. Second, we use the system of equations (47)-(48), derived under assumption $\lambda > 0$, to get equation (49) that implicitly defines $\xi_{\delta}$. We then show that if $\lambda$ is sufficiently small, there exists a unique solution to this equation that is smooth in $\lambda$ around 0. Moreover, the solution to this equation coincides with the solution derived in Step 1 when $\lambda = 0$. In the third step, we linearize the solution of equation (49) around $\lambda = 0$ and prove Propositions 7 and 8.

**Step 1: Solving the model when $\lambda = 0$.**

When $\lambda = 0$, prior and signals about the ESG component $\tilde{\delta}$ are infinitely imprecise. Therefore, the price cannot be informative about $\tilde{\delta}$ in any equilibrium so that $p_{\delta} = 0$ and $\tilde{p} = p_0 + p_z \tilde{z} + n \tilde{n} = p_0 + p_n (\xi_z \tilde{z} + \tilde{n})$. Green investors do not trade the stock because its payoff is infinitely risky to them. As a result, the equilibrium price coefficients are shaped by trading activities of traditional and noise investors only. In particular, demand for the stock from traditional investors is

$$D^t (\tilde{z}, \tilde{p}) = m_t \frac{1}{\gamma} \frac{\tilde{z} \frac{\tau_s}{\tau + \tau_s} + p_z \frac{1}{\tau + \tau_s} \frac{\tilde{p} - p_0 - p_z \tilde{z} \frac{\tau_s}{\tau + \tau_s}}{p_z^2 + p_z \tilde{z} \frac{\tau_s}{\tau + \tau_s}}}{\frac{1}{\tau + \tau_s} - \left( \frac{(\tau_s \gamma m_t \tilde{p})}{p_z^2 + p_z \tilde{z} \frac{\tau_s}{\tau + \tau_s}} \right)^2}.$$

The market clearing condition is $D^t (\tilde{z}, \tilde{p}) + \tilde{n} = 1$. By matching the price coefficients, it is straightforward to show that $p_n > 0$, $p_z > 0$, $p_0 < 0$, $\xi_z = \frac{\tau_s}{\gamma} m_t$. As mentioned earlier, the price cannot be informative about $\tilde{\delta}$, therefore we have $p_{\delta} = 0$, $\xi_{\delta} = 0$.

**Step 2: Equation for $\xi_{\delta}$ when $\lambda > 0$.**

When $\lambda > 0$, we can use system (47)-(48) to get a quintic equation of $\xi_{\delta}$, analogous to equation (33) in the baseline model

$$f (\xi_{\delta}) = \xi_{\delta}^5 - \left( \frac{\tau_s}{\gamma} m_g \right) \lambda \frac{\beta_{\delta}}{\lambda \beta_z^2 + \beta_{\delta}^2} \xi_{\delta}^4 + 2 \lambda \kappa \xi_{\delta}^3 - 2 \left( \frac{\tau_s}{\gamma} m_g \right) \lambda \kappa \frac{\beta_{\delta}}{\lambda \beta_z^2 + \beta_{\delta}^2} \xi_{\delta}^2 + \left[ \lambda^2 \kappa^2 + \left( \frac{\tau_s}{\gamma} m_t \right)^2 \lambda^2 \kappa \frac{\beta_{\delta}}{\lambda \beta_z^2 + \beta_{\delta}^2} \right] \xi_{\delta} - \left( \frac{\tau_s}{\gamma} m_g \right) \lambda^3 \kappa^2 \frac{\beta_{\delta}}{\lambda \beta_z^2 + \beta_{\delta}^2} = 0.$$
which can be rewritten as

\[ f(\xi) = \left( \xi - \frac{\tau_s m_g}{\gamma} \right) \lambda \frac{\beta_s}{\lambda \beta_s^2 + \beta_s^2} \left( \xi^2 + \lambda \kappa \right)^2 + \left( \frac{\tau_s}{\gamma} m_t \right)^2 \lambda^2 \kappa \frac{\beta_s^2}{\lambda \beta_s^2 + \beta_s^2} \xi = 0. \]

If \( \lambda = 0 \), this equation has a unique solution \( \xi = 0 \), which coincides with the one derived in Step 1. If \( \lambda > 0 \), there always exists a positive solution because \( f(0) < 0 \) and \( f(\infty) > 0 \). Moreover, all solutions are below \( \frac{\tau_s m_g \lambda \beta_s}{\lambda \beta_s^2 + \beta_s^2} \). When \( \lambda \) is sufficiently small, the solution is unique. Indeed, differentiate (49) and observe that for \( \xi \in \left( 0, \frac{\tau_s m_g \lambda \beta_s}{\lambda \beta_s^2 + \beta_s^2} \right) \),

\[ \frac{\partial f}{\partial \xi} > -4 \left( \frac{\tau_s m_g}{\gamma} \frac{\lambda \beta_s}{\lambda \beta_s^2 + \beta_s^2} \right)^4 - 4 \left( \frac{\tau_s m_g}{\gamma} \frac{\lambda \beta_s}{\lambda \beta_s^2 + \beta_s^2} \right)^2 \lambda \kappa + \lambda^2 \kappa^2 + \left( \frac{\tau_s}{\gamma} m_t \right)^2 \frac{\lambda^2 \kappa \beta_s^2}{\lambda \beta_s^2 + \beta_s^2}. \]

If \( \lambda \) is sufficiently small, the last positive term is larger in absolute terms than the first two negative terms combined. Therefore, \( f(\xi) \) is strictly increasing on the relevant interval, which guarantees that there exists a unique solution \( \xi(\lambda) \). Moreover, the function \( \xi(\lambda) \) is smooth in the neighborhood of zero because \( f(\xi, \lambda) \) is smooth in the neighborhood of \((0, 0)\).

**Step 3: Linearization.**

Because \( \xi(\lambda) \) is smooth around \( \lambda = 0 \), we can use its Taylor series to approximate it around this point. Write \( \xi = \xi_1 \lambda + o(\lambda) \), where \( \xi_1 \) does not depend on \( \lambda \), and plug it in (49). Omitting higher order terms, we obtain \( \xi_1 = \frac{(\frac{\tau_s}{\gamma} m_g)^\kappa}{\beta_s((\frac{\tau_s}{\gamma} m_t)^\gamma + \kappa)} > 0. \)

Similarly, we have \( \xi_z = \xi_{z,0} + \xi_{z,1} \lambda + o(\lambda) \). Using equation (47), we get \( \xi_{z,0} = \frac{\tau_s}{\gamma} m_t \) and \( \xi_{z,1} = \xi_{z,1} \frac{1}{\beta_z} \left[ \beta_z - \frac{(\frac{\tau_s}{\gamma} m_g)}{(\frac{\tau_s}{\gamma} m_t)^\gamma + \kappa} \right] \). The linear term of \( \xi_z \) is negative if and only if \( \beta_z < \frac{(\frac{\tau_s}{\gamma} m_g)}{(\frac{\tau_s}{\gamma} m_t)^\gamma + \kappa} \), which proves part (i) of Proposition 7.

Next, we linearize price informativeness, given by (25)-(26). For \( PI_g \), we have

\[ PI_{g,0} + PI_{g,1} \lambda = (\tau + \tau_s) \frac{\xi_{z,0} \lambda + \lambda \kappa}{\xi_{z,0} \beta_s^2 + \beta_s^2} \Rightarrow PI_{g,0} = 0, \quad PI_{g,1} = (\tau + \tau_s) \frac{1}{\beta_s^2} > 0. \]
For $PI_t$, we have

$$PI_{t,0} + PI_{t,1} \lambda = (\tau + \tau_s) \frac{\xi_2 \mu \lambda + 2 \xi_1 \xi_{2,0} \lambda^2 + \xi_{2,1} \lambda^2 + \lambda \kappa}{\xi_{2,1} \lambda^2 + \lambda \kappa} \Rightarrow$$

$$PI_{t,0} = (\tau + \tau_s) \frac{\left(\frac{\omega}{\tau_s} \mu_t \right)^2 + \kappa}{\kappa} , PI_{t,1} = (\tau + \tau_s) \frac{2 \xi_{2,0} \xi_{2,1}}{\kappa \beta_5} \left(\beta_z - \frac{3}{2} \left(\frac{\omega}{\tau_s} m_g \right) \left(\frac{\omega}{\tau_s} m_t \right) \right) .$$

Clearly, $PI_{t,1} < 0$ if and only if $\beta_z - \frac{3}{2} \left(\frac{\omega}{\tau_s} m_g \right) \left(\frac{\omega}{\tau_s} m_t \right) < 0$, which proves part (ii) of Proposition 7.

Finally, recall that $CoC = \frac{\gamma}{m_t PI_t + m_g PI_g}$ and so $\frac{dCoC}{d\lambda} \propto -\left( m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda} \right)$, where $\propto$ denotes proportionality up to a positive term. Using $PI_{g,1}$ and $PI_{t,1}$ derived above, we have

$$m_t \frac{dPI_t}{d\lambda} + m_g \frac{dPI_g}{d\lambda} = m_t PI_{t,1} + m_g PI_{g,1} + o(1) =$$

$$\frac{\tau + \tau_s}{\beta_5^2} \frac{2 m_g \left(\frac{\omega}{\tau_s} m_t \right)^2}{\left(\frac{\omega}{\tau_s} m_t \right)^2 + \kappa} \left[ \beta_z - \frac{3}{2} \left(\frac{\omega}{\tau_s} m_t \right) \left(\frac{\omega}{\tau_s} m_g \right) + \frac{1}{2} \left(\frac{\omega}{\tau_s} m_t \right)^2 + \kappa \right] + o(1).$$

If the expression in the brackets is negative, $CoC$ increases in $\lambda$ for a sufficiently small $\lambda$. This proves Proposition 8.

### D Empirical appendix

#### D.1 Data

Our data comes from several sources. Carbon emissions are from S&P Global Trucost spanning between 2010 and 2021. We use the sum of Scope 1 and location-based Scope 2 emissions as the measure of total emissions. Our unit is 10,000 tonnes of carbon dioxide equivalent (tCO$_2$e). We follow Bolton and Kacperczyk (2021) to winsorize emission intensities at 2.5%. Holdings by US active ESG equity mutual funds is from Li et al. (2022), who first identify active ESG funds by reading the “Principle Investment Strategies” section of fund prospectuses and then use monthly holding data from Morningstar. Accounting variables and stock prices are from Compustat and CRSP. We exclude firms in financial services. Nominal values from Compustat and firm market capitalization are inflation-adjusted using the CPI index for all urban consumers (CPIAUCSL) from the
Bureau of Labor Statistics. Financial ratios are winsorized at 1%. We use two-digit SIC codes to form industries.

D.2 Robustness

In this section, we show the robustness of our results in Section 2 to using a different measure of pricing ratio (Appendix D.2.1), different sample selection (Appendix D.2.2), and controlling for future financial performance (Appendix D.2.3).

D.2.1 Alternative pricing ratio

We replicate our exercise in Section 2 except that we now use log ratio of market capitalization to book equity, i.e., $\log\frac{M_{i,t}}{B_{i,t}}$, instead of $\log\frac{M_{i,t}}{A_{i,t}}$ in regression (1). This takes into account the difference in leverage across firms. Table 2 shows that our results are robust to this variation.

<table>
<thead>
<tr>
<th></th>
<th>$H=\text{ESG_ownership}$</th>
<th>$H=\text{ESG_funds}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{i,t} \times \log \left(\frac{M_{i,t}}{B_{i,t}}\right)$</td>
<td>-0.070* (0.035)</td>
<td>-0.063** (0.029)</td>
</tr>
<tr>
<td></td>
<td>-0.067* (0.036)</td>
<td>-0.065** (0.030)</td>
</tr>
<tr>
<td>$H_{i,t}$</td>
<td>0.049 (0.031)</td>
<td>0.032 (0.041)</td>
</tr>
<tr>
<td></td>
<td>0.052 (0.033)</td>
<td>0.053 (0.048)</td>
</tr>
<tr>
<td>$\log \left(\frac{M_{i,t}}{B_{i,t}}\right)$</td>
<td>0.083* (0.048)</td>
<td>0.167** (0.075)</td>
</tr>
<tr>
<td></td>
<td>0.100 (0.062)</td>
<td>0.182** (0.089)</td>
</tr>
<tr>
<td>$C_{i,t}/A_{i,t}$</td>
<td>0.975*** (0.007)</td>
<td>0.975*** (0.008)</td>
</tr>
<tr>
<td></td>
<td>0.974*** (0.008)</td>
<td>0.974*** (0.008)</td>
</tr>
<tr>
<td>Controls</td>
<td>N Y</td>
<td>N Y</td>
</tr>
<tr>
<td>Industry/Year FE</td>
<td>Y Y</td>
<td>Y Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.947 0.948</td>
<td>0.947 0.948</td>
</tr>
<tr>
<td>Obs</td>
<td>7,182 7,033</td>
<td>7,182 7,033</td>
</tr>
</tbody>
</table>

Table 2: ESG ownership and information content of asset prices: alternative pricing ratio. The table reports the estimates of specification (1) but use log market-to-book ratio $\log MB_{i,t}$ as predictor. Controls include ratio of EBITDA to assets, log market capitalization, market leverage, CAPM $\beta$, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.
D.2.2 Alternative sample

In the main text, we require a firm to have at least 9 years of data on total emission between 2010 and 2021 (i.e., 75% of emissions data should be non-missing). In this section, we experiment with different cutoffs and report our results in Table 3. Columns (1) and (2) include firms with no missing observations; columns (3) and (4) include firms with at least 10 observations; columns (5) and (6) include firms with at least 8 observations. Our estimates remain similar to those in the baseline specification.

<table>
<thead>
<tr>
<th>Dependent variable: $C_{i,t+1}/A_{i,t}$</th>
<th>N=12</th>
<th>N≥10</th>
<th>N≥8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>$ESG_{ownership_{i,t}}$ × log(M$<em>{i,t}/A</em>{i,t}$)</td>
<td>-0.035</td>
<td>-0.049*</td>
<td>-0.040*</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.028)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>$ESG_{ownership_{i,t}}$</td>
<td>0.021</td>
<td>-0.011</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.025)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>$ESG_{#funds_{i,t}}$ × log(M$<em>{i,t}/A</em>{i,t}$)</td>
<td>-0.051**</td>
<td>-0.057**</td>
<td>-0.054***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.023)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$ESG_{#funds_{i,t}}$</td>
<td>-0.005</td>
<td>-0.021</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.030)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>log(M$<em>{i,t}/A</em>{i,t}$)</td>
<td>0.092*</td>
<td>0.175***</td>
<td>0.135**</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.065)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>$C_{i,t}/A_{i,t}$</td>
<td>0.975***</td>
<td>0.975***</td>
<td>0.976***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>

| Controls | Y | Y | Y | Y | Y | Y |
| Industry/Year FE | Y | Y | Y | Y | Y | Y |
| $R^2$ | 0.947 | 0.948 | 0.947 | 0.947 | 0.951 | 0.951 |
| Obs | 5,349 | 5,349 | 6,446 | 6,446 | 7,419 | 7,419 |

Table 3: ESG ownership and information content of asset prices: alternative sample. The table reports the estimates of specification (1) by varying the required number of observations on emissions for a firm to enter the sample. In the baseline, we require at least 9 observations. Columns (1) and (2) require at least 12 (no missing observations). Columns (3) and (4) require at least 10. Columns (5) and (6) require at least 8. Controls include ratio of EBITDA to assets, log market capitalization, market leverage, CAPM $\beta$, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.
D.2.3 Controlling for future financial performance

We conduct an exercise where we include into the right hand side of specification (1) not only current but also future financial performance. In particular, we run

\[
\frac{C_{i,t+1}}{A_{i,t}} = \beta_0 + \beta_1 H_{i,t} \times \log \frac{M_{i,t}}{A_{i,t}} + \beta_2 H_{i,t} + \beta_3 \log \frac{M_{i,t}}{A_{i,t}} + \beta_4 \frac{C_{i,t}}{A_{i,t}} + \beta_5 \frac{E_{i,t+1}}{A_{i,t}} + Controls_{i,t} + Industry_i + Year_t + \varepsilon_{i,t},
\]

where controls are listed in the caption of Table 4 (recall that one of the control variables is \(\frac{E_{i,t}}{A_{i,t}}\)). This specification assumes that market participants at time \(t\) have already received perfect information about future financial performance \(E_{i,t+1}\). It, therefore, investigates whether asset prices are still negatively associated with future emissions. In other words, we are interested in whether asset prices of firms held by ESG funds provide additional information about future emissions that is orthogonal to financial information. Our results suggest this is indeed the case.

D.3 Price informativeness about financial performance

In this section, we investigate the relationship between asset prices and future financial performances. Following our specification in Section 2 of the main text, we estimate the following panel regression and report our results in Table 5:

\[
\frac{E_{i,t+1}}{A_{i,t}} = \beta_0 + \beta_1 H_{i,t} \times \log \frac{M_{i,t}}{A_{i,t}} + \beta_2 H_{i,t} + \beta_3 \log \frac{M_{i,t}}{A_{i,t}} + \beta_4 \frac{E_{i,t}}{A_{i,t}} + Controls_{i,t} + Industry_i + Year_t + \varepsilon_{i,t},
\]

where \(E_{i,t}\) is firm \(i\)’s financial performance in year \(t\). Control variables are listed in the table caption. As columns (1) and (2) suggest, when we use EBITDA for \(E\), we do not find the relationship between current prices and future earnings to be different across firms by their ESG fund ownership. In columns (3) and (4), we use sales for \(E\), and we find that current prices are more negatively associated with future sales when using the number of ESG funds holding the firm as the independent variable.
Table 4: ESG ownership and information content of asset prices: controlling for future earnings. The table reports the estimates of specification (50). Columns (1) and (2) use EBITDA for E. Columns (3) and (4) use sales for E. Controls include current financial performance $E_{i,t}/A_{i,t}$, log market capitalization, market leverage, CAPM $\beta$, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.

D.4 Holding regressions

We show in this appendix that active ESG funds in our sample are averse to holding high emitters. Specifically, we run the following panel regression:

$$H_{i,t+1} = \beta_0 + \beta_1 \frac{C_{i,t}}{A_{i,t}} + \beta_2 H_{i,t} + \beta_3 Controls_{i,t} + Industry_i + Year_t + \varepsilon_i. \quad (52)$$

Our results are shown in Table 6. Columns (1) and (2) use for $H$ the proportion of ESG holdings $ESG\_ownership$. Columns (3) and (4) use for $H$ log 1 plus the number of ESG funds $ESG\_#funds$. Column (5) and (6) estimate Poisson regressions where dependent variable $H_{i,t+1}$ is the raw number of ESG funds that is not log-transformed. We include
Table 5: ESG ownership and information content of asset prices about future earnings. The table reports the estimates of specification (51). Columns (1) and (2) use EBITDA for $E_i,t$ and Columns (3) and (4) use sales for $E_i,t$. Controls include current emission intensities $C_{i,t}/A_{i,t}$, log market capitalization, market leverage, CAPM $\beta$, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.

<table>
<thead>
<tr>
<th>Model</th>
<th>Dependent variable: $E_{i,t+1}/A_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E=EBITDA$</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>$ESG_{ownership_{i,t}} \times \log(M_{i,t}/A_{i,t})$</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
</tr>
<tr>
<td>$ESG_{ownership_{i,t}}$</td>
<td>-0.000</td>
</tr>
<tr>
<td>$ESG_{#funds_{i,t}} \times \log(M_{i,t}/A_{i,t})$</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
</tr>
<tr>
<td>$ESG_{#funds_{i,t}}$</td>
<td>-0.003**</td>
</tr>
<tr>
<td>$\log(M_{i,t}/A_{i,t})$</td>
<td>0.034***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
</tr>
<tr>
<td>$E_{i,t}/A_{i,t}$</td>
<td>0.679***</td>
</tr>
<tr>
<td></td>
<td>(0.153)</td>
</tr>
</tbody>
</table>

| Controls | Y | Y | Y | Y |
| Industry/Year FE | Y | Y | Y | Y |
| $R^2$ | 0.715 | 0.716 | 0.965 | 0.965 |
| Obs | 7,033 | 7,033 | 7,085 | 7,085 |

Table 5: ESG ownership and information content of asset prices about future earnings. The table reports the estimates of specification (51). Columns (1) and (2) use EBITDA for $E_i,t$ and Columns (3) and (4) use sales for $E_i,t$. Controls include current emission intensities $C_{i,t}/A_{i,t}$, log market capitalization, market leverage, CAPM $\beta$, log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors are clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.

a variety of firm controls that are listed in the table caption together with industry fixed effects.

The estimates for $\beta_1$ are negative and statistically significant. These results confirm that high emission stocks observe low ESG fund ownership. Columns (2), (4) and (6) show that after controlling for current ESG fund holdings, the relationship between current emissions and ESG fund holdings next year remains negative. This suggests that high emissions are also associated with decreases in ESG fund ownership in the following year. It is worth noting that we do not find such a negative relationship when using 13f institutional holdings as the dependent variable, in line with the findings by Bolton and Kacperczyk (2021). Furthermore, our results in Table 1 are also robust to controlling for 13f holdings and the interaction term between 13f holdings and current prices.
Table 6: Carbon emissions and ESG ownership. The table reports the estimates of specification (52). Columns (1) and (2) use ESG\textsubscript{ownership} for \(H\) and estimate OLS. Columns (3) and (4) use ESG\textsubscript{#funds} for \(H\) and estimate OLS. Columns (5) and (6) use the raw number of ESG funds that is not log-transformed, i.e., \(\exp(ESG\textsubscript{#funds}) - 1\) for \(H\) and run Poisson regressions. Controls include ratio of EBITDA to assets, log ratio of market capitalization to assets, log market capitalization, market leverage, CAPM \(\beta\), log property, plant and equipment, ratio of capital expenditures to assets, sales growth, stock return volatility, and the Amihud’s (2002) ratio. Standard errors clustered at the industry level. */**/*** denotes 10%/5%/1% statistical significance.

<table>
<thead>
<tr>
<th>(\frac{C_{i,t}}{A_{i,t}})</th>
<th>(H_{i,t})</th>
<th>Controls</th>
<th>(\text{Industry/Yr FE})</th>
<th>(\text{(Pseudo) R}^2)</th>
<th>(\text{Obs})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{OLS (1)})</td>
<td>(\text{OLS (2)})</td>
<td>(\text{OLS (3)})</td>
<td>(\text{OLS (4)})</td>
<td>(\text{Poisson (5)})</td>
<td>(\text{Poisson (6)})</td>
</tr>
<tr>
<td>-0.002**</td>
<td>-0.001</td>
<td>-0.012***</td>
<td>-0.003***</td>
<td>-0.024***</td>
<td>-0.017***</td>
</tr>
<tr>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.003)</td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>0.734***</td>
<td>0.762***</td>
<td>0.064***</td>
<td>(0.029)</td>
<td>(0.013)</td>
<td>(0.007)</td>
</tr>
</tbody>
</table>

\(C_{i,t}/A_{i,t}\) and \(H_{i,t}\) are expressed as coefficients with standard errors in parentheses. Y indicates the inclusion of the control. Obs: number of observations.

**E General information structure**

In the main text, we consider an analytically tractable case when traditional and green investors have access to information of the same quality. In this section, we explore the role of information structure for our main results. In particular, we establish that our results about the existence of multiple equilibria in the trading game and the nature of these equilibria are robust to general assumptions about information available to investors.

First, we allow \(\tilde{z}\) and \(\tilde{\delta}\) to have different ex ante variances, \(\tau_{\tilde{z}}^{-1}\) and \(\tau_{\tilde{\delta}}^{-1}\), respectively.

Second, traditional and green investors receive informative signals about \(\tilde{z}\) and \(\tilde{\delta}\) of potentially different precisions. In particular, investor \(i\) of type \(j \in \{t, g\}\) receives two private signals, \(s^i_{\tilde{z}} = \tilde{z} + \tilde{\varepsilon}^i_{\tilde{z}}\) and \(s^i_{\tilde{\delta}} = \tilde{\delta} + \tilde{\varepsilon}^i_{\tilde{\delta}}\), where \(\tilde{\varepsilon}^i_{\tilde{z}} \sim N \left(0 , \left(\tau^i_{\tilde{z}}\right)^{-1}\right)\) and \(\tilde{\varepsilon}^i_{\tilde{\delta}} \sim N \left(0 , \left(\tau^i_{\tilde{\delta}}\right)^{-1}\right)\). Given their preferences, we assume that traditional (green) investors receive some useful information about \(\tilde{z}\) (\(\tilde{\delta}\)), namely, \(\tau^t_{\tilde{z}} > 0\) and \(\tau^g_{\tilde{\delta}} > 0\). Other signals can be in principle uninformative, \(\tau^t_{s_{\tilde{z}}} \geq 0\) and \(\tau^g_{s_{\tilde{\delta}}} \geq 0\). Finally, we maintain our baseline assumptions: masses of traditional and green investors are positive, \(m_t > 0\) and \(m_g > 0\); investors’ risk aversion parameter is \(\gamma > 0\); traditional investors care only about \(\tilde{z}\) and green investors care about \(\beta_{\tilde{z}}\tilde{z} + \beta_{\tilde{\delta}}\tilde{\delta}\), where \(\beta_{\tilde{z}} \geq 0\) and \(\beta_{\tilde{\delta}} > 0\); noise traders’ demand is \(\tilde{n} \sim N \left(0 , \tau_{\tilde{n}}^{-1}\right)\).
Under general information structure, the system of equations (30)-(31) for $\xi_z$ and $\xi_\delta$ becomes

$$\xi_z = m_t \tau^t_{s_z} + m_g \tau^g_{s_z} \frac{\beta_z \left( \xi^2_z + \frac{\tau^t_{s_z} + \tau^g_{s_z}}{\tau_n} \right) - \xi_\delta \xi_z \beta_\delta}{(\xi_z \beta_\delta - \xi_\delta \beta_z)^2 + (\beta_z^2 (\tau_\delta + \tau^g_{s_z}) + \beta_\delta^2 (\tau_z + \tau^g_{s_z})) \frac{1}{\tau_n}},$$  \tag{53}

$$\xi_\delta = -m_t \tau^t_{s_\delta} \frac{\xi_\delta \xi_z}{\xi^2_z + \frac{\tau^t_{s_z} + \tau^g_{s_\delta}}{\tau_n}} + m_g \tau^g_{s_\delta} \frac{\beta_\delta \left( \xi^2_\delta + \frac{\tau^t_{s_\delta} + \tau^g_{s_z}}{\tau_n} \right) - \xi_\delta \xi_z \beta_z}{(\xi_\delta \beta_z - \xi_z \beta_\delta)^2 + (\beta_\delta^2 (\tau_z + \tau^g_{s_\delta}) + \beta_z^2 (\tau_\delta + \tau^g_{s_z})) \frac{1}{\tau_n}}. \tag{54}$$

In (53)-(54), we set $\gamma = 1$. This is without loss of generality because it is equivalent to redefining the masses of traditional and green investors.

**Proposition 10.** Fix $m_t > 0$, $m_g > 0$, $\gamma > 0$, $\beta_z \geq 0$, $\beta_\delta > 0$, $\tau^t_{s_z} > 0$, $\tau^t_{s_\delta} \geq 0$, $\tau^g_{s_z} \geq 0$, $\tau^g_{s_\delta} > 0$. For any $\tau_n > 0$, an equilibrium with a linear price $\tilde{p} = p_0 + p_\tilde{z} \tilde{z} + p_\delta \delta + p_n \tilde{n}$ exists. Moreover, for a sufficiently large $\tau_n$ multiple equilibria exist if one of the following conditions is satisfied:

(i) $\tau^t_{s_z} > 0$ and $\tau^g_{s_z} > 0$;

(ii) $\tau^t_{s_z} > 0$, $\tau^g_{s_z} = 0$, and either $\frac{4 \beta_\delta^2 m_t^2 \tau^t_{s_z} \tau^g_{s_z}}{m^2_g (\tau^g_{s_z})^2} < 1$ or $\beta_z > 0$;

(iii) $\tau^t_{s_\delta} = 0$, $\tau^g_{s_z} > 0$, and $\frac{4 m_g \tau^g_{s_z} \left( \tau^g_{s_z} m_g + \beta_z m_t \tau^t_{s_z} \right)}{\beta^2_\delta m^2_t (\tau^t_{s_z})^2} < 1$;

(iv) $\tau^t_{s_z} = 0$, $\tau^g_{s_\delta} = 0$, $\beta_z > 0$ and $\frac{4 \beta_\delta m_g \tau^g_{s_\delta}}{\beta^2_\delta m_t \tau^t_{s_z}} < 1$.

Below, we first discuss Proposition 10 and then formally prove it at the end of this section.

Proposition 10 emphasizes the importance of the information structure for the existence of multiple equilibria in the trading stage. In particular, they arise when investors have access to information about fundamentals that they value differently. To see it clearly, it is instructive to consider a special case when green investors care only about the $\tilde{\delta}$-component, i.e. $\beta_z = 0$. For a sufficiently small exogenous noise (large $\tau_n$), multiple equilibria always arise as long as traditional and green investors receive *some* informative signals about $\tilde{\delta}$ and $\tilde{z}$, respectively. In an equilibrium that resembles the T-equilibrium, the price is closely associated with $\tilde{z}$ and is thus very informative to traditional investors. This incentivizes them to trade the stock intensively. In particular, they actively trade
against their \( \tilde{\delta} \)-signals, virtually offsetting the impact of green investors who trade in the opposite direction. The price is, therefore, weakly associated with \( \tilde{\delta} \). Analogously, there is an equilibrium that resembles the G-equilibrium, where the price is closely associated with \( \tilde{\delta} \).

Notice that the multiplicity is possible even if only one investor group receives signals about the factor they do not value, e.g., \( \tau_{s}^{t} > 0 \) and \( \tau_{s}^{g} = 0 \) (the case with \( \tau_{s}^{t} = 0 \) and \( \tau_{s}^{g} > 0 \) is analogous). In the absence of relevant signals about \( \tilde{z} \), green investors are not able to offset traditional investors’ trading along the \( \tilde{z} \)-dimension. The price is always informative to traditional investors because the price coefficient \( \xi_{z} \) is shaped solely by their trading activities. The multiplicity is still possible due to trading in the opposite directions along the \( \tilde{\delta} \)-dimension. It requires, however, that the mass of traditional investors is small and their private signals are not precise relative to those of green investors, i.e., \( \frac{4\beta^{2}m^{2}r_{z}^{t}r_{z}^{g}}{m^{2}r_{\tilde{z}}^{t}r_{\tilde{z}}^{g}} < 1 \). If this is not the case, traditional investors dominate the trading along the \( \tilde{\delta} \)-dimension and the price is uniquely determined. Note that if green investors care about the \( \tilde{z} \)-component, multiple equilibria are always possible for a sufficiently small noise. If \( \beta_{z} > 0 \), preferences of green and traditional investors are partially aligned. Green investors benefit to some extent from traditional investors’ trading as they can learn about \( \tilde{z} \) from the price. The price is less noisy to them, and they trade more aggressively based on their \( \tilde{\delta} \)-signals.

Finally, the equilibrium is always unique if investors are informed only about the factors they care about, i.e., \( \tau_{s}^{t} = \tau_{s}^{g} = 0 \). In this case, there is no trading in the opposite directions because the investors’ information sets are orthogonal. This case is studied in Rahi and Zigrand (2018) and Rahi (2021). As in the previous case, multiple equilibria might arise if \( \beta_{z} > 0 \). In this case, signals received by green investors are not perfectly aligned with what they value and, therefore, they benefit from the information about \( \tilde{z} \) contained in the price.

Overall, Proposition 10 shows that, under fairly general assumptions on the information structure, the price might not be uniquely pinned down if the stock is traded by investors with heterogeneous valuations. Equilibria differ in terms of which investor group most actively trades the stock and which factors the price is mostly informative about. There are two key requirements for the multiplicity to emerge. First, investors of one group need to possess some information about the fundamental that investors of the other group value. That allows investors with heterogeneous preferences to trade against each other based on the same information. Second, the amount of exogenous noise should
be small; otherwise, the price is always an imprecise signal to all rational investors.

Proof of Proposition 10.

As in other proofs, this one involves many tedious yet straightforward algebraic manipulations, which we frequently perform via Matlab Symbolic Math Toolbox and do not show.

The first part of the proof involves the reduction of (53)-(54) to a polynomial equation either for $\xi_\delta$ or $\xi_z$. Depending on the values of signal precisions, this equation is either cubic or have a higher odd order. For cubic equations, we investigate the number of roots using the sign of the discriminant. For higher order equations, the analysis is conceptually similar to the proof of Lemma 3. In particular, we prove that for a sufficiently large $\tau_n$, there are at least three distinct real roots by showing that the polynomial changes its sign at least three times.

Getting a polynomial equation for either $\xi_z$ or $\xi_\delta$ from the system (53)-(54) involves different steps when $\beta_z = 0$ and $\beta_z > 0$, so we analyze these two cases separately. Each case is further split into four subcases that jointly cover all possible values of signal precisions. In some of those subcases, we introduce new notation. Since subcases are independent from one another, the additional notation is case-specific, that is, we might use the same notation in different subcases to denote different objects.

Case 1: $\beta_z = 0$.

Note that we do not impose the restriction that $\beta_z^2 + \beta_\delta^2 = 1$ as in our baseline model, which makes it possible for the two groups of investors to be exposed to different risk levels. The system (53)-(54) simplifies to

$$
\begin{align*}
\xi_z &= m_t \tau_{sz} - \hat{m}_g \tau_{sz}^g \frac{\xi_\delta \xi_z}{\xi_z^2 + \frac{\tau_z + \tau_{sz}^g}{\tau_n}}, \\
\xi_\delta &= \hat{m}_g \tau_{sz}^g - m_t \tau_{sz}^l \frac{\xi_\delta \xi_z}{\xi_\delta^2 + \frac{\tau_\delta + \tau_{sz}^l}{\tau_n}},
\end{align*}
$$

(55)

where we denote $\hat{m}_g = \frac{1}{\rho_g} m_g$.

Case 1.1: $\tau_{sz}^l = \tau_{sz}^g = 0$.

If investors receive signals only about fundamentals they care about, the equilibrium in the trading stage is trivially unique: $\xi_z = m_t \tau_{sz}^l$ and $\xi_\delta = \hat{m}_g \tau_{sz}^g$.

Case 1.2: $\tau_{sz}^l = 0$ and $\tau_{sz}^g > 0$.

If only green investors receive informative signals about $\hat{\delta}$, their trading activity solely determines the corresponding price coefficient, $\xi_\delta = \hat{m}_g \tau_{sz}^g$. $\xi_z$ solves the following equa-
\[ \xi_z^3 - \xi_z^2 \left[ m_t \tau_{s_z} \right] + \xi_z \left[ \frac{\tau_{z} + \tau_{s_z}^g}{\tau_n} + \hat{m}_g^2 \tau_{s_z} \tau_{s_z}^g \right] - m_t \tau_{s_z} \tau_{s_z}^g = 0. \]  

(56)

This equation has at least one real root because it is cubic, and the real root(s) must be positive since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of \( \frac{1}{\tau_n} \):

\[ D = \sum_{i=0}^{3} d_i \left( \frac{1}{\tau_n} \right)^i, \]

where \( d_0 = \hat{m}_g^4 m_t^2 \left( \tau_{s_z}^g \right)^2 - 4 \hat{m}_g^6 \left( \tau_{s_z}^g \right)^3 \). For a sufficiently large \( \tau_n \), \( D > 0 \) if \( d_0 > 0 \).

Therefore, for a sufficiently large \( \tau_n \), (56) has three distinct real roots if

\[ \frac{4 \hat{m}_g^2 \tau_{s_z}^g \tau_{s_z}^g}{m_t^2 (\tau_{s_z}^i)^2} = \frac{4 m_t^2 \tau_{s_z}^g \tau_{s_z}^g}{\beta_2^2 m_t^2 (\tau_{s_z}^i)^2} < 1. \]

**Case 1.3:** \( \tau_{s_z}^i > 0 \) and \( \tau_{s_z}^g = 0 \).

This case is symmetric to **Case 1.2**. There are three solutions to (55) if \( \tau_n \) is sufficiently large and

\[ \frac{4 m_t^2 \tau_{s_z}^i \tau_{s_z}^i}{m_t^2 (\tau_{s_z}^g)^2} = \frac{4 \beta_2^2 m_t^2 \tau_{s_z}^g \tau_{s_z}^g}{m_t^2 (\tau_{s_z}^g)^2} < 1. \]

**Case 1.4:** \( \tau_{s_z}^i, \tau_{s_z}^g > 0 \).

Since the first equation of (55) is linear in \( \xi_{s_z} \), we can straightforwardly write \( \xi_{s_z} = \xi_{s_z}(\xi_z) \). Plugging it in the second equation of the system, we obtain the following equation for \( \xi_z \):

\[ f(\xi_z) = \sum_{i=0}^{9} a_i \xi_z^i = 0. \]  

(57)

Moreover, \( a_0 = 1 \) and \( a_0 = a_{0,3} \left( \frac{1}{\tau_n} \right)^3 \), where \( a_{0,3} < 0 \) does not depend on \( \tau_n \). Then there exists at least one positive real root. Let’s now show that there exists at least three positive real roots for a sufficiently large \( \tau_n \). Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.
We can write

\[ a_0 = a_{0,3} \left( \frac{1}{\tau_n} \right)^3, \]

\[ a_1 = a_{1,2} \left( \frac{1}{\tau_n} \right)^2 + a_{1,3} \left( \frac{1}{\tau_n} \right)^3, \]

\[ a_2 = a_{2,2} \left( \frac{1}{\tau_n} \right)^2 + a_{2,3} \left( \frac{1}{\tau_n} \right)^3, \]

\[ a_3 = a_{3,1} \frac{1}{\tau_n} + a_{3,2} \left( \frac{1}{\tau_n} \right)^2 + a_{3,3} \left( \frac{1}{\tau_n} \right)^3, \]

where \( a_{i,j} \neq 0 \) are coefficients that do not depend on \( \tau_n \). Moreover, \( a_{0,3} < 0 \) and \( a_{1,2} > 0 \).

Then, evaluating \( f(\cdot) \) at \(-\frac{a_{0,3}}{a_{1,2} \tau_n} + c_1 \frac{1}{\tau_n} > 0 \) for some \( c_1 > 0 \), we obtain

\[ f \left( -\frac{a_{0,3}}{a_{1,2} \tau_n} + c_1 \frac{1}{\tau_n} \right) = a_{1,2} c_1 \left( \frac{1}{\tau_n} \right)^3 + o \left( \left( \frac{1}{\tau_n} \right)^3 \right). \]

For a sufficiently large \( \tau_n \), \( f \left( -\frac{a_{0,3}}{a_{1,2} \tau_n} + c_1 \frac{1}{\tau_n} \right) > 0 \).

Next, we can write (57) also as a polynomial of \( \frac{1}{\tau_n} \):

\[ f (\xi_z) = \sum_{i=0}^{3} b_i(\xi_z) \left( \frac{1}{\tau_n} \right)^i, \]

where

\[ b_0(\xi_z) = \xi_z \left( \xi_z - m_t \tau_s^l \right)^2 \left( \xi_z - m_t \tau_s^l \right)^2 + m_g^2 \tau_s^l \left( \tau_s^l \right)^2 + m_t^2 \tau_s^l \left( \tau_s^l \right)^2 \tau_s^l \xi_z \left( \xi_z - m_t \tau_s^l \right). \]

Then, evaluating \( f(\cdot) \) at \( m_t \tau_s^l - \left( \frac{1}{\tau_n} \right)^{1/2} \), we obtain

\[ f \left( m_t \tau_s^l - \left( \frac{1}{\tau_n} \right)^{1/2} \right) = -m_g^2 m_t \left( \tau_s^l \right)^2 \tau_s^l \left( m_t \tau_s^l \right)^5 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right). \]

Therefore, for a sufficiently large \( \tau_n \), \( f \left( m_t \tau_s^l - \left( \frac{1}{\tau_n} \right)^{1/2} \right) < 0 \) and \( m_t \tau_s^l - \left( \frac{1}{\tau_n} \right)^{1/2} > -\frac{a_{0,3}}{a_{1,2} \tau_n} + c_1 \frac{1}{\tau_n} > 0 \). Furthermore, because \( a_9 > 0 \), for any \( \tau_n > 0 \) \( f(\xi_z) > 0 \) if \( \xi_z \) is sufficiently large. Hence, we have shown that (57) has at least three (positive real)
solutions for $\xi_z$ if $\tau_n$ is sufficiently large.

**Case 2:** $\beta_z > 0$.

We now work with the system (53)-(54).

**Case 2.1:** $\tau^t_{s_s} = \tau^g_{s_z} = 0$.

The price coefficient $\xi_z$ is $m_t \tau^t_{s_s}$. $\xi_\delta$ solves

$$
\xi^2_\delta[\beta^2_z] - \xi^2_\delta \left[ 2 \beta_z \beta_\delta m_t \tau^t_{s_s} \right] + \xi_\delta \left[ \left( \beta^2_z (\tau_\delta + \tau^g_{s_z}) \right) + \beta^2_\delta \tau_z \right] \frac{1}{\tau_n} + m_g m_t \tau^g_{s_s} \tau^t_{s_s} \beta_z + \left( \beta_\delta m_t \tau^t_{s_s} \right)^2 \right] - m_g \tau^g_{s_s} \beta_\delta \left( \left( m_t \tau^t_{s_s} \right)^2 + \frac{\tau_z}{\tau_n} \right) = 0.
$$

(58)

This equation has at least one real root because it is cubic, and the real root(s) must be positive since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be written as a polynomial of $\frac{1}{\tau_n}$:

$$
D = \sum_{i=0}^{3} d_i \left( \frac{1}{\tau_n} \right)^i,
$$

where $d_0 = \beta^4_z m^2_g m^2_t (\tau^t_{s_s})^3 (\tau^g_{s_z})^2 \left( m_t \tau^t_{s_s} \beta^2_\delta - 4 \beta_z m_g \tau^g_{s_s} \right)$. For a sufficiently large $\tau_n$, $D > 0$ if $d_0 > 0$. Therefore, for a sufficiently large $\tau_n$, (58) has three distinct real roots if

$$
\frac{4 \beta_z m_g \tau^g_{s_z}}{\beta^2_\delta m_t \tau^t_{s_s}} < 1.
$$

**Case 2.2:** $\tau^t_{s_s} = 0$ and $\tau^g_{s_z} > 0$.

Notice that $\xi_\delta \beta_\delta \tau^g_{s_z} + \xi_z \beta_z \tau^t_{s_s}$ is constant, so $\xi_z(\xi_\delta)$ is a linear function. Plugging it back to (54), we obtain the following equation for $\xi_\delta$:

$$
f(\xi_\delta) = \sum_{i=0}^{3} a_i \xi^i_\delta = 0, \quad (59)
$$

where $a_1, a_3 > 0$ and $a_0, a_2 < 0$. This equation has a real root because it is cubic. It has only positive real roots since the coefficients of odd/even powers are positive/negative. It has three distinct real roots when its discriminant is positive. The discriminant can be
written as a polynomial of \( \frac{1}{\tau_n} \):

\[
D = \sum_{i=0}^{3} d_i \left( \frac{1}{\tau_n} \right)^i,
\]

where

\[
d_0 = \frac{m_g^2}{\beta_z^2} (m_g \tau_{s_z}^g + \beta_z m_t \tau_{s_z}^t)^2 \left( \tau_{s_z}^g \beta_z^2 + \tau_{s_z}^g \beta_z^2 \right)^2 \left( \beta_z^2 m_t^2 \left( \tau_{s_z}^t \right)^2 - 4 \tau_{s_z}^g \tau_{s_z}^g m_g^2 - 4 \beta_z m_g m_t \tau_{s_z}^g \tau_{s_z}^t \right).
\]

For a sufficiently large \( \tau_n \), \( D > 0 \) if \( d_0 > 0 \). Therefore, for a sufficiently large \( \tau_n \), (59) has three distinct real roots if

\[
\frac{4 \tau_{s_z}^g \tau_{s_z}^g m_g^2 + 4 \beta_z m_g m_t \tau_{s_z}^g \tau_{s_z}^t}{\beta_z^2 m_t^2 \left( \tau_{s_z}^t \right)^2} = \frac{4 m_g \tau_{s_z}^g \left( \tau_{s_z}^g m_g + \beta_z m_t \tau_{s_z}^t \right)}{\beta_z^2 m_t^2 \left( \tau_{s_z}^t \right)^2} < 1.
\]

**Case 2.3:** \( \tau_{s_z}^t > 0 \) and \( \tau_{s_z}^g = 0 \).

The price coefficient \( \xi_z \) is \( m_t \tau_{s_z}^t \). \( \xi_z \) solves

\[
f(\xi_z) = \sum_{i=1}^{5} a_i \xi_z^i = 0.
\]

Moreover, \( a_5 = \beta_z^2 \) and \( a_0 = a_{0,1} \frac{1}{\tau_n} + a_{0,2} \left( \frac{1}{\tau_n} \right)^2 \), where \( a_{0,1}, a_{0,2} < 0 \) do not depend on \( \tau_n \). Then there exists at least one positive real root. Let’s now show that there exists at least three positive real roots for a sufficiently large \( \tau_n \). Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.

We can write

\[
a_0 = a_{0,1} \frac{1}{\tau_n} + a_{0,2} \left( \frac{1}{\tau_n} \right)^2,
\]

\[
a_1 = a_{1,0} + a_{1,1} \frac{1}{\tau_n} + a_{1,2} \left( \frac{1}{\tau_n} \right)^2,
\]

where \( a_{i,j} \neq 0 \) are coefficients that do not depend on \( \tau_n \). Moreover, \( a_{0,1} < 0 \) and \( a_{1,0} > 0 \).

76
Then, evaluating $f(\cdot)$ at $-\frac{a_{0,1}}{a_{1,0}} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} > 0$ for some $c_1 > 0$, we obtain

$$f \left( -\frac{a_{0,1}}{a_{1,0}} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} \right) = a_{1,0} c_1 \frac{1}{\tau_n} + o \left( \frac{1}{\tau_n} \right).$$

For a sufficiently large $\tau_n$, $f \left( -\frac{a_{0,1}}{a_{1,0}} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} \right) > 0$.

Next, we can write (60) also as a polynomial of $\frac{1}{\tau_n}$:

$$f (\xi) = \sum_{i=0}^{2} b_i (\xi) \left( \frac{1}{\tau_n} \right)^i,$$

where

$$b_0(\xi) = \xi \left( \xi^2 + m_t^2 \tau_{s_z}^t \right) \left( \beta \xi - \beta m_t \tau_{s_z}^t \right)^2 + m_t m_g \tau_{s_z}^g \xi \left( \beta \xi - \beta m_t \tau_{s_z}^t \right).$$

Then, evaluating $f(\cdot)$ at $\frac{1}{\beta z} \left( \beta \xi - \beta m_t \tau_{s_z}^t \right)$, we obtain

$$f \left( \frac{1}{\beta z} \left( \beta m_t \tau_{s_z}^t \right) \right) = -m_t m_g \tau_{s_z}^g \xi \left( \beta \xi - \beta m_t \tau_{s_z}^t \right)^2 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right).$$

Therefore, for a sufficiently large $\tau_n$, $f \left( \frac{1}{\beta z} \left( \beta m_t \tau_{s_z}^t \right) \right) < 0$ and, at the same time, $\frac{1}{\beta z} \left( \beta m_t \tau_{s_z}^t \right) > -\frac{a_{0,1}}{a_{1,0}} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} > 0$. Furthermore, because $a_5 > 0$, for any $\tau_n > 0$ $f(\xi) > 0$ if $\xi$ is sufficiently large. Hence, we have shown that (60) has at least three (positive real) solutions for $\xi$ if $\tau_n$ is sufficiently large.

**Case 2.4: $\tau_{s_z}^t, \tau_{s_z}^g > 0$.**

Notice that $\xi \beta \tau_{s_z}^g + \xi \beta \tau_{s_z}^g$ is linear in $\xi_z$, so we can straightforwardly write $\xi_z = \xi_z (\xi)$. Plugging it back to (54), we obtain the following equation for $\xi$:

$$f(\xi) = \sum_{i=1}^{9} a_i \xi_i = 0. \quad (61)$$

Moreover, $a_9 > 0$ and $a_0 < 0$. Then there exists at least one positive real root. Let’s now show that there exists at least three real roots for a sufficiently large $\tau_n$. Our approach is analogous to the proof of Lemma 3, so we keep the proof brief.

77
We can write

\[ a_0 = a_{0.4} \left( \frac{1}{\tau_n} \right)^3 + a_{0.5} \left( \frac{1}{\tau_n} \right)^4, \]

\[ a_1 = a_{1.3} \left( \frac{1}{\tau_n} \right)^2 + a_{1.4} \left( \frac{1}{\tau_n} \right)^3 + a_{1.5} \left( \frac{1}{\tau_n} \right)^4, \]

\[ a_2 = a_{2.3} \left( \frac{1}{\tau_n} \right)^2 + a_{2.4} \left( \frac{1}{\tau_n} \right)^3, \]

\[ a_3 = a_{3.2} \left( \frac{1}{\tau_n} \right) + a_{3.3} \left( \frac{1}{\tau_n} \right)^2 + a_{3.4} \left( \frac{1}{\tau_n} \right)^3, \]

\[ a_4 = a_{4.2} \left( \frac{1}{\tau_n} \right) + a_{4.3} \left( \frac{1}{\tau_n} \right)^2, \]

where \( a_{i,j} \neq 0 \) are coefficients that do not depend on \( \tau_n \). Moreover, \( a_{0.4} < 0 \) and \( a_{1.3} > 0 \).

Then, evaluating \( f(\cdot) \) at \( -a_{0.4} \frac{1}{a_{1.3}} + c_1 \frac{1}{\tau_n} > 0 \) for some \( c_1 > 0 \), we obtain

\[ f \left( -a_{0.4} \frac{1}{a_{1.3}} + c_1 \frac{1}{\tau_n} \right) = a_{1.3} c_1 \left( \frac{1}{\tau_n} \right)^3 + o \left( \left( \frac{1}{\tau_n} \right)^3 \right). \]

For a sufficiently large \( \tau_n \), \( f \left( -a_{0.4} \frac{1}{a_{1.3}} + c_1 \frac{1}{\tau_n} \right) > 0 \).

Next, we can write (61) also as a polynomial of \( \frac{1}{\tau_n} \):

\[ f (\xi) = \sum_{i=0}^{4} b_i(\xi)(\frac{1}{\tau_n})^i, \]

where \( b_0(\xi) \) has a root at \( \xi = \hat{\xi} = \beta \left[ m_s \tau_{s} \tau_{s} - \beta k_m \left( \tau_{s} - \tau_{s} \right) \left( \tau_{s} - \tau_{s} \right) \right]. \) Note that under our benchmark assumptions, \( \tau_{s} \tau_{s} - \tau_{s} \tau_{s} = 0 \) and \( \hat{\xi} > 0 \). Moreover, \( \hat{\xi} > 0 \) as long as traditional (green) investors are relatively better informed about \( \ddot{z} \)-component (\( \ddot{\delta} \)-component). Therefore, we consider \( \hat{\xi} \geq 0 \) as a more empirically relevant case. However, for the sake of completeness, we also study the case \( \hat{\xi} < 0 \) separately.

**Case 2.4.1:** \( \hat{\xi} > 0 \). Evaluate \( b_0(\cdot) \) at \( \hat{\xi} = \left( \frac{1}{\tau_n} \right)^{1/2} \) to obtain

\[ b_0 \left( \hat{\xi} - \left( \frac{1}{\tau_n} \right)^{1/2} \right) = -c_2 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right), \]

where \( c_2 \) is a positive coefficient which does not depend on \( \tau_n \). Then, evaluating \( f(\cdot) \) at
the same point, we obtain
\[
f \left( \hat{\xi}_\delta - \left( \frac{1}{\tau_n} \right)^{1/2} \right) = -c_2 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right).
\]

For a sufficiently large \( \tau_n \), the above expression is negative and, at the same time, \( \hat{\xi}_\delta - \left( \frac{1}{\tau_n} \right)^{1/2} > -\frac{a_0 a_1}{4} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} > 0 \). Furthermore, because \( a_0 > 0 \), for any \( \tau_n > 0 \) \( f (\xi_\delta) > 0 \) if \( \xi_\delta \) is sufficiently large. Hence, we have shown that (61) has at least three (positive real) solutions for \( \xi_\delta \) if \( \tau_n \) is sufficiently large.

**Case 2.4.2:** \( \hat{\xi}_\delta < 0 \). Evaluate \( b_0(\cdot) \) at \( \hat{\xi}_\delta - \left( \frac{1}{\tau_n} \right)^{1/2} \) to obtain
\[
b_0 \left( \hat{\xi}_\delta + \left( \frac{1}{\tau_n} \right)^{1/2} \right) = c_2 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right),
\]
where \( c_2 \) is the same positive coefficient as in Case 2.4.1. Then, evaluating \( f (\cdot) \) at the same point, we obtain
\[
f \left( \hat{\xi}_\delta + \left( \frac{1}{\tau_n} \right)^{1/2} \right) = c_2 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right).
\]

For a sufficiently large \( \tau_n \), the above expression is positive and, at the same time, \( \hat{\xi}_\delta + \left( \frac{1}{\tau_n} \right)^{1/2} < 0 < -\frac{a_0 a_1}{4} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n} \). Furthermore, because \( a_0 > 0 \), for any \( \tau_n > 0 \) \( f (\xi_\delta) < 0 \) if \( \xi_\delta \) is sufficiently large in absolute terms and negative. Hence, we have shown that (61) has at least three real solutions for \( \xi_\delta \) if \( \tau_n \) is sufficiently large (recall that \( f(0) = a_0 < 0 \)).

**Case 2.4.3:** \( \hat{\xi}_\delta = 0 \).

In this case, \( b_0(\cdot) \) can be written as
\[
b_0(\xi_\delta) \equiv A\xi_\delta^6 \sum_{i=0}^{3} b_{i,\delta} \xi_\delta^i,\]
where \( A > 0, b_{0,3} > 0, b_{0,2} < 0, b_{0,1} > 0, b_{0,0} > 0 \). Then there exists \( \hat{\xi}_\delta < 0 \) that solves \( b_0(\xi_\delta) = 0 \) such that
\[
b_0 \left( \hat{\xi}_\delta + \left( \frac{1}{\tau_n} \right)^{1/2} \right) = c_3 \left( \frac{1}{\tau_n} \right)^{1/2} + o \left( \left( \frac{1}{\tau_n} \right)^{1/2} \right),
\]
where $c_3$ is a positive constant. Moreover, at this point

$$f\left(\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2}\right) = c_3 \left(\frac{1}{\tau_n}\right)^{1/2} + o\left(\left(\frac{1}{\tau_n}\right)^{1/2}\right).$$

For a sufficiently large $\tau_n$, the above expression is positive and, at the same time, $\hat{\xi}_\delta + \left(\frac{1}{\tau_n}\right)^{1/2} < 0 < -\frac{a_0 a_4}{a_1 a_3} \frac{1}{\tau_n} + c_1 \frac{1}{\tau_n}$. Furthermore, because $a_0 > 0$, for any $\tau_n > 0$ $f(\hat{\xi}_\delta) < 0$ if $\xi_\delta$ is sufficiently large in absolute terms and negative. Hence, we have shown that (61) has at least three real solutions for $\xi_\delta$ if $\tau_n$ is sufficiently large (recall that $f(0) = a_0 < 0$).

\section*{F Investors with homogeneous preferences}

The key assumption we make throughout the paper is that there are two groups of investors with heterogeneous stock valuations. Because of preference heterogeneity, they use information about the same fundamentals differently and trade in opposite directions, which might give rise to multiple equilibria that differ in the relative price informativeness about the two fundamentals. We show the robustness of this result to general assumptions on the information structure in Appendix E.

The goal of this appendix is to show that preference heterogeneity is an essential ingredient for equilibrium multiplicity. In particular, we explore a model that features two groups of investors that have homogeneous preferences but might have different information about the two fundamentals. The key difference between our setting and Goldstein and Yang (2015) is that we allow investors of both groups to receive informative signals about both fundamentals. As we discuss in Appendix E, this is crucial to support multiple equilibria in the trading stage when investors’ preferences are heterogeneous. Our key result here is that equilibrium in the trading stage is unique when preferences are homogeneous.

We consider the same framework as described in Section 3 with several differences. First, we assume that both groups of investors have the same stock valuation, $\beta_z \tilde{z} + \beta_\delta \tilde{\delta}$. For consistency, we keep denoting the two groups using $t$ and $g$ subscripts. The masses of the two groups are $m_t$ and $m_g$. Without loss of generality, we set the utility weights $\beta_z = \beta_\delta = 1$ and the risk aversion parameter $\gamma = 1$. Further, we assume that $t$-investors ($g$-investors) specialize in particular types of information and, thus, receive signals about $\tilde{z}$ and $\tilde{\delta}$ with precisions of $\tau_s (\lambda \tau_s)$ and $\lambda \tau_s (\tau_s)$, respectively. Without loss of generality,
we assume $\lambda \in [0, 1]$. The priors for $\tilde{z}$ and $\tilde{\delta}$ are assumed to be the same, $\tau_z = \tau_\delta = \tau$.\footnote{These assumptions on the information structure can be further relaxed (at the expense of tractability but without changing the final result) by allowing for different prior precisions and more general signal precisions. The analyses are available upon request.}

Market clearing implies the following system of equations for $\xi_z$ and $\xi_\delta$:

$$
\begin{align*}
\xi_z &= \tau_s \left[ m_t \frac{\xi_\delta^2 + \frac{\tau + \lambda \tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1 + \lambda)) \frac{1}{\tau_n}} + mg \lambda \frac{\xi_\delta^2 + \frac{\tau + \lambda \tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1 + \lambda)) \frac{1}{\tau_n}} \right], \\
\xi_\delta &= \tau_s \left[ m_t \lambda \frac{\xi_z^2 + \frac{\tau + \lambda \tau_s}{\tau_n} - \xi_z \xi_\delta}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1 + \lambda)) \frac{1}{\tau_n}} + mg \frac{\xi_z^2 + \frac{\tau + \lambda \tau_s}{\tau_n} - \xi_\delta \xi_z}{(\xi_z - \xi_\delta)^2 + (2\tau + \tau_s(1 + \lambda)) \frac{1}{\tau_n}} \right].
\end{align*}
$$

Denote $x \equiv \xi_\delta - \xi_z$. It is easy to see that $\xi_z$ and $\xi_\delta$ are uniquely pinned down for a given $x$. Furthermore, the system can be simplified to the following quintic equation for $x$:

$$
f(x)g(x) = (1 - \lambda)^2 (1 + \lambda) \tau_n \tau_s^3 m_g m_t x, \tag{62}
$$

where

$$
\begin{align*}
f(x) &= x \left( x^2 \tau_n + 2\tau + \tau_s(1 + \lambda) \right) + x \tau_n \tau_s^2 \left( \lambda \left( m_g^2 + m_t^2 \right) + m_g m_t \left( 1 + \lambda^2 \right) \right) - \\
&\quad \tau \tau_s \left( m_g - m_t \right) \left( 1 - \lambda \right), \\
g(x) &= x^2 \tau_n + 2\tau + \tau_s(1 + \lambda).
\end{align*}
$$

Clearly, (62) has a unique solution $x = 0$ when $\lambda = 1$. Suppose now that $\lambda < 1$ and $m_g > m_t$ (case of $m_g \leq m_t$ can be considered analogously). Our goal is to show that (62) has a unique solution.

We first show that there exists a unique solution to (62) on $x \geq 0$. Since $f(0) < 0$ and $f(x)$ is an increasing and convex function, there exists a unique $x > 0$ such that $f(x) = 0$ and that $f(x) > 0$ if and only if $x > x$. Moreover, $f(x)g(x)$ is an increasing convex function on $x \geq x$. Therefore, there exists exactly one solution to (62) on $x \geq 0$.

We then verify that there is no solution on $x < 0$. First, $f(0)g(0) < 0$. Second, $f(x)g(x)$ is increasing and concave on $x < 0$. Finally, the derivative of $f(x)g(x)$ at 0 is $f'(0)g(0) + f(0)g'(0) > (1 + \lambda^2)(1 + \lambda)\tau_n \tau_s^3 m_g m_t > \left( 1 + \lambda \right)^2 \tau_n \tau_s^3 m_g m_t$. So the right-hand side of (62) is always above the left-hand side on $x < 0$. We, therefore, have established the following proposition.
Proposition 11. If investors have homogeneous preferences, there exists a unique equilibrium with a linear price.

We conclude that the equilibrium multiplicity in the trading game requires investors to have heterogeneous stock valuations. Otherwise, the trading behaviors of investors are aligned, and the price is always simultaneously informative to both investor groups.

G Heterogeneous ESG preferences

In our baseline model, we assume that green investors have homogeneous preferences and uniformly value the total ESG output. In reality, individual investors may assign distinct utility weights to various components of ESG. In this appendix, we explore the extension that allows for heterogeneity in ESG preferences among green investors and investigate implications for investors’ trading strategies and the equilibrium pricing function. Due to the complexity of a fully general model with heterogeneous ESG preferences, we resort to making specific assumptions and solving the model numerically. Further research can explore this model at a deeper level.

In particular, we assume that the stock payoff consists of three components: a financial component $\tilde{z}$, an environmental component $\tilde{\delta}$, and a social component $\tilde{\eta}$. All three components are independent and identically distributed normal variables, $\tilde{z}, \tilde{\delta}, \tilde{\eta} \sim N(0, \tau^{-1})$. There are three groups of rational investors: a mass $m_t = \frac{m}{2}$ of traditional investors, a mass $m_e = \frac{(1-\theta)m}{2}$ of environmental investors, and a mass $m_s = \frac{\theta m}{2}$ of social investors. That is, the economy features equal masses of traditional and ESG investors. Among ESG investors, a fraction $\theta$ are social investors, and the rest are environmental investors. Each rational investor values only one payoff component but observes three private signals $\tilde{s}_i^z \sim N(\tilde{z}, \tau_s^{-1}), \tilde{s}_i^\delta \sim N(\tilde{\delta}, \tau_s^{-1}),$ and $\tilde{s}_i^\eta \sim N(\tilde{\eta}, \tau_s^{-1})$. All other model features are equivalent to those of our baseline model. Note that when $\theta = 0$ or 1, the model reduces to the case consider in Section 4.1.

Analogously to the equilibrium characterization of our baseline model, we first analyze the trading intensities of rational investors. Each rational investor now has three trading intensities with respect to her three private signals. Considering traditional investors as
the following parametrization:

\[ m_\theta \]

of the case in which the variance of the noise traders’ demand, \( \tau \), In particular, the normalized price coefficients, \( \theta \), the aggregate trading intensities of all three groups of rational investors:

\[ \xi_z = \text{hump-shaped.} \]

In the following, we solve this system of equations numerically. First, we investigate the case in which the variance of the noise traders’ demand, \( \tau_n^{-1} \), is large. We consider the following parametrization: \( m = 1, \gamma = 1, \tau = 1, \sigma = 10, \) and \( \tau_n = 1. \) Under this parametrization, equilibrium is unique for any fraction of social investors in ESG investors \( \theta \in [0, 1]. \) Figure 10 plots the three normalized price coefficients as functions of \( \theta. \) The effect of \( \theta \) on \( \xi_\delta \) and \( \xi_\eta \) is intuitive. As \( \theta \) increases, the composition of ESG investors shifts toward social investors, resulting in a higher price association with the social component \( \tilde{\eta} \) and a lower price association with the environmental component \( \tilde{\delta}. \) Interestingly, the relationship of the financial price coefficient \( \xi_z \) and \( \theta \) is hump-shaped. The mass of traditional investors does not vary with \( \theta, \) and their trading intensity with respect to their financial signals, \( i_z^t \), is constant. What drives the change in \( \xi_z \) is the trading against their financial signals by ESG investors. When \( \theta = 0.5, \) the aggregate preference heterogeneity among ESG investors reaches its maximum, with masses of environmental and social investors being equal. At this point, trades by environmental and social investors introduce substantial amounts of noise to each other. As a result, the stock becomes risky to ESG investors, and they trade less actively on their private
signals. In particular, they trade less actively against their financial signals, which in turn pushes $\xi_z$ up.

![Unique equilibrium: Price coefficients ($\theta$)](image)

Figure 10: Price coefficients $\xi_z$, $\xi_d$ and $\xi_\eta$ as functions of the fraction of social investors in ESG investors $\theta$. Parametrization: $m = 1$, $\gamma = 1$, $\tau = 1$, $\tau_s = 10$, $\tau_n = 1$. Equilibrium is unique for all values of $\theta \in [0, 1]$.

Second, we explore the case in which the variance of noise traders’ demand is small (Figure 11). We set $\tau_n = 5$ and leave other parameters unchanged. Recall that when the fraction of social investors among ESG investors $\theta$ is either 0 or 1, the model reduces to the special case considered in Section 4.1. As we establish in that section, if $\tau_n$ is sufficiently large, there are three equilibria. We find that, under our parametrization, the equilibrium multiplicity is preserved if $\theta$ is close to either 0 or 1, i.e., when $\theta < \underline{\theta}$ and $\theta > \bar{\theta}$. However, for moderate values of $\theta \in (\underline{\theta}, \bar{\theta})$, equilibrium is unique. Intuitively, when $\theta$ is small (large), the economy is populated with similar masses of traditional and environmental (social) investors, and either group of investors can dominate the trading, leading to multiple equilibria. When $\theta$ is moderate, there is a large heterogeneity among ESG investors, and they always face a substantial amount of noise when trading. Consequently, neither group of ESG investors can dominate the trading, and the only possible equilibrium is $T$-equilibrium, in which the price is mostly driven by the financial component.

In summary, our numerical example shows that having two groups of ESG investors with heterogeneous preferences tends to dilute the focus of ESG investment. This dilution makes the stock riskier to trade for ESG investors in general. Consequently, it reduces ESG investors’ trading intensities against financial information, leading to a higher price association with the financial payoff component. Moreover, such a dilution also makes it easier for traditional investors to dominate the trading, thereby reducing the likelihood of equilibrium multiplicity.

It is worth emphasizing that our results in this appendix are based on specific numeri-
Figure 11: Price coefficients $\xi_z$ (panel A), $\xi_\delta$ (panel B) and $\xi_\eta$ (panel C) as functions of the fraction of social investors in ESG investors $\theta$. Parametrization: $m = 1$, $\gamma = 1$, $\tau = 1$, $\tau_s = 10$, $\tau_n = 5$. Vertical dashed lines mark equilibrium multiplicity region: equilibrium is unique if $\theta \in [\theta, \bar{\theta}]$, and there are three equilibria otherwise.

cal values and simplifying assumptions. Consequently, we consider them as an initial step in highlighting the potential importance of modeling ESG investors in a more nuanced manner. We leave this for future research.