Information Sharing in Financial Markets

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01/24/23

Abstract

We study information sharing between strategic investors who are informed about asset fundamentals. We demonstrate that a coarsely informed investor optimally chooses to share information if his counterparty investor is well informed. By doing so, the coarsely informed investor invites the other investor to trade against his information, thereby reducing his price impact. Paradoxically, the well informed investor loses from receiving information because of the resulting worsened market liquidity and the more aggressive trading by the coarsely informed investor. Our analysis sheds light on phenomena such as private communications among investors and public information sharing on social media.

Keywords: Information sharing, trading against error, trading profits, asset markets

JEL: D82, G14, G18

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1 Introduction

Information sharing is ubiquitous to financial markets. In an early survey by Shiller and Pound (1989), a majority of institutional investors attribute their recent trades to discussions with peers. More recently, Hong et al. (2005) and Pool et al. (2015) provide evidence suggesting that mutual fund managers trade based on local word-of-mouth communication in the asset-management community. In today's environment, communication continues in different forms on the Internet, in social media outlets such as Twitter, Seeking Alpha, StockTwits, and Reddit, or in private Internet communities such as SumZero and Value Investors Club.[1] It is thus not surprising that communications among investors are thought to have a profound influence on trading strategies and financial market outcomes, as summarized by this quote from Shiller (2015, p.180): "Word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations."

Despite large empirical and anecdotal evidence, the theoretical basis for understanding information sharing in financial markets needs further development. Sharing information is costly to investors, as it might reduce their informational advantage. Hence, it is important to understand what benefit investors get from sharing information and under what circumstances they will choose to share information. Of particular interest is whether those who share information are the most informed. From the point of view of informational efficiency, one would hope that this is the case, but these traders might not have the incentive to do so. Anecdotally, it also seems that much of the information shared in financial markets comes from traders who are not particularly sophisticated. In this paper, we develop a model that sheds new light on these issues. We show how information sharing can be beneficial for some traders because of how it affects the behavior of other traders, and, most importantly, that the traders who choose to share information are the less informed. We explore the implications that this endogenous information sharing has for investor profits and market quality.

Our model is based on the Kyle (1985) framework, extended to allow investors to choose whether to share their private information before trading. The market has one risky asset, traded by informed investors, noise traders, and a competitive market maker. Our basic model considers the parsimonious case of two investors, denoted by H and L, where H has perfect information about the fundamentals of the asset while L receives noisy information. In extensions, we consider a less-than-perfectly informed H trader and also multiple traders. Before trading, H and L simultaneously decide whether to share their private information with each other. Then, they trade on the endowed information and the shared information, if any, and the price is set by a market maker at the conditional expected value of the asset given the total order flow.

Our key result is that in equilibrium, the L investor chooses to share his information with

[1]See https://sumzero.com/ and https://valueinvestorsclub.com/ for details. Crawford et al. (2017) and Crawford et al. (2018) contain a detailed introduction to the two social networking websites.

the H investor, while the H investor does not. Hence, information flows against the direction of information efficiency. This result is in stark contrast to what is typically assumed in the information-sharing literature. In the existing theories, when information is transmitted, it is assumed to be either from informed to uninformed investors (e.g., Indjejikian et al., 2014; Kovbasyuk and Pagano, 2015; Ljungqvist and Qian, 2016; Liu, 2017) or to be mutually exchanged among investors (e.g., Colla and Mele, 2010; Ozsoylev and Walden, 2011; Han and Yang, 2013; Manela, 2014; Chen et al., 2015). Our model demonstrates the importance of asking who has a greater incentive to share information and endogenizing the direction of information flow. The key mechanism behind the result is that the less informed investor benefits from sharing the information and having the more informed investor trade against him, offsetting his price impact.

To understand this result, it is useful to break down how the receiver of information uses it in the market, and how this, in turn, affects the sender of information. There are two effects. First, the “forecasting-fundamental effect” makes the receiver more informed about the asset fundamental and pushes him to trade on the shared information in the same direction as the sender. Second, the “trading-against-error effect” helps the receiver forecast the component in the sender's order flow that is based on noise and pushes him to trade on the shared information in the opposite direction of the sender. The first effect makes sharing information undesirable since it reduces the informational advantage that the sender has. The second effect makes sharing information desirable since it reduces the price impact that the sender will have when trading on his signal. Since H is more informed than L, the first effect dominates for H, and the second effect dominates for L, generating the result that in equilibrium, L will share information and H will not. This result is shown most clearly in our baseline model, in which H is perfectly informed, but as mentioned above, we show it is robust in a setting where H also has noisy information but is more informed than L.

Ex post, when L shares information with H, both of them benefit from the way that H is using this information, i.e., from the trading against error. For H, trading against error is an opportunity to benefit more from the expected difference between fundamental value and price. For L, trading against error by H reduces L's own price impact, given that H trades against L's information. This is why L chooses to share the information and H chooses to trade against it. However, considering the overall ex ante effect, we show that H is actually made worse off by the fact that L shares information with him. This is due to the indirect effect that L's information sharing has on L's trading strategy and on the market maker's pricing rule.

Specifically, in response to information sharing, L trades more aggressively since the price is less sensitive to his information, and the market maker sets prices to be more sensitive to overall order flow because of an increase in informed trades (even though informed trade is partially offset by less aggressive trading by H). These changes harm H, making him overall worse off despite the positive ex post direct effect. By contrast, the indirect effect just strengthens the overall benefit to L. The problem for H is that he lacks the commitment ability to ignore the information once it is shared, as it is optimal for him to use it ex post. This, in turn, makes him worse off ex ante. We go deeper into the issue of lack of commitment in an extension that introduces multiple H investors.

Thinking beyond the direction of information sharing and the implications for individual investors, it is important to explore the implications for key market variables. We do so by comparing an economy with no information sharing to one where information is shared as in the equilibrium described above. We show that, even though the information is shared by less informed investors, information sharing still leads to an overall increase in the informativeness of the price (i.e., market efficiency). This is a combination of the fact that L is trading more aggressively on his signal and H is trading against the noise, implying that overall more fundamental information is injected into the price. A direct implication of this effect is that information sharing leads to lower market liquidity, as the market maker's adverse-selection concern is strengthened. Considering the effect on volume, we find that it can go both ways. When he shares information, L's trading volume increases, whereas both H and the market maker reduce their trading volume. Overall, we find that the first effect dominates when L's information is highly imprecise, as then sharing information leads him to trade a lot more aggressively.

Our baseline model derives the above results in a simple framework. Yet, to show the robustness and explore other dimensions that shape information sharing, we provide a battery of extensions and variations of the baseline model. First, while our baseline model considers a perfectly informed H investor, this is clearly a significant simplification. Hence, we extend the model to include two differentially imperfectly informed investors. We show that the more informed investor never shares information, and the less informed shares as long as his information is sufficiently less precise. Second, in the main model we consider a decision on information sharing that is made before investors observe their signals. We consider an extension with an information-sharing decision made after the realization of the signal. This is a more complicated setting because of signaling issues, but we generally confirm that the main intuition of our model still holds.

Third, as mentioned above, our model shows that the H investor is overall worse off in the information-sharing economy. An important question then is why he cannot commit not to listen to the information. In a more realistic model with multiple H investors, we show that even if they had the ability to commit they would individually choose not to do so. There is essentially a coordination problem among the H investors that prevents choosing commitment. Fourth, our main model assumes that information is only shared with investors, but an important question for some applications is what happens if it was shared publicly and the market maker observed it as well. We provide an extension where this is the form of information sharing and show that the results remain as long as different agents interpret the information slightly differently with transmission noise. Finally, we provide three additional extensions in the online appendix and briefly describe them in the main text: endogenous information acquisition, multiple H and L investors, and three types of differentially informed investors. In all these alternative settings, we find that our key result – that less informed investors choose to share information with more informed investors – continues to hold for a robust set of parameter values and that the result is driven by H trading against the information shared by L.

After analyzing the model and its implications, we discuss the connection of the model to empirical evidence and popular commentary on financial markets. We argue that our model can apply to the well-documented market chatter in financial markets, where investors privately communicate with each other. We also discuss how our model, and in particular one of the extensions described above, applies to the recently growing public information sharing on social media. In both cases, our theory speaks to the question of who shares information with whom and makes the unique prediction, which is consistent with descriptions of the financial market, that information tends to flow from the less informed investors.

The remainder of the paper is organized as follows. We now discuss the relation of our paper to the literature. Section 2 presents the baseline model. In Section 3, we solve the model and describe the main implications of information sharing in equilibrium. Section 4 presents several extensions and variations of the baseline model and briefly describes several others that are contained in the online appendix. In Section 5, we discuss the link of the model to real-world financial markets and empirical evidence on them. Section 6 concludes. Proofs are relegated to the appendix.

Related Literature Our paper contributes to the literature on information sharing in financial markets. Many studies take information sharing as given and examine its implications (e.g., Duffie and Manso, 2007; Duffie et al., 2009, 2010; Colla and Mele, 2010; Ozsoylev and Walden, 2011; Han and Yang, 2013; Manela, 2014; Boyarchenko et al., 2021). Those studies do not answer the question of why investors share information in the first place.

The existing literature has also offered various possible reasons for information sharing. Benabou and Laroque (1992) argue that insiders can use privileged information to manipulate markets. Stein (2008) posits that an agent shares ideas in the hope that his counterpart will be able to take it one step further, and will then bounce the more fully developed idea right back to him. Ljungqvist and Qian (2016) suggest that arbitrageurs with short positions may reveal their information to accelerate price correction, thereby circumventing limits to arbitrage. The idea that information revelation can be used to accelerate price correction is particularly relevant for investors with short-term incentives (e.g., Kovbasyuk and Pagano, 2015; Liu, 2017; Schmidt, 2019). In addition, by injecting noise into the information spread, an investor gains an advantage over uninformed followers (Van Bommel, 2003) or other informed competitors (Indjejikian et al., 2014); by disclosing a mixture of fundamental information and his position, an investor induces market makers to move the asset price in a manner favorable to him (Pasquariello and Wang, 2016). Foucault and Lescourret (2003) show that information sharing is possible between traders with different types of information (fundamental vs. non-fundamental information). In a contemporaneous paper, Balasubramaniam (2021) shows that competing traders share information when they disagree much with each other.

None of the existing papers on information sharing asks the question of who shares information with whom. Our study fills this void and predicts that information is transmitted from the less informed investor to the more informed investor. We think this is a fundamental point, especially given observations that information shared in financial markets may not be of high quality. In addition, another new implication of our model is that the information sender is better off, whereas the receiver becomes worse off after information sharing. In contrast, in the existing explanations, both should be better off from information sharing at the expense of third parties (e.g., Indjejikian et al., 2014; Foucault and Lescourret, 2003). Finally, our theory does not require that the information sender owns initial positions or has short-term incentives. Unlike other explanations in which the investor "talks for her book" (e.g., Pasquariello and Wang, 2016; Schmidt, 2019), in our model the investor does not have any book yet and instead reveals information to help build it.

The trading-against-error effect also connects our theory to the literature on private information about noise trading (e.g., Ganguli and Yang, 2009; Marmora and Rytchkov, 2018; Farboodi and Veldkamp, 2020). Both L's trades and noise trading look like "dumb money" to H, but the dumb money in our setting is driven by the error term in the less informed investor's signal, as opposed to pure noise trading. The literature on noise-trading information does not address the key question we ask (who shares information with whom) and so does not offer our novel results that the less informed share with the more informed and that the information receiver (sender) becomes worse off (better off).

2 Model

We consider a Kyle model (Kyle, 1985) and extend its analysis to allow for information sharing between investors. The economy has three dates: $t = 0, 1, 2$. Figure 1 describes the timeline of the economy. There is a single risky asset with a date-2 liquidation value $\tilde{v}$, where $\tilde{v} \sim N(0, 1)$.[2] The financial market operates on date 1, and it is populated by three groups of agents: one market maker, noise traders, and two risk-neutral rational investors. As standard in the literature, the market maker sets the price based on the weak market-efficiency rule and noise traders submit exogenous random market orders. The two rational investors are endowed with private information about the fundamental $\tilde{v}$ of the risky asset,

[2] The normalization that $\tilde{v}$ has a zero mean and a unit standard deviation is without loss of generality. If we instead assume $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$ (with $\bar{v} \in \mathbb{R}$ and $\sigma_v > 0$), then all our results would hold as long as we reinterpret the information precision levels as signal-to-noise ratios.

and their information is of different precision levels. On $t = 0$, information can be shared between the two rational investors.

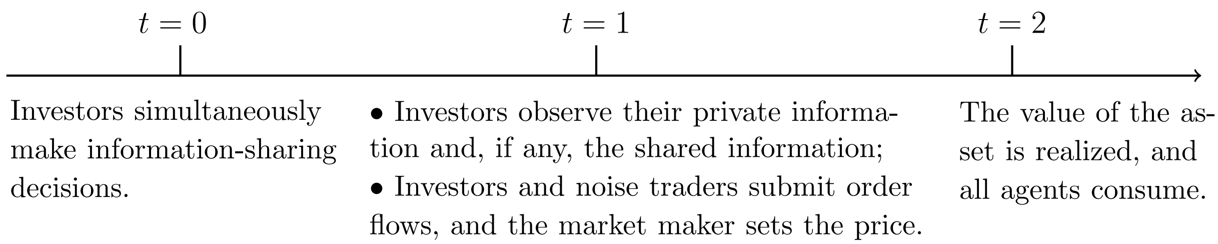


Figure 1: Timeline

The two rational investors, denoted by H and L, differ in their information quality: H owns more precise information about the fundamental than L. To illustrate the mechanism transparently, we now assume that H perfectly observes $\tilde{v}$. In one extended setting analyzed in Section 4.1, we will assume that H has imperfect information about $\tilde{v}$ and show that the result continues to hold as long as L's information is sufficiently imprecise. L is coarsely informed about $\tilde{v}$, and he can only observe a noisy private signal as follows:

$$\tilde{y} = \tilde{v} + \tilde{e}, \text{ where } \tilde{e} \sim N(0, \rho^{-1}). \tag{1}$$

The parameter $\rho \in (0, \infty)$ governs the quality of L's private information.

On $t = 0$, H and L simultaneously decide whether to share their private information to maximize their respective expected trading profits. For investor $i \in \{H, L\}$, we use $A_i \in \{S, \varnothing\}$ to denote information-sharing decisions, where $A_i = S$ means that the investor fully shares information with the other investor, whereas $A_i = \varnothing$ means that the investor keeps it secret. For instance, if on date 0, L decides to share information with H (i.e., $A_L = S$), then H perfectly observes $\tilde{y}$ on date 1.[3] We assume that the date-0 information-sharing decisions become common knowledge at the beginning of date 1, so that we can apply backward induction in Section 3.1 to compute the equilibrium of our economy.

In the main model, for tractability reasons, we have assumed that investors can commit themselves to an information-sharing policy before receiving private information. The other-

[3] We have also considered an alternative setup that allows for partial information sharing between investors and found that our results remain unchanged. In this alternative setup, we follow the industrial-organization literature on information sharing among firms (e.g., Vives, 1984; Gal-Or, 1985; Darrough, 1993) and the literature on disclosure by firms or regulators (e.g., Diamond, 1985; Morris and Shin, 2002; James and Lawler, 2011; Goldstein and Yang, 2017), and specify information sharing as follows: Suppose that H shares with L a garbled signal $\tilde{s}_H = \tilde{v} + \tilde{\varepsilon}_H$, where $\tilde{\varepsilon}_H \sim N\left(0, \tau_H^{-1}\right)$ and that L shares with H a garbled signal $\tilde{s}_L = \tilde{y} + \tilde{\varepsilon}_L$, where $\tilde{\varepsilon}_L \sim N\left(0, \tau_L^{-1}\right)$. The precision levels τ_H and τ_L of the shared information are controlled by H and L, respectively, and can range between 0 and ∞; that is, $\tau_i \in [0, \infty]$, for $i \in \{H, L\}$. If $\tau_i = 0$, investor i's shared information is not informative at all, or equivalently investor i does not share any information. If $\tau_i = \infty$, investor i fully shares information. Thus, our baseline model essentially assumes that τ_i takes only two values, 0 and ∞.

wise ex post setup (i.e., investors make information-sharing decisions after receiving private information) is less tractable because of signalling issues. In Section 4.2, we have analyzed a setting with ex post information sharing and generally confirm that the key insights of our main model continue to hold.

Trading occurs on $t = 1$, and we use $\tilde{p}$ to denote the equilibrium asset price. Conditional on the endowed private information, as well as the shared information (if any), investor $i \in \{H, L\}$ places market order $\tilde{x}_i$ to maximize the expected trading profit as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})|\mathcal{F}_i], \tag{2}$$

where $\mathcal{F}_i$ indicates investor i's information set. For instance, if L shares information with H but H does not share information with L (i.e., $A_L = S$ and $A_H = \varnothing$), then the two investors' information sets are respectively $\mathcal{F}_L = \{\tilde{y}\}$ and $\mathcal{F}_H = \{\tilde{v}, \tilde{y}\}$. Noise traders place market order $\tilde{u}$, where $\tilde{u} \sim N(0, \sigma_u^2)$ (with $\sigma_u > 0$) and $\tilde{u}$ is independent of all other random shocks. The total order flow faced by the market maker is $\tilde{\omega} = \tilde{x}_H + \tilde{x}_L + \tilde{u}$. The market maker sets price $\tilde{p}$ according to the weak-efficiency rule,

$$\tilde{p} = E(\tilde{v}|\tilde{\omega}). \tag{3}$$

3 Equilibrium and Implications

In this section, we first characterize the equilibrium in Section 3.1. Since we assume that the date-0 information-sharing decisions become observable at the beginning of date 1, we will apply backward induction to compute the equilibrium. Section 3.2 examines the implications of information sharing for investor profits and market quality.

3.1 Equilibrium Characterization

3.1.1 Equilibrium at the Trading Stage

Given investors' information-sharing decisions A_L and A_H on date 0, different trading subgames follow on date 1, depending on whether the investors share their respective private information. The investors' expected profits evaluated on the date-1 subgame equilibrium will serve as their payoffs of the date-0 information-sharing game. As standard in the literature, in each subgame, we consider linear equilibrium in which investors' optimal trading strategies and the market maker's equilibrium pricing rule are linear functions. We next discuss each subgame separately.

Subgame 1: Neither Investor Shares Information ($A_L = A_H = \varnothing$).

When neither investor shares information, each investor only observes the endowed information. Hence, $\mathcal{F}_L = \{\tilde{y}\}$ and $\mathcal{F}_H = \{\tilde{v}\}$. This setting is the classical Kyle (1985) trading game with two deferentially informed investors and the equilibrium derivation is standard. We conjecture the following trading strategies of investors and the pricing rule of the market maker: $\tilde{x}_L = \beta_y \tilde{y}$, $\tilde{x}_H = \alpha_v \tilde{v}$, and $\tilde{p} = \lambda \tilde{\omega}$, where β_y, α_v, and λ are endogenous constants. We compute those constants by examining the agents' maximization problems.

Using the conjectured H's trading strategy and the market maker's pricing rule, we can compute L's conditional expected profit in equation (2) as follows:

$$E[\tilde{x}_L(\tilde{v} - \tilde{p})|\tilde{y}] = \tilde{x}_L \left[\frac{\rho}{1+\rho}\tilde{y} - \lambda \left(\tilde{x}_L + \alpha_v \frac{\rho}{1+\rho}\tilde{y} \right) \right].$$

Maximizing the expected profit yields L's implied trading rule, $\tilde{x}_L = \frac{(1-\lambda\alpha_v)\rho}{2\lambda(1+\rho)}\tilde{y}$. Comparing this implied trading rule with the conjectured L's trading strategy, we have $\beta_y = \frac{(1-\lambda\alpha_v)\rho}{2\lambda(1+\rho)}$. Similarly, using the conjectured L's trading strategy and the market maker's pricing rule, we can compute H's conditional expected trading profits as follows:

$$E[\tilde{x}_H \left(\tilde{v} - \tilde{p}\right)|\tilde{v}] = \tilde{x}_H \left[\tilde{v} - \lambda(\tilde{x}_H + \beta_y \tilde{v})\right].$$

We then compute H's optimal trading rule and compare it with the conjectured strategy for H, yielding $\alpha_v = \frac{1-\lambda\beta_y}{2\lambda}$. Finally, using the conjectured trading strategies, the market maker sets the price according to equation (3) as follows: $\tilde{p} = \frac{\alpha_v+\beta_y}{(\alpha_v+\beta_y)^2+\beta_y^2\frac{1}{\rho}+\sigma_u^2}\tilde{\omega}$. This implies that $\lambda = \frac{\alpha_v+\beta_y}{(\alpha_v+\beta_y)^2+\beta_y^2\frac{1}{\rho}+\sigma_u^2}$. We can combine this equation with $\beta_y = \frac{(1-\lambda\alpha_v)\rho}{2\lambda(1+\rho)}$ and $\alpha_v = \frac{1-\lambda\beta_y}{2\lambda}$ to characterize the equilibrium trading and pricing rules and the two investors' resulting expected profits, which are summarized in the following lemma, where the superscript "$\varnothing\varnothing$" denotes the fact that neither L nor H shares information.

Lemma 1 (Neither investor shares information)**.** *Suppose that neither investor shares information on date 0 (i.e., $A_L = A_h = \varnothing$). In the date-1 trading equilibrium, the two investors' trading strategies are $\tilde{x}_L = \beta_y^{\varnothing\varnothing}\tilde{y}$ and $\tilde{x}_H = \alpha_v^{\varnothing\varnothing}\tilde{v}$, respectively, where*

$$\beta_y^{\varnothing\varnothing} = \frac{\rho\sigma_u}{\sqrt{4+\rho(5+2\rho)}} \text{ and } \alpha_v^{\varnothing\varnothing} = \frac{(2+\rho)\sigma_u}{\sqrt{4+\rho(5+2\rho)}}, \tag{4}$$

and the market maker's pricing rule is $\tilde{p} = \lambda^{\varnothing\varnothing}\tilde{\omega}$, where

$$\lambda^{\varnothing\varnothing} = \frac{\sqrt{4+\rho(5+2\rho)}}{(4+3\rho)\sigma_u}. \tag{5}$$

The two investors' date-0 unconditional expected profits evaluated at the date-1 trading equi-

librium are, respectively:

$$\pi_L^{\varnothing\varnothing} = \frac{\rho(1+\rho)\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} \text{ and } \pi_H^{\varnothing\varnothing} = \frac{(2+\rho)^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}. \tag{6}$$

Subgame 2: L Shares Information and H Does Not ($A_L = S$ and $A_H = \varnothing$).

If L shares information whereas H does not, the two investors' information sets become $\mathcal{F}_L = \{\tilde{y}\}$ and $\mathcal{F}_H = \{\tilde{v}, \tilde{y}\}$. In Section 3.1.2, we will show that these sharing decisions will form the equilibrium at the date-0 information-sharing stage. We follow similar steps as in Subgame 1 and derive the date-1 trading equilibrium in Subgame 2. The results are summarized in the following lemma, where the superscript "$S\varnothing$" denotes the fact that L shares but H does not share information.

Lemma 2 (L shares information but H does not)**.** *Suppose that only L shares information on date 0 (i.e., $A_L = S$ and $A_H = \varnothing$). In the date-1 trading equilibrium, the two investors' trading strategies are $\tilde{x}_L = \beta_y^{S\varnothing}\tilde{y}$ and $\tilde{x}_H = \alpha_v^{S\varnothing}\tilde{v} + \alpha_y^{S\varnothing}\tilde{y}$, respectively, where*

$$\beta_y^{S\varnothing} = \frac{2\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}},\ \alpha_v^{S\varnothing} = \frac{3\sigma_u\sqrt{1+\rho}}{\sqrt{9+8\rho}}, \text{ and } \alpha_y^{S\varnothing} = -\frac{\rho\sigma_u}{\sqrt{(1+\rho)(9+8\rho)}}, \tag{7}$$

and the market maker's pricing rule is $\tilde{p} = \lambda^{S\varnothing}\tilde{\omega}$, where

$$\lambda^{S\varnothing} = \frac{\sqrt{9+8\rho}}{6\sigma_u\sqrt{1+\rho}}. \tag{8}$$

The two investors' date-0 unconditional expected profits evaluated at the date-1 trading equilibrium are, respectively:

$$\pi_L^{S\varnothing} = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}} \text{ and } \pi_H^{S\varnothing} = \frac{(9+4\rho)\sigma_u}{6\sqrt{(1+\rho)(9+8\rho)}}. \tag{9}$$

One notable finding in Lemma 2 is that H trades against the information shared by L (i.e., $\alpha_y^{S\varnothing} < 0$), which underlies the key mechanism that drives L's information-sharing incentives in our setting. This result can be best seen from the first-order condition (FOC) of the H investor's profit maximization problem. Inserting the market maker's pricing rule into equation (2), we can compute H's conditional expected profit as $\tilde{x}_H E\left(\tilde{v} - \lambda^{S\varnothing}\tilde{x}_L|\tilde{v},\tilde{y}\right) - \lambda^{S\varnothing}\tilde{x}_H^2$. Taking the FOC, we obtain that

$$\tilde{x}_H = \underbrace{\frac{1}{2\lambda^{S\varnothing}}E\left(\tilde{v}|\tilde{v},\tilde{y}\right)}_{\text{Forecasting fundamental}} \quad \underbrace{-\frac{1}{2}E\left(\tilde{x}_L|\tilde{v},\tilde{y}\right)}_{\text{Trading against error}}. \tag{10}$$

The above equation clearly demonstrates how investor H can use the shared information $\tilde{y}$ to improve his investment decisions. The first term states that, in principle, H can use $\tilde{y}$ to forecast asset fundamental $\tilde{v}$, pushing H to trade on the shared information in the same direction as L. We label this term as the "forecasting-fundamental effect." Since in our baseline model, H knows $\tilde{v}$ perfectly, $\tilde{y}$ does not generate additional value through this term, and thus, this forecasting-fundamental effect is inactive in the baseline model.[4]

The second term in equation (10) states that H can use $\tilde{y}$ to forecast L's order flow $\tilde{x}_L$, which pushes H to trade on the shared information in the opposite direction of L.[5] We refer to this term as the "trading-against-error effect" because it captures that H trades against the error component $\tilde{e}$ in L's signal $\tilde{y}$. Intuitively, investor H can use his knowledge of $\tilde{v}$ and $\tilde{y}$ to back out $\tilde{e}$ and thus, information set $\{\tilde{v}, \tilde{y}\}$ is equivalent to information set $\{\tilde{v}, \tilde{e}\}$. So, we can re-express H's trading strategy as follows:

$$\tilde{x}_H = \alpha_v^{S\varnothing}\tilde{v} + \alpha_y^{S\varnothing}\tilde{y} = \left(\alpha_v^{S\varnothing} + \alpha_y^{S\varnothing}\right)\tilde{v} + \alpha_y^{S\varnothing}\tilde{e}. \tag{11}$$

The term $\alpha_y^{S\varnothing}\tilde{e}$ with $\alpha_y < 0$ in (11) captures the fact that investor H trades against the error $\tilde{e}$. Intuitively, investor L cannot disentangle fundamental $\tilde{v}$ from error $\tilde{e}$ in his signal $\tilde{y}$, while investor H can. The error-driven trading by L resembles "dumb money" to investor H, and trading against dumb money earns H extra profits. Specifically, given fundamentals $\tilde{v}$, when $\tilde{e}$ is positive, investor L will buy more, thereby pushing up asset price; H knows that this price increase is not driven by fundamentals, so he will sell accordingly. Conversely, when $\tilde{e}$ is negative, investor L will sell the extra risky asset, pressing down the price; at this low price, H will buy since he knows that the asset fundamental has not changed.

Subgame 3: H Shares Information ($A_H = S$)

In principle, there are two subgames when H shares information, depending on whether L shares his information. Nonetheless, it turns out that the equilibrium in these two subgames is the same because H owns perfect information about the asset fundamental, and once he shares it with L, L no longer uses his own noisy signal $\tilde{y}$ in predicting the asset fundamental and thus does not trade on $\tilde{y}$. Thus, regardless of L's information-sharing decision, once H shares information, the trading game degenerates to the classical Kyle (1985) setting with two perfectly informed traders. We summarize the trading equilibrium in the following lemma and use the subscript "$\cdot S$" to indicate the fact that H shares information while L

[4]Formally, in the baseline model, we have $E(\tilde{v}|\tilde{v}, \tilde{y}) = E(\tilde{v}|\tilde{v}) = \tilde{v}$ and thus, $\frac{\partial E(\tilde{v}|\tilde{v},\tilde{y})}{\partial \tilde{y}} = 0$. In Section 4.1, we will explore an extension in which investor H has imperfect information about $\tilde{v}$. There, the information shared by L is also helpful for H to predict $\tilde{v}$, thereby making the forecasting-fundamental effect active. Still, our result continues to hold as long as H's information is sufficiently precise, and the result is still driven by the fact that H trades against the information shared by L.

[5]We can show that $-\frac{1}{2}E(\tilde{x}_L|\tilde{v}, \tilde{y}) = -\frac{1}{2}E(\tilde{x}_L|\tilde{y}) = -\frac{\beta_y^{S\varnothing}}{2}\tilde{y}$. Since $\beta_y^{S\varnothing} > 0$ (L always trades on his information), we have $\alpha_y^{S\varnothing} = \frac{\partial}{\partial \tilde{y}}\left[-\frac{1}{2}E(\tilde{x}_L|\tilde{v}, \tilde{y})\right] = -\frac{\beta_y^{S\varnothing}}{2} < 0$.

either shares or does not share information.

Lemma 3 (H shares information)**.** *Suppose H shares information on date 0 (i.e., $A_H = S$). In the date-1 trading equilibrium, regardless of L's date-0 information-sharing decision, the two investors' date-1 trading strategies are $\tilde{x}_L = \beta_v^{\cdot S}\tilde{v}$ and $\tilde{x}_H = \alpha_v^{\cdot S}\tilde{v}$, where*

$$\beta_v^{\cdot S} = \alpha_v^{\cdot S} = \frac{\sigma_u}{\sqrt{2}}, \tag{12}$$

and the market maker's pricing rule is $\tilde{p} = \lambda^{\cdot S}\tilde{\omega}$, where

$$\lambda^{\cdot S} = \frac{\sqrt{2}}{3\sigma_u}. \tag{13}$$

The two investors' date-0 unconditional expected profits evaluated at the date-1 trading equilibrium are, respectively:

$$\pi_L^{\cdot S} = \pi_H^{\cdot S} = \frac{\sigma_u}{3\sqrt{2}}. \tag{14}$$

3.1.2 Equilibrium at the Information-Sharing Stage

We now go back to date 0 to analyze investors' information-sharing decisions. The equilibrium profits summarized in Lemmas 1–3 describe investors' payoffs in this information-sharing game. We plot the payoff matrix of this game in Figure 2. In each entry of the payoff matrix, the first input represents L's payoff, while the second input represents H's payoff.

		H	
		Not share ($\varnothing$)	Share (S)
L	Not Share ($\varnothing$)	$\frac{\rho(1+\rho)\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}, \underline{\frac{(2+\rho)^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}}$	$\underline{\frac{\sigma_u}{3\sqrt{2}}}, \frac{\sigma_u}{3\sqrt{2}}$
	Share (S)	$\underline{\frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}}}, \underline{\frac{(9+4\rho)\sigma_u}{6\sqrt{(1+\rho)(9+8\rho)}}}$	$\underline{\frac{\sigma_u}{3\sqrt{2}}}, \frac{\sigma_u}{3\sqrt{2}}$

This figure plots the two investors' payoffs in the date-0 information-sharing game, which are the unconditional expected trading profits evaluated at the date-1 trading game equilibrium. The first (second) input in each cell represents the payoff of investor L (H). The underline indicates each investor's best response given the other investor's information-sharing decision.

Figure 2: Payoff matrix of the date-0 information sharing game

By comparing H's expected profit across Lemmas 1–3, we notice that H's payoff is higher if he keeps secret no matter whether L shares information (i.e., $\pi_H^{\varnothing\varnothing} > \pi_H^{\cdot S}$ and $\pi_H^{S\varnothing} > \pi_H^{\cdot S}$). As a result, H's dominant strategy is not to share information. This result is intuitive and expected: Once H shares his information with L, L becomes perfectly informed about the asset fundamental as well; the two perfectly informed investors compete very fiercely in the financial market, which significantly lowers H's trading profit. In other words, the forecasting-fundamental effect dominates for H, discouraging him from sharing information.

Given that H chooses not to share information in equilibrium, we use the expressions of $\pi_L^{\varnothing\varnothing}$ and $\pi_L^{S\varnothing}$ in Lemmas 1 and 2 to examine L's best response. We find that $\pi_L^{S\varnothing} > \pi_L^{\varnothing\varnothing}$; that is, L makes a higher trading profit when sharing information. So, in our economy, information transmits from the less informed investor to the more informed one, rather than in the opposite direction as commonly considered in the literature (e.g., Benabou and Laroque, 1992; Van Bommel, 2003; Indjejikian et al., 2014; Ljungqvist and Qian, 2016; Schmidt, 2019). This result is surprising, and its driving force is the novel trading-against-error effect discussed above.

To understand the intuitions for this result, we note that information sharing has two effects on L's profit. First, trading against error by H implies that H trades against L's information, which reduces L's price impact and thus directly benefits L. Second, there is an indirect effect induced by further changes in investors' trading strategies and the market maker's pricing rule.[6] Specifically, in response to information sharing, L trades more aggressively since H's trading against L's information makes the price less sensitive to L's information. In addition, the more aggressive trading by L forces H to trade less aggressively. These changes in investors' trading aggressiveness benefit investor L. After L shares information, the overall order flow contains more informed trades (even though informed trade is partially offset by less aggressive trading by H). The market maker faces more serious adverse selection risk and so steepens the pricing schedule, which hurts both investors L and H. Nonetheless, this negative effect due to price impact is weaker than the positive effect due to trading aggressiveness so that overall, L is better off from changes in trading and pricing rules. The following proposition summarizes the above discussions.

Proposition 1 (Equilibrium information sharing in financial markets)**.** *On date 0, there exists a unique information-sharing equilibrium in which L shares his private information, whereas H does not (i.e., $A_L^* = S$ and $A_H^* = \varnothing$). On date 1, investors' equilibrium trading rules and the market maker's equilibrium pricing rule are characterized by Lemma 2.*

[6] We can formalize these two effects as follows. Fixing L's trading rule at $\beta_y^{\varnothing\varnothing}$ and the market maker's price impact at $\lambda^{\varnothing\varnothing}$, we compute H's optimal trading rule in response to information sharing: $\tilde{x}_H^{direct} = \arg\max_{\tilde{x}_H} E\left[\tilde{x}_H\left(\tilde{v} - \lambda^{\varnothing\varnothing}\left(\tilde{x}_H + \beta_y^{\varnothing\varnothing}\tilde{y}\right)\right)|\tilde{v},\tilde{y}\right] = \frac{1}{2\lambda^{\varnothing\varnothing}}\tilde{v} - \frac{\beta_y^{\varnothing\varnothing}}{2}\tilde{y}$. With this optimal trading rule by H (but the same trading rule by L and pricing rule by the market maker), L's profit becomes $\pi_L^{direct} = E\left[\beta_y^{\varnothing\varnothing}\tilde{y}\left(\tilde{v} - \lambda^{\varnothing\varnothing}\tilde{x}_H^{direct} - \lambda^{\varnothing\varnothing}\beta_y^{\varnothing\varnothing}\tilde{y}\right)\right]$. The direct effect is then given by $\pi_L^{direct} - \pi_L^{\varnothing\varnothing}$, which can be shown to be positive. The indirect effect is captured by $\pi_L^{S\varnothing} - \pi_L^{direct}$, where $\pi_L^{S\varnothing}$ is L's equilibrium profit computed when both investors and the market maker optimize in response to information sharing. We can show that the indirect effect is also positive: $\pi_L^{direct} - \pi_L^{\varnothing\varnothing} > 0$.

3.2 Implications of Information Sharing

We now examine the effect of information sharing on investors' profits and market quality by comparing our economy with endogenous information sharing (as described by Proposition 1) to a benchmark economy with no information sharing (as described by Lemma 1).

The most surprising profit result is that H actually becomes worse off in equilibrium, although he is receiving additional information. Like the analysis of L's profit, information sharing also has a direct and an indirect effect on H's profit. First, by observing and trading against L's shared information, H makes more profits from identifying more trading opportunities from the expected difference between fundamental value and price, holding constant investor L's trading rule and the market maker's pricing rule. This direct effect on H's profit has the same sign as the direct effect on L's profit, but for a different reason. Second, unlike the indirect effect on L's profit, the indirect effect on H's profit is negative: As we discussed in the previous subsection, due to the trading-against-error effect, information sharing causes the L investor to trade more aggressively, the H investor to trade less aggressively, and the market maker to steepen the pricing schedule, all of which adversely affect H's trading profit.

We can show that the negative indirect effect dominates the positive direct effect so that H becomes worse off in equilibrium.[7] Intuitively, L's information quality is relatively poor, so he does not pose a serious threat to H's trading. The direct benefit of H knowing L's information and predicting L's order flow is limited. By contrast, H, as the most informed investor, grabs the most profit share from the market, and thus, any adjustment in the price schedule by the market maker will significantly harm the H investor, which implies that the negative indirect effect is relatively significant.

The problem for H is that once the information is shared, he finds it optimal to use it ex post. Ex ante, if investor H could commit not to use the received information from L, he would have been better off. However, the inability to make this commitment changes the equilibrium behavior, generating negative effects on H's profit. In Section 4.3, we will further examine this commitment issue by considering an extended economy with multiple H-investors and find that in that extension, H-investors may choose not to commit even if they can make a credible commitment.

As discussed in the previous subsection, information sharing benefits investor L, which is why the L investor endogenously chooses to share information in equilibrium. When H and L are combined, the whole investor side makes more profits after information sharing. In other words, what L gains exceeds what H loses. For example, when $\rho = \sigma_u = 1$, by sharing information, L's profit increases by 32.7%, H's profit drops by 4.1%, and the two investors' total profit increases by 2.6%.

Thinking beyond the direction of information sharing and its implications for investor

[7] We can conduct a similar exercise as in Footnote 6 and formalize the direct and indirect effects on H's profit. The direct effect is given by $\pi_H^{direct} - \pi_H^{\varnothing\varnothing}$, which is positive. The indirect effect is $\pi_H^{S\varnothing} - \pi_H^{direct}$, which is negative and dominant, so that the total effect $\pi_H^{S\varnothing} - \pi_H^{\varnothing\varnothing}$ is negative.

profits, it is important to examine the implications for key market quality variables, such as market efficiency, market liquidity, and trading volume. We measure market efficiency using the precision of the asset payoff conditional on asset price, i.e., $m \equiv \frac{1}{Var(\tilde{v}|\tilde{p})}$. After information sharing, the total order flow becomes more correlated with the fundamental, so the price aggregates more fundamental information. This increase in price informativeness arises from the fact that L is trading more aggressively on his signal and H is trading against the noise. Market liquidity is captured by Kyle's λ, an inverse measure of market depth: More liquid markets have a smaller λ. A direct implication of the increasing informed trades in the total order flow is that the market maker raises the price impact to manage the increasing adverse selection risk, dampening market liquidity.

Finally, we follow Vives (2010) and measure the total volume traded, denoted by TV, by the sum of the expected absolute value of the trading from different agents in the model divided by 2: $TV = \frac{1}{2}\Big(E\big[|\tilde{x}_H| + |\tilde{x}_L| + |\tilde{\omega}| + |\tilde{u}|\big]\Big)$. When examining each agent's trading volume, we find that after information sharing, since L trades more aggressively and H trades less aggressively, L's trading volume increases whereas H's volume decreases; that is, $E\left[|\tilde{x}_L|\right]$ increases but $E(|\tilde{x}_H|)$ decreases. Meanwhile, the market maker's trading volume decreases; namely, $E(|\tilde{\omega}|)$ decreases. In terms of the total trading volume TV, when L's information precision ρ is sufficiently low, L's signal has a significant amount of noise and hence the trading-against-error effect is strong; so, L trades much more aggressively after information sharing, thereby driving up the total trading volume. However, when ρ is sufficiently high, the trading-against-error effect is diminished, and the increase in L's trading volume is mild. As a result, the total trading volume decreases after information sharing. Proposition 2 summarizes the above results.

Proposition 2 (Implications of information sharing)**.** *Compared with the benchmark economy without information sharing, in our economy with endogenous information sharing:*

(1) *L is better off, H is worse off, and their combined profit is higher; that is,* $\pi_L^{S\varnothing} > \pi_L^{\varnothing\varnothing}$, $\pi_H^{S\varnothing} < \pi_H^{\varnothing\varnothing}$, *and* $\pi_H^{S\varnothing} + \pi_L^{S\varnothing} > \pi_H^{\varnothing\varnothing} + \pi_L^{\varnothing\varnothing}$.

(2) *Market efficiency is higher and market liquidity is lower; that is,* $m^{S\varnothing} > m^{\varnothing\varnothing}$ *and* $\lambda^{S\varnothing} > \lambda^{\varnothing\varnothing}$. *When ρ is sufficiently low ($\rho \to 0$), trading volume increases (i.e., $TV^{S\varnothing} > TV^{\varnothing\varnothing}$); when ρ is sufficiently high ($\rho \to \infty$), trading volume decreases (i.e., $TV^{S\varnothing} < TV^{\varnothing\varnothing}$).*

4 Extensions and Variations

Our baseline model derives the key insights in a simple framework. In this section, we consider various extensions and variations to demonstrate the robustness of our key results and explore other dimensions that shape information sharing.

4.1 Imperfectly Informed H-Investor

To show our results most clearly, in the baseline model, H is assumed to have perfect information about the asset fundamental. We here relax this assumption and consider the more general case in which information can be transmitted between imperfectly informed investors. We consider two investors, denoted by 1 and 2, who are endowed with private information about $\tilde{v}$ with different precision levels. Specifically, investor i receives the following private signal:

$$\tilde{y}_i = \tilde{v} + \tilde{e}_i, \text{ with } \tilde{e}_i \sim N(0, \rho_i^{-1}) \text{ and } \rho_i \in (0, +\infty], \text{ for } i \in \{1, 2\},$$

where $\{\tilde{v}, \tilde{e}_1, \tilde{e}_2\}$ are mutually independent. Parameter ρ_i controls investor i's information quality. If $\rho_1 > \rho_2$, investor 1 is more informed than investor 2. The baseline model is nested by assuming that one investor's information precision is infinity and the other investor' information precision is finite. All of our other assumptions remain unchanged from the baseline model. The following proposition summarizes the equilibrium information sharing in this extended economy.

Proposition 3 (Information sharing between imperfectly informed investors)**.** *Consider two investors endowed with private information with different precision levels. Assume that investor i is weakly more informed, i.e., $\rho_i \geq \rho_j$, where $i, j \in \{1, 2\}$ and $i \neq j$. There exists a unique information-sharing equilibrium in which*

(1) *Investor i never shares information, i.e., $A_i^* = \varnothing$;*

(2) *If $\rho_i \geq \hat{\rho}_i \equiv 2(\rho_j + 1)$, then investor j shares information, i.e., $A_j^* = S$; otherwise, investor j does not share information, i.e., $A_j^* = \varnothing$.*

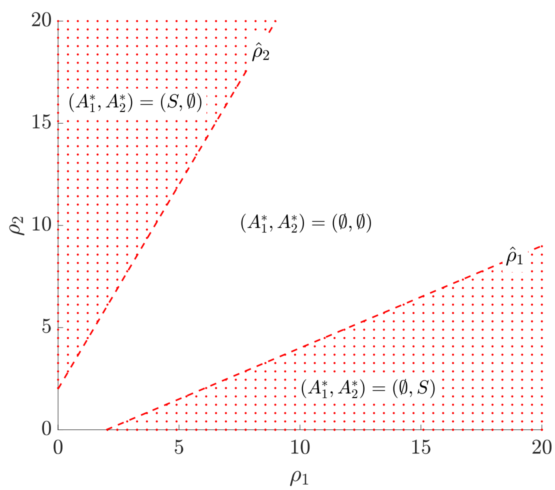


Figure 3: Information sharing between imperfectly informed investors

Figure 3 graphically illustrates Proposition 3 by plotting the two investors' equilibrium information-sharing behaviors against the precision levels of their endowed information. Neither investor shares information when the two investors' information precision levels are close to each other. When one investor's information is sufficiently better than the other investor's information, the better informed investor keeps secret, and the less informed investor starts to share information with the other investor.

Similar to the baseline model, information sharing by the less informed investor is driven by the possibility that the more informed investor trades against the information shared by the less informed investor. Without loss of generality, let us consider the case in which $\rho_1 \geq \rho_2$ so that investor 2 (as the L-investor) shares his information with investor 1 (as the H-investor). After information sharing, the two investors' equilibrium trading strategies are respectively as follows: $\tilde{x}_1 = \alpha_{y_1}\tilde{y}_1 + \alpha_{y_2}\tilde{y}_2$ and $\tilde{x}_2 = \beta_y \tilde{y}_2$. Similar to equation (10) in the baseline model, we can examine the more informed investor 1's profit maximization problem and compute his optimal trading as follows:

$$\tilde{x}_1 = \underbrace{\frac{1}{2\lambda}E\left(\tilde{v}|\tilde{y}_1,\tilde{y}_2\right)}_{\text{Forecasting fundamental}} \quad \underbrace{-\frac{1}{2}E\left(\tilde{x}_2|\tilde{y}_1,\tilde{y}_2\right)}_{\text{Trading against error}}. \tag{15}$$

The more informed investor 1 uses information $\tilde{y}_2$ both for predicting asset fundamentals $\tilde{v}$ and for predicting the less informed investor 2's order flow $\tilde{x}_2$. These two ways of using $\tilde{y}_2$ affect investor 1's optimal trading $\tilde{x}_1$ in opposite ways: $\frac{\partial}{\partial \tilde{y}_2}\frac{1}{2\lambda}E\left(\tilde{v}|\tilde{y}_1,\tilde{y}_2\right) = \frac{1+\rho_1}{2\lambda(1+\rho_1+\rho_2)^2} > 0$ (the "forecasting fundamental effect"); and $\frac{\partial}{\partial \tilde{y}_2}\left[-\frac{1}{2}E\left(\tilde{x}_2|\tilde{y}_1,\tilde{y}_2\right)\right] = -\frac{\beta_y}{2} < 0$ (the "trading-against-error" effect).

Investor 1 (the more informed investor) trades against information $\tilde{y}_2$ shared by investor 2 (the less informed investor) if and only if he uses $\tilde{y}_2$ primarily for predicting investor 2's order flow $\tilde{x}_2$ (i.e., $\alpha_{y_2} < 0$ if and only if $\frac{\partial}{\partial \tilde{y}_2}\frac{1}{2\lambda}E\left(\tilde{v}|\tilde{y}_1,\tilde{y}_2\right) < \left|\frac{\partial}{\partial \tilde{y}_2}\left[-\frac{1}{2}E\left(\tilde{x}_2|\tilde{y}_1,\tilde{y}_2\right)\right]\right|$). This will be true when ρ_1 is sufficiently high such that investor 1's own information $\tilde{y}_1$ already provides a very accurate estimation about $\tilde{v}$. Formally, we can compute that

$$\begin{aligned}
\alpha_{y_1} &= \frac{\rho_1}{2\lambda(1+\rho_1+\rho_2)} > 0, \\
\alpha_{y_2} &= \frac{\rho_2(2+2\rho_2-\rho_1)}{6\lambda(1+\rho_1)(1+\rho_1+\rho_2)} < 0 \text{ iff } \rho_1 > \hat{\rho}_1 \equiv 2(1+\rho_2), \\
\beta_y &= \frac{\rho_2}{3\lambda(1+\rho_2)} > 0.
\end{aligned}$$

Thus, $\alpha_{y_2} < 0$ if and only if $\rho_1 > \hat{\rho}_1 \equiv 2(1+\rho_2)$. Note that the threshold $\hat{\rho}_1$ takes the same value as the threshold in Proposition 3, which determines whether investor 2 optimally shares his information in equilibrium. Taken together, similar to the baseline model, the less informed investor shares his information if and only if the more informed investor trades against the shared information.

4.2 Ex Post Information Sharing

In the baseline model, for tractability reasons, investors are assumed to make information-sharing decisions before observing their private information (see Figure 1). This assumption removes any signaling motives of the information sender. In this subsection, we consider an alternative setting in which, each investor decides whether or not to share information after observing the realization of their private information. All of our other assumptions remain unchanged from the baseline model.

We use D_i to denote investor i's information-sharing set of signal realizations, where $i \in \{H, L\}$. Take investor L as an example. When the realization of L's private signal $\tilde{y}$ belongs to set D_L, L truthfully shares $\tilde{y}$ with H; otherwise, L does not reveal any of his information. In principle, D_i can take any form. In our analysis, we focus only on the "corner" equilibrium in which D_i is either empty (i.e., investor i never shares information after observing the signal realization) or the entire real line (i.e., investor i always shares information after observing the signal realization). This is because if an equilibrium involves "interior" information-sharing sets (i.e., investor i shares information for some signal realizations but does not for other signal realizations), then the equilibrium linearity breaks down, which precludes analytical tractability.

In this ex post information sharing setting, we must also deal with off-equilibrium beliefs. We take the passive-belief approach that is commonly adopted in the signaling literature (e.g., McAfee and Schwartz, 1994). That is, upon observing a deviation from an investor's equilibrium information-sharing decision, other market participants do not update their beliefs regarding the distribution of the deviant investor's private signal. One justification is that they interpret the deviation as a tremble and assume that trembles are uncorrelated with the investor's information.

The following proposition summarizes the findings under ex post information sharing.

Proposition 4 (Ex post information sharing)**.** *In the economy in which investors make information-sharing decisions after observing the realizations of their private signals, the following statements hold:*

(1) *That neither L nor H shares information cannot be sustained in equilibrium (i.e., "someone must share information in equilibrium").*

(2) *That H shares his information cannot be sustained in equilibrium (i.e., "H never shares information in equilibrium").*

(3) *There exists an equilibrium in which L always fully shares information, whereas H never shares information (i.e., "the equilibrium in the baseline model with ex ante information sharing continues to be an equilibrium in this variation economy").*

Part (1) of Proposition 4 states that in the ex post information-sharing setting, if an equilibrium exists, there must be information transmitted between the two investors. That is, silence cannot be sustained in equilibrium. Parts (2) and (3) further state that relative to H, L has more incentives to share information. Thus, information transmits generically from L to H. Intuitively, as any information sharing by the more informed investor only dissipates his information advantage, H never shares information. By contrast, due to the trading-against-error effect, there always exists an equilibrium in which L shares information with H regardless of the signal realization. The mechanism remains the same as in the baseline model, namely, the trading-against-error effect.

4.3 H: "I Am Not Listening"

In the baseline model, H is overall worse off in the information-sharing economy. An important observation then is that if possible, H would commit not to listen to the information. For instance, investors choose not to register for the online investment community SumZero. Alternatively, they choose not to participate in the investment conferences, which have emerged as a hallmark event in the investment management industry (Luo, 2018). However, when there are multiple H investors, it is unclear whether they will commit because they do not fully internalize the negative effect of trading against the shared information.

To examine this issue, we extend the model with a number M of H-investors and allow them to simultaneously decide, at the beginning of date 0, whether to invest in a costless technology that credibly commits not to use any shared information at the trading stage. After the commitment investment decisions are made, L decides whether to share information with all H-investors. Other features of the model remain unchanged.

Although we cannot characterize all equilibria in this extended economy, we can identify two important findings about commitment adoption. First, when there are more than three H investors, it is not an equilibrium that every H investor adopts the commitment technology. Intuitively, when other H investors adopt the commitment technology, one H investor can deviate to using the information shared by L and enjoy its incremental value without fully accounting for its impact on L's trading and the market maker's price schedule. Second, for the same reason, we can show that when there are sufficiently many H investors, it is always an equilibrium in which each H does not adopt the commitment technology on date 0 and hence will use L's shared information on date 1. In addition, these H investors would be better off if they all could commit not to use the information shared by L. Taken together, there is essentially a coordination problem among the H investors that prevents the adoption of the commitment technology.

Proposition 5. *Suppose that there are a number M of Hs and one L and that each H simultaneously chooses whether to adopt a costless commitment technology. The following statements hold:*

(1) *When $M > 3$, there does not exist an equilibrium in which L shares information and every H commits not to use the shared information.*

(2) *There exists a constant $\hat{M} > 0$ such that when $M > \hat{M}$, there exists an equilibrium in which L shares information and all Hs use L's shared information. In addition, Hs' profits would be higher had they all committed not to use the shared information.*

4.4 Publicly Shared Information

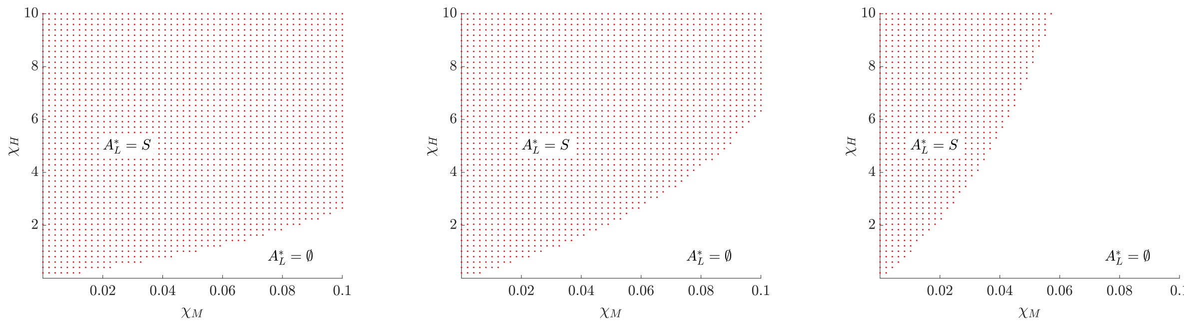
In the baseline model, information sharing can occur only between the two rational investors. This treatment captures the empirical setting of private communication among investors (sometimes called "market chatters" in practice; see Section 5.1 for more discussions). As mentioned in the Introduction, there are other realistic settings of information communication, such as social media communications, in which a large number of market participants observe the shared information. So it is important to ask what happens if the information is shared publicly. Since the market maker in our model can be interpreted as a reduced form to aggregate the general public, we now extend our baseline model to allow the market maker to observe the shared information.

Our analysis focuses on L's information-sharing behavior. To nest our baseline model, we assume that investors and the market maker have different capabilities in interpreting the same signal. We follow Myatt and Wallace (2012) and introduce "receiver noise" to the shared signal. Specifically, if investor L shares information $\tilde{y}$, investor H and the market maker then observe, respectively,

$$\tilde{q}_H = \tilde{y} + \tilde{\zeta}_H \text{ and } \tilde{q}_M = \tilde{y} + \tilde{\zeta}_M,$$

where $\tilde{\zeta}_H \sim N(0, \chi_H^{-1})$ and $\tilde{\zeta}_M \sim N(0, \chi_M^{-1})$ with $\chi_H \in [0, \infty]$ and $\chi_M \in [0, \infty]$. The random variables $\{\tilde{v}, \tilde{y}, \tilde{\zeta}_H, \tilde{\zeta}_M\}$ are mutually independent. The two constants χ_H and χ_M respectively capture the signal-interpretation capabilities of the H-investor and the market maker. The baseline model corresponds to $\chi_H = \infty$ (i.e., H can perfectly interpret L's shared information) and $\chi_M = 0$ (i.e., the market maker cannot process L's shared information). We use Figure 4 to characterize L's equilibrium information-sharing behavior. The shaded area indicates the region in which L shares information ($A_L^* = S$), and the blank area is the one in which L does not share ($A_L^* = \varnothing$).

We observe that information sharing by the coarsely informed investor remains a prevalent phenomenon in this extended economy. In particular, when H has a superior ability than the market maker in interpreting the shared information (i.e., low χ_M and high χ_H), L is willing to share information despite the potential information leakage to the market maker. The trading-against-error effect still drives the result; that is, by sharing private information and inviting H to trade against it, L has his order flow partially offset and obtains a better



(a) $\rho = 3$ (b) $\rho = 5$ (c) $\rho = 10$

This figure plots the regimes of L's equilibrium information-sharing behavior in the parameter space of (χ_M, χ_H) for different values of ρ in different panels. We set $\sigma_u = 1$. The shaded area indicates that L shares information in equilibrium (i.e., $A_L^* = S$), whereas the blank area indicates that L does not share information in equilibrium (i.e., $A_L^* = \varnothing$).

Figure 4: Publicly shared information

execution price. The condition of low χ_M and high χ_H is likely to hold in practice due to the specialization of different agents (the market maker focuses on making the market, whereas strategic investors specialize in collecting information to inform trading). In addition, Figure 4 shows that as L owns more precise information (ρ increases), he is less likely to share information. This is because, with less error in the endowed information, L benefits less from the trading-against-error effect in information sharing.

4.5 Other Extensions

In addition to the extensions and variations that are described above, we provide three additional extensions in the online appendix: (i) endogenous information acquisition by the L investor; (ii) multiple H investors and L investors; and (iii) three differentially informed investors. Overall, our results are robust in these extensions. Specifically, there exists endogenous information sharing by less informed investors in a wide range of parameter values, and the information-sharing incentives are driven by the more informed investor trading against the shared information.

We now summarize some new results in these three extensions. In the extension of endogenous information acquisition, we find that information sharing has an ambiguous effect on L's information acquisition. On the one hand, relative to the benchmark without information sharing, by sharing information L can make more profits and thus afford to acquire more information to inform his trading decisions. On the other hand, L's private information becomes more similar to H's after acquiring more precise information, which tends to depress L's information-acquisition incentives. In the extension of multiple coarsely informed L-investors and multiple well informed H-investors, the L-investors' information-sharing incentives are low when there are many L-investors and a few H-investors. In the extension with three differentially informed investors (one perfectly informed investor H and two coarsely informed investors L_1 and L_2), the least informed investor always has the strongest incentive to share information, because the least informed investor can gain the most from the trading-against-error effect.

5 Applications

We now use our analysis of the baseline model in Section 2 and the extension in Section 4.4 to shed light on private and public information communication among investors.

5.1 Market Chatters: Private Communication among Investors

In practice, market participants are known to swap information privately among themselves, and they are often called "market chatters" (see, e.g., Zaloom, 2003). More broadly, word-of-mouth communication is prevalent in transmitting information in financial markets (e.g., Shiller and Pound, 1989; Hong et al., 2005; Pool et al., 2015). Thanks to emerging technologies, professional investors also develop their own private websites, such as Value Investors Club and SumZero, to share information. Crawford et al. (2018) find that the recommendation posted on SumZero experiences a sustained reaction after being posted and triggers price drift in the direction of the recommendation, suggesting that the posts contain valuable private information. Our analysis of the baseline model in Section 2 directly speaks to the incentives and consequences of this kind of private information communication.

As we discussed in the Introduction, existing theories on information sharing do not ask the fundamental question of who shares information with whom. Our theory offers a novel perspective to understanding investors' information-sharing incentives and makes unique predictions about who shares information with whom. According to our theory, an investor with coarse information optimally chooses to share information with well informed investors and induces them to offset his own order flow. This mechanism differs dramatically from most existing theories, which predict either that information flows from informed to uninformed (e.g., Indjejikian et al., 2014; Ljungqvist and Qian, 2016) or that information is assumed to be exchanged among investors (e.g., Han and Yang, 2013; Chen et al., 2015).

There also exists some suggestive evidence that is consistent with our prediction. For instance, Crawford et al. (2017) find that predominantly small hedge fund managers share information in Value Investors Club. Given that the smaller funds have limited resources and access to information (Bhattacharya et al., 2018), this empirical finding squares with our key prediction that coarsely informed investors are more likely to share their information. Moreover, some empirical findings are broadly aligned with our key mechanism, namely, the trading-against-error effect. Specifically, Cowgill and Zitzewitz (2015) find that Google's prediction markets exhibit an optimism bias and more experienced traders trade against this identified inefficiency, which has the same flavor as the trading-against-error effect.

5.2 Public Communication on Social Media

Social media is landscape-shifting, with its relevance in financial markets only growing (SEC, 2012). While there is increasing interest in analyzing and utilizing investment opinions expressed on social media, the evidence regarding their usefulness is mixed. For instance, Antweiler and Frank (2004) and Das and Chen (2007) find that the volume of messages on message boards, such as Yahoo! or Raging Bull, is associated with stock return volatility. However, they do not detect a strong relationship between opinions transmitted through social media and stock returns. In fact, one common view is that due to their openness and lack of regulation, social media outlets provide uninformed actors an avenue to easily spread erroneous information among market participants (see Frieder and Zittrain (2007) and Hanke and Hauser (2008) for related evidence). By contrast, more recently, Chen et al. (2014) find that the views expressed in Seeking Alpha articles and commentaries predict stock returns over the ensuing three months and earnings surprises. Jame et al. (2016) show that crowdsourced earnings forecasts on the Estimize platform provide incrementally value-relevant information to predict earnings. Bartov et al. (2017) document that tweets just before a firm's earnings announcement predict its earnings and announcement returns.

As commented by Antweiler and Frank (2004), in order to understand whether social media posts contain information or are pure noise, one needs a theory to understand why market participants post messages on social media outlets. Our extension in Section 4.4 offers such a theory and weighs in on the empirical debate. Our theory predicts that the investment opinions on social media do contain fundamental information, but at the same time, the information must be very noisy. Specifically, each investor who shares their investment opinions on social media is represented by the coarsely informed investor L in our model, whereas the sophisticated investor who extracts investment signals from social media (such as a hedge fund that actively analyzes tweets or r/wallstreetbets) is the well informed investor H. As such, the opinions expressed on social media can be seen as being transmitted from the coarsely informed investors to the well informed ones.

Our model also sheds new light on the increasingly popular trading strategies based on the sentiment extracted from social media. For example, a growing number of hedge funds are buying the data feeds from Dataminr, which applies advanced analytics to the entire Twitter "fire hose" to detect events likely to move the market.[8] Our theory suggests that for investors who are not well informed, such sentiment data might better inform their trading decisions and increase trading profits. However, suppose the investors have already been well informed about the fundamental of a firm, an industry, or the economy. In that case, a

[8] "How investors are using social media to make money," December 7, 2015, *Fortune*.

subscription to the social media data feeds and trade on them can backfire because market liquidity may worsen in response to trading based on social media posts (see Propositions 2 and 5).

6 Conclusion

Information sharing is prevalent in financial markets. In this paper, we propose a theory to examine the incentives and consequences of information sharing by investors. The existing theories do not ask the question of who shares information with whom and why, and our theory fills this void. Our theory predicts that an investor with coarse information optimally chooses to share information with a well informed investor, because the former benefits from the latter trading against the shared information, which therefore dampens the former's price impact. By contrast, the well informed investor never shares information because doing so only dissipates his informational advantage and erodes his profit accordingly.

Relative to the economy without information sharing, the less informed investor makes more profits as he is less concerned about his price impact and trades more aggressively. The more informed investor paradoxically earns fewer profits despite seeing more information because he trades less aggressively on his own information and the market maker steepens the price schedule. Market efficiency improves, market liquidity worsens, and depending on the quality of the less informed investor's information, total trading volume can be higher or lower. Overall, our model offers a novel and complementary explanation for why investors share information in financial markets. The equilibrium exhibits the unique feature that information transmits from the less informed investor to the more informed investor.

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Appendix: Proofs

Proof of Lemma 1

See the main text.

Proof of Lemma 2

If only L shares information, the two investors' information sets are $\mathcal{F}_L = \{\tilde{y}\}$ and $\mathcal{F}_H = \{\tilde{v}, \tilde{y}\}$. We conjecture that their respective trading strategies are $\tilde{x}_L = \beta_y \tilde{y}$ and $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_y \tilde{y}$. L's conditional expected profit is $E[\tilde{x}_L(\tilde{v} - \tilde{p})|\tilde{y}] = \tilde{x}_L \left[\frac{\rho}{1+\rho}\tilde{y} - \lambda(\tilde{x}_L + \alpha_v \frac{\rho}{1+\rho}\tilde{y} + \alpha_y \tilde{y})\right]$. Maximizing the expected profit yields L's optimal trading rule $\tilde{x}_L = \beta_y \tilde{y}$, where

$$\beta_y = \frac{[1 - \lambda(\alpha_v + \alpha_y)]\rho - \lambda\alpha_y}{2\lambda(1+\rho)}. \tag{A1}$$

H's conditional profit is $E[\tilde{x}_H(\tilde{v} - \tilde{p})|\tilde{v}] = \tilde{x}_H [\tilde{v} - \lambda(\tilde{x}_H + \beta_y \tilde{y})]$. Maximizing the expected profit yields H's optimal trading rule $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_y \tilde{y}$, where

$$\alpha_v = \frac{1}{2\lambda} \text{ and } \alpha_y = -\frac{\beta_y}{2}. \tag{A2}$$

The market maker's pricing rule is

$$\lambda = \frac{\alpha_v + \alpha_y + \beta_y}{(\alpha_v + \alpha_v + \beta_y)^2 + (\beta_y + \alpha_y)^2 \frac{1}{\rho} + \sigma_u^2}. \tag{A3}$$

Using equations (A1), (A2), and (A3) we can characterize investors' optimal trading rules in (7) and the pricing rule in (8). Further, inserting (7) and (8) into the two investors' profits and taking expectations, we obtain the two investors' unconditional expected profits in (9).

Proof of Lemma 3

When H shares information $\tilde{v}$, L will not use his endowed information $\tilde{y}$ to predict the asset fundamental any more. Therefore, regardless of L's sharing decision, the signal $\tilde{y}$ is of no use to H either. The trading game thus degenerates to the classical Kyle (1985) setting with two perfectly informed traders, and we can easily obtain the equilibrium outcomes as stated in Lemma 3.

Proof of Proposition 1

Based on Lemmas 1–3,

$$g_1(\rho) \equiv \pi_H^{\varnothing\varnothing} - \pi_H^{\cdot S} = \frac{\sigma_u}{6}\left(\frac{6(2+\rho)^2}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} - \sqrt{2}\right) > 0,$$

where the inequality follows $g_1'(\rho) = -\frac{(\rho+2)\left(17\rho^2+38\rho+24\right)\sigma_u}{2(3\rho+4)^2(2\rho^2+5\rho+4)^{3/2}} < 0$ and $g_1(+\infty) = 0$. Further,

$$g_2(\rho) \equiv \pi_H^{S\varnothing} - \pi_H^{\cdot S} = \frac{\sigma_u}{6}\left(\frac{9+4\rho}{\sqrt{9+17\rho+8\rho^2}} - \sqrt{2}\right) > 0,$$

where the inequality holds because $g_2'(\rho) = -\frac{(76\rho+81)\sigma_u}{12(\rho+1)^{3/2}(8\rho+9)^{3/2}} < 0$ and $g_2(+\infty) = 0$. Therefore, regardless of L's sharing decision, H's dominant strategy is not to share information. Next, given that H does not share information, L always wants to share information because

$$f(\rho) \equiv \frac{\pi_L^{S\varnothing}}{\pi_L^{\varnothing\varnothing}} = \frac{2(4+3\rho)\sqrt{4+5\rho+2\rho^2}}{3(1+\rho)\sqrt{(1+\rho)(9+8\rho)}} > 1,$$

where the inequality follows because $f'(\rho) = \frac{-\rho\left(50\rho^2+224\rho+335\right)-164}{3(\rho+1)^{5/2}(8\rho+9)^{3/2}\sqrt{\rho(2\rho+5)+4}} < 0$ and $f(+\infty) = 1$. Therefore, the unique equilibrium is that L shares information but H does not.

Proof of Proposition 2

We have shown that $\pi_L^{S\varnothing} > \pi_L^{\varnothing\varnothing}$ in the proof of Proposition 1. According to Lemmas 1–2, we know that

$$\pi_H^{S\varnothing} - \pi_H^{\varnothing\varnothing} = \frac{1}{6}\sigma_u\left(\frac{9+4\rho}{\sqrt{9+17\rho+8\rho^2}} - \frac{6(2+\rho)^2}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}\right) < 0,$$

where the inequality follows because

$$\begin{aligned}\left((9+4\rho)(4+3\rho)\sqrt{4+5\rho+2\rho^2}\right)^2 - \left(6(2+\rho)^2\sqrt{9+17\rho+8\rho^2}\right)^2 \\ = -\rho\left(1296+3044\rho+2611\rho^2+970\rho^3+132\rho^4\right) < 0.\end{aligned}$$

Further,

$$\left(\pi_L^{S\varnothing}+\pi_H^{S\varnothing}\right)-\left(\pi_L^{\varnothing\varnothing}+\pi_H^{\varnothing\varnothing}\right)=\sigma_u\left(\frac{\sqrt{9+8\rho}}{\sqrt{1+\rho}}-\frac{6\sqrt{4+5\rho+2\rho^2}}{4+3\rho}\right)>0,$$

where the inequality follows because $(9+8\rho)(4+3\rho)^2-36(1+\rho)(4+5\rho+2\rho^2)=\rho(20+21\rho)>0$. This completes the proof of Part (1) of Proposition 2.

For market liquidity,

$$\lambda^{S\varnothing}-\lambda^{\varnothing\varnothing}=\frac{1}{6\sigma_u}\left(\frac{\sqrt{9+8\rho}}{\sqrt{1+\rho}}-\frac{6\sqrt{4+5\rho+2\rho^2}}{4+3\rho}\right)>0,$$

where the inequality holds because $(4+3\rho)^2(\sqrt{9+8\rho})-36(1+\rho)(4+5\rho+2\rho^2)=\rho(20+21\rho)>0$. For market efficiency, in the benchmark economy without information sharing, we compute $m^{\varnothing\varnothing}=\frac{4+3\rho}{2+\rho}$, and in our economy with endogenous information sharing, we compute $m^{S\varnothing}=\frac{6(1+\rho)}{3+2\rho}$. A direct comparison yields $m^{S\varnothing}>m^{\varnothing\varnothing}$.

Finally, we discuss trading volume. The trading volume of H in the benchmark economy and that in the economy with endogenous information sharing are respectively $TV_H^{\varnothing\varnothing}=\sqrt{\frac{2}{\pi}}\frac{(2+\rho)\sigma_u}{\sqrt{4+5\rho+2\rho^2}}$ and $TV_H^{S\varnothing}=\sqrt{\frac{2}{\pi}}\frac{\sigma_u\sqrt{9+4\rho}}{\sqrt{9+8\rho}}$, and it can be shown that

$$TV_H^{S\varnothing}-TV_H^{\varnothing\varnothing}=\sqrt{\frac{\pi}{2}}\sigma_u\left(\frac{\sqrt{9+4\rho}}{\sqrt{9+8\rho}}-\frac{2+\rho}{\sqrt{4+5\rho+2\rho^2}}\right)<0,$$

where the inequality follows because $(9+4\rho)(4+5\rho+2\rho^2)-(2+\rho)^2(9+8\rho)=-\rho(7+3\rho)<0$. Similarly, the trading volume of L in the benchmark economy and that in the economy with endogenous information sharing are respectively $TV_L^{\varnothing\varnothing}=\sqrt{\frac{2}{\pi}}\frac{\sqrt{\rho(1+\rho)}\sigma_u}{\sqrt{4+5\rho+2\rho^2}}$ and $TV_L^{S\varnothing}=\sqrt{\frac{2}{\pi}}\frac{\sqrt{2\rho}\sigma_u}{\sqrt{9+8\rho}}$, and

$$TV_L^{S\varnothing}-TV_L^{\varnothing\varnothing}=\sqrt{\frac{2}{\pi}}\sigma_u\left(2\frac{\sqrt{\rho}}{\sqrt{9+8\rho}}-\frac{\sqrt{\rho(1+\rho)}}{\sqrt{4+5\rho+2\rho^2}}\right)>0,$$

where the inequality follows because $4\rho(4+5\rho+2\rho^2)-\rho(1+\rho)(9+8\rho)=\rho(7+3\rho)>0$. Further, the trading volume of the market maker in the benchmark economy and that in the economy with endogenous information sharing are respectively $TV_M^{\varnothing\varnothing}=2\frac{\sqrt{(1+\rho)(4+3\rho)}\sigma_u}{\sqrt{\pi}\sqrt{4+5\rho+2\rho^2}}$ and

$TV_M^{S\varnothing} = \frac{2\sqrt{3(3+4\rho)}\sigma_u}{\sqrt{\pi}\sqrt{9+8\rho}}$, and

$$TV_M^{S\varnothing} - TV_M^{\varnothing\varnothing} = \frac{2}{\sqrt{\pi}}\left(\frac{\sqrt{3(3+4\rho)}}{\sqrt{9+8\rho}} - \frac{\sqrt{4+7\rho+3\rho^2}}{4+5\rho+2\rho^2}\right) < 0,$$

where the inequality follows because $3(3+4\rho)(4+5\rho+2\rho^2) - (4+7\rho+3\rho^2)(9+8\rho) = -\rho(2+5\rho) < 0$. Last, for total trading volume,

$$g_3(\rho) \equiv TV^{S\varnothing} - TV^{\varnothing\varnothing} = \frac{2\sqrt{\rho} + \sqrt{6(4+3\rho)} + \sqrt{9+4\rho}}{2\sqrt{9+8\rho}} - \frac{\sqrt{\rho(1+\rho)} + 2 + \rho + \sqrt{8+14\rho+6\rho^2}}{2\sqrt{4+5\rho+2\rho^2}}.$$

It is easy to show that when $\rho \to 0$, $g_3(\rho) > 0$ and $g_3(\rho) \to 0$. And when $\rho \to \infty$, $g_3(\rho) < 0$ and $g_3(\rho) \to 0$.

Proof of Proposition 3

As in the baseline model, we discuss the following four subgames. We use the superscript A_1A_2 to indicate different subgames, where $A_i \in \{S, \varnothing\}$ and $i \in \{1, 2\}$.
Subgame 1: Both Investors Share Information. When both investors share their information, they have the same information set: $\mathcal{F}_1 = \mathcal{F}_2 = \{\tilde{y}_1, \tilde{y}_2\}$, and their trading strategies are $\tilde{x}_1 = \alpha_1\tilde{y}_1 + \alpha_2\tilde{y}_2$ and $\tilde{x}_2 = \beta_1\tilde{y}_1 + \beta_2\tilde{y}_2$. For investor 1, the conditional expected profit is

$$E[\tilde{x}_1(\tilde{v} - \tilde{p})|\tilde{y}_1, \tilde{y}_2] = \tilde{x}_1\left(\frac{\rho_1\tilde{y}_1 + \rho_2\tilde{y}_2}{1+\rho_1+\rho_2} - \lambda(\tilde{x}_1 + \beta_1\tilde{y}_1 + \beta_2\tilde{y}_2)\right). \tag{A4}$$

Maximizing the profit yields the optimal trading rule $\tilde{x}_1 = \alpha_1\tilde{y}_1 + \alpha_2\tilde{y}_2$, where

$$\alpha_1 = \frac{1}{2}\left(\frac{\rho_1}{\lambda(1+\rho_1+\rho_2)} - \beta_1\right), \ \alpha_2 = \frac{1}{2}\left(\frac{\rho_2}{\lambda(1+\rho_1+\rho_2)} - \beta_2\right). \tag{A5}$$

Similarly, investor 2's optimal trading rule can be computed as $\tilde{x}_2 = \beta_1\tilde{y}_1 + \beta_2\tilde{y}_2$, where

$$\beta_1 = \frac{1}{2}\left(\frac{\rho_1}{\lambda(1+\rho_1+\rho_2)} - \alpha_1\right), \ \beta_2 = \frac{1}{2}\left(\frac{\rho_2}{\lambda(1+\rho_1+\rho_2)} - \alpha_2\right). \tag{A6}$$

The market maker sets pricing rule $\lambda = \frac{\alpha_1+\alpha_2+\beta_1+\beta_2}{(\alpha_1+\alpha_2+\beta_1+\beta_2)^2+(\alpha_1+\beta_1)^2/\rho_1+(\alpha_2+\beta_2)^2/\rho_2+\sigma_u^2}$. Together with equations (A5) and (A6), we solve the market maker's pricing rule as

$$\lambda^{SS} = \frac{\sqrt{2(\rho_1+\rho_2)}}{3\sigma_u\sqrt{1+\rho_1+\rho_2}}, \tag{A7}$$

and investors' trading rules are determined accordingly:

$$\alpha_1^{SS} = \beta_1^{SS} = \frac{\rho_1}{3\lambda^{SS}(1+\rho_1+\rho_2)} \text{ and } \alpha_2^{SS} = \beta_2^{SS} = \frac{\rho_2}{3\lambda^{SS}(1+\rho_1+\rho_2)}. \tag{A8}$$

Inserting (A7) and (A8) into investor 1's profit (A4) and taking expectation yields investor 1's unconditional expected profit, and similarly, we can derive investor 2's profit as well:

$$\pi_1^{SS} = \pi_2^{SS} = \frac{\sigma_u\sqrt{\rho_1+\rho_2}}{3\sqrt{2(1+\rho_1+\rho_2)}}. \tag{A9}$$

Subgame 2: Neither Investor Shares Information. When neither investor shares information, the two investors' information sets are $\mathcal{F}_1 = \{\tilde{y}_1\}$ and $\mathcal{F}_2 = \{\tilde{y}_2\}$, and their trading strategies are $\tilde{x}_1 = \alpha_1\tilde{y}_1$ and $\tilde{x}_2 = \beta_2\tilde{y}_2$. Following a similar derivation as in Subgame 1, we derive the market maker's optimal pricing rule

$$\lambda^{\varnothing\varnothing} = \frac{\sqrt{(2\rho_2^2+5\rho_2+4)\,\rho_1^2 + (5\rho_2^2+8\rho_2+4)\,\rho_1 + 4\rho_2\,(\rho_2+1)}}{(4\,(\rho_2+1)+\rho_1\,(3\rho_2+4))\,\sigma_u},$$

and the two investors' optimal trading rules:

$$\alpha_1^{\varnothing\varnothing} = \frac{\rho_1\,(\rho_2+2)}{\lambda^{\varnothing\varnothing}\,(4\,(\rho_2+1)+\rho_1\,(3\rho_2+4))} \text{ and } \beta_2^{\varnothing\varnothing} = \frac{(\rho_1+2)\,\rho_2}{\lambda^{\varnothing\varnothing}\,(4\,(\rho_2+1)+\rho_1\,(3\rho_2+4))}.$$

And the two investors' unconditional expected trading profits are as follows:

$$\pi_1^{\varnothing\varnothing} = \frac{\rho_1(1+\rho_1)(2+\rho_2)^2\sigma_u}{(4+4(\rho_1+\rho_2)+3\rho_1\rho_2)\sqrt{2\rho_1^2\rho_2^2+(\rho_1+\rho_2)\,(4+4(\rho_1+\rho_2)+5\rho_1\rho_2)}}, \tag{A10}$$

$$\pi_2^{\varnothing\varnothing} = \frac{\rho_2(1+\rho_2)(2+\rho_1)^2\sigma_u}{(4+4(\rho_1+\rho_2)+3\rho_1\rho_2)\sqrt{2\rho_1^2\rho_2^2+(\rho_1+\rho_2)\,(4+4(\rho_1+\rho_2)+5\rho_1\rho_2)}}. \tag{A11}$$

Subgame 3: Only Investor 1 Shares Information. Investor 1's information set is $\mathcal{F}_1 = \{\tilde{y}_1\}$ and trading strategy is $\tilde{x}_1 = \alpha_1\tilde{y}_1$. Investor 2's information set is $\mathcal{F}_2 = \{\tilde{y}_1, \tilde{y}_2\}$ and trading strategy is $\tilde{x}_2 = \beta_1\tilde{y}_1 + \beta_2\tilde{y}_2$. Again, following the similar derivation as in previous subgames, we compute the market maker's optimal trading rule

$$\lambda^{S\varnothing} = \frac{\sqrt{8\rho_1^2+8\,(\rho_2+1)\,\rho_1+9\rho_2}}{6\sqrt{(\rho_1+1)\,(\rho_1+\rho_2+1)}\sigma_u},$$

and the two investors' optimal trading rules:

$$\alpha_1^{S\varnothing} = \frac{\rho_1}{3\lambda^{S\varnothing}(\rho_1+1)},\ \beta_1^{S\varnothing} = \frac{\rho_1(2\rho_1-\rho_2+2)}{6\lambda^{S\varnothing}(\rho_1+1)(\rho_1+\rho_2+1)},\ \text{and}\ \beta_2^{S\varnothing} = \frac{\rho_2}{2\lambda^{S\varnothing}(\rho_1+\rho_2+1)}.$$

And the two investors' conditional profits are as follows:

$$\pi_1^{S\varnothing} = \frac{2\sigma_u\rho_1\sqrt{(1+\rho_1)(1+\rho_1+\rho_2)}}{3(1+\rho_1)\sqrt{8\rho_1^2+8\rho_1(1+\rho_2)+9\rho_2}}, \tag{A12}$$

$$\pi_2^{S\varnothing} = \frac{(4\rho_1^2+9\rho_2+4\rho_1(1+\rho_2))\,\sigma_u}{6\sqrt{(1+\rho_1)(1+\rho_1+\rho_2)}\sqrt{8\rho_1^2+8\rho_1(1+\rho_2)+9\rho_2}}. \tag{A13}$$

Subgame 4: Only Investor 2 Shares Information. Investor 1's information set is $\mathcal{F}_1 = \{\tilde{y}_1, \tilde{y}_2\}$ and trading strategy is $\tilde{x}_1 = \alpha_1\tilde{y}_1 + \alpha_2\tilde{y}_2$. Investor 2's information set is $\mathcal{F}_2 = \{\tilde{y}_2\}$ and trading strategy is $\tilde{x}_2 = \beta_2\tilde{y}_2$. Similarly, we can derive the market maker's optimal trading rule:

$$\lambda^{\varnothing S} = \frac{\sqrt{8\rho_2(\rho_2+1)+\rho_1(8\rho_2+9)}}{6\sqrt{(\rho_2+1)(\rho_1+\rho_2+1)}\sigma_u},$$

and the two investors' optimal trading rules:

$$\alpha_1^{\varnothing S} = \frac{\rho_1}{2\lambda^{\varnothing S}(\rho_1+\rho_2+1)}, \alpha_2^{\varnothing S} = \frac{\rho_2(-\rho_1+2\rho_2+2)}{6\lambda^{\varnothing S}(\rho_2+1)(\rho_1+\rho_2+1)},\ \text{and}\ \beta_2^{\varnothing S} = \frac{\rho_2}{3\lambda^{\varnothing S}(\rho_2+1)}.$$

The two investors' profits are as follows:

$$\pi_1^{\varnothing S} = \frac{(4\rho_2^2+9\rho_1+4\rho_2(1+\rho_1))\,\sigma_u}{6\sqrt{(1+\rho_2)(1+\rho_1+\rho_2)}\sqrt{8\rho_2^2+8\rho_2(1+\rho_1)+9\rho_1}}, \tag{A14}$$

$$\pi_2^{\varnothing S} = \frac{2\sigma_u\rho_2\sqrt{(1+\rho_2)(1+\rho_1+\rho_2)}}{3(1+\rho_2)\sqrt{8\rho_2^2+8\rho_2(1+\rho_1)+9\rho_1}}. \tag{A15}$$

We know that given that investor 2 shares information, investor 1 will not do so because

$$\pi_1^{SS} - \pi_1^{\varnothing S} = \frac{\sigma_u}{6\sqrt{1+\rho_1+\rho_2}}\left(\sqrt{2(\rho_1+\rho_2)} - \frac{4\rho_2(1+\rho_2)+\rho_1(9+4\rho_2)}{\sqrt{(1+\rho_2)}\sqrt{8\rho_2(1+\rho_2)+\rho_1(9+8\rho_2)}}\right) < 0,$$

where the inequality follows because

$$\begin{aligned}&2(\rho_1+\rho_2)(1+\rho_2)(8\rho_2(1+\rho_2)+\rho_1(9+8\rho_2)) - (4\rho_2(1+\rho_2)+\rho_1(9+4\rho_2))^2 \\ &\qquad = -\rho_1(38\rho_2(1+\rho_2)+\rho_1(63+38\rho_2)) < 0.\end{aligned}$$

Similarly, we know that $\pi_2^{SS} < \pi_2^{S\varnothing}$; that is, if investor 1 shares information, investor 2 will not do so. Taken together, in equilibrium at most one investor shares information.

Furthermore,

$$\begin{aligned}
\pi_1^{S\varnothing} - \pi_1^{\varnothing\varnothing} &= \rho_1\sigma_u\sqrt{1+\rho_1} \times \Bigg(\frac{2\sqrt{1+\rho_1+\rho_2}}{3(1+\rho_1)\sqrt{8\rho_1^2+9\rho_2+8\rho_1(1+\rho_2)}} - \\
&\quad \frac{\sqrt{1+\rho_1}(2+\rho_2)^2}{(4(1+\rho_2)+\rho_1(4+3\rho_2))\sqrt{4\rho_2(1+\rho_2)+\rho_1^2(4+5\rho_2+2\rho_2^2)+\rho_1(4+8\rho_2+5\rho_2^2)}}\Bigg) \\
&= \frac{\rho_1\sigma_u\sqrt{1+\rho_1}}{\left(\begin{pmatrix} 3(1+\rho_1)(4(1+\rho_2)+\rho_1(4+3\rho_2))\sqrt{8\rho_1^2+9\rho_2+8\rho_1(1+\rho_2)} \\ \times\sqrt{4\rho_2(1+\rho_2)+\rho_1^2(4+5\rho_2+2\rho_2^2)+\rho_1(4+8\rho_2+5\rho_2^2)} \end{pmatrix} \times \begin{pmatrix} 2\sqrt{1+\rho_1+\rho_2}(4(1+\rho_2)+\rho_1(4+3\rho_2)) \\ \times\sqrt{4\rho_2(1+\rho_2)+\rho_1^2(4+5\rho_2+2\rho_2^2)+\rho_1(4+8\rho_2+5\rho_2^2)} \\ +3(1+\rho_1)^{3/2}(2+\rho_2)^2\sqrt{8\rho_1^2+9\rho_2+8\rho_1(1+\rho_2)} \end{pmatrix}\right)} \\
&\quad \times (\rho_2 - 2(1+\rho_1)) \times \begin{pmatrix} 2\left(51\rho_2^3+244\rho_2^2+400\rho_2+224\right)\rho_1^4 \\ +\left(75\rho_2^4+748\rho_2^3+2296\rho_2^2+2912\rho_2+1344\right)\rho_1^3 \\ +\left(293\rho_2^4+1852\rho_2^3+4144\rho_2^2+3944\rho_2+1344\right)\rho_1^2 \\ +\left(389\rho_2^4+1928\rho_2^3+3380\rho_2^2+2352\rho_2+448\right)\rho_1 \\ +\rho_2\left(175\rho_2^3+726\rho_2^2+1044\rho_2+520\right) \end{pmatrix},
\end{aligned}$$

which is positively related to $\rho_2 - 2(1+\rho_1)$. Similarly, $\pi_2^{\varnothing S} - \pi_2^{\varnothing\varnothing}$ is positively related to $\rho_1 - 2(1+\rho_2)$. Overall, we obtain the following equilibrium results:

(1) If $\rho_2 \geq 2(1+\rho_1)$, the unique equilibrium is that only investor 1 shares information;

(2) If $\rho_1 \geq 2(1+\rho_2)$, the unique equilibrium is that only investor 2 shares information;

(3) Otherwise, the unique equilibrium is that neither investor shares their information.

Proof of Proposition 4

We first prove Part (1) of this proposition. Upon the realizations of H's and L's private signals: $v = \tilde{v}$ and $y = \tilde{y}$, suppose that neither investor shares information. Following the proof of Lemma 1, we can show the two investors' conditional expected trading profits as follows:

$$E[\tilde{x}_L^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y] = \frac{\rho^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}y^2,$$
$$E[\tilde{x}_H^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v] = \frac{(2+\rho)^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}v^2.$$

Now suppose that L deviates and shares his observation of y. Then (i) H updates his belief and trades accordingly, and (ii) the market maker knows that L shares information (but not the specific realization) and adjusts the pricing rule accordingly. L's profit thus becomes

$$E[\tilde{x}_L^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y] = \frac{2\rho^2\sigma_u\sqrt{1+\rho}}{3(1+\rho)^2\sqrt{9+8\rho}}y^2.$$

Since $E[\tilde{x}_L^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y] > E[\tilde{x}_L^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y]$ for any realization of $\tilde{y}$, L always has the incentive to deviate from the conjectured equilibrium.

For Part (2) of the proposition, suppose that upon observing the realization of his private signal, H shares it. Then following the proof of Lemma 3, regardless of L's sharing decision, we can show H's conditional expected trading profits as follows:

$$E[\tilde{x}_H^{SS}(\tilde{v}-\tilde{p})|\tilde{v}=v] = E[\tilde{x}_H^{\varnothing S}(\tilde{v}-\tilde{p})|\tilde{v}=v] = \frac{\sigma_u}{3\sqrt{2}}v^2.$$

Now we study H's deviation in the following two cases. First, consider that in the conjectured equilibrium L does not share information. Since after H's deviation, neither the market maker nor L updates beliefs about the distribution of H's private signal, we can compute H's conditional expected trading profits after deviation as follows:

$$E[\tilde{x}_H^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v] = \frac{(2+\rho)^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}v^2.$$

Since $E[\tilde{x}_H^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v] > E[\tilde{x}_H^{\varnothing S}(\tilde{v}-\tilde{p})|\tilde{v}=v]$ for any realization of $\tilde{v}$, H has the incentive to deviate if L does not share information.

Second, consider that L also shares information in the conjectured equilibrium. Again, since after H's deviation, neither the market maker nor L updates beliefs about the distribution of H's private signal, we compute H's conditional expected profit after deviation as follows:

$$E[\tilde{x}_H^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v] = \frac{(\rho+(3+2\rho)^2v^2)\sigma_u\sqrt{1+\rho}}{6(1+\rho)^2\sqrt{9+8\rho}}.$$

Since $E[\tilde{x}_H^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v]-E[\tilde{x}_H^{SS}(\tilde{v}-\tilde{p})|\tilde{v}=v]=\frac{\sigma_u}{6}\left(\frac{\rho}{(1+\rho)^{3/2}\sqrt{9+8\rho}}+\left(\frac{(3+2\rho)^2}{(1+\rho)^{3/2}\sqrt{9+8\rho}}-\sqrt{2}\right)v^2\right)>0$ for any realization of $\tilde{v}$, H always has incentives to deviate and not share information. Taken together, regardless of L's sharing decision, H always has incentives to deviate. So the conjectured equilibrium involving H sharing information cannot be sustained.

We finally prove Part (3) of this proposition in the following two steps. Suppose that upon observing the realization of their private signals, L shares information whereas H does not. Then following the proof of Lemma 2, the two investors' conditional expected profits can be computed as follows:

$$E[\tilde{x}_L^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y]=\frac{2\rho^2\sigma_u\sqrt{1+\rho}}{3(1+\rho)^2\sqrt{9+8\rho}}y^2,$$
$$E[\tilde{x}_H^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v]=\frac{(\rho+(3+2\rho)^2v^2)\sigma_u\sqrt{1+\rho}}{6(1+\rho)^2\sqrt{9+8\rho}}.$$

First, given that L shares information, we show that H will not deviate from the no-sharing decision for any given realization of $\tilde{v}$. Suppose not and H deviates and shares his observation of v. Then (i) L updates his belief about the asset fundamental and trades on the shared information accordingly, and (ii) the market maker knows that H shares his information. H's profits then become:

$$E[\tilde{x}_H^{SS}(\tilde{v}-\tilde{p})|\tilde{v}=v]=\frac{\sigma_u}{3\sqrt{2}}v^2.$$

As shown in the second case in the proof of Part (2) of this proposition, since $E[\tilde{x}_H^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{v}=v]>E[\tilde{x}_H^{SS}(\tilde{v}-\tilde{p})|\tilde{v}=v]$ for any realization of $\tilde{v}$, H will not deviate from his no-sharing strategy. Second, given that H does not share his information, we will show that L will not deviate from sharing information for any realization of $\tilde{y}$. Suppose not and L deviates and chooses not to share his information. Then, (i) H holds passive beliefs along this off-equilibrium path, i.e., believing that L's private information $\tilde{y}$ still follows the original distribution; and (ii) the market maker observes that L does not share his information but instead holds passive beliefs (not updating beliefs about the distribution of L's private signal), so L's profits become

$$E[\tilde{x}_L^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y]=\frac{\rho^2\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}}y^2.$$

We can show that for any $\tilde{y}$, $E[\tilde{x}_L^{\varnothing\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y]<E[\tilde{x}_L^{S\varnothing}(\tilde{v}-\tilde{p})|\tilde{y}=y]$, so L will not deviate from sharing information. Taken together, it can be sustained as an equilibrium that L shares information whereas H never shares regardless of the realizations of their private signals.

Proof of Proposition 5

To prepare the proof, we first examine the trading equilibrium in which Hs do not share their information and L shares his information. In this way, we focus on Hs' reading behavior. Assume that among the H investors, M_1 of them choose to listen to the information shared by investor L. We consider the following symmetric linear trading equilibrium: the H investor who uses the shared information trades $\tilde{x}_i = \alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}$ units of the risky asset, where $i \in \{1, ..., M_1\}$; the H investor who commits not to use the shared information trades $\tilde{x}_k = \alpha_{v_2}\tilde{v}$ units of the risky asset, where $k \in \{M_1+1, ..., M\}$; L trades $\tilde{x}_L = \beta\tilde{y}$ units of the risky asset; and the market paper sets a linear pricing rule, $\tilde{p} = \lambda\tilde{\omega}$.

Consider H investor $i \in \{1, ..., M_1\}$ who uses the shared information. With the information set $\{\tilde{v}, \tilde{y}\}$, his conditional expected profit is as follows:

$$E[\tilde{x}_i(\tilde{v} - \tilde{p})|\tilde{v}, \tilde{y}] = \tilde{x}_i \left(\tilde{v} - \lambda \left(\tilde{x}_i + (M_1 - 1)(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{y}) + (M - M_1)\alpha_{v_2}\tilde{v} + \beta\tilde{y}\right)\right).$$

Maximizing profits yields the investor's optimal trading rule $\tilde{x}_i = \alpha_{v_i}\tilde{v} + \alpha_{L_i}\tilde{y}$ with

$$\alpha_{v_i} = \frac{1}{2\lambda}\left(1 - (M_1 - 1)\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right) \text{ and } \alpha_{L_i} = -\frac{1}{2}\left((M_1 - 1)\alpha_{L_1} + \beta\right). \quad \text{(A16)}$$

For H investor $k \in \{M_1 + 1, ..., M\}$ who commits not to use the shared information, with the information set $\{\tilde{v}\}$, his conditional expected profit is as follows:

$$E[\tilde{x}_k(\tilde{v} - \tilde{p})|\tilde{v}] = \tilde{x}_k \left(\tilde{v} - \lambda \left(\tilde{x}_k + M_1(\alpha_{v_1}\tilde{v} + \alpha_{L_1}\tilde{v}) + (M - M_1 - 1)\alpha_{v_2}\tilde{v} + \beta\tilde{v}\right)\right).$$

Maximizing the profit yields the investor's optimal trading rule $\tilde{x}_k = \alpha_{v_k}\tilde{v}$ with

$$\alpha_{v_k} = \frac{1}{2\lambda}\left(1 - \beta\lambda - M_1\lambda(\alpha_{L_1} + \alpha_{v_1}) - (M - M_1 - 1)\lambda\alpha_{v_2}\right). \quad \text{(A17)}$$

For investor L, his conditional expected trading profits are as follows:

$$E[\tilde{x}_L(\tilde{v} - \tilde{p})|\tilde{y}] = \tilde{x}_L \left(\frac{\rho}{1+\rho}\tilde{y} - \lambda\left(\tilde{x}_L + M_1\left(\alpha_{v_1}\frac{\rho}{1+\rho}\tilde{y} + \alpha_{L_1}\tilde{y}\right) + (M - M_1)\alpha_{v_2}\frac{\rho}{1+\rho}\tilde{y}\right)\right).$$

Maximizing investor L's profits yields the optimal trading strategy $\tilde{x}_L = \beta\tilde{y}$, with

$$\beta = -\frac{M_1}{2}\alpha_{L_1} + \frac{\rho}{2\lambda(1+\rho)}\left(1 - M_1\lambda\alpha_{v_1} - (M - M_1)\lambda\alpha_{v_2}\right). \quad \text{(A18)}$$

Imposing symmetric equilibrium $\alpha_{v_i} = \alpha_{v_1}$, $\alpha_{L_i} = \alpha_{L_1}$, and $\alpha_{v_k} = \alpha_{v_2}$, the interaction of the reaction functions (A16)–(A18) yields the optimal trading strategies as specified below:

$$\alpha_{v_1} = \frac{(2+M_1)(1+\rho)}{\lambda\left((1+M)(2+M_1)+(2+M)(1+M_1)\rho\right)}, \tag{A19}$$

$$\alpha_{L_1} = -\frac{\rho}{\lambda\left((1+M)(2+M_1)+(2+M)(1+M_1)\rho\right)}, \tag{A20}$$

$$\alpha_{v_2} = \frac{2+M_1+\rho+M_1\rho}{\lambda\left((1+M)(2+M_1)+(2+M)(1+M_1)\rho\right)}, \tag{A21}$$

$$\beta = \frac{(1+M_1)\rho}{\lambda\left((1+M)(2+M_1)+(2+M)(1+M_1)\rho\right)}. \tag{A22}$$

Using the weak efficiency rule, the market maker's optimal pricing rule is as follows:

$$\lambda = \frac{M_1(\alpha_{v_1}+\alpha_{L_1})+(M-M_1)\alpha_{v_2}+\beta}{(M_1(\alpha_{v_1}+\alpha_{L_1})+(M-M_1)\alpha_{v_2}+\beta)^2+(M_1\alpha_{L_1}+\beta)^2/\rho+\sigma_u^2}.$$

Inserting the optimal trading strategies into λ we can derive the equilibrium pricing rule $\tilde{p} = \lambda\tilde{\omega}$ with

$$\lambda = \frac{\sqrt{M(2+M_1+\rho+M_1\rho)^2+\rho+\rho^2+M_1\rho(3+M_1+(2+M_1)\rho)}}{\left((1+M)(2+M_1)+(2+M)(1+M_1)\rho\right)\sigma_u}. \tag{A23}$$

Now, inserting the optimal trading rules (A19)–(A22) and the optimal pricing rule (A23) into the investors' expected trading profits and taking expectations yields their respective unconditional profits as follows:

$$\pi_i(M_1,M) = (\rho+1)\left(M_1^2(\rho+1)+2M_1(\rho+2)+\rho+4\right)\sigma_u\Gamma^{-1}, \tag{A24}$$

$$\pi_k(M_1,M) = (M_1\rho+M_1+\rho+2)^2\sigma_u\Gamma^{-1}, \tag{A25}$$

$$\pi_L(M_1,M) = (M_1+1)^2\rho(\rho+1)\sigma_u\Gamma^{-1}, \tag{A26}$$

where

$$\Gamma = ((M+2)(M_1+1)\rho+(M+1)(M_1+2)) \\ \times\sqrt{M(M_1\rho+M_1+\rho+2)^2+\rho(M_1((M_1+2)\rho+M_1+3)+\rho+1)}.$$

Proof of Part (1) We discuss if $M_1 = 0$ (that all Hs commit not to use L's shared information) is an equilibrium. When $M_1 = 0$, based on equations (A25), the profits of Hs are as follows:

$$\pi_k(0,M) = \frac{(2+\rho)^2\sigma_u}{(2+2\rho+M(2+\rho))\sqrt{M(3+2\rho)^2+\rho(5+4\rho)}}. \tag{A27}$$

If one H deviates and uses the shared information, according to equation (A24), his profits will become

$$\pi_i(1,M) = \frac{(1+\rho)(9+4\rho)\sigma_u}{(3+4\rho+M(3+2\rho))\sqrt{M(3+2\rho)^2+\rho(5+4\rho)}}.$$

Therefore, $M_1 = 0$ is not an equilibrium if $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$. Further, we know that

$$\frac{\partial}{\partial M}\left(\frac{\pi_i(1,M)}{\pi_k(0,M)}\right) = \frac{\rho\pi_i(1,M)}{\pi_k(0,M)} \frac{\begin{pmatrix} M^2\left(18\rho^4+121\rho^3+303\rho^2+335\rho+138\right) \\ +M\left(56\rho^4+299\rho^3+577\rho^2+469\rho+132\right) \\ +2(\rho+1)^2\left(24\rho^2+56\rho+33\right)\end{pmatrix}}{\begin{pmatrix} 2(M(\rho+2)+2(\rho+1))\left(M(\rho+2)^2+\rho(\rho+1)\right) \\ \times(M(2\rho+3)+4\rho+3)\left(M(2\rho+3)^2+\rho(4\rho+5)\right)\end{pmatrix}} > 0$$

and when $M = 4$,

$$\frac{\pi_i(1,4)}{\pi_k(0,4)} = \frac{2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}}{3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}} > 1,$$

where the inequality holds because

$$\begin{aligned}&\left(2(\rho+1)(3\rho+5)(4\rho+9)\sqrt{5\rho^2+17\rho+16}\right)^2 - \left(3(\rho+2)^2(4\rho+5)\sqrt{20\rho^2+53\rho+36}\right)^2 \\ &\quad = \rho\left(240\rho^6+2744\rho^5+13071\rho^4+33004\rho^3+46336\rho^2+34132\rho+10260\right) > 0.\end{aligned}$$

Therefore, if $M > 3$, we must have $\frac{\pi_i(1,M)}{\pi_k(0,M)} > 1$; that is, $M_1 = 0$ cannot be sustained in equilibrium.

Proof of Part (2) To prove Part (2) of the proposition, we first characterize the condition under which that L shares information and all H-investors use the shared information can be sustained in equilibrium. We then prove that in this equilibrium Hs' profits would be higher had they all committed not to use the shared information.

First, we discuss when $M_1 = M$ (that all Hs use L's shared information) can be an equilibrium. When $M_1 = M$, based on equation (A24), the profits of Hs are as follows:

$$\pi_i(M,M) = \frac{\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)\sigma_u}{(M^2+3M+2)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}. \tag{A28}$$

If one H deviates and chooses not to use the shared information, according to equation (A25), his profits become

$$\pi_k(M-1,M) = \frac{(M\rho+M+1)^2\sigma_u}{\left(M^2(\rho+1)+2M(\rho+1)+1\right)\sqrt{M^3(\rho+1)^2+M^2\left(\rho^2+3\rho+2\right)+M(\rho+1)-\rho}}.$$

Therefore, $M_1 = M$ is an equilibrium if $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$. Further,

$$\begin{aligned}\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) &= \frac{\rho\pi_k(M-1,M)}{\pi_i(M,M)} \\ &\times\frac{\begin{pmatrix} 4M^{10}(\rho+1)^4+M^9(\rho+1)^3(23\rho+31)+M^8(\rho+1)^2\left(29\rho^2+89\rho+64\right) \\ -M^7(\rho+1)^2\left(97\rho^2+244\rho+143\right)-M^6\left(397\rho^4+2092\rho^3+4005\rho^2+3326\rho+1016\right) \\ -M^5\left(624\rho^4+3715\rho^3+7907\rho^2+7213\rho+2397\right)-M^4\left(532\rho^4+3590\rho^3+8551\rho^2+8651\rho+3166\right) \\ -M^3\left(258\rho^4+1966\rho^3+5321\rho^2+6128\rho+2551\right)-M^2\left(68\rho^4+562\rho^3+1785\rho^2+2484\rho+1246\right) \\ -M\left(8\rho^4+62\rho^3+260\rho^2+523\rho+340\right)-2\left(3\rho^2+25\rho+20\right)\end{pmatrix}}{\begin{pmatrix} 2\left(M^2+3M+2\right)(M\rho+M+1)\times\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right) \\ \times\left(M^3(\rho+1)+M^2(3\rho+4)+M(3\rho+4)+\rho\right) \\ \times\left(M^3(\rho+1)^2+M^2\left(\rho^2+3\rho+2\right)+M(\rho+1)-\rho\right)\end{pmatrix}}.\end{aligned}$$

So, $\lim_{M\to\infty}\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) > 0$ and $\lim_{M\to 1}\frac{\partial}{\partial M}\left(\frac{\pi_k(M-1,M)}{\pi_i(M,M)}\right) < 0$. We further know that $\lim_{M\to\infty}\frac{\pi_k(M-1,M)}{\pi_i(M,M)} = 1$ and $\lim_{M\to 1}\frac{\pi_k(M-1,M)}{\pi_i(M,M)} = \frac{6(\rho+2)^2\sqrt{8\rho^2+17\rho+9}}{(3\rho+4)(4\rho+9)\sqrt{2\rho^2+5\rho+4}} > 1$. Thus, there must exist $\hat{M}_1 > 0$ such that when $M > \hat{M}_1$, $\frac{\pi_k(M-1,M)}{\pi_i(M,M)} < 1$; that is, $M_1 = M$ can be sustained as an equilibrium.

Finally, to sustain the conjectured equilibrium, we need to make sure that (i) L will not deviate to not sharing information, and (ii) none of the H investors will deviate to sharing information. In the conjectured equilibrium, setting $M_1 = M$ yields L's expected profits as follows:

$$\pi_L = \frac{(M+1)\rho\sigma_u}{(M+2)\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2}}.$$

Following the similar derivation we did at the beginning of the proof, we can derive that if

L deviates and chooses not to share information, his unconditional expected profit becomes

$$\pi_L^{\text{deviate}} = \frac{\rho(\rho+1)\sigma_u}{(M(\rho+2)+2(\rho+1))\sqrt{M(\rho+2)^2+\rho(\rho+1)}}.$$

Define $f_1(M) \equiv \frac{\pi_L}{\pi_L^{deviate}}$. We show that $f_1'(M) > 0$ and $f_1(1) = \frac{2(3\rho+4)\sqrt{2\rho^2+5\rho+4}}{3(\rho+1)\sqrt{8\rho^2+17\rho+9}} > 1$. Therefore, for $M \geq 1$, $\pi_L > \pi_L^{\text{deviate}}$ always holds; that is, L will not deviate.

In the conjectured equilibrium, one H's expected profit is $\pi_i(M,M)$ as shown in (A28). If, instead, he deviates and shares his information with L, then all $M+1$ investors know the asset fundamental $\tilde{v}$, and H's profit from deviation becomes $\pi_H^{\text{deviate}} = \frac{\sigma_u}{(M+2)\sqrt{M+1}}$. Define $f_2(M) = \frac{\pi_i(M,M)}{\pi_H^{\text{deviate}}}$. We show that $f_2'(M) < 0$ and $\lim_{M\to\infty} f_2(M) = 1$, so $f_2(M) > 1$. Thus, for all $M \geq 1$, $\pi_i(M,M) > \pi_H^{\text{deviate}}$; that is, H will not deviate. Overall, L shares information, and none of Hs shares information but all Hs use the shared information can be sustained in equilibrium.

Last, we show that while $M_1 = M$ can be sustained as an equilibrium when $M > \hat{M}_1$, it is always dominated by $M_1 = 0$ in terms of H's profit. That is, Hs would have been better off had they all committed not to use the shared information.

An H investor's profit when no Hs use L's shared information $\pi_k(0,M)$ and when all Hs use the shared information $\pi_i(M,M)$ are given, respectively, by (A27) and (A28). We know that

$$\frac{\partial}{\partial M}\frac{\pi_k(0,M)}{\pi_i(M,M)} = \rho(\rho+2)^2$$
$$\times \frac{\begin{pmatrix} -M^7(\rho+1)^2\left(4\rho^2+15\rho+14\right) - M^6\left(22\rho^4+119\rho^3+242\rho^2+217\rho+72\right) \\ -M^5\left(33\rho^4+165\rho^3+314\rho^2+268\rho+86\right) + M^4\left(37\rho^4+185\rho^3+368\rho^2+336\rho+116\right) \\ +M^3\left(181\rho^4+805\rho^3+1372\rho^2+1060\rho+312\right) + M^2\left(237\rho^4+949\rho^3+1380\rho^2+844\rho+176\right) \\ +2M\rho\left(70\rho^3+247\rho^2+293\rho+116\right) + 32\rho(\rho+1)^3 \end{pmatrix}}{\begin{pmatrix} 2(M(\rho+2)+2(\rho+1))^2\left(M^2(\rho+1)+2M(\rho+2)+\rho+4\right)^2 \\ \times\sqrt{\rho\left(M^2(\rho+1)+M(2\rho+3)+\rho+1\right)+M(M\rho+M+\rho+2)^2} \times \left(M(\rho+2)^2+\rho(\rho+1)\right)^{3/2} \end{pmatrix}}.$$

So, there exists a constant $\hat{M}_2 > 0$ such that when $M > \hat{M}_2$, $\frac{\partial}{\partial M}\frac{\pi_k(0,M)}{\pi_i(M,M)} < 0$. Further, as $M \to +\infty$, $\frac{\pi_k(0,M)}{\pi_i(M,M)} \to 1$. When $M = 2$,

$$\frac{\pi_k(0,2)}{\pi_i(2,2)} = \frac{6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}}{(2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}} > 1,$$

where the inequality holds because

$$\left(6(\rho+2)^2\sqrt{27\rho^2+59\rho+32}\right)^2 - \left((2\rho+3)(9\rho+16)\sqrt{3\rho^2+9\rho+8}\right)^2 = \\ \rho\left(612\rho^4+4137\rho^3+10431\rho^2+11608\rho+4800\right) > 0.$$

Therefore, when $M > \hat{M}_2$, $\frac{\pi_k(0,M)}{\pi_i(M,M)} > 1$; that is, the H investors would be better off had all of them committed not to use the shared information. Overall, when $M > \hat{M} \equiv \max\{\hat{M}_1, \hat{M}_2\}$, Part (2) of the proposition holds.

Online Appendix

S1 Equilibrium Characterization in Section 4.4

In this extended economy, we are interested in examining if L still shares information once the shared information can be observed by the market maker, but the market maker and H have different capabilities in interpreting the information. Specifically, if L shares information $\tilde{y}$, H and the market maker respectively observe

$$\tilde{q}_H = \tilde{y} + \tilde{\zeta}_H \text{ and } \tilde{q}_M = \tilde{y} + \tilde{\zeta}_M,$$

whereas if H shares his information $\tilde{v}$, L and the market maker respectively observe

$$\tilde{q}_L = \tilde{v} + \tilde{\xi}_L \text{ and } \tilde{q}'_M = \tilde{v} + \tilde{\xi}_M,$$

with $\tilde{\xi}_L, \tilde{\zeta}_H \sim N(0, \chi_H^{-1})$ and $\tilde{\xi}_M, \tilde{\zeta}_M \sim N(0, \chi_M^{-1})$. The random variables $\{\tilde{v}, \tilde{y}, \tilde{\zeta}_H, \tilde{\zeta}_M, \tilde{\xi}_L, \tilde{\xi}_M\}$ are mutually independent. We then follow the baseline model to first consider the date-1 subgame equilibrium and then determine the equilibrium information-sharing strategy.

Subgame 1: Neither Investor Shares Information ($A_L = A_H = \varnothing$). The subgame becomes the no-information-sharing benchmark studied in Lemma 1, in which the two investors' unconditional trading profits are given by $\pi_H^{\varnothing\varnothing}$ and $\pi_L^{\varnothing\varnothing}$.

Subgame 2: L Shares Information and H Does Not ($A_L = S$ and $A_H = \varnothing$). Investor H receives signal $\tilde{q}_H = \tilde{y} + \tilde{\zeta}_H$ and we conjecture his linear trading strategy to be $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_L \tilde{q}_H$. L's trading strategy is $\tilde{x}_L = \beta_y \tilde{y}$. In addition to the total order flow $\tilde{\omega}$, the market maker observes $\tilde{q}_M = \tilde{y} + \tilde{\zeta}_M$ and adopts a linear pricing rule $\tilde{p} = \lambda_\omega \tilde{\omega} + \lambda_L \tilde{q}_M$. Note that L could not observe $\tilde{q}_H$ or $\tilde{q}_M$ because the noise terms $\tilde{\zeta}_H$ and $\tilde{\zeta}_M$ are receiver noises.

For investor H, his conditional trading profit on $t = 1$ is:

$$E\left[\tilde{x}_H(\tilde{v} - \tilde{p})|\tilde{v}, \tilde{q}_H\right] = \tilde{x}_H \left(\tilde{v} - \lambda_\omega \left(\tilde{x}_H + \beta_y \frac{\rho\tilde{v} + \chi_H \tilde{q}_H}{\rho + \chi_H}\right) - \lambda_L \frac{\rho\tilde{v} + \chi_H \tilde{q}_H}{\rho + \chi_H}\right), \tag{S1}$$

Maximizing the profit yields investor H's optimal trading rule: $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_L \tilde{q}_H$ with

$$\alpha_v = \frac{\chi_H + \rho\left(-\lambda_L\right) + \rho - \rho\lambda_w\beta_y}{2\lambda_w\left(\chi_H + \rho\right)}, \tag{S2}$$

$$\alpha_L = -\frac{\chi_H\left(\lambda_L + \lambda_w\beta_y\right)}{2\lambda_w\left(\chi_H + \rho\right)}. \tag{S3}$$

Similarly, L's conditional trading profit is

$$E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y}\right]=\tilde{x}_L\left(\frac{\rho}{1+\rho}\tilde{y}-\lambda_\omega\left(\tilde{x}_L+\alpha_v\frac{\rho}{1+\rho}\tilde{y}+\alpha_L\tilde{y}\right)-\lambda_L\tilde{y}\right), \tag{S4}$$

Maximizing the profit yields investor L's optimal trading rule $\tilde{x}_L=\beta_y\tilde{y}$ with

$$\beta_y=\frac{\rho\left(1-\alpha_v\lambda_w\right)-\left(\rho+1\right)\left(\lambda_L+\alpha_L\lambda_w\right)}{2(\rho+1)\lambda_w}. \tag{S5}$$

The market maker's pricing rule is $\tilde{p}=E[\tilde{v}|\tilde{\omega},\tilde{q}_M]=\lambda_\omega\tilde{\omega}+\lambda_L\tilde{q}_M$, with λ_ω and λ_L computed as follows:

$$\lambda_\omega = \chi_H\left(\rho\alpha_L+\alpha_v\left(\chi_M+\rho\right)+\rho\beta_y\right)\Gamma^{-1}, \tag{S6}$$

$$\lambda_L = \chi_M\left(-\chi_H\alpha_L\alpha_v+\chi_H\left(\rho\sigma_u^2-\alpha_v\beta_y\right)+\rho\alpha_L^2\right)\Gamma^{-1}, \tag{S7}$$

where

$$\Gamma=\alpha_L^2\left((\rho+1)\chi_H+(\rho+1)\chi_M+\rho\right)+2\chi_H\alpha_L\left(\rho\alpha_v+(\rho+1)\beta_y\right) + \chi_H\left(\sigma_u^2\left((\rho+1)\chi_M+\rho\right)+\alpha_v^2\left(\chi_M+\rho\right)+2\rho\alpha_v\beta_y+(\rho+1)\beta_y^2\right).$$

Based on equations (S2), (S3), (S5), (S6), and (S7), we are able to compute optimal trading rules $\{\alpha_v,\alpha_L,\beta_y\}$ and the optimal pricing rule $\{\lambda_\omega,\lambda_L\}$. We then replace them with the two investors' profit functions (S1) and (S4) to compute their respective unconditional trading profits $\pi_H^{S\varnothing}$ and $\pi_L^{S\varnothing}$.

Subgame 3: H Shares Information and L Does Not ($A_L=\varnothing$ and $A_H=S$). Investor L receives signal $\tilde{q}_L=\tilde{v}+\tilde{\xi}_L$ and we conjecture his linear trading strategy to be $\tilde{x}_L=\beta_y\tilde{y}+\beta_H\tilde{q}_L$. H's trading strategy is $\tilde{x}_H=\alpha_v\tilde{v}$. In addition to the total order flow $\tilde{\omega}$, the market maker observes $\tilde{q}'_M=\tilde{v}+\tilde{\xi}_M$ and adopts a linear pricing rule $\tilde{p}=\lambda_\omega\tilde{\omega}+\lambda_H\tilde{q}'_M$.

For investor L, his conditional trading profit on $t=1$ is

$$E\left[\tilde{x}_L(\tilde{v}-\tilde{p})|\tilde{y},\tilde{q}_L\right]=\tilde{x}_L\left(\frac{\rho\tilde{y}+\chi_H\tilde{q}_L}{1+\rho+\chi_H}-\lambda_\omega\left(\tilde{x}_L+\alpha_v\frac{\rho\tilde{y}+\chi_H\tilde{q}_L}{1+\rho+\chi_H}\right)-\lambda_H\frac{\rho\tilde{y}+\chi_H\tilde{q}_L}{1+\rho+\chi_H}\right),$$

Maximizing the profit yields investor L's optimal trading strategy $\tilde{x}_L=\beta_y\tilde{y}+\beta_H\tilde{q}_L$ with

$$\beta_y=\frac{\rho(1-\lambda_H-\alpha_v\lambda_w)}{2\lambda_w\left(1+\chi_H+\rho\right)},$$

$$\beta_H=-\frac{\chi_H(1-\lambda_H-\alpha_v\lambda_w)}{2\lambda_w\left(1+\chi_H+\rho\right)}.$$

Similarly, investor H's conditional trading profit on $t = 1$ is

$$E\left[\tilde{x}_H(\tilde{v} - \tilde{p})|\tilde{v}\right] = \tilde{x}_H\left(\tilde{v} - \lambda_\omega\left(\tilde{x}_H + \beta_y\tilde{v} + \beta_H\tilde{v}\right) - \lambda_H\tilde{v}\right),$$

Maximizing the profit yields investor H's optimal trading strategy: $\tilde{x}_H = \alpha_v\tilde{v}$ with

$$\alpha_v = \frac{1 - \lambda_H + \lambda_w(\beta_H + \beta_y)}{2\lambda_w}.$$

The market maker's pricing rule is $\tilde{p} = E[\tilde{v}|\tilde{\omega}, \tilde{q}'_M] = \lambda_\omega\tilde{\omega} + \lambda_H\tilde{q}'_M$, with

$$\begin{aligned}
\lambda_\omega &= \chi_H\rho\left(\alpha_v + \beta_y + \beta_H\right)\Gamma^{-1}, \\
\lambda_H &= \chi_M\left(\rho\beta_H^2 + \chi_H(\beta_y^2 + \rho\sigma_u^2)\right)\Gamma^{-1},
\end{aligned}$$

where

$$\begin{aligned}
\Gamma = \rho\beta_H^2\left(\chi_H + \chi_M + 1\right) + &+2\rho\beta_H\chi_H\left(\alpha_v + \beta_y\right) \\
&+ \chi_H\left(\rho\left(\chi_M + 1\right)\sigma_u^2 + \beta_y^2\left(\chi_M + \rho + 1\right) + \rho\alpha_v^2 + 2\rho\alpha_v\beta_y\right).
\end{aligned}$$

As in Subgame 1, we can compute the optimal trading strategies $\{\beta_y, \beta_H, \alpha_v\}$, the optimal pricing rule $\{\lambda_w, \lambda_H\}$, and the two investors' unconditional trading profits $\pi_H^{\varnothing S}$ and $\pi_L^{\varnothing S}$.

Subgame 4: Both Investors Share Information ($A_L = A_H = S$). We conjecture the two investors' trading strategies to be $\tilde{x}_H = \alpha_v\tilde{v} + \alpha_L\tilde{q}_H$ and $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{q}_L$, and the market maker's pricing rule to be $\tilde{p} = \lambda\tilde{\omega} + \lambda_L\tilde{q}_M + \lambda_H\tilde{q}'_M$. Next, investor L's conditional trading profit on $t = 1$ is

$$E\left[\tilde{x}_L(\tilde{v} - \tilde{p})|\tilde{y}, \tilde{q}_L\right] = \tilde{x}_L\left(\frac{\rho\tilde{y} + \chi_H\tilde{q}_L}{1 + \rho + \chi_H} - \lambda_\omega\left(\tilde{x}_L + (\alpha_v + \alpha_L)\frac{\rho\tilde{y} + \chi_H\tilde{q}_L}{1 + \rho + \chi_H}\right) - \lambda_L\tilde{y} - \lambda_H\frac{\rho\tilde{y} + \chi_H\tilde{q}_L}{1 + \rho + \chi_H}\right),$$

Maximizing the profit yields investor L's optimal trading strategy $\tilde{x}_L = \beta_y\tilde{y} + \beta_H\tilde{q}_L$ with

$$\begin{aligned}
\beta_y &= -\frac{\rho\lambda_H + \rho((\alpha_L + \alpha_v)\lambda_w - 1) + \lambda_L(1 + \rho + \chi_H)}{2\lambda_w(1 + \rho + \chi_H)}, \\
\beta_H &= \frac{(1 - \lambda_H - (\alpha_L + \alpha_v)\lambda_w)\chi_H}{2\lambda_w(1 + \rho + \chi_H)}.
\end{aligned}$$

For investor H, his conditional trading profit on $t = 1$ is

$$E\left[\tilde{x}_H(\tilde{v} - \tilde{p})|\tilde{v}, \tilde{q}_H\right] = \tilde{x}_H\left(\tilde{v} - \lambda_\omega\left(\tilde{x}_H + \beta_y\frac{\rho\tilde{v} + \chi_H\tilde{q}_H}{\rho + \chi_H} + \beta_H\tilde{v}\right) - \lambda_L\frac{\rho\tilde{v} + \chi_H\tilde{q}_H}{\rho + \chi_H} - \lambda_H\tilde{v}\right),$$

Maximizing the profit yields investor H's optimal trading strategy $\tilde{x}_H = \alpha_v \tilde{v} + \alpha_L \tilde{q}_H$ with

$$\alpha_v = \frac{1 - \lambda_H - \beta_H \lambda_w - \frac{\rho(\lambda_L + \beta_L \lambda_w)}{\rho + \chi_H}}{2\lambda_w},$$
$$\alpha_L = -\frac{(\lambda_L + \beta_y \lambda_w)\chi_H}{2\lambda_w(\rho + \chi_H)}.$$

The market maker's pricing rule is $\tilde{p} = E[\tilde{v}|\tilde{\omega}, \tilde{q}_M, \tilde{q}'_M] = \lambda_w \tilde{\omega} + \lambda_L \tilde{q}_M + \lambda_H \tilde{q}'_M$, where

$$\lambda_w = \chi_H \left(\chi_H^2 (-\alpha_L) + \chi_H (\beta_H + (-2\rho - 1)\alpha_L + \alpha_v) + \rho (\beta_H + \alpha_v + \beta_y)\right) \Gamma^{-1},$$
$$\lambda_L = \chi_H \begin{pmatrix} \rho (\beta_H^2 + \alpha_L^2) + \chi_H^2 \alpha_L (\alpha_L + \beta_y) \\ +\chi_H (\alpha_L ((\rho - 1)(\beta_H + \alpha_v) + (\rho + 1)\beta_y) - \beta_y (\beta_H + \alpha_v) + (\rho + 1)\alpha_L^2 + \rho \sigma_u^2) \end{pmatrix} \Gamma^{-1},$$
$$\lambda_H = \chi_H \begin{pmatrix} \beta_H^2 (\chi_H + \rho) + \alpha_L^2 ((\rho + 2)\chi_H + \rho) + \chi_H \left(\sigma_u^2 (\chi_H + \rho) + \beta_y^2\right) \\ +\chi_H \alpha_L (\beta_H (\chi_H + \rho) + \alpha_v (\chi_H + \rho) + (\rho + 2)\beta_y) \end{pmatrix} \Gamma^{-1},$$

where

$$\begin{aligned}\Gamma^{-1} = {} & \beta_H^2 (\chi_H (2\chi_H + 3\rho + 1) + \rho) + \alpha_L^2 (\chi_H (2\chi_H + 3\rho + 2) + \rho) \\ & + 2\chi_H \alpha_L (\rho \beta_H + \beta_y (\chi_H + \rho + 1) + \rho \alpha_v) + 2\beta_H \chi_H^2 (\alpha_v (\chi_H + \rho) + \rho \beta_y) \\ & + \chi_H \left(\sigma_u^2 (\chi_H (\chi_H + 2\rho + 1) + \rho) + \alpha_v^2 (\chi_H + \rho) + \beta_y^2 (\chi_H + \rho + 1) + 2\rho \alpha_v \beta_y\right).\end{aligned}$$

Again, as before we can solve for the optimal trading strategies $\{\beta_y, \beta_H, \alpha_v, \alpha_L\}$ and the optimal trading rule $\{\lambda_w, \lambda_H, \lambda_L\}$ and compute the two investors' unconditional trading profits π_H^{SS} and π_L^{SS}.

In the numerical analysis, we verify that H will not deviate from the no-information-sharing strategy.

S2 Extension: Information Acquisition by L

In this section, we consider investor L's information-acquisition decision. Before the two investors' information sharing on $t = 0$, to acquire information of precision level ρ, investor L needs to incur a cost according to a linear cost function, $c \cdot \rho$, where c is a positive constant. L chooses the precision level ρ to maximize his expected trading profit net of the information-acquisition cost. We then study the effect of information sharing on L's information-acquisition incentives by comparing this extended model to its benchmark economy in which there is endogenous information production but no information sharing (i.e., L can produce information and $A_L = A_H = \varnothing$).

In the benchmark economy in which there is no information sharing and L can decide how much information to produce, based on equations (6), L's expected trading profit net of the information-acquisition cost can be expressed as follows:

$$\pi_L^{\varnothing\varnothing} - c \cdot \rho = \frac{\rho(1+\rho)\sigma_u}{(4+3\rho)\sqrt{4+5\rho+2\rho^2}} - c \cdot \rho.$$

Maximizing the net profit yields L's optimal information-acquisition decision, ρ^0, which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{32+84\rho+69\rho^2+19\rho^3}{2(4+3\rho)^2(4+5\rho+2\rho^2)^{3/2}}. \tag{S8}$$

When information sharing is permitted, according to Proposition 1, we know that in equilibrium L shares his information whereas H does not share. Based on equation (9), L's expected profit net of the information-acquisition cost can be calculated as follows:

$$\pi_L^{S\varnothing} - c \cdot \rho = \frac{2\rho\sigma_u}{3\sqrt{(1+\rho)(9+8\rho)}} - c \cdot \rho.$$

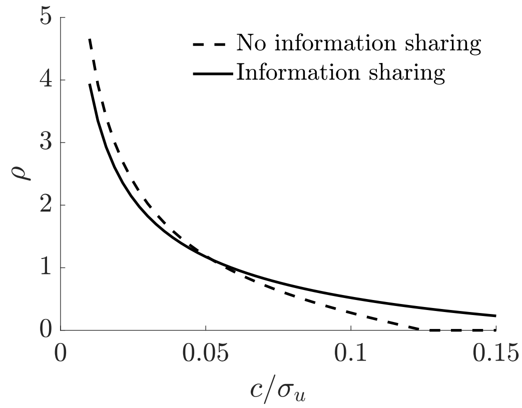
Again, maximizing the net profit yields the optimal information-acquisition decision, ρ^*, which is uniquely determined by the following equation:

$$\frac{c}{\sigma_u} = \frac{18+17\rho}{3(9+17\rho+8\rho^2)^{3/2}}. \tag{S9}$$

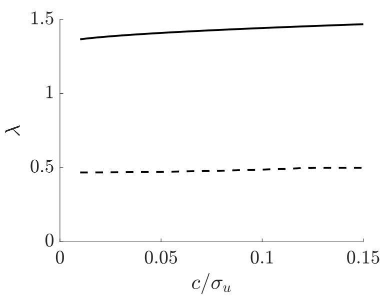
Then, a comparison of ρ^* with ρ^0 yields the following proposition.

Proposition S1. *Assume that information acquisition is costly for investor L. There exists a constant $\hat{c}$, where $\hat{c} \approx 0.0520$, such that relative to the economy without information sharing, when information sharing is permitted, if $c/\sigma_u > (<)\, \hat{c}$, L acquires more (less) information; that is, $\rho^* > (<)\, \rho^0$.*

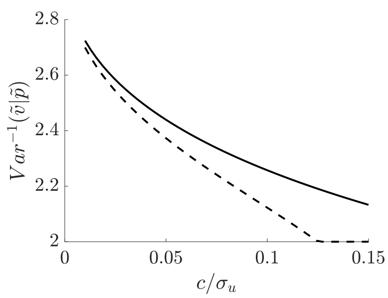
Relative to the benchmark economy without information sharing, when information sharing is permitted, L's information acquisition is determined by the following trade-off. On the one hand, as discussed above, by sharing information with H, L better hides his informed order flows, thereby trading more aggressively even though his information remains as noisy as before. This trading-against-error effect depresses L's incentives to acquire information. On the other hand, with higher trading profits after sharing his information, L can afford to acquire more information about the fundamental and make more informed trading decisions. This in turn encourages investor L to acquire more information.



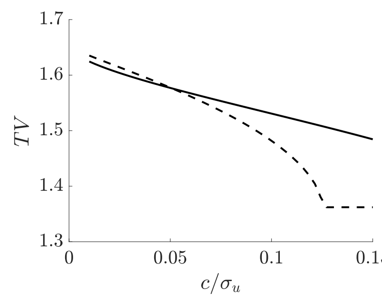
(a) Information precision



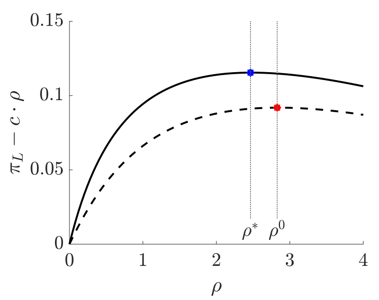
(b1) Market liquidity



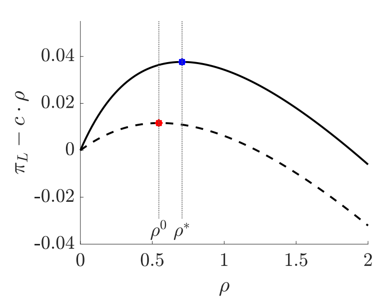
(b2) Market efficiency



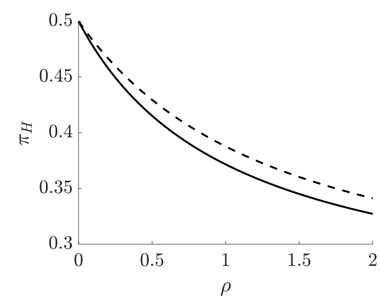
(b3) Trading volume



(c1) L's net profit ($c = 0.02$)



(c2) L's net profit ($c = 0.08$)



(c3) H's total profit

Figure S1: Costly information acquisition for investor L

Proposition S1 formalizes the above trade-off faced by L when making information-acquisition decisions. The relative strength of the two forces depends on the primitives of the model, namely, the cost of acquiring information and the noise trading volatility. Ceteris paribus, L's net gains of information acquisition decrease with the information-acquisition cost c and increase with the noise trading volatility σ_u, so c and σ_u have opposite effects on L's incentives to acquire information. Nevertheless, the optimal information acquisition only depends on c/σ_u.

If c/σ_u is low, L has acquired a great deal of information. He is less concerned about improving the forecasting ability of the fundamental, but cares more about hiding his informed order flows. When information sharing is permitted, L would like to induce H to trade against his information and help offset his informed order flow. Therefore, L is less incentivized to acquire information. However, if c/σ_u is high, L only acquires a limited amount of information. When information sharing is permitted, with higher profits, L would like to produce more information to further improve his trading decisions. Panel (a) of Figure S1 graphically illustrates the information-acquisition result.

How does L's information-acquisition behavior affect market quality? We plot market liquidity, market efficiency, and trading volume across the extended economy and its benchmark economy in Panels (b1)–(b3) of Figure S1, respectively. We find that regardless of L's information acquisition, the economy with information sharing always features lower market liquidity and higher market efficiency. Again, with L sharing information, H trades against it, which reduces the noise in the total order flow and induces the market maker to raise the price impact. Therefore, market liquidity decreases. Further, despite that L may acquire less information when information sharing is permitted, the intensive trading by the two investors renders the total order flow more correlated with the fundamental, which always improves market efficiency. Finally, consistent with the baseline model, whether trading volume increases or decreases depends on the precision of L's private information, which is further determined by the primitive parameter c/σ_u. Specifically, if c/σ_u is low so that L has acquired very precise information, trading volume decreases after information sharing, whereas if c/σ_u is high so that L only acquires imprecise information, trading volume increases due to L's aggressive trading.

To take a further look at L's optimal information-acquisition decisions, we plot how L's information production ρ affects his expected trading profit net of the information-acquisition cost $\pi_L - c \cdot \rho$ in Panels (c1) and (c2) of Figure S1 under the cost $c = 0.02$ and $c = 0.08$, respectively. The other parameter is $\sigma_u = 1$. The solid (dashed) line denotes the economy with (without) information sharing. Consistent with Proposition S1, with a fixed σ_u, if c is low (high), L acquires less (more) information when information sharing is permitted, namely, $\rho^* < (>)\, \rho^0$. More importantly, Panels (c1) and (c2) show that allowing L's endogenous information acquisition can only reinforce his gains from the information sharing. In other words, when information sharing is permitted, by choosing ρ^* investor L makes higher profits than those in the situation where the information precision is exogenously given. Finally, regardless of L's information-acquisition cost c or information production ρ, relative to the benchmark economy without information sharing, H always makes lower profits, as shown in Panel (c3) of Figure S1.

Proof of Proposition S1

Denote the right-hand-side of equations (S8) and (S9) as $f_{\varnothing}(\rho)$ and $f_S(\rho)$, respectively. It is easy to see that $f'_{\varnothing}(\rho) < 0$ and $f'_S(\rho) < 0$. Therefore, as c/σ_u increases, investor L acquires less information in both cases with and without information sharing; that is, both ρ^* and ρ^0 decrease in c/σ_u. Further, setting $f_{\varnothing}(\rho) = f_S(\rho)$ yields $\rho = \hat{\rho} \approx 1.1307$ and we know that $f_{\varnothing}(\hat{\rho}) = f_S(\hat{\rho}) \equiv \hat{c} \approx 0.0520$. Therefore, the function $f_S(\rho) - f_{\varnothing}(\rho) = 0$ has a unique root at $\rho = \hat{\rho}$.

Since $\lim_{\rho\to 0} f_S(\rho) - f_{\varnothing}(\rho) = 7/72 > 0$, if $c/\sigma_u > \hat{c}$ so that both ρ^0 and ρ^* are small, we know that $\rho^* > \rho^0$. Similarly, since $\lim_{\rho\to\infty} f_S(\rho) - f_{\varnothing}(\rho) = 0$, when $c/\sigma_u < \hat{c}$, both ρ^0 and ρ^* are large, and we know that $\rho^* < \rho^0$.

S3 Extension: Multiple Ls and Hs

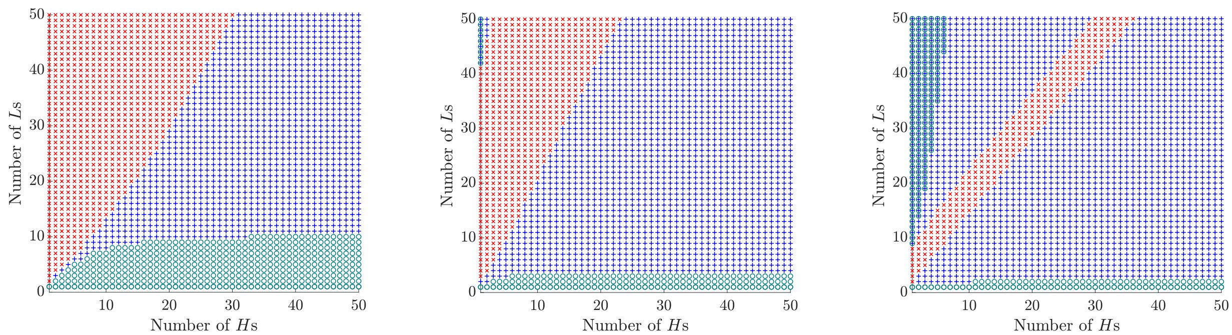
There are two groups of investors: (i) a number M of Hs, denoted by H_1,...,H_M, and (ii) a number N of Ls, denoted by L_1,...,L_N, where M and N are integers. All Hs observe $\tilde{v}$ whereas investor L_n, where $n \in \{1, ..., N\}$, only observes a noisy signal:

$$\tilde{y}_n = \tilde{v} + \tilde{\eta} + \tilde{e}_n, \text{ with } \tilde{\eta} \sim N(0, \xi^{-1}) \text{ and } \tilde{e}_n \sim N(0, \rho^{-1}). \tag{S10}$$

The signal structure (S10) introduces common noise $\tilde{\eta}$ into Ls' information and thus allows for information correlation among them. One can interpret that the common noise represents the sentiment among the coarsely informed investors. As in Section 4.3, we focus on Ls' information-sharing behavior; moreover, once an L shares information, this signal becomes observed by all Hs and the other peer Ls. The baseline model is nested by setting $M = N = 1$ and $\xi = \infty$.

We defer the equilibrium characterization to Section S3.1 and now use numerical analyses to first demonstrate that information sharing remains a prevalent phenomenon despite the presence of a large number of Ls. Denote N_1 as the number of Ls who share information. Figure S2 plots three types of Ls' information-sharing behavior in the parameter space of M and N for different values of ξ: (i) None of Ls shares information (i.e., $N_1^* = 0$, marked by "x"), (ii) all Ls share information (i.e., $N_1^* = N$, marked by "o"), and (iii) a fraction of Ls share information (i.e., $0 < N_1^* < N$, marked by "+").

Panels (a) and (b) of Figure S2 show that when there are a large number of Ls but a small number of Hs (i.e., high N but low M), none of Ls is willing to share information in equilibrium. This is intuitive because in the presence of multiple Ls, revealing private information to other peer Ls dissipates an L's informational advantage and with a small



(a) $\xi = 50$ (b) $\xi = 5$ (c) $\xi = 0.5$

This figure plots the regimes of Ls' information-sharing behavior in the parameter space of (M, N) for different values of ξ in different panels. The parameter values are $\sigma_u = 1$ and $\rho = 1$. Denote N_1 the number of Ls sharing information in equilibrium. We use "x" to indicate that none of Ls shares private information (i.e., $N_1^* = 0$), "o" to indicate that all Ls share private information (i.e., $N_1^* = N$), and "+" to indicate that an interior fraction of Ls share information (i.e., $0 < N_1^* < N$).

Figure S2: Multiple Ls and Hs

number of Hs, the trading-against-error effect is diminished; both forces discourage Ls from sharing their information.

However, as shown in Panel (c) of Figure S2, even with a large number of Ls but a small number of Hs, Ls' information-sharing incentives can be restored when the common noise $\tilde{\eta}$ is an important component (i.e., low ξ). Now, sharing information and inviting Hs to trade against the common noise $\tilde{\eta}$ becomes the common interest among Ls, thereby sustaining the coordination-type equilibrium in which several Ls are willing to share private information despite the potential loss of informational advantage to peers.

S3.1 Equilibrium Characterization

We now characterize the equilibrium. Suppose that among the number N of L investors, N_1 of them choose to share information ($A_{L_1} = ... = A_{L_{N_1}} = S$), and the rest do not share any information ($A_{L_{N_1+1}} = ... = A_{L_N} = \varnothing$). We then conjecture the following linear symmetric trading strategies: Investor H_k trades $\tilde{x}_{H,k} = \alpha_v \tilde{v} + \alpha_L (\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $k \in \{1, ..., M\}$; investor L_i who shares information trades $\tilde{x}_{L,i} = \beta (\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $i \in \{1, ..., N_1\}$; and investor L_j that does not share information trades $\tilde{x}_{L,j} = \gamma_0 \tilde{y}_j + \gamma (\tilde{y}_1 + ... + \tilde{y}_{N_1})$, where $j \in \{N_1 + 1, ..., N\}$. Note that all Ls who share information have the same information set $\{\tilde{y}_1, ..., \tilde{y}_{N_1}\}$, and investor L_j who does not share information has her unique information $\tilde{y}_j$ in addition to the common information set $\{\tilde{y}_1, ..., \tilde{y}_{N_1}\}$. We also consider a linear pricing rule $\tilde{p}=\lambda\tilde{\omega}$, where the total order flow $\tilde{\omega}=\sum_{k=1}^{M}\tilde{x}_{H,k}+\sum_{i=1}^{N_1}\tilde{x}_{L,i}+\sum_{j=N_1+1}^{N}\tilde{x}_{L,j}+\tilde{u}$.

We next derive each investor's optimal trading strategy. For investor H_k, where $k\in\{1,...,M\}$, given his information set $\{\tilde{v},\tilde{y}_1,...,\tilde{y}_{N_1}\}$, the conditional expected profit is

$$
\begin{aligned}
&E\left[\tilde{x}_{H,k}\left(\tilde{v}-\tilde{p}\right)|\tilde{v},\tilde{y}_1,...,\tilde{y}_{N_1}\right]=\tilde{x}_{H,k}\\
&\times\left(\tilde{v}-\lambda\left(\begin{array}{c}\tilde{x}_{H,k}+(M-1)\,\alpha_v\tilde{v}+(M-1)\,\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)+N_1\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\+\left(N-N_1\right)\left(\gamma_0\tilde{v}+\gamma_0\frac{\frac{\rho}{1+\rho}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\xi+\frac{\rho}{1+\rho}N_1}+\gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right)\end{array}\right)\right).
\end{aligned}
$$

Maximizing the profit yields H_k's optimal trading strategy $\tilde{x}_{H,k}=\alpha_{v,k}\tilde{v}+\alpha_{L,k}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, where

$$
\begin{aligned}
\alpha_{v,k}&=\frac{1-\left((M-1)\,\alpha_v+\left(N-N_1\right)\gamma_0\right)\lambda}{2\lambda}, &\text{(S11)}\\
\alpha_{L,k}&=-\frac{1}{2}\left((M-1)\,\alpha_L+N\gamma+\frac{\left(N-N_1\right)\gamma_0\frac{\rho}{1+\rho}}{\xi+\frac{\rho}{1+\rho}N_1}+N_1\left(\beta-\gamma\right)\right). &\text{(S12)}
\end{aligned}
$$

For L_i who shares information, where $i\in\{1,...,N_1\}$, under the information set $\{\tilde{y}_1,...,\tilde{y}_{N_1}\}$, the conditional expected profit is

$$
\begin{aligned}
&E\left[\tilde{x}_{L,i}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_1,...,\tilde{y}_{N_1}\right]=\tilde{x}_{L,i}\\
&\times\left(\frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}N_1}-\lambda\left(\begin{array}{c}M\alpha_v\frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}N_1}+M\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\ \tilde{x}_i+\left(N_1-1\right)\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\\+\left(N-N_1\right)\left(\gamma_0\frac{\rho\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\frac{\xi}{1+\xi}+N_1\rho}+\gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right)\end{array}\right)\right).
\end{aligned}
$$

Maximizing the profit yields L_i's optimal trading strategy $\tilde{x}_{L,i}=\beta_i\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, with

$$
\beta_i=\frac{1}{2\lambda}\left(\begin{array}{c}\left(1-M\alpha_v\lambda\right)\frac{\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}N_1}-M\alpha_L\lambda\\-\left(N_1-1\right)\beta\lambda-\left(N-N_1\right)\lambda\left(\gamma+\gamma_0\frac{\rho}{\frac{\xi}{1+\xi}+N_1\rho}\right)\end{array}\right). \qquad \text{(S13)}
$$

For L_j who does not shares information, where $j\in\{N_1+1,...,N\}$, under the information set $\{\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\}$, the conditional expected profit is

$$E\left[\tilde{x}_{L,j}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\right] = \tilde{x}_{L,j} \times \left(\begin{array}{c} \frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} \\ -\lambda \left(\begin{array}{c} M\alpha_v \frac{\frac{\rho\xi}{\rho+\xi}\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} + M\alpha_L\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right) + N_1\beta\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right) \\ +\tilde{x}_{L,j} + \left(N-N_1-1\right)\left(\gamma_0 \frac{\rho\left(\tilde{y}_j+\tilde{y}_1+...+\tilde{y}_{N_1}\right)}{\frac{\xi}{1+\xi}+(N_1+1)\rho} + \gamma\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)\right) \end{array} \right) \end{array} \right).$$

Maximizing the profit yields L$_j$'s optimal trading strategy $\tilde{x}_{L,j} = \gamma_{0,i}\tilde{y}_j + \gamma_i\left(\tilde{y}_1+...+\tilde{y}_{N_1}\right)$, where

$$\gamma_{0,i} = \frac{1}{2\lambda}\left(\left(1-M\alpha_v\lambda\right)\frac{\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}\left(N_1+1\right)} - \left(N-N_1-1\right)\gamma_0\lambda\frac{\rho}{\frac{\xi}{1+\xi}+\left(N_1+1\right)\rho}\right), \tag{S14}$$

$$\gamma_i = \frac{1}{2}\left(\begin{array}{c} \frac{1}{\lambda}\frac{(1-M\alpha_v\lambda)\frac{\rho\xi}{\rho+\xi}}{1+\frac{\rho\xi}{\rho+\xi}(N_1+1)} + N_1\left(\gamma-\beta+\frac{\rho\gamma_0}{\frac{\xi}{1+\xi}+(N_1+1)\rho}\right) \\ -\left(N-1\right)\frac{(N_1+1)\rho\gamma+\rho\gamma_0+\frac{\gamma\xi}{1+\xi}}{\frac{\xi}{1+\xi}+(N_1+1)\rho} - M\alpha_L \end{array} \right). \tag{S15}$$

Imposing symmetric trading strategies on equations (S11)–(S15), i.e., $\alpha_{v,k} = \alpha_v, \alpha_{L,k} = \alpha_L, \beta_i = \beta, \gamma_{0,i} = \gamma_0, \gamma_i = \gamma$, we obtain the equilibrium trading strategies $\{\alpha_v, \alpha_L, \beta, \gamma_0, \gamma\}$ as functions of $(N_1, \lambda; M, N)$.

The pricing rule is $\tilde{p} = \lambda\tilde{\omega}$, where

$$\lambda = \frac{M\alpha_v + MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)}{\left(\begin{array}{c} \left(M\alpha_v + MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)\right)^2 \\ +\left(MN_1\alpha_L + N_1^2\beta + \gamma\left(N-N_1\right)N_1 + \gamma_0\left(N-N_1\right)\right)^2\frac{1}{\xi} \\ +\left(M\alpha_L + N_1\beta + \left(N-N_1\right)\gamma\right)^2\frac{N_1}{\rho} + \gamma_0^2\frac{N-N_1}{\rho} + \sigma_u^2 \end{array} \right)}.$$

Together with equilibrium trading strategies, we can solve for the equilibrium pricing rule as a function of $(N_1; M, N)$.

Next, given $(N_1; M, N)$, we can compute the expected profit of Ls who share their private information, denoted by $\pi_{L,S}\left(N_1; M, N\right)$ and that of Ls who do not share information, $\pi_{L,\varnothing}\left(N_1; M, N\right)$:

$$\begin{aligned} \pi_{L,S}\left(N_1; M, N\right) &= E\left[E\left[\tilde{x}_{L,i}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_1, ..., \tilde{y}_{N_1}\right]\right], \\ \pi_{L,\varnothing}\left(N_1; M, N\right) &= E\left[E\left[\tilde{x}_{L,j}\left(\tilde{v}-\tilde{p}\right)|\tilde{y}_j, \tilde{y}_1, ..., \tilde{y}_{N_1}\right]\right]. \end{aligned}$$

Therefore, the equilibrium number of Ls who share information is solved as follows.

- If $\pi_{L,S}(N; M, N) \geq \pi_{L,\varnothing}(N-1; M, N)$, it can be sustained in equilibrium that all Ls share information, i.e., $N_1^* = N$;

- If $\pi_{L,S}(N-1; M, N) \geq \pi_{L,\varnothing}(N-2; M, N)$ and $\pi_{L,\varnothing}(N-1; M, N) \geq \pi_{L,S}(N; M, N)$, then $N_1^* = N-1$ can be sustained in equilibrium; ...

- If $\pi_{L,S}(n; M, N) \geq \pi_{L,\varnothing}(n-1; M, N)$ and $\pi_{L,\varnothing}(n; M, N) \geq \pi_{L,S}(n+1; M, N)$, then $N_1^* = n$ can be sustained in equilibrium; ...

- If $\pi_{L,\varnothing}(0; M, N) \geq \pi_{L,S}(1; M, N)$, then it can be sustained in equilibrium that none of Ls shares information, i.e., $N_1^* = 0$.

Finally, it is easy to verify that none of Hs will deviate and share their information.

S4 Extension: Heterogeneous Information Precision

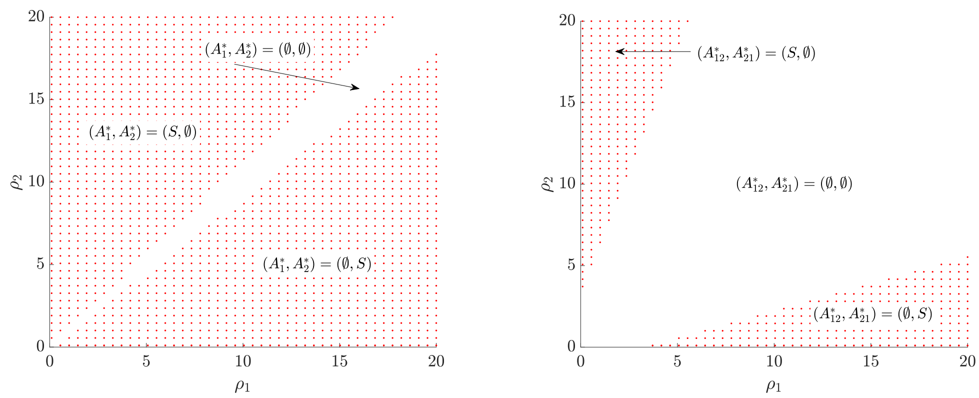
While Section S3 has considered the economy with multiple coarsely informed investors, their private information is of the same precision. In this section, we further study the case in which these multiple coarsely informed investors own information of different precision levels. We focus on the simplest case in which there are one perfectly informed investor H and two coarsely informed investors L_1 and L_2.

In this extended economy, H learns the fundamental value $\tilde{v}$ and investor L_i, where $i \in \{1, 2\}$, only observes a noisy signal about the fundamental:

$$\tilde{y}_i = \tilde{v} + \tilde{e}_i, \text{ with } \tilde{e}_i \sim N(0, \rho_i^{-1}).$$

We consider the following two cases. First, as in Sections 4.3 and S3, we assume that the information, once shared, is observed by all investors $\{H, L_1, L_2\}$. Therefore, investor L_i decides whether or not to share $\tilde{y}_i$ with all rational investors including H and L_j, where $j \neq i$. Panel (a) of Figure S3 plots the information-sharing behavior of the two coarsely informed investors L_1 and L_2 in this case. We find that no Ls share information when their information is of similar quality, and only the L with relatively coarser information is willing to share information. The intuition is the same as we analyzed in Section 4.1.

Second, we allow Ls to make selective sharing; that is, if L_1 shares information with H, the third party, including the other investor L_2 or the market maker, is unable to observe it. We find that both Ls are willing to share their private information with H and the intuition directly follows from the trading-against-error effect, i.e., $A_{1H}^* = A_{2H}^* = S$. We then use Panel (b) of Figure S3 to examine the information sharing between the two coarsely



(a) Non-selective sharing

(b) Selective sharing

Figure S3: Information sharing by L_1 and L_2

informed investors. Again, there is no information sharing between Ls when their information is of similar precision, and information only flows from the investor with lower information precision to the one with higher information precision. Moreover, compared with Panel (a), the regime of no information flows between the coarsely informed investors is much larger. This is because sharing private information with peer investor L_j greatly dissipates L_i's informational advantage; now with selective information sharing possible, L_i is able to alleviate this concern by withholding his information from the peer coarsely informed investor.