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Government guarantees and financial stability ☆

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Abstract

Banks are intrinsically fragile because of their role as liquidity providers. This results in under-provision of liquidity. We analyze the effect of government guarantees on the interconnection between banks' liquidity creation and likelihood of runs in a global-game model, where banks' and depositors' behavior are endogenous and affected by the amount and form of guarantee. The main insight of our analysis is that

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guarantees are welfare improving because they induce banks to improve liquidity provision, although that sometimes increases the likelihood of runs or creates distortions in banks' behavior.

JEL classification: G21; G28

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1. Introduction

Government guarantees to financial institutions are common all over the world. The recent financial crisis has led to renewed interest and debate about their role and their desirability. On the one hand, government guarantees are thought to have a positive role in preventing panic among investors, and hence help stabilize the financial system. On the other hand, they may distort banks' incentives, thus leading to an increase in financial fragility (see, e.g., Calomiris, 1990; Demirgüç-Kunt and Detragiache, 1998; Gropp et al., 2014, and Acharya and Mora, 2015).

In light of this trade-off, evaluating the overall effects of government guarantees to banks requires a framework in which the behavior of banks and their investors interacts with the amount and form of guarantees. Such a model is known to be notoriously rich and hard to solve. It needs to endogenize the probability of runs and how it is affected by banks' risk choices and government guarantees. It also needs to endogenize banks' risk choices and how they vary with the guarantee, taking into account investors' expected run behavior. We make technical progress in this paper by putting all these ingredients together in a tractable model, which generates some surprising results on the effects of government guarantees. Most notably, we show that the increase in bank fragility resulting from the introduction of government guarantees may sometimes be a desirable consequence.

To conduct our analysis, we build on the model developed in Goldstein and Pauzner (2005), where depositors' withdrawal decisions are uniquely determined using the global-game methodology. Our novelty is to add a government to this model to study how the government guarantee policy interacts with the banking contract and the probability of a run. In our model, there are two periods. Banks raise funds from risk-averse consumers in the form of deposits and invest them in risky projects whose return depends on the fundamentals of the economy. Depositors derive utility from consuming both a private and a public good. At the interim date, each depositor learns whether he needs to consume early or not and receives an imperfect signal regarding the fundamentals of the bank. Impatient depositors withdraw at that point and patient ones decide when to withdraw based on the information received. In deciding whether to run or not, depositors compare the payoff they would get from going to the bank prematurely and waiting until maturity. These payoffs depend on the fundamentals and the expectation about the proportion of depositors running.

As in Goldstein and Pauzner (2005), the equilibrium outcome is that runs occur when the fundamentals are below a unique threshold, which depends on the banking contract. Within the range where they occur, they can be classified into *panic-based runs* or *fundamental-based runs*. The former type of run is one that is generated by the self-fulfilling belief of depositors that other depositors will run. The latter type of run happens when the signal on the fundamentals is low enough to make running a dominant strategy for depositors. As in Diamond and Dybvig (1983), there is perfect competition with free entry in the banking sector, and so banks offer a contract that maximizes depositors' expected utility. Unlike in Diamond and Dybvig (1983), however, banks recognize the implications that the contract has for the possibility of a run and take them into account when deciding on the contract. As a result, in the decentralized equilibrium without guarantees banks reduce the amount of liquidity they offer to depositors demanding early withdrawal so that there is under provision of liquidity insurance. Yet, the deposit contract offered by banks is such that inefficient fundamental-based and panic-based runs occur in equilibrium. While banks internalize the cost of the runs, the benefit from risk sharing is large enough to lead them to offer contracts that entail some inefficient runs.

We then analyze whether the provision of public guarantees through the transfer of resources from the public good to the banking sector can alleviate the inefficiencies of the decentralized allocation in terms of inefficient runs and lack of protection against project risk. Essentially, by ensuring depositors receive a minimum payment irrespective of the specific bank outcome, guarantees can play two distinct roles: They ameliorate the coordination failure among depositors, thus reducing the probability of panic-based runs; and they protect late depositors against project failure. To highlight their distinct roles, we start by considering the possibility of different levels of guarantees, depending on the bank outcome. We first analyze a simple form of guarantees which, in the spirit of Diamond and Dybvig, addresses only panic runs due to depositors' coordination failure. Second, we consider a guarantee scheme that protects depositors against bank project risk, even in the absence of runs. Finally, we consider a standard deposit insurance scheme, which fulfils all these roles by guaranteeing depositors to receive the same payment irrespective of the bank outcome.

In the first scheme, depositors are guaranteed to receive a minimum payment only if the bank project is successful irrespective of what the other depositors do. By eliminating the negative externality that a run imposes, this scheme prevents panic runs with a mere announcement effect and thus it does not entail any public disbursement. Hence, it does not lead to distortions in the bank's choice of the deposit contract. However, unlike in Diamond and Dybvig (1983), fundamental runs still occur in our framework, as depositors are not protected against bank project risk.

An important result is that, under this guarantee scheme, banks increase deposit rates to early withdrawers, and so perform more liquidity transformation. This increases the probability of fundamental-based runs, thus possibly leading to an increase in the overall run probability relative to the decentralized equilibrium. The result that crises may become more likely in the presence of public intervention, which is consistent with the evidence in Demirgüç-Kunt and Detragiache (1998), can be interpreted as one form of increased risk taking following the introduction of guarantees (e.g., Calomiris, 1990). However, even if banks increase the amount they offer for early withdrawals so that the overall likelihood of runs can be higher than in the decentralized equilibrium, the introduction of guarantees still improves welfare as banks are not acting against depositors' interests. Hence, the model demonstrates the need for caution in interpreting often-mentioned empirical results.

Although it eliminates panics and improves welfare, the first guarantee scheme does not protect depositors against bank project risk, that is the risk that the bank project fails at the final date in the absence of a run. Thus, we next investigate whether, in addition to eliminating panics, it is optimal to provide some insurance to depositors against such risk by guaranteeing them a minimum payment at the final date when the bank project fails. We show that, under certain conditions, offering some guarantee against project risk is desirable. However, given that the government makes transfers to banks when their projects fail, intervention is now costly and there are distortions in banks' behavior. As before, banks increase deposit rates in response to the guarantee, but choose to under provide liquidity insurance as they do not internalize the government's costs. This is where the intuition of moral hazard often featured in the public debate – according to which banks' incentives are distorted by guarantees – starts to show up in our model. Interestingly, however, while it is commonly thought that banks set deposit rates too high in response to guarantees, our framework shows that the distortion can go in the opposite direction.

Both schemes described above entail contingent guarantee payments, which is not in line with real-world deposit insurance schemes, in which depositors always receive a minimum guaranteed payment irrespective of bank outcome. In this case, we show that all trade-offs of the previous scheme remain valid in that, despite entailing bank moral hazard, deposit insurance is still welfare improving because it reduces the run probability and induces banks to perform better liquidity transformation. However, as it is not possible to adjust the guaranteed amount to the different bank outcomes, the deposit insurance scheme reduces – but does not eliminate – panic runs. In addition, the distortion in banks' behavior depends now on the size of the guarantee. When this is low, banks under provide liquidity insurance, as before. By contrast, when the guarantee is high, banks tend to over provide liquidity insurance.

In summary, a careful analysis of the effects of government guarantees shows that they have an important role helping the financial system to provide risk sharing to investors while mitigating the problems associated with coordination failures and bank project risk. The common criticism against guarantees – that they may be a source of financial fragility – ignores that some risk taking by banks due to liquidity transformation is desirable as banks under provide liquidity when they are concerned about run risk. Guarantees, in turn, relax these concerns allowing banks to provide greater liquidity transformation, which is welfare improving.

Our analysis focuses on the effect of guarantees on the interconnection between banks' liquidity creation and likelihood of runs, so that all risk taking in our model is captured on the liability side. In doing this, we disregard other possible aspects of government guarantees such as, for example, the choice of assets by banks. It is possible that extending the model further will uncover undesirable aspects of government guarantees. Also, we make several simplifying assumptions on the form of the banking contract and government guarantees, which keep the analysis tractable, but might prevent additional implications from being revealed. Still, our framework, to the best of our knowledge, is the first one that allows studying the endogenous probability of runs and the endogenous risk choice by banks and how they interact with each other and with the government's guarantee policy.

Our paper provides a step towards understanding the interconnection between guarantees, fragility and bank's behavior. In the seminal Diamond and Dybvig (1983) analysis, the introduction of deposit insurance eliminates panic runs so that banks fully perform their role as liquidity providers. However, this approach has two important shortcomings.[1] First, deposit insurance has no effect on banks' behavior as it only works as an equilibrium selection device. Second, as it has a simple announcement effect, deposit insurance entails no disbursement for the government. This no longer holds when runs can also occur as a result of deteriorating bank fundamentals (see evidence in Gorton, 1988; Calomiris and Gorton, 1991 and Calomiris and Mason, 2003). In such cases, deposit insurance typically does not fully prevent runs, entails actual costs of paying for failed banks and distorts banks' behavior (e.g., Calomiris, 1990, and Cooper and Ross, 2002). The richness of our model, where runs can be both fundamental and panic driven and the probability of runs is endogenously determined as a function of the parameters of the demand deposit

[1] Similar environments where runs are driven by agents' expectations and public intervention is desirable to eliminate the panic equilibrium are analyzed in subsequent papers including, recently, Cooper and Kempf (2016).

contract, allows us to overcome these two shortcomings and fully analyze the interconnections between government guarantees, depositors' withdrawal decisions and banks' behavior.

Our public intervention differs from bailouts, which represent a form of ex post intervention aimed at mitigating the negative consequences of a crisis rather than preventing it. Despite these differences, our analysis shares some features with the literature on bailouts (see, among others, Farhi and Tirole, 2012; Nosal and Ordonez, 2016; Keister, 2016 and Keister and Narasiman, 2016), in that these contributions also analyze how the (anticipation of) bailouts may adversely affect banks' risk taking incentives, and ultimately the desirability of public intervention.

Among these contributions, the closest papers to ours are Keister (2016) and Keister and Narasiman (2016). In both contributions the anticipation of a bailout introduces a trade-off: on the one hand, it induces banks to engage in more liquidity creation, thus increasing depositors' incentives to run; on the other hand, it improves investors' payoffs, thus reducing their incentives to run if they expect others to do the same. Whether the bailout improves welfare and leads to more or less fragility depends on which of these two effects dominates.

In both papers, the occurrence of runs depends on the realization of a sunspot variable, whose probability is exogenous and not affected by the anticipation of bailouts, and there is always a no run equilibrium irrespective of the bailout policy chosen by the government. An advantage of our model is that the probability of runs is fully endogenous, and so we are able to better characterize the interconnection between fragility, public guarantees and bank behavior. Our results that guarantees enable banks to perform more welfare-improving liquidity transformation, which is true even if the probability of crisis increases, and the characterization of the direction of distortions caused by guarantees are not present in the other papers.

The ability to endogenize the probability of panic- and fundamental-driven runs and derive unique equilibria in context where agents have private information on some random variables relies on the use of global games as in the literature originating with Carlsson and van Damme (1993).[2] The closest contribution to ours in this literature is Goldstein and Pauzner (2005). As our model builds on theirs, it shares the same technical challenge of characterizing the existence of a unique equilibrium in a context in which there are no global strategic complementarities.

As our analysis shows, having a unique equilibrium and being able to disentangle the various effects of a specific policy is key to evaluating its desirability, effectiveness and costs. In line with this, global games techniques have been increasingly used in recent years to analyze relevant policy questions concerning financial regulation and public intervention (e.g., Bebchuk and Goldstein, 2011; Choi, 2014; Vives, 2014 and Eisenbach, 2017).

The paper proceeds as follows. Section 2 describes the model without government intervention. Section 3 derives the decentralized equilibrium. Section 4 analyzes the guarantee schemes. It first characterizes schemes where the guaranteed amounts are contingent to bank outcome; and then a standard deposit insurance scheme with fixed payments. Section 5 uses a parametric example to illustrate the properties of the model. Section 6 contains discussion and concluding remarks. All proofs are contained in the appendix.

[2] Applications of global games in finance include Morris and Shin (1998, 2004); Goldstein and Pauzner (2004); Corsetti et al. (2004); Goldstein (2005), and Rochet and Vives (2004). See also Morris and Shin (2003) for a survey on the theory and application of global games.

2. The basic model

Our model is based on Goldstein and Pauzner (2005), augmented to include a government for the purpose of studying guarantee policies. There are three dates ($t = 0, 1, 2$), a continuum $[0, 1]$ of banks and a continuum $[0, 1]$ of consumers in every bank.

Banks raise one unit of funds from consumers in exchange for a deposit contract as specified below, and invest in a risky project. For each unit invested at date 0, the project returns 1 if liquidated at date 1 and a stochastic return $\tilde{R}$ at date 2 given by

$$\tilde{R} = \begin{cases} R > 1 & \text{w.p. } p(\theta) \\ 0 & \text{w.p. } 1 - p(\theta). \end{cases}$$

The variable θ, which represents the state of the economy, is uniformly distributed over $[0, 1]$. We assume that $p(\theta) = \theta$ and $E_\theta[p(\theta)]R > 1$, which implies that the expected long term return of the project is superior to the short term return.

Each consumer is endowed with one unit at date 0 and nothing thereafter. At date 0, each consumer deposits his endowment at the bank. The bank promises a fixed payment $c_1 > 1$ to depositors withdrawing at date 1. Alternatively, depositors can choose to wait until date 2 and receive a risky payoff $\widetilde{c}_2$, as specified below.

Consumers are ex ante identical but can be of two types ex post: each of them has a probability λ of being an early consumer (impatient) and consuming at date 1, and a probability $1 - \lambda$ of being a late consumer (patient) and consuming at either date (we usually refer to them as early depositors and late depositors, respectively). Consumers privately learn their type at date 1.

The government has an endowment g, which, for the moment, it can only use to provide public goods to consumers in addition to the deposit payments they obtain from banks. Consumers' preferences are then given by

$$U(c, g) = u(c) + v(g),$$

where $u(c)$ represents the utility from the consumption of the payments obtained from banks and $v(g)$ is the utility from the consumption of the public good provided by the government. In what follows, we will refer to $u(c)$ and $v(g)$ also as private and public utility, respectively.[3] The function $U(c, g)$ satisfies $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and the relative risk aversion coefficient, $-cu''(c)/u'(c)$, is greater than one for any $c \geq 1$. In addition, we focus on the case where the government's endowment g is small enough that $u'(1) < v'(g)$. This ensures that the government has no incentives to make direct transfers to consumers.

The state of the economy θ is realized at the beginning of date 1, but is publicly revealed only at date 2. After θ is realized at date 1, each consumer receives a private signal x_i of the form

$$x_i = \theta + \varepsilon_i, \tag{1}$$

where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, +\varepsilon]$. After the signal is realized, consumers decide whether to withdraw at date 1 or wait until date 2.

There is perfect competition among banks, so that they choose the deposit contract $(c_1, \widetilde{c}_2)$ at date 0 that maximizes depositors' expected utility. As usual in the financial crisis literature (e.g.

[3] Consumers receive the same amount of public good irrespective of their type. As with the good provided by the bank, early consumers enjoy the public good at date 1 while late consumers enjoy it at either date. Given there is no discounting, the timing of the provision does not matter for the late types.

Diamond and Dybvig, 1983 and numerous papers thereafter), the deposit contract involves a non-contingent date 1 payment c_1 and a date 2 payment $\widetilde{c}_2$ equal to the return of the non-liquidated units of the bank project divided by the number of remaining late depositors. The payment c_1 must be lower than the amount $\frac{1-\lambda c_1}{1-\lambda}R$ that each late depositor receives at date 2 when only the λ early depositors withdraw early and the project succeeds. Otherwise, the deposit contract is never incentive compatible and late consumers always have an incentive to withdraw early and thus generate a run. The bank satisfies consumers' withdrawal demands at date 1 by liquidating the project. If the liquidation proceeds are not enough to repay the promised c_1 to the withdrawing depositors, each of them receives a pro-rata share of the liquidation proceeds.[4]

The timing of the model is as follows. At date 0, each bank chooses the promised payment c_1. At date 1, after realizing their type and receiving the private signal about the state of the fundamentals θ, depositors decide whether to withdraw early or wait until date 2. At date 2, the bank project return is realized and waiting late depositors receive a pro-rata share. The model is solved backwards.

3. The decentralized equilibrium without guarantees

In this section we derive the decentralized allocation following Goldstein and Pauzner (2005), where we use the superscript D to denote the equilibrium variables. We start by analyzing depositors' withdrawal decisions at date 1 for a given fixed payment c_1.

Early consumers always withdraw at date 1 to satisfy their consumption needs. By contrast, late consumers decide whether to withdraw at date 1 based on the signal x_i they receive since this provides information on both θ and other depositors' actions. Upon receiving a high signal, a late consumer attributes a high posterior probability to a positive bank project return R at date 2 and infers that the others have also received a high signal. This lowers his belief about the likelihood of a run and thus his own incentive to withdraw at date 1. Conversely, when the signal is low, the opposite happens and a late consumer has a high incentive to withdraw early. This suggests that late consumers withdraw at date 1 when the signal is low enough, and wait until date 2 when the signal is sufficiently high.

To show this formally, we first examine two regions of extremely bad and extremely good fundamentals, where each late consumer's action is based on the realization of the fundamentals irrespective of his beliefs about the others' behavior. We start with the lower region.

Lower dominance region. When θ is very low, running is a dominant strategy: upon receiving his signal, a late consumer is certain that the expected utility from waiting until date 2, $\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)$, is lower than that from withdrawing at date 1, $u(c_1)$, even if only the early depositors were to withdraw ($n=\lambda$). We then denote by $\underline{\theta}(c_1)$ the value of θ that solves

$$u(c_1)=\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right), \tag{2}$$

that is

$$\underline{\theta}(c_1)=\frac{u(c_1)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)}. \tag{3}$$

[4] The assumption that depositors' repayments follow a pro-rata share rule rather than a sequential service constraint as in Goldstein and Pauzner (2005) simplifies the analysis without affecting the qualitative results.

We refer to the interval $\left[0, \underline{\theta}(c_1)\right)$ as the lower dominance region, where runs are only driven by bad fundamentals. For the lower dominance region to exist for any $c_1 \geq 1$, there must be feasible values of θ for which all late depositors receive signals that assure them to be in this region. Since the noise contained in the signal x_i is at most ε, each late depositor withdraws at date 1 if he observes $x_i < \underline{\theta}(c_1) - \varepsilon$. It follows that all depositors receive signals that assure them that θ is in the lower dominance region when $\theta < \underline{\theta}(c_1) - 2\varepsilon$. Given that $\underline{\theta}$ is increasing in c_1, the condition for the lower dominance region to exist is satisfied for any $c_1 \geq 1$ if $\underline{\theta}(1) > 2\varepsilon$.

Upper dominance region. The upper dominance region of θ corresponds to the range $\left(\overline{\theta}, 1\right]$ in which fundamentals are so good that waiting is a dominant strategy. We construct this region by assuming that in the range $(\overline{\theta}, 1]$ the project is safe, i.e., $p(\theta) = 1$, and yields the same return $R > 1$ at dates 1 and 2. Given $c_1 < \frac{1-\lambda c_1}{1-\lambda} R \leq R$, this ensures that the bank does not need to liquidate more units than the n depositors withdrawing at date 1. Then, upon observing a signal indicating that the fundamentals θ are in the upper dominance region, a late consumer is certain to receive his payment $\frac{1-\lambda c_1}{1-\lambda} R$ at date 2, irrespective of his beliefs on other depositors' actions, and thus he has no incentives to run. Similarly to before, the upper dominance region exists if there are feasible values of θ for which all late depositors receive signals that assure them to be in this range. This is the case if $\overline{\theta} < 1 - 2\varepsilon$.

The intermediate region. When the signal indicates that θ is in the intermediate range $\left[\underline{\theta}(c_1), \overline{\theta}\right]$, a depositor's decision to withdraw early depends on the realization of θ as well as on his beliefs regarding other late depositors' actions. To see how, we first calculate a late depositor's utility differential between withdrawing at date 2 and at date 1 as given by

$$v(\theta, n) = \begin{cases} \theta u\left(\frac{1-nc_1}{1-n} R\right) - u(c_1) & \text{if } \lambda \leq n \leq \widehat{n} \\ 0 - u(\frac{1}{n}) & \text{if } \widehat{n} \leq n \leq 1, \end{cases} \tag{4}$$

where n represents the proportion of depositors withdrawing at date 1 and

$$\widehat{n} = 1/c_1 \tag{5}$$

is the value of n at which the bank exhausts its resources if it pays $c_1 \geq 1$ to all withdrawing depositors. For $n \leq \widehat{n}$, a waiting late depositor obtains $\frac{1-nc_1}{1-n} R$ with probability θ while an early withdrawer obtains c_1. By contrast, for $n \geq \widehat{n}$ the bank liquidates its entire project at date 1. Late depositors receive nothing when waiting until date 2 and the pro-rata share $1/n$ when withdrawing early.

As Fig. 1 illustrates, the function $v(\theta, n)$ decreases in n for $n \leq \widehat{n}$ and increases with it afterwards, crossing zero once for $n \leq \widehat{n}$ and remaining always below afterwards. Thus, the model exhibits the property of *one-sided strategic complementarity* as in Goldstein and Pauzner (2005) and there exists a unique equilibrium in which a late depositor runs if and only if his signal is below the threshold $x^*(c_1)$. At this signal value, a late depositor is indifferent between withdrawing at date 1 and waiting until date 2. The following result holds.

Proposition 1. *The model has a unique equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1)$ and do not run above. At the limit, as $\varepsilon \to 0$, $x^*(c_1)$ simplifies to*

$$\theta^*(c_1) = \frac{u(c_1)\left[1 - \lambda c_1\right] + c_1 \int_{n=\widehat{n}}^{1} u(\frac{1}{n})}{c_1 \int_{n=\lambda}^{\widehat{n}} u\left(\frac{1-nc_1}{1-n} R\right)}. \tag{6}$$

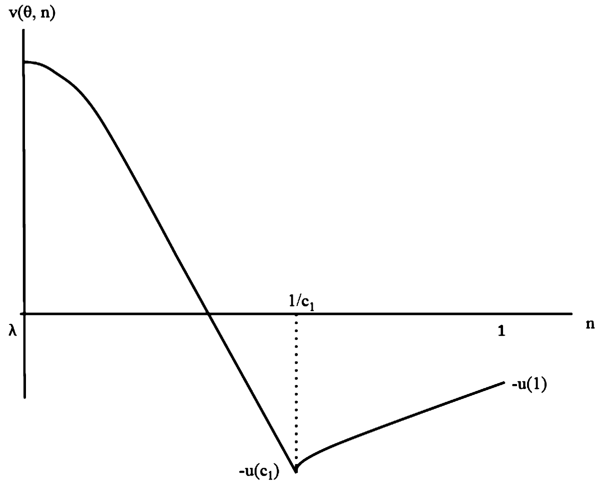

Fig. 1. Depositors' utility differential in the decentralized economy. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1. The function is decreasing in n for $\lambda \leq n < \frac{1}{c_1}$ and increasing for $\frac{1}{c_1} \leq n < 1$. It crosses zero only once for $n < \frac{1}{c_1}$ and remains below zero afterwards.

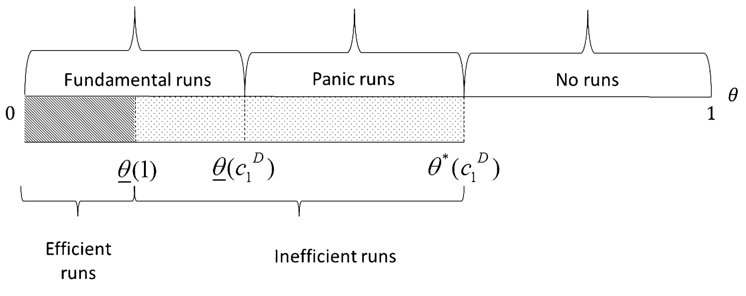

Fig. 2. Depositors' withdrawal decision. The figure characterizes depositors' withdrawal decision as a function of the fundamentals of the economy θ. Depositors run if $\theta < \theta^*(c_1^D)$ and do not run otherwise. In the region in which they run, two types of crisis can be distinguished. If $\theta < \underline{\theta}(c_1^D)$, runs are fundamentals-driven. If $\underline{\theta}(c_1^D) < \theta < \theta^*(c_1^D)$, runs are panic-driven. While all panic runs are inefficient, fundamental runs are inefficient only in the range $\underline{\theta}(1) < \theta < \underline{\theta}(c_1^D)$. Otherwise they entail an efficient liquidation of the banks' asset.

The proposition states that even in the intermediate region a late depositor's action depends uniquely on the signal he receives as this provides information both on the fundamentals θ and the other depositors' actions. For θ in the interval $\left[\underline{\theta}(c_1), \theta^*(c_1)\right)$ there is strategic complementarity in consumers' withdrawal decisions: If $c_1 > 1$, the bank has to liquidate more than one unit for each withdrawing depositor. This implies that late depositors' date 2 payoff is decreasing in the proportion n of early withdrawing depositors and so their incentives to run increases with n. In the limit case when $\varepsilon \to 0$, all late depositors behave alike as they receive approximately the same signal and take the same action. This implies that only complete runs, where all late depositors withdraw at date 1, occur. In what follows, we will focus on this limit case.

Proposition 1 implies that a run occurs for any $\theta < \theta^*(c_1)$, but for different reasons as also illustrated in Fig. 2. For θ in the interval $\left[0, \underline{\theta}(c_1)\right)$ runs are *fundamental-based*: Late depositors withdraw early because they expect the fundamentals to be bad so that running is a dominant strategy. For θ in the interval $\left[\underline{\theta}(c_1), \theta^*(c_1)\right)$ runs are *panic-based*: Late depositors withdraw because they expect others to do the same. The two types of runs differ significantly in terms of efficiency. Panic runs are always inefficient as they result from a coordination failure among depositors. By contrast, fundamental runs can be efficient if they lead to the early liquidation of unprofitable investments. For each unit that the bank liquidates at date 1 to repay the withdrawing depositors, the return R is foregone with probability θ. Liquidating the project is then inefficient for any $\theta > \underline{\theta}(1)$ since the utility $u(1)$ that a depositor obtains from the liquidated unit is lower than the expected utility $\theta u(R)$ he would obtain from the same unit if invested until date 2. If $c_1 > 1$, then $\underline{\theta}(1) < \underline{\theta}(c_1) < \theta^*(c_1)$. Thus, fundamental runs are efficient in the range $[0, \underline{\theta}(1))$ but inefficient in the range $[\underline{\theta}(1), \underline{\theta}(c_1))$.

The likelihood of both types of run – as given by the thresholds $\underline{\theta}(c_1)$ and $\theta^*(c_1)$ – is affected by the deposit payment c_1 as follows.

Corollary 1. *Both thresholds $\underline{\theta}(c_1)$ and $\theta^*(c_1)$ are increasing in c_1 $\left(i.e., \frac{\partial \underline{\theta}(c_1)}{\partial c_1} > 0 \text{ and } \frac{\partial \theta^*(c_1)}{\partial c_1} > 0\right)$ with $\frac{\partial \theta^*(c_1)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1)}{\partial c_1}$.*

The corollary suggests that both run thresholds increase with the deposit rates offered by banks, although their sensitivity is different. The reason is that the higher c_1, the lower is the payoff $\widetilde{c}_2$ accrued by the late depositors at date 2 and thus the stronger is the incentive for each late depositor to withdraw early. The panic threshold $\theta^*(c_1)$ is more sensitive to changes in c_1 than the fundamental threshold $\underline{\theta}(c_1)$ because in the case of panic runs an increase in c_1 also changes the beliefs that each depositor has on the others' behavior and on the damage that their withdrawals will cause to the remaining investors' returns. This reinforces each late depositor's incentive to run, thus making $\theta^*(c_1)$ more sensitive to changes in c_1 than $\underline{\theta}(c_1)$.

Given depositors' withdrawal decisions at date 1, we compute the optimal deposit contract c_1. At date 0 each bank chooses c_1 to maximize depositors' expected utility as given by

$$\underset{c_1}{Max} \int_0^{\theta^*(c_1)} u(1)\, d\theta + \int_{\theta^*(c_1)}^{1} \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \int_0^1 v(g) d\theta. \tag{7}$$

The first term represents depositors' expected utility from depositing at the bank for $\theta < \theta^*(c_1)$ when each depositor obtains 1 in the occurrence of a run. The second term is depositors' expected utility for $\theta \geq \theta^*(c_1)$ when the bank continues until date 2: The λ early consumers obtain c_1, while the $(1-\lambda)$ late depositors receive $\frac{1-\lambda c_1}{1-\lambda} R$ at date 2 with probability θ. The last term is depositors' utility from the public good. Since the entire government's endowment g is used to provide the public good, depositors' utility $v(g)$ is unaffected by the occurrence of runs. We have the following result.

Proposition 2. *The optimal deposit contract $c_1^D > 1$ in the decentralized solution solves*

$$\lambda \int_{\theta^*(c_1)}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta +$$
$$-\frac{\partial \theta^*(c_1)}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right] = 0 \tag{8}$$

In choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (8), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, which is captured by the second term in (8), is the loss in expected utility $\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right]$ due to the increased probability of runs, as measured by $\frac{\partial\theta^*(c_1)}{\partial c_1}$. At the optimum, the bank chooses $c_1^D > 1$ even if this entails panic runs. The reason is that when $c_1 = 1$, the difference between early and late depositors' expected payment is maximal. A slight increase of c_1 provides a large benefit in terms of risk sharing given that depositors have a relative risk aversion coefficient greater than 1. Furthermore, at $c_1 = 1$ the loss in terms of expected utility in the case of a run approaches zero.[5] Thus, increasing c_1 slightly above 1 entails a second-order cost and a first-order benefit and so it is always optimal. The optimal c_1^D is chosen so that runs occur only for $\theta < \theta^*(c_1) < \overline{\theta}$. If this was not the case and runs occurred for any θ, consumers would obtain a utility $u(1)$, which would be lower than the utility they reach with the optimal c_1^D.

The bank's choice of $c_1^D > 1$ entails a trade-off in our model. On the one hand, c_1 represents the amount of risk sharing banks offer to depositors. On the other hand, given that the probability of runs is endogenous and is linked to the parameters of the deposit contract, c_1 represents a form of risk as it determines banks' exposure to runs. Hence, the higher payment c_1, the greater is banks' liquidity creation but also their fragility. Since banks anticipate the effect of a higher c_1 on their fragility, they reduce deposit rates thus scaling down liquidity creation.

The decentralized allocation entails several inefficiencies. First, as $c_1^D > 1$, both inefficient fundamental and panic runs occur. Second, depositors are exposed to project risk in that they obtain zero with probability $1-\theta^*(c_1^D)$ when the project fails at date 2. Third, the anticipation of runs affects banks' ability to provide liquidity insurance to early depositors. To illustrate these frictions, we characterize the allocation reached by a social planner that can enforce efficient runs and use public funds to protect depositors against bank project risk, while being restricted as banks to offer only non-contingent demandable deposit contracts. Formally, the social planner chooses c_1 and $\overline{c}_f$ to maximize depositors' expected utility as given by

$$\int_0^{\underline{\theta}(1)} u(1)\, d\theta + \int_{\underline{\theta}(1)}^{1} \left[\lambda u(c_1) + (1-\lambda)\left(\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta)\, u\left(\overline{c}_f\right)\right)\right] d\theta \tag{9}$$
$$+ \int_0^{\underline{\theta}(1)} v(g)\, d\theta + \int_{\underline{\theta}(1)}^{1} \left[\theta v(g) + (1-\theta)\, v\left(g-(1-\lambda)\overline{c}_f\right)\right] d\theta.$$

The expression differs from (7) in two respects: the run threshold is now given by $\underline{\theta}(1)$ instead of $\theta^*(c_1)$ and depositors obtain a positive payment $\overline{c}_f$ with probability $1-\theta$ when the project fails. This implies a reduction in the provision of public good when no runs occur, as evident in the last term of (9).

[5] When $c_1 = 1$, the term $\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(1)\right]$ simplifies to $(1-\lambda)\left[\theta^*(1)u(R) - u(1)\right] = 0$ with $\theta^*(1) = \left[\frac{u(1)}{u(R)}\right]$.

The optimal values for c_1^{SP} and $\overline{c}_f^{SP}$ solve

$$\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(c_1) - \theta R u' \left(\frac{1-\lambda c_1}{1-\lambda} R \right) \right] d\theta = 0, \tag{10}$$

and

$$\int_{\underline{\theta}(1)}^{1} (1-\lambda)(1-\theta) \left[u'\left(\overline{c}_f\right) - v'\left(g - (1-\lambda)\overline{c}_f\right) \right] d\theta = 0.$$

The solution for c_1^{SP} improves risk sharing relatively to the decentralized allocation, i.e., $c_1^{SP} > c_1^D$, and offers some protection against project risk, i.e., $\overline{c}_f^{SP} > 0$ when $u'(0) - v'(g) > 0$.

4. Introducing government guarantees

One way to improve the decentralized allocation derived above is through the provision of government guarantees, which consist in a minimum payment to depositors when the bank is unable to repay them the promised amount: at date 1 if there is a run and all depositors receive a pro-rata share of 1; at date 2 if the bank project succeeds but there is a panic run; at date 2 if the bank project fails irrespective of the proportion of early withdrawers. This implies that guarantees can play three distinct roles: Allowing a transfer of public resources to depositors at date 1; ameliorating the coordination failure among depositors, thus reducing the probability of panic-based runs; and protecting late depositors against bank project failure.

To illustrate these different roles, we start by allowing the government to offer different levels of guarantees depending on bank outcome. In particular, we first analyze a simple form of guarantees which, in the spirit of Diamond and Dybvig, addresses only panic runs due to depositors' coordination failure. Second, we consider a guarantee scheme that protects depositors against bank project risk, even in the absence of runs. Third, we show that providing a transfer of public resources to depositors at date 1 is never optimal. Finally, we analyze a standard deposit insurance scheme, which fulfils all these roles as it guarantees depositors to receive the same payment irrespective of the bank outcome.

In all cases, the introduction of guarantees modifies the timing of the model as follows. At date 0, the government chooses the amount to guarantee $\overline{c}$ and then the bank chooses c_1. At date 1, after learning their types and receiving the signal about the state of fundamentals θ, depositors decide whether to withdraw early or wait until date 2. As before, for each guarantee scheme considered, we solve the model backward. We first characterize depositors' withdrawal decisions for given $\overline{c}$ and c_1 and obtain the thresholds $\underline{\theta}(c_1, \overline{c})$ and $\theta^*(c_1, \overline{c})$ for the fundamental and panic runs, respectively. Then, we characterize the bank's choice of c_1, for given $\overline{c}$, and finally the government's choice of $\overline{c}$. In all guarantee schemes, the government finances the promised guarantee $\overline{c}$ through the transfer of resources from its endowment g to the banking sector.

4.1. Guarantees against panic runs

We first analyze a scheme aimed at reducing the occurrence of panic runs by addressing the coordination problem among depositors. Under this scheme, depositors are guaranteed to receive a minimum payment if the bank project is successful at date 2 irrespective of what the other depositors do. We show that this guarantee eliminates panics and induces banks to offer higher deposit rates. In turn, this increases the probability of fundamental runs and possibly even the probability of runs overall. We use the superscript P to denote the equilibrium variables under this guarantee scheme.

As highlighted in the analysis of the decentralized economy, panic runs arise in our model because of the strategic complementarity between depositors' actions. The greater the number of depositors withdrawing at date 1, the more units of the project the bank needs to liquidate prematurely. This, in turn, increases the incentive for a late depositor to run since his repayment in the case he waits is reduced. The coordination failure among depositors leads to a panic-driven run for θ in the interval $[\underline{\theta}(c_1), \theta^*(c_1))$.

Consider now that the government promises depositors to receive a minimum repayment $\overline{c}_s > 0$ when the bank project succeeds at date 2. This reduces late depositors' incentives to withdraw at date 1 for θ in the range $[\underline{\theta}(c_1), \theta^*(c_1))$ because a depositor's date 2 payoff when the bank project succeeds becomes less dependent on the other depositors' withdrawal decisions. As in Diamond and Dybvig (1983), this scheme has a pure announcement effect, without entailing any disbursement for the government. As a result, the government finds it optimal to set $\overline{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ so that panic runs are eliminated.

While eliminating panics, this scheme fails to address fundamental-based runs as it does not protect depositors against bank project risk. Thus, late depositors still choose to run when they expect bad fundamentals, that is when $\theta < \underline{\theta}(c_1)$, where $\underline{\theta}(c_1)$ is determined in (3). Importantly though, the elimination of panic runs affects the bank's choice of c_1 and thus the probability of fundamental runs in equilibrium will be different from the decentralized economy. This is what we turn to next.

Given that runs occur now for $\theta < \underline{\theta}(c_1)$, each bank chooses c_1 to maximize depositors' expected utility as given by

$$\underset{c_1}{Max} \int_0^{\underline{\theta}(c_1)} u(1)d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta + \int_0^1 v(g)d\theta \tag{11}$$

The terms in (11) have the same meaning as in (7), with the difference that runs are now only fundamental-driven and thus the relevant threshold is $\underline{\theta}(c_1)$ instead of $\theta^*(c_1)$. Since the insurance scheme does not entail any cost, the government still provides g units of public good and depositors still obtain public utility $v(g)$ as in the decentralized allocation without guarantees. As there, the solution to the problem in (11) must be such that $\underline{\theta}(c_1) < \overline{\theta}$ at the equilibrium c_1. This gives the following result.

Proposition 3. *The optimal deposit contract $c_1^P > c_1^D$ in the case of a guarantee scheme against panic runs solves*

$$\lambda \int_{\underline{\theta}(c_1)}^1 \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta +$$
$$-\frac{\partial \underline{\theta}(c_1)}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\underline{\theta}(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right] = 0. \tag{12}$$

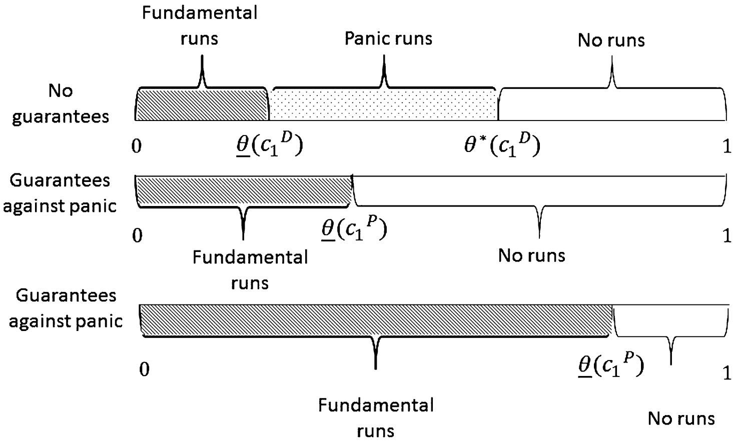

Fig. 3. Guarantees against panic runs and financial stability. The figure shows the effect of the introduction of the guarantee scheme against panic runs on the stability of the banking sector. The guarantee scheme removes completely the occurrence of panic-driven runs, but fundamental runs become more likely as a result of an increase in the repayment offered by banks to early withdrawing depositors. If the increase in c_1 is very large, then the overall probability of runs can be larger in the economy with guarantees than without it (i.e., $\underline{\theta}(c_1^P) > \theta^*(c_1^D)$).

As in the decentralized economy, the bank chooses the deposit contract that trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (12), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (12), is the loss in expected utility $\left[\lambda u(c_1) + (1-\lambda)\underline{\theta}(c_1) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$ due to the increased probability of fundamental runs as measured by $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$. The solution c_1^P is now larger than c_1^D in the decentralized economy as given by the solution to (8). The reason is that both the run threshold $\underline{\theta}(c_1)$ and its sensitivity to changes in c_1, as represented by $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$, are lower than $\theta^*(c_1)$ and $\frac{\partial \theta^*(c_1)}{\partial c_1}$ as shown in Corollary 1. As a result, the marginal benefit of an increase in c_1 in terms of better risk sharing is higher than the one in the decentralized economy, while its marginal cost is lower. Thus, the bank has an incentive to choose a higher c_1 and improve liquidity creation relative to the case without government intervention.

The proposition has important implications in terms of bank stability, as illustrated in Fig. 3. The guarantee scheme eliminates panic runs, but it increases the probability of fundamental runs due to the increased deposit rate. That is $\underline{\theta}(c_1^P) > \underline{\theta}(c_1^D)$, given Corollary 1 and $c_1^P > c_1^D$.

If the difference between c_1^P and c_1^D is large enough, it can even happen that the bank becomes more fragile than in the decentralized solution, that is $\underline{\theta}(c_1^P) > \theta^*(c_1^D)$. The bank chooses to do so if the benefit of the increased c_1 in terms of improved risk sharing outweighs the cost in terms of higher probability of runs. After comparing the expected overall utilities with and without guarantees in equilibrium as given in (11) and (7), and rearranging the terms, we can express the condition for this to happen as follows:

$$\int_{\underline{\theta}(c_1^P)}^{1} \left[\left(\lambda u(c_1^P) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1^P}{1-\lambda} R\right)\right)\right. \tag{13}$$

$$-\left(\lambda u(c_1^D)+(1-\lambda)\theta u\left(\frac{1-\lambda c_1^D}{1-\lambda}R\right)\right)\Bigg]d\theta >$$

$$\int_{\theta^*(c_1^D)}^{\underline{\theta}(c_1^P)}\left[\lambda u(c_1^D)+(1-\lambda)\theta u\left(\frac{1-\lambda c_1^D}{1-\lambda}R\right)-u\left(1\right)\right]d\theta.$$

The term on the LHS represents the benefit in terms of greater risk sharing deriving from a larger c_1 in the range $\theta > \underline{\theta}(c_1^P)$ where no run occurs both in the decentralized economy and in the one with guarantees. The term on the RHS represents instead the loss in terms of foregone risk sharing when in the range $\theta^*(c_1^D) \le \theta < \underline{\theta}(c_1^P)$ a run occurs in the presence of guarantees because of the higher c_1. Characterizing when (13) holds is not straightforward as several effects are at play. We will provide an example in which this happens and guarantees lead to greater overall instability in Section 5.

To sum up, the analyzed guarantee scheme eliminates panic runs and thus induces banks to provide more liquidity, which in turn increases fundamental runs. Importantly, such increased fragility is not driven by banks' attempt to maximize the value of the guarantee as there is no disbursement of the public good in equilibrium. In other words, there is no moral hazard problem on the side of the banks. As a result, despite the increased bank fragility, this guarantee scheme is welfare improving as it allows better risk sharing than in the decentralized equilibrium.

4.2. Guarantees against fundamental project risk

We now consider that, in addition to eliminating panics, the guarantee scheme protects depositors against the bank project risk. Under this scheme depositors are also promised a minimum payment at date 2 if the bank project fails and no run has occurred. As a consequence, the probability of fundamental runs is reduced but, as guarantees are now provided in equilibrium when the project fails, they are costly in terms of reduced public good provision. Differently from the scheme that only eliminates panics, the choice of the deposit contract now features a moral hazard problem as banks do not internalize the cost that guarantees entail for the government. We use the superscript F to denote the equilibrium variables with guarantees against fundamental project risk.

Consider now that the government promises a minimum repayment $\overline{c}_f > 0$ at date 2 with probability $1-\theta$ when the project fails in addition to the payment $\overline{c}_s > 0$ when the project succeeds. First, it is immediate to see that preventing panic runs is still desirable as they destroy resources in the range $[\underline{\theta}(c_1), \theta^*(c_1))$. Thus, setting $\overline{c}_s = \frac{1-\lambda c_1}{1-\lambda}R$ is still optimal.

Given this, we start by characterizing late depositors' withdrawal decisions at date 1 for given c_1 and $\overline{c}_f$ in the absence of panics. Late depositors decide to run at date 1 when θ is below the threshold $\underline{\theta}^F\left(c_1, \overline{c}_f\right)$ which solves

$$u\left(c_1\right)=\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+\left(1-\theta\right)u\left(\overline{c}_f\right).$$

The terms have the same meaning as in (2) with the difference that depositors now receive utility $u(\overline{c}_f)$ at date 2 when, with probability $1-\theta$, the bank project fails and the guarantee $\overline{c}_f$ is paid. The solution is then equal to

$$\underline{\theta}^F\left(c_1, \overline{c}_f\right)=\frac{u\left(c_1\right)-u\left(\overline{c}_f\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}. \tag{14}$$

It is immediate to see that the threshold of fundamental runs increases in c_1 and decreases in $\overline{c}_f$ (i.e., $\frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial c_1} > 0$ and $\frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial \overline{c}_f} < 0$).

We now turn to date 0 and analyze the bank's choice of c_1 and the government's choice of $\overline{c}_f$. We start with the former. Given $\overline{c}_f$ and anticipating depositors' withdrawal decisions, each bank chooses c_1 to maximize depositors' expected utility as given by

$$\underset{c_1}{Max} \int_0^{\underline{\theta}^F(c_1,\overline{c}_f)} u(1)\, d\theta \tag{15}$$
$$+ \int_{\underline{\theta}^F(c_1,\overline{c}_f)}^{1} \left[\lambda u(c_1) + (1-\lambda)\left(\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + (1-\theta)\, u\left(\overline{c}_f\right)\right)\right] d\theta$$
$$+ \int_0^{\underline{\theta}^F(c_1^*,\overline{c}_f)} v(g)\, d\theta + \int_{\underline{\theta}^F(c_1^*,\overline{c}_f)}^{1} \left[\theta v(g) + (1-\theta)\, v\left(g - (1-\lambda)\,\overline{c}_f\right)\right] d\theta$$

The expression in (15) is similar to the one for the social planner in (9), except that the run threshold is now given by $\underline{\theta}^F\left(c_1, \overline{c}_f\right)$ instead of $\underline{\theta}(1)$. The first term in (15) represents depositors' expected utility when a run occurs for $\theta < \underline{\theta}^F\left(c_1, \overline{c}_f\right)$ and depositors obtain $u(1)$. The second term in (15) is the expected utility for $\theta \geq \underline{\theta}^F\left(c_1, \overline{c}_f\right)$ when there is no run: Early depositors obtain the promised repayment and late depositors receive either the promised payment with probability θ when the project succeeds or the guarantee level $\overline{c}_f$ with probability $1-\theta$ when the project fails. The last two terms in (15) represent the utility from the public good. For $\theta < \underline{\theta}^F\left(c_1, \overline{c}_f\right)$ the government uses g entirely to provide the public good so that depositors obtain utility $v(g)$. The same happens with probability θ for $\theta \geq \underline{\theta}^F\left(c_1, \overline{c}_f\right)$. By contrast, with probability $1-\theta$ for $\theta \geq \underline{\theta}^F\left(c_1, \overline{c}_f\right)$ the government uses $(1-\lambda)\,\overline{c}_f$ of its endowment to provide the guarantee and thus depositors only obtain $v\left(g-(1-\lambda)\,\overline{c}_f\right)$ as utility from the public good.

As they are atomistic, banks do not internalize the cost of the guarantee in their choice of c_1 and, consequently, the last two terms in (15) depend on the equilibrium choice c_1^* of other banks rather than the individual bank's choice of c_1. We obtain the following.

Proposition 4. *The deposit contract $c_1^F > c_1^P$ against the bank fundamental project risk solves*

$$\lambda \int_{\underline{\theta}^F(c_1,\overline{c}_f)}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta \tag{16}$$
$$-\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\,\underline{\theta}^F\left(c_1,\overline{c}_f\right) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right.$$
$$\left. + (1-\lambda)\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right) u\left(\overline{c}_f\right) - u(1)\right] = 0$$

and it is increasing in the amount of guarantee, i.e., $\frac{dc_1^F}{d\overline{c}_f} > 0$.

As usual, in choosing the promised payment to early depositors the bank trades off the marginal benefit of a higher c_1 with its marginal cost. The former, as represented by the first term in (16), is the better risk sharing obtained from the transfer of consumption from late to early depositors. The latter, as captured by the second term in (16), is the loss in expected utility due to the increased probability of fundamental runs as measured by $\frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial c_1}$.

The proposition shows also that the optimal c_1 increases with $\overline{c}_f$. Thus, protecting depositors also against project risk induces banks to provide even more risk sharing to early depositors than when only panics are eliminated. The increased liquidity provision is, however, not necessarily welfare improving. The reason is that now providing the guarantee is costly in equilibrium when the government reduces the provision of the public to increase depositors' payment in the event of project failure. This creates distortions in the bank's choice of the deposit rate. Anticipating this effect, the government chooses to provide a positive level of the guarantee only if its benefits in terms of liquidity provision and insurance against project failure outweigh its costs in terms of lower public good provision. To see when this is the case, we now turn to the government's choice of $\overline{c}_f$.

Given the bank's choice $c_1^F\left(\overline{c}_f\right)$, at date 0 the government chooses $\overline{c}_f$ to maximize depositors' total expected utility, which is given by the expression in (15) evaluated at $c_1 = c_1^F(\overline{c}_f)$. Note that this objective function differs from that of the social planner in (9) as the government choose $\overline{c}_f$ while taking c_1 as set by the bank and depositors' withdrawal decisions as represented by the threshold $\underline{\theta}^F\left(c_1, \overline{c}_f\right)$ in (14).

Proposition 5. *If $u'(0) - v'(g) > 0$, the government chooses $\overline{c}_f^F > 0$, which solves:*

$$\begin{aligned}
&\int_{\underline{\theta}^F(c_1,\overline{c}_f)}^{1} (1-\lambda)(1-\theta)\left[u'\left(\overline{c}_f\right) - v'\left(g-(1-\lambda)\overline{c}_f\right)\right] d\theta \\
&-\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial \overline{c}_f}\Bigg[\lambda u\left(c_1\right) + (1-\lambda)\underline{\theta}^F\left(c_1,\overline{c}_f\right) u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) \\
&+\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)u\left(\overline{c}_f\right) - u(1)\Bigg] \\
&-\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial \overline{c}_f}\left[\underline{\theta}^F\left(c_1,\overline{c}_f\right)v(g) + \left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)v\left(g-(1-\lambda)\overline{c}_f\right) - v(g)\right] \qquad (17) \\
&-\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\frac{dc_1}{d\overline{c}_f}\left[\underline{\theta}^F\left(c_1,\overline{c}_f\right)v(g) + \left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)v\left(g-(1-\lambda)\overline{c}_f\right) - v(g)\right] \\
&= 0,
\end{aligned}$$

where $c_1 = c_1^F(\overline{c}_f)$.

The proposition shows that the government chooses $\overline{c}_f^F > 0$ to protect late depositors against bank project failure if the marginal benefit in terms of increased private utility outweighs the marginal cost in terms of reduced utility from the public good. Given the concavity of the utility functions, this condition will be satisfied unless the government is very constrained in its endowment g.

The choice of the optimal amount of $\overline{c}_f^F$ depends on the amount of public good g. There are three effects to consider. First, the guarantee is essentially a transfer from the public to the private good, so that a marginal change in the amount provided affects the marginal utilities from consumption as reflected in the first term in (17). Second, an increase in the guaranteed amount has the direct effect of reducing the probability of runs through the term $\frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial \overline{c}_f}$. As reflected in the second and third term in (17), this increases the utility from private consumption, while decreasing that from public consumption due to the government's payment of the guarantee when the bank project fails. Third, a higher guaranteed payment has an indirect effect on the utility from the public good through the bank's choice of c_1, as captured in the last term in (17). In particular, the increased probability of a run induced by a higher c_1 increases the utility from public good, which is not internalized by the bank.

As mentioned above, the provision of a positive amount of guarantee $\overline{c}_f$ induces distortions in bank behavior. To investigate these in more detail, we first derive the equilibrium in the case when the government would choose both c_1 and $\overline{c}_f$ and then we compare it with the one characterized in Propositions 4 and 5. In this case, the government chooses c_1 and $\overline{c}_f$ to maximize (15) for given depositors' withdrawal decisions, but taking explicit account of the disbursement needed to provide the guarantee when choosing c_1. We denote the optimal choice of c_1 as c_{1G}^F and we obtain the following.

Proposition 6. *For a given $\overline{c}_f$, banks choose an inefficiently low c_1, that is $c_1^F < c_{1G}^F$, where c_{1G}^F solves*

$$\begin{aligned}
&\lambda \int_{\underline{\theta}^F(c_1,\overline{c}_f)}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta \\
&- \frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial c_1}\Bigg[\lambda u(c_1) \\
&+ (1-\lambda)\left(\underline{\theta}^F(c_1,\overline{c}_f)\, u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + \left(1-\underline{\theta}^F(c_1,\overline{c}_f)\right) u(\overline{c}_f)\right) - u(1)\Bigg] \\
&- \frac{\partial \underline{\theta}^F(c_1,\overline{c}_f)}{\partial c_1}\left[\underline{\theta}^F(c_1,\overline{c}_f)\, v(g) + (1-\underline{\theta}^F(c_1,\overline{c}_f))\, v\left(g-(1-\lambda)\overline{c}_f\right) - v(g)\right] = 0. \quad (18)
\end{aligned}$$

The proposition suggests that the distortion in the bank's choice of c_1 leads banks to be less exposed to runs than would be optimal. This result, which is opposite to common wisdom, hinges on the fact that the government internalizes that the production of public good is reduced in the case there is no run and the bank project fails, while the public good is fully provided when there is a run. This idea relates to the concept of prompt corrective action: Liquidating banks early rather than intervening them later when banks' assets turn out to be unsuccessful may be desirable when it allows minimizing the costs associated with public intervention.

4.3. Guarantees as a transfer at date 1

So far we have shown that the government finds it optimal to offer levels of guarantees $\overline{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ and $\overline{c}_f > 0$ at date 2 to eliminate panic runs and protect depositors against bank project risk. Now we consider whether it would be desirable to also guarantee some minimum repayment at date 1 in the event of a run to increase depositors' payoffs. We show this is not the case as long as the government can offer different payments depending on bank outcome.

When the government guarantees depositors a payment $\overline{c}_1 > 1$ at date 1 when a run occurs, depositors' overall expected utility is given by

$$\underset{\overline{c}_1}{Max} \int_0^{\underline{\theta}^F(c_1,\overline{c}_f)} u(\overline{c}_1)\, d\theta$$
$$+ \int_{\underline{\theta}^F(c_1,\overline{c}_f)}^{1} \left[\lambda u(c_1) + (1-\lambda)\left(\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta)\, u\left(\overline{c}_f\right)\right)\right] d\theta$$
$$+ \int_0^{\underline{\theta}^F(c_1^*,\overline{c}_f)} v(g-\overline{c}_1+1)\, d\theta + \int_{\underline{\theta}^F(c_1^*,\overline{c}_f)}^{1} \left[\theta v(g) + (1-\theta)\, v\left(g-(1-\lambda)\overline{c}_f\right)\right] d\theta \qquad (19)$$

The expression is the same as in (15) with the difference that for $\theta < \underline{\theta}^F\left(c_1, \overline{c}_f\right)$ depositors now obtain $u(\overline{c}_1)$ from the private good and $v(g-\overline{c}_1+1)$ from the public good. We obtain the immediate following result.

Proposition 7. *The government chooses not to offer any guarantee at date* 1*, that is* $\overline{c}_1 < 1$.

As it does not have any effect on the probability of runs, guaranteeing depositors in the case of a run is equivalent to provide a direct transfer of public good to the private sector. This is not optimal in our model since, by imposing the condition $u'(1) < v'(g)$ in Section 2, we restrict our attention to the case when the government's endowment g is such that transfers of public to the private agents can be desirable only if they help reduce the occurrence of runs.

4.4. Standard deposit insurance

We now consider a standard deposit insurance scheme where depositors receive a minimum payment when the bank is unable to repay them. Formally, this means $\overline{c}_s = \overline{c}_f = \overline{c}_1 = \overline{c}$. Within this scheme, providing a guarantee reduces panics, protects depositors against bank project risk and provides a transfer at date 1 if the government finds it optimal to set $\overline{c} > 1$.[6] We show that all trade-offs analyzed in the previous sections remain in place and that providing the guarantee can still be welfare-enhancing.

To study this new form of guarantee, we follow the same steps as in the previous sections and we use the superscript DI to denote the equilibrium variables. We first characterize the equilibrium in depositors' withdrawal decisions for given c_1 and $\overline{c}$. To make the analysis meaningful, we focus on the parameter space where $\overline{c} < c_1$ in equilibrium so that depositors are not fully insured and still have incentives to run. It follows that panic runs can now occur, and thus we need to characterize the run threshold as in Section 3. The upper dominance region is as in the

[6] The condition $u'(1) < v'(g)$ works against the optimality of $\overline{c} > 1$ as it rules out transfers at date 1 when guarantees are conditional on bank outcome, as shown in Section 4.3. However, unlike that case, with standard deposit insurance, the government may find it optimal to set $\overline{c} > 1$ in order to reduce panics.

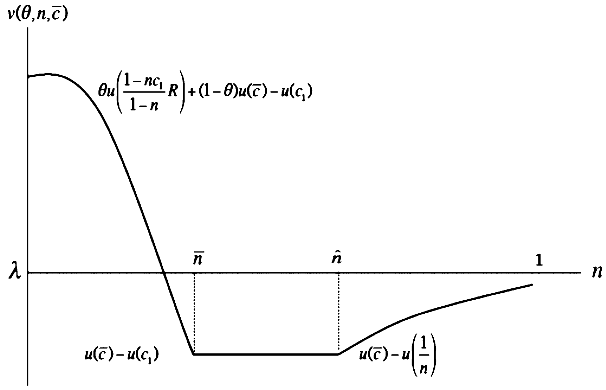

Fig. 4a. Depositor's utility differential with a guarantee against panic and fundamental failures when $\bar{c} \leq 1$. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1 for a given guarantee $\bar{c}$ chosen by the government. The function is decreasing in n for $\lambda \leq n < \bar{n}$, constant in the range $\bar{n} \leq n < \hat{n}$ and increasing for $\hat{n} \leq n \leq 1$. It crosses zero only once for $n < \bar{n}$ and remains below zero afterwards.

decentralized economy, while the upperbound of the lower dominance region $\underline{\theta}(c_1, \bar{c})$ is the same as in (14) with $\bar{c}$ instead of $\bar{c}_f$. In the intermediate region, a late depositor's utility differential between withdrawing at date 2 versus date 1, denoted $v(\theta, n, \bar{c})$, is given by

$$v(\theta,n,\bar{c}) = \begin{cases} \theta u\left(\frac{1-nc_1}{1-n}R\right) + (1-\theta)u\left(\bar{c}\right) - u(c_1) & \text{if } \lambda \leq n \leq \bar{n} \\ u\left(\bar{c}\right) - u(c_1) & \text{if } \bar{n} \leq n \leq \widehat{n} \\ u\left(\bar{c}\right) - u(\frac{1}{n}) & \text{if } \widehat{n} \leq n \leq \min\{1, \widetilde{n}\} \\ u\left(\bar{c}\right) - u(\bar{c}) & \text{if } \min\{1, \widetilde{n}\} \leq n \leq 1 \end{cases}, \quad (20)$$

where $\bar{n} = \frac{R-\bar{c}}{Rc_1-\bar{c}}$, $\widehat{n} = \frac{1}{c_1}$ and $\widetilde{n} = \frac{1}{\bar{c}}$. The expression for $v(\theta, n, \bar{c})$ has four intervals. In the first interval, for $\lambda \leq n \leq \bar{n}$, depositors waiting until date 2 receive $\frac{1-nc_1}{1-n}R > \bar{c}$ with probability θ and $\bar{c}$ with probability $1-\theta$, while those withdrawing early obtain c_1. As n reaches $\bar{n}$, banks' repayment to depositors at date 2 falls below $\bar{c}$ so that late depositors always receive $\bar{c}$ for $n \geq \bar{n}$. Depositors withdrawing at date 1 receive the promised repayment c_1 as long as $n \leq \widehat{n}$, that is as long as the bank has enough resources to pay c_1 from the liquidation of the project at date 1. As n grows further (i.e., for any $\widehat{n} \leq n \leq \min\{1, \widetilde{n}\}$), the bank liquidates its entire project for a value of 1 and each depositor receives the pro-rata share $\frac{1}{n}$ when withdrawing at date 1. Finally, for any $\min\{1, \widetilde{n}\} \leq n \leq 1$, the pro-rata share $\frac{1}{n}$ falls below $\bar{c}$ and depositors withdrawing at date 1 start receiving the guarantee $\bar{c}$. This case occurs only if $\bar{c} > 1$.

The functions $v(\theta, n, \bar{c})$ are illustrated in Figs. 4a and 4b for the case $\bar{c} \leq 1$ and $\bar{c} > 1$, respectively. When $\bar{c} \leq 1$, the function $v(\theta, n, \bar{c})$ crosses zero once and remains strictly below zero afterwards. By contrast, when $\bar{c} > 1$, it crosses zero for $n < \widehat{n}$, it stays below zero for $\widehat{n} \leq n \leq \widetilde{n}$ and it is equal to zero for $\widetilde{n} \leq n \leq 1$. Despite this, in both cases there still exists a unique threshold equilibrium.

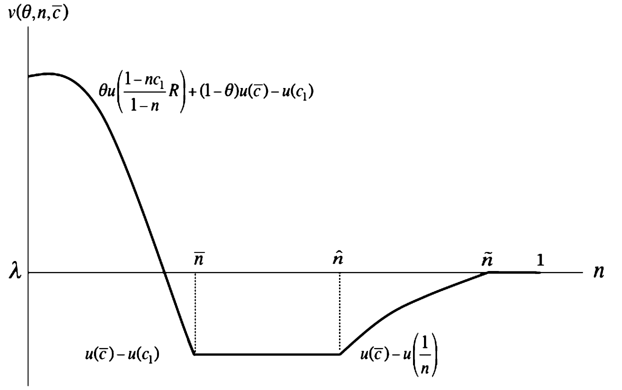

Fig. 4b. Depositor's utility differential with a guarantee against panic and fundamental failures when $\overline{c} > 1$. The figure shows how the utility differential for a late depositor between withdrawing at date 2 versus date 1 changes with the number n of depositors withdrawing at date 1 for a given guarantee $\overline{c}$ chosen by the government. The function is decreasing in n for $\lambda \leq n < \overline{n}$, constant in the range $\overline{n} \leq n < \hat{n}$, increasing for $\hat{n} \leq n \leq \tilde{n}$ and again constant in the range $\tilde{n} \leq n \leq 1$. It crosses zero only once for $n < \overline{n}$ and takes value zero in the interval $\tilde{n} \leq n \leq 1$.

As in the decentralized economy, the threshold signal $x^*(c_1, \overline{c})$ can be found as the solution to the indifference condition that equates a depositor's expected utility from withdrawing early with the one from waiting until date 2.[7] We have the following result.

Proposition 8. *The model with a standard deposit insurance scheme has a unique threshold equilibrium in which late depositors run if they observe a signal below the threshold $x^*(c_1, \overline{c})$ and do not run above. In the limit as $\varepsilon \to 0$, the equilibrium threshold $x^*(c_1, \overline{c})$ simplifies to*

$$\theta^*(c_1, \overline{c}) = \frac{\int_{n=\lambda}^{\widehat{n}} u\left(c_1\right) + \int_{n=\widehat{n}}^{min(\widetilde{n},1)} u(\frac{1}{n}) - \int_{n=\lambda}^{min(\widetilde{n},1)} u\left(\overline{c}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\overline{c}\right)\right]}. \tag{21}$$

The threshold $\theta^*(c_1, \overline{c})$ is increasing in c_1 and decreasing in $\overline{c}$, i.e., $\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1} > 0$ and $\frac{\partial\theta^*(c_1,\overline{c})}{\partial \overline{c}} < 0$.

The proposition characterizes the equilibrium threshold $\theta^*(c_1, \overline{c})$ as a function of the deposit contract c_1 chosen by the bank and the level of guarantees $\overline{c}$ set by the government. The expression for $\theta^*(c_1, \overline{c})$ depends on whether the level of guarantees $\overline{c}$ is above or below 1 as this determines when depositors enjoy the guarantee. As in the decentralized economy, for a given $\overline{c}$, a higher c_1 leads to more runs as it increases depositors' payoff at date 1, while lowering that at date 2. By contrast, for a given c_1, a higher $\overline{c}$ reduces $\theta^*(c_1, \overline{c})$ as it increases the expected payment that late depositors receive if they wait until date 2. This represents the positive direct effect of government intervention on bank fragility.

[7] In what follows, we restrict our attention to threshold equilibria. As in the previous section, we show that the model has a unique threshold equilibrium, but, unlike the decentralized equilibrium, we do not prove that this is the only possible equilibrium of the model.

Having characterized depositors' withdrawal decisions, we can now turn to date 0 and analyze the bank's decision. Given $\overline{c}$ and anticipating depositors' withdrawal decisions, each bank chooses c_1 to maximize depositors' expected utility:

$$\begin{aligned}&\underset{c_1}{Max} \int_0^{\theta^*(c_1,\overline{c})} u(\max(1,\overline{c}))d\theta \\ &+ \int_{\theta^*(c_1,\overline{c})}^{1} \left[\lambda u(c_1) + (1-\lambda)\left(\theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + (1-\theta)\,u\left(\overline{c}\right)\right)\right] d\theta \\ &+E\left[v\left(g, c_1^*, \overline{c}\right)\right] \end{aligned} \tag{22}$$

where c_1^* denotes the equilibrium value of c_1 chosen by all banks, and $E\left[v\left(g, c_1^*, \overline{c}\right)\right]$ is the expected utility from the public good as given by

$$E\left[v\left(g, c_1^*, \overline{c}\right)\right] = \int_0^{\theta^*(c_1^*,\overline{c})} v\left(g\right) d\theta + \int_{\theta^*(c_1^*,\overline{c})}^{1} \left[\theta v(g) + (1-\theta)v(g-(1-\lambda)\overline{c})\right] d\theta \tag{23}$$

when $\overline{c} \leq 1$, and by

$$E\left[v\left(g, c_1^*, \overline{c}\right)\right] = \int_0^{\theta^*(c_1^*,\overline{c})} v\left(g-\overline{c}+1\right) d\theta + \int_{\theta^*(c_1^*,\overline{c})}^{1} \left[\theta v(g) + (1-\theta)v\left(g-(1-\lambda)\overline{c}\right)\right] d\theta \tag{24}$$

when $\overline{c} > 1$.

The bank's problem is similar to the one in (15), with only a few differences. First, the run threshold is now $\theta^*(c_1, \overline{c})$ instead of $\underline{\theta}^F(c_1, \overline{c}_f)$. Second, if $\overline{c} > 1$ depositors receive $u(\overline{c})$ instead of $u(1)$ and utility from the public good $v\left(g-\overline{c}+1\right)$ instead of $v(g)$ when there is a run at date 1.

As in the decentralized equilibrium, the solution for c_1 is such that $\theta^*(c_1, \overline{c}) < \overline{\theta}$ at the equilibrium choice of c_1. This maximizes depositors' expected utility as runs do not always occur. We have the following result.

Proposition 9. *The deposit contract $c_1^{DI} > c_1^D$ in the case of a standard deposit insurance scheme solves*

$$\begin{aligned}&\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \\ &-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \left(1-\theta^*(c_1,\overline{c})\right)u\left(\overline{c}\right)\right)\right. \\ &\left.-u(max(1,\overline{c}))\right] = 0, \end{aligned} \tag{25}$$

and is increasing in the amount of the guarantee, i.e., $\frac{dc_1^{DI}}{d\overline{c}} > 0$.

The choice of c_1^{DI} is similar to before: The bank still trades off the marginal benefit of a higher c_1 in terms of better risk sharing with its marginal cost in terms of increased probability of panic runs; and the equilibrium c_1^{DI} is increasing in the guaranteed amount.

Given the bank's choice of c_1^{DI}, at date 0 the government chooses $\overline{c}^{DI}$ to maximize depositors' total expected utility, which is given by the expression in (22) evaluated at $c_1 = c_1^{DI}(\overline{c})$. The problem here is very complex and does not admit a sharp characterization. Still, we can show that under certain conditions, the government finds it optimal to provide a positive amount of guarantee. We have the following result.

Proposition 10. *If $u'(0) - v'(g) > 0$, the government chooses $\overline{c}^{DI} > 0$, as characterized in the Appendix.*

The proposition suggests that the government still finds it optimal to offer a positive level of guarantee if the marginal benefit in terms of increased private utility outweighs the marginal cost in terms of reduced utility from the public good. This holds despite the fact that the scheme does not allow specific bank outcomes to determine the level of the guarantee.

As before, the equilibrium entails distortions in bank behavior, which depend on the size of the guarantee, i.e., $\overline{c} \lesseqgtr 1$. Following the same steps as in Section 4.2 and denoting c_{1G}^{DI} the optimal choice of c_1 of the government when it chooses both c_1 and $\overline{c}$, we obtain the following.

Proposition 11. *For a given $\overline{c}$, banks choose an inefficiently low c_1 (i.e., $c_1^{DI} < c_{1G}^{DI}$) if $\overline{c} \leq 1$ and if $\overline{c} > 1$ when $\left[\theta^*(c_1^{DI}, \overline{c})v(g) + (1 - \theta^*(c_1^{DI}, \overline{c}))v(g - (1 - \lambda)\overline{c}) - v(g - \overline{c} + 1)\right] < 0$ and an inefficiently high one (i.e., $c_1^{DI} > c_{1G}^{DI}$) otherwise, where c_{1G}^{DI} is characterized in the Appendix.*

As shown in the proposition, the distortion in the bank's choice of c_1 can now go either way, as the bank can choose either to under or over provide liquidity insurance. The direction of the distortion depends on whether the government ends up paying depositors more at date 2 when the bank project fails or when there is a run. The former case, which entails the same distortion as the one analyzed in Section 4.2, holds if $\overline{c} \leq 1$ and when $\left[\theta^*(c_1, \overline{c})v(g) + (1 - \theta^*(c_1, \overline{c}))v(g - (1 - \lambda)\overline{c}) - v(g - \overline{c} + 1)\right] < 0$ if $\overline{c} > 1$. The latter emerges otherwise and is more in line with common wisdom as it entails $c_1^{DI} > c_{1G}^{DI}$.

5. A numerical example

In this section we illustrate the properties of the model using a numerical example. The goal is to demonstrate that our main results hold in a reasonable parameter space and provide a comparison of the different guarantee schemes analyzed in terms of liquidity creation, bank fragility and welfare.

In the example depositors' utility functions from the private good $u(c)$ and from the public good $v(g)$ are given by

$$u(c) = \frac{(c+t)^{1-\sigma}}{1-\sigma} - \frac{(t)^{1-\sigma}}{1-\sigma},$$

and

$$v(g) = \frac{(g+t)^{1-\sigma}}{1-\sigma} - \frac{(t)^{1-\sigma}}{1-\sigma},$$

Table 1
$g = 0.7$.

	$\underline{\theta}$ θ^*	c_1 c_2	$\overline{c}_s$ $\overline{c}_f$	$E\left[u\left(c_1, c_2, \overline{c}\right)\right]$ $E[v(g,\overline{c})]$	$\Delta SW\left(c_1, c_2, g, \overline{c}\right)$ (%)
Decentralized economy	0.451436 0.463204	1.0076 4.98372	0 0	0.0139202 0.00861532	–
Guarantees against panic runs	0.488273 $\underline{\theta}$	1.10762 4.7694	$\frac{1-\lambda c_1}{1-\lambda}R$ 0	0.013945 0.00861532	0.11
Guarantees against fundamental project risk	0.380187 $\underline{\theta}$	1.12573 4.7058	$\frac{1-\lambda c_1}{1-\lambda}R$ 0.329	0.0145561 0.008157575	0.78
Deposit insurance	0.355442 0.373496	1.01445 4.96905	0.27 0.27	0.0144034 0.0082353	0.45

respectively. The two functions are variations of a standard CRRA function with σ being the risk aversion coefficient. The parameter t ensures that the assumption $u(0) = v(0) = 0$ is always satisfied, and it can be interpreted as the consumption that depositors enjoy from resources not invested in the bank. We maintain $p(\theta) = \theta$, with the upper dominance region corresponding to $\theta = 1$, and set the parameters as follows $\sigma = 3$; $R = 5$; $\lambda = 0.3$, $t = 4$ and $g = 0.7$.

In Table 1, we report the decentralized allocation and the equilibrium in all guarantee schemes analyzed. Each scheme is labelled according to the corresponding paper section. The columns of the tables report, in order, the probability of panic and fundamental runs (θ^*, $\underline{\theta}$), the equilibrium values for the deposit contract (c_1, c_2), the equilibrium levels of guarantee $\overline{c}_s$ and $\overline{c}_f$, the expected utility from the private and public good ($E\left[u\left(c_1, c_2, \overline{c}\right)\right]$ and $E[v(g, \overline{c})]$) and the percentage change in social welfare, as given by the sum $E\left[u\left(c_1, c_2, \overline{c}\right)\right] + E[v(g, \overline{c})]$, in the various interventions relative to the decentralized economy.

Table 1 shows that all types of guarantees lead to higher deposit rates ($c_1^F = 1.12573 > c_1^P = 1.10762 > c_1^{DI} = 1.01445 > c_1^D = 1.0076$) and thus greater risk sharing. With guarantees against panics only, panic runs are eliminated, but the overall probability of runs ($\underline{\theta}^P\left(c_1^P\right) = 0.488273$) is higher than in the decentralized solution ($\theta^*(c_1^D) = 0.463162$) because of the higher deposit rate. This occurs neither when guarantees also protect against fundamental project risk ($\underline{\theta}^F\left(c_1^F, \overline{c}_f^F\right) = 0.380187$) nor with deposit insurance ($\theta^*(c_1^{DI}, \overline{c}^{DI}) = 0.373496$). All guarantee schemes entail higher welfare than the decentralized solution, but the guarantee against fundamental project risk performs better than the others ($\Delta SW^F = 0.78 > \Delta SW^{DI} = 0.45 > \Delta SW^P = 0.11$) as it allows the government to better target the different bank outcomes.

6. Discussion and concluding remarks

In this paper we develop a model where both panic and fundamental runs are possible and both banks' and depositors' decisions are endogenously determined. We show that government guarantees are beneficial in that they improve depositors' welfare, as a result of the induced greater risk-sharing, even if this comes sometimes at the cost of greater fragility. This result holds also in the context of guarantee schemes that entail an actual disbursement for the government and so distortions in banks' risk-taking.

The paper offers a convenient framework to evaluate the implications of government guarantees because it allows the probability of runs and bank's behavior to be endogenous. The framework builds on some assumptions regarding the type of contract banks offer and the set of feasible actions of the various players involved (i.e., depositors, banks and the government) that are typical in the standard literature on financial crises. Below, we discuss these assumptions. Attempting to relax them and enrich the framework is, in our view, a fruitful path for future research.

First, our framework assumes that banks offer non-contingent deposit contracts, which cannot be ex post renegotiated. Similarly, the amounts promised in the guarantee schemes we consider are non-contingent and non-renegotiable as is typical in deposit insurance schemes. This framework enables us, differently from the previous literature, to fully endogenize investors' runs decisions, banks' risk choices, and the interaction between them and with the guarantees regime. Extending this framework to consider the optimal contracts would certainly be an interesting direction. However, the tractability would clearly be a constraint.

Second, for simplicity, we do not allow banks to store liquidity between the intermediate and the final date as a way to insure themselves against the possibility that their project fails at the final date. The only choice banks make is the amount they offer to early withdrawing investors. As Ahnert and Elamin (2015) show, if storage is possible, banks will make use of it at the intermediate date. This reduces depositors' incentives to run and may improve allocation. However, as storage is costly, banks do not find it optimal to fully self-insure. As a consequence, banks are still subject to project risk and both fundamental and panic runs occur in equilibrium. This still leaves a potential role for public guarantees as a way to further reduce runs and offer better protection to depositors against project failure. Investigating this issue is an interesting direction for future research.

Finally, still in line with the existing literature on public intervention, we consider that public funds are always sufficient for the provision of guarantees, although their size affects the optimal amount of guarantees that the government offers in equilibrium. In other words, depositors are sure to receive the transfers announced by the government. The events in the recent Eurozone crisis proved that guarantees may not be always feasible and their provision may even have negative implications for the solvency of the country, their credibility and in turn their effectiveness. In a recent paper, Leonello (2018) analyzes these issues in a model where the government providing the guarantees is fragile and has access to limited resources. She studies the role that guarantees play for banks and sovereign stability and their interaction, but her analysis abstract from moral hazard considerations. Allowing for the possibility of sovereign default and removing the assumptions of full credibility and feasibility of the guarantees, while still analyzing their impact on banks' risk-taking incentives, would require modifying our framework significantly and could pose tractability concerns. However, the basic trade-off between fragility and liquidity creation triggered by the provision of the guarantee that we highlight in this paper would still be present in the extended framework.

Appendix A

Proof of Proposition 1. The proof follows Goldstein and Pauzner (2005). The arguments in their proof establish that there is a unique equilibrium in which depositors run if and only if the signal they receive is below a common signal $x^*(c_1)$. The number n of depositors withdrawing at date 1 is equal to the probability of receiving a signal x_i below $x^*(c_1)$ and, given that depositors' signals are independent and uniformly distributed over the interval $[\theta-\varepsilon, \theta+\varepsilon]$, it is:

$$n(\theta, x^*(c_1)) = \begin{cases} 1 & \text{if } \theta \le x^*(c_1)-\varepsilon \\ \lambda + (1-\lambda)\left(\frac{x^*(c_1)-\theta+\varepsilon}{2\varepsilon}\right) & \text{if } x^*(c_1)-\varepsilon \le \theta \le x^*(c_1)+\varepsilon \\ \lambda & \text{if } \theta \ge x^*(c_1)+\varepsilon \end{cases} \tag{26}$$

When θ is below $x^*(c_1)-\varepsilon$, all patient depositors receive a signal below $x^*(c_1)$ and run. When θ is above $x^*(c_1)+\varepsilon$, all late depositors wait until date 2 and only the λ early consumers withdraw early. In the intermediate interval, when θ is between $x^*(c_1)-\varepsilon$ and $x^*(c_1)+\varepsilon$, there is a partial run as some of the late depositors run. The proportion of late consumers withdrawing early decreases linearly with θ as fewer agents observe a signal below the threshold.

Having characterized the proportion of agents withdrawing for any possible value of the fundamentals θ, we can now compute the threshold signal $x^*(c_1)$. A patient depositor who receives the signal $x^*(c_1)$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1)$ can be then found as the solution to

$$f(\theta, c_1) = \int_{n=\lambda}^{\frac{1}{c_1}} \left[\theta(n) u\left(\frac{1-nc_1}{1-n}R\right) - u(c_1)\right] + \int_{n=\frac{1}{c_1}}^{1} \left[u(0) - u\left(\frac{1}{n}\right)\right] = 0, \tag{27}$$

where, from (26), $\theta(n) = x^*(c_1)+\varepsilon-2\varepsilon\frac{(n-\lambda)}{1-\lambda}$. Equation (27) follows from (4) and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits until date 2. Note that in the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1)$, and we denote it as $\theta^*(c_1)$. Solving (27) with respect to $\theta^*(c_1)$ gives the threshold as in the proposition. □

Proof of Corollary 1. The expression for $\frac{\partial \underline{\theta}(c_1)}{\partial c_1}$ can be obtained simply differentiating $\underline{\theta}(c_1)$, as given in (3) with respect to c_1. This gives

$$\frac{\partial \underline{\theta}(c_1)}{\partial c_1} = \frac{u'(c_1) + \underline{\theta}(c_1)\left(\frac{\lambda R}{1-\lambda}\right) u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)} > 0 \tag{28}$$

since $u'(c) > 0$.

To prove that $\theta^*(c_1)$ is increasing in c_1, we apply the implicit function theorem to the expression for $f(\theta^*, c_1)$ as given by (27) in the proof of Proposition 1. We obtain

$$\frac{\partial \theta^*(c_1)}{\partial c_1} = -\frac{\frac{\partial f(\theta^*, c_1)}{\partial c_1}}{\frac{\partial f(\theta^*, c_1)}{\partial \theta^*}}.$$

It is easy to see that $\frac{\partial f(\theta^*, c_1)}{\partial \theta} > 0$. Thus, the sign of $\frac{\partial \theta^*(c_1)}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*, c_1)}{\partial c_1}$, where

$$\frac{\partial f(\theta^*, c_1)}{\partial c_1} = -\frac{1}{c_1^2}\left[\theta^*(c_1) u\left(\frac{1-\frac{1}{c_1}c_1}{1-\frac{1}{c_1}}R\right) - u(c_1)\right] + \frac{1}{c_1^2}\left[u(0) - u(c_1)\right]$$

$$-\int_{n=\lambda}^{\frac{1}{c_1}} \left[u'(c_1)+\theta^*(c_1)\left(\frac{nR}{1-n}\right)u'\left(\frac{1-nc_1}{1-n}R\right)\right]$$

$$=-\int_{n=\lambda}^{\frac{1}{c_1}} \left[u'(c_1)+\theta^*(c_1)\left(\frac{nR}{1-n}\right)u'\left(\frac{1-nc_1}{1-n}R\right)\right]<0.$$

Thus,

$$\frac{\partial\theta^*(c_1)}{\partial c_1}=\frac{\int_{n=\lambda}^{\frac{1}{c_1}}\left[u'(c_1)+\theta^*(c_1)\left(\frac{nR}{1-n}\right)u'(\frac{1-nc_1}{1-n}R)\right]}{\int_{n=\lambda}^{\frac{1}{c_1}}u\left(\frac{1-nc_1}{1-n}R\right)}>0. \tag{29}$$

To complete the proof, we need to show that $\frac{\partial\theta^*(c_1)}{\partial c_1}>\frac{\partial\underline{\theta}(c_1)}{\partial c_1}$. To see this, substitute the expression for each derivative from (28) and (29). After a few manipulations, we can rewrite the condition $\frac{\partial\theta^*(c_1)}{\partial c_1}>\frac{\partial\underline{\theta}(c_1)}{\partial c_1}$ as follows

$$u'(c_1)\int_{n=\lambda}^{\frac{1}{c_1}}u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+\int_{n=\lambda}^{\frac{1}{c_1}}\theta^*(c_1)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\left(\frac{nR}{1-n}\right)u'\left(\frac{1-nc_1}{1-n}R\right)$$

$$>u'(c_1)\int_{n=\lambda}^{\frac{1}{c_1}}u\left(\frac{1-nc_1}{1-n}R\right)+\int_{n=\lambda}^{\frac{1}{c_1}}\underline{\theta}(c_1)u\left(\frac{1-nc_1}{1-n}R\right)u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\left(\frac{\lambda R}{1-\lambda}\right),$$

which holds since $\frac{1-\lambda c_1}{1-\lambda}R>\frac{1-nc_1}{1-n}R$ and $\frac{nR}{1-n}>\frac{\lambda R}{1-\lambda}$ for any $n>\lambda$ and $\theta^*(c_1)>\underline{\theta}(c_1)$. This completes the proof of the corollary. □

Proof of Proposition 2. Differentiating (7) with respect to c_1 gives the deposit contract c_1^D as the solution to (8).

To show that $c_1^D>1$, we evaluate (8) at $c_1=1$. From (6), at $c_1=1$ the threshold $\theta^*(c_1)$ simplifies to

$$\theta^*(1)=\frac{(1-\lambda)u(1)}{(1-\lambda)u(R)},$$

and, from (3), it is then

$$\theta^*(1)=\underline{\theta}(1).$$

Thus, when $c_1=1$, (8) can be rewritten as follows:

$$\lambda\int_{\underline{\theta}(1)}^{1}\left[u'(1)-\theta Ru'(R)\right]d\theta-\left.\frac{\partial\underline{\theta}(c_1)}{\partial c_1}\right|_{c_1=1}(1-\lambda)\left[\underline{\theta}(1)u(R)-u(1)\right]$$

The second term is equal to zero because of the definition of $\underline{\theta}(c_1)$ in (3), and thus the expression simplifies to

$$\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta.$$

Since the relative risk aversion coefficient is bigger than 1, it follows

$$1 \cdot u'(1) > Ru'(R),$$

so that $\lambda \int_{\underline{\theta}(1)}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta > 0$ and thus $c_1^D > 1$. □

Proof of Proposition 3. Denote $FOC_{c_1}^P(c_1)$ as the first order condition in (12) which implicitly determines the deposit contract c_1^P chosen by the banks. To show that $c_1^P > c_1^D$, we need to compare (8) with (12) and show that $FOC_{c_1}^P(c_1)$ evaluated at $c_1 = c_1^D$ is greater than (8) evaluated at $c_1 = c_1^D$, which is equal to zero. The first term in each expression only differs in the lower extreme of the integrals and it is easy to see that the first term in (8) is smaller than that in (12) since $\theta^*(c_1) > \underline{\theta}(c_1)$. Thus, we only need to compare $\frac{\partial \theta^*(c_1^D)}{\partial c_1}\left[\lambda u(c_1^D) + (1-\lambda)\theta^*(c_1^D) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$ with $\frac{\partial \underline{\theta}(c_1^D)}{\partial c_1}\left[\lambda u(c_1^D) + (1-\lambda)\underline{\theta}(c_1^D) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$ and show that the former is larger than the latter. It is easy to see that

$$\left[\lambda u(c_1^D) + (1-\lambda)\theta^*(c_1^D) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right]$$
$$> \left[\lambda u(c_1^D) + (1-\lambda)\underline{\theta}(c_1^D) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u(1)\right],$$

since $\theta^*(c_1^D) > \underline{\theta}(c_1^D)$. Moreover, $\frac{\partial \theta^*(c_1^D)}{\partial c_1} > \frac{\partial \underline{\theta}(c_1^D)}{\partial c_1}$ holds from Corollary 1. Thus, the proposition follows. □

Proof of Proposition 4. Denote $FOC_{c_1}^F$ as the first order condition in (16), which implicitly determines the deposit contract chosen by the banks. To show that $\frac{dc_1^F}{d\overline{c}_f} > 0$ and so that $c_1^F > c_1^P$, we use the implicit function theorem as follows:

$$\frac{dc_1^F}{d\overline{c}_f} = -\frac{\frac{\partial FOC c_1^F}{\partial \overline{c}_f}}{\frac{\partial FOC c_1^F}{\partial c_1}}.$$

Since c_1^F is an interior solution, $\frac{\partial FOC c_1^F}{\partial c_1} < 0$ and the sign of $\frac{dc_1^F}{d\overline{c}_f}$ is equal to the sign of $\frac{\partial FOC c_1^F}{\partial \overline{c}_f}$, with

$$\begin{aligned}\frac{\partial FOC c_1^F}{\partial \overline{c}_f} = &-\frac{\partial \underline{\theta}^F\left(c_1, \overline{c}_f\right)}{\partial c_1 \partial \overline{c}_f}\left[\lambda u\left(c_1\right) + (1-\lambda)\underline{\theta}^F\left(c_1, \overline{c}_f\right) u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right. \\ &\left. + (1-\lambda)\left(1 - \underline{\theta}^F\left(c_1, \overline{c}_f\right)\right) u\left(\overline{c}_f\right) - u\left(1\right)\right] \\ &-\lambda \frac{\partial \underline{\theta}^F\left(c_1, \overline{c}_f\right)}{\partial \overline{c}_f}\left[u'\left(c_1\right) - \underline{\theta}^F\left(c_1, \overline{c}_f\right) R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] \\ &-\frac{\partial \underline{\theta}^F\left(c_1, \overline{c}_f\right)}{\partial c_1}\frac{\partial \underline{\theta}^F\left(c_1, \overline{c}_f\right)}{\partial \overline{c}_f}(1-\lambda)\left[u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) - u\left(\overline{c}_f\right)\right].\end{aligned} \tag{30}$$

Using the expression for $\underline{\theta}^F\left(c_1,\overline{c}_f\right)$ as given in (14), we can compute

$$\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1\partial\overline{c}_f}=\frac{\frac{\lambda R}{1-\lambda}u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}+u'\left(\overline{c}_f\right)\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}. \tag{31}$$

Substituting (31) into (30) and rearranging, we obtain

$$\begin{aligned}\frac{\partial FOCc_1^F}{\partial\overline{c}_f}=&-\left[\frac{\frac{\lambda R}{1-\lambda}u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}+\frac{u'\left(\overline{c}_f\right)\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\right]\left[u\left(c_1\right)-u\left(1\right)\right]\\&-\lambda\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}\left[u'\left(c_1\right)-\underline{\theta}^F\left(c_1,\overline{c}_f\right)Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]\\&-\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}\left(1-\lambda\right)\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)\right],\end{aligned} \tag{32}$$

since, given the definition of $\underline{\theta}^F\left(c_1,\overline{c}_f\right)$, it holds that

$$\begin{aligned}&\left[\lambda u\left(c_1\right)+\left(1-\lambda\right)\underline{\theta}^F\left(c_1,\overline{c}_f\right)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right.\\&\left.+\left(1-\lambda\right)\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)u\left(\overline{c}_f\right)-u\left(1\right)\right]=u\left(c_1\right)-u\left(1\right).\end{aligned}$$

Given $\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}>0$ and $\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}<0$, all terms in (32) are positive besides

$$-\frac{u'\left(\overline{c}_f\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\left[u\left(c_1\right)-u\left(1\right)\right] \tag{33}$$

in the first row in (32). From (16), we have that

$$\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\left[u\left(c_1\right)-u\left(1\right)\right]=\lambda\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\left[u'\left(c_1\right)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right],$$

and we can rewrite (33) as follows

$$-\lambda\frac{u'\left(\overline{c}_f\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\left[u'\left(c_1\right)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right].$$

It follows that, for $\frac{\partial FOCc_1^F}{\partial\overline{c}_f}>0$, it suffices to show that

$$\begin{aligned}&\lambda\frac{u'\left(\overline{c}_f\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\left[u'\left(c_1\right)-\theta Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]<\\&\lambda\left|\frac{\partial\underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial\overline{c}_f}\right|\left[u'\left(c_1\right)-\underline{\theta}^F\left(c_1,\overline{c}_f\right)Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]\end{aligned}$$

Using

$$\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial \overline{c}_f}=-\frac{u'\left(\overline{c}_f\right)\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)},$$

we can rearrange the inequality above first as follows

$$\lambda\frac{u'\left(\overline{c}_f\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\left[u'\left(c_1\right)-\theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]<$$
$$\lambda\frac{u'\left(\overline{c}_f\right)\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u\left(\overline{c}_f\right)}\left[u'\left(c_1\right)-\underline{\theta}^F\left(c_1,\overline{c}_f\right)Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right],$$

and after a few manipulations as

$$\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)>\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}\underline{\theta}^F\left(c_1,\overline{c}_f\right)Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right),$$

which always holds and so $\frac{dc_1^F}{d\overline{c}_f}>0$. The inequality $c_1^F>c_1^P$ follows directly from $\frac{dc_1^F}{d\overline{c}_f}>0$, as c_1^P is computed for $\overline{c}_f=0$. Thus, the proposition follows. □

Proof of Proposition 5. The expression (17) in the proposition is obtained by taking the derivative of (15) with respect to $\overline{c}_f$ and applying the Envelope theorem as c_1 is chosen optimally by the bank as the solution to (16). To prove that the government chooses a positive level of guarantees, we show that the first order condition (17) evaluated for $\overline{c}_f=0$ is positive.

Evaluating (17) for $\overline{c}_f=0$, we obtain

$$\int_{\underline{\theta}^F\left(c_1,\overline{c}_f\right)}^{1}(1-\lambda)\left[u'(0)-v'(g)\right]d\theta \tag{34}$$
$$-\left.\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial \overline{c}_f}\right|_{\overline{c}_f=0}\left[\lambda u\left(c_1\right)+(1-\lambda)\underline{\theta}^F\left(c_1,\overline{c}_f\right)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(1)\right],$$

as $\left[\underline{\theta}^F(c_1,0)\,v(g)+\left(1-\underline{\theta}^F(c_1,0)\,v(g)\right)-v(g)\right]=0$. Given that $\left.\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial \overline{c}_f}\right|_{\overline{c}_f=0}=-\frac{u'(0)}{u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)}(1-\underline{\theta}^F(c_1,0))<0$, the second term in (34) is positive. Thus, a sufficient condition for (34) to be positive and, in turn, for $\overline{c}_f>0$ is $u'(0)-v'(g)>0$. The proposition follows. □

Proof of Proposition 6. Comparing (16) with (18), it is easy to see that they only differ in the last term in (18), that is

$$-\frac{\partial \underline{\theta}^F\left(c_1,\overline{c}_f\right)}{\partial c_1}\left[\underline{\theta}^F\left(c_1,\overline{c}_f\right)v(g)+\left(1-\underline{\theta}^F\left(c_1,\overline{c}_f\right)\right)v\left(g-(1-\lambda)\overline{c}\right)-v(g)\right],$$

which is positive for given c_1 and $\overline{c}$. Thus, the expression in (16) is smaller than that in (18), implying $c_1^F < c_{1G}^F$. The proposition follows. □

Proof of Proposition 7. Differentiating (19) with respect to $\overline{c}_1$ gives

$$\int_0^{\underline{\theta}^F(c_1,\overline{c}_f)} \left[u'(\overline{c}_1) - v'(g - \overline{c}_1 + 1)\right] d\theta. \tag{35}$$

The expression in (35) is negative when evaluated at $\overline{c}_1 = 1$ since, by assumption, $u'(1) - v'(g) < 0$. The proposition follows. □

Proof of Proposition 8. The proof proceeds in steps. First, we characterize the threshold $\theta^*(c_1, \overline{c})$. Then, we analyze its properties. In each step of the proof, we distinguish two cases depending on whether $\overline{c}$ is larger or smaller than 1.

Characterization of the threshold $\theta^*(c_1, \overline{c})$

Consider first the case where $\overline{c} \leq 1$. The proof is analogous to the one for Proposition 2. A patient depositor who receives the signal $x^*(c_1, \overline{c})$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1, \overline{c})$ can be then found as the solution to

$$f(\theta, c_1, \overline{c}) = \int_{n=\lambda}^{\overline{n}} \left[\theta(n) u\left(\frac{1 - nc_1}{1 - n} R\right) + (1 - \theta(n)) u(\overline{c}) - u(c_1)\right] + \int_{n=\overline{n}}^{\widehat{n}} [u(\overline{c}) - u(c_1)]$$
$$+ \int_{n=\widehat{n}}^{1} \left[u(\overline{c}) - u\left(\frac{1}{n}\right)\right] = 0, \tag{36}$$

where, still from (26), $\theta(n) = x^*(c_1, \overline{c}) + \varepsilon - 2\varepsilon \frac{(n-\lambda)}{1-\lambda}$. Equation (36) follows from (20) in the case $\overline{c} \leq 1$ and requires that a late depositor's expected utility when he withdraws at date 1 is equal to that when he waits until date 2. At the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1, \overline{c})$, and the threshold $\theta^*(c_1, \overline{c})$ solves (36).

The case where $\overline{c} > 1$ is more involved since we need first to show that, despite the fact that the function $v(\theta, n, \overline{c})$ is zero in the range $\widetilde{n} \leq n \leq 1$, a unique threshold equilibrium exists. We then split this part of the proof in two parts. First, we prove that a unique threshold equilibrium exists and then, we compute the equilibrium threshold.

Existence of a unique threshold equilibrium when $\overline{c} > 1$

The proof follows Goldstein and Pauzner (2005). Recall that the proportion of depositors running $n(\theta, x^*)$ when they behave according to the same threshold strategy x^* is given by (26). Denote as $\Delta(x_i, \dot{n}(\theta))$ an agent's expected difference in utility between withdrawing at date 2 rather than at date 1 when he holds beliefs $\dot{n}(\theta)$ regarding the number of depositors running. The function $\Delta(x_i, \dot{n}(\theta))$ is given by

$$\Delta(x_i, \dot{n}(\theta)) = \frac{1}{2\varepsilon} \int_{x_i-\varepsilon}^{x_i+\varepsilon} E_n[v(\theta, \dot{n}(\theta))]\, d\theta.$$

Since for any realization of θ, the proportion of depositors running is deterministic, we can write $n(\theta)$ instead of $\dot{n}(\theta)$ and the function $\Delta(x_i, n(\theta))$ simplifies to

$$\Delta(x_i, \dot{n}(\theta)) = \frac{1}{2\varepsilon} \int_{x_i-\varepsilon}^{x_i+\varepsilon} v(\theta, n(\theta)) d\theta.$$

Notice that when all depositors behave according to the same threshold strategy x^*, $\dot{n}(\theta) = n(\theta, x^*)$ defined in (26). The following lemma states a few properties of the function $\Delta(x_i, \dot{n}(\theta))$.

Lemma 1. *i) The function $\Delta(x_i, \dot{n}(\theta))$ is continuous in x_i; ii) for any $a > 0$, $\Delta(x_i + a, \dot{n}(\theta) + a)$ is non-decreasing in a, iii) $\Delta(x_i + a, \dot{n}(\theta) + a)$ is strictly increasing in a if there is a positive probability that $n < \overline{n}$ and $\theta < \overline{\theta}$.*

Proof of Lemma 1. The proof follows Goldstein and Pauzner (2005). The function $\Delta(.)$ is continuous in x_i as a change in x_i only changes the limits of integration in the computation of $\Delta(.)$. The function $\Delta(x_i + a, \dot{n}(\theta) + a)$ is non-decreasing in a since, as a increases, depositors see the same distribution of n but expect θ to be higher. Since $v(\theta, n)$ is non-decreasing in θ, $\Delta(.)$ is non-decreasing in a. In order for $\Delta(x_i + a, \dot{n}(\theta) + a)$ to be strictly increasing in a, we need that $\theta < \overline{\theta}$ and that there is a positive probability that $n < \overline{n}$. This is the case because, when $n < \overline{n}$ and $\theta < \overline{\theta}$, $v(\theta, n)$ is strictly increasing in θ, and, thus, $\Delta(x_i + a, \dot{n}(\theta) + a)$ is strictly increasing in a. □

A threshold equilibrium with the threshold signal x^* exists, if and only if no depositor finds it optimal to run if he receives a signal higher than x^* and to wait if he receives a signal below x^*:

$$\Delta(x_i, n(\theta, x^*)) < 0 \ \forall x_i < x^*; \tag{37}$$

$$\Delta(x_i, n(\theta, x^*)) > 0 \ \forall x_i > x^*. \tag{38}$$

By continuity, a depositor must be indifferent between withdrawing at date 1 rather than date 2 when he receives the signal $x_i = x^*$

$$\Delta(x^*, n(\theta, x^*)) = 0. \tag{39}$$

In the lower and upper dominance regions, $\Delta(x^*, n(\theta, x^*)) < 0$ and $\Delta(x^*, n(\theta, x^*)) > 0$, respectively. Thus, by continuity of $\Delta(x^*, n(\theta, x^*))$ in x_i, there exists some x^* at which it equals to zero. To prove that the x^* is unique, we use the property stated in Lemma 1 that $\Delta(x^*, n(\theta, x^*))$ is strictly increasing in x_i in the range $\theta \in \left[x^* - \varepsilon, x^* + \varepsilon\right]$ since from (26), there is always a positive probability that $n < \overline{n}$ in that range. Thus, there is only one value of x^*, which is a candidate to be a threshold equilibrium. To show that it is indeed an equilibrium we have to show that no depositor has an incentive to deviate. This means that we have to show that, given that (39) holds, (37) and (38) also hold.

Let's start from (37). Decompose the intervals $[x_i - \varepsilon, x_i + \varepsilon]$ and $\left[x^* - \varepsilon, x^* + \varepsilon\right]$ over which the integrals in $\Delta(x_i, n(\theta, x^*))$ and $\Delta(x^*, n(\theta, x^*))$ are defined into a common part $c = [x_i - \varepsilon, x_i + \varepsilon] \bigcap \left[x^* - \varepsilon, x^* + \varepsilon\right]$ and two disjoint parts $d_i = \frac{[x_i - \varepsilon, x_i + \varepsilon]}{c}$ and $d^* = \frac{\left[x^* - \varepsilon, x^* + \varepsilon\right]}{c}$. We can then rewrite the integrals $\Delta(x_i, n(\theta, x^*))$ and $\Delta(x^*, n(\theta, x^*))$ as follows:

$$\Delta(x_i, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*))$$

$$\Delta(x^*, n(\theta, x^*)) = \frac{1}{2\varepsilon} \int\limits_{\theta \in c} v(\theta, n(\theta, x^*)) + \frac{1}{2\varepsilon} \int\limits_{\theta \in d^*} v(\theta, n(\theta, x^*))$$

For any $\theta \in d_i$, $n = 1$ since $\theta \le x^* - \varepsilon$. Thus, $v(\theta, n(\theta, x^*)) = 0$ and, in turn $\Delta(x_i, n(\theta, x^*)) = 0$ in that interval. In order to show that $\Delta(x_i, n(\theta, x^*)) < 0$, we need to show that $\frac{1}{2\varepsilon} \int_{\theta \in c} v(\theta, n(\theta, x^*)) < 0$. This is the case because (39) holds and the fundamentals in the range d^* are better than those in the range d_i, which implies that $\frac{1}{2\varepsilon} \int_{\theta \in d^*} v(\theta, n(\theta, x^*)) > \frac{1}{2\varepsilon} \int_{\theta \in d_i} v(\theta, n(\theta, x^*)) = 0$. The proof for (38) is analogous.

The equilibrium threshold when $\overline{c} > 1$

Having proved the existence of a unique threshold equilibrium, we can now compute $x^*(c_1, \overline{c})$. A patient depositor who receives the signal $x^*(c_1, \overline{c})$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $x^*(c_1, \overline{c})$ can be then found as the solution to

$$f(\theta, c_1, \overline{c}) = \int\limits_{n=\lambda}^{\overline{n}} \left[\theta(n) u\left(\frac{1 - nc_1}{1 - n} R\right) + (1 - \theta(n)) u(\overline{c}) - u(c_1)\right] + \int\limits_{n=\overline{n}}^{\widehat{n}} [u(\overline{c}) - u(c_1)]$$
$$+ \int\limits_{n=\widehat{n}}^{\widetilde{n}} \left[u(\overline{c}) - u\left(\frac{1}{n}\right)\right] + \int\limits_{n=\widetilde{n}}^{1} [u(\overline{c}) - u(\overline{c})] = 0. \tag{40}$$

As before, $\theta(n) = x^*(c_1, \overline{c}) + \varepsilon - 2\varepsilon \frac{(n-\lambda)}{1-\lambda}$ and, in the limit, when $\varepsilon \to 0$, $\theta(n) \to x^*(c_1, \overline{c})$ and the threshold $\theta^*(c_1, \overline{c})$ solves (40).

Properties of the threshold $\theta^*(c_1, \overline{c})$

We now move on to analyze the properties of the threshold $\theta^*(c_1, \overline{c})$. To prove that $\theta^*(c_1, \overline{c})$ is increasing in c_1, we use the implicit function theorem and obtain

$$\frac{\partial \theta^*(c_1, \overline{c})}{\partial c_1} = -\frac{\frac{\partial f(\theta^*, c_1, \overline{c})}{\partial c_1}}{\frac{\partial f(\theta^*, c_1, \overline{c})}{\partial \theta^*}},$$

where the expression for $f(\theta^*, c_1, \overline{c})$ is given in (36) for the case $\overline{c} \le 1$ and (40) for the case $\overline{c} > 1$. It is easy to see that the denominator is positive since

$$\frac{\partial f(\theta^*, c_1, \overline{c})}{\partial \theta^*} = \int\limits_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1 - nc_1}{1 - n} R\right) - u(\overline{c})\right] > 0,$$

for both cases $\overline{c} \le 1$ and $\overline{c} > 1$. Thus, the sign of $\frac{\partial \theta^*(c_1, \overline{c})}{\partial c_1}$ is given by the opposite sign of $\frac{\partial f(\theta^*, c_1, \overline{c})}{\partial c_1}$. After some manipulations, we obtain:

$$\frac{\partial f(\theta^*, c_1, \overline{c})}{\partial c_1} = -\int\limits_{n=\lambda}^{\widehat{n}} u'(c_1) - \int\limits_{n=\lambda}^{\overline{n}} \theta^*(c_1, \overline{c}) u'\left(\frac{1 - nc_1}{1 - n} R\right)\left(\frac{nR}{1 - n}\right) < 0,$$

for both cases $\overline{c} \le 1$ and $\overline{c} > 1$. This implies

$$\frac{\partial \theta^*(c_1, \overline{c})}{\partial c_1} = \frac{\int_{n=\lambda}^{\widehat{n}} u'(c_1) + \int_{n=\lambda}^{\overline{n}} \theta^*(c_1, \overline{c}) u'\left(\frac{1 - nc_1}{1 - n} R\right)\left(\frac{nR}{1 - n}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1 - nc_1}{1 - n} R\right) - u(\overline{c})\right]} > 0. \tag{41}$$

We now turn to the effect of $\overline{c}$ on the threshold. To prove that $\theta^*(c_1,\overline{c})$ is decreasing in $\overline{c}$, we again use the implicit function theorem and obtain

$$\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}} = -\frac{\frac{\partial f(\theta^*,c_1,\overline{c})}{\partial\overline{c}}}{\frac{\partial f(\theta^*,c_1,\overline{c})}{\partial\theta}}.$$

The denominator is as before and it is positive. Thus, the sign of $\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}$ is given by the opposite sign of $\frac{\partial f(\theta^*,c_1,\overline{c})}{\partial\overline{c}}$.

We start from the case $\overline{c} \le 1$. Taking the derivative of (36) with respect to $\overline{c}$, after some manipulations, we obtain:

$$\frac{\partial f(\theta^*,c_1,\overline{c})}{\partial\overline{c}} = \int\limits_{n=\lambda}^{1} u'(\overline{c}) - \int\limits_{n=\lambda}^{\overline{n}} \theta^*(c_1,\overline{c})u'(\overline{c}) > 0,$$

which implies that

$$\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}} = -\frac{\int_{n=\lambda}^{1} u'(\overline{c}) - \int_{n=\lambda}^{\overline{n}} \theta^*(c_1,\overline{c})u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\overline{c}\right)\right]} < 0. \tag{42}$$

The case $\overline{c} > 1$ is analogous. Taking the derivative of (40) with respect to $\overline{c}$, after a few manipulations, we obtain

$$\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}} = -\frac{\int_{n=\lambda}^{\tilde{n}} u'(\overline{c}) - \theta^*(c_1,\overline{c}) \int_{n=\lambda}^{\overline{n}} u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u\left(\overline{c}\right)\right]} < 0.$$

Thus, the proposition follows. □

Proof of Proposition 9. We consider first the case $\overline{c} \le 1$. Denote $FOC^{DI}_{c_1}(c_1,\overline{c})$ as the first order condition that implicitly determines c_1^{DI}. This is given by (25) evaluated at $\overline{c} \le 1$ and, thus equal to

$$\begin{aligned}
&\lambda \int\limits_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\overline{c})\right]\right. \\
&\left. +(1-\lambda)u(\overline{c}) - u(1)\right] = 0.
\end{aligned} \tag{43}$$

To compute $\frac{dc_1}{d\overline{c}}$ the we use the implicit function theorem. Thus, $\frac{dc_1}{d\overline{c}} = -\frac{\frac{\partial FOC^{DI}_{c_1}(c_1,\overline{c})}{\partial\overline{c}}}{\frac{\partial FOC^{DI}_{c_1}(c_1,\overline{c})}{\partial c_1}}$. Since c_1^{DI} is an interior solution, $\frac{dc_1^{DI}}{d\overline{c}} > 0$ if and only if $\frac{\partial FOC^{DI}_{c_1}(c_1,\overline{c})}{\partial\overline{c}} > 0$. We have

$$\frac{\partial FOC^{DI}_{c_1}(c_1,\overline{c})}{\partial\overline{c}} = -\lambda\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1) - \theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]$$

$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1\partial\overline{c}}\left[\lambda u(c_1)+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]+(1-\lambda)u(\overline{c})-u(1)\right]
$$
$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}(1-\lambda)\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]
$$
$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}(1-\lambda)\left(1-\theta^*(c_1,\overline{c})\right)u'(\overline{c}).
$$

Recall that $\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}>0$ and $\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}<0$. Deriving $\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}$, as given in (41), with respect to $\overline{c}$, after a few manipulations, the cross derivative $\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1\partial\overline{c}}$ becomes

$$
\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1\partial\overline{c}}=\frac{1}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u\left(\overline{c}\right)\right]}
$$
$$
\times\left\{\frac{-R(c_1-1)}{(Rc_1-\overline{c})^2}\theta^*(c_1,\overline{c})\left(\frac{\overline{n}R}{1-\overline{n}}\right)u'\left(\frac{1-\overline{n}c_1}{1-\overline{n}}R\right)\right.
$$
$$
\left.+\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\int\limits_{n=\lambda}^{\overline{n}}u'\left(\overline{c}\right)+\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\int\limits_{n=\lambda}^{\overline{n}}u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)\right\}
$$

Substituting the expression for $\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1\partial\overline{c}}$ into that for $\frac{\partial FOC_{c_1}(c_1^{DI},\overline{c})}{\partial\overline{c}}$, after a few manipulations, we obtain:

$$
\frac{\partial FOC_{c_1}(c_1,\overline{c})}{\partial\overline{c}}=-\lambda\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1)-\theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] \tag{44}
$$
$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\frac{\int_{n=\lambda}^{\overline{n}}u'\left(\frac{1-nc_1}{1-n}R\right)\left(\frac{nR}{1-n}\right)}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u\left(\overline{c}\right)\right]}\left[\lambda u(c_1)\right.
$$
$$
\left.+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]-(1-\lambda)u(\overline{c})-u(1)\right]
$$
$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[(1-\lambda)\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]+(1-\lambda)(1-\theta^*(c_1,\overline{c}))u'(\overline{c})\right]
$$
$$
-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\frac{\int_{n=\lambda}^{\overline{n}}u'\left(\overline{c}\right)}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u\left(\overline{c}\right)\right]}\left[\lambda u(c_1)\right.
$$
$$
\left.+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]-(1-\lambda)u(\overline{c})-u(1)\right]
$$
$$
+\frac{\frac{R(c_1-1)}{(Rc_1-\overline{c})^2}\theta^*(c_1,\overline{c})\left(\frac{\overline{n}R}{1-\overline{n}}\right)u'\left(\frac{1-\overline{n}c_1}{1-\overline{n}}R\right)}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u\left(\overline{c}\right)\right]}\left[\lambda u(c_1)\right.
$$
$$
\left.+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]-(1-\lambda)u(\overline{c})-u(1)\right].
$$

All the terms in the expression above are positive beside the bracket

$$-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[(1-\lambda)\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]+(1-\lambda)(1-\theta^*(c_1,\overline{c}))u'(\overline{c})\right] \tag{45}$$

and

$$-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\frac{\int_{n=\lambda}^{\overline{n}}u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}\left[\lambda u(c_1)\right.$$
$$\left.+(1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]-(1-\lambda)u(\overline{c})-u(1)\right] \tag{46}$$

We first show that (45) is positive. In order for this to be true, we need to show that the term in the square bracket is negative. Substituting the expression for $\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}$ from (42), after a few manipulations, the term in the square bracket simplifies to

$$\frac{-\int_{n=\lambda}^{1}u'(\overline{c})+\theta^*(c_1,\overline{c})\int_{n=\lambda}^{\overline{n}}u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]$$
$$+\int_{n=\lambda}^{1}(1-\theta^*(c_1,\overline{c}))u'(\overline{c}),$$

which can be rearranged as

$$-\int_{n=\lambda}^{1}u'(\overline{c})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}+\theta^*(c_1,\overline{c})\int_{n=\lambda}^{\overline{n}}u'(\overline{c})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}$$
$$+\int_{n=\lambda}^{1}(1-\theta^*(c_1,\overline{c}))u'(\overline{c}).$$

After a few manipulations, the expression above can be rewritten as follows

$$u'(\overline{c})\left[\begin{array}{c}-(\overline{n}-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}-(1-\overline{n})\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}+\\+(1-\lambda)(1-\theta^*(c_1,\overline{c}))\end{array}\right].$$

To show that (45) is positive it suffices to show that

$$(\overline{n}-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{1}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}>(1-\lambda)(1-\theta^*(c_1,\overline{c})).$$

Rewriting the condition above as follows

$$(1-\lambda)(1-\theta^*(c_1,\overline{c}))\frac{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)-u(\overline{c})\right]}{\int_{n=\lambda}^{\overline{n}}\left[u\left(\frac{1-nc_1}{1-n}R\right)-u(\overline{c})\right]}>(1-\lambda)(1-\theta^*(c_1,\overline{c})),$$

it is easy to see that it always holds since $\frac{1-\lambda c_1}{1-\lambda}R>\frac{1-nc_1}{1-n}R$ for any $n>\lambda$.

In order to prove that $\frac{c_1^{DI}}{d\overline{c}} > 0$, we are left to show that (46) is dominated by some other term in (44). Thus, we show that (46) is smaller than the positive term $-\lambda \frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1) - \theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]$. To do this, first, recall that from (43) it holds

$$\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\theta^*(c_1,\overline{c})\left[u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) - u(\overline{c})\right] + (1-\lambda)u(\overline{c}) - u(1)\right]$$
$$= \lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta.$$

Thus, a sufficient condition for $\frac{dc_1^{DI}}{d\overline{c}} > 0$ is that

$$-\lambda \frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[u'(c_1) - \theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] >$$
$$\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \frac{\int_{n=\lambda}^{\overline{n}} u'(\overline{c})}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u(\overline{c})\right]}.$$

Substituting the expression for $\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}$, after a few manipulations, the condition above becomes

$$\lambda\left[u'(c_1) - \theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]\left[\int_{\lambda}^{1} u'(\overline{c}) - \theta^*(c_1,\overline{c})\int_{\lambda}^{\overline{n}} u'(\overline{c})\right] >$$
$$\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \int_{n=\lambda}^{\overline{n}} u'(\overline{c}),$$

which can be simplified to

$$\left[u'(c_1) - \theta^*(c_1,\overline{c})Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right]u'(\overline{c})\left[(1-\lambda) - (\overline{n}-\lambda)\theta^*(c_1,\overline{c})\right] >$$
$$(1-\theta^*(c_1,\overline{c}))\left[u'(c_1) - E[\theta \mid \theta > \theta^*(c_1,\overline{c})]Ru'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right](\overline{n}-\lambda)u'(\overline{c}).$$

Since $\theta^*(c_1,\overline{c}) < E[\theta \mid \theta > \theta^*(c_1,\overline{c})]$ and $\left[(1-\lambda) - (\overline{n}-\lambda)\theta^*(c_1,\overline{c})\right] > (1-\theta^*(c_1,\overline{c}))(\overline{n}-\lambda)$, the condition above holds and $\frac{dc_1^{DI}}{d\overline{c}} > 0$. The proof for the case $\overline{c} > 1$ is analogous. The inequality $c_1^{DI} > c_1^{D}$ follows directly from $\frac{dc_1^{DI}}{d\overline{c}} > 0$ as c_1^{D} is characterized in Proposition 2 for $\overline{c} = 0$. Thus, the proposition follows. □

Proof of Proposition 10. For convenience, as in the previous proofs, we distinguish the case $\overline{c} \leq 1$ from $\overline{c} > 1$. Given that the expression (22) is differentiable at $\overline{c} = 1$, the optimal level of guarantee $\overline{c}^{DI}$ corresponds to the solution to

$$\int_{\theta^*(c_1,\overline{c})}^{1} (1-\lambda)(1-\theta)\left[u'(\overline{c}) - v'(g-(1-\lambda)\overline{c})\right] d\theta$$

$$
\begin{aligned}
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\lambda u(c_1)+(1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+\left(1-\theta^*(c_1,\overline{c})\right)u\left(\overline{c}\right)\right)-u(1)\right]\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g)+(1-\theta^*(c_1,\overline{c}))v\left(g-(1-\lambda)\overline{c}\right)-v(g)\right] \qquad (47)\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\frac{\partial c_1}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g)+(1-\theta^*(c_1,\overline{c}))v\left(g-(1-\lambda)\overline{c}\right)-v(g)\right]=0,
\end{aligned}
$$

in the case $\overline{c}\leq 1$ and to

$$
\begin{aligned}
&\int_0^{\theta^*(c_1,\overline{c})}\left[u'(\overline{c})-v'\left(g-\overline{c}+1\right)\right]d\theta+\int_{\theta^*(c_1,\overline{c})}^1(1-\lambda)(1-\theta)\left[u'\left(\overline{c}\right)-v'\left(g-(1-\lambda)\overline{c}\right)\right]d\theta\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\lambda u(c_1)+(1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+(1-\theta^*(c_1,\overline{c}))u\left(\overline{c}\right)\right)-u(\overline{c})\right]\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g)+(1-\theta^*(c_1,\overline{c}))v\left(g-(1-\lambda)\overline{c}\right)-v\left(g-\overline{c}+1\right)\right] \qquad (48)\\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\frac{\partial c_1}{\partial\overline{c}}\left[\theta^*(c_1,\overline{c})v(g)+(1-\theta^*(c_1,\overline{c}))v\left(g-(1-\lambda)\overline{c}\right)-v\left(g-\overline{c}+1\right)\right]=0,
\end{aligned}
$$

when $\overline{c}>1$.

In order to prove that the government chooses a positive level of guarantees, we show that the first order condition (47) is positive for $\overline{c}=0$. Evaluating (47) for $\overline{c}=0$, we obtain

$$
\begin{aligned}
&\int_{\theta^*(c_1,0)}^1(1-\lambda)\left(1-\theta\right)\left[u'\left(0\right)-v'(g)\right]d\theta\\
&-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\lambda u(c_1)\right.\\
&\left.+(1-\lambda)\left(\theta^*(c_1,0)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+\left(1-\theta^*\left(c_1,0\right)\right)u\left(0\right)\right)-u(1)\right]\\
&-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)\right]\\
&-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\right|_{\overline{c}=0}\frac{\partial c_1}{\partial\overline{c}}\left[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)\right],
\end{aligned}
$$

with $\theta^*(c_1,0)$ being equal to the threshold $\theta^*(c_1)$ in the decentralized economy, where $\overline{c}=0$.

As $\left[\theta^*(c_1,0)v(g)+(1-\theta^*(c_1,0))v\left(g\right)-v(g)\right]=0$, the expression simplifies further to

$$
\begin{aligned}
&\int_{\theta^*(c_1,0)}^1(1-\lambda)\left(1-\theta\right)\left[u'\left(0\right)-v'(g)\right]d\theta \qquad (49)\\
&-\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}\left[\lambda u(c_1)\right.\\
&\left.+(1-\lambda)\left(\theta^*(c_1,0)u\left(\frac{1-\lambda c_1}{1-\lambda}R\right)+\left(1-\theta^*\left(c_1,0\right)\right)u\left(0\right)\right)-u(1)\right].
\end{aligned}
$$

Since $\overline{n} = \widehat{n}$ when $\overline{c} = 0$, $\left.\frac{\partial\theta^*(c_1,\overline{c})}{\partial\overline{c}}\right|_{\overline{c}=0}$ is equal to

$$-\frac{\int_{n=\lambda}^{1} u'(0) - \int_{n=\lambda}^{\widehat{n}} \theta^*(c_1,0)u'(0)}{\int_{n=\lambda}^{\widehat{n}} \left[u\left(\frac{1-nc_1}{1-n}R\right) - u(0)\right]} < 0,$$

and it follows that the second term in (49) is positive. Thus, a sufficient condition for (49) to be positive and, in turn, $\overline{c} > 0$, is $u'(0) - v'(g) > 0$. The proposition follows. □

Proof of Proposition 11. Taking the derivative of (22) with respect to c_1, we obtain c_{1G}^{DI}. This corresponds to the solution to

$$\begin{aligned}
&\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \left(1-\theta^*(c_1,\overline{c})\right)u(\overline{c})\right) - u(1)\right] \\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g)\right] = 0
\end{aligned} \tag{50}$$

when $\overline{c} \le 1$, and

$$\begin{aligned}
&\lambda \int_{\theta^*(c_1,\overline{c})}^{1} \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda}R\right)\right] d\theta \\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^*(c_1,\overline{c})u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \left(1-\theta^*(c_1,\overline{c})\right)u(\overline{c})\right) - u(\overline{c})\right] \\
&-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g-\overline{c}+1)\right] = 0
\end{aligned} \tag{51}$$

when $\overline{c} > 1$. Evaluating (25) taking $\overline{c} \le 1$ and comparing it with (50), it is easy to see that they only differ in the last term of (50) that is

$$-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g)\right]$$

Since $\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g)\right] < 0$, for given c_1 and $\overline{c}$, the expression in (25) is smaller than that in (50), thus implying that $c_1^{DI} < c_{1G}^{DI}$.

Evaluate now (25) taking $\overline{c} > 1$ and compare it with (51). They only differ in the last term in (51), which is equal to

$$-\frac{\partial\theta^*(c_1,\overline{c})}{\partial c_1}\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g-\overline{c}+1)\right]$$

The bracket $\left[\theta^*(c_1,\overline{c})v(g) + (1-\theta^*(c_1,\overline{c}))v(g-(1-\lambda)\overline{c}) - v(g-\overline{c}+1)\right]$ can be either positive or negative. Using the same argument as in the case with $\overline{c} \le 1$, it follows that $c_1^{DI} < c_{1G}^{DI}$ if the bracket is negative, and $c_1^{DI} > c_{1G}^{DI}$ if it is positive. Thus, the proposition follows. □

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